

1 Doubts

$\lfloor \cdot \rfloor$ represents floor function (whereever mentioned)

1. Consider,

$$f(x) = \begin{cases} |4x - 5| \cdot \lfloor x \rfloor & x > 1 \\ \lfloor \cos \pi x \rfloor & x \leq 1 \end{cases}$$

Is f continous at $x = 0$?

2. Consider,

$$g(x) = \begin{cases} \frac{x^{1/3} - 1}{x^{1/2} - 1} & x > 0 \\ \frac{\ln x}{x - 1} & \frac{1}{2} < x < 1 \end{cases}$$

Is g a function?

3. Consider,

$$f(x) = \frac{x}{x+1} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \dots \infty$$

Discuss continuity at $x = 0$.

4. If f is even and $f'(0)$ exists, then $f'(0)$ is?

5. Consider,

$$f(x) = \lfloor 2 + 5|n| \sin x \rfloor \quad \forall x \in (0, \pi), n \in \mathbb{Z}$$

f has exactly 9 points of non-differentiability then find set of all values of n .

6. If $f'(x)$ is given then how to identify differentiability of f ?

7. Disprove,

If

$$f(x) = \sum_{n=1}^x x = \underbrace{x + x + x + \dots + x}_{x \text{ times}}$$

then

$$f'(x) = \sum_{n=1}^x 1 = \underbrace{1 + 1 + 1 + \dots + 1}_{x \text{ times}} = x$$

\therefore

$$\frac{d}{dx}(f(x)) = \frac{d}{dx}(x^2) = x$$

8. [Math StackExchange Question on Continuity of Composite Functions](#)

9. [Math StackExchange Proof on Roots of Odd Degree Polynomials](#)

Doubt in answer by [Shuchang](#) in Method of IVT.

10. Let $f(x) = x^2$ and $g(x) = \sum_{n=1}^x x$

are both f and g identical?

11. Prove or Disprove, If $f(x)$ is a real continuous function for all $x \in \mathbb{R}$ and f is symmetric about two different lines perpendicular to axis of x (say $x = a$ and $x = b$, $a > b$), i.e.

$$f(a - x) = f(a + x)$$

and

$$f(b - x) = f(b + x)$$

then f is periodic with period $2(a - b)$

12. Prove or Disprove,

$$\lim_{x \rightarrow 0^+} \frac{\cos^{-1}(1 - x^2)}{x} = \sqrt{2}$$

13. The range of

$$f(x) = 8^{\cos x} + 8^{\sin x}$$

14. Consider,

$$f(x) = \begin{cases} x + b & x < 0 \\ \cos x & x \geq 0 \end{cases}$$

Find b such that f is differentiable at $x = 0$.

15. Can a even continuous function have range \mathbb{R} ?