Exercise session on polynomial interpolation

October 12, 2020

Exercise 1

Consider the function $f(x) = |\sin(x)|$ on the interval $[-2, 4]$.

(a) Determine the Lagrange interpolating polynomial built from the nodes

$$[-2, -1.3, -\pi/4, 0, 1, \pi/2, 2, 3, 4]$$

and evaluate it on a mesh of constant step $h = 0.01$ on the interval $[-2, 4]$.

(b) Determine the composite linear interpolation of the same data using the function `interp1` with `linear` option and evaluate it on the same mesh.

(c) Determine the composite cubic interpolation of the same data using the function `interp1` with `pchip` option and evaluate it on the same mesh.

(d) Determine the composite spline interpolation of the same data using the function `interp1` with `spline` option and evaluate it on the same mesh.

For all approximations, plot the interpolating polynomials and the exact function and compute the relative error of the approximation of f in the 2 and infinity norm on a mesh of constant step $h = 0.01$ on the interval $[-2, 4]$.

Exercise 2

Repeat the previous exercise for the function $f(x) = 10\exp(-2x^2)$ and the nodes

$$[-2, -1, -1.3, -\pi/4, -0.2, 0, 1, \pi/2, 2, 2.5, 3, 3.5, 4].$$

Exercise 3

Consider the nodes of an equispaced grid of 50 subintervals over $[0, 10]$ and compute at these locations the function $f(x) = \cos(x) + 0.3\epsilon_g$, where ϵ_g denotes a random Gaussian variable with zero mean and standard deviation 1 (use the function `randn`).

(a) Determine the Lagrange interpolating polynomial built from the nodes and data computed above. Evaluate it on a mesh of constant step $h = 0.01$ on the interval $[0, 10]$.

(b) Determine the composite linear interpolation of the same data using the function `interp1` with `linear` option and evaluate it on the same mesh.

(c) Determine the composite cubic interpolation of the same data using the function `interp1` with `pchip` option and evaluate it on the same mesh.

(d) Determine the composite spline interpolation of the same data using the function `interp1` with `spline` option and evaluate it on the same mesh.

Compare the quality of the results by measuring on the mesh of constant step $h = 0.01$ on the interval $[0, 10]$ the difference between the interpolations performed and the values of the function $g(x) = \cos(x)$.

Exercise 4

Consider the following experimental data, in which σ represents the stress and ε represents the deformation:

σ $[1000 \times \mathrm{kg_F/cm^2}]$	ε [cm/cm]
0.1800	0.0005
0.3000	0.0010
0.5000	0.0013
0.6000	0.0015
0.7200	0.0020
0.7500	0.0045
0.8000	0.0060
0.9000	0.0070
1.0000	0.0085

We know from physics that stress and deformation are related by a relationship $\varepsilon = f(\sigma)$, called constitutive equation, and we want to compute an approximation of the function f in order to estimate deformations ε that correspond to stress values σ for which we do not have experimental results. For this purpose, we can apply polynomial interpolation techniques. Write a script `constitutive.m` that computes:

(a) Lagrange polynomial interpolation of the data using `polyfit` and `polyval`;

(b) composite linear interpolation using the function `interp1` with `linear` option;

(c) composite cubic interpolation using the function `interp1` with `cubic` option;

(d) represents on the same figure the interpolation polynomials obtained in points (a), (b) and (c) with coloured lines and the experimental data with black circles;

(e) evaluate the deformation ε that corresponds to the stress $\sigma = 0.4 \cdot 1000 \times \mathrm{kg_F/cm^2}$ using the Lagrange polynomial interpolation obtained in (a).