Session Types

Logical Foundations of Concurrent Computation

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(Part 1.1)

About Myself

- Associate Professor and Leader, Fundamental Computing Group
- Brief bio: PhD in 2010 (Bologna), then Postdoc in Lisbon (2010-2014). Joined Groningen in 2014. Currently main supervisor for two PhD students.
- **Research Interests**:
  Semantics of concurrency, process models, type systems, relative expressiveness

This Course: Session Types / Propositions as Sessions

An introduction to **session types** for concurrency, and to the logical foundations of message-passing computation.

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  Not only: sessions have elegant **logical foundations**!
- **Linear Logic**: A resource-oriented view on propositions / session protocols.
  Linear resources can be used exactly once.

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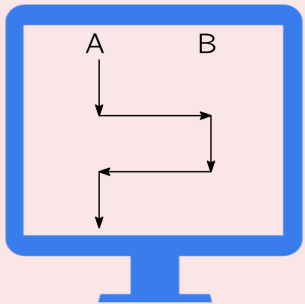
Your questions and feedback are warmly welcome!

Outline

When is a Program Correct?

Sequential Programs



"Programs produce outputs that are consistent with their input"

Concurrent Programs?

Message-Passing Concurrent Programs?

- Software components (**services**) distributed across networks

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An (imperfect) analogy:

When is a Program Correct?

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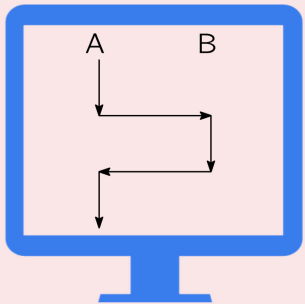

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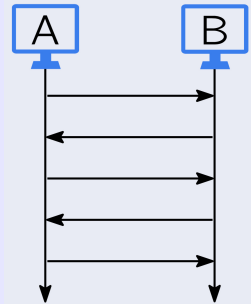

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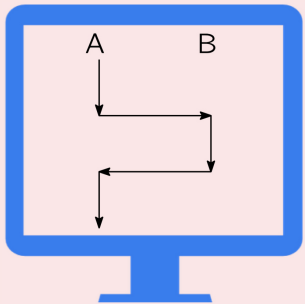

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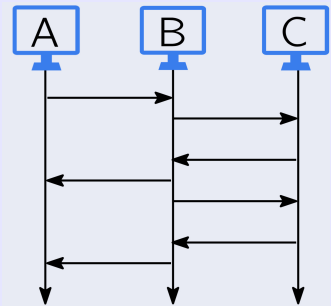

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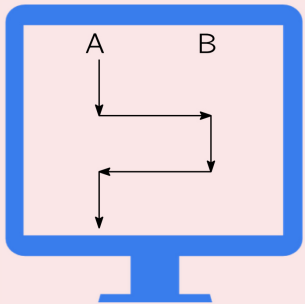

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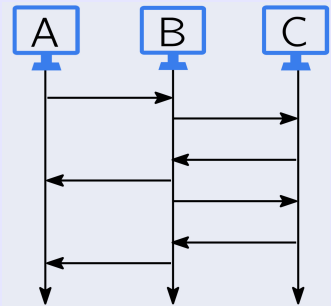

"Programs always respect their intended **protocols**"

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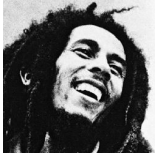

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Note: The structure of messaging matters!

Example: A Two-Buyer Protocol

Desiderata for the implementations of Alice, Bob, and Seller:

- **Fidelity** – they **follow the intended protocol**.
  - Alice never ask Bob twice within the same conversation
  - Alice doesn't continue the transaction if Bob can't contribute
  - Alice chooses among the options provided by Seller

Example: A Two-Buyer Protocol

Desiderata for the implementations of Alice, Bob, and Seller:

- **Fidelity** – they **follow the intended protocol**.
- **Safety** – they don't feature **communication errors**.
  - Seller always returns an integer when Alice requests a quote

Example: A Two-Buyer Protocol

Desiderata for the implementations of Alice, Bob, and Seller:

- **Fidelity** – they **follow the intended protocol**.
- **Safety** – they don't feature **communication errors**.
- **Deadlock-Freedom** – they do not **"get stuck"** while running the protocol.
  - Alice eventually receives an answer from Bob on his contribution.

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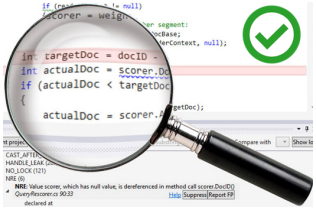
Correctness

- A **non-trivial notion**, which follows from the interplay of these properties.
- **Hard to enforce** due to actions being "scattered around" in programs.

→ A session specifies a protocol's structure, enabling program verification. It stipulates **what** and **when** should be exchanged (along a channel)

Type Systems

- Can detect bugs before programs are run
- Attached to many programming languages
- Implement a specific notion of correctness
  A program is either correct or incorrect



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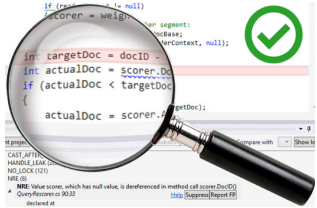

Sequential Languages

- **Data** type systems classify values in a program
- Examples: Integers, strings, etc

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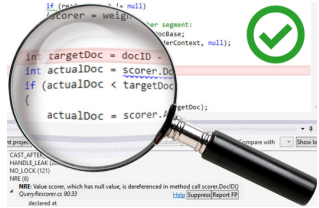

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Concurrent Languages

- **Behavioral** type systems classify protocols in a program
- Example: “first send username, then receive true/false, finally close”
- A typical bug: sending messages in the wrong order

How to Design Type Systems? Logic to the Rescue!

-

How to Design Type Systems? Logic to the Rescue!

A landmark result in programming language theory:

- **Propositions as types** (Curry, 1935; Howard, 1969)

Propositions in Intuitionistic Logic	$\leftrightarrow$	Types
Proofs in natural deduction	$\leftrightarrow$	Sequential programs (λ-calculus)
Proof simplification	$\leftrightarrow$	Program evaluation (β-reduction)

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- What about concurrent programs? A **resource-oriented** view!

How to Design Type Systems? Logic to the Rescue!

- **Propositions as sessions**

Propositions in **Linear Logic** (LL)	$\leftrightarrow$	**Session types**
Proofs in LL	$\leftrightarrow$	Interacting processes
Cut elimination in LL	$\leftrightarrow$	process communication

Outline

Protocols as Session Types

Session types specify protocols in terms of

- communication actions: send and receive
- choices: offers and selections
- recursion

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Session protocols are attached to **interaction devices**:

- communication channels in programs (think Go and Rust)
- TCP-IP sockets
- ...

Protocols as Session Types

A formal syntax for protocols:

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Notice:

- Sequential communication patterns (no built-in concurrency)
- U stands for basic values (e.g. `int`) but also other sessions S

The Two-Buyer Protocol, Revisited

Recall the protocol between Alice, Bob, and Seller:

1. Alice sends a book title to Seller, who sends a quote back.
2. Alice checks whether Bob can contribute in buying the book.
3. Alice uses the answer from Bob (yes/no) to interact with Seller, either:
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Session Types for The Two-Buyer Protocol

Two independent protocols, with Alice "leading" the interactions:

1. A session type for Seller (in its interaction with Alice):

$$S_{\mathsf{SA}} = ?\texttt{book};\ !\texttt{quote};\ \&\begin{cases}\text{buy}: & ?\texttt{paym};\ ?\texttt{address};\ !\texttt{ok};\ \text{end}\\ \text{cancel}: & ?\texttt{thanks};\ !\texttt{bye};\ \text{end}\end{cases}$$

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2. A session type for Alice (in its interaction with Bob):

$$S_{\mathsf{AB}} = \texttt{!cost}; \& \begin{cases} \text{share}: & \texttt{?address}; \texttt{!ok}; \text{end} \\ \text{close}: & \texttt{!bye}; \text{end} \end{cases}$$

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Implementations for Alice, Bob, and Seller should be **compatible**.

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Duality relates session types with opposite behaviors:

- The dual of sending is receiving (and vice versa)
- Branching is the dual of selection (and vice versa)

The dual of session type S is written $\overline{S}$.

Session Types for The Two-Buyer Protocol

Example:

- Recall that S_{AB} describes Alice's viewpoint in her interaction with Bob:

$$S_{\mathsf{AB}} = \ !\texttt{cost}; \& \begin{cases} \text{share}: & ?\texttt{address}; \ !\texttt{ok}; \ \text{end} \\ \text{close}: & !\texttt{bye}; \ \text{end} \end{cases}$$

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- Also, Alice's implementation should conform to both $\overline{S_{\mathsf{SA}}}$ and S_{AB}.

Session Type Duality, Formally

Given a (finite) session type S, its dual type $\overline{S}$ is inductively defined as follows:

$$\begin{aligned}
\overline{!U;S} &= ?U;\overline{S} \\
\overline{?U;S} &= !U;\overline{S} \\
\overline{\&\{l_i : S_i\}_{i\in I}} &= \oplus\{l_i : \overline{S_i}\}_{i\in I} \\
\overline{\oplus\{l_i : S_i\}_{i\in I}} &= \&\{l_i : \overline{S_i}\}_{i\in I} \\
\overline{\mathsf{end}} &= \mathsf{end}
\end{aligned}$$

Notice:

- For recursive types, duality is defined **coinductively** (the dual of $\mu t.S$ is *not* $\mu t.\overline{S}$)

Many Classes of Session Types

There are many classes/variants of session types:

- We overviewed **binary** sessions, where types describe two-party protocols. (We used two separate types to handle Alice, Bob, and Seller.)
- With **multiparty** sessions, types describe protocols involving more than two participants. There is a single **global type** and multiple **local types**.

Many Classes of Session Types (contd.)

There are many classes/variants of session types:

- We saw a syntax of types that describes **regular behaviors**: both '$?U; S$' and '$!U; S$' are protocols, in which ';' composes an action with a protocol.
- In **context-free** session types, we can freely compose two protocols: '$S_1; S_2$'. This is arguably the most expressive class of protocols (to date).

Many Classes of Session Types (contd.)

There are many classes/variants of session types:

- Session types can be attached to languages with **synchronous** and **asynchronous** communication.
  - Synchronous: A message is received as soon as it is sent
  - Asynchronous: Message reception is not immediate

  These operational differences are reflected in types. The real world is asynchronous but synchronous is easier to define.

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- Session types are **paradigm-independent**, and can be adapted to functional, object-oriented, and actor-based languages with concurrency.

Taking Stock

Up to here:

- Correctness for communicating programs
- Sessions as protocol specifications
- A formal syntax for session types
- Example: A two-buyer protocol
- Protocol compatibility in terms of duality for session types

Coming next:

- Linear logic, in its intuitionistic variant