Wavelet Transform for Image Processing

Zhang Jinrui[1]
He Jiashun
Meng Jingyuan
Jiang Zishen
Mo Zian

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[1]alternative email:zhangjr1022@mails.jlu.edu.cn

Background and Motivation

- Image preprocessing is a crucial step in modern computer vision and machine learning.
- Models often require input images to have a fixed size and consistent format.
- Our goal is to convert any input image into a standardized grayscale format of size $2^N \times 2^N$.
- This not only simplifies downstream processing but also ensures compatibility with wavelet transform techniques, which rely on consistent input dimensions.

Wavelet Transform Overview

- The wavelet transform is a powerful mathematical tool for analyzing signals at multiple scales.
- Unlike the Fourier transform, which only reveals frequency content, wavelets provide both frequency and spatial localization.
- This is particularly useful in image processing where textures, edges, and fine structures need to be captured at different resolutions.
- Common wavelet families include Haar, Daubechies, and Coiflets — all benefit from inputs with dimensions that are powers of two.

Image Processing Workflow

1. **Image Loading and Grayscale Conversion**: The image is loaded using a Python imaging library and converted to grayscale. This reduces data complexity and focuses analysis on structural content.
2. **Rescaling with Aspect Ratio Preservation**: The image is resized using high-fidelity interpolation to ensure one side reaches the target length (2^N), without distortion.
3. **Centered Cropping**: The resized image is cropped to $2^N \times 2^N$, ensuring input uniformity without compromising important visual features.
4. **Matrix Output**: The final image is converted into a matrix format suitable for mathematical operations, storage, and machine learning integration.

Theoretical Justification

- Many wavelet algorithms are optimized for inputs whose dimensions are powers of two — enabling efficient recursive decomposition.
- Improper dimensions can introduce edge effects or truncation artifacts, degrading transform results.
- LANCZOS interpolation is used due to its ability to preserve edges and textures during resizing, which is critical for wavelet-based analysis.
- Converting to a 2D matrix makes the image directly usable for NumPy operations and compatible with machine learning frameworks like TensorFlow and PyTorch.

Application Scenarios

- **Educational Tools**: Demonstrate how images are converted, processed, and decomposed in the frequency domain.
- **Scientific Research**: Used in experiments where consistent image input is necessary, such as style transfer or anomaly detection.
- **Data Preprocessing**: Helps in preparing inputs for neural networks, especially for tasks that benefit from texture analysis like medical imaging or remote sensing.
- **Feature Engineering**: Enables extraction of localized frequency features for traditional machine learning models.

Extension Opportunities

- **Multi-channel Support**: Extend to RGB or multi-spectral image processing for richer feature representations.
- **Batch Automation**: Apply the transformation to entire datasets, allowing integration into large-scale pipelines.
- **Visualization Tools**: Incorporate graphical interfaces to preview the original vs. transformed image and to display wavelet decomposition layers.
- **Parameter Tuning**: Allow users to experiment with different interpolation methods or wavelet bases interactively.

Conclusion

- The image conversion program serves as a foundational tool that bridges raw image data and advanced analysis techniques.
- Its adherence to standardized formats makes it ideal for educational purposes, algorithm development, and production pipelines.
- Future development can make it even more powerful by supporting color analysis, batch operations, and intuitive visual feedback.

Standard Haar Decomposition

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- sDF
- sdf

sfdfg

- sdlfkj
- sDF
- sdf

sadsgkjh

- sdlfkj
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sdf

- sdlfkj
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- sdf

Basic Idea

- $$\sum_{k=1}^{n} \hat{I}_k^4 \geq (\sum_{k=1}^{n} \hat{I}_k^2)^2 = (\sum_{k=1}^{n} I_k^2)^2$$
- $$\min_{\hat{I}_k \in I^2 st. \sum_{k=1}^{n} \hat{I}_k^2 = \sum_{k=1}^{n} I_k^2} (-\sum_{k=1}^{n} \hat{I}_k^4)$$

Basic Idea

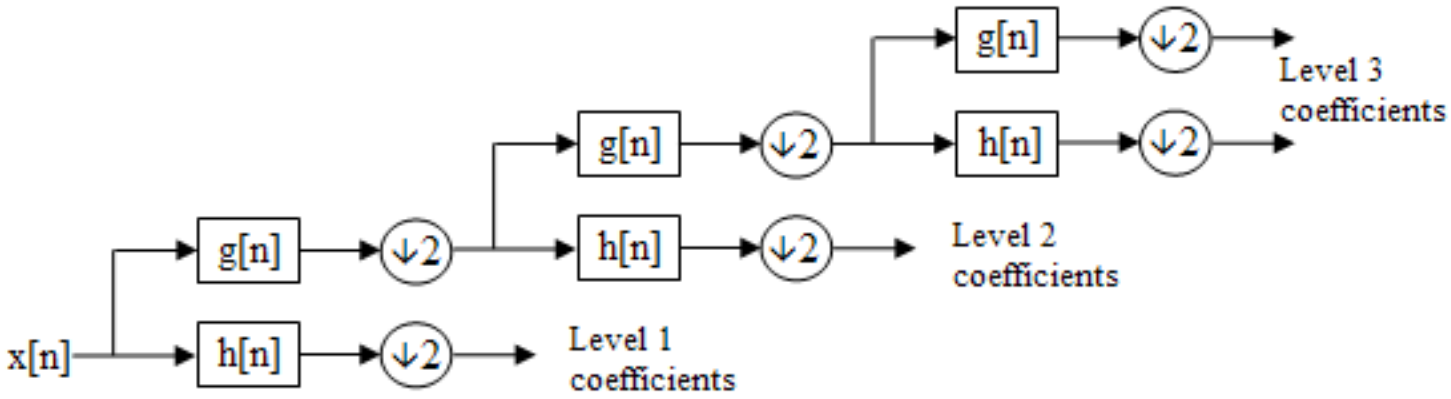

Figure: Cascading

- $$\min_{\Phi}(-\sum_{k=1}^{n}\hat{l}_k^4)$$
- Condense the energy to as less coefficients as possible.

Results

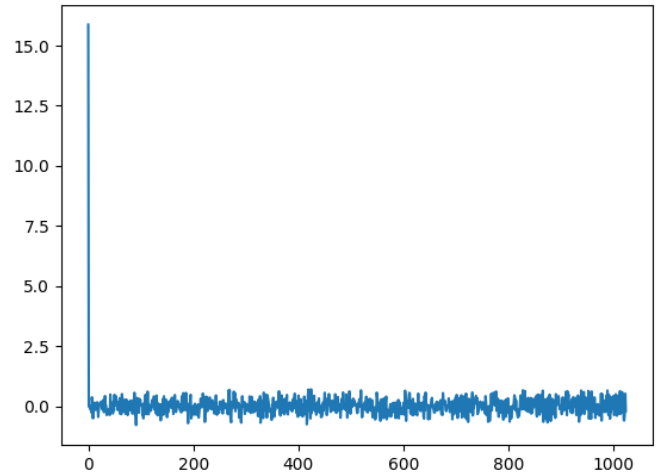

Figure: Haar 2x2 filter bank random input. Compression Rate 0.2568

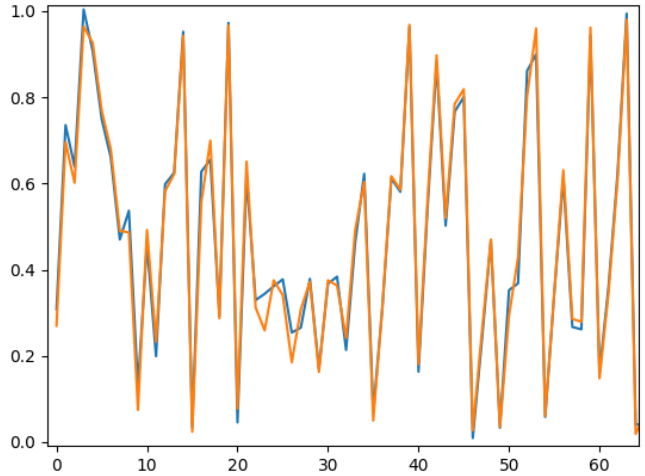

Figure: Haar 2x2 filter bank random input.Total average energy loss 0.0009

- Random sequence would have large information entropy so it have a low compression rate.

Results

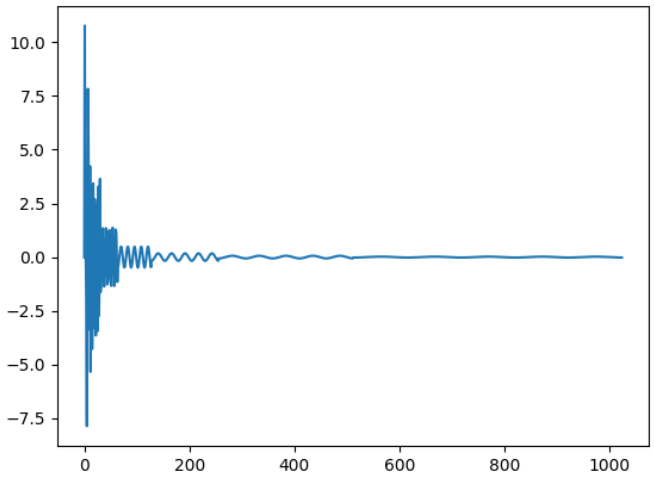

Figure: Haar 2x2 filter bank sin input frequency. Compression Rate 0.9287

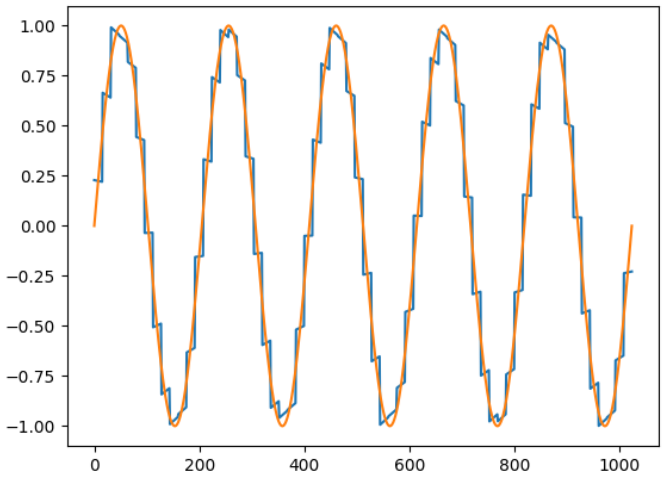

Figure: Haar 2x2 filter bank sin input reconstruction.Total average energy loss 0.0104

- A smooth signal have small information entropy, so will have a high compression rate.

Results

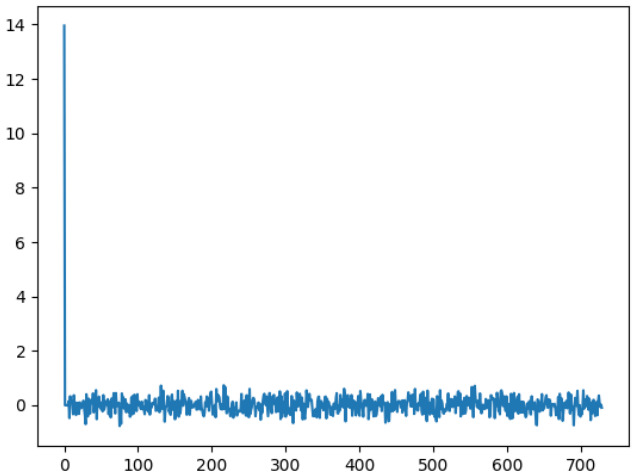

Figure: Haar 3x3 filter bank random input. Compression Rate 0.1906

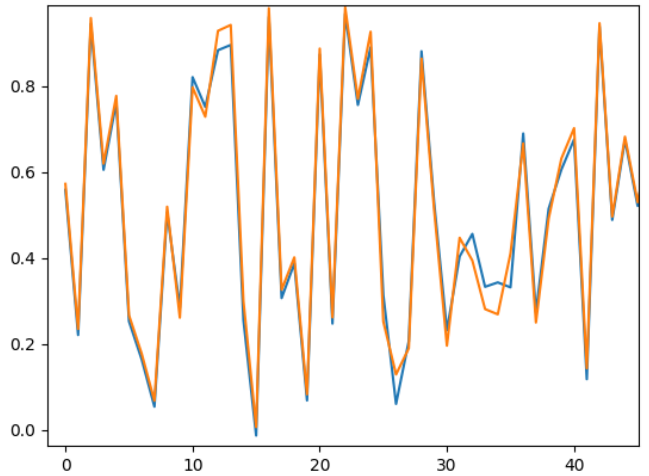

Figure: Haar 3x3 filter bank random input.Total average energy loss 0.0008

- same random sequence same reason.

Results

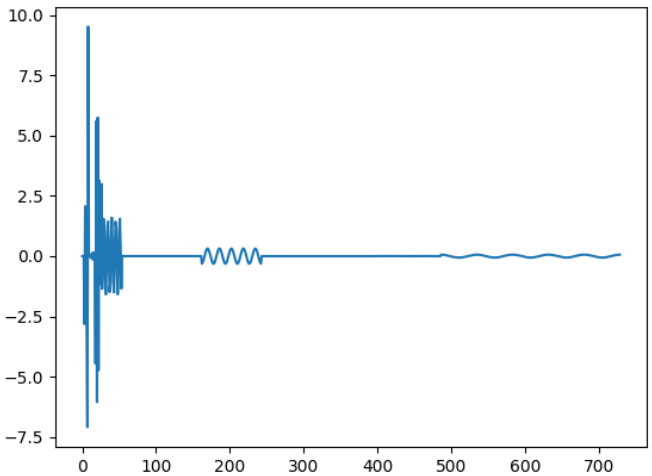

Figure: Haar 3x3 filter bank sin input frequency. Compression Rate 0.7750

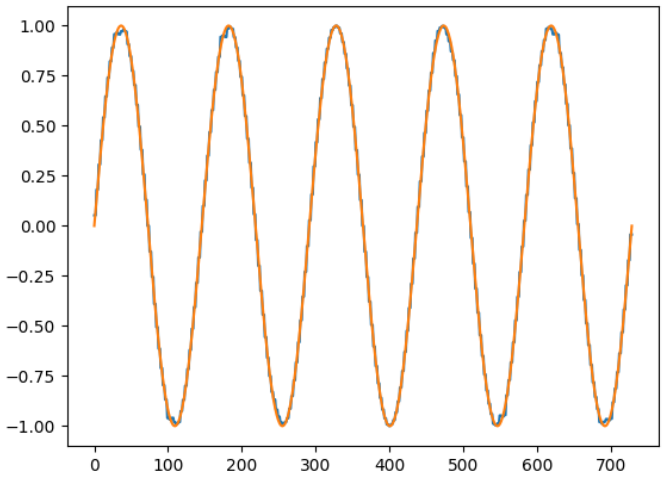

Figure: Haar 3x3 filter bank sin input reconstruction.Total average energy loss 0.0007

- same smooth signal same reason.

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