Exercise session on numerical solution of ODEs, 2

October 30, 2020

Exercise 1

Consider the initial value problem

$$y' = -y + \exp t \sin t \qquad t \in [0,\, 5]$$

$$y(0) = 10,$$

whose exact solution is

$$y_{ex}(t) = \frac{51}{5}\exp(-t) - \frac{1}{5}\exp(t)(\cos(t) - 2\sin(t)).$$

Solve the initial value problem with a) the three stage Runge Kutta method b) the Adams Bashforth three step method, using $N = 5000$ time steps. Use the commands `tic, toc` to estimate the computational cost of each method and say which one requires less time to compute the solution. Use the exact solution to provide the supplementary initial conditions for the multistep method. Compute the absolute and relative errors at the final time. Compute the absolute and relative errors in the infinity norm over the whole solution interval. Say which method is the most accurate for this problem. Repeat the computation with half the number of time steps and compute the empirical estimate of the convergence order for the two methods, checking if the result is coherent with the theory.

Exercise 2

(a) Repeat exercise 1 using the Adams Bashforth three step method, using the Heun method to provide the supplementary initial conditions.

(b) Repeat exercise 1 using 1) the four stage Runge Kutta method 2) the Adams Bashforth four step method.

(c) Repeat exercise 1 using $N = 50000$ and $N = 10^5$ steps. Compute the empirical estimate of the convergence order for the two methods, checking if the result is coherent with the theory and, if not, explaining why.

Exercise 3

Consider the initial value problem

$$
\begin{aligned}
&y' = -50y + 4y^2 + t \qquad t \in [0,\, 1] \\
&y(0) = 2,
\end{aligned}
$$

- Compute a reference solution using the MATLAB solver `ode15s` with a relative error tolerance of 10^{-10} and a maximum time step $h = 10^{-4}$.
- Compute a numerical solution using the explicit Euler method with $N = 15$ time steps and check that the method is unstable. Determine on the basis of the theory the minimum number of steps such that a stable solution can be obtained (hint: assume $\lambda = 50$).
- Compute a numerical solution using the implicit Euler method with $N = 15$ time steps and a tolerance value of 10^{-6} for the nonlinear solver. Use the reference solution to compute the maximum absolute error on the whole interval. Estimate the required computational time using the `tic` and `toc` commands and compare it to that of the explicit Euler method with minimum number of steps such that a stable solution can be obtained.