POLITECNICO

MILANO 1863

School of Civil, Environmental and Land Management Engineering
M.Sc program in Civil Engineering for Risk Mitigation
Computational Mechanics

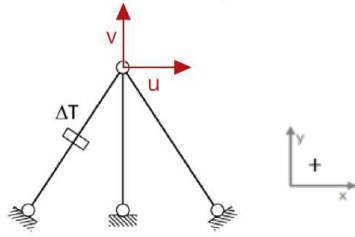*Prof. Gabriella Bolzon*
Presented by: Jasem Avaz Nasab

Exercises- Winter set
A.Y:21-22

Supplementary Exercises – set E - winter set - SOLUTION

Exercise 1
NODE NUMBERING & FREE DEGREES OF FREEDOM (sketch)

$n = \dfrac{55}{48}$

STRAIN ENERGY EXPRESSION

$$E(u,v) = 0.2843 \times u^2 \frac{EA}{b} + 0.8096 \times v^2 \frac{EA}{b} - 0.6575 \times uEA\alpha\Delta T - 0.7534 \times vEA\alpha\Delta T + 0.7604 \times EAb\alpha^2\Delta T^2$$

EQUILIBRIUM EQUATIONS

$$\frac{dE(u,v)}{du} = 0 \;;\; 0.5686 \times u\frac{EA}{b} - 0.6575 \times EA\alpha\Delta T = 0$$

$$\frac{dE(u,v)}{dv} = 0 \;;\; 1.6192 \times v\frac{EA}{b} - 0.7534 \times EA\alpha\Delta T = 0$$

DISPLACEMENT VALUES   **DEFORMED CONFIGURATION** (quoted sketch)

$u = 1.1565 \times b\alpha\Delta T$

$v = 0.4653 \times b\alpha\Delta T$

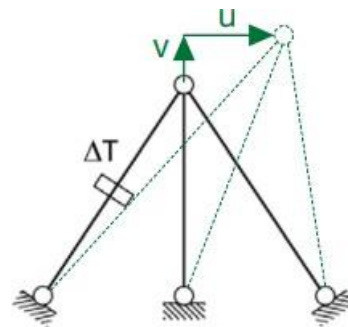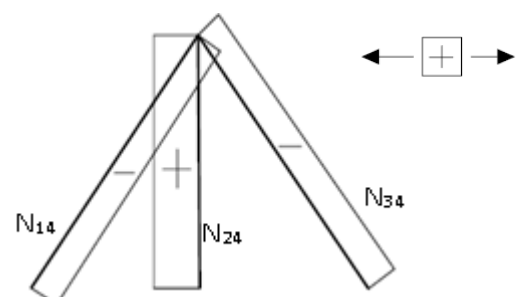**DIAGRAM OF INTERNAL FORCES**

$N_{14} = -0.2695 \times EA\alpha\Delta T$
$N_{24} = 0.4061 \times EA\alpha\Delta T$
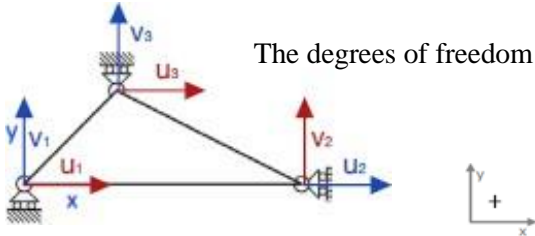$N_{34} = -0.2695 \times EA\alpha\Delta T$

Exercise 2
NODE NUMBERING & FREE DEGREES OF FREEDOM (sketch)

$m = \frac{1}{10}$

DISPLACEMENT FIELD AS A FUNCTION OF FREE DISPLACEMENTS

The degrees of freedom are highlighted in red

$$\begin{bmatrix} Sx \\ Sy \end{bmatrix} = \begin{bmatrix} u_1 \times \left(1 - \frac{x}{3b} - \frac{2y}{3b}\right) + u_3 \times \frac{y}{b} \\ v_2 \times \left(\frac{x}{3b} - \frac{y}{3b}\right) \end{bmatrix}$$

STRAIN ENERGY EXPRESSION

$$E = \frac{5Eh \times (20u_1^2 - 8u_1v_2 - 36u_3u_1 + 18u_3v_2 + 11v_2^2 + 27u_3^2}{432}$$

EQUILIBRIUM EQUATIONS

$Eh(0.462u_1 - 0.0925v_2 - 0.417u_3) = 0.5125b^2hp$
$Eh(-0.0925u_1 + 0.2546v_2 + 0.2083u_3) = 0.9875b^2hp$
$Eh(-0.417u_1 + 0.2083v_2 - 0.625u_3) = 0.525b^2hp$

DISPLACEMENT VALUES

$u_1 = 3.798 \frac{b^2p}{E}$

$v_2 = 3.438 \frac{b^2p}{E}$

$u_3 = 2.226 \frac{b^2p}{E}$

STRAIN EXPRESSIONS

$\varepsilon_x = -1.266 \frac{bp}{E}$

$\varepsilon_y = -1.146 \frac{bp}{E}$

$\gamma_{xy} = 0.840 \frac{bp}{E}$

STRESS EXPRESSIONS

$\sigma_x = -1.725 \, bp$
$\sigma_y = -1.625 \, bp$
$\sigma_{xy} = 0.350 \, bp$
$\sigma_z = -0.67 \, bp$

DEFORMED CONFIGURATION (quoted sketch)