

Definite Integration Doubts

D1.

$$\int_{-1}^1 \frac{d}{dx} \left(\tan^{-1} \frac{1}{x} \right) dx$$

Why isn't the above integral equal to $\int_{-1}^1 d \left(\tan^{-1} \frac{1}{x} \right) = \left[\tan^{-1} \frac{1}{x} \right]_{-1}^1 = \frac{\pi}{2}$

D2. Consider,

$$I = \int_{-1}^1 \sqrt{1+x^2} dx$$

Let, $u = 1 + x^2 \implies du = 2x dx$

also, $x = -1 \implies u = 2, x = 1 \implies u = 2$

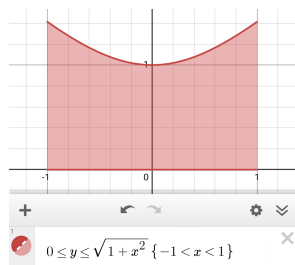
Now, I becomes,

$$\int_2^2 \frac{\sqrt{u}}{2\sqrt{u-1}} du$$

Since, upper limit and lower limit are same,

$$\boxed{I = 0}$$

But from graph,



Red region represents I , Clearly, $I > 0$

Why is this substitution wrong?

D3. How to evaluate $\int_0^a [x^n] dx$?