Exercise session on numerical integration,1

October 15, 2020

Exercise 1

Consider the definite integral

$$I = \int_0^2 (x^2 + x)\, dx = \frac{14}{3}.$$

Compute a numerical approximation of I with the simple midpoint formula and with the simple trapezoidal formula. Compute the absolute and relative errors obtained with each formula. Repeat the computation for

$$\int_0^1 (x^2 + x)\, dx = \frac{5}{6} \qquad \int_0^{\frac{1}{2}} (x^2 + x)\, dx = \frac{1}{6}.$$

Compare the error behaviour with that predicted by the theory.

Exercise 2

Repeat the previous exercise for the integrals

$$\int_0^2 \sqrt{x}\, dx = \frac{4\sqrt{2}}{3} \qquad \int_0^1 \sqrt{x}\, dx = \frac{2}{3} \qquad \int_0^{\frac{1}{2}} \sqrt{x}\, dx = \frac{1}{3\sqrt{2}}.$$

Exercise 3

Consider the definite integrals

$$I = \int_0^1 (x^3 + x^2 - 1)\, dx = -\frac{5}{12}.$$

Compute a numerical approximation of I with the simple Simpson formula and with the simple two-point Gauss Legendre formula. Compute the absolute and relative errors obtained with each formula. Compare the results with the theoretical expectations. Repeat the computation using the midpoint and trapezoidal formulas. Explain the difference in the results. Repeat the same computation for the integrals of the same function on $[0, \frac{1}{2}]$ and $[0, \frac{1}{4}]$, computing a reference exact solution using the command `integral`.

Exercise 4

Repeat exercise 3 for the definite integral

$$I = \int_0^1 \exp x \, dx.$$

Exercise 5

Consider the definite integral

$$I = \int_0^{\frac{1}{2}} \cos(x) \, dx = \sin\left(\frac{1}{2}\right).$$

Compute a numerical approximations of I with the midpoint formula, the trapezoidal formula, the simple Simpson formula, the simple two-point Gauss Legendre formula. Compute the absolute errors obtained with each formula. Compare the results with the theoretical expectations.