1. A^TA可逆 $\Leftrightarrow$ $A^TAx=0$只有0解 $\Leftrightarrow$ $(Ax)^TAx=x^TA^TAx=0$

$\Leftrightarrow$ 令 $Ax=y$，则 $y^Ty=0$ 即 $Ax=0$

又∵ $Ax=0$ 只有零解 $\Leftrightarrow$ A列满秩

2. ∵ $\frac{\partial J}{\partial\theta_p}=\sum_{i=1}^{m}(\theta^Tx^{(i)}-y^{(i)})\cdot x_p^{(i)}$，$\frac{\partial^2 J}{\partial\theta_p\partial\theta_q}=\sum_{i=1}^{m}x_p^{(i)}\cdot x_q^{(i)}$

$$\therefore \nabla_\theta J=\begin{bmatrix}\frac{\partial J}{\partial\theta_1}\\ \frac{\partial J}{\partial\theta_2}\\ \vdots\end{bmatrix}=X^T(X\theta-Y),\quad H=\begin{bmatrix}\sum_{i=1}^{m}x_1^{(i)}x_1^{(i)}, & \sum_{i=1}^{m}x_1^{(i)}x_2^{(i)}, & \cdots\\ \sum_{i=1}^{m}x_2^{(i)}\cdot x_1^{(i)}, & \cdots & \\ \vdots & & \end{bmatrix}$$

$$=X^TX$$

$$\therefore \theta_{t+1}=\theta_t-H^{-1}\cdot\nabla_\theta J=\theta_t-(X^TX)^{-1}\cdot X^T(X\theta_t-Y)$$

$$=\theta_t-(X^TX)^{-1}\cdot(X^TX)\cdot\theta_t+(X^TX)^{-1}\cdot X^T\cdot y$$

$$=(X^TX)^{-1}\cdot X^T\cdot y$$

3. 已知 $X=\begin{bmatrix}1, & 2005\\ 1, & 2006\\ 1, & 2007\\ 1, & 2008\\ 1, & 2009\end{bmatrix}$，$Y=\begin{bmatrix}12\\19\\29\\37\\45\end{bmatrix}$

可得 $\theta=(X^TX)^{-1}X^TY=\begin{bmatrix}-16830.4\\ 8.4\end{bmatrix}$

∴ $y=8.4x-16830.4$

∴ 当$x=2012$时，$y=70.4$，约为70

4、 $\because g'(z)=g(z)[1-g(z)] \Rightarrow \frac{\partial}{\partial\theta_p}h(x)=h(x)[1-h(x)]\cdot x_p$.

$\therefore \frac{\partial}{\partial\theta_p}J(\theta)=-\frac{1}{m}\sum_{i=1}^{m}\frac{1}{h(y^{(i)}x^{(i)})}\cdot h(y^{(i)}x^{(i)})\cdot[1-h(y^{(i)}x^{(i)})]\cdot y^{(i)}\cdot x_p^{(i)}$

$=-\frac{1}{m}\sum_{i=1}^{m}[1-h(y^{(i)}x^{(i)})]\cdot y^{(i)}\cdot x_p^{(i)}$

$\therefore H_{pq}=\frac{\partial}{\partial\theta_p\partial\theta_q}J(\theta)=-\frac{1}{m}\sum_{i=1}^{m}-h(y^{(i)}x^{(i)})[1-h(y^{(i)}x^{(i)})]\cdot y^{2(i)}\cdot x_p^{(i)}\cdot x_q^{(i)}$

$=\frac{1}{m}\sum_{i=1}^{m}h(y^{(i)}x^{(i)})\cdot(1-h(y^{(i)}x^{(i)}))x_p^{(i)}x_q^{(i)}$

$=\frac{1}{m}\sum_{i=1}^{m}h(x^{(i)})[1-h(x^{(i)})]\cdot x_p^{(i)}\cdot x_q^{(i)}$

$\therefore H=\frac{1}{m}\sum_{i=1}^{m}h(x^{(i)})[1-h(x^{(i)})]\,x^{(i)}\cdot x^{(i)T}$

$\therefore z^THz=\frac{1}{m}\sum_{i=1}^{m}h(x^{(i)})[1-h(x^{(i)})]\,z^Tx^{(i)}\cdot x^{(i)T}\cdot z$

$\because h(x^{(i)})\in(0,1)$ $\therefore h(x^{(i)})[1-h(x^{(i)})]>0$, $\forall i$

$\therefore$ 只考虑 $\sum_{i=1}^{m}z^Tx^{(i)}x^{(i)T}z$ 即可.

又$\because$ $x^{(i)T}z\in R$ 且 $z^Tx^{(i)}=(x^{(i)T}z)^T$

$\therefore \sum_{i=1}^{m}z^Tx^{(i)}x^{(i)T}z=\sum_{i=1}^{m}(x^{(i)T}z)^T(x^{(i)T}z)=\sum_{i=1}^{m}(x^{(i)T}z)^2\geq 0$

$\therefore z^THz\geq 0$