

2. Counting Problems

#1. Unique subsets = #subsets with one U + #subsets with two U's
+ #subsets with three U's

$$\begin{aligned} \text{unusual} &= 1 + \binom{4}{3} + \binom{4}{2} \\ &= 1 + 4 + 6 \\ &= \underline{11} \end{aligned}$$

Unique strings: #subsets w/ one U $\times 5!$
+ #subsets w/ two U's $\times \frac{5!}{2!}$
+ #subsets w/ three U's $\times \frac{5!}{3!}$

$$\begin{aligned} &= 5! + 4 \times \frac{5!}{2!} + 6 \times \frac{5!}{3!} \\ &= 120 + 2 \times 120 + 120 \\ &= \underline{480} \end{aligned}$$

#2. num of 2 pairs = #num of pair combinations
 \times #num of suit combinations for two pairs

$$\begin{aligned} &= \binom{13}{2} \times \binom{4}{2} \times \binom{4}{2} \\ &= 78 \times 6 \times 6 \\ &= 2808 \end{aligned}$$

$$\begin{aligned} &2808 \times \text{\#num of remaining cards not of same value} \\ &= 2808 \times 44 \\ &= \underline{123552} \end{aligned}$$

#3. #dist. where fighting couple receives no song
+ #dist. where fighting couple receives one song

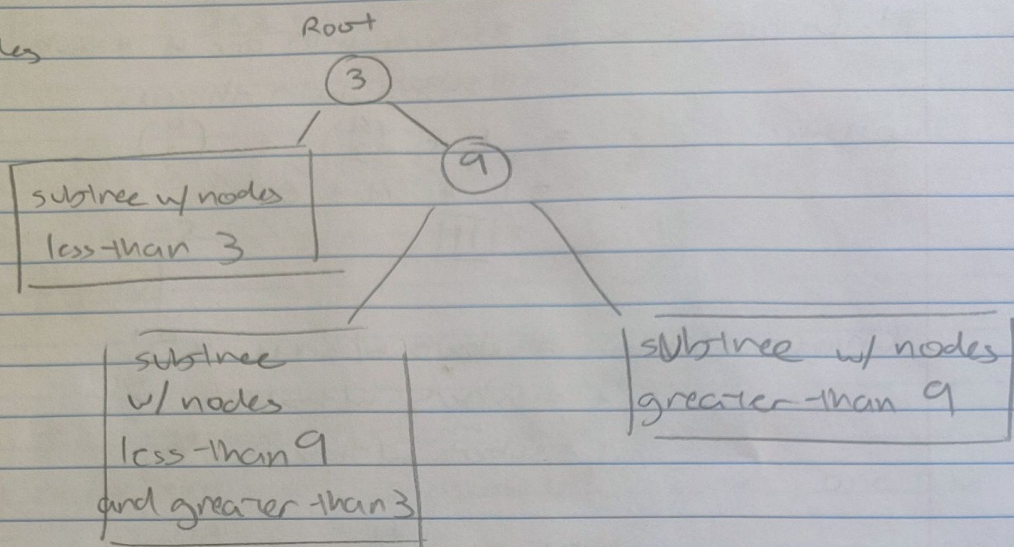
$$= \binom{16 + 6 - 1}{6 - 1} + \binom{15 + 6 - 1}{6 - 1} = 20349 + 15504$$

↑ split all
16 songs
among the other
6 couples

↑ split the
remaining 15
songs among the
other couples
given that
the fighting couple
gets one

$$= \underline{35853}$$

#4. 12 nodes



subtrees w/ nodes less than 3 = 2

↑
To order 2 nodes in BST

subtrees w/ nodes greater than 9

↑

order 3 nodes in BST

$$= (\text{left tree has } 2)(\text{right tree has } 0)$$

$$+ (\text{left tree has } 1)(\text{right tree has } 1)$$

$$+ (\text{left tree has } 0)(\text{right tree has } 2)$$

$$= 2 \times 1 + 1 \times 1 + 1 \times 2$$

$$= 5$$

To order 4 nodes in a BST, we can use the same subtree logic

$$= (\text{left tree has } 3)(\text{right tree has } 0) + (\text{left tree has } 2)(\text{right tree has } 1)$$

$$+ (\text{right tree has } 1)(\text{left tree has } 2) + (\text{left tree has } 0)(\text{right tree has } 3)$$

$$= 5 \times 1 + 2 \times 1 + 1 \times 2 + 1 \times 5$$

$$= 14$$

subtrees w/ nodes less than 9 and greater than 3 = (left tree has 4)(right tree has 0)

$$+ (\text{left tree has } 3)(\text{right tree has } 1)$$

$$+ (\text{left tree has } 2)(\text{right tree has } 2)$$

$$+ (\text{left tree has } 1)(\text{right tree has } 3)$$

$$+ (\text{left tree has } 0)(\text{right tree has } 4)$$

↑

order 5 nodes in BST

$$= 14 \times 1 + 5 \times 1 + 2 \times 2 + 1 \times 5 + 1 \times 14$$

$$= 42$$

Total BST's w/ root = 3, root right child = 9

$$= 2 \times 5 \times 42$$

$$= \boxed{420}$$

#5. # patient combinations w/ 4 nurses
 + # patient combinations w/ 3 nurses, one is on break

$$= \binom{10-1}{4-1} + \binom{10-1}{3-1}$$

\uparrow
 4 nurses, all nurses
 treat at least
 one patient

\uparrow
 3 nurses, one
 is on break,
 all nurses treat
 at least one
 patient

$$= 84 + 36$$

$$= \boxed{120}$$