

Exercise session on the numerical solution of linear systems

November 19, 2020

Exercise 1

Build a matrix \mathbf{A} of dimension 400×400 that has all elements equal to -6 on the main diagonal, all elements equal to -1 on the first subdiagonal and superdiagonal and all elements equal to 1 on the fourth subdiagonal and superdiagonal (use the command `diag`).

- (a) Check that the matrix is diagonally dominant by rows.
- (b) Check that the matrix is symmetric and positive definite.
- (c) Compute the vector $\mathbf{b} = \mathbf{A}\mathbf{x}_{ex}$, where $\mathbf{x}_{ex} = [1, -1, \dots, 1, -1]^T$, $\mathbf{x}_{ex} \in \mathbf{R}^{400}$.
- (d) Solve the system $\mathbf{Ax} = \mathbf{b}$ using by the Cholesky factorization method.
- (e) Compute the absolute and relative error of the computed solution with respect to \mathbf{x}_{ex} in the l^2 and l^∞ norm and compare them with the *a priori* and *a posteriori* error estimates.

Exercise 2

Consider the $n \times n$ matrix \mathbf{H}_n , where \mathbf{H}_n the Hilbert matrix of dimension n , built using the command `hilb`. For $n = 1, \dots, 20$, check if the matrix is diagonally dominant by rows and if the matrix is symmetric and positive definite; then

- (a) compute the vector $\mathbf{b} = \mathbf{Ax}_{ex}$, where $\mathbf{x}_{ex} = [2, 2, \dots, 2, 2]^T$, $\mathbf{x}_{ex} \in \mathbf{R}^n$;
- (b) solve the system $\mathbf{Ax} = \mathbf{b}$ using by the Cholesky factorization method;
- (c) compute the absolute and relative error of the computed solution with respect to \mathbf{x}_{ex} in the l^2 and l^∞ norm and compare them with the *a priori* and *a posteriori* error estimates.
- (d) report the errors and the *a priori* error estimates on a semilogarithmic plot as a function of $n = 1, \dots, 20$.

Exercise 3

Build a matrix \mathbf{B} of dimension 300×300 that has all elements equal to 4 on the main diagonal, all elements equal to -1 on the first subdiagonal, all elements equal to 1 on the first superdiagonal, all elements equal to 0.5 on the tenth subdiagonal and all elements equal to -0.3 on the ninth superdiagonal (use the command `diag`). Consider the 300×100 matrix \mathbf{C} that consists of the first 100 columns of \mathbf{B} and build the 100×100 matrix $\mathbf{A} = \mathbf{C}^T \mathbf{C}$.

- (a) check that the matrix \mathbf{A} is symmetric and positive definite
- (b) compute the vector $\mathbf{b} = \mathbf{A}\mathbf{x}_{ex}$, where $\mathbf{x}_{ex} = [1, 2, \dots, 99, 100]^T$, $\mathbf{x}_{ex} \in \mathbf{R}^n$;
- (c) solve the system $\mathbf{A}\mathbf{x} = \mathbf{b}$ using by the Cholesky factorization method;
- (d) compute the absolute and relative error of the computed solution with respect to \mathbf{x}_{ex} in the l^2 and l^∞ norm and compare them with the *a priori* and *a posteriori* error estimates.