Sprague-Grundy values of Subtraction and All-But Games

21 grudnia 2024

Project Overview

Definition 1. *Let us denote by* `SUBTRACTION(S)` *a game that is played with a heap of counters, where* $S \subset \mathbb{N} \setminus \{0\}$ *and the move is to remove any number of elements, provided that number is in* S*. The last player to move wins.*

Definition 2. *Let us denote by* `ALLBUT(S)` *a game that is played with a heap of counters, where* $S \subset \mathbb{N}$ *and the move is to remove any number of elements, provided that number is positive and* ***NOT*** *in* S*. The last player to move wins.*

Definition 3. *Let* $g : \mathbb{N} \cup \{0\} \mapsto \mathbb{N} \cup \{0\}$*. We call function* g *periodic if there exists* $l \geqslant 0$ *and* $p > 0$ *such that* $\forall n \geqslant l : \ g(n+p) = g(n)$*, where* p *is a period and* l *is a pre-period.*

Definition 4. *Let* $g : \mathbb{N} \cup \{0\} \mapsto \mathbb{N} \cup \{0\}$*. We call function* g *arithmetic periodic if there exists* $l, d \geqslant 0$ *and* $p > 0$ *such that* $\forall n \geqslant l : \ g(n+p) = g(n) + d$*, where* p *is a period,* l *is a pre-period and* d *is so called saltus.*

The primary aim of this project is to study the periodicity of Sprague-Grundy values associated with `ALLBUT`(S) and `SUBTRACTION`(S) games.

Theoretical Considerations

Check the periodicity and the arithmetic periodicity of Sprague-Grundy values of `ALLBUT`(S) and `SUBTRACTION`(S) games for finite sets S. Provide the corresponding formal proofs as well as the estimations for the periods in the documentation. These results serve as a foundation for the implementation and case analysis phase below.

Implementation and Analysis

Based on the results of the previous section, implement the algorithm that, for a given finite set S, calculates the period/arithmetic period and the pre-period for games `SUBTRACTION`(S) and `ALLBUT`(S). Comment on its correctness using theoretical considerations obtained in the previous section. Use the algorithm to provide interesting examples and compare the theoretical estimation of the period to the computed results in you case study analysis. Perhaps you can state some interesting hypothesis or research problems?

Literature Overview

Create a literature overview by examining existing research on periodicity and period estimation in subtraction and all-but games or other combinatorial games. Clearly state what is known and what is an open problem in this topic.

Deadline and the Submission Method

The deadline for the project is 20.01.2025. The results will be presented by the teams after the submission, during the scheduled meeting. The documentation should contain theoretical considerations, the results (examples) and analysis obtained using the algorithm and the literature overview. It should be uploaded to Teams as pdf files as well as the code of the algorithm before the end of the deadline. The team may consists of three or less students.

The deadline for the project is 20.01.2025. Results will be presented by the teams after submission during a scheduled meeting. The documentation should include theoretical considerations, the literature overview, analysis obtained using the algorithm including conclusions and hypothesis. Submissions should be uploaded to Teams in PDF format, along with the algorithm code, before the deadline. Each team may consist of up to three students. Teams may consist of students enrolled in the *Game Theory* or *Teoria Gier* lectures.