Homework 3

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You may submit handwritten or typed answer. Write your answers clearly and justify all your computations. You may cite any result shown in class.

1. (1 point) Sum of two independent random variables.

   Let X and Y denote the outcomes of two independent die rolls. Use the formula for the p.m.f. of sums to compute the following two probabilities: $\mathrm{P}(X + Y = 7)$ and $\mathrm{P}(X + Y = 11)$.

2. (2 points) Joint distribution of random variables.

   Let X and Y be jointly distibuted random variables, with $0 \leq X, Y \leq 1$. Suppose that the joint p.d.f. of X and Y is given by:

   $$f_{X,Y}(x, y) = \frac{6}{7}(x + y)^2 \tag{1}$$

   (a) Compute the marginal p.d.f. f_X of X.

   (b) Compute the probability $\mathrm{P}(X > Y/2)$.

   (c) Compute the expectation $\mathbb{E}\, XY$.

3. (2 points) Algebraic properties of the expectation.

   Let $X \sim \mathcal{N}(1, 2)$ and $Y \sim \mathcal{N}(-1, 2)$ be independent random variables. Compute the following quantity: $\mathbb{E}(2x - y)^2$. Hint: there is no need for integration. Recall the algebraic properties of the expectation, and the alternate formula for the variance.

4. (2 points) Normal approximation.

   Suppose that a food truck serves one customer at a time. The average time to serve a customer is 5 minutes. Use a normal approximation to compute the probability that serving 50 customers takes longer than 4.5 hours. (You may suppose that the serving time for a single customer follows an exponential distribution with the appropriate parameter).

5. (3 points) Maximum likelihood estimator of the Poisson distribution.

   Suppose that $X_1, \ldots, X_n$ are i.i.d Poisson random variables with parameter λ.

   (a) Write down the log-likelihood of the data.

   (b) Compute the m.l.e. estimator of λ.

   (c) Compute the mean-squared error of the m.l.e. estimator of λ