POLITECNICO
MILANO 1863

School of Civil, Environmental and Land Management Engineering
M.Sc program in Civil Engineering for Risk Mitigation
Computational Mechanics

Prof. Gabriella Bolzon
Presented by: Jasem Avaz Nasab

Base Exercises
A.Y:21-22

Exercise 1 - SOLUTION

SURNAME: ***AVAZ NASAB*** NAME: ***JASEM***
STUDENT IDENTIFICATION NUMBER: ***9 9 6 3 1 0*** PERSON CODE: ***10753087***
a b c d e f

EXPRESSIONS OF THE VOLUME FORCES

$m = 8.5$ $n = 9$

$F_x = 8.5\frac{x}{l} - 9\frac{y}{l}$ $F_y = -8.5\frac{x}{l} + 9\frac{y}{l}$

VALUES OF EQUIVALENT NODAL FORCES

triangle

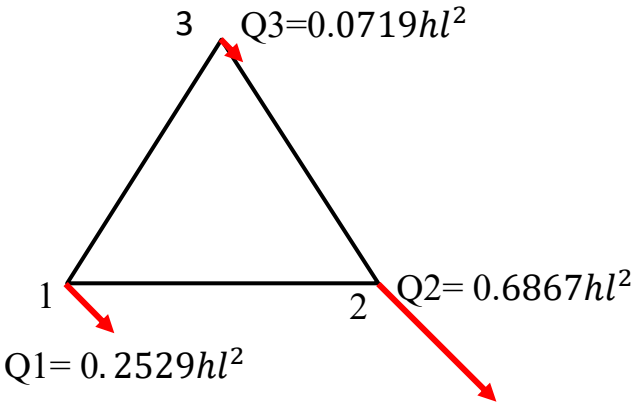
Q1x= 0.1788 hl^2
Q1y=-0.1788 hl^2
Q2x= 0.4856 hl^2
Q2y=-0.4856 hl^2
Q3x= 0.0509 hl^2
Q3y=-0.0509 hl^2

square

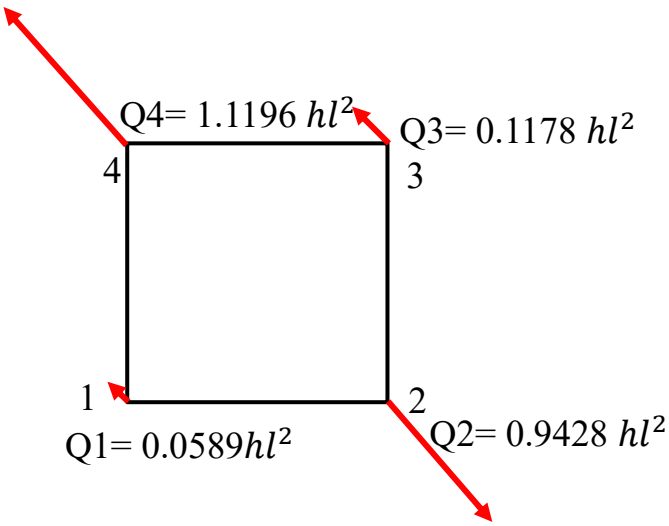
Q1x=-0.0417 hl^2
Q1y= 0.0417 hl^2
Q2x= 0.6667 hl^2
Q2y=-0.6667 hl^2
Q3x=-0.0833 hl^2
Q3y= 0.0833 hl^2
Q4x=-0.7917 hl^2
Q4y= 0.7917 hl^2

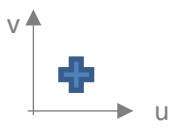
SKETCH OF EQUIVALENT NODAL FORCES

triangle



square





Exercise 2 - SOLUTION

SURNAME: NAME: ***JASEM***

STUDENT IDENTIFICATION NUMBER: ***9 9 6 3 1 0*** PERSON CODE: ***AVAZ NASAB***

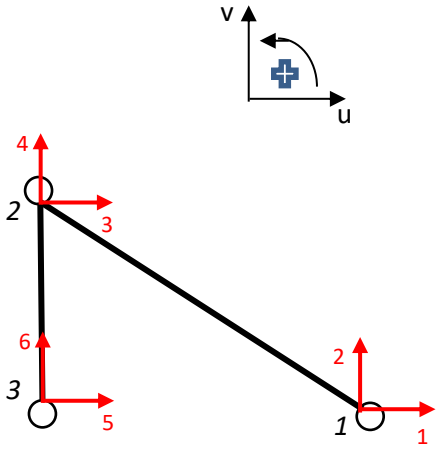
a b c d e f

n_1 = **1.9** n_2 = **1.1**

STRUCTURES a) and b)

WHOLE STIFFNESS MATRIX – matrix rank = ***2***

$$K_{whole} = \frac{EA}{b}\begin{bmatrix} 0.365 & -0.192 & -0.365 & 0.192 & 0 & 0 \\ -0.192 & 0.101 & 0.192 & -0.101 & 0 & 0 \\ -0.365 & 0.192 & 0.365 & -0.192 & 0 & 0 \\ 0.192 & -0.101 & -0.192 & 1.101 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}$$



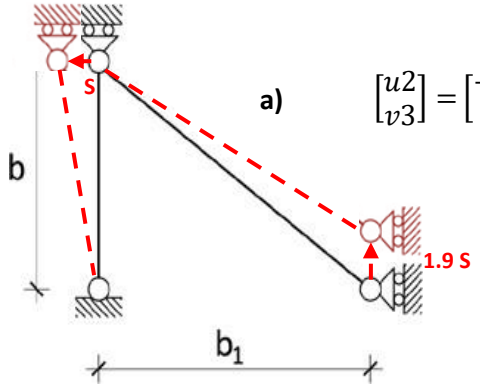
CONFIGURATION a)

REDUCED STIFFNESS MATRIX – matrix rank = ***1***

$$K_{reduced} = \frac{EA}{b}\begin{bmatrix} 0.101 & 0.192 \\ 0.192 & 0.365 \end{bmatrix}$$

possible rigid body motions (y/n) = ***yes***

numerical and graphical representation of the rigid body motion, if any



$$\begin{bmatrix} u2 \\ v3 \end{bmatrix} = \begin{bmatrix} -0.526 \\ 1 \end{bmatrix}$$

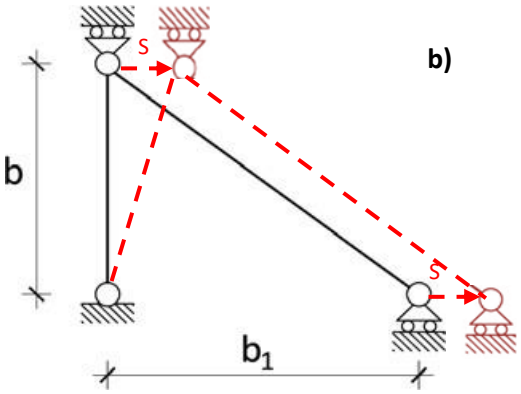
CONFIGURATION b)

REDUCED STIFFNESS MATRIX – matrix rank = ***1***

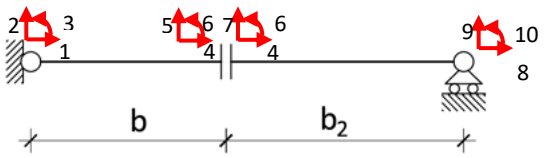
$$K_{reduced} = \frac{EA}{b}\begin{bmatrix} 0.365 & -0.365 \\ -0.365 & 0.365 \end{bmatrix}$$

possible rigid body motions (y/n) = ***yes***

numerical and graphical representation of the rigid body motion, if any



$$\begin{bmatrix} u2 \\ u3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$



STRUCTURES c) and d)

WHOLE STIFFNESS MATRIX – matrix rank = 6

$$K_{whole} = \begin{bmatrix} \frac{EA}{b} & 0 & 0 & -\frac{EA}{b} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI}{b^3} & \frac{6EI}{b^2} & 0 & -\frac{12EI}{b^3} & \frac{6EI}{b^2} & 0 & 0 & 0 & 0 \\ 0 & \frac{6EI}{b^2} & \frac{4EI}{b} & 0 & -\frac{6EI}{b^2} & \frac{2EI}{b} & 0 & 0 & 0 & 0 \\ -\frac{EA}{b} & 0 & 0 & \frac{1.909EA}{b} & 0 & 0 & 0 & -\frac{0.909EA}{b} & 0 & 0 \\ 0 & -\frac{12EI}{b^3} & -\frac{6EI}{b^2} & 0 & \frac{12EI}{b^3} & -\frac{6EI}{b^2} & 0 & 0 & 0 & 0 \\ 0 & \frac{6EI}{b^2} & \frac{2EI}{b} & 0 & -\frac{6EI}{b^2} & \frac{7.636EI}{b} & \frac{4.958EI}{b^2} & 0 & -\frac{4.959EI}{b^2} & \frac{1.818EI}{b} \\ 0 & 0 & 0 & 0 & 0 & \frac{0.535EI}{b^2} & \frac{9.016EI}{b^3} & 0 & -\frac{9.016EI}{b^3} & \frac{0.535EI}{b^2} \\ 0 & 0 & 0 & -\frac{0.909EA}{b} & 0 & 0 & 0 & \frac{0.909EA}{b} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -\frac{4.959EI}{b^2} & -\frac{9.016EI}{b^3} & 0 & \frac{9.016EI}{b^3} & -\frac{4.959EI}{b^2} \\ 0 & 0 & 0 & 0 & 0 & \frac{1.818EI}{b} & \frac{4.959EI}{b^2} & 0 & -\frac{4.959EI}{b^2} & \frac{3.636EI}{b} \end{bmatrix}$$

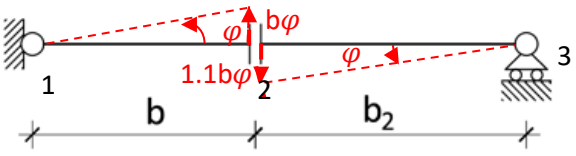
CONFIGURATION c)

*REDUCED STIFFNESS MATRIX – matrix rank = **6***

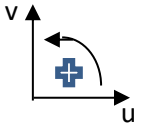
*possible rigid body motions (y/n) = **yes***

numerical and graphical representation of the rigid body motion, if any

$$K_{red,c} = \begin{bmatrix} \frac{4EI}{b} & 0 & -\frac{6EI}{b^2} & \frac{2EI}{b} & 0 & 0 & 0 \\ 0 & \frac{1.909EA}{b} & 0 & 0 & 0 & -\frac{0.909EA}{b} & 0 \\ -\frac{6EI}{b^2} & 0 & \frac{12EI}{b^3} & -\frac{6EI}{b^2} & 0 & 0 & 0 \\ \frac{2EI}{b} & 0 & -\frac{6EI}{b^2} & \frac{7.636EI}{b} & \frac{4.959EI}{b^2} & 0 & \frac{1.818EI}{b} \\ 0 & 0 & 0 & \frac{4.959EI}{b^2} & \frac{9.016EI}{b^3} & 0 & \frac{4.959EI}{b^2} \\ 0 & -\frac{0.909EA}{b} & 0 & 0 & 0 & \frac{0.909EA}{b} & 0 \\ 0 & 0 & 0 & \frac{1.818EI}{b} & \frac{4.959EI}{b^2} & 0 & \frac{3.636EI}{b} \end{bmatrix}$$



$$\begin{bmatrix} \varphi 1 \\ u2 \\ v21 \\ \varphi 2 \\ v23 \\ u3 \\ \varphi 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \\ -1.1 \\ 0 \\ 1 \end{bmatrix}$$



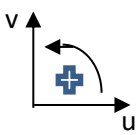
CONFIGURATION d)

*REDUCED STIFFNESS MATRIX – matrix rank = **6***

*possible rigid body motions (y/n) = **no***

numerical and graphical representation of the rigid body motion, if any

$$K_{red,d} = \begin{bmatrix} \frac{4EI}{b} & 0 & -\frac{6EI}{b^2} & \frac{2EI}{b} & 0 & 0 \\ 0 & \frac{1.909EA}{b} & 0 & 0 & 0 & -\frac{0.909EA}{b} \\ -\frac{6EI}{b^2} & 0 & \frac{12EI}{b^3} & -\frac{6EI}{b^2} & 0 & 0 \\ \frac{2EI}{b} & 0 & -\frac{6EI}{b^2} & \frac{7.636EI}{b} & \frac{4.959EI}{b^2} & 0 \\ 0 & 0 & 0 & \frac{4.959EI}{b^2} & \frac{9.016EI}{b^3} & 0 \\ 0 & -\frac{0.909EA}{b} & 0 & 0 & 0 & \frac{0.909EA}{b} \end{bmatrix}$$



Exercise 3 - SOLUTION

SURNAME: ***AVAZ NASAB*** NAME: ***JASEM***

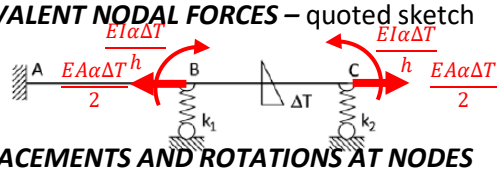
STUDENT IDENTIFICATION NUMBER: **9 9 6 3 1 0** PERSON CODE: ***10753087***

a b c d e f

n_1 = **3** n_2 = **20**

NUMERICAL SOLUTION

EQUIVALENT NODAL FORCES – quoted sketch



DISPLACEMENTS AND ROTATIONS AT NODES

horizontal displacement at node B: uB = 0

vertical displacement at node B: vB = -0.032658 $\Delta TL^2/h$

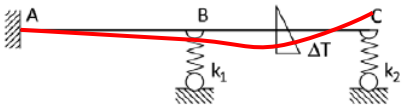
rotation at node B: φB = -0.120019 αΔTL/h

horizontal displacement at node C: uC = αΔTL/4

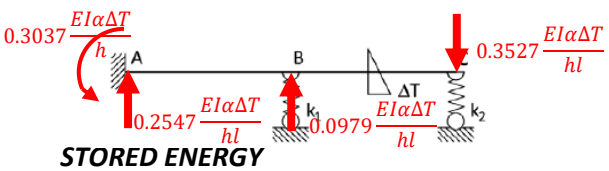
vertical displacement at node C: vC = 0.017636 $\alpha\Delta TL^2/h$

rotation at node C: φC = 0.335892 αΔTL/h

DEFORMED SHAPE – quoted graph



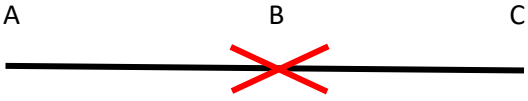
REACTION FORCES – quoted sketch



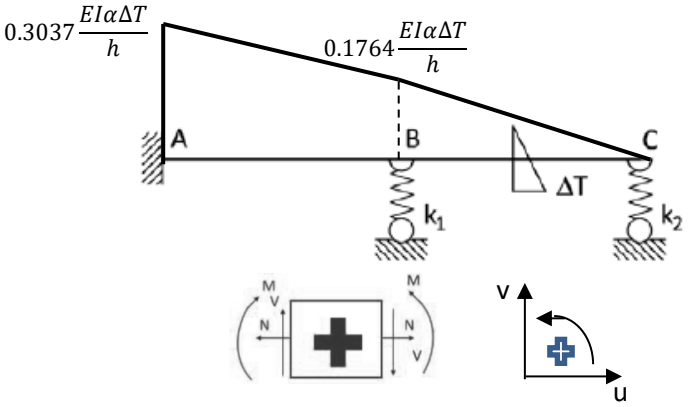
STORED ENERGY

$22.044 * 10^{-3}LEI(\alpha\Delta T/h)^2$

AXIAL FORCE DISTRIBUTION – quoted graph



BENDING MOMENT DISTRIBUTION – quoted graph



ANALYTICAL SOLUTION

DISPLACEMENTS AND ROTATIONS AT NODES

horizontal displacement at node B: uB = 0

vertical displacement at node B: vB = -0.032658 $\Delta TL^2/h$

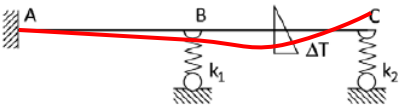
rotation at node B: φB = -0.120019 αΔTL/h

horizontal displacement at node C: uC = αΔTL/4

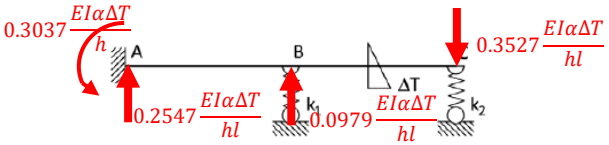
vertical displacement at node C: vC = 0.017636 $\alpha\Delta TL^2/h$

rotation at node C: φC = 0.335892 αΔTL/h

DEFORMED SHAPE – quoted graph



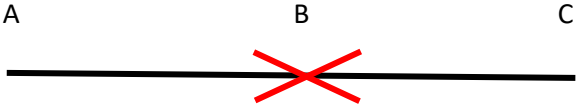
REACTION FORCES – quoted sketch



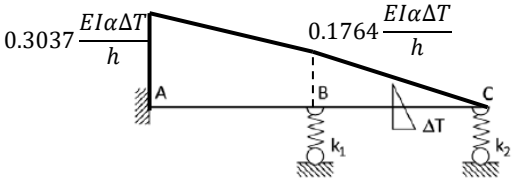
STORED ENERGY

$22.044 * 10^{-3}LEI(\alpha\Delta T/h)^2$

AXIAL FORCE DISTRIBUTION – quoted graph



BENDING MOMENT DISTRIBUTION – quoted graph



SHEAR FORCE DISTRIBUTION – quoted graph