Let's assume that T is an ω-categorical theory with countable model M and that $X : Mod(T) \to Set$ is a Δ-functor. For every $x \in X(M)$, we know there is an $a_x \in M$ (of some length) which is a support for x i.e. if $f, g : M \to N$ such that $f(a_x) = g(a_x)$ then $X(f)(x) = X(g)(x)$.

Now for each $x \in X(M)$, consider a relation R_x which will be defined for all $N \in Mod(T)$.

$$R_x^N = \{(f(a_x), X(f)(x)) \in N \times X(N) : f : M \to N\}.$$

Consider the class of structures

$$C = \{(N, X(N), \{R_x^N : x \in X(M)\}) : N \in Mod(T)\}.$$

Claim: C is an elementary class.

It suffices to show that if $\tilde{N}_i = (N_i, X(N_i), \{R_x^{N_i} : x \in X(M)\}) \in C$ for $i \in I$ and U is an ultrafilter on I then

$$R_x^{\prod_U \tilde{N}_i} = \prod_U R_x^{\tilde{N}_i}.$$

By Los, the inclusion right to left is clear. In the other direction, imagine that $(\bar{a}, \bar{x})$ is in the left side. So there is some $f : M \to \prod_U N_i$ such that $f(a_x) = \bar{a}$ and $X(f)(x) = \bar{x}$. Using ω-categoricity and Los, ultrafilter often, $\bar{a}_i$ has the same type as a_x. Let $f_i : M \to N_i$ send a_x to $\bar{a}_i$ for those i's (M is the prime model of T so such a map exists). Now the map $(\prod_U f_i) \circ \Delta$ agrees with f on a_x and so $\prod_U X(f_i) \circ \Delta$ agrees with $X(f)$ on a_x. This means that ultrafilter often $X(f_i)(x) = x_i$ which is what we wanted. $\square$

Now we get to invoke conceptual completeness. Consider the forgetful functor $F : C \to Mod(T)$ sending $\tilde{N}$ to N. This is certainly onto on objects; the only thing to check is that if $f : N \to N^U$ is an elementary map then this lifts to an embedding between $\tilde{N}$ and $\prod_U \tilde{N}$. On $X(N)$, let's have it act as $X(f)$. Suppose now that $R_x(n, y)$ holds in $\tilde{N}$. There is some elementary $g : M \to N$ such that $g(a_x) = n$ and $X(g)(x) = y$. But then $f(g(a_x)) = f(n)$ and $X(f)(g(x)) = X(f)(y)$ and so $R_x(f(a_x), X(f)(x))$ holds in $\prod_U \tilde{N}$.

In the other direction, if $R_x(f(n), X(f)(x))$ holds in $\tilde{N}^U$ then consider the map $g : M \to N^U$ which witnesses this. Without loss, since M is the prime model of T we may assume that g maps M into $f(N)$. Let $h = f^{-1} \circ g$ and apply the previous argument to see that $R_x(n, y)$ holds in N.

Now we have the assumptions of conceptual completeness and we conclude that the sort X is equivalent to one in T^{eq} and that all the R_x's are definable in T^{eq}.