Exercise session on the numerical solution of linear systems

November 16, 2020

Exercise 1

Build a matrix $\mathbf{A}$ of dimension 200×200 that has all elements equal to 5 on the main diagonal and all elements equal to -1 on the first and second subdiagonal and superdiagonal (use the command `diag`).

(a) Write a script to check that the matrix is diagonally dominant by rows.

(b) Write a script to check that the matrix is symmetric and positive definite.

(c) Compute the vector $\mathbf{b} = \mathbf{A}\mathbf{x}_{ex}$, where $\mathbf{x}_{ex} = [1, 1, \ldots, 1, 1]^T$, $\mathbf{x}_{ex} \in \mathbf{R}^{200}$.

(d) Solve the system $\mathbf{A}\mathbf{x} = \mathbf{b}$ using by the LU factorization method. Check that pivoting is not performed.

(e) Compute the absolute and relative error of the computed solution with respect to $\mathbf{x}_{ex}$ in the l^2 and l^∞ norm.

Exercise 2

Consider the $n \times n$ matrix $\alpha\mathbf{I} + \mathbf{H}_n$, where $\mathbf{H}_n$ the Hilbert matrix of dimension n, build using the command `hilb`. For the case $\alpha = 1$, $n = 20$, check if the matrix is diagonally dominant by rows and if the matrix is symmetric and positive definite; then

(a) compute the vector $\mathbf{b} = \mathbf{A}\mathbf{x}_{ex}$, where $\mathbf{x}_{ex} = [-1, 1, \ldots, 1, -1]^T$, $\mathbf{x}_{ex} \in \mathbf{R}^{20}$;

(b) solve the system $\mathbf{A}\mathbf{x} = \mathbf{b}$ using by the LU factorization method;

(c) compute the absolute and relative error of the computed solution with respect to $\mathbf{x}_{ex}$ in the l^2 and l^∞ norm. Repeat the exercise for the case $\alpha = 10^{-8}$.

Exercise 3

Using the command `vander`, build the 10×10 Vandermonde matrix associated to the vector $\mathbf{v} = [0.0001, 2, 3, 4, 5, 6, 7, 8, 9, 10]^T$. Compute the vector $\mathbf{b} = \mathbf{A}\mathbf{x}_{ex}$, where $\mathbf{x}_{ex} = [10, 9, \ldots, 2, 1]^T$, $\mathbf{x}_{ex} \in \mathbf{R}^{10}$.

(a) Solve the system $\mathbf{Ax} = \mathbf{b}$ using the LU factorization method; check if pivoting has been performed;

(b) compute the absolute and relative error of the computed solution with respect to $\mathbf{x}_{ex}$ in the l^2 and l^∞ norm.