Advanced Programming Languages for AI H02A8

Gerda Janssens
Departement computerwetenschappen
200A.01.26
http://people.cs.kuleuven.be/~gerda

Overview Lecture 1

- About APLAI
- Intro: Constraint (Logic) Programming
  1. with ECLiPSe
  2. Prolog
  3. Constraint programming terminology
  4. Running examples

Starting Point

- Knowledge of Prolog
- Background in AI
  - constraint propagation
  - search
  - condition-action rules

Aim

Study more programming languages and tools useful in the AI context

Selection for 12-13

Constraint (Logic) Programming

ECLiPSe (ILOG, OPL, Cosytec)

Rule Based Systems

Constraint Handling Rules (CHR)

Jess (rule engine, Java, Business rules)

(Local search)

Format 12-13

- Lectures (2 studypoints)
  - Different systems/languages
  - Different approaches
  - APLAI slides + wiki on Toledo
  - Additional material: Howto use it?? http://4c.ucc.ie/~hsimonis/ELearning/index.htm
- Assignment (2 studypoints)
  - Groups of 2
  - Minimal requirements + optional part

12-13 Assignment (separate slides/text)

- For the problems at hand,
- Given the different approaches studied in APLAI,
- Explore !!!

- Minimal requirements: some tasks
- Optional: an additional task

Constraint (Logic) Programming

with ECLiPSe

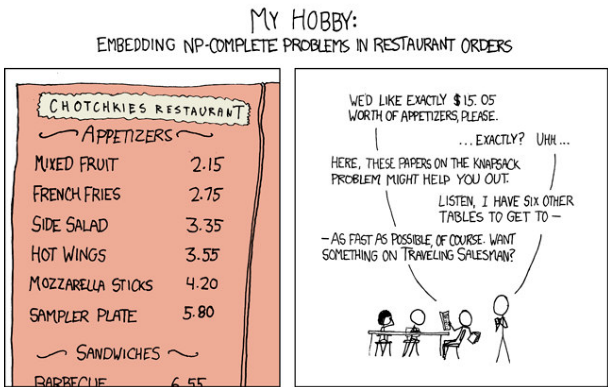
http://eclipseclp.org

constraints embedded in Prolog

Topics

- What constraint satisfaction problems and constrained optimisation problems are and how they are solved using constraint programming techniques.
- How to generate these problems in ECLiPSe
- What support ECLiPSe provides for general and domain specific methods
- How to solve constraint satisfaction and constrained optimisation problems in ECLiPSe

xkcd problem



Starting Point

- Prolog issues
  - Unification, backtracking search, arithmetic
  - Passive / active constraints
- Constraint issues
  - Constraint propagation
  - Search: backtracking search and branch and bound search

1. ECLIPSE

ECLiPSe history

- ECRC (1984) development of advanced reasoning techniques for practical problems
- CHIP (1988) incorporated constraint satisfaction into LP by using finite domain variables and relying on top-down search techniques
- ECLiPSe (1991) = CHIP + (database and parallel programming facilities)

ECLiPSe history

- 1997 interface to an external linear and mixed integer programming package
- 1999 commercial rights to IC-Parc (London)
- 2004 bought by Cisco Systems
- Still freely available for teaching and research (production planning, transportation, scheduling, bioinformatics, optimisation of contracts …);
  commercial optimisation SW by Cisco

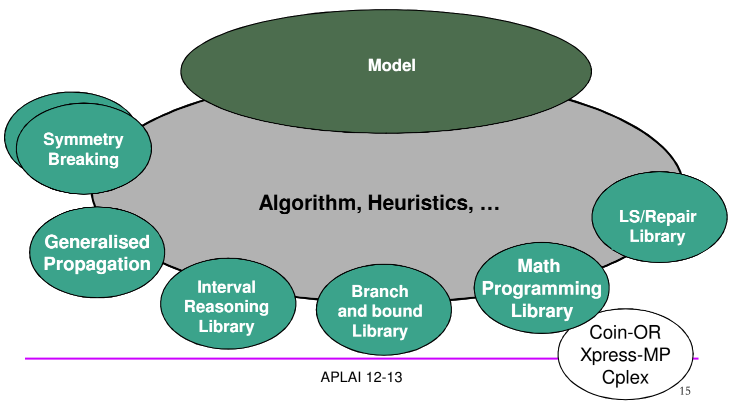
Motivation

ECLiPSe attempts to support - in some form or other - the most common techniques used in solving Constraint (Optimization) Problems:

- CP – Constraint Programming
- MP – Mathematical Programming
- LS – Local Search
- and combinations of those

ECLiPSe is built around the CLP (Constraint Logic Programming) paradigm

ECLiPSe for Modelling and Solving



ECLiPSe Usage: www.eclipseclp.org

Applications

- Developing problem solvers
- Embedding and delivery

Research

- Teaching
- Prototyping solution techniques

Mozilla Public License

- can be freely used for any purpose

CLP Books

- Constraint Logic Programming using ECLiPSe, Krzysztof Apt and Mark Wallace. Cambridge University Press, 2007. ISBN-13 978-0-521-86628-6
  Plaatsingsnummer: **6 681.3*D32 ECLI/2007**
  A practical introduction to constraint programming and to ECLiPSe
- Principles of Constraint Programming, Krzysztof R. Apt, Cambridge University Press, 2003. ISBN 0 521 82583 0

Online material

- Explore the website: examples, mauals (tutorial, reference, libraries)
- http://eclipseclp.org/reports/index.html
  mentions books, introductory material, applications

CLP Books

- Programming with Constraints: an Introduction, Kim Marriott and Peter J. Stuckey, MIT Press, 1998.
- The OPL Optimization Programming Language, Pascal Van Hentenryck, MIT Press, 1999. An industrial implementation of OPL is available from the international software company ILOG.
  http//www.ilog.fr/products/optimization

Books

- Constraint-Based Local Search, Pascal Van Hentenryck and Laurent Michel, MIT Press, 2005. ISBN-10:0-262-22077-6
  combinatorial optimization problems; combines constraint programming and local search; a programming language, COMET, that supports both modeling and search abstractions in the spirit of constraint programming.

2. PROLOG

Some Prolog links

- FAQ van Prolog:
  http://www.logic.at/prolog/faq/faq.html
- Online tutorials:
  Bartak , Fisher, and P. Blackburn et al.
- Books on Prolog
- The World Wide Web Virtual Library: Logic Programming

Back to Prolog: unification

```
[eclipse 2]: p(k(Z,f(X,b,Z)))=p(k(h(X),f(g(a),Y,Z))).

[eclipse 3]: p(k(Z,f(X,b,Z)))=p(k(h(X),f(g(Z),Y,Z))).
```

Use Martelli-Montanari algorithm

What with occur check??

What on failure of unification?

append.pl

```
% app(Xs, Ys, Zs) :- Zs is the result of
%     concatenating the lists Xs and Ys.

app([], Ys, Ys).
app([X | Xs], Ys, [X | Zs]) :- app(Xs, Ys, Zs).
```

Backtracking

```
[eclipse 1]: [append].
append.pl  compiled traceable 276 bytes in 0.00 seconds

Yes (0.00s cpu)
[eclipse 2]: append(Xs,Ys,[1,2,3]).

Xs = []
Ys = [1, 2, 3]
Yes (0.00s cpu, solution 1, maybe more) ? ;

Xs = [1]
Ys = [2, 3]
Yes (0.00s cpu, solution 2, maybe more) ? ;

Xs = [1, 2]
Ys = [3]
Yes (0.00s cpu, solution 3, maybe more) ? ;

Xs = [1, 2, 3]
Ys = []
Yes (0.00s cpu, solution 4)
[eclipse 3]:
```


Pure Prolog and declarative programming

- Declarative interpretation (what is being computed;meaning): a Prolog program is a set of formulas (first order logic; semantics)
- Procedural interpretation (how computation takes place; method): mgu, SLD resolution
- Declarative programming: Programs can be interpreted in a natural way as formulas in some logic. Then the results of the program computations follow logically from the program. Towards executable specifications.

Arithmetic in Prolog

- Infinitely many integer constants, also floats

  `0  -3  19     0.0 -1.344  3.333`
- Given a tree (node/3, empty): sum of its values

```
sum(empty,0).
sum(node(L,R,W), Total):-
                sum(L,TotalL),
                sum(R,TotalR),
                Total is TotalL + TotalR + W.
```

- `is/2` **the arithmetic evaluator**
  - **evaluates** the operand at the rhs
  - **unifies** the result with the operand at the lhs
  - has to be at the right place

=/2 and is/2

- `[eclipse 1]: X is 1 + 2 .`
- `[eclipse 2]: X = 1 + 2 .       % ?- X = pair(1,2).`
- `[eclipse 3]: X = 1 + 2, Y is X .`

```
+  -(binary) -(unary) * //  mod
The relation between integer division // and modulus
  operation mod is as follows:
      X =:= (X mod Y) + (X // Y) * Y
```

Arithmetic comparison predicates

- Less than <
- Less than or equal =<
- Equality =:=
- Disequality =\=
- Greater than or equal >=
- Greater than >

Ordered/2

```
% ordered(Xs) :- Xs is an =<-ordered list of numbers.
ordered([]).
ordered([H | Ts]) :- ordered(H, Ts).

% ordered(H, Ts) :- [H|Ts] is an =<-ordered list of numbers.
ordered(_, []).
ordered(H, [Y | Ts]) :- H =< Y, ordered(Y, Ts).


[eclipse 3]: ordered([1,X,6]).
instantiation fault in 1 =< X
Abort
[eclipse 4]: ordered([X]).          %remedy??
```


Arithmetic constraints: passive

- Compare f(a,X) = f(Y,b) with 3*X < Y +2
- Constraints: restrict the values of variables
- Using comparison predicates : arithmetic constraints
- Evaluation of a constraint
  - Can affect the variables in the constraint: active
  - Cannot affect them: passive
- =/2 w.r.t. </2 (only for testing!!!)

What if </2 were an active arithmetic constraint???

```
[eclipse 3]: ordered([2,X,2]).
X = 2
```

This behaviour is possible when using the ECLiPSe's ic library.

3. CONSTRAINT PROGRAMMING

Constraint Programming: a primer

- What constraints do you already know?
- A constraint is an atomic formula.
- We assume that each constraint has at least one variable.
- Each interpretation associates with a constraint c a subset of the Cartesian product of the domains of its variables.
- The constraint $X < Y$ over the set of integers
- with as interpretation $\{ (a,b) \mid a,b \in Z \text{ and } a < b\}$

Satisfiable and solved constraints

- An assignment of values to the constraint variables satisfies a constraint (is a solution to a constraint) if the used sequence of values belongs to its interpretation.
- A constraint is satisfiable if some assignment to its variables satisfies it and is unsatisfiable if no such assignment exists.
- Over the set of reals, $X > X+Y$ is satisfiable; over the set of natural numbers, it is unsatisfiable.
- A constraint is solved if all assignments of values to its variables satisfy it. $X + Y > 0$ over the natural numbers.

Constraint satisfaction problem

- A constraint satisfaction problem, a CSP, is a finite sequence of variables, each ranging over a possibly different domain, and a finite set of constraints, each on a subsequence of the considered variables.
- The variables in a CSP: decision variables.
- CSPs are considered in the context of an interpretation.
- A solution to a CSP is an assignment of values to its variables that satisfies each constraint.

Consistent and solved CSPs

- A CSP is consistent (or feasible) if it has a solution and inconsistent (or infeasible) if it does not.
- A CSP is solved if each of its constraints is solved.
- A CSP is failed if one of its domains is empty or one of its constraints is unsatisfiable.
- If a CSP is failed then it is inconsistent, but not necessarily the other way around.
- Two CSPs are equivalent if they have the same set of solutions.

Common classes of constraints

- Uniquely determined by the considered predicates and function symbols.
- Interpretation is clear from the context.
- Classes:
  - Equality, disequality
  - Boolean
  - Linear
  - Arithmetic

Equality and disequality

- Two predicate symbols, equality = and disequality ≠
- Variables are interpreted over arbitrary domains
- What do you know about the constraints
  $x = x$ and $x \neq x$
  $x = y$ and $x \neq y$
- What is the solution(s) of the CSP
  $< x = y, y \neq z, z \neq u;$
  $x \in \{a,b,c\}, y \in \{a,b,d\}, z \in \{a,b\}, u \in \{b\}>$

Boolean constraints

- Constants: true and false
- Function symbols: negation ~, conjunction ^, disjunction V.
- The resulting terms (s,t) are Boolean expressions
- Predicate symbol: =/2; s,t Boolean expressions
  s = t s is shorthand for s = true
- The variables are interpreted over the set of truth values {0,1}; the interpretation of the connectives is given by the standard truth tables.
- (~x) ^ (y V z) = true with as interpretation …

Linear constraints

- Over the set of integers, or the set of reals, or over a subset of one of these sets (usually an interval).
- A fixed set of numeric constants representing reals or integers.
- Linear constraint, s op t, where
  op either <, ≤, =, ≠, ≥, or > (for reals)
  s and t linear expressions such as
  4.x + 3.y –y

Arithmetic constraints

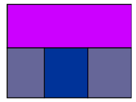
- Only difference w.r.t. linear constraints lies in the use of the multiplication symbol.
- Now also 4.x + x.y –x < y.(z+3) – 3.u

4. RUNNING EXAMPLES

CSP examples: todo model them

- Map colouring
- SEND+MORE = MONEY
- N-queens problem

Map colouring



- A finite set of regions
- A (smaller) set of colours
- A neighbour relation between pairs of regions

Associate a colour with each region so that no two neighbours have the same colour!

Decision variables? Domains? Constraints?

SEND+MORE = MONEY

- Cryptarithmetic problem: digits are replaced by letters …
- Replace each letter by a different digit such that the sum is ok.
- Modeled by which kind of constraints???

N-queens problem

- Place n queens on the n x n chess board, where $n \geq 3$, so that they do not attack each other.
- Representation 1: using n^2 0/1 variables x_ij, where $i,j \in [1..n]$, representing one field
- Representation 2: using linear constraints and n variables x_i with domain [1..n]. x_i denotes the position of the queen in the ith column.

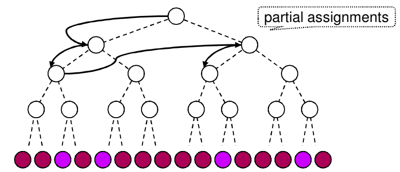
COP problems: examples

- Also a cost function mapping the values of the x_i variables to a real number
- Optimal: here minimal
- Huge area:
  - The knapsack problem
  - A coins problem
  - The facility location problem

Solving CSPs and COPs

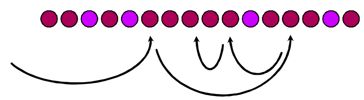
- Domain specific methods are preferred
  - Systems of linear equations over reals : methods from linear algebra
  - Systems of linear inequalities over reals: methods from the area of Linear Programming
- Also general methods are needed
  - Developed in area of Constraint Programming
  - Based on search
    - Local search
    - Top-down search

Exploring search spaces



CLP Tree search:

- constructive
- partial/total assignments
- systematic
- complete or incomplete



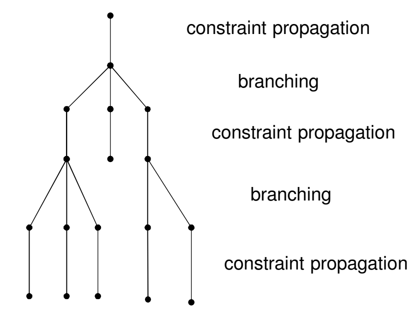
"Local" search:

- move-based (trajectories)
- only total assignments
- usually random element
- incomplete

Top-down search for CSPs

- Combined with a branching (splitting) strategy and constraint propagation.
- Role of branching?
- To split a CSP into 2 or more CSP's whose union is equivalent to the initial CSP
- Role of propagation?
- To transform a CSP into one that is equivalent but simpler
- Alternating propagation and branching

Search tree generated on the fly



Leaves: CSP's that are solved or obviously inconsistent (e.g. empty domain)
Backtracking search

Organisation of backtracking search

- Ordering the decision variables according to some variable choice heuristics
- Branching through splitting the domain of a variable, e.g. labelling splits a finite domain into the singleton domains
- Some value choice heuristics for the ordering of the values in each domain, e.g. bisection which is used for interval domains or finite set domains.

Domain reduction by propagation

- Removal of values that do not participate in any solution
- One reduction can trigger other reductions such that reduction can be further improved.
- Implied constraints: do not alter the set of solutions, but can lead to a smaller search tree as they can be used during propagation
- Complete seach

Branch and bound search for COPs

- Wanted: solution with a minimal value of the cost function
- During backtracking search keep the currently best value of the cost function
- Prune the seach tree by identifying nodes under which no solution with a smaller cost can be present

Towards Constraint Programming

- Programming language with support for generating and solving CSPs and COPs.
- Support for CSP variables, unknowns as in mathematics, and their domains by built-in facilities
- Support for general methods such as backtracking and branch and bound search, various variable and value choice heuristics.
- Specialised, domain specific methods in the form of constraint solvers integrated with the general methods