Sample questions

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The midterm will have 6 questions. All questions will be marked and your grade will be computed from the 5 best questions. You will have access to an A4 (letter) sized note sheet, front and back. Each question is worth 20 marks. Some questions may have a part marked with a star – it is intended to indicate that this part may be somewhat more challenging and less rewarding in terms of credit. The total points of such parts will be less than 10.

1. Probability.

   Alice has two coins: coin A, which is a fair coin, and coin B, which is a double-headed coin.

   (a) (2 points) Let C_A be the event that Alice chooses coin A, and C_B the event that Alice chooses coin B.
   What is $\mathrm{P}(C_A) + \mathrm{P}(C_B)$ (no justification needed).

   (b) (3 points) Suppose that Alice picks a coin and tosses it, and let H_1 be the event that the coin lands on heads.
   Write down $\mathrm{P}(H_1 \mid C_A)$ and $\mathrm{P}(H_1 \mid C_B)$ from the problem description.

   (c) (6 points) Suppose that Alice picked the coin at random, that is, $\mathrm{P}(C_A) = 0.5$. Compute $\mathrm{P}(H_1)$.

   (d) (6 points) Now, suppose that the coin lands on heads. Compute the probability $\mathrm{P}(C_A \mid H_1)$ that Alice chose the coin A.

   (e)* (3 points) Suppose that Alice uses the same coin, and tosses it again. Let H_2 be the event that it lands on heads. Compute $\mathrm{P}(H_2 \mid H_1)$.

2. Jointly distributed random variable.

   Let X and Y be continuous random variables with $0 \le X \le 2$ and $0 \le Y \le 2$. Suppose that X and Y have joint p.d.f.

   $$f_{XY}(x, y) = C(x + y^2) \tag{5}$$

   where C is some real number.

   (a) (3 points) Compute the value of C.

   (b) (5 points) What is the marginal p.d.f. f_X of X?

   (c) (6 points) Compute $\mathbb{E}\, X^2$

   (d) (6 points) Compute $\mathbb{E}\, XY$

3. Discrete random variable

   Let X have binomial distribution with parameters $n = 16$ and $p = 1/4$. Let Y have binomial distribution with parameters $n = 16$ and $p = 3/4$. Suppose that X and Y are independent.

   Hint: there is no need to use the p.m.f. of the binomial ditribution for the questions below.

   (a) (2 points) Compute $\mathbb{E}\, X$, $\mathbb{E}\, Y$ and $\mathbb{E}\, X + Y$

   (b) (2 points) What is the maximal value of $X + Y$?

   (c) (4 points) What is $\mathbb{E}\, XY$?

   (d) (8 points) Compute $\mathbb{E}(X + Y)^2$

   (e) (4 points) Compute $\mathbb{E}(Y - 12)(X - 4)$.