3. Probability

#1.

P(no student will have to answer more than one question)

$$= \frac{\text{\# of ways Prof chooses 8 students}}{\text{Total num. of ways Prof can distribute 8 questions}}$$

$$= \frac{\binom{15}{8}}{\binom{8+15-1}{15-1}} = \frac{6435}{319770}$$

$$= \boxed{\frac{13}{646} \approx 0.0201}$$

#2. #(3 digit even integers w/ 2 odd first digits)
+ #(4 digit even integers w/ 2 odd first digits)
+ #(5 digit even integers w/ 2 odd first digits)
/ 100 000

$$= \frac{\underbrace{5\times4}_{\text{2 odd digits}}\times\underbrace{5}_{\text{even last digit}} + 5\times4\times5\times\underbrace{7}_{\text{possible choices for other integer}} + 5\times4\times5\times\underbrace{7\times6}_{\text{possible choices for other two integers}}}{100\ 000}$$

$= 0.05$

$$P(\text{exactly 5 in 8}) = \binom{8}{5}(0.05)^5(1-0.05)^3$$
$$\approx \boxed{1.5\times10^{-5}}$$

#3. $P(A) = P(\text{2 dice show 4+}) + P(\text{3 dice show 4+})$

$$= \binom{3}{2}\times\left(\tfrac{3}{6}\right)^2\left(\tfrac{3}{6}\right) + \binom{3}{3}\times\left(\tfrac{3}{6}\right)^3$$

$$= 3\times\frac{1}{8} + \frac{1}{8}$$

$$= \frac{1}{2}$$

$P(B) = 6 \times \left(\frac{1}{6}\right)^3$

$= \frac{1}{36}$

$P(A \cap B) = P(\text{all 3 dice show } 4+)$

$= 3 \times \left(\frac{1}{6}\right)^3$

$= \frac{1}{72}$

$P(A) \times P(B) = \frac{1}{2} \times \frac{1}{36} = \frac{1}{72}$

$\boxed{\text{A and B are independent}}$

#4 $P(\text{flush}) = \dfrac{4 \times \binom{13}{5}}{\binom{52}{5}} = \dfrac{33}{16660}$

$\frac{16660}{33} \approx 504.848$

This means that Paul is expected to play about 505 hands of poker to get a flush

#15. $P(\text{superstar played} \mid \text{win 4 or 5})$

$= \dfrac{P(\text{win 4 or 5 w/ star})}{P(\text{win 4 or 5 w/o star}) + P(\text{win 4 or 5 w/ star})}$

$= \dfrac{0.75 \times \binom{5}{4}(0.7)^4(0.3)}{0.25 \times \binom{5}{4}(0.5)^5 + 0.75 \times \binom{5}{4}(0.7)^4(0.3)}$

$= \boxed{0.874}$