Definite Integration

1 Definite Integral

Let $f(x)$ be a function defined in the closed interval $[a, b]$ and $F(x)$ be its anti-derivative, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

is called the definite integral of the function $f(x)$ over the interval $[a, b]$, a and b are called limits of integration, lower and upper limit respectively.

2 Geometrical Interpretation of Definite Integral

If $f(x) > 0 \;\forall\; x \in [a, b]$, then $\int_a^b f(x)\,dx$ is numerically equal to the area bounded by the curves $y = f(x), y = 0, x = a, x = b$

In general, $\int_a^b f(x)\,dx$ represents the net signed area (or algebraic sum of areas) i.e area below the axis of x is counted as $-ve$ and that above is counted as $+ve$

3 Definite Integration by *u-sub*

To evaluate definite integral of type,

$$I = \int_a^b f(x)g'(x)\,dx$$

Let

$$u = g(x) \implies du = g'(x)\,dx$$

Now, I trasforms to,

$$I = \int_{g(a)}^{g(b)} f(u)\,du$$

Important Note

- For the substitution to be valid, it must be continuous in the interval of integration, i.e. If $u = g(x)$, then $g(x)$ must be continuous in $[a, b]$.

4 Properties of Definite Integration

4.1 Definite Integration is independent of the change of variable

$$\int_a^b f(x)\,dx = \int_a^b f(u)\,du$$

4.2 If limits of definite integral are flipped, then its value only differs in sign

$$\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$$

5 King's Rule

$$\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$$

6 Integration of Piecewise Functions

6.1

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$c \in \mathbb{R}$$

6.2

$$\int_0^a f(x)\,dx = \int_0^{a/2} f(x)\,dx + \int_0^{a/2} f(a-x)\,dx$$

7 Integration of Even, Odd Functions

$$\int_{-a}^a f(x)\,dx = \begin{cases} 0 & f(x) = -f(-x) \text{ Odd symmetric about } x=0 \\ 2\displaystyle\int_0^a f(x)\,dx & f(x) = f(-x) \text{ Even symmetric about } x=0 \end{cases}$$

8 Integration in case of Even, Odd Symmetries

$$\int_a^b f(x)\,dx = \begin{cases} 0 & f(a+x) = -f(b-x) \quad \text{Odd symmetric} \\ & \text{or } f\left(\dfrac{a+b}{2} - x\right) = -f\left(\dfrac{a+b}{2} + x\right) \textit{ about } x = \dfrac{a+b}{2} \\ 2\displaystyle\int_a^{(a+b)/2} f(x)\,dx & f(a+x) = f(b-x) \quad \text{Even symmetric} \\ & \text{or } f\left(\dfrac{a+b}{2} - x\right) = f\left(\dfrac{a+b}{2} + x\right) \textit{ about } x = \dfrac{a+b}{2} \end{cases}$$