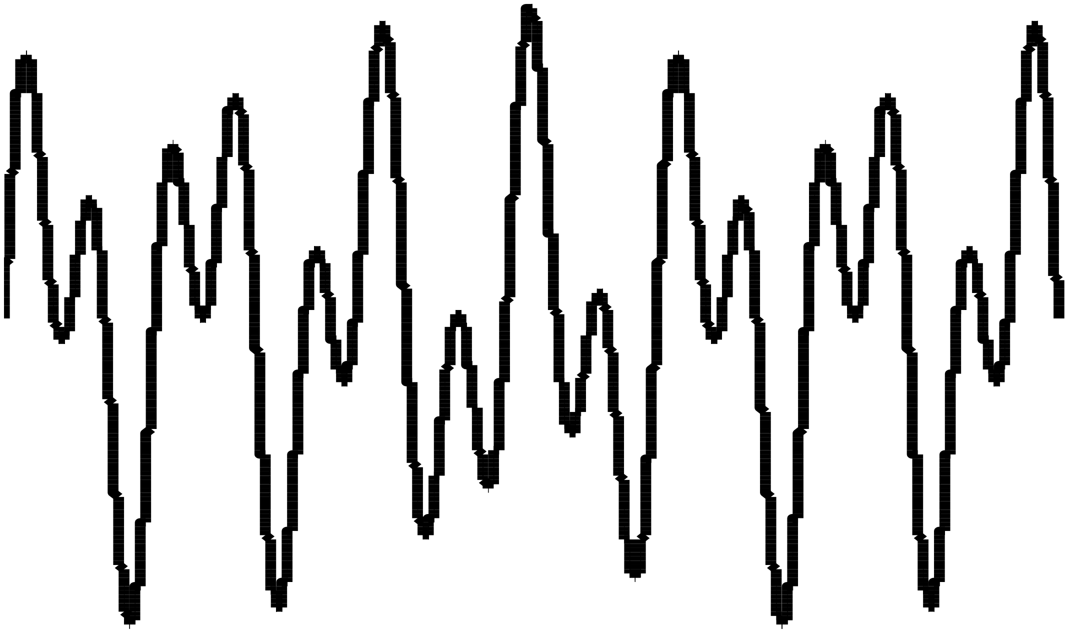
ROYAL INSTITUTE
OF TECHNOLOGY

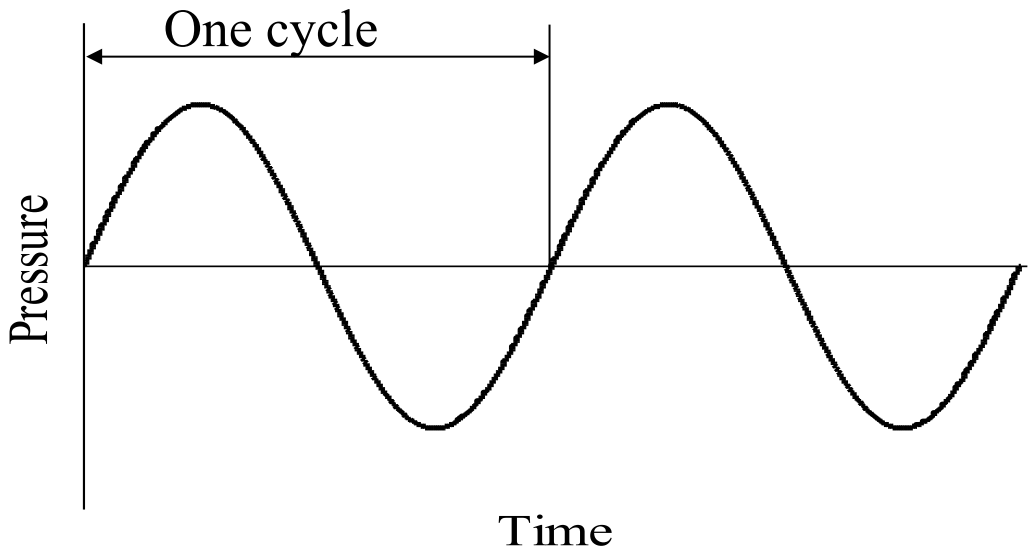
Video Computing

Prof. Haibo Li

How to Characterize a Signal?



A Typic Signal



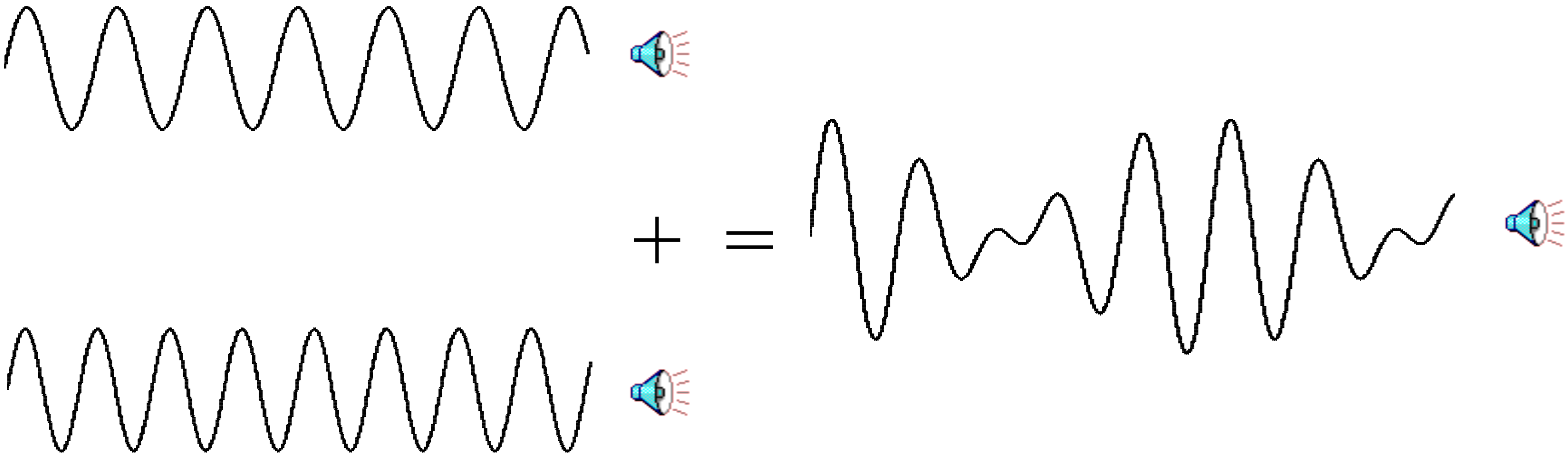
Frequency = number of cycles per second

unit of frequency: Hertz (Hz)

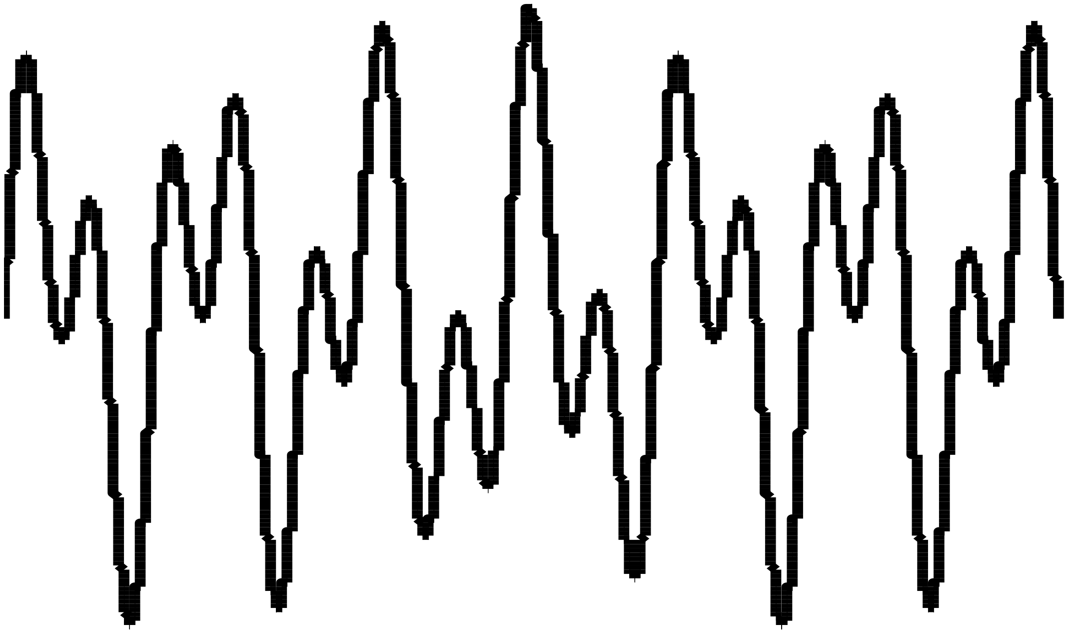
e.g. 400 Hz = 400 cycles per second

Signal Synthesis

Consider the situation in which we have two tones of different frequencies presented simultaneously.



How to Characterize a Signal?



ROYAL INSTITUTE OF TECHNOLOGY

Fourier told us.........

JOURNAL DE L'ÉCOLE SOCIÉTAIRE

FAISANT SUITE AU JOURNAL

LE PHALANSTÈRE ou LA RÉFORME INDUSTRIELLE (1832-1834).

LA

PHALANGE

JOURNAL DE LA SCIENCE SOCIALE

DÉCOUVERTE ET CONSTITUÉE

PAR

CHARLES FOURIER.

Industrie, Politique, Sciences, Art et Littérature.

TOME I.

IDÉE D'UN PHALANSTÈRE.

Habitation d'une PHALANGE de 400 à 500 familles associées en fonctions de *Agriculture, ménage, fabriques, éducation, art, sciences, etc.;*

Remplaçant, dans l'ORDRE SOCIÉTAIRE, les 400 à 500 constructions incohérentes, *maisons, masures, granges, étables, etc.,*

d'une bourgade de 1800 à 2000 habitants dans l'ORDRE MORCELÉ actuel.

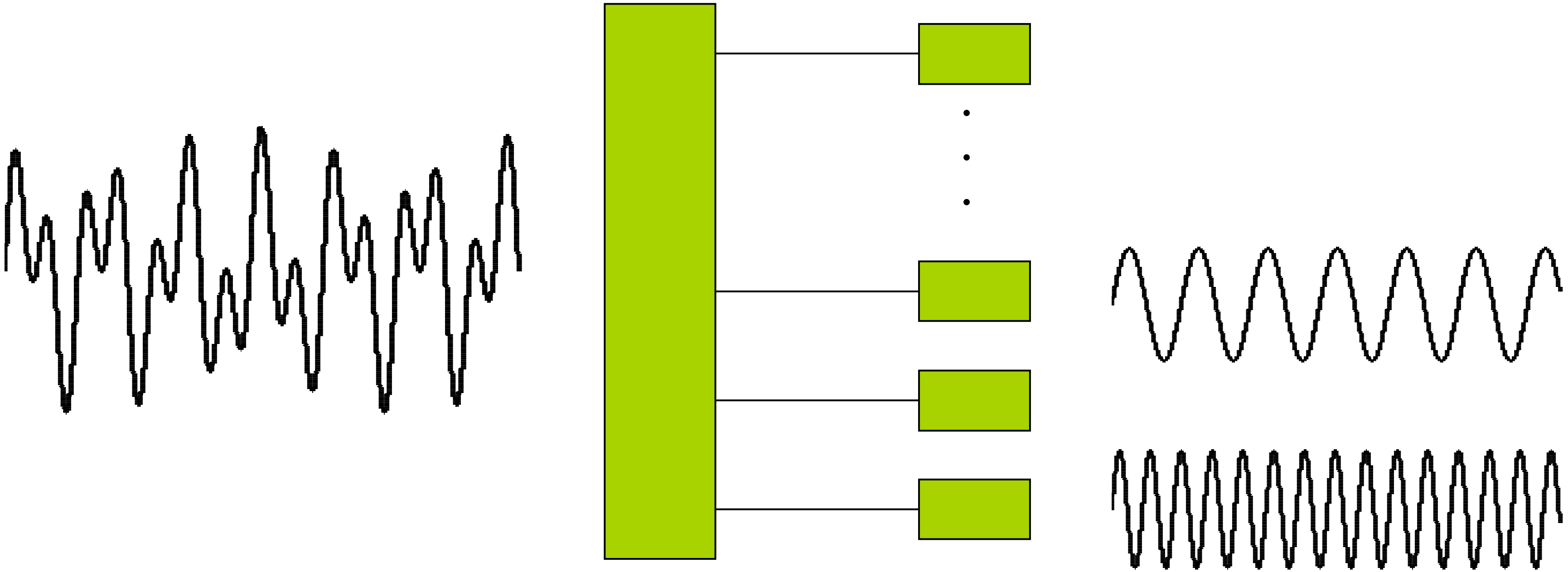
PARIS

AU BUREAU DE LA PHALANGE, RUE JACOB, N° 54.

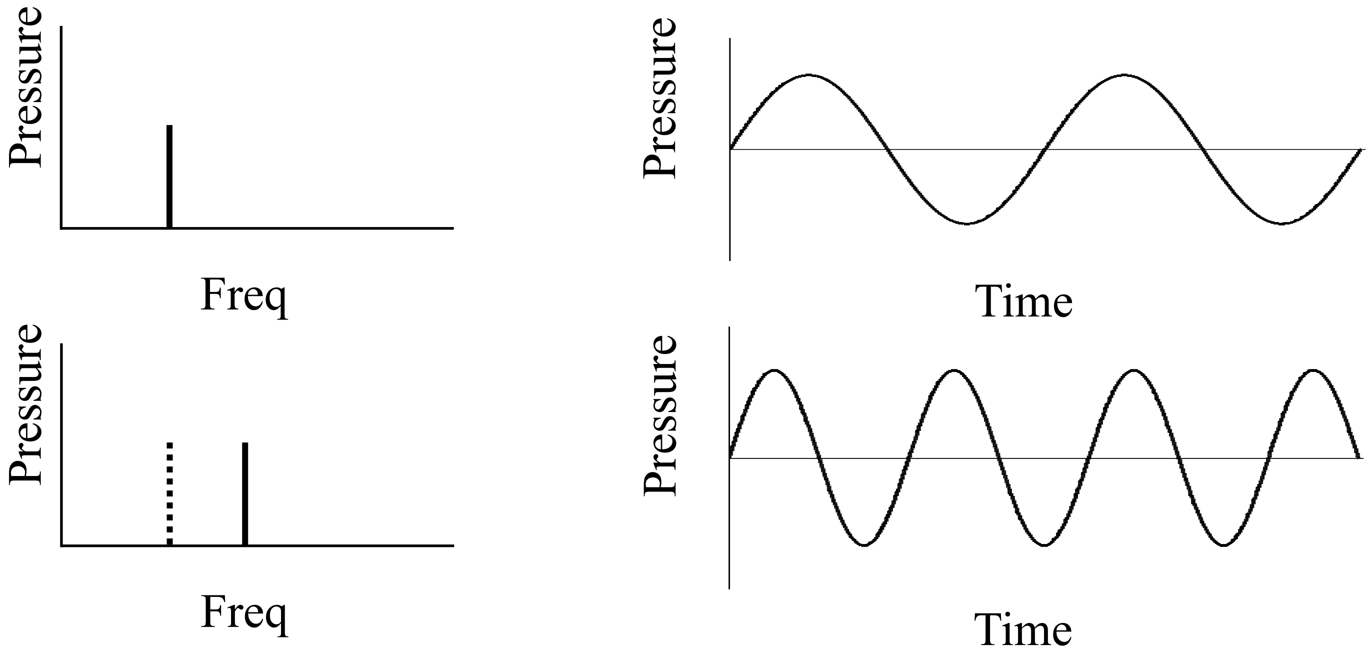
1836-1837

Signal Analysis

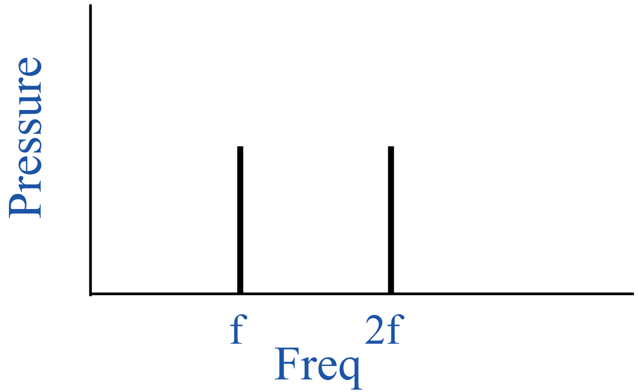
A preliminary analysis of a sound mixture into individual frequency components

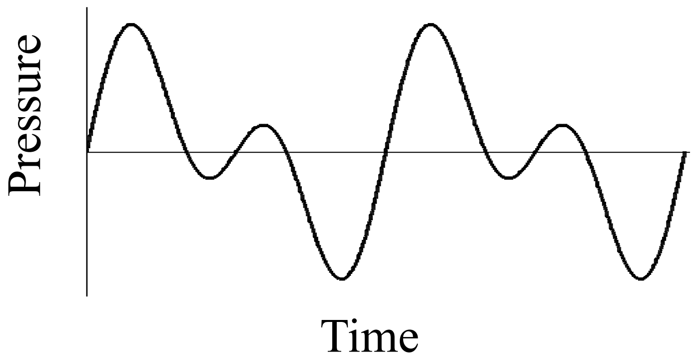


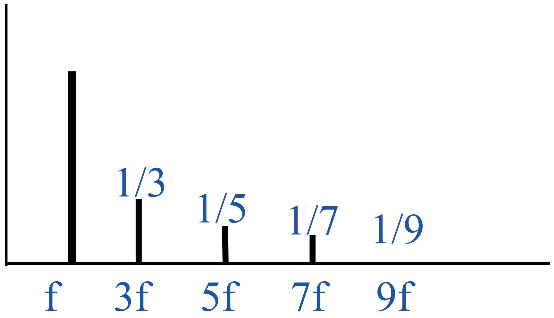
Fourier Transform

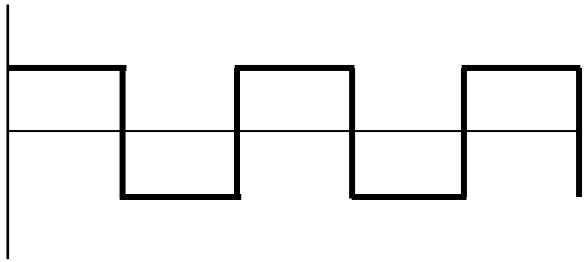


Fourier Transform

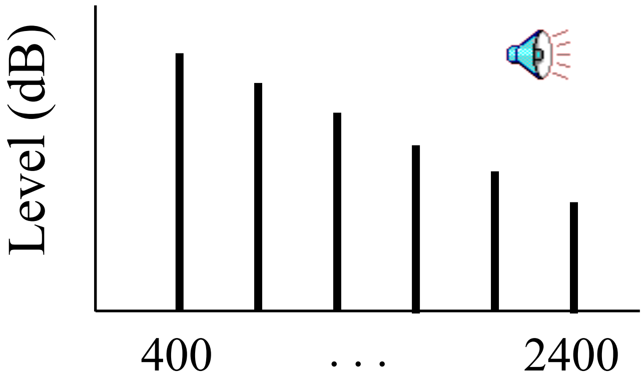


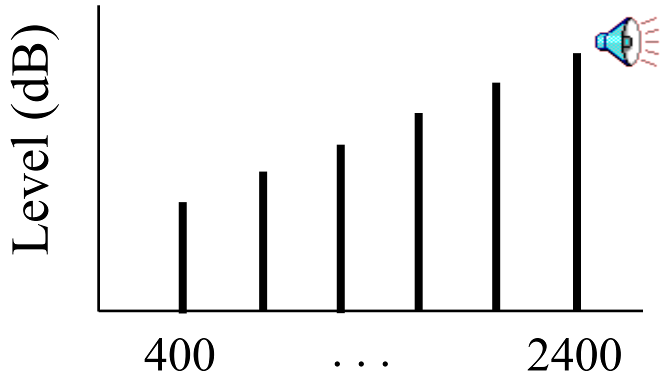




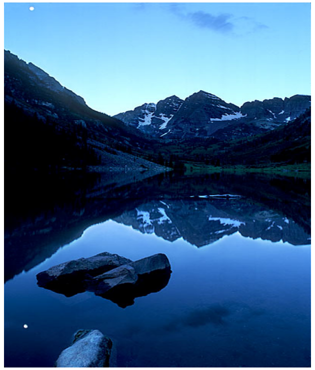


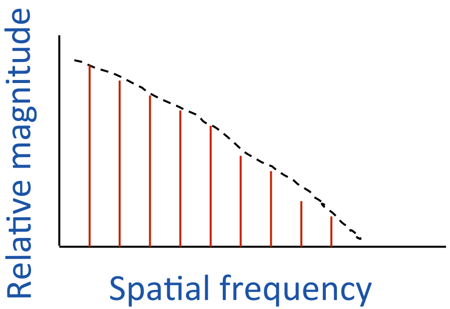
Signal Characterization





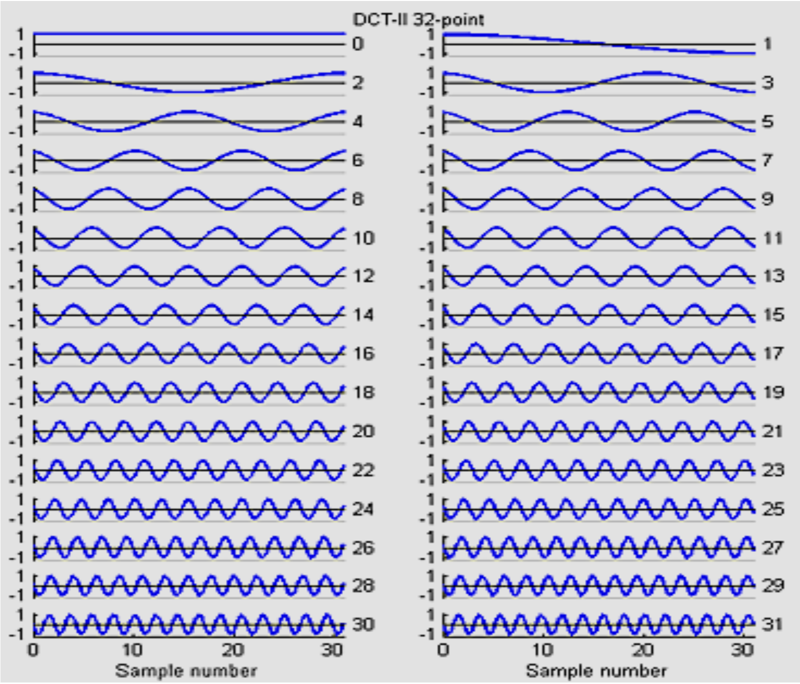
Natural Signals





KTH VETENSKAP OCH KONST
ROYAL INSTITUTE OF TECHNOLOGY

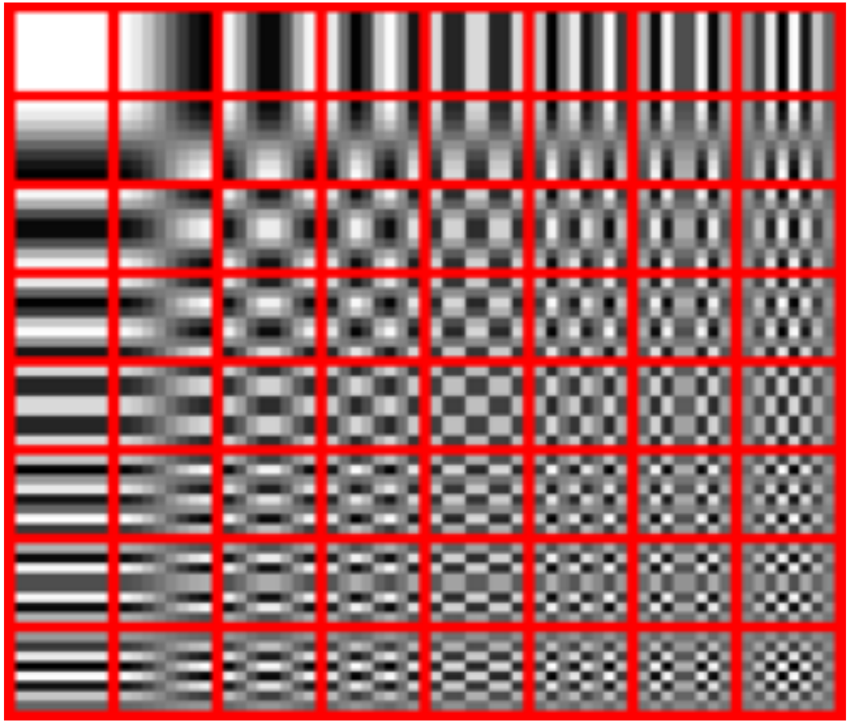
Fourier Transform of a Picture



DCT = discrete Cosine Transform (real functions)

ROYAL INSTITUTE OF TECHNOLOGY

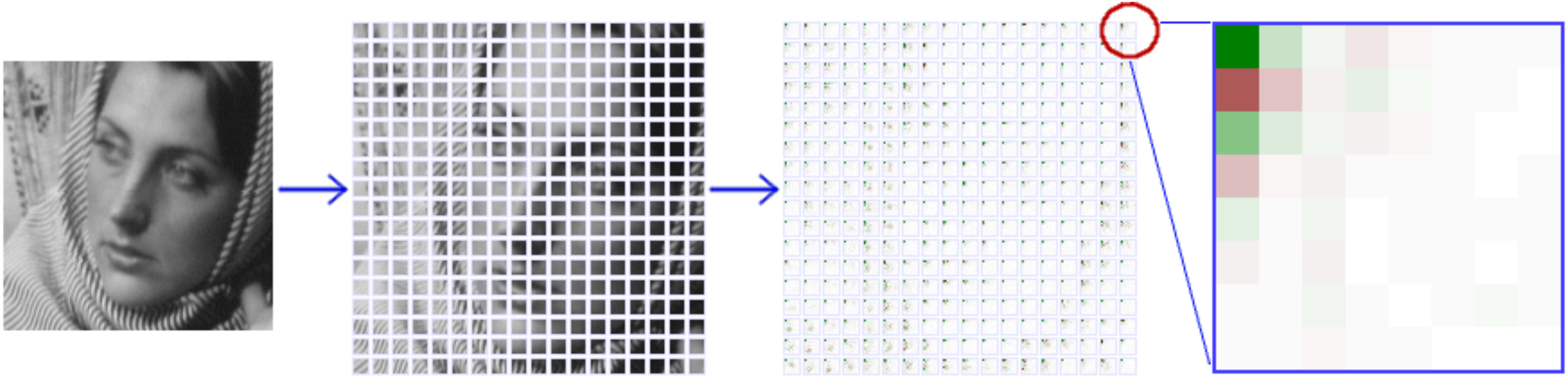
Image Compression: DCT



$$\alpha_p \alpha_q \cos\frac{\pi(2m+1)p}{2M} \cos\frac{\pi(2n+1)q}{2N}, \qquad \begin{array}{l} 0 \le p \le M-1 \\ 0 \le q \le N-1 \end{array}$$

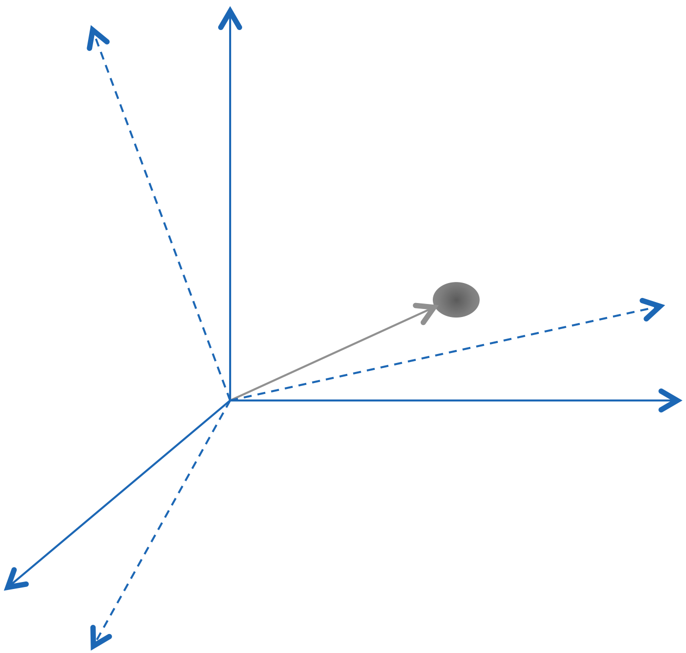
ROYAL INSTITUTE OF TECHNOLOGY

JPEG Image Compression



Original image — *Pixel blocks* — *DCT coefficient blocks* — *Single coefficient block*

Signal Representation



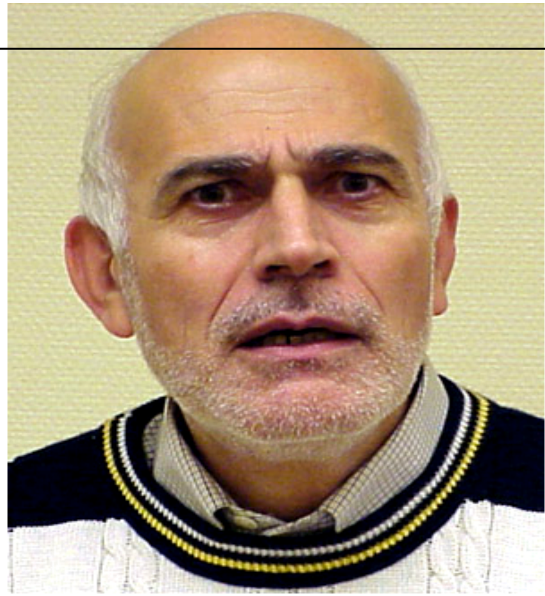
$x = a_0\mathbf{e}_0 + a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + \ldots$

Two questions

- **how to select basis $\mathbf{e}_i$**
- **how to calculate a_i**

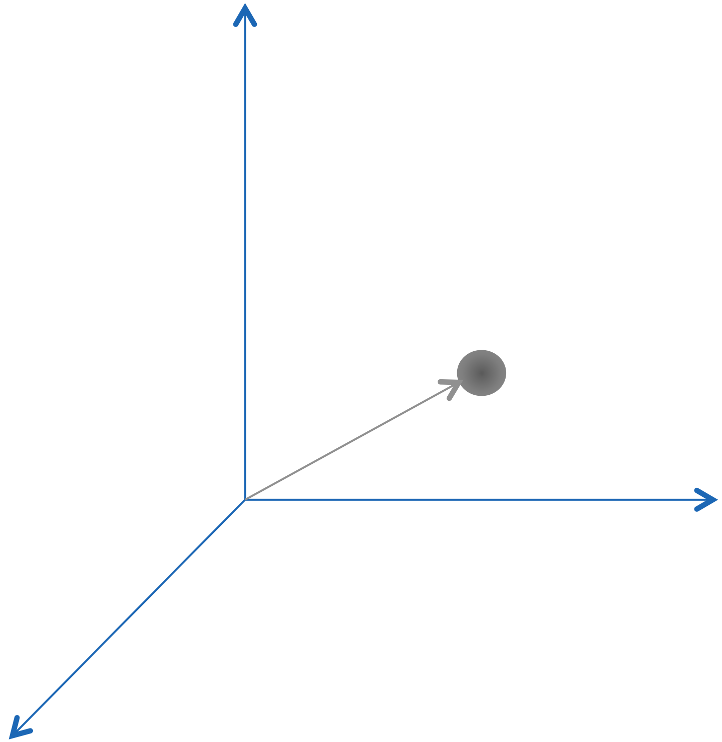
Vector Representation

123 112 122 124 ....
120 119 122 122 ....
112 130 140 112 ....

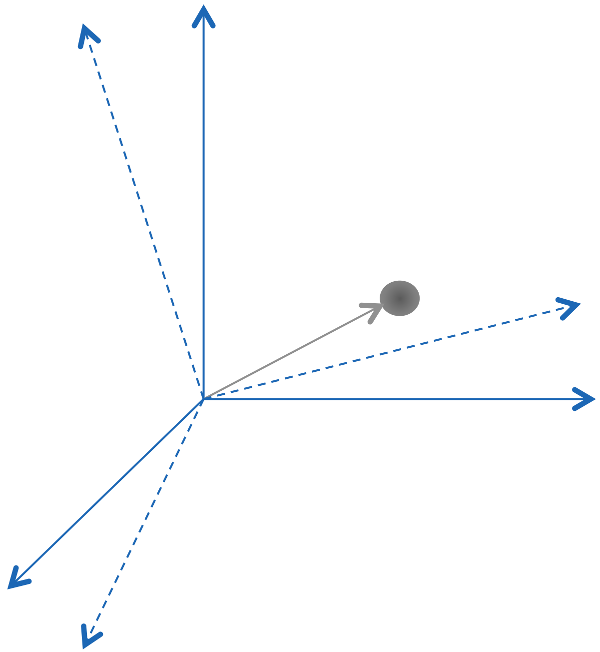


Media Signals

x = 

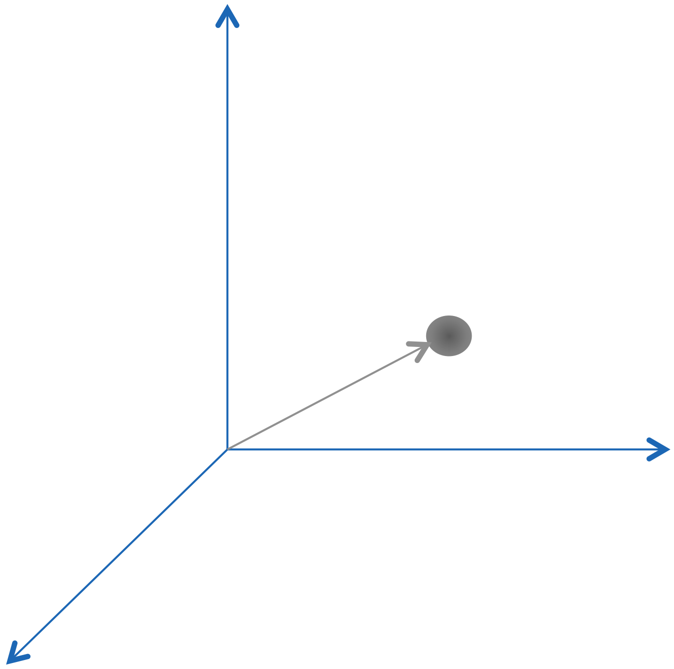


Signal Representation



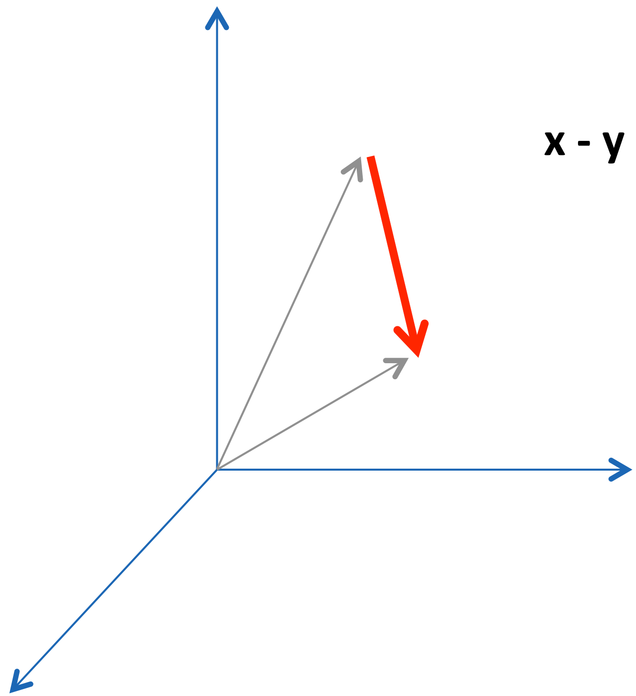
Vector Operations

Signal Representation



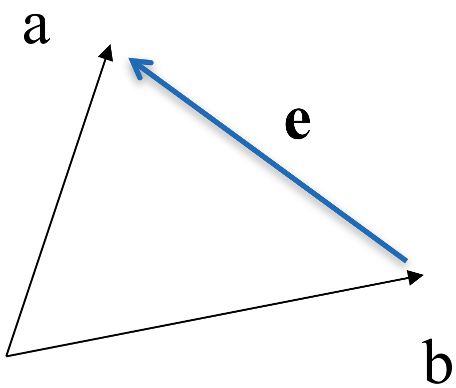
magnitude= norm

Signal Representation



distance

Vector Distance

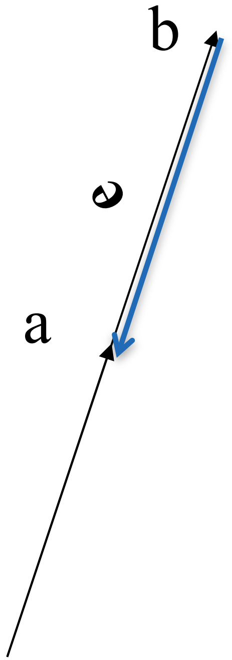


a

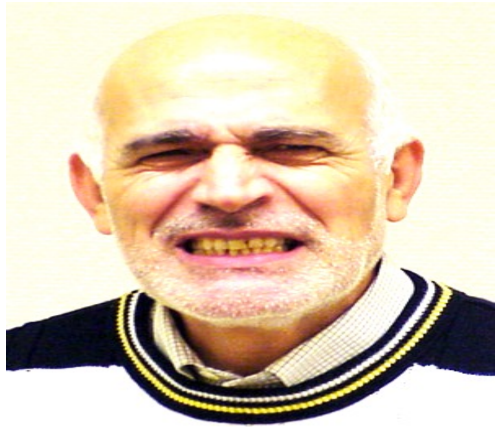
b

$$\mathbf{e} = \mathbf{a}-\mathbf{b}$$

Vector Distance



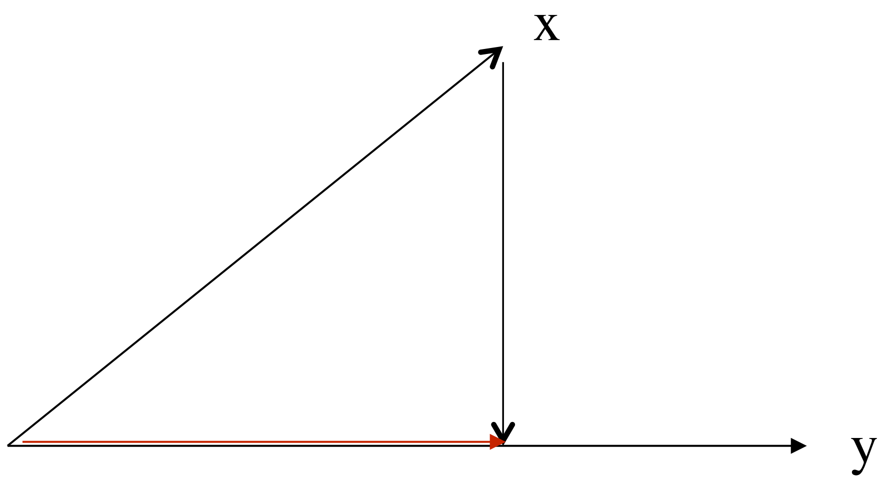
a



b

$$\mathbf{e} = \mathbf{a} - \mathbf{b}$$

Signal Representation



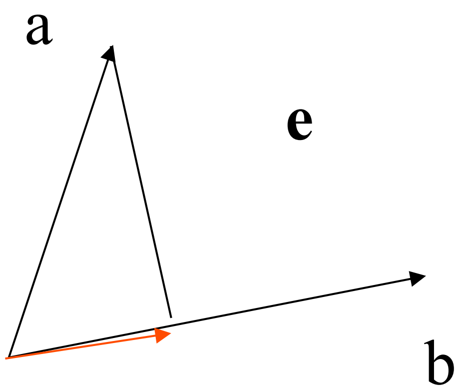
projection = xy^T

Vector Approximation

$$e = \mathbf{a} - \tilde{\mathbf{a}}$$

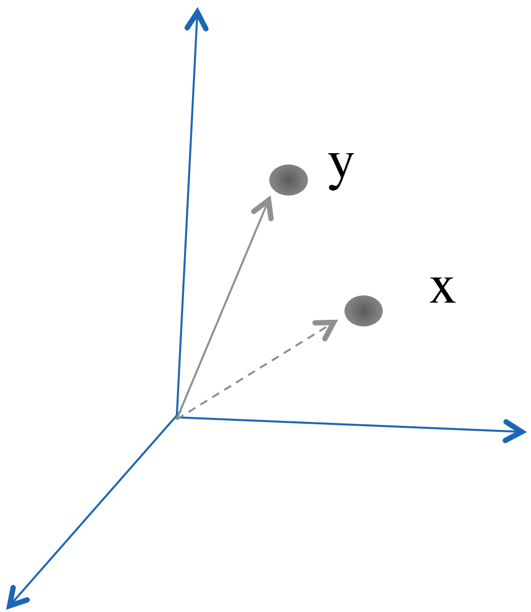
a

b



$$\tilde{\mathbf{a}} = k\mathbf{b}$$

Signal Representation

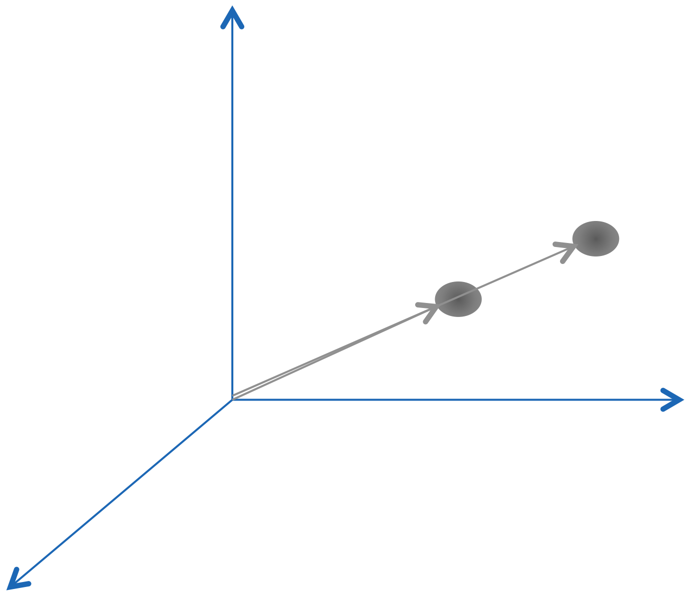


Linear Transform

y = Ax

ROYAL INSTITUTE OF TECHNOLOGY

Signal Representation

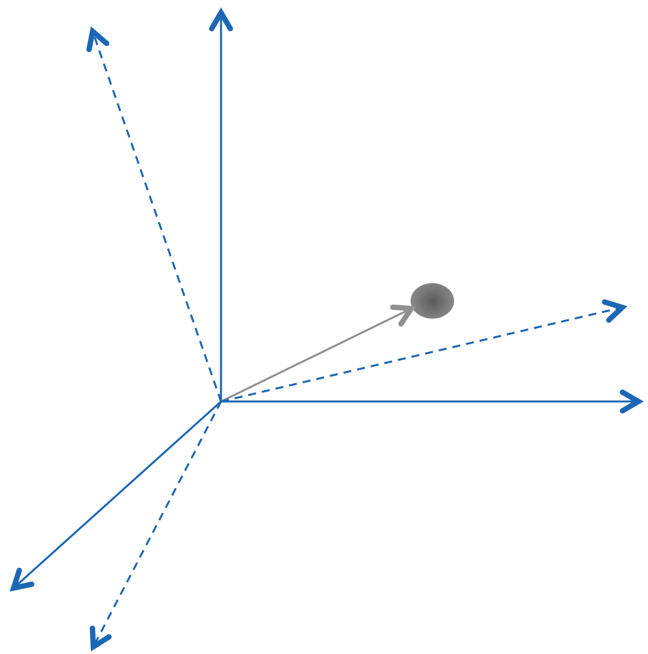


Linear Transform

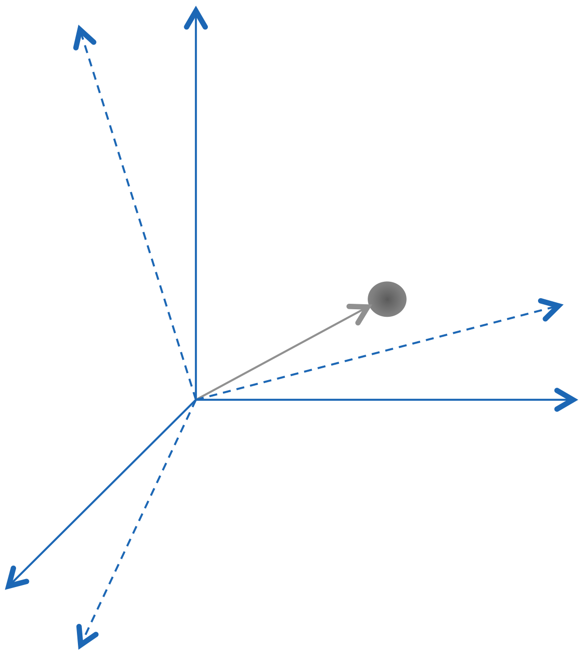
$$\mathbf{y} = \mathbf{A}\mathbf{x} = \lambda\mathbf{x}$$

Eigen value
and
Eigen vector

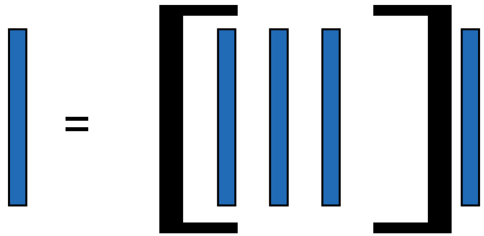
Signal Representation



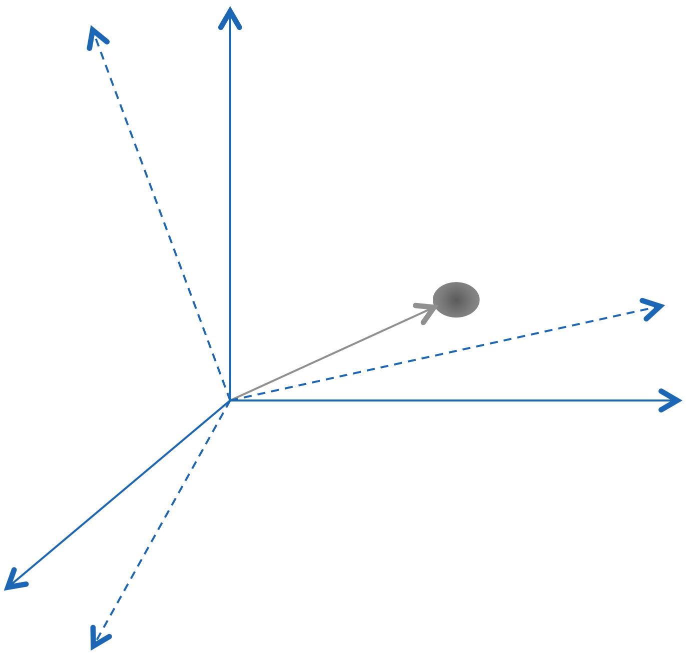
Signal Representation



$$x = a_0\mathbf{e}_0 + a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + \ldots$$



Signal Representation



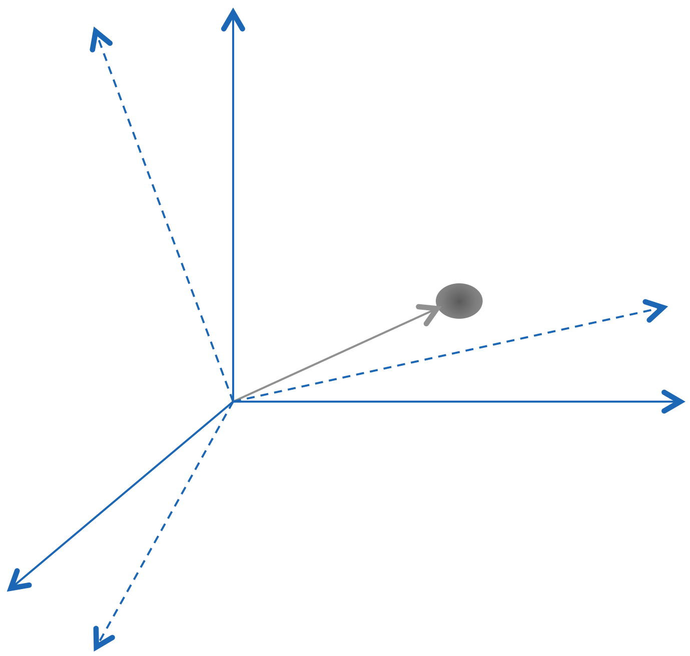
$x = a_0\mathbf{e}_0 + a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + \ldots$

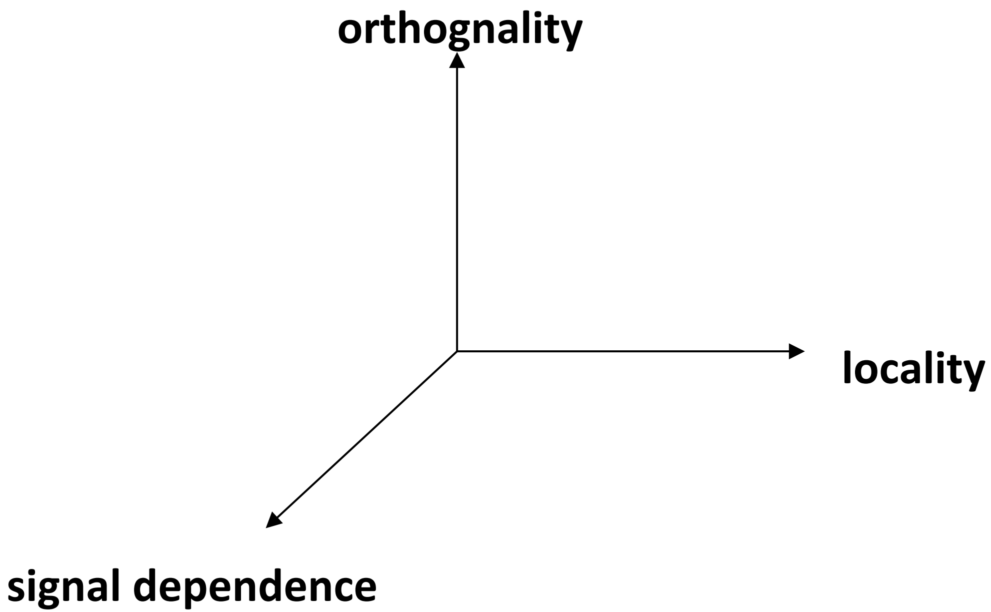
Two questions

- **how to select basis $\mathbf{e}_i$**
- **how to calculate a_i**

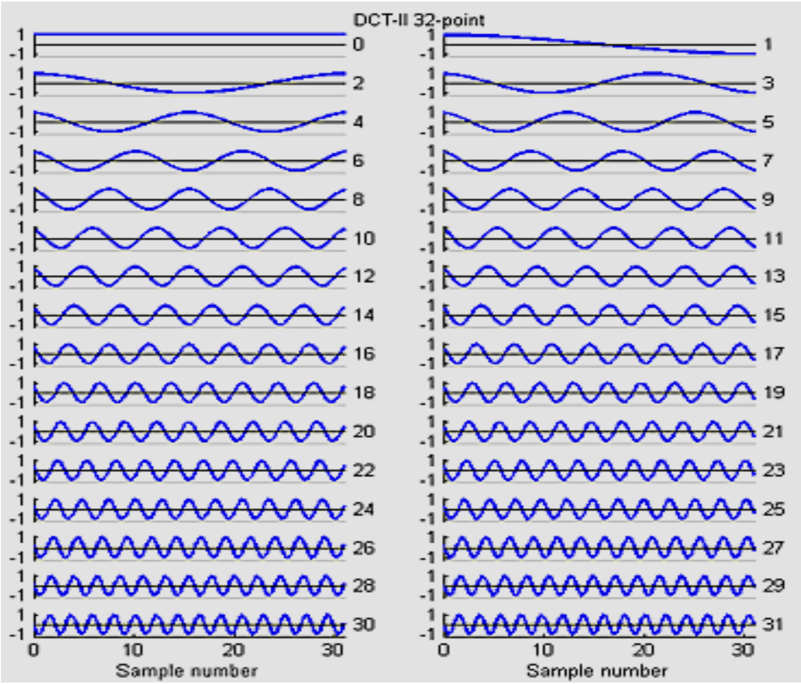
Basis functions

Attributes of basis functions





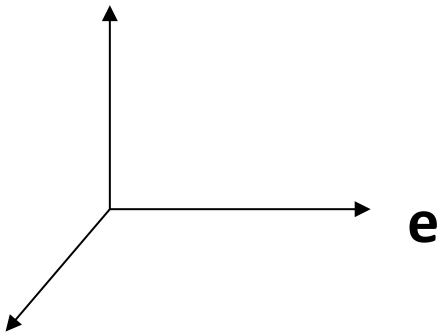
Fourier Transform

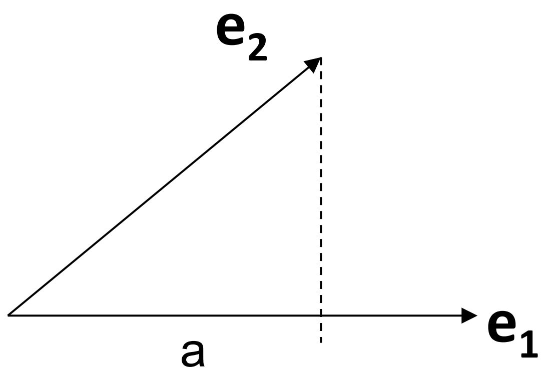


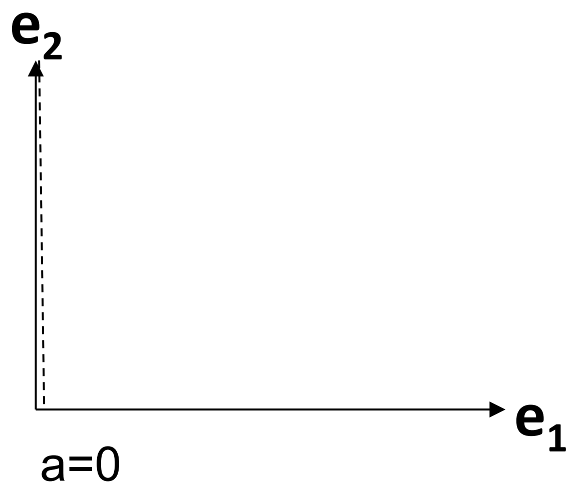
ROYAL INSTITUTE OF TECHNOLOGY

Orthogonality

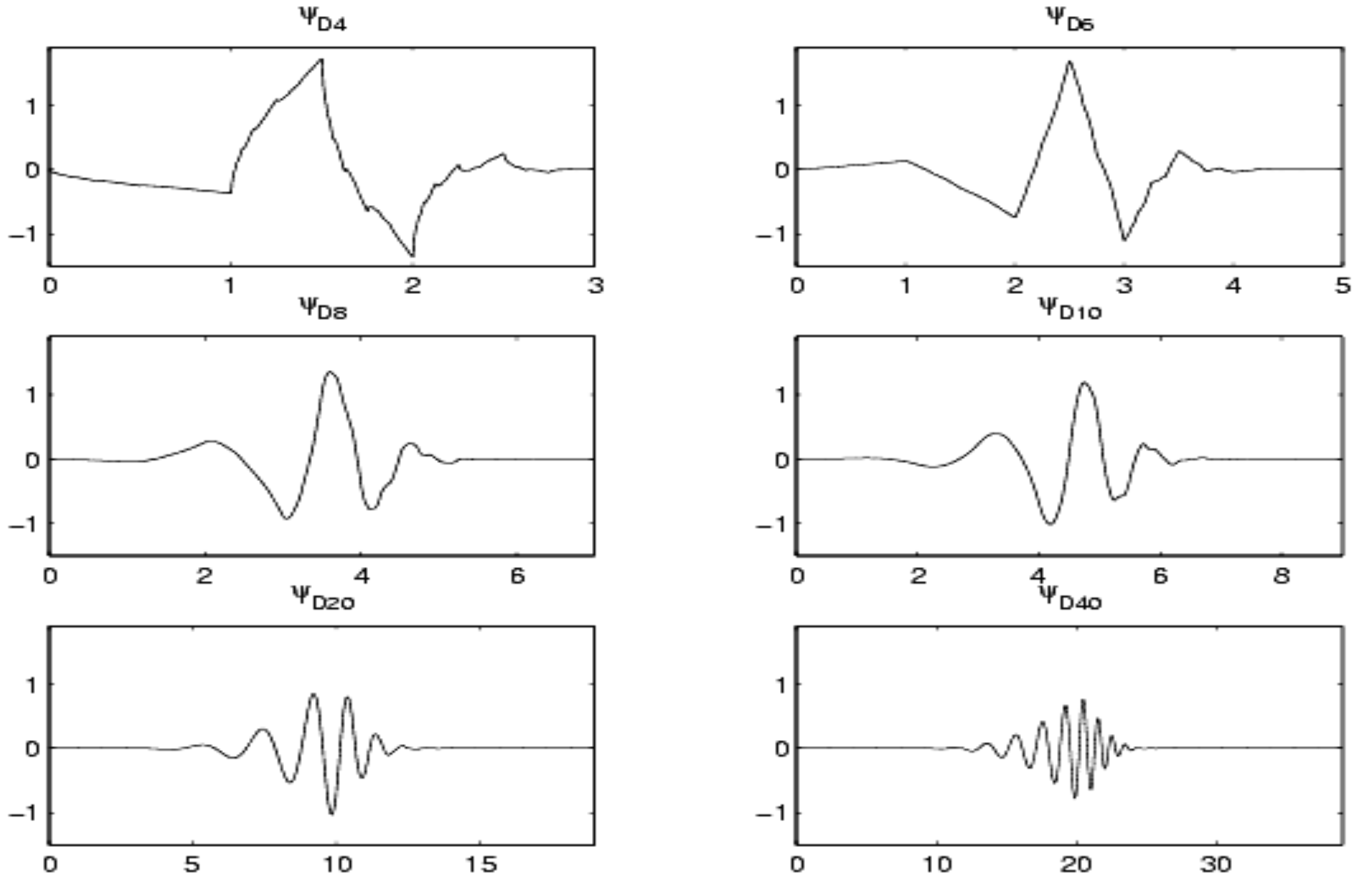
$x = a_0\mathbf{e}_0 + a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + \ldots.$





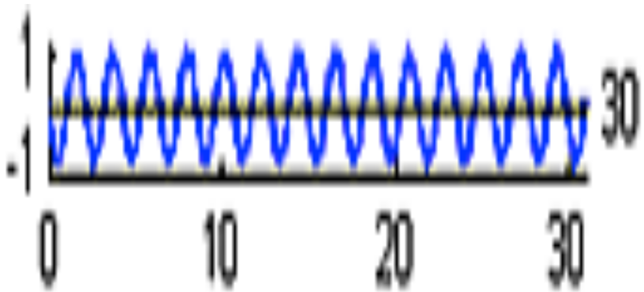


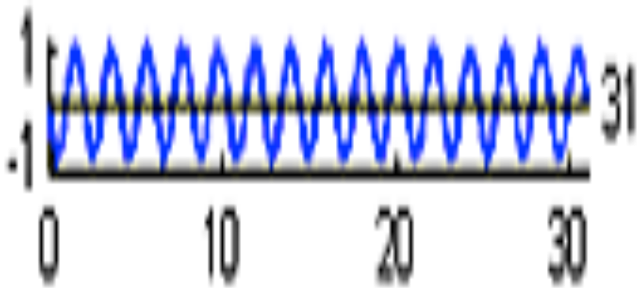
Wavelet



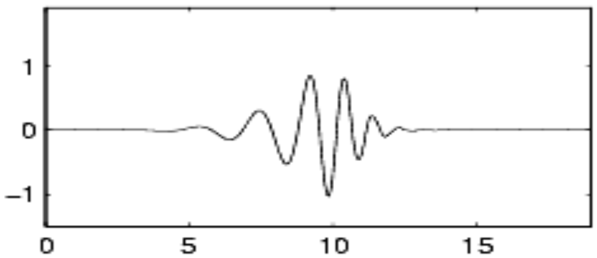
Locality

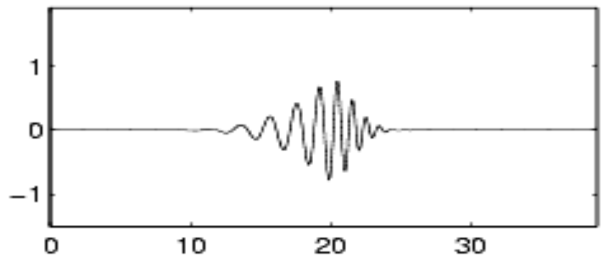
Global support



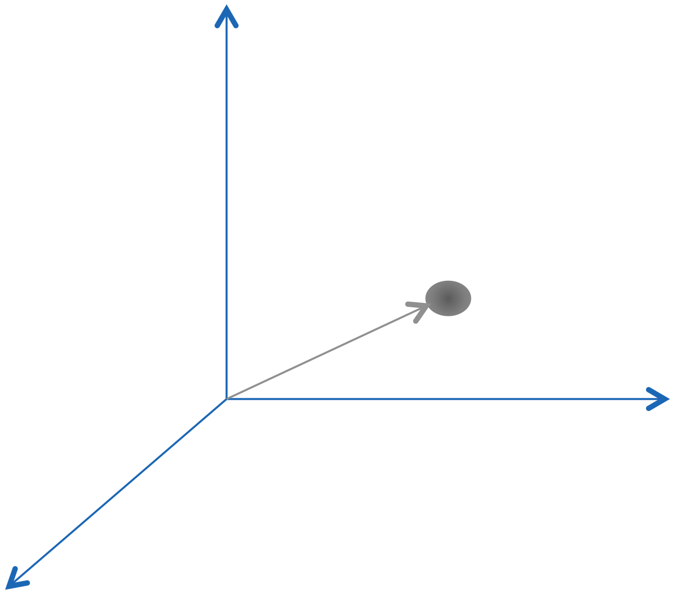


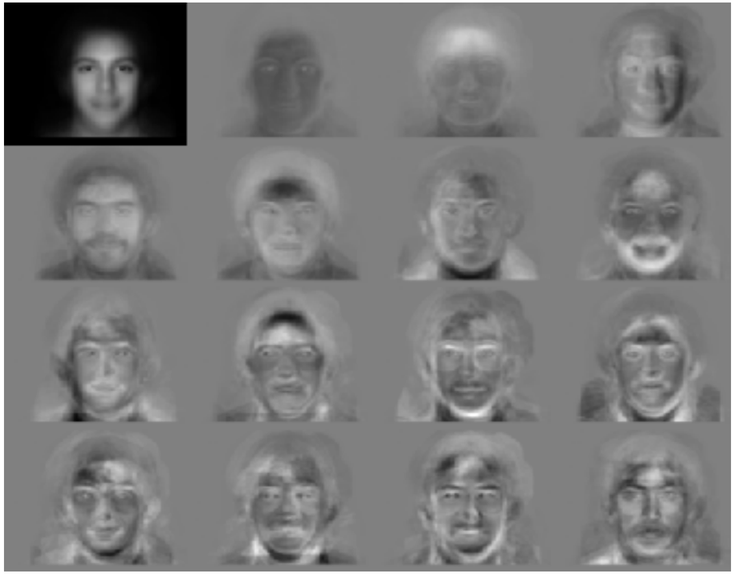
Local support



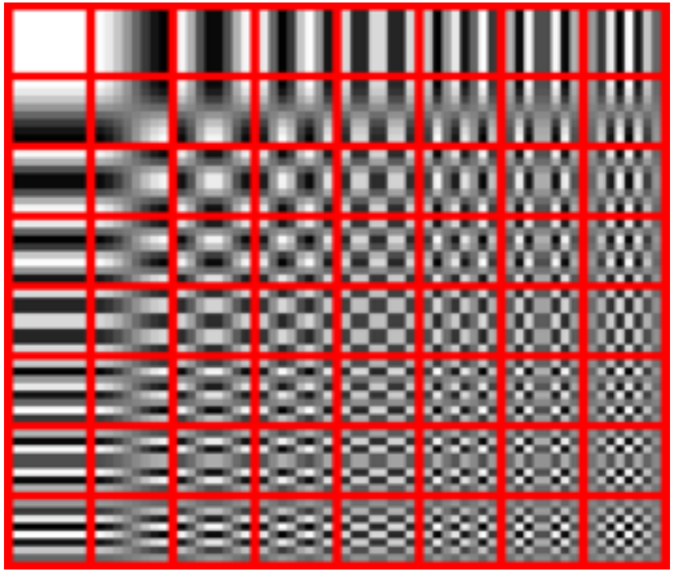


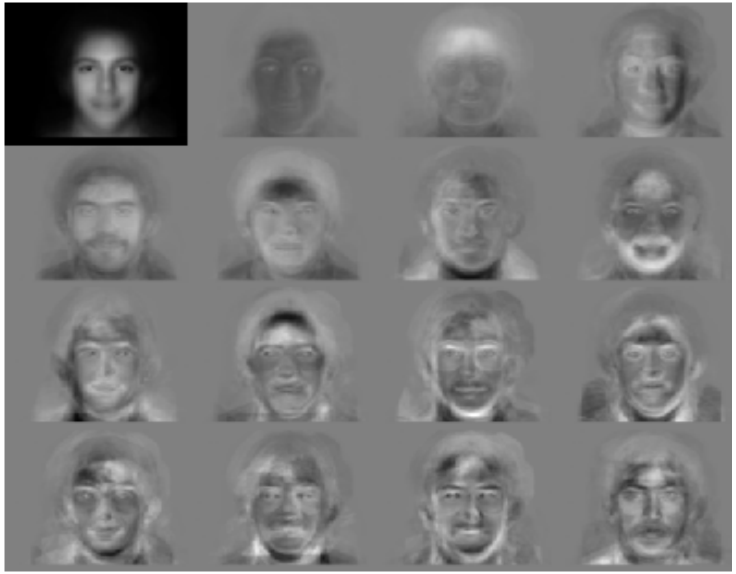
Eigenfaces





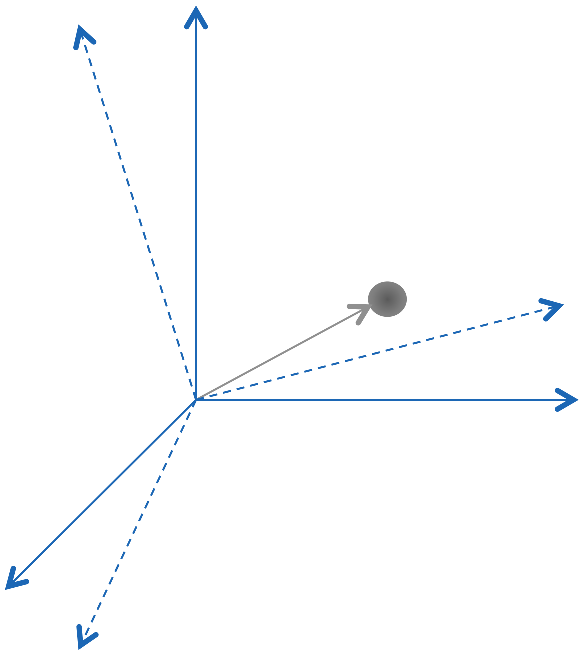
Signal-dependency





Computing weights

Signal Representation



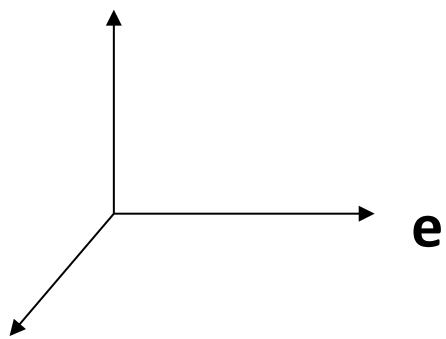
$$x = a_0\mathbf{e}_0 + a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + \ldots$$

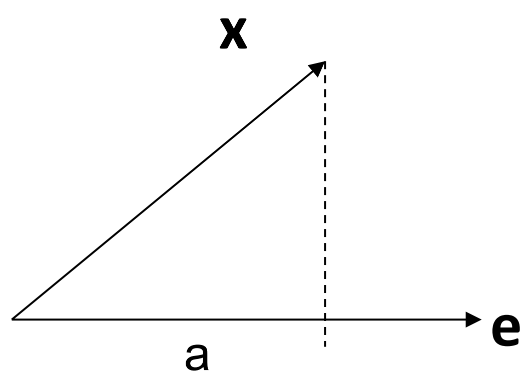
Two questions

- **how to select basis $\mathbf{e}_i$**
- **how to calculate a_i**

Orthogonal case

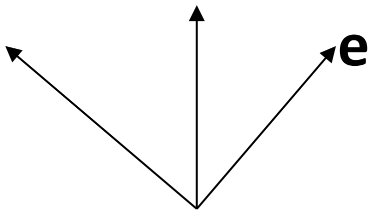
$x = a_0\mathbf{e}_0 + a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + \ldots.$





Nonorthogonal case

$x = a_0\mathbf{e}_0 + a_1\mathbf{e}_1 + a_2\mathbf{e}_2 + \ldots$

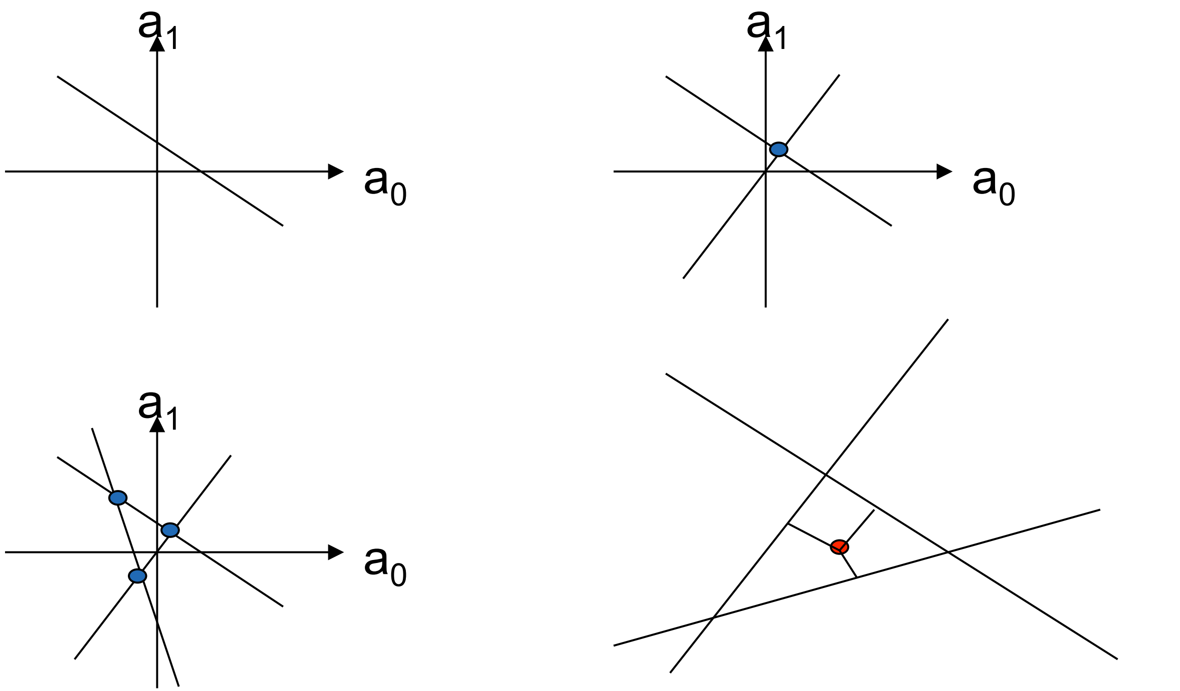


Least-squred (LS) solution!

ROYAL INSTITUTE OF TECHNOLOGY

Least-Squared Solution

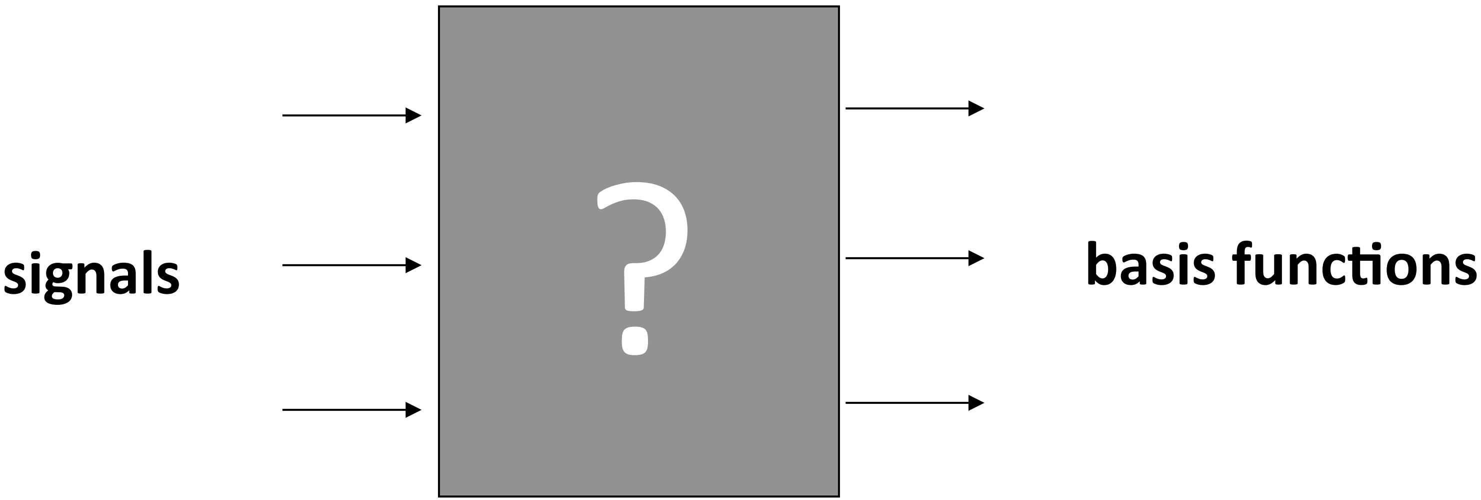
$x = a_0\mathbf{e}_0 + a_1\mathbf{e}_1$



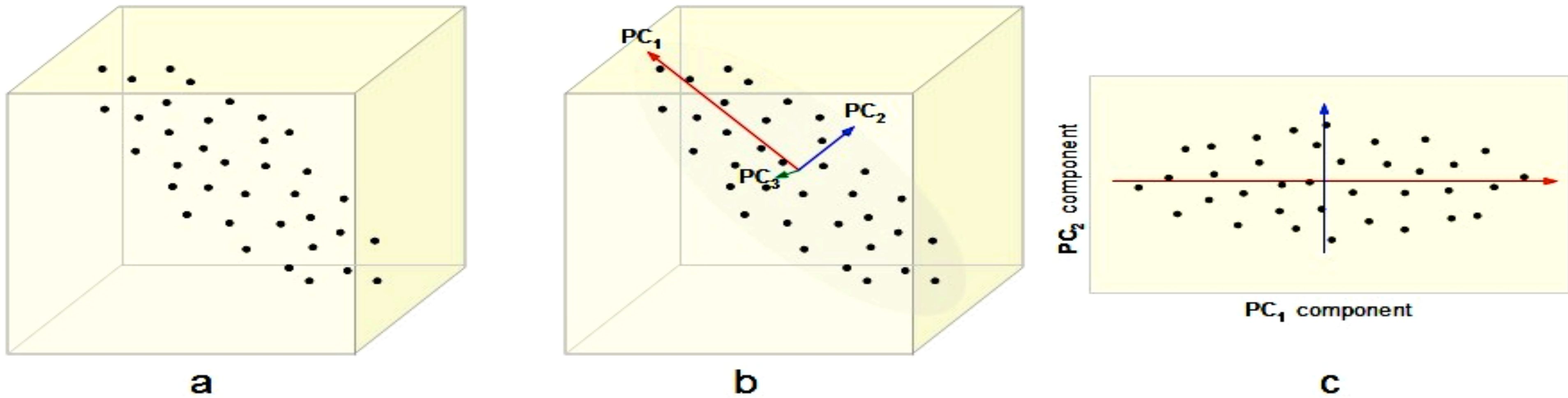
Least-squred (LS) solution!

Computing basis functions

ROYAL INSTITUTE
OF TECHNOLOGY

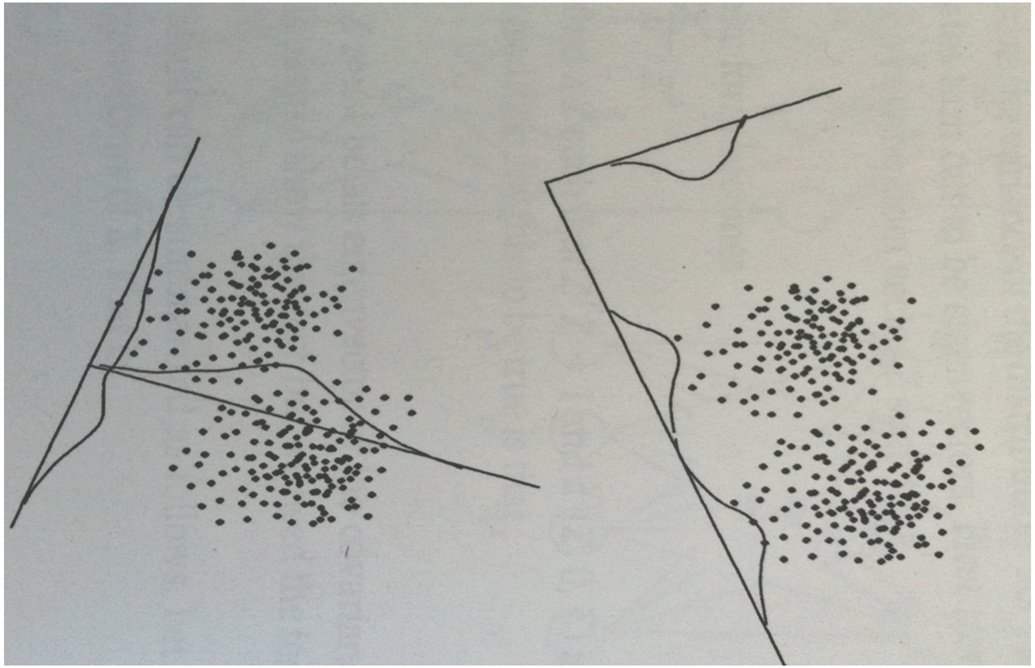


Basis functions from variances



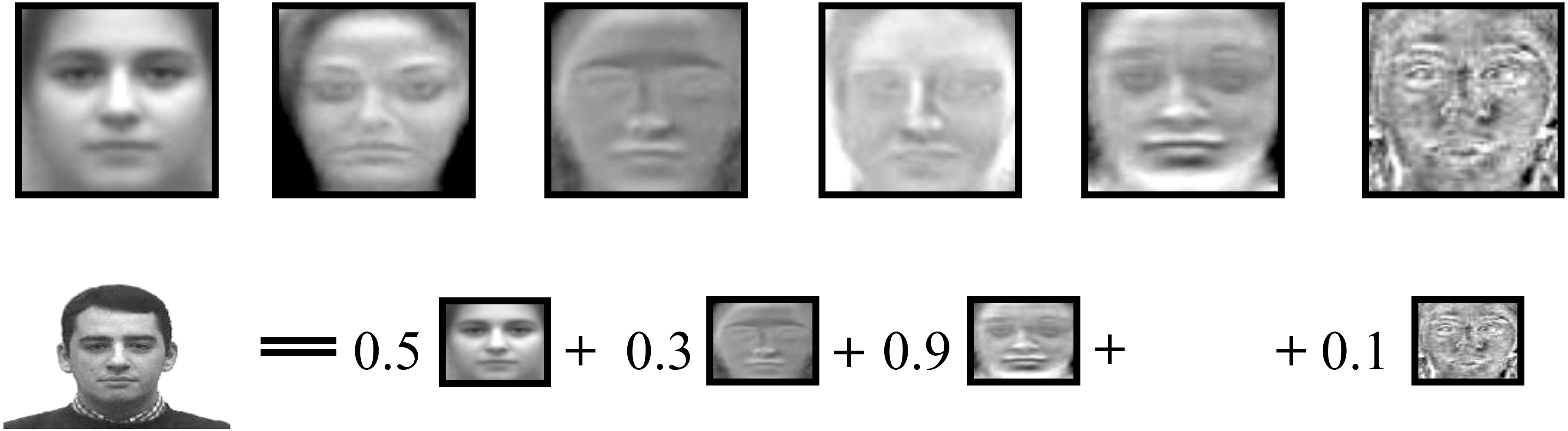
Principal Component Analysis (PCA)

Coordinate System



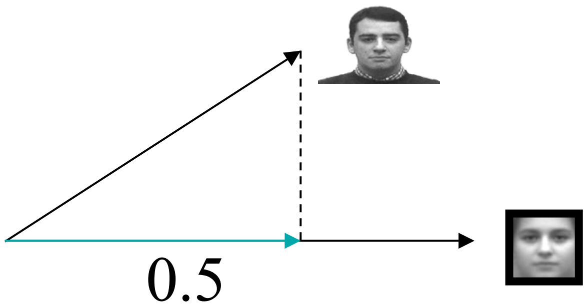
KTH VETENSKAP OCH KONST
ROYAL INSTITUTE OF TECHNOLOGY

Basic faces



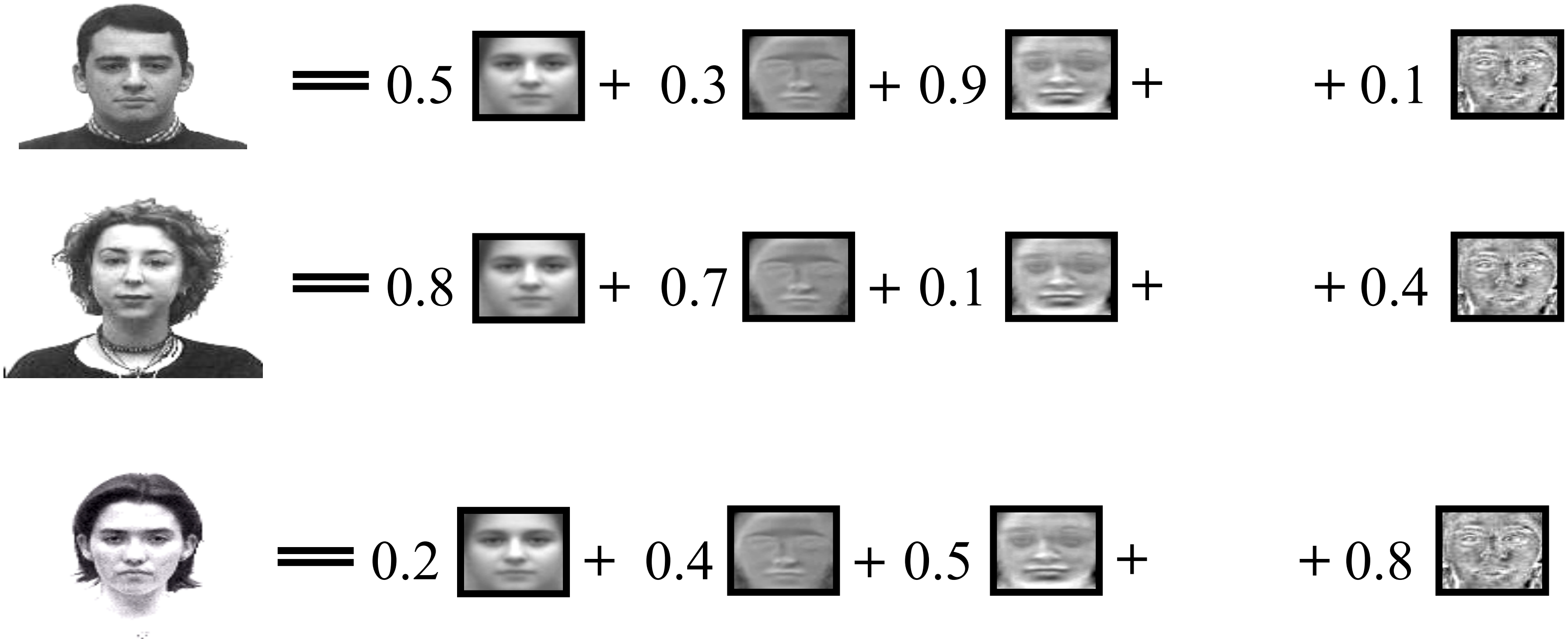
Surprisingly, the number of basic faces is only around 100!

Face Representation

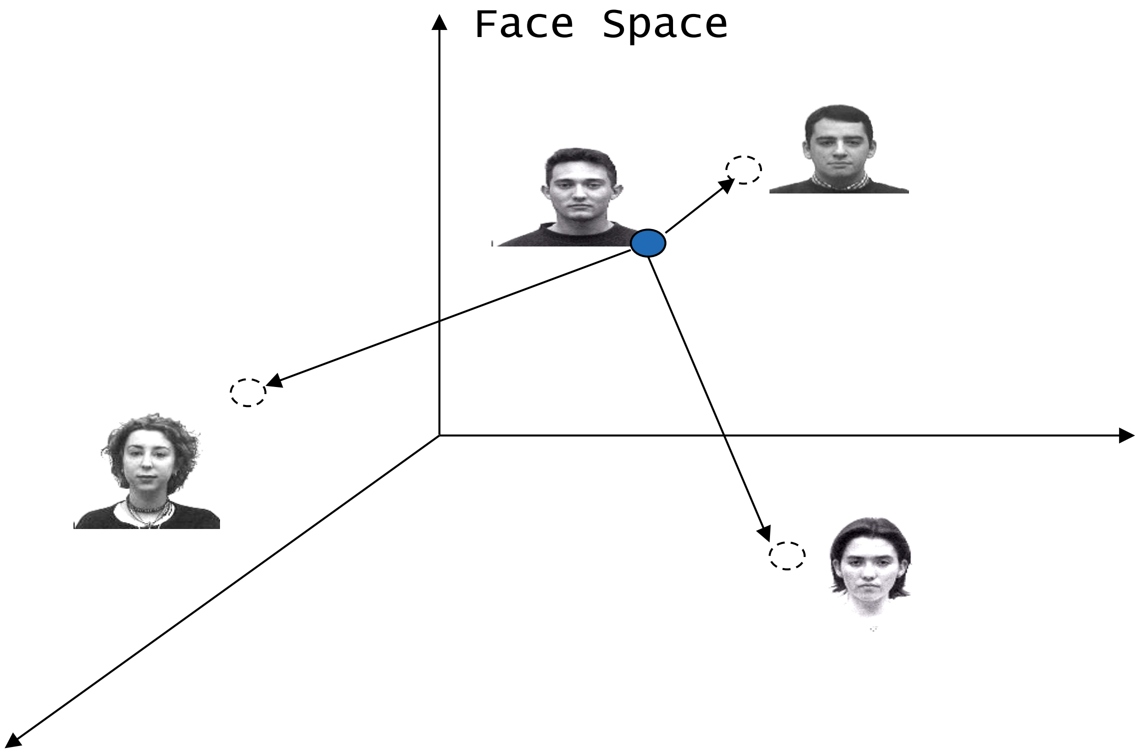


KTH VETENSKAP OCH KONST
ROYAL INSTITUTE OF TECHNOLOGY

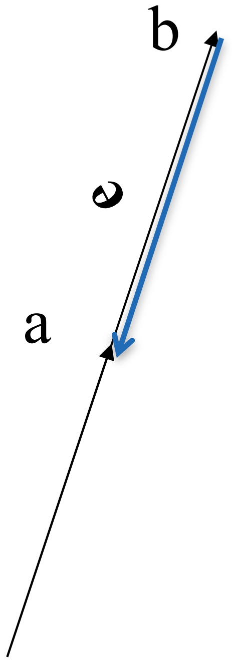
Face codes



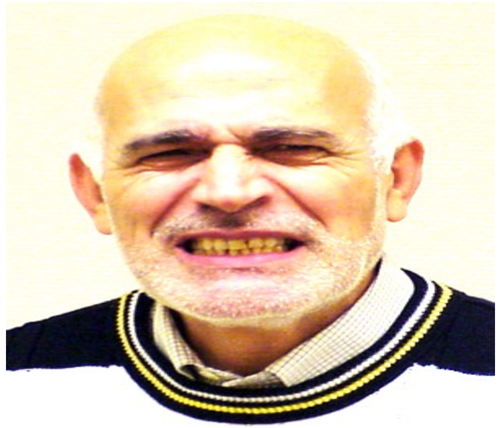
Face recognition = Classification



Vector Distance



a

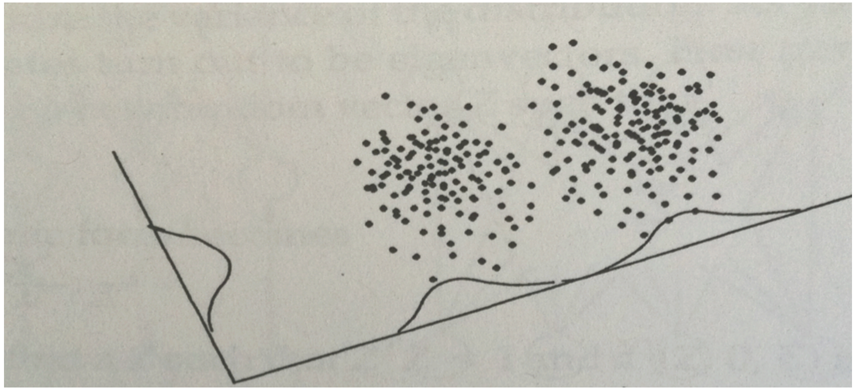


b

Normalization: $\mathbf{a} = x\mathbf{b} + y$ Least squared solution

Random Vectors

Consider vectors are drawn from some random distribution that captures the natural variations in the world. A random vector X is specified by a probability density function p(X)



- Lab 1

Group 7, Group 9

- Lab2

Group 1, Group 3, Group 9, Group 10, Group 17