Energy Conservation in Discrete Wavelet Transform (DWT) and Its Application to Image Compression

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outline

1. Research background and objectives
2. Theoretical analysis
3. Experimental Design and Code Implementation
4. Conclusions and Future Work

Research background and objectives

Advantages of Wavelet Transform

- Multiresolution analysis
- frequency localization capability

Importance of Energy Conservation

- Ensures information integrity in signal processing
- Theoretical basis for image compression, denoising

Research Objectives

- Validate energy conservation mathematically
- Quantify energy retention via image compression experiments

Theoretical analysis

Theoretical Foundations of DWT

- Scaling function (ϕ) and wavelet function (ψ)
- Multi-scale decomposition: Approximation (cA) and Detail (cH, cV, cD) subbands
- Orthogonal Wavelet Basis Properties:

$$\langle \psi_{j,k}, \psi_{m,n} \rangle = \delta_{j,m} \, \delta_{k,n}, \qquad j, k, m, n \in \mathbb{Z}$$

Critical Conditions

- Use of orthogonal wavelets (e.g., Haar, Db1)
- No signal truncation or padding-induced errors

Energy Conservation Formula

Energy Conservation Formula for Orthogonal DWT

- For orthogonal wavelet bases:

$$\sum_{i=1}^{M}\sum_{j=1}^{N}|x[i,j]|^2 = \sum_{k,l}|d_k[l]|^2 + \sum_{m}|a_J[m]|^2$$

- For Haar wavelet bases:

$$\begin{aligned}\sum_{i,j}|x[i,j]|^2 = \sum_{i,j}|cA[i,j]|^2 + \sum_{i,j}|cH[i,j]|^2 \\ + \sum_{i,j}|cV[i,j]|^2 + \sum_{i,j}|cD[i,j]|^2\end{aligned}$$

Experimental Design and Code Implementation

Workflow

- Image preprocessing
  - grayscale conversion
  - resizing to power-of-2 dimensions
- DWT decomposition("dwt2"function)
- Energy calculation and conservation validation
- Image reconstruction("idwt2"function)

Parameters

- Test image:"Lena"(52*52)
- Wavelet basis:Haar

Code and Implementation

matlab

```matlab
[cA, cH, cV, cD] = dwt2(A, 'haar');
energy_transformed = sum(cA(:).^2) +
sum(cH(:).^2) + sum(cV(:).^2) +
sum(cD(:).^2);
```

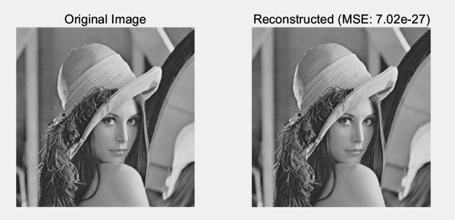
Figura 1: "Haar"basis

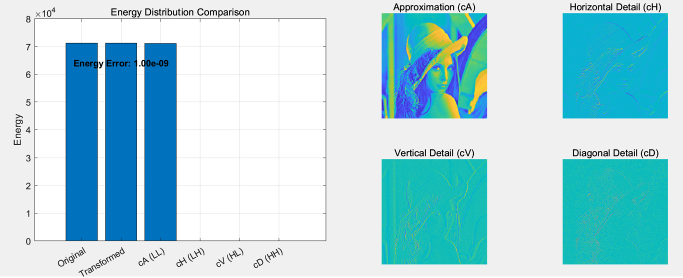
```matlab
%% 5. Verify Energy Conservation
energy_error = abs(energy_original - energy_transformed);
fprintf('Verify Energy Conservation:\n');
fprintf(' |E_original - E_transformed| = %.8e\n', energy_error);
if energy_error < 1e-8
    fprintf('Energy Conservation holds\n');
else
    fprintf('Energy Conservation violated\n');
end

```

Figura 2: Verify Energy Conservation

Code and Implementation





Conclusions and Future Work

Conclusions

- DWT strictly satisfies energy conservation when using orthogonal wavelet bases
- Proper image size alignment and orthogonal basis selection are critical factors

Future Work

- Energy error analysis for non-orthogonal wavelet frames
- Study of energy distribution in multi-scale decomposition (multi-level DWT)