Indefinite Integration

1 Fundamental Definition of Indefinite Integration

If f and F and functions such that $\dfrac{d}{dx}(F(x)) = f(x)$ then F is anti-derivative of f w.r.t. x symbolically,

$$\int f(x)\,dx = F(x) + C$$

where C is the constant of Integration

2 Anti-Derivatives of Some Standard Functions

i. $\displaystyle\int k \cdot f(x)\,dx = k \cdot \int f(x)\,dx \qquad \forall\ k \in \mathbb{R}$

ii. $\displaystyle\int [f_1(x) \pm f_2(x) \pm f_3(x) \pm \ldots \pm f_n(x)]\,dx \;=\; \int f_1(x)\,dx \pm \int f_2(x)\,dx \pm \int f_3(x)\,dx \pm \ldots \int f_n(x)\,dx \qquad \forall\, n \in \mathbb{N}$

iii. $\displaystyle\int x^n\,dx = \frac{x^{n+1}}{n+1} + C \qquad \forall\ n \in \mathbb{R} - \{1\}, x \in \mathbb{R}$

iv. $\displaystyle\int \frac{1}{x}\,dx = \ln|x| + C \qquad \forall\ x \in \mathbb{R} - \{0\}$

v. $\displaystyle\int e^x\,dx = e^x + C \qquad \forall\ x \in \mathbb{R}$

vi. $\displaystyle\int a^x\,dx = \frac{a^x}{\ln a} + C \qquad \forall\ a \in \mathbb{R}^+ - \{1\}, x \in \mathbb{R}$

vii. $\displaystyle\int \sin x\,dx = \cos x + C \qquad \forall\ x \in \mathbb{R}$

viii. $\displaystyle\int \cos x\,dx = \sin x + C \qquad \forall\ x \in \mathbb{R}$

ix. $\int \sec^2 x \, dx = \tan x + C \qquad \forall \; x \in \mathbb{R} - \left\{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\right\}$

x. $\int \csc^2 x \, dx = -\cot x + C \qquad \forall \; x \in \mathbb{R} - \{n\pi : n \in \mathbb{Z}\}$

xi. $\int \sec x \tan x \, dx = \sec x + C \qquad \forall \; x \in \mathbb{R} - \left\{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\right\}$

xii. $\int \csc x \cot x \, dx = -\csc x + C \qquad \forall \; x \in \mathbb{R} - \{n\pi : n \in \mathbb{Z}\}$

xiii. $\int \cot x \, dx = \ln|\sin x| + C \qquad \forall \; x \in \mathbb{R} - \{n\pi : n \in \mathbb{Z}\}$

xiv. $\int \tan x \, dx = -\ln|\cos x| + C \qquad \forall \; x \in \mathbb{R} - \left\{(2n+1)\frac{\pi}{2} : n \in \mathbb{Z}\right\}$

xv. $\int \sec x \, dx = \ln|\sec x + \tan x| + C \qquad \forall \; \in$

xvi. $\int \csc x \, dx = \ln|\csc x - \cot x| + C \qquad \forall \; \in$

xvii. $\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1}\left(\frac{x}{a}\right) + C \qquad \forall \; a \in \mathbb{R} - \{0\}$

xviii. $\int \frac{-1}{\sqrt{a^2 - x^2}} \, dx = \cos^{-1}\left(\frac{x}{a}\right) + C \qquad \forall \; a \in \mathbb{R} - \{0\}$

xix. $\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \qquad \forall \; a \in \mathbb{R} - \{0\}$

xx. $\int \frac{-1}{a^2 + x^2} \, dx = \frac{1}{a} \cot^{-1}\left(\frac{x}{a}\right) + C \qquad \forall \; a \in \mathbb{R} - \{0\}$

xxi. $\int \frac{1}{x\sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \sec^{-1}\left(\frac{x}{a}\right) + C \qquad \forall \; a \in \mathbb{R} - \{0\}$

xxii. $\int \frac{-1}{x\sqrt{x^2 - a^2}} \, dx = \frac{1}{a} \csc^{-1}\left(\frac{x}{a}\right) + C \qquad \forall \; a \in \mathbb{R} - \{0\}$

xxiii. $\int \sqrt{x} \, dx = \frac{2x\sqrt{x}}{3} + C$

Star Points

1. If $f(x)$ is continuous $\forall \, x \in D_f$ and,

$$\int f(x)\, dx = F(x) + C \text{ on hold}$$

2. If $f(x)$ is discontinuous at $x = x_1$, then its anti-derivative can be continuous at $x = x_1$.

3. Anti-derivative of a periodic function may not be periodic.