No.

Date:

JESISKA LORENZA / XII - IPA / 16

Aturan pembagian Turunan 2 fungsi

$y = \frac{u}{v} \rightarrow y' = \frac{u'v - uv'}{v^2}$

1. $y = \frac{6 + 3x^2}{2x^2 + 5}$ ($u = 6 + 3x^2$, $v = 2x^2 + 5$)

$u' = 6x$

$v' = 4x$

$y' = \frac{u'v - uv'}{v^2}$

$= \frac{6x(2x^2 + 5) - (6 + 3x^2)(4x)}{(2x^2 + 5)^2}$

$= \frac{12x^3 + 30x - 24x + 12x^3}{(2x^2 + 5)^2}$

$= \frac{6x}{(2x^2 + 5)}$

2. $y = \frac{2x + 3}{2x - 5}$ ($u = 2x + 3$, $v = 2x - 5$)

$u' = 2$

$v' = 2$

$y' = \frac{u'v - uv'}{v^2}$

$= \frac{2(2x - 5) - (2x + 3)(2)}{(2x - 5)^2}$

$= \frac{-16}{(2x-5)^2} = \frac{0x - 16}{(2x-5)^2}$

$y = \frac{ax + b}{(cx + d)} \rightarrow y' = \frac{ad - bc}{(cx + d)^2}$

3. $f(x) = \frac{2x + 3}{2x - 5}$ ($a = 2$, $b = 3$, $c = 2x$, $d = -5$)

$= \frac{2(-5) - 3(2)}{(2x-5)^2}$ (-10, -6)

$= \frac{-16}{(2x-5)^2}$

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4. $f(x) = \dfrac{x}{x-4} = \dfrac{1x+0}{1x-4} = \dfrac{1(-4)-0(1)}{(x-4)^2} = \dfrac{-4}{(x-4)^2}$

5. $f(x) = \dfrac{1}{x-4}$

$f'(x) = \dfrac{-1x+1}{1x-4} = \dfrac{-1(-4)-(1)(1)}{(x-4)^2} = \dfrac{3}{(x-4)^2}$

6. $y = \cot x = \dfrac{\cos x}{\sin x}$ ($\cos x \to u$, $\sin x \to v$)

$u' = -\sin x$

$v' = \cos x$

$y' = \dfrac{u'v - uv'}{v^2} = \dfrac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$ ($-\sin^2 x$, $-\cos^2 x$)

$= \dfrac{-(\sin^2 x + \cos^2 x)}{(\sin x)^2}$

$= -\left(\dfrac{1}{\sin x}\right)^2$

$= -\csc^2 x$

$y = \dfrac{a\sin x + b\cos x}{c\sin x + d\cos x} \to y' = \dfrac{ad - bc}{(c.\sin x + d\cos x)^2}$

$u = a\sin x + b\cos x \to u' = a\cos x - b\sin x$

$v = c\sin x + d\cos x \to v' = c\cos x - d\sin x$

$y' = \dfrac{u'v - uv'}{v^2}$

$y' = \dfrac{(a\cos x - b\sin x)(c\sin x + d\cos x) - (a.\sin x + b\sin x)(c.\cos x - d\sin x)}{(c\sin x + d\cos x)^2}$

$y' = \dfrac{ac\sin x\cos x + ad\cos^2 x - bc\sin^2 x - bd\sin x\cos x - [ac\sin x\cos x - ad\sin^2 x + bc\cos^2 x - bd\sin x\cos x]}{(c\sin x + d\cos x)^2}$

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$$y' = \frac{ac\sin x\cos x + ad\cos^2 x - bc\sin^2 x - bd\sin x\cos x - ac\sin x\cos x + ad\sin^2 x - bc\cos^2 x + bd\sin x\cos x}{(c\sin x + d\cos x)^2}$$

$$y' = \frac{ad(\cos^2 x + \sin^2 x) - bc(\sin^2 x + \cos^2 x)}{(c\sin x + d\cos x)^2}$$

$$\boxed{y' = \frac{ad - bc}{(c\sin x + d\cos x)^2}}$$

contoh pemakaian ↓

1. $y = \frac{\cos x}{\cos x + \sin x}$

$$y' = \frac{0.\sin x + 1\cos x}{1\sin x + 1\cos x} = \frac{0(1) - 1(1)}{(1\sin x + 1\cos x)^2} = \frac{-1}{(\sin x + \cos x)^2}$$

2. $y = \frac{2\sin x - 1\cos x}{1\sin x + 3\cos x}$

$$y' = \frac{6 - (-1)}{(\sin x + 3\cos x)^2} = \frac{7}{(\sin x + 3\cos x)^2}$$

3. $y = \frac{\cos x - \sin x}{\cos x + \sin x}$

$$y' = \frac{-1\sin x + 1\cos x}{1\sin x + 1\cos x} = \frac{-1 - 1}{(\sin x + \cos x)^2} = \frac{-2}{(\sin x + \cos x)^2}$$

4. $y = \frac{1 - \sin x}{\sin x - 3}$

$$y' = \frac{-1\sin x + 1}{1\sin x - 3} = \frac{-1 - 1}{(\sin x + \cos x)^2} = \frac{-2}{(\sin x + \cos x)^2}$$

5. $y = \frac{4\cos x - 2\sin x}{5\cos x + 3\sin x}$

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$y' = \frac{-2\sin x + 9\cos x}{3\sin x + 5\cos x} = \frac{-10 - 12}{(3\sin x + 5\cos x)^2} = \frac{-22}{(3\sin x + 5\cos x)^2}$

6. $y = \frac{\sin x}{\sin x + \cos x}$

$y' = \frac{1\sin x + 0\cos x}{1\sin x + 1\cos x}$

$y' = \frac{1 - 0}{(1\sin x + 1\cos x)^2}$

$y' = \frac{1}{(\sin x + \cos x)^2}$

CATATAN VIDEO

→ Turunan Pembagian dua fungsi

$y = \frac{u}{v} \rightarrow y' = \frac{u'v - uv'}{v^2}$

→ contoh soal

1. Turunan dari $f(x) = \frac{1 + \tan 3x}{5x}$ adalah

$f(x) = \frac{1 + \tan 3x}{5x}$ (u = pembilang, v = penyebut)

$u' = 3\sec^2 3x$

$v' = 5$

$f'(x) = \frac{u'v - uv'}{v^2}$

$= \frac{(3\sec^2 3x)(5x) - (1 + \tan 3x)(5)}{(5x)^2}$

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$$f'(x) = \frac{15x \sec^2 3x - 5 \tan 3x - 5}{25x^2}$$

2. Turunan dari $f(x) = \frac{\cos x + 1}{1 - \cos x}$ adalah

$u = \cos x + 1 \rightarrow u' = -\sin x$

$v = 1 - \cos x \rightarrow v' = \sin x$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

$$= \frac{(-\sin x)(1 - \cos x) - (\cos x + 1)(\sin x)}{(1 - \cos x)^2}$$

$$= \frac{-\sin x + \sin x \cos x - (\sin x \cos x + \sin x)}{(1 - \cos x)^2}$$

$$= \frac{-2 \sin x + 0}{(1 - \cos x)^2}$$

$$= \frac{-2 \sin x}{(1 - \cos x)^2}$$

3. Turunan dari $f(x) = \frac{1 - \sin x}{\sin x - 3}$ adalah

$u = 1 - \sin x \rightarrow u' = -\cos x$

$v = \sin x - 3 \rightarrow v' = \cos x$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

$$= \frac{(-\cos x)(\sin x - 3) - (1 - \sin x)(\cos x)}{(\sin x - 3)^2}$$

$$= \frac{-\sin x \cos x + 3 \cos x - \cos x + \sin x \cos x)}{(\sin x - 3)^2}$$

$$= \frac{2 \cos x}{(\sin x - 3)^2}$$

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4. Turunan dari $f(x) = \frac{\sin x - \cos x}{\cos x + \sin x}$ adalah

$u = \sin x - \cos x \rightarrow u' = \cos x - (-\sin x)$

$v = \cos x + \sin x \rightarrow v' = -\sin x + \cos x$

$f'(x) = \frac{u'v - uv'}{v^2}$

$f'(x) = \frac{(\cos x + \sin x)(\cos x + \sin x) - (\sin x - \cos x)(-\sin x + \cos x)}{(\cos x + \sin x)^2}$

$f'(x) = \frac{\cos^2 x + 2\sin x \cos x + \sin^2 x - (-\sin^2 x + 2\sin x \cos x - \cos^2 x)}{\cos^2 x + 2\sin x \cos x + \sin^2 x}$

$f'(x) = \frac{1 + 2\sin x \cos x - (-1) \cdot 2\sin x \cos x}{1 + 2\sin x \cos x}$

$f'(x) = \frac{2}{1 + \sin 2x}$

5. Jika $f(x) = \frac{\cos x + 2}{\sin x}$ dengan $\sin x \neq 0$, maka $f'(\frac{\pi}{2})$ adalah

$u = \cos x + 2 \rightarrow u' = -\sin x$

$v = \sin x \rightarrow v' = \cos x$

$f'(x) = \frac{u'v - uv'}{v^2}$

$= \frac{(-\sin x)(\sin x) - (\cos x + 2)(\cos x)}{(\sin x)^2}$

$= \frac{-\sin^2 x + \cos^2 x - 2\cos x}{\sin^2 x}$

$f'(\frac{\pi}{2}) = \frac{-1 + 0 - 0}{1^2}$

$= -1$

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6. Jika $f(x) = \frac{\sin 2x - \cos x}{\cos 4x}$, maka $f'(\frac{\pi}{4})$ adalah

$u = \sin 2x - \cos x \rightarrow u' = 2\cos 2x + \sin x$

$v = \cos 4x \rightarrow v' = -4\sin 4x$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

$$f'(x) = \frac{(2\cos 2x + \sin x)(\cos 4x) - (\sin 2x - \cos x)(-4\sin 4x)}{(\cos 4x)^2}$$

$$f'(\tfrac{\pi}{4}) = \frac{(2\cos\tfrac{\pi}{2} + \sin\tfrac{\pi}{4})(\cos\pi) - (\sin\tfrac{\pi}{2} - \cos\tfrac{\pi}{4})(-4\sin\pi)}{(\cos\pi)^2}$$ ($\sin\pi \rightarrow 0$)

$$= \frac{(2 \cdot 0 + \frac{1}{2}\sqrt{2})(-1)}{(-1)^2}$$

$$= -\frac{1}{2}\sqrt{2}$$

7. Turunan dari $f(x) = \frac{\cot(x^2+2)}{\sec(x^2+2)}$ adalah

$u = \cot(x^2+2) \rightarrow u' = -2x\csc^2(x^2+2)$

$v = \sec(x^2+2) \rightarrow v' = 2x\sec(x^2+2)\tan(x^2+2)$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

$$f'(x) = \frac{-2x\csc^2(x^2+2)\cdot\sec(x^2+2) - \cot(x^2+2)\cdot 2x\sec(x^2+2)\tan(x^2+2)}{\sec^2(x^2+2)}$$ (minus $(2x\sec(x^2+2)\times 1)$)

$$= \frac{-2x\sec(x^2+2)\left[\csc^2(x^2+2) + 1\right]}{\sec^2(x^2+2)}$$

$$= \frac{-2x\left[\csc^2(x^2+2) + 1\right]}{\sec(x^2+2)}$$

LATIHAN SOAL VIDEO

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Cari turunan pertama fungsi berikut!

1. $f(x) = \frac{\cos x}{\sin x}$

$u = \cos x \rightarrow u' = -\sin x$

$v = \sin x \rightarrow v' = \cos x$

$f'(x) = \frac{u'v - uv'}{v^2}$

$= \frac{(-\sin x)(\sin x) - (\cos x)(\cos x)}{(\sin x)^2}$

$= \frac{-\sin^2 x - \cos^2 x}{\sin x^2}$ (dikali -1)

$= \frac{\sin^2 x + \cos^2 x}{\sin x^2}$

$= \frac{1}{\sin x^2}$

2. $f(x) = \frac{1 - \sin x}{2 + \cos x}$

$u = 1 - \sin x \rightarrow u' = -\cos x$

$v = 2 + \cos x \rightarrow v' = -\sin x$

$f'(x) = \frac{u'v - uv'}{v^2}$

$= \frac{(-\cos x)(2 + \cos x) - (1 - \sin x)(-\sin x)}{(2 + \cos x)^2}$

$= \frac{-2\cos x - \cos^2 x - (-\sin x + \sin^2 x)}{(2 + \cos x)^2}$

$= \frac{-2\cos x - \cos^2 x + \sin x + \sin^2 x}{(2 + \cos x)^2}$

3. $f(x) = \frac{\sin x}{\sin x + \cos x}$

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$u = \sin x \rightarrow u' = \cos x$

$v = \sin x + \cos x \rightarrow v' = \cos x - \sin x$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

$$f'(x) = \frac{(\cos x)(\sin x + \cos x) - (\sin x)(\cos x - \sin x)}{(\sin x + \cos x)^2}$$

$$= \frac{\cancel{\cos x \sin x} + \cos^2 x - \cancel{\sin x \cos x} + \sin^2 x}{(\sin x + \cos x)^2}$$

$$= \frac{\cos^2 x + \sin^2 x}{(\sin x + \cos x)^2}$$

$$= \frac{1}{(\sin x + \cos x)^2}$$

9. $$f(x) = \frac{1 + \sin x}{\sqrt{x}}$$

$u = 1 + \sin x \rightarrow u' = \cos x$

$v = \sqrt{x} \rightarrow v' = \frac{1}{2\sqrt{x}}$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

$$= \frac{(\cos x)(\sqrt{x}) - (1 + \sin x)\left(\frac{1}{2\sqrt{x}}\right)}{(\sqrt{x})^2}$$

$$= \frac{\sqrt{x}\cos x - \frac{1}{2\sqrt{x}} - \frac{1}{2\sqrt{x}}\sin x}{x}$$

$$= \frac{\frac{2x\cos x - 1 - \sin x}{2\sqrt{x}}}{x}$$

$$= \frac{2x\cos x - 1 - \sin x}{2x\sqrt{x}}$$

5. $$f(x) = \frac{x^2 + \tan 2x}{3x + 2\tan x}$$

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$u = x^2 + \tan 2x \rightarrow u' = 2x + 2\sec^2 2x$

$v = 3x + 2\tan x \rightarrow v' = 3 + 2\sec^2 x$

$f'(x) = \dfrac{u'v - uv'}{v^2}$

$f'(x) = \dfrac{(2x + 2\sec^2 2x)(3x + 2\tan x) - (x^2 + \tan 2x)(3 + 2\sec^2 x)}{(3x + \tan x)^2}$

$f'(x) = \dfrac{6x^2 + 4x\tan x + 6\sec^2 2x + 4\sec^2 2x \tan x - x^2 - 3\tan 2x - 2\tan 2x \sec^2 x}{(3x + \tan x)^2}$

$f'(x) = \dfrac{5x^2 + 4x\tan x + 6\sec^2 2x + 4\sec^2 2x + \tan x - 3\tan 2x - 2\tan 2x \sec^2 x}{(3x + \tan x)^2}$

6. $f(x) = \dfrac{4\cos x - 2\sin x}{5\cos x + 3\sin x}$

$u = 4\cos x - 2\sin x \rightarrow -4\sin x - 2\cos x$

$v = 5\cos x + 3\sin x \rightarrow -5\sin x + 3\cos x$

$f'(x) = \dfrac{u'v - uv'}{v^2}$

$f'(x) = \dfrac{(-4\sin x - 2\cos x)(5\cos x + 3\sin x) - (4\cos x - 2\sin x)(-5\sin x + 3\cos x)}{(5\cos x + 3\sin x)^2}$

$f'(x) = \dfrac{-13\sin 2x - 12\sin^2 x - 10\cos^2 x + 13\sin 2x - 2\cos^2 x + 10}{(5\cos x + 3\sin x)^2}$

$= \dfrac{-12\sin^2 x - 12\cos^2 x + 10}{(5\cos x + 3\sin x)^2}$

7. $f(x) = \dfrac{x\sin x}{\cos x + \sin x}$

$u = x\sin x \rightarrow u' = x\cos x$

$v = \cos x + \sin x \rightarrow v' = -\sin x + \cos x$

$f'(x) = \dfrac{u'v - uv'}{v^2}$

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$$f'(x) = \frac{(x\cos x)(\cos x + \sin x) - (x\sin x)(-\sin x + \cos x)}{(\cos x + \sin x)^2}$$

$$f'(x) = \frac{x\cos^2 x + x\cos x\sin x + x\sin^2 x - x\sin x\cos x}{(\cos x + \sin x)^2}$$

$$f'(x) = \frac{x\cos^2 x + x\sin^2 x}{(\cos x + \sin x)^2}$$

$$f'(x) = \frac{x(\cos^2 x + \sin^2 x)}{(\cos x + \sin x)^2} \longrightarrow x(1)$$

$$f'(x) = \frac{x}{(\cos + \sin x)^2}$$

8. $$f(x) = \frac{9x^2\sin^2 x + 9}{x\sin x}$$

$u = 9x^2\sin^2 x + 9 \rightarrow u' = 18x(\cos^2 x)(2\sin x)$

$v = x\sin x \rightarrow v' = x\cos x$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

$$f'(x) = \frac{18x(\cos^2 x)(2\sin x)(x\sin x) - (9x^2\sin^2 x + 9)(x\cos x)}{(x\sin x)^2}$$

$$f'(x) = \frac{36x^2\cos^2 x\sin^2 x - 9x^3\sin^2 x\cos x - 9x\cos x}{(x\sin x)^2}$$

9. $$f(x) = \sqrt{\frac{\sec x + \tan x}{\sec x - \tan x}}$$

$$f(x) = \frac{\sqrt{(\sec x + \tan x)}}{\sqrt{(\sec x - \tan x)}}$$

$u = \sqrt{\sec x + \tan x} \Rightarrow u' = \frac{1}{2}(\sec x.\tan x + \sec^2 x)^{-\frac{1}{2}}$

$v = \sqrt{\sec x - \tan x} \Rightarrow v' = \frac{1}{2}(\sec x.\tan x - \sec^2 x)^{-\frac{1}{2}}$

$$f'(x) = \frac{u'v - uv'}{v^2}$$

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$$f'(x) = \left(\frac{1}{2\sqrt{\sec x \tan x + \sec^2 x}}\right)\left(\sqrt{\sec x - \tan x}\right) - \left(\sqrt{\sec x + \tan x}\right)\left(\frac{1}{2\sqrt{\sec x \tan x - \sec^2 x}}\right)$$

$$\frac{\left(\sqrt{\sec x - \tan x}\right)^2}{\sec x - \tan x}$$

$$= \frac{\sqrt{\sec x - \tan x}}{2\sqrt{\sec x \tan x + \sec^2 x}} - \frac{\sqrt{\sec x + \tan x}}{2\sqrt{\sec x \tan x - \sec^2 x}}$$

$$\sec x - \tan x$$