1 Doubts

 $\lfloor \cdot \rfloor$ represents floor function (whereever mentioned)

1. Consider,

$$f(x) = \begin{cases} |4x - 5| \cdot \lfloor x \rfloor & x > 1\\ \lfloor \cos \pi x \rfloor & x \le 1 \end{cases}$$

Is f continous at x = 0?

2. Consider,

$$g(x) = \begin{cases} \frac{x^{1/3} - 1}{x^{1/2 - 1}} & x > 0\\ \frac{\ln x}{x - 1} & \frac{1}{2} < x < 1 \end{cases}$$

Is g a function?

3. Consider,

$$f(x) = \frac{x}{x+1} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \dots \infty$$

Discuss continuity at x = 0.

- 4. If f is even and f'(0) exists, then f'(0) is?
- 5. Consider,

$$f(x) = |2 + 5|n|\sin x \mid \forall \ x \in (0, \pi), n \ \forall \in \mathbb{Z}$$

f has exactly 9 points of non-differentiability then find set of all values of n.

- 6. If f'(x) is given then how to identify differentiability of f?
- 7. Disprove,

If

$$f(x) = \sum_{n=1}^{x} x = \underbrace{x + x + x + \dots + x}_{x \text{ times}}$$

then

$$f'(x) = \sum_{n=1}^{x} 1 = \underbrace{1 + 1 + 1 + \dots + 1}_{x \text{ times}} = x$$

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$$\frac{d}{dx}(f(x)) = \frac{d}{dx}(x^2) = x$$

- 8. Math StackExchange Question on Continuity of Composite Functions
- 9. Math StackExachange Proof on Roots of Odd Degree Polynomials Doubt in answer by Shuchang in Method of IVT.
- 10. Let $f(x) = x^2$ and $g(x) = \sum_{n=1}^{x} x^n$

are both f and g identical?

11. Prove or Disprove, If f(x) is a real continuous function for all $x \in \mathbb{R}$ and f is symmetric about two different lines perpendicular to axis of x (say x = a and x = b, a > b), i.e.

$$f(a-x) = f(a+x)$$

and

$$f(b-x) = f(b+x)$$

then f is periodic with period 2(a-b)

12. Prove or Disprove,

$$\lim_{x \to 0^+} \frac{\cos^{-1} \left(1 - x^2\right)}{x} = \sqrt{2}$$

13. The range of

$$f(x) = 8^{\cos x} + 8^{\sin x}$$

14. Consider,

$$f(x) = \begin{cases} x+b & x < 0\\ \cos x & x \ge 0 \end{cases}$$

Find b such that f is differentiable at x = 0.