JESISKA LORENZA / XII - IPA / 16

No.

Date:

LIMIT 9

1. Tentukan $\lim_{x \to 0} \left(\frac{1}{\tan x} - \frac{1}{\sin x} \right)$

$y = \sin x \to y' = \cos x$

$y = \cos x \to y' = -\sin x$

$y = \tan x \to y' = \sec^2 x$

$= \lim_{x \to 0} \left(\frac{0}{\sec^2 x} - \frac{0}{\cos x} \right)$

$= 0 - 0$

$= 0$

2. Tentukan $\lim_{x \to \frac{\pi}{4}} (1 - \cos 2x)^{\sec 2x}$

$= \lim_{x \to \frac{\pi}{4}} \left(1 - \cos 2\left(\frac{\pi}{4}\right)\right)^{\sec 2x}$

$= 1^{\sec 2x}$

$= 1$

3. Tentukan $\lim_{x \to 0} \frac{\sin 7x + \tan 3x - \sin 5x}{\tan 9x - \tan 3x - \sin x}$

bentuk $\frac{0}{0}$

bagi pembilang dan penyebut dengan x

$= \lim_{x \to 0} \frac{\frac{\sin 7x}{x} + \frac{\tan 3x}{x} - \frac{\sin 5x}{x}}{\frac{\tan 9x}{x} - \frac{\tan 3x}{x} - \frac{\sin x}{x}}$

$= \lim_{x \to 0} \frac{7 + 3 - 5}{9 - 3 - 1}$

$= \frac{5}{5}$

$= 1$

Date:

4. Tentukan $\lim\limits_{x \to 0} \dfrac{1-\cos^2 x}{x^2 . \cot(x+\frac{\pi}{3})}$

$1 - \cos^2 ax = \sin^2 ax$

$$\lim_{x \to 0} \frac{1-\cos^2 x}{x^2 . \cot(x+\frac{\pi}{3})} = \lim_{x \to 0} \frac{\sin^2 x}{x^2 \cot(x+\frac{\pi}{3})}$$

$$= \lim_{x \to 0} \frac{\sin x}{x} . \frac{\sin x}{x} . \frac{1}{\cot(x+\frac{\pi}{3})}$$

$$= 1 . 1 . \frac{1}{\cot(0+\frac{\pi}{3})}$$

$$= 1 . \frac{1}{\frac{1}{\sqrt{3}}}$$

$$= \sqrt{3}$$

5. Tentukan $\lim\limits_{x \to \frac{\pi}{4}} \left(x - \frac{\pi}{4}\right) \sec 2x$

$\cos x = \sin\left(\frac{\pi}{2} - x\right)$

$$\lim_{x \to \frac{\pi}{4}} \left(x - \frac{\pi}{4}\right) \sec 2x = \lim_{x \to \frac{\pi}{4}} \frac{\left(x - \frac{\pi}{4}\right)}{\cos 2x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\left(x - \frac{\pi}{4}\right)}{\sin\left(\frac{\pi}{2} - 2x\right)}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\left(x - \frac{\pi}{4}\right)}{\sin -2\left(x - \frac{\pi}{4}\right)}$$

$$= -\frac{1}{2}$$

6. Tentukan $\lim\limits_{x \to 3} \dfrac{1-\cos^2(x-3)}{x^2-6x+9}$

$1 - \cos^2 ax = \sin^2 ax$

$$\lim_{x \to 3} \frac{1-\cos^2 x}{x^2-6x+9} = \lim_{x \to 3} \frac{\sin^2 (x-3)}{(x-3)^2}$$

Scanned with CamScanner

No.

Date:

$$= \lim_{x \to 3} \frac{\sin(x-3)}{(x-3)} \cdot \frac{\sin(x-3)}{(x-3)}$$

$$= 1 . 1$$

$$= 1$$

7. Tentukan $\lim_{x \to 0} \frac{\sin 8x + \sin 2x}{4x \cos 3x}$

$$\sin x + \sin y = 2 \sin \tfrac{1}{2}(x+y) \cdot \cos \tfrac{1}{2}(x-y)$$

$$\lim_{x \to 0} \frac{\sin 8x + \sin 2x}{4x \cdot \cos 3x} = \lim_{x \to 0} \frac{2 \sin \frac{1}{2}(8x+2x) \cdot \cos \frac{1}{2}(8x-2x)}{4x \cos 3x}$$

$$= \lim_{x \to 0} \frac{2 \sin 5x \cdot \cos 3x}{4x \cos 3x}$$

$$= \lim_{x \to 0} 2 \cdot \frac{\sin 5x}{4x} \cdot \frac{\cos 3x}{\cos 3x}$$

$$= 2 \cdot \frac{5}{4} \cdot 1$$

$$= \frac{5}{2}$$

8. Tentukan $\lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x}$

$$\cos ax = 2\cos^2 \tfrac{1}{2} ax - 1$$

$$\lim_{x \to 0} \frac{\sqrt{2} - \sqrt{1 + \cos x}}{\sin^2 x} = \lim_{x \to 0} \frac{\sqrt{2} - \sqrt{(2\cos^2 \frac{1}{2} x - 1)}}{\sin^2 x}$$

$$= \lim_{x \to 0} \frac{\sqrt{2} - \sqrt{2} \cdot \sqrt{(\cos^2 \frac{1}{2} x)}}{\sin^2 x}$$

$$= \lim_{x \to 0} \frac{\sqrt{2}\,(1 - \cos \frac{1}{2} x)}{\sin^2 x}$$

$$1 - \cos ax = 2 \sin^2 \tfrac{1}{2} ax$$

Scanned with CamScanner

No.

Date:

$$= \lim_{x \to 0} \frac{\sqrt{2}\left(2\sin^2 \frac{1}{2}\left(\frac{1}{2}\right)x\right)}{\sin^2 x}$$

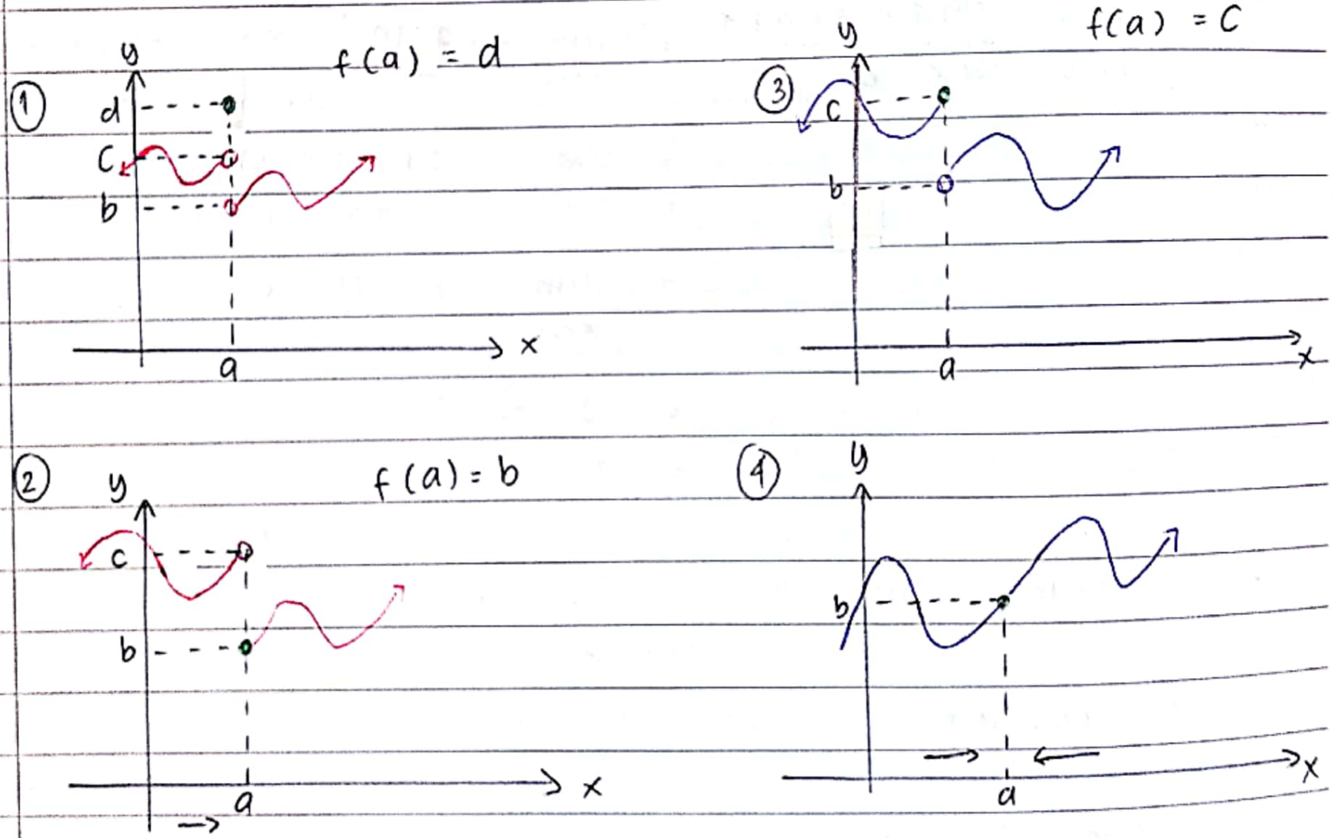
$$= \lim_{x \to 0} \frac{\sqrt{2}\left(2\sin^2 \frac{1}{4}x\right)}{\sin^2 x}$$

$$= \sqrt{2} \cdot 2 \cdot \frac{1}{4} \cdot \frac{1}{4}$$

$$= \frac{1}{8}\sqrt{2}$$

Tidak ada latihan soal yang diberikan di video

KEKONTINUAN FUNGSI



Syarat fungsi kontinu, $f(x)$ kontinu di $x = a$

1) $f(a)$ terdefinisi
2) $\lim_{x \to a} f(x)$ ada
3) $\lim_{x \to a} f(x) = f(a)$

No.

Date:

contoh 1

(1) $f(x) = \begin{cases} x+1, & x < 2 \\ x^2-1, & x \geq 2 \end{cases}$ apakah kontinu di $x = 2$?

$c = 2 \rightarrow f(2) = 2^2 - 1 = 3$

$\lim_{x \to 2^-} f(x) = \lim_{x \to 2} (2+1) = 3$

$\lim_{x \to 2^+} f(x) = \lim_{x \to 2} (2+1) = 3$

$\Big\} \lim_{x \to 2} f(x) = 3$

$\lim_{x \to 2} f(x) = f(2)$ ✓ (kontinu di $x = 2$)

contoh 2

(2) $f(x) = \begin{cases} \frac{x^2-1}{x-1} & x \neq 1 \\ 1 & x = 1 \end{cases}$ kontinu di $x = 1$?

$f(1) = 1$

$\lim_{x \to 1} \frac{x^2-1}{x-1} = \lim_{x \to 1} \frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}} = 2$

$\underbrace{\lim_{x \to 1} f(1)}_{2} \neq \underbrace{f(1)}_{1}$

$2 \neq 1$

diskontinu

contoh 3

(3) $f(x) = \begin{cases} \frac{(2x+3)(x-2)}{(x-2)}, & x \neq 2 \\ 5, & x = 2 \end{cases}$

$f(2) = 5$

Date:

$$\lim_{x \to 2} \frac{(2x+3)\cancel{(x-2)}}{\cancel{(x-2)}} = \lim_{x \to 2} (2x+3)$$

$$= 2.2+3$$

$$= 7$$

$$\underbrace{\lim_{x \to 2} f(x)}_{7} \neq \underbrace{f(2)}_{5}$$

$$7 \neq 5 \quad \text{(Diskontinu)}$$

contoh 4 :

4 $f(x) = \cos(x)$ apakah kontinu ?

- $x = c$, $f(c)$ terdefinisi $\rightarrow f(c) = \cos(c)$
- $h = x - c \rightarrow x = h + c$, $x \rightarrow c$

$h \rightarrow 0$ $\quad \rightarrow \lim_{x \to c} f(x)$ ada

$\lim_{h \to 0} \cos\underline{(h+c)}$ ada

$$\lim_{h \to 0} [\underbrace{\cos h}_{1} . \underbrace{\cos c}_{\cos c} - \underbrace{\sin h}_{0} . \sin c]$$

$$\lim_{h \to 0} (\cos c - 0) = \cos c$$

$$\lim_{h \to 0} (\cos c - 0) = f(c) = \lim_{x \to c} f(x)$$

$$\cos c = \cos c$$

kontinu

contoh 5 :

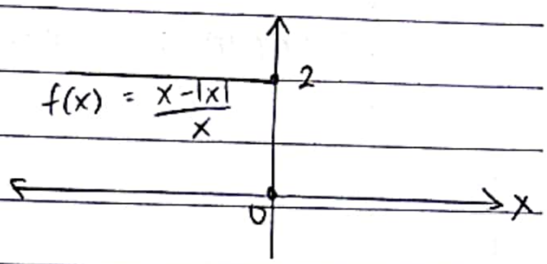
5 $f(x) = \begin{cases} \dfrac{x - |x|}{x} , & x < 0 \\ 2 , & x = 0 \end{cases}$ $\quad |x| \begin{cases} x , & x \geq 0 \\ -x , & x < 0 \end{cases}$

No.

Date:

- $f(0) = 2$
- untuk $x < 0 \rightarrow f(x) = \dfrac{x-(-x)}{x} = \dfrac{2x}{x} = 2$

$\hookrightarrow \lim\limits_{x \to 0} f(x) = 2$



- $\lim\limits_{x \to 0} f(x) = f(0) = 2$

contoh 6

$$f(x) = \begin{cases} ax+3 & , \text{ untuk } x \le 2 \\ x^2+1 & , \text{ untuk } 2 < x \le 4 \\ 5-bx & , \text{ untuk } x > 4 \end{cases}$$

f(x) kontinu, cari a + b !

$x = 2$
$$\begin{cases} \lim\limits_{x \to 2^-} f(x) = \lim\limits_{x \to 2^+} f(x) \\ \lim\limits_{x \to 2^-} ax+3 = \lim\limits_{x \to 2^+} x^2+1 \end{cases}$$

$2a + 3 = 5$

$2a = 2$

$a = 1$

$x = 4$
$$\begin{cases} \lim\limits_{x \to 4^-} f(x) = \lim\limits_{x \to 4^+} f(x) \\ \lim\limits_{x \to 4^-} x^2+1 = \lim\limits_{x \to 4^+} (5-bx) \end{cases}$$

$4^2 + 1 = 5 - b(4)$

$17 = 5 - 4b$

$12 = -4b$

$b = -3$

Jadi $a + b$

$= 1 + (-3)$

$= -2$ //

Catatan Nyimak di halaman selanjutnya

Scanned with CamScanner

No:

Date:

Catatan Nyimak

Putaran 1 : Senin, 30 Agustus 08 : 15 - 08 : 28

Putaran 2 : Senin, 6 September 05 : 46 - 05 : 59

Putaran 3 : Senin, 6 september 06 : 01 - 06 : 14