Common distributions

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Common distributions

In order to model data that arises in statistics, it will be useful to have a collection of common distributions with well know properties.

Parametrized families

These distributions will often be a family of similar distribution with some parameters that specify the exact distribution.

Common discrete distributions

- Bernoulli
- Binomial
- Poisson

Bernoulli distribution

The bernoulli distribution is the distribution of the coin flip.

Parameter

The bernoulli distribution has a single parameter p, with $0 \leq p \leq 1$.
It represents the probability of a head.

p.m.f.

The p.m.f. of the bernoulli distribution is given by:

$$\begin{aligned} \mathrm{P}(X = 0) &= 1 - p \\ \mathrm{P}(X = 1) &= p \end{aligned}$$

Mean and variance

$$\begin{aligned} \mu &= p \\ \sigma^2 &= p(1 - p) \end{aligned}$$

Binomial distribution

The binomial distribution is the distribution of a series of coin flip.

Parameter

- p The probability of a single head. $0 \leq p \leq 1$.
- n The number of flips. Positive integer.

p.m.f.

$$\mathrm{P}(X = k) = \binom{n}{k} p^k (1-p)^{1-k} \tag{1}$$

Mean and variance

$$\mu = np$$
$$\sigma^2 = np(1-p)$$

Poisson distribution

The Poisson distribution is often used to parametrize counts.

Parameter

$\lambda > 0$ The rate of occurrence per unit of time.

p.m.f.

$$\mathrm{P}(X = k) = \frac{1}{k!}\lambda^k e^{-\lambda} \tag{2}$$

Mean and variance

$$\mu = \lambda$$
$$\sigma^2 = \lambda$$

Common continuous distributions

- Uniform
- Normal
- Exponential

Uniform distribution

A "random" number on an interval.

Parameter

a, b with $b > a$ the ends of the interval.

p.d.f.

$$f_X(x) = \frac{1}{b-a} \text{ for } a \leq x \leq b. \tag{3}$$

Mean and variance

$$\mu = \frac{a+b}{2}$$
$$\sigma^2 = \frac{(b-a)^2}{12}$$

Normal distribution

The normal distribution is commonly used to model continuous quantities.

Parameters

μ The mean of the normal

σ^2 The variance of the normal

p.d.f.

$$f_X(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\pi\sigma^2}(x-\mu)^2} \tag{4}$$

Mean and variance

The normal distribution has mean μ and variance σ^2.

The exponential distribution is commonly used to model waiting times.

Parameter

$\lambda > 0$ The rate.

p.d.f.

$$f_X(x) = \lambda e^{-\lambda x} \text{ for } x \geq 0 \tag{5}$$

Mean and variance

$$\mu = \frac{1}{\lambda}$$
$$\sigma^2 = \frac{1}{\lambda^2}$$