Modeling and Analysis of Thermoelectric Modules

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Abstract - **The objective of this work was to develop a SPICE compatible equivalent circuit of a thermoelectric module (TEM). A methodology is developed for extracting the parameters of the proposed model from manufacturers' data of Thermoelectric Coolers (TEC) and Thermoelectric Generators (TEG). The model could be helpful for analyzing the drive requirements of TECs and loading effects of TEGs. The present model is compatible with PSPICE or other electric circuit simulators. An important feature of the model is its ability to generate small signal transfer functions that can be used to design feedback networks for temperature control applications.**

I. INTRODUCTION

A thermoelectric module (TEM) is a solid-state energy converter. It normally consists of an array of pellets from dissimilar semiconductor material (p and n type), which are joined, thermally in parallel and electrically in series. The TEM can be used for cooling, heating, and energy generation [1] - [3]. As a thermoelectric cooler (TEC), the TEM already found applications in thermal management and control of microelectronic devices such as diode lasers and CPUs. As thermoelectric generator (TEG), the TEM could be used to produce electric power in remote locations when temperature gradients are available [2].

The objective of this work was to develop a SPICE compatible equivalent circuit of a TEM. Equivalent circuit is a convenient tool for electronic engineers. It helps to present the problem in electronic circuit terms, helps to understand its functionality, and facilitates the solving of cooling or power-generation problems without the need for expertise in thermal engineering.

II. PRINCIPLES OF OPERATION

There are five main physical processes taking place in thermoelectric module: **Thermal convection** - the phenomenon named by Fourier process, described by physical constant k (W/Km), which is determined by thermal conductivity and geometry of the pellet. Θ (K/W) is a thermal resistance of the couple

$$\Theta = \frac{1}{k}\frac{h}{A} \tag{1}$$

$$T = \Theta q \tag{2}$$

where h/A is geometry factor, h – height of the pellet (m), A – cross-section area (m^2), T – temperature (K), and q – heat (W). **Joule heating** is the physical process of heat dissipation on the resistive elements. The electrical resistance R of a couple of pellets is:

$$R = \rho\frac{h}{A} \tag{3}$$

$$q_j = I^2 R \tag{4}$$

where ρ - resistivity of the material (Ω m), q_j – Joule heating (W), I – electric current (A). **Peltier cooling/heating** – the phenomenon of absorption (or dissipation) of heat by a junction between two dissimilar materials when electrical current flows throw the junction. The heat q_p absorbed/dissipated by the junction is:

$$q_p = \pi I \tag{5}$$

where π (V) is a temperature dependent Peltier coefficient corresponding to a specific pair of materials. **Seebeck power generation** is a process by which heating (or cooling) of the junction of two dissimilar materials generates an electrical potential of the junction:

$$\pi = \alpha T \tag{6}$$

where α (V/K) is a Seebeck coefficient.

The potential difference the two junctions of the pellet will be:

$$U = \pi_a - \pi_e = \alpha(T_a - T_e) = \alpha\Delta T \tag{7}$$

where $\pi_{a/e}$ is a potential of absorbing/emitting junctions and $T_{a/e}$ is a temperature of absorbing/emitting junctions.

The additional thermoelectric phenomena – **Thompson phenomenon**, which is described by the Thompson coefficient $\tau = d\alpha/dT$ (V/K^2). The effect of this phenomenon is small [1], [3] and is therefore neglected in this work.

Fig. 1 shows a section of a TEM, which operates in the mode of thermoelectric cooler, when the power supply is connected to electric port and heat is pumped from the cold side to the hot side.
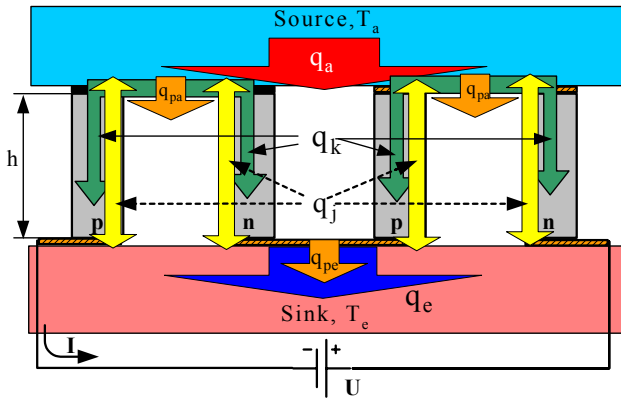
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Fig. 1. Energy equilibrium for thermoelectric cooler.

Fig. 2. Proposed equivalent circuit of a thermoelectric module for steady state. Bold lines are used for the thermal parts.

Following the first law of thermodynamics, one can express the energy equilibrium at both sides of the thermoelectric module that are defined as the absorbing (a) and emitting (e) junctions. For absorbing side, one can write:

$$q_a = \frac{\Delta T}{\Theta_m} + \alpha_m T_a I - \frac{I^2 R_m}{2} \qquad (8)$$

and for the emitting side:

$$q_e = \frac{\Delta T}{\Theta_m} + \alpha_m T_e I + \frac{I^2 R_m}{2} \qquad (9)$$

$$\alpha_m = \alpha N \qquad (10)$$

$$R_m = R N \qquad (11)$$

$$\Theta_m = \Theta / N \qquad (12)$$

where q_a is a heat absorbing at a-side, q_e – heat emitting at e-side, N – number of couples, T_a and T_e – temperatures of (a-) and (e-) sides in K, and $\Delta T = T_e - T_a$.

The electrical part of the module is described as electrical resistance R_m and an electrical potential difference U:

$$U = \alpha_m T_e - \alpha_m T_a = \alpha_m \Delta T \qquad (13)$$

It is a common practice in one-dimensional heat transfer problems [4] to apply an equivalent electrical circuit scheme. This approach was adopted in this study to describe the TEM system in which several energy types exist. All non-electrical processes are described in terms of electrical analogies, and transformers (or dependent sources) represent their interconnections. By this, the equivalent circuit of thermo-electrical system of TEM can built as a pure electrical circuit. The proposed equivalent circuit topology of the model (Fig. 2) is based on equations (8), (9), and (13) for a- and e-junctions [5].

III. CALCULATION OF THE PARAMETERS OF THE MODEL FROM THE MANUFACTURER'S DATASHEETS

Manufacturers of TECs ([6], [7], [8], and others) use the following parameters to specify their product:

ΔT_{max} - is the largest temperature differential (K) that can be obtained between the hot and cold ceramic plates of a TEM for the given level of hot-side temperature T_h,

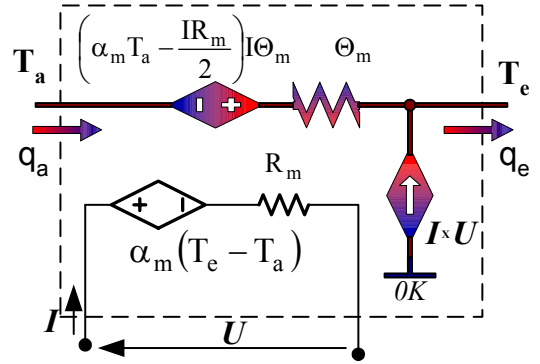I_{max} is the input current (A) which will produce the maximum possible ΔT across a TEC, and

U_{max} is the DC voltage (V) that will deliver the maximum possible ΔT at the supplied I_{max}.

Applying (8), (9), and (13) one can use the set of data: T_h, ΔT_{max}, U_{max}, I_{max} for calculating the parameters of proposed model:

Using the relations (8), (9), and (13), one can derive the characteristic parameters:

$$\Delta T_{max} = T_h + \frac{\left(1 - \sqrt{1 + 2 T_h Z}\right)}{Z} \qquad (14)$$

$$I_{max} = \frac{\sqrt{1 + 2 T_h Z} - 1}{\alpha_m \Theta_m} \qquad (15)$$

$$U_{max} = \alpha_m T_h \qquad (16)$$

where Z is figure of merit of the TEM. $Z = \alpha_m^2 \Theta_m / R_m$.

Applying (14) – (16), one can use the set of data: T_h, ΔT, U_{max}, I_{max} for calculating the parameters of the proposed model:

$$\alpha_m = \frac{U_{max}}{T_h} \qquad (17)$$

$$R_m = \frac{U_{max}}{I_{max}} \frac{\left(T_h - \Delta T_{max}\right)}{T_h} \qquad (18)$$

$$\Theta_m = \frac{\Delta T_{max}}{I_{max} U_{max}} \frac{2 T_h}{\left(T_h - \Delta T_{max}\right)} \qquad (19)$$

Manufacturers of TEGs normally specify the electrical properties rather than the thermal ones. The quoted parameters include electric power, open-circuit voltage, maximum power (for matched load), efficiency for matched load, maximum efficiency etc. The temperature range of power generator is usually grater than that of a cooling module and therefore one cannot neglect the dependence of the parameters on temperature. That is why some manufacturers provide data for different temperatures.

The datasheet of TEGs often includes power at matched load W_m (load is matched to internal resistance), load voltage at matched load U_m, open circuited voltage U_{oc}, and maximum efficiency η_{opt}.

Using this data, one can calculate the parameters of the equivalent circuit directly from datasheets:

$$R_m = \frac{U_m^2}{W_m} \qquad (20)$$

$$\alpha_m = \frac{2U_m}{\Delta T} \qquad (21)$$

$$\Theta_m = \frac{2\Delta T \eta_{opt}(2 - \eta_{opt}) R_m}{(\Delta T - \eta_{opt} T_a)^2 \, \alpha_m^2} \qquad (22)$$

IV. EXAMPLES

In this chapter, several examples of model application are presented:

1. The TB-127-1.4-1.2 – is one of thermoelectric cooling modules available from Kryotherm [6]. From manufacturer's datasheets: ΔT_{max}= 70 K, I_{max}=7.6 A, U_{max}=15.9 V, under condition that T_h=300 K,

Applying (16) – (18) one can calculate the model parameters: α_m=0.053 V/K, R_m=1.6 Ω, Θ_m=1.5 K/W.

Fig. 3 shows the result of application of the DC-sweep simulation that reconstructs the performance plot of the TEM TB-127-1.4-1.2. Dashed line is simulation result. Original performance plot was copied from the software, placed on the Internet by Kryotherm.

2. The HZ-20 is manufactured by Hi-Z Technology [9]. The data from datasheets: for hot side temperature $T_h = 230^oC$ and cold side temperature $T_c = 30\ ^oC$, $W_m = 19$ W, $U_m = 2.38$ V, $\eta_{opt} = 4.5\%$. From (20) – (22): $\alpha_m = 0.0238$ V/K, $\Theta_m = 0.589$ K/W, $R_m = 0.298$ Ω.

Fig. 4 shows the DC-sweep simulation of the HZ-20 module. The plot looks identical to the one given in the datasheet of the module.

V. COMPARISON OF THE EXPERIMENTAL DATA WITH SIMULATION USING THE MODEL

The laboratory measurements of physical TEM were compared with computer simulations to be certain that the model permits to simulate the processes taking place in TEM. The experiment was carried out using the TEM TB-127-1.4-1.2 (Kryotherm) that had the dimensions of 40mmx40mm. The module was inserted between two massive aluminum plates (40mmx40mmx5mm) with implemented thermocouples for temperature measurement. Both plates were insulated thermally from the ambient air. The TEM was first used as a cooler and a DC voltage was applied to it (Fig. 5). When the temperature difference between the plates reached a predetermined value, the supply voltage was turned off. From that point, TEM was continuing its operation as a generator and its output voltage U measured for different loads.

To simulate the experimental conditions by proposed model, one needs first to calculate:

1. The parameters of the model (16) – (18).
2. Lumped heat capacity of the aluminum plates. This can be calculated by:

$$C_{al} = c\rho V \qquad (23)$$

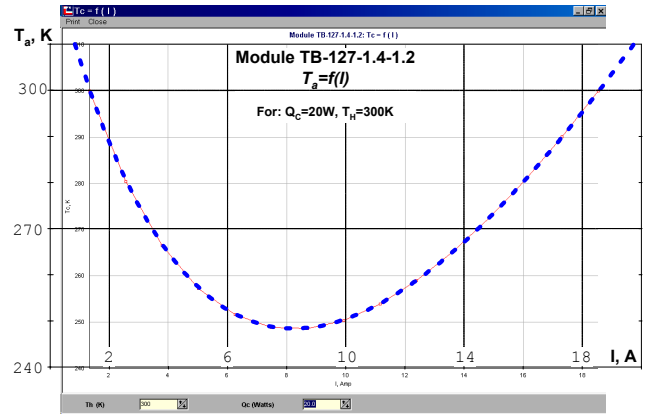where c – specific heat (kJ/kg K), ρ - density (kg/m³), and V –

Fig. 3. Performance of TEM TB-127-1.4-1.2. Dashed line is simulation result obtained by proposed model; solid line is the published performance plot by manufacturer.
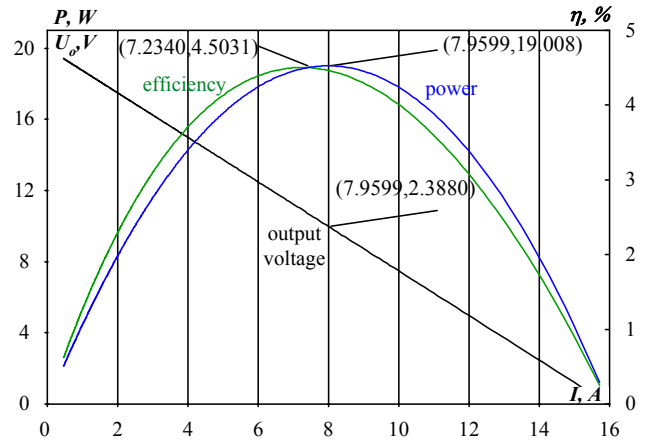

Fig. 4. DC-sweep simulation of performance of the Thermoelectric Generator HZ-20 made by Hi-Z Technology, Inc.
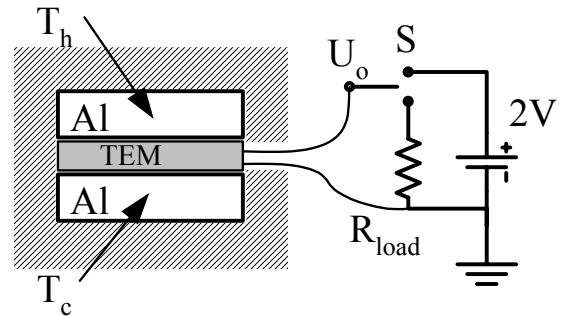

Fig. 5. The experimental setup.

volume (m³). In this specific case, the C_{al} is about 19 J/K.

3. Thermal insulation. Even though the system is thermally insulated, small heat leakage still exists. The value of the thermal resistance can be calculated from steady state measurements when applying a low input power.

In steady state (in our case after about five hrs), the thermal resistance Θ_{iso} can be calculated from T_h, T_c and T_{room} by:

$$R_{iso} = \frac{\alpha_m^2 \Theta_m^2 (T_c + T_h + 2\cdot 273)^2 (T_c - T_{room})}{(T_c - T_h)^2 (2R_m + \alpha_m^2 \Theta_m (T_c + T_h + 2\cdot 273))} \qquad (24)$$
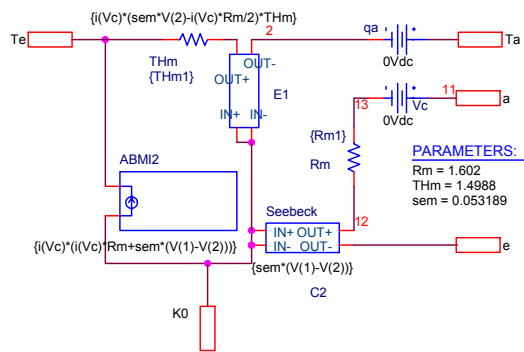
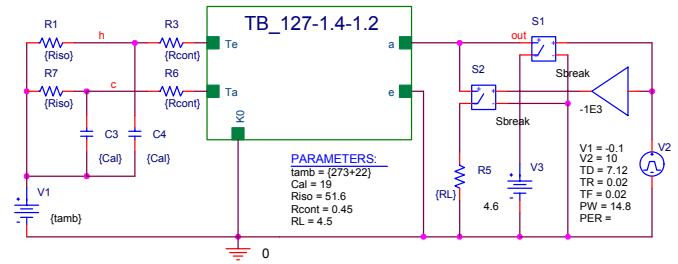Fig. 6.   PSPICE model of TEM TB-127-1.2-1.4.

Fig. 7.   PSPICE model for simulating the experiment. R_{iso} is a thermal resistance of the thermal insulation, C_{al} – thermal capacity of the aluminum plates. R_{cont} – thermal resistance of the thermal contact between TEM and the plate.

4. The thermal resistance of a contact between the TEM and the plate can be estimated from datasheets of the thermal interface material (silicon grease in our case).

The scheme for PSPICE simulation is shown on Fig. 6 and Fig. 7.
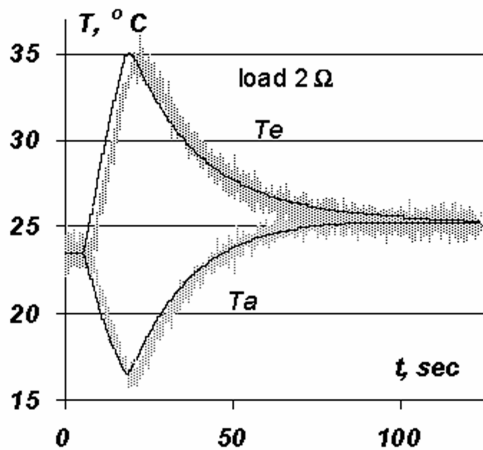
Fig. 8 shows the experimental results together with computer simulation. The good agreement clearly shows that the model is valid not only for steady state conditions (DC) but also for simulating dynamic behavior.
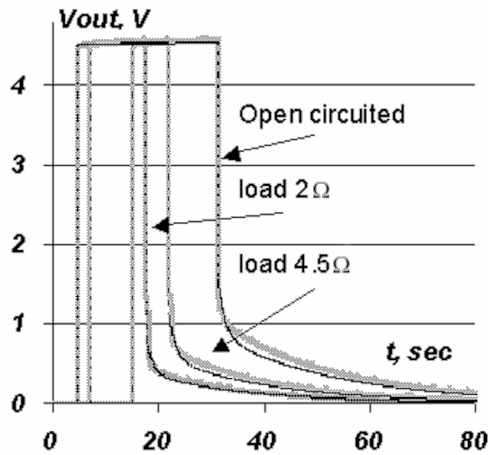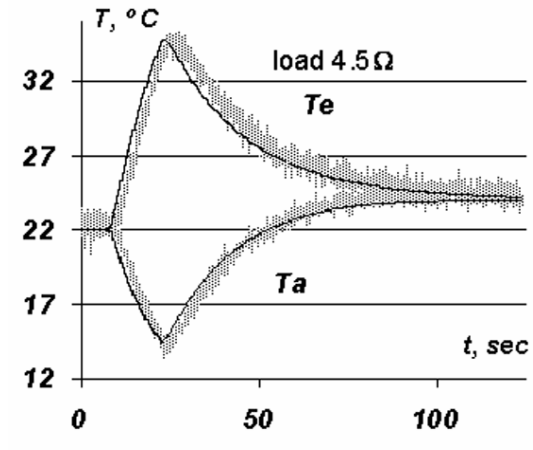

(a)
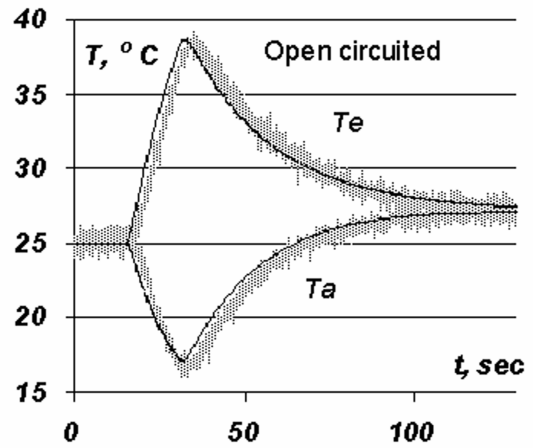
(b)

(c)

(d)

Fig. 6.   The behavior of the TEM under sequence of powering and loading on electric port. Experimental data shown in gray line, simulation results are black line. (a) voltage on the electric terminal. (b), (c), and (d) – Temperatures of absorbing (T_a) and emitting (T_e) sides of TEM, for open-circuited electric terminal, loaded by 2 Ω resistor, and loaded with 4.5 Ω resistor respectively.

VI. Conclusions

In this study, equivalent circuits are used to describe TEM systems in which several energy types exist, when all non-electrical processes are emulated by electrical analogies, and their interconnections are represented by transformers or dependent sources. The model on Fig 2 is a two-port electrical system when one of the ports is an equivalent circuit of the thermal part. Consequently, the model can be implemented as a block in any electrical scheme.

The study shows how the manufacturer's data for Thermoelectric Cooler (TEC) as well as for Thermoelectric Generator (TEG) can be used to extract the parameters of the proposed model.

The model could be helpful for analyzing the drive requirements of TECs and loading effects of TEGs. Another important application of proposed model is when the performance of the TEM needs to be analyzed under specific conditions such as heat leakage, non-ideal thermal insulation etc. Using the model one can analyze not only existing modules, but also specify an optimal module for a specific problem.

The present model is compatible with PSPICE or other electric circuit simulators for DC, AC, and TRANSIENT simulation types and will thus be an excellent tool for solving problems of temperature control.

Several examples of successful utilization of the model are presented. The paper is based on data of many different manufacturers that were used to reproduce accurately the performance of commercial TEMs.

An important feature of the model is its ability to generate small signal transfer functions that can be used to design feedback network in temperature control applications.

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