Extra Topics

1 Modulo Operator (Arithmetic Remainder)

If $x \in \mathbb{R}^+$ and $n \in \mathbb{N}$, we can uniquely write $x = mn + r$, where $m \in \mathbb{W}$ and $r \in [0, n)$.
We define

$$x \bmod n = r$$

e.g. $10.5 \bmod 3 = 1.5$

2 Every Function can be expressed as sum of two Even and Odd Symmetric Functions about $x = a$

Let $f(x)$ be any general function.
Let $E(x)$ be a function Even Symmetric about $x = a$ and
$O(x)$ be a function Odd Symmetric about $x = a$
$\therefore$

$$E(a+x) = E(a-x)$$
$$O(a+x) = -O(a-x)$$

such that,

$$f(x) = E(x) + O(x)$$

Hence,

$$E(x) = \frac{f(x) + f(2a-x)}{2}$$
$$O(x) = \frac{f(x) - f(2a-x)}{2}$$

$$f(x) = \underbrace{\frac{f(x) + f(2a-x)}{2}}_{\text{Even Symmetric Part}} + \underbrace{\frac{f(x) - f(2a-x)}{2}}_{\text{Odd Symmetric Part}}$$

3 If a function is Odd Symmetric about $x = a$ then it must vanish at $x = a$ (if defined)

Let $O(x)$ be a function Odd Symmetric about $x = a$
$\therefore O(a + x) = -O(a - x)$
Pluging $x = 0$, We get,

$$\boxed{O(a) = 0}$$

4 Some Important Series

i. $$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$

ii. $$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$

iii. $$\sum_{r=1}^{n} r^3 = \left[\frac{n(n+1)}{2}\right]^2$$

iv. $$\sum_{k=1}^{n} ar^k = a\left(\frac{1-r^n}{1-r}\right)$$

v. $$\sum_{r=0}^{n-1} \sin(\alpha + r\beta) = \frac{\sin n\,{}^{\beta}\!/_{2}}{\sin {}^{\beta}\!/_{2}} \cdot \sin\left[\alpha + (n-1)\beta\right]$$

vi. $$\sum_{r=0}^{n-1} \cos(\alpha + r\beta) = \frac{\sin n\,{}^{\beta}\!/_{2}}{\sin {}^{\beta}\!/_{2}} \cdot \cos\left[\alpha + (n-1)\beta\right]$$

vii. $$\sum_{r=1}^{\infty} \frac{(-1)^{r+1}}{r^2} = \frac{\pi^2}{12}$$

viii. $$\sum_{r=1}^{\infty} \frac{1}{r^2} = \frac{\pi^2}{6}$$

ix. $$\sum_{r=0}^{\infty} \frac{1}{(2r+1)^2} = \frac{\pi^2}{8}$$

x. $\displaystyle\sum_{r=1}^{\infty} \frac{1}{(2r)^2} = \frac{\pi^2}{24}$