JESISKA LORENZA / XII - IPA / 16

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Turunan Fungsi Eksponen

$f(x) = a^x \rightarrow f'(x) = a^x \ln a$

$f(x) = a^{g(x)} \rightarrow f'(x) = g'(x) \cdot a^{g(x)} \ln a$

Turunan Fungsi Bilangan Natural (e)

$f(x) = e^x \rightarrow f'(x) = e^x$

$f(x) = e^{g(x)} \rightarrow f'(x) = g'(x) \cdot e^{g(x)}$

Contoh soal

1. Turunan dari

a. $f(x) = 2^x$

$f'(x) = 2^x \ln 2$

b. $f(x) = 5^{2x+3}$

$f'(x) = 2 \cdot 5^{2x+3} \ln 5$

c. $f(x) = 3^{3x^2-2x+1}$

$f'(x) = (6x-2)\, 3^{3x^2-2x+1} \ln 3$

d. $f(x) = \pi^{\sin^2 x}$

$f'(x) = 2 \sin x \cdot \cos x \cdot \pi^{\sin^2 x} \ln \pi$

$f'(x) = \sin 2x \cdot \pi^{\sin^2 x} \cdot \ln \pi$

e. $f(x) = 3^{\ln x}$

$f'(x) = \frac{1}{x}\, 3^{\ln x} \ln 3$

2. Tentukan turunan dari :

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a. $f(x) = e^{2x+3}$

$f'(x) = 2e^{2x+3}$

b. $f(x) = 3e^{-4x+8}$

$f'(x) = 3(-4)\,e^{-4x+8}$

$f'(x) = -12e^{-4x+8}$

c. $f(x) = e^{-\frac{1}{2}x^2}$

$f'(x) = \left(-\frac{1}{2}\right) 2x\, e^{-\frac{1}{2}x^2}$

$f'(x) = -x \,.\, e^{-\frac{1}{2}x^2}$

d. $f(x) = e^{\ln x}$

$f'(x) = \frac{1}{x}\, e^{\ln x}$

e. $f(x) = e^{e^x}$

$f'(x) = e^x \quad e^{e^x}$

$f'(x) = e^{x + e^x}$

3. Tentukan turunan dari $y = x^x$

$\ln y = \ln x^x$

$\ln y = x \ln x$

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(x . \ln x)$

$\frac{1}{y} \cdot \frac{dy}{dx} = 1 . \ln x + x\left(\frac{1}{x}\right)$

$\frac{dy}{dx} = y(\ln x + 1)$

$y' = x^x(\ln x + 1)$

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4. Tentukan turunan dari $y = (x+5)^{2x}$

$\ln y = \ln (x+5)^{2x}$

$\ln y = 2x \ln (x+5)$

$\frac{d}{dx}(\ln y) = \frac{d}{dx}(2x \ln (x+5))$

$\frac{1}{y} \cdot \frac{dy}{dx} = 2 . \ln (x+5) + 2x \left(\frac{1}{x+5}\right)$

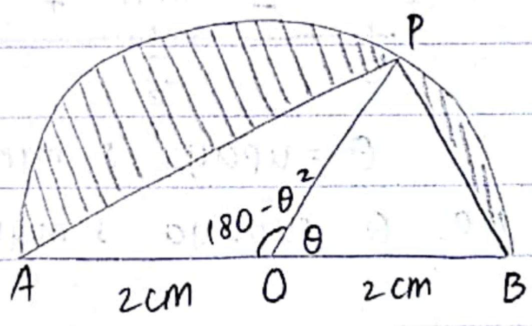
$\frac{dy}{dx} = y \left[2 . \ln (x+5) + \frac{2x}{x+5}\right]$

$y' = (x+5)^{2x} \left[2 . \ln (x+5) + \frac{2x}{x+5}\right]$

SOAL CERITA PENERAPAN

1. Diketahui $\frac{1}{2}$ lingkaran dengan pusat O dan d = AB, r = 2cm
Titik P terletak pada $\frac{1}{2}$ lingkaran dengan $\angle POB = \theta$ radian.
Misalkan juga luas arsiran adalah S

a. Tentukan luas $\triangle OPB$ dalam θ
b. Buktikan luas $\triangle OPB = \triangle OPA$
c. Buktikan $S = 2(\pi - 2\sin\theta)$
d. Tentukan θ supaya S min
e. Tentukan θ supaya S maks



a. luaas $\triangle OPB$

$L = \frac{1}{2}(OP)(OB)\sin\theta$

$L = \frac{1}{2}(2)(2)\sin\theta$

$L(\theta) = 2\sin\theta$

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b. Luas $\triangle OPA$

$L = \frac{1}{2} (OP)(OA) \sin(\pi - \theta)$

$L = \frac{1}{\cancel{2}} (\cancel{2})(2) \sin\theta$

$L(\theta) = 2 \sin\theta$

$L \triangle OPA = L \triangle OPB = 2 \sin\theta$

c. Luas S

$S = \frac{1}{2} \pi r^2 - L \triangle OPB - L \triangle OPA$

$S = \frac{1}{\cancel{2}} \pi (2)^{\cancel{2}} - 2\sin\theta - 2\sin\theta$

$S = 2\pi - 4\sin\theta$

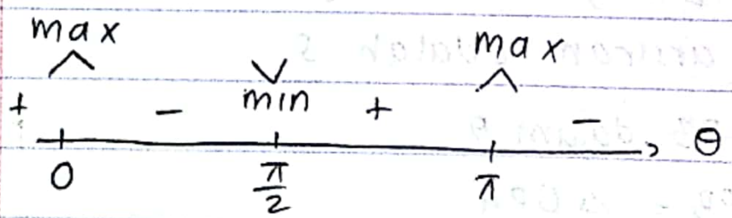
$S(\theta) = 2(\pi - 2\sin\theta)$

d. Agar S min $\rightarrow S'(\theta) = 0$

$S'(\theta) = 0 - 4\cos\theta = 0$

$\cos\theta = 0$ untuk $0 \le \theta \le \pi$

$\theta = \frac{\pi}{2}$



θ supaya S min $= \frac{\pi}{2}$, $S_{min} = 2\pi - 4$

e. θ supaya S maks $= 0$ atau π, $S_{max} = 2\pi$

LATIHAN SOAL

1. Tentukan turunan pertama fungsi

a. $f(x) = 10^{2x-1}$

$f'(x) = 2 \cdot 10^{2x-1} \ln 10$

b. $f(x) = (\sqrt{2})^{4x-3}$

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$f'(x) = 4 \cdot (\sqrt{2})^{4x-3} \ln \sqrt{2}$

c. $f(x) = e^{\sqrt{x}}$

$f'(x) = \frac{1}{2\sqrt{x}} \cdot e^{\sqrt{x}} \ln e$

d. $f(x) = e^{4-2x^2}$

$f'(x) = -4x \cdot e^{4-2x^2} \ln e$

e. $f(x) = \underset{u}{e^{-x}} \cdot \underset{v}{\ln x}$

$u' = -1 e^{-x} \ln e$

$v' = \frac{1}{x}$

$f'(x) = u'v + uv'$

$f'(x) = (-e^{-x} \ln e)(\ln x) + (e^{-x}) \cdot \frac{1}{x}$

$f'(x) = -e^{-x} \ln e \ln x + \frac{1}{x} \cdot e^{-x}$

f. $f(x) = (2x-5)^{x+5}$

$\ln y = \ln (2x-5)^{x+5}$

$\ln y = (x+5) \ln (2x-5)$

$\frac{d}{dx} \ln y = \frac{d}{dx} ((x+5) \ln (2x-5))$

$\frac{1}{y} \cdot \frac{d}{dx} = 1 \ln (2x-5) + (x+5) \left(\frac{1}{2x-5}\right)$

$\frac{dy}{dx} = y \left[\ln (2x-5) + \frac{x+5}{2x-5} \right]$

$y' = (2x-5)^{x+5} \left[\ln (2x-5) + \frac{x+5}{2x-5} \right]$

$f'(x) = (2x-5)^{x+5} \left[\ln (2x-5) + \frac{x+5}{2x-5} \right]$

g. $f(x) = (x^2-1)^{\frac{1}{x}}$

$\ln y = \ln (x^2-1)^{\frac{1}{x}}$

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$\ln y = \frac{1}{x} \ln (x^2-1)$

$\frac{d}{dx}(\ln y) = \frac{d}{dx}\left(\frac{1}{x}\ln(x^2-1)\right)$

$\frac{1}{y} \cdot \frac{d}{dx} = -\frac{1}{x^2}\ln(x^2-1) + \frac{1}{x} \cdot \frac{1}{x^2-1}$

$\frac{d}{dx} = y\left[-\frac{1}{x^2}\ln(x^2-1) + \left(\frac{1}{x(x^2-1)}\right)\right]$

$y' = (x^2-1)^{\frac{1}{x}}\left[-\frac{1}{x^2}\ln(x^2-1) + \frac{1}{x(x^2-1)}\right]$

$f'(x) = (x^2-1)^{\frac{1}{x}}\left[-\frac{1}{x^2}\ln(x^2-1) + \frac{1}{x(x^2-1)}\right]$

h. $f(x) = e^{\frac{1}{x^2}} + \frac{1}{e^{x^2}}$

$f(x) = e^{\frac{1}{x^2}} + e^{x^{-2}}$

$f'(x) = -\frac{2}{x^3} \cdot e^{\frac{1}{x^2}} \ln e + \left(-\frac{2}{x^3} \cdot e^{x^{-2}} \ln e\right)$

$f'(x) = -\frac{2}{x^3}\left(e^{\frac{1}{x^2}} \ln e + e^{x^{-2}} \ln e\right)$

i. $f(x) = e^{2x^2-x}$

$f'(x) = (4x-1)\, e^{2x^2-x} \ln e$

j. $f(x) = x^{x^x}$

$f'(x) = x^x \; x^{x^x}$

$f'(x) = x^{x+x^x}$