山东大学　计算机科学与技术　学院

机器学习（双语）课程实验报告

学号：201705130113	姓名：　黄瑞哲	班级：计科 17.3
实验题目：Logistic 回归		
实验学时：2	实验日期：　2019.10.7	

实验目的：

一、掌握 Logistic 回归

二、掌握牛顿迭代法

三、能够区分牛顿迭代法和梯度下降法的优劣

硬件环境：

Intel Core i5-8300H @ 2.3GHz

软件环境：

Windows10 Pro 1903

Python 3.7

Visual Studio Code 1.38.1

Sublime Text 3

MinGW-w64

实验步骤与内容：

一、读取数据画出散点图

二、利用梯度下降法求 theta 以及迭代过程中的损失函数值

$$\nabla_\theta L = \frac{1}{m}\sum_{i=1}^{m}(h_\theta(x^{(i)}) - y^{(i)})x^{(i)}$$

并且绘制决策边界，对[20, 80]做预测。

三、利用牛顿下降法求 theta

1. 牛顿法原理

$$\theta^{(t+1)} = \theta^{(t)} - H^{-1}\nabla_\theta L$$

2. 求海森矩阵

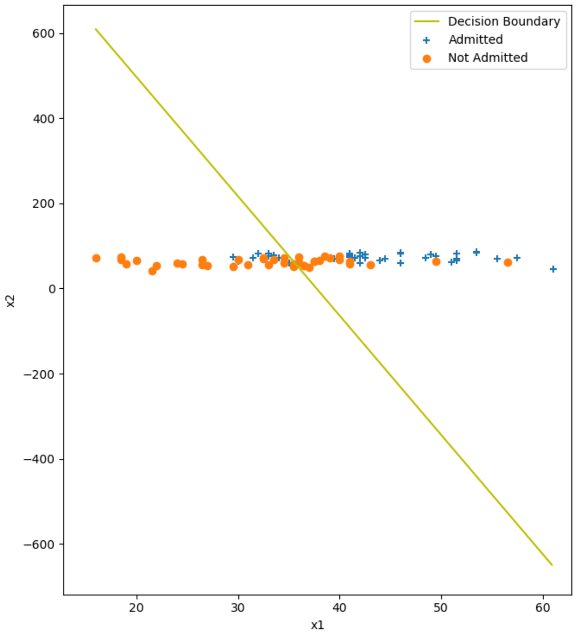
$$H = \frac{1}{m}\sum_{i=1}^{m}\left[h_\theta(x^{(i)})\left(1 - h_\theta(x^{(i)})\right)x^{(i)}\left(x^{(i)}\right)^T\right]$$

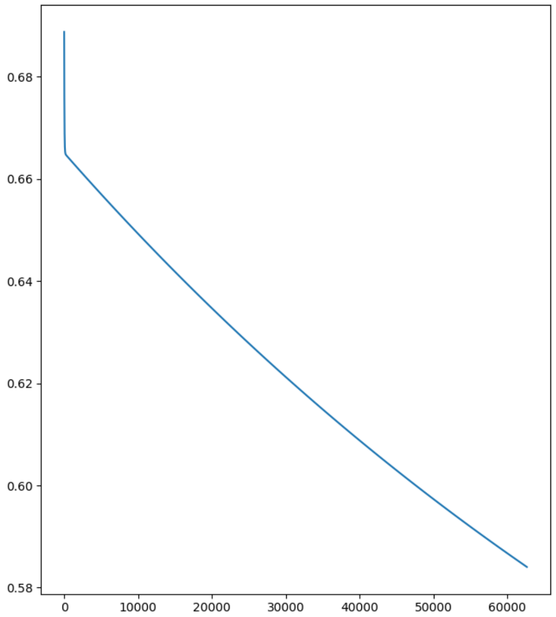
3. 绘制决策边界并且对[20, 80]做预测

结论分析与体会：

对于梯度下降，

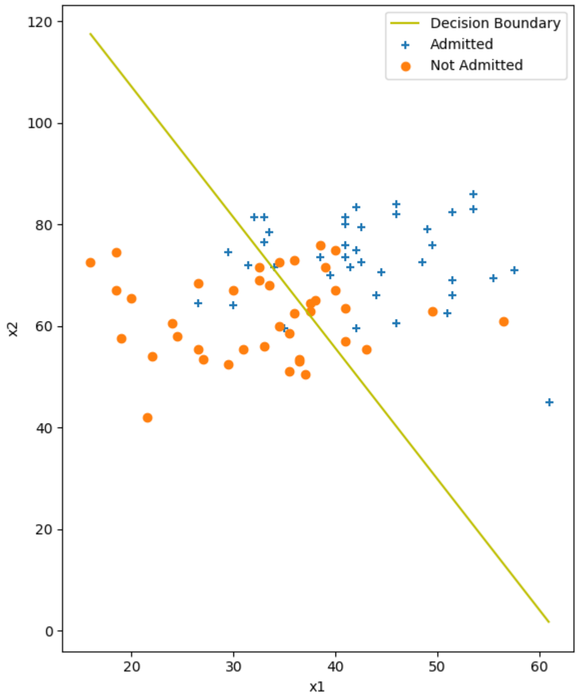
当学习率为 0.0009 时

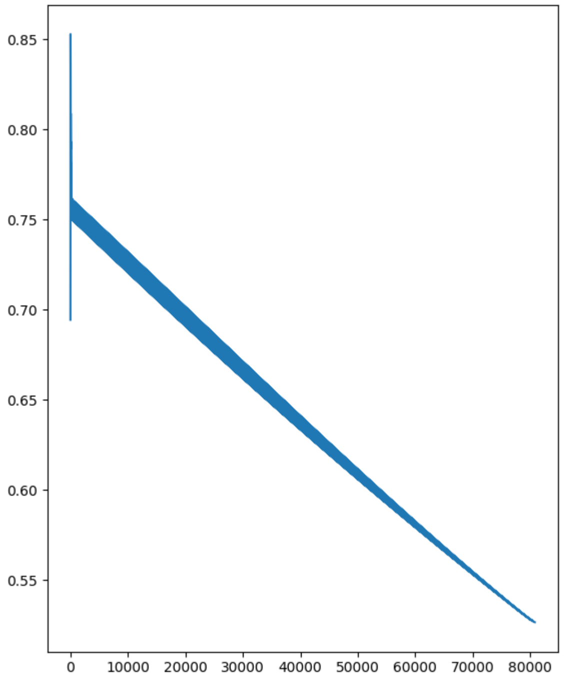




发现最终的损失函数只收敛到了 0.58 左右，而迭代了 60000 多次。

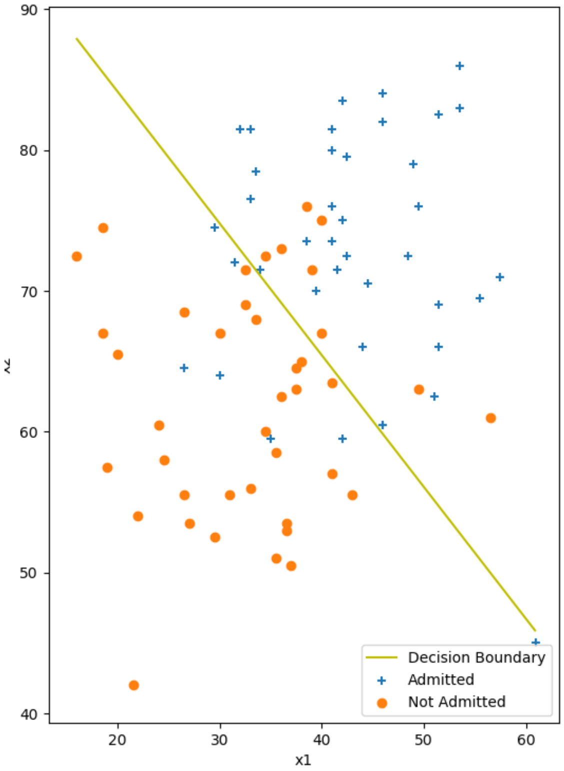
把学习率调整至 0.0015 后

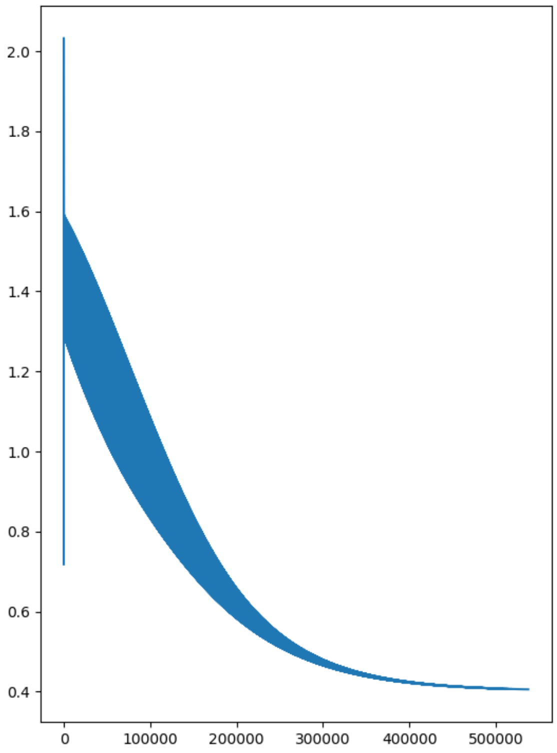




损失函数收敛至 0.5，迭代 80000 多次

将学习率调整至 0.0025 后





发现学习过程中损失函数不断震荡，最终收敛至 0.4 左右，但是迭代了 500000 次，相比于之前的效果更佳但是消耗的时间更多。

得到的 theta 值为

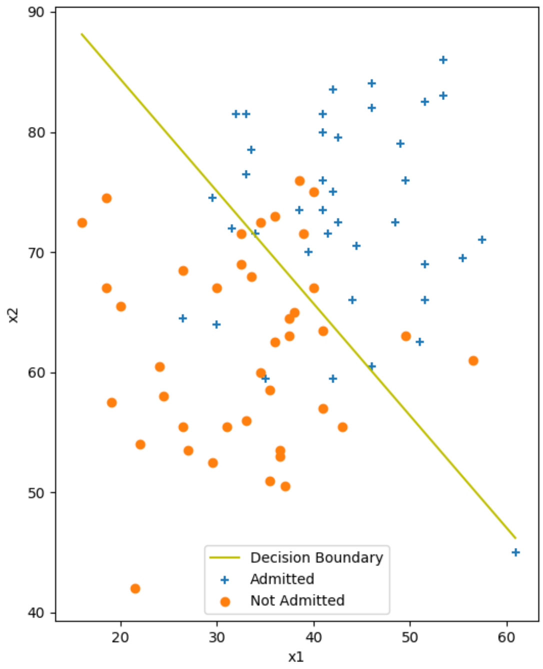
```
[[-16.1544      0.146911    0.157049]]
```

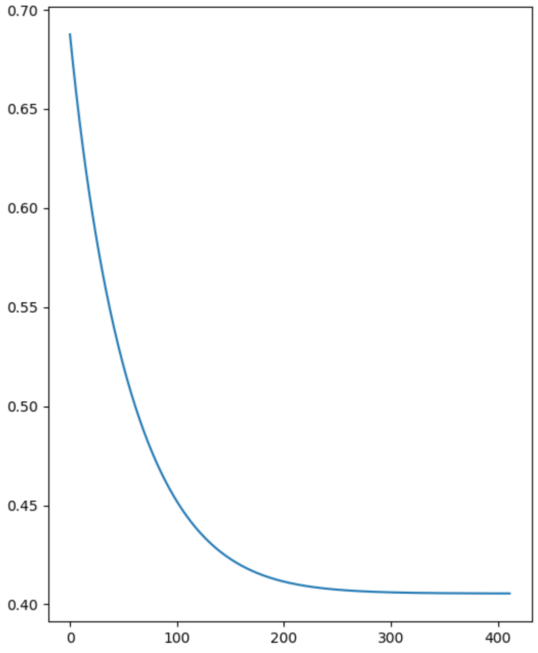
预测的概率为

```
[[0.65751957]]
```

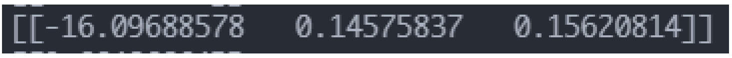
在学习的过程中，由于需要迭代的次数过多，python 不能在短时间内计算出答案，于是使用 C++进行计算，计算时间在 5s 左右，最终把得到的结果在 python 中可视化。

使用牛顿迭代法后

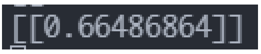




发现只迭代了 400 多次就收敛到了 0.4，迭代次数相比于梯度下降法大大减小，节省了不少的时间，得到的效果也很好。

得到的 theta 值为

```
[[-16.09688578   0.14575837   0.15620814]]
```

预测的概率为

```
[[0.66486864]]
```

附录：程序源代码

Python

```python
import numpy as np


def sig(x): return 1 / (1 + np.exp(-x))


def J(feature, label, weights):
    ret = 0
    m = np.shape(feature)[0]
    for i in range(m):
        h = sig(feature[i] * weights)
        if h <= 0:
            h = 0.0000001
        elif h >= 1:
            h = 0.9999999
        ret += -label[i, 0] * np.log(h) - (1 - label[i, 0]) * np.log(1 - h)
    return ret / m


def load_data():
    feature = []
    label = []
    with open('exp2/data/ex2x.dat') as f:
        for each_line in f.readlines():
            feature_temp = []
            feature_temp.append(1)
            for data in each_line.strip().split():
                feature_temp.append(float(data))
            feature.append(feature_temp)
    with open('exp2/data/ex2y.dat') as f:
        for each_line in f.readlines():
            label_temp = []
            for data in each_line.strip().split():
                label_temp.append(float(data))
            label.append(label_temp)
    return np.mat(feature), np.mat(label)
```

```
def load_result(filename):
    w = np.mat(np.zeros((3, 1)))
    cost = []
    with open(filename) as f:
        alpha, w[0, 0], w[1, 0], w[2, 0], _ = f.readline().split()
        for each_line in f.readlines():
            cost.append(each_line.strip())
    return alpha, w, np.mat(cost).T


# 梯度下降 太慢
def fit(feature, label, alpha, epsilon=1e-7):
    m, n = np.shape(feature)
    # w = np.mat([-4.96253179, 0.07103084, 0.03521583]).T
    w = np.mat(np.zeros((n, 1)))
    val = 0
    iteration = 0
    cost = []
    while True:
        iteration += 1
        h = sig(feature * w)
        err = label - h
        w = w + alpha * feature.T * err / m
        last = val
        val = J(feature, label, w)
        cost.append(val)
        if abs(val - last) < epsilon:
            break
    return w, np.mat(cost).T, iteration


def plot_fit(feature, label, **kwrags):
    x_pos = []
    y_pos = []
    x_neg = []
    y_neg = []
    for (x, y) in zip(feature, label):
        if y[0] == 1:
            x_pos.append(x[0, 1])
            y_pos.append(x[0, 2])
        else:
            x_neg.append(x[0, 1])
            y_neg.append(x[0, 2])
    file = kwrags.get('filename', None)
    weights = kwrags.get('weights', [])
    cost = kwrags.get('cost', [])
    alpha = kwrags.get('alpha', -1)
    if file:
        alpha, weights, cost = load_result(file)
    print(weights.T)
    print(1 - predict(np.mat([1, 20, 80]), weights))  # probability of [20, 80] will not be admiited
    import matplotlib.pyplot as plt
    plt.figure(str(alpha) if alpha != -1 else "Newton")
    plt.subplot(1, 2, 1)
    plt.scatter(x_pos, y_pos, marker='+', label='Admitted')
    plt.scatter(x_neg, y_neg, marker='o', label='Not Admitted')
    x = np.arange(min(feature[:, 1]), max(feature[:, 1]), 0.1)
    y = (-weights[0, 0] - weights[1, 0] * x) / weights[2, 0]
    plt.plot(x, y, 'y-', label='Decision Boundary')
    plt.xlabel('x1')
    plt.ylabel('x2')
    plt.legend()
    plt.subplot(1, 2, 2)
    plt.plot(np.linspace(0, len(cost), len(cost)), cost)
    plt.show()


def predict(data, w):
    h = sig(data * w)
    # m = np.shape(h)[0]
    # for i in range(m):
    # h[i, 0] = 0.0 if h[i, 0] < 0.5 else 1.0
    return h


def newton(feature, label, epsilon=1e-6):
    m, n = np.shape(feature)
    w = np.mat(np.zeros((n, 1)))
    val = 0
    iteration = 0
    cost = []
    while True:
        iteration += 1
        H = np.mat(np.zeros((n, n)))
        for i in range(m):
            h = sig(feature[i] * w)
            H += (h * (1 - h))[0, 0] * feature[i].T * feature[i]
        err = label - sig(feature * w)
```

```python
        w = w + H.I * feature.T * err / m
        last = val
        val = J(feature, label, w)
        cost.append(val[0, 0])
        if abs(val - last) < epsilon:
            break
    return w, cost, iteration


if __name__ == '__main__':
    feature, label = load_data()
    '''
    exe1 梯度下降法 C++计算结果，Python 可视化
    '''
    # # w, theta, iteration = fit(feature, label, 0.0012)
    #   result_file   =   ["0.000900.txt",   "0.001200.txt",   "0.001500.txt",
"0.001800.txt", "0.001900.txt", "0.002000.txt", "0.002500.txt"]
    # for file in result_file:
    #     plot_fit(feature, label, filename="exp2/data/"+file)

    '''
    exe2 牛顿迭代
    '''
    w, cost, iteration = newton(feature, label)
    plot_fit(feature, label, weights=w, cost=cost)
```

C++

```cpp
#include <algorithm>
#include <cassert>
#include <cmath>
#include <cstdio>
#include <cstring>
#include <iostream>
#include <map>
#include <set>
#include <string>
#include <vector>
#include <queue>
#include <fstream>
#define inf 0x3f3f3f3f
#define cases(t) for (int cas = 1; cas <= int(t); ++cas)
typedef long long ll;
typedef double db;
using namespace std;
```

```
#ifdef NO_ONLINE_JUDGE
#define LOG(args...) do { cout << #args << " -> "; err(args); } while (0)
void err() { cout << endl; }
template<typename T, typename... Args> void err(T a, Args... args) { cout << a <<
' '; err(args...); }
#else
#define LOG(...)
#endif

const db eps = 1e-6;

db x[80][3];
db y[80];
db w[3], ww[3];

db sig(db x) {
    return 1.0 / (1.0 + exp(-x));
}

db cal() {
    db ret = 0;
    for (int i = 0; i < 80; ++i) {
        db h = 0;
        for (int k = 0; k < 3; ++k)    h += w[k] * x[i][k];
        h = sig(h);
        ret += -y[i] * log(h) - (1 - y[i]) * log(1 - h);
    }
    return ret / 80;
}

vector<db> fit(db alpha) {
    for (int i = 0; i < 3; ++i) w[i] = ww[i] = 0;
    db val = 0, last;
    vector<db> cost;
    while (1) {
        for (int j = 0; j < 3; ++j) {
            db J = 0;
            for (int i = 0; i < 80; ++i) {
                db h = 0;
                for (int k = 0; k < 3; ++k) h += ww[k] * x[i][k];
                J += (sig(h) - y[i]) * x[i][j];
            }
            w[j] -= alpha * J / 80;
        }
        for (int i = 0; i < 3; ++i)    ww[i] = w[i];
        last = val;
        val = cal();
        cost.push_back(val);
        // LOG(iteration, val - last);
        if (fabs(val - last) <= eps)   break;
    }
    return cost;
}

int main() {
    ifstream fin("data/ex2x.dat");
    for (int i = 0; i < 80; ++i) {
        x[i][0] = 1;
        fin >> x[i][1] >> x[i][2];
    }
    fin.close();
    fin.open("data/ex2y.dat");
    for (int i = 0; i < 80; ++i) {
        fin >> y[i];
    }
    fin.close();
    // for (int i = 0; i < 80; ++i) {
    //     cout << x[i][0] << ' ' << x[i][1] << ' ' << x[i][2] << ' ' << y[i] <<
endl;
    // }
    db alpha = 0.002;
    vector<db> ret = fit(alpha);
    // LOG(alpha, w[0], w[1], w[2]);
    string name = "data/" + to_string(alpha) + ".txt";
    ofstream fout(name.c_str());
    fout << alpha << " " << w[0] << " " << w[1] << " " << w[2] << " " << cal() <<
endl;
    for(auto v:ret)  fout << v << endl;
    return 0;
}
/*
alpha, w[0], w[1], w[2], fit(alpha) -> 0.0004 -0.00924545 0.0459596 -0.0229827
535
alpha, w[0], w[1], w[2], fit(alpha) -> 0.0005 -0.0110848 0.0466746 -0.0233549 514
alpha, w[0], w[1], w[2], fit(alpha) -> 0.0006 -0.43249 0.0491012 -0.0185842 17245
alpha, w[0], w[1], w[2], fit(alpha) -> 0.0007 -1.10884 0.0519389 -0.0103293 39377
alpha, w[0], w[1], w[2], fit(alpha) -> 0.0008 -1.66343 0.0543808 -0.00361771 53419
alpha, w[0], w[1], w[2], fit(alpha) -> 0.0009 -2.1325 0.0565278 0.00201761 62666
alpha, w[0], w[1], w[2], fit(alpha) -> 0.001 -2.53826 0.0584452 0.00686168 68898
```

```
alpha, w[0], w[1], w[2], fit(alpha) -> 0.0012 -3.21363 0.0617594 0.0148612 76044
alpha, w[0], w[1], w[2], fit(alpha) -> 0.0015 -4.10757 0.066715 0.0258728 80821
alpha, w[0], w[1], w[2], fit(alpha) -> 0.0018 -8.70063 0.0939862 0.0776859 183547
*/
```