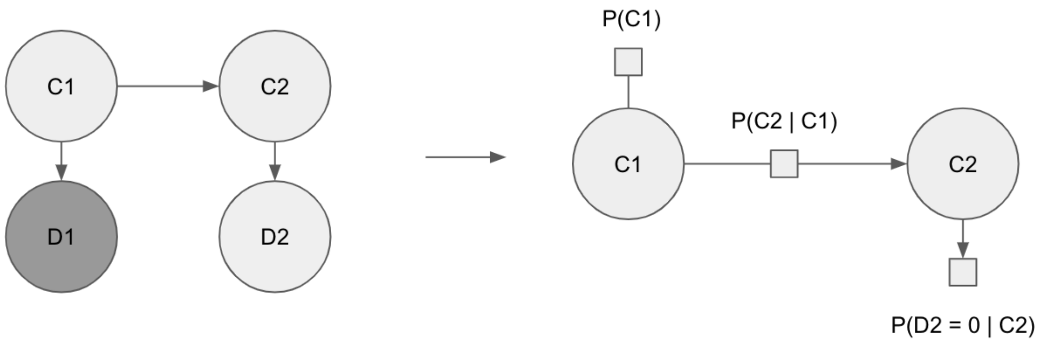
Question 1.a, Car Tracking, CS221



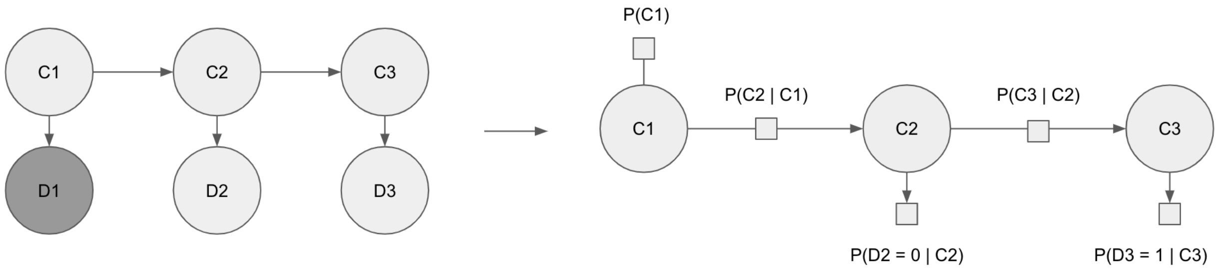
The factor graph is shown above.

$P(C_2|D_2 = 0) \propto p(d_2 = 0|c_2) \sum_{c_1} p(c_1) \cdot p(c_2|c_1)$

$P(C_2 = 1|D_2 = 0) \propto p(d_2 = 0|c_2 = 1) \sum_{c_1} p(c_1) \cdot p(c_2 = 1|c_1) = \eta \cdot [0.5\epsilon + 0.5(1 - \epsilon)] = 0.5\eta$

$P(C_2 = 0|D_2 = 0) \propto p(d_2 = 0|c_2 = 0) \sum_{c_1} p(c_1) \cdot p(c_2 = 0|c_1) = (1-\eta) \cdot [0.5(1-\epsilon) + 0.5\epsilon] = 0.5(1-\eta)$

After normalization, $P(C_2 = 1|D_2 = 0) = \eta$



$P(C_2|D_2=0, D_3=1) \propto p(d_2=0|c_2) \cdot \sum_{c_1} p(c_1) \cdot p(c_2|c_1) \cdot \sum_{c_3} p(c_3|c_2) \cdot p(d_3=1|c_3)$

$P(C_2=1|D_2=0, D_3=1) \propto p(d_2=0|c_2=1) \sum_{c_1} p(c_1) \cdot p(c_2=1|c_1) \cdot \sum_{c_3} p(c_3|c_2=1) \cdot p(d_3=1|c_3) = \eta \cdot [0.5\epsilon + 0.5(1-\epsilon)] \cdot [\epsilon\eta + (1-\epsilon)(1-\eta)] = 0.5\eta \cdot [\epsilon\eta + (1-\epsilon)(1-\eta)]$

$P(C_2=0|D_2=0, D_3=1) \propto p(d_2=0|c_2=0) \sum_{c_1} p(c_1) \cdot p(c_2=0|c_1) \cdot \sum_{c_3} p(c_3|c_2=0) \cdot p(d_3=1|c_3) = (1-\eta) \cdot [0.5(1-\epsilon) + 0.5\epsilon] = 0.5(1-\eta) \cdot [(1-\epsilon)\eta + \epsilon(1-\eta)] = 0.5(1-\eta) \cdot [(1-\epsilon)\eta + \epsilon(1-\eta)]$

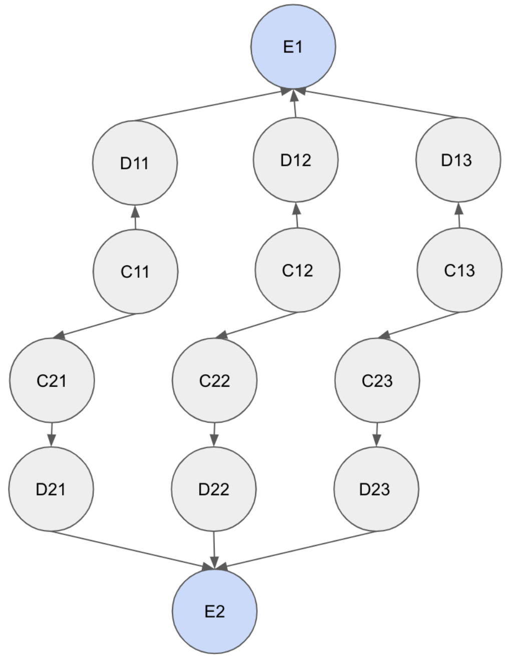
After normalization, $P(C_2=1|D_2=0, D_3=1) = \frac{\eta \cdot [\epsilon\eta + (1-\epsilon)(1-\eta)]}{\eta \cdot [\epsilon\eta + (1-\epsilon)(1-\eta)] + (1-\eta) \cdot [(1-\epsilon)\eta + \epsilon(1-\eta)]}$

- $P(C_2|D_2 = 0) = \eta = 0.2$

  $P(C_2|D_2 = 0, D_3 = 1) = \frac{\eta\cdot[\epsilon\eta+(1-\epsilon)(1-\eta)]}{\eta\cdot[\epsilon\eta+(1-\epsilon)(1-\eta)]+(1-\eta)\cdot[(1-\epsilon)\eta+\epsilon(1-\eta)]} = 0.4157$

- $D_3 = 1$ increases the probability of $C_2 = 1$. Given $D_3 = 1$, means the car is at position 1 at the time $t = 3$, thus it's more likely that $C_3 = 1$ based on local conditional distribution and thus it's more likely that $C_2 = 1$.

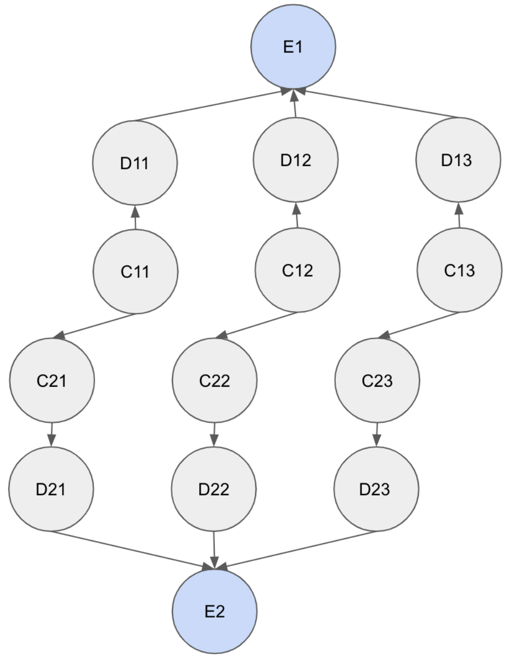
- To make $P(C_2|D_2 = 0) = P(C_2|D_2 = 0, D_3 = 1)$, we'll have $\epsilon = 0.1899$, which means we lower the $p(c_t|c_{t-1})$ for $c_t = c_{t-1}$. In this case, $P(C_3 = 1|C_2 = 1)$ will be lower.



The Bayesian network is shown above.

Since C_{11} and C_{12} are independent, $P(C_{11} = c_{11}, C_{12} = c_{12}|E_1 = e_1) = P(C_{11} = c_{11}|E_1 = e_1)P(C_{12} = c_{12}|E_1 = e_1) \propto p(c_{11})p_n(e_{11}; ||a_1 - c_{11}||, \sigma^2) \cdot p(c_{12})p_n(e_{12}; ||a_1 - c_{12}||, \sigma^2)$

Assume there is one assignment C_i that obtain the maximum value. As we have K cars, and $p(1i)$ is the same for all i, we can at least get $K!$ of assignment by getting permutations on C_i.



The Bayesian network is shown above.
If we eliminate from $D_{11}, C_{11}, C_{21}, C_{31} \cdots, C_{T1}, D_{T1}$, we'll have factors connecting to every E_t. Thus, the tree width T.

We could use Forward-Backward algorithm.

$$p(c_{ti}|e_1, \cdots, e_T) \propto S_{ti}(c_{ti}) \tag{1}$$

where $S_{ti}(c_{ti}) = F_{ti}(c_{ti})B_{ti}(c_{ti})$

$$F_{ti}(c_{ti}) = \sum_{c_{(t-1)i}} F_{(t-1)i}(c_{(t-1)i}w(c_{(t-1)i}, c_{ti})) \tag{2}$$

$$B_{ti}(c_{ti}) = \sum_{c_{(t+1)i}} B_{(t+1)i}(c_{(t+1)i}w(c_{ti}, c_{(t+1)i})) \tag{3}$$

Algorithm:

- for each i in K:
- compute $F_{1i}, F_{2i}, \cdots F_{Li}$
- compute $B_{Li}, B_{(L-1)i}, \cdots B_{1i}$
- compute S_{ti} for each t and normalize