Functions

1 Function

- Let A and B be two non-empty sets and f be a relation which associates each element of set A with unique element of set B, f is called a function from A to B.
- If any line $\parallel$ to axis of y intersects graph of $y = f(x)$ at more than once then, f is not a function.

1.1 Domain

Let $f : A \to B$ be a function then set A which consits all those elements for which image under f is well-defined, is called *Domain* of f.

1.1.1 Suggestions on finding domain of f

i. If $f(x) = \sqrt[2n]{g(x)}$ then Dom $f \in g(x) \geq 0$

ii. If $f(x) = \dfrac{1}{g(x)}$ then Dom $f \in \mathbb{R} - \{x : x = a, g(a) = 0\}$

iii. If $f(x) = \log_{h(x)} g(x)$ then Dom $f \in \{x : x = a, g(a) > 0\} \bigcap \{x : x = a, h(a) > 0 \vee h(a) \neq 1\}$

iv. If $f(x) = g(x) + h(x)$ then Dom $f \in$ Dom $g \bigcap$ Dom h

1.2 Co-domain

Set B is Co-domain of f.

1.3 Range

Range of f is set of images of elements in domain A under f.

2 Periodic Functions

A function $f(x)$ is said to be periodic if there exist a +ve real number λ such that

$$f(x + \lambda) = f(x)$$

The smallest of all such λ is called the fundamental periodic or "periodic" of f.

2.1 Period of some Standard functions

Function	Periodic
$\sin^n x, \cos^n x, \sec^n x, \csc^n x$	$\begin{cases} \pi & n \in even \\ 2\pi & n \in odd \ \vee fraction \end{cases}$
$\tan^n x, \cot^n x$	$\pi n \in$ even $\vee$ odd
$\lvert\sin x\rvert, \lvert\cos x\rvert, \lvert\tan x\rvert, \lvert\cot x\rvert, \lvert\sec x\rvert, \lvert\csc x\rvert$	π

3 Odd and Even Functions

Odd Functions

A function f is odd if,

$$f(-x) = -f(x)$$

i.e. symmetric about origin.

Even Functions

A function f is even if,

$$f(x) = f(-x)$$

i.e. symmetric about axis of y.

Properties

i. Product of two even or two odd is even.

ii. Product of odd and even is odd.

iii. Every function can be expressed as sum of a odd and even function.

iv. Derivative of odd is even and Derivative of even is odd.

v. Even function or odd function when squared becomes even.

vi. Only function which is both even and odd is $f(x) = 0$