Exercise session on numerical integration, 2

October 16, 2020

Exercise 1

Write MATLAB functions implementing the composite midpoint, trapezoidal, Simpson and Gauss Legendre formulae as defined in the lecture notes for a generic input function $f(x)$. Check the correctness of the implementation verifying that

(a) these functions are integrating exactly polynomials of appropriate degree for each formula

(b) these function provide the right empirical convergence order when approximating the definite integral

$$I = \int_0^{\frac{\pi}{2}} \exp(-x)\cos(x)\,dx = \frac{1}{2}\left(1 + \exp(-\pi/2)\right)$$

with $m = 20, 40, 80, 160$ subintervals.

Exercise 2

Consider the definite integral

$$I = \int_0^5 \cos(4x)\exp(-x) \;\; d\,x.$$

(a) Compute a reference value for I using the function `integral`;

(b) repeat part b) of Exercise 1 in this case.

Exercise 3

Consider the definite integral

$$I = \int_0^1 \left[\beta x^3 \sin(\beta x) - 3x^2\cos(\beta x) - \beta\sin(\beta x)\right]\,dx,$$

whose exact value is -1 for all values of $\beta > 0$.

(a) In the case $\beta = 10$, compute a numerical approximation of I with the composite Gauss Legendre formula using 20 subintervals. Compute the resulting absolute error. Repeat the computations with 40 subintervals and estimate empirically the convergence order of the method.

(b) Repeat the computations in the case $\beta = 100$. Explain if the results are coherent with the theory and why.

Exercise 4

Consider the definite integral

$$I = \int_0^1 x^{\frac{1}{5}}\,dx = \frac{5}{6}.$$

Compute a numerical approximations of I with the midpoint formula, the trapezoidal formula, the Simpson formula and the Gauss Legendre formula, using in each case both 40 and 80 subintervals. Compute the absolute errors obtained with each formula. Compare the results with the theoretical expectations.