ATM S 559 Homework 1

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Introduction and Overview

This is a simplified attempt at modeling global ocean temperature anomalies using the Held model[1] discussed in class. This exploration will investigate the temperature anomalies predicted by the Held model in four test cases:

[1] Isaac M Held, Michael Winton, Ken Takahashi, Thomas Delworth, Fanrong Zeng, and Geoffrey K Vallis. Probing the fast and slow components of global warming by returning abruptly to preindustrial forcing. *Journal of Climate*, 23(9):2418–2427, 2010

- **(Case 1)** This is the "standard" case where the ocean heat uptake effacacy $\epsilon = 1$, $\gamma = 0.5$.
- **(Case 2)** $\epsilon = 3$, and γ remains unchanged.
- **(Case 3)** $\gamma = 2.5$ while $\epsilon = 1$.
- **(Case 4)** $\epsilon = 0.5$, and $\gamma = 0.5$.

The model involves two coupled equations (found in the paper or the homework assignment) which are solved for T_{ml} and T_{do}, the upper ocean and deep ocean temperature anomalies, respectively. The equations were solved for temperature anomalies using a forward Euler method, described in appendix A. From these temperatures we can also find the flux imbalance,

$$N(t) = -\beta T_{ml} - (\epsilon - 1)\gamma(T_{ml} - T_{do}) + F(t)\,, \tag{1}$$

which has units of W$\cdot$m^{-2}. Additionally, the equilibrium climate response is defined in terms of the radiative forcing and the heat flux damping term as $T_{eq} = F(t)/\beta$. In all cases $F(t)$ is held constant at 3.75 W$\cdot$m^{-2} to approximate a sudden doubling of CO_2 in the atmosphere.

Temperature Anomalies and Flux Imbalance

Case 1

The standard case illustrates a climate response where the ocean heat uptake efficacy is unity. In this scenario, $T_{eq} = 3.125\,K$, but equilibrium was not reached, even at 2000 years. Figure 1 shows the response of T_{ml} and T_{do} to the sudden increase in radiative forcing as a result of doubling CO_2 as well as $N(t)$ as a function of T_{ml}. Note that the mixing layer initially heats rapidly before assuming a more gradual slope, while the deep ocean uptakes heat slowly over time.

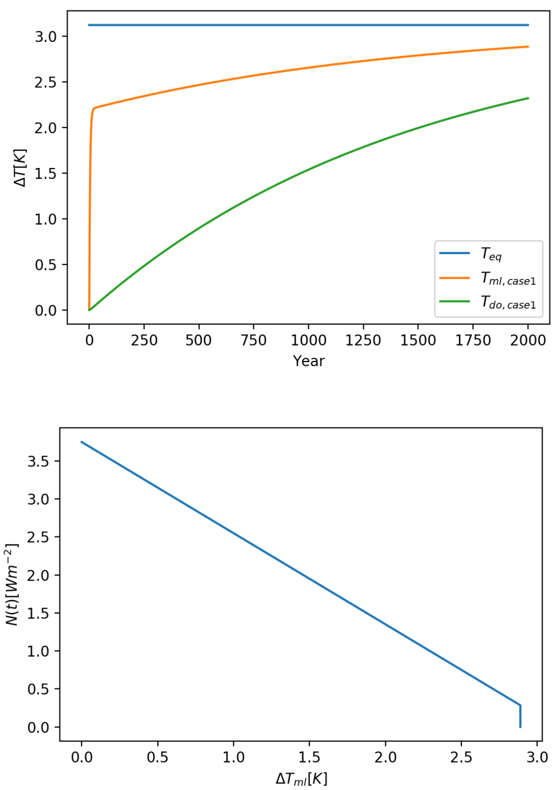
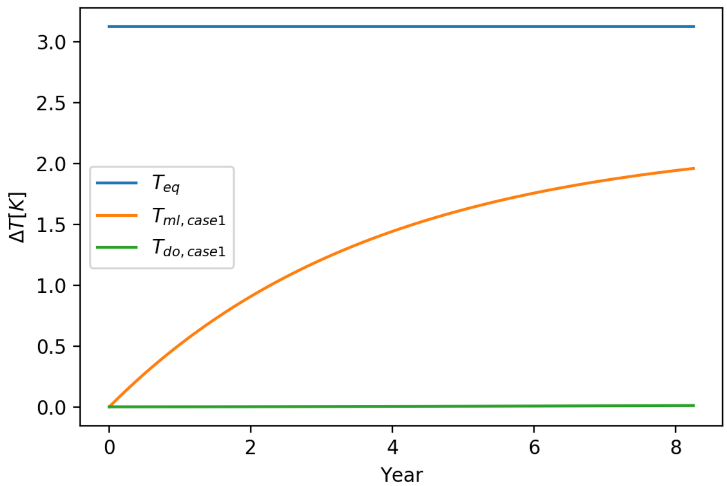


Figure 1: Case 1 temperature anomalies and flux imbalance for the "standard" case of $\epsilon = 1$ and $\gamma = 0.5$.

The plot of $N(t)$ has an unexpected drop on the right of the plot which I will address in the conclusion.

Figure 2 shows the initial fast response of T_{ml}, climbing about 2 K within the first decade.

Figure 2: The initial rapid climb of T_{ml} in response to radiative forcing.



Case 2

In case 2, the increased efficacy causes T_{ml} to jump to a lower initial value before gradually increasing toward T_{eq}. Unlike case 1 where $N(t)$ vs. T_{ml} had a constant slope of $-\beta$, $N(T)$ vs. T_{ml} for case 2 has two slopes in two regions. This is the impact of the coupling of the mixing and deep ocean layers on $N(t)$ when $\epsilon \neq 0$.

If only the slope of the first interval, β_0, is used to estimate T_{infer}, then T_{infer} will be an underestimate of T_{eq}. In case 2 $T_{infer}/T_{eq} \approx 0.56$, whereas the GFDL figure model from homework 1 has $T_{infer}/T_{eq} \approx 0.53$. The kink in the plot in case 2 appears around year 800. This is much larger than $\tau = c/\beta$, which is closer to 6.5 years. I have not figured out why this is the case. The severity of the kink seems to be the result of the coupling term between the mixing layer and deep ocean, perhaps because of the lag of the deep ocean to respond to the radiative forcing.

Case 3 and Case 4

These two cases were run to investigate how ϵ and γ affect the results. Figure 4 is the result of increasing γ to 2. This doubling of γ causes a smaller initial jump in T_{ml}, but T_{ml} shows a faster approach

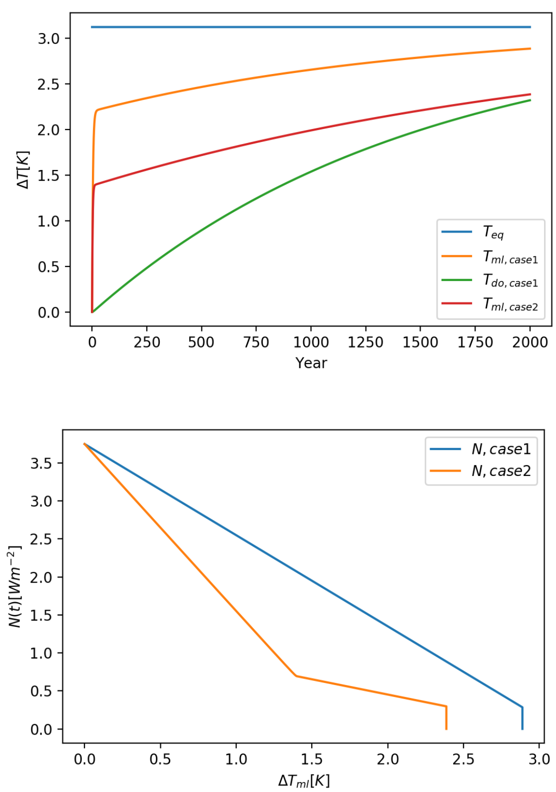
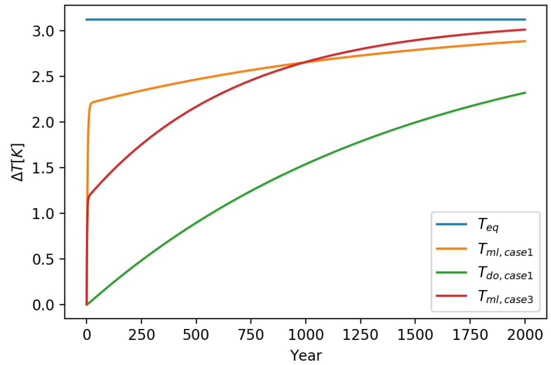


Figure 3: Case 2 temperature anomalies and flux imbalance when $\epsilon = 3$ and $\gamma = 0.5$.



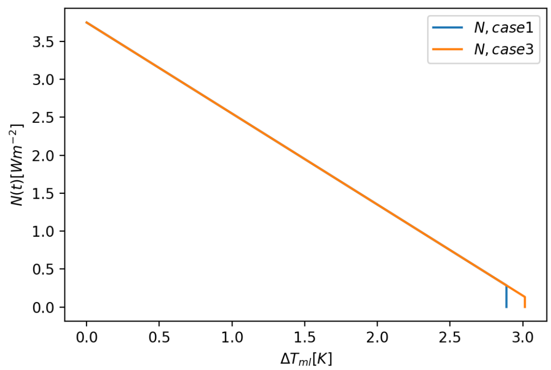


Figure 4: Case 3 temperature anomalies and flux imbalance when $\epsilon = 1$ and $\gamma = 2$.

toward T_{eq} than the baseline case. Increasing γ shifts location of the unexplained drop in $N(t)$.

Decreasing ϵ to 0.5 had little effect on T_{ml} compared with case 1, but it did change the behavior of $N(t)$. Figure 5 shows an increased (less steep) slope in $N(t)$ over the baseline case, and the kink in the plot is shifted to the right before the slope decreases.

Summary and Conclusions

It was interesting to explore this simple model. Changing parameters helped me build some intuition about radiative forcing and the slow response of the deep ocean. In the immediate future, I would like to understand why $N(t)$ suddenly dropped to zero, which I suspect was a numerical mistake on my part. I would also like to vary $F(t)$, perhaps as a function of a simple ice cover model.

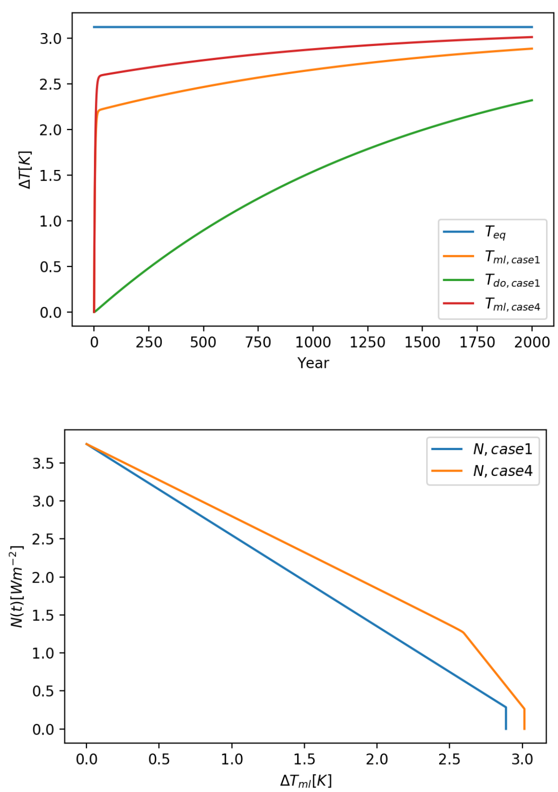


Figure 5: Case 4 temperature anomalies and flux imbalance when $\epsilon = 0.5$ and $\gamma = 0.5$.

References

[1] Isaac M Held, Michael Winton, Ken Takahashi, Thomas Delworth, Fanrong Zeng, and Geoffrey K Vallis. Probing the fast and slow components of global warming by returning abruptly to preindustrial forcing. *Journal of Climate*, 23(9):2418–2427, 2010.

Appendix A: Numerical Method

A forward Euler method was used to solve the equation in the held model such that,

$$T_{ml}^{n+1} = T_{ml}^{n} - \frac{\Delta t \beta T_{ml}^{n}}{c_{ml}} - \frac{\Delta t \epsilon \gamma (T_{ml}^{n} - T_{do}^{n})}{c_{ml}} + \frac{\Delta t F(t)}{c_{ml}} \tag{2}$$

$$T_{do}^{n+1} = T_{do}^{n} \frac{\Delta t \gamma}{c_{do}} (T_{ml}^{n} - T_{do}^{n})\,. \tag{3}$$

Appendix B: Python Code

The following script solves the Held model equations for T using a forward Euler method. The values of ϵ and γ were changed for each run.

```
#
# Jesse Dumas
# ATM S 559 HW 1: Held model
#
import numpy as np
import matplotlib.pyplot as plt
import pandas as pd

beta = 1.2
epsilon = 1.0
gamma = 0.5
c_p =  3985 # J/kg-K
depth_ml = 50 # m
depth_do = 4000 # m
rho = 1025 # kg/m^3
c_ml = rho * c_p * depth_ml
c_do = rho * c_p * depth_do
n_step = 12 # steps/y
years = 2000 # years
number_of_steps = n_step * years # Total number of model steps
radiative_forcing = 3.75 # W/m^2
```

```
T_eq = (radiative_forcing / beta) * np.ones(number_of_steps)
tau = c_ml / beta
dt = 365.0 * 24 * 3600 / n_step

T_ml2 = np.zeros(number_of_steps)
T_do2 = np.zeros(number_of_steps)

for i in np.arange(0, number_of_steps-1):
    T_ml2[i+1] = T_ml2[i] - (dt * beta * (T_ml2[i]) / c_ml) - (dt * epsilon * gamma * (T_ml2[i] -\
    T_do2[i]) / c_ml) + dt * radiative_forcing/c_ml
    T_do2[i+1] = (dt * gamma / c_do) * (T_ml2[i] - T_do2[i]) + T_do2[i]

N = np.zeros(number_of_steps)
for iii in np.arange(0, number_of_steps - 1):
    N[iii] = -beta * T_ml2[iii] - (epsilon - 1) * gamma * (T_ml2[iii] - T_do2[iii]) +\
    radiative_forcing

T_ml3 = np.zeros(number_of_steps)
T_do3 = np.zeros(number_of_steps)

epsilon = 0.5

for i in np.arange(0, number_of_steps-1):
    T_ml3[i+1] = T_ml3[i] - (dt * beta * (T_ml3[i]) / c_ml) - (dt * epsilon * gamma * (T_ml3[i] -\
    T_do3[i]) / c_ml) + dt * radiative_forcing/c_ml

    T_do3[i+1] = (dt * gamma / c_do) * (T_ml3[i] - T_do3[i]) + T_do3[i]

N2 = np.zeros(number_of_steps) # flux imbalance
for iii in np.arange(0, number_of_steps - 1):
    N2[iii] = -beta * T_ml3[iii] - (epsilon - 1) * gamma * (T_ml3[iii] - T_do3[iii]) +\
    radiative_forcing

# make plots
ysteps2=np.arange(0,number_of_steps) * 1.0 / n_step

plt.figure(2)

plt.plot(ysteps2, T_eq)
plt.plot(ysteps2, T_ml2)
plt.plot(ysteps2, T_do2)
plt.plot(ysteps2, T_ml3)
plt.ylabel("$\Delta T [K]$")
plt.xlabel("Year")
```

```
plt.legend(("$T_{eq}$", "$T_{ml, case 1}$", "$T_{do, case 1}$", "$T_{ml, case 4}$" ),loc="lower right")

plt.savefig("TvYear4.png", dpi=200)

plt.figure(3)

plt.plot(T_ml2, N)
plt.plot(T_ml3, N2)
plt.ylabel("$N(t) [W m^{-2}]$")
plt.xlabel("$\Delta T_{ml} [K]$")
plt.legend(("$N, case 1$", "$N, case 4$"),loc="upper right")

plt.savefig("NvT4.png", dpi=200)
```