1 Doubts

$\lfloor \cdot \rfloor$ represents floor function (whereever mentioned)

1. Consider,
$$f(x) = \begin{cases} |4x-5| \cdot \lfloor x \rfloor & x > 1 \\ \lfloor \cos \pi x \rfloor & x \leq 1 \end{cases}$$
Is f continous at $x = 0$?

2. Consider,
$$g(x) = \begin{cases} \dfrac{x^{1/3}-1}{x^{1/2-1}} & x > 0 \\ \dfrac{\ln x}{x-1} & \dfrac{1}{2} < x < 1 \end{cases}$$
Is g a function?

3. Consider,
$$f(x) = \frac{x}{x+1} + \frac{x}{(x+1)(2x+1)} + \frac{x}{(2x+1)(3x+1)} + \ldots \infty$$
Discuss continuity at $x = 0$.

4. If f is even and $f'(0)$ exists, then $f'(0)$ is?

5. Consider,
$$f(x) = \lfloor 2 + 5|n| \sin x \rfloor \ \forall \ x \in (0, \pi), n \ \forall \in \mathbb{Z}$$
f has exactly 9 points of non-differentiability then find set of all values of n.

6. If $f'(x)$ is given then how to identify differentiability of f?

7. Disprove,
If
$$f(x) = \sum_{n=1}^{x} x = \underbrace{x + x + x + \ldots + x}_{x \text{ times}}$$
then
$$f'(x) = \sum_{n=1}^{x} 1 = \underbrace{1 + 1 + 1 + \ldots + 1}_{x \text{ times}} = x$$

$\therefore$

$$\frac{d}{dx}(f(x)) = \frac{d}{dx}\left(x^2\right) = x$$

8. Math StackExchange Question on Continuity of Composite Functions

9. Math StackExachange Proof on Roots of Odd Degree Polynomials
   Doubt in answer by Shuchang in Method of IVT.

10. Let $f(x) = x^2$ and $g(x) = \sum_{n=1}^{x} x$

    are both f and g identical?

11. Prove or Disprove, If $f(x)$ is a real continous function for all $x \in \mathbb{R}$ and f is symmetric about two different lines perpendicular to axis of x (say $x = a$ and $x = b$, $a > b$), i.e.

    $$f(a - x) = f(a + x)$$

    and

    $$f(b - x) = f(b + x)$$

    then f is periodic with period $2(a - b)$

12. Prove or Disprove,

    $$\lim_{x \to 0^+} \frac{\cos^{-1}\left(1 - x^2\right)}{x} = \sqrt{2}$$

13. The range of

    $$f(x) = 8^{\cos x} + 8^{\sin x}$$

14. Consider,

    $$f(x) = \begin{cases} x + b & x < 0 \\ \cos x & x \geq 0 \end{cases}$$

    Find b such that f is differentiable at $x = 0$.