[redacted]

3-20

$\emptyset$ = empty set, { }, no elements inside

S = sample space, all possible outcomes

A = event A

B = event B

$\cap$ = intersection, "and"

$\cup$ = union, "or"

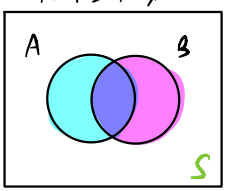
$A \cap B$ =

$A \cup B$ =

Inclusion-Exclusion Principle

not disjoint

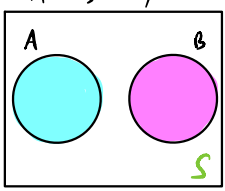
$A \cap B \neq \emptyset$



$P(A \text{ or } B) = ?$

disjoint

$A \cap B = \emptyset$



$P(A \text{ or } B) = ?$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B)$

$P(A \cap B) = 0$

Example (Last ACT question we did!)

You have 10 cards. The first 5 are labeled A, and the last 5 are labeled B. Also, the first 2 are red, the next 2 are green, the next 2 are blue, the next 2 are orange, and the last 2 are pink.

$P(A) = \frac{1}{2}$

$P(\text{red}) = \frac{1}{5}$

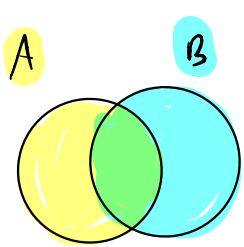
$P(A \text{ and red}) = \frac{1}{10}$

If you draw randomly, what is the probability that you draw a card labeled A or a red card?

$$\frac{6}{10} = P(A) + P(\text{red}) - P(A \text{ and red})$$

$$\frac{5}{10} + \frac{2}{10} - \frac{1}{10}$$

A or B

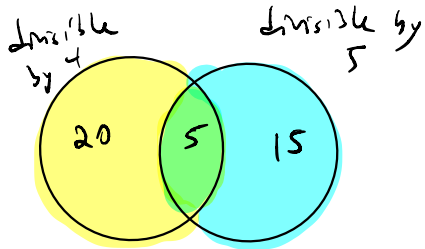


How many numbers between 1 and 100, inclusive, are divisible by 4 or 5 but not both?

25 numbers divisible by 4

20 numbers divisible by 5

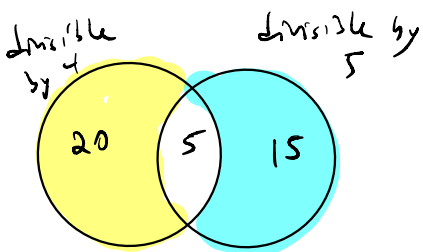
5 numbers divisible by 4 and 5



$$25 + 20 - \underset{(*)}{2 \times 5} = 35$$

Or $20 + 15 = 35$

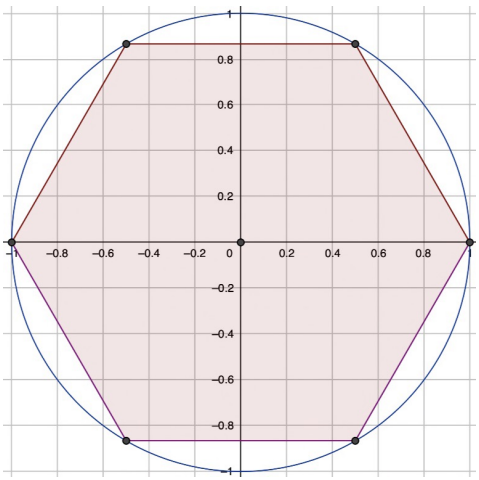
(*) Subtract the intersection 2×5 twice because we want



but $25 + 20$ actually adds the intersection of 5 numbers *twice*.

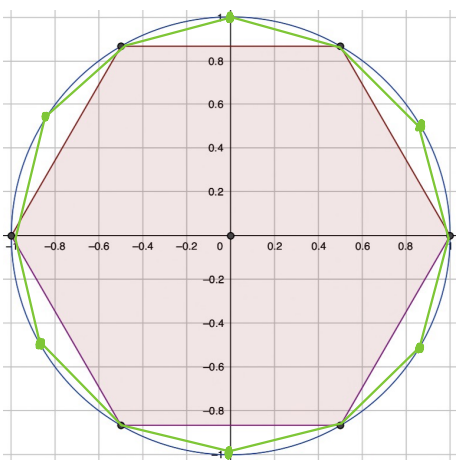
π day! Here's the Archimedean method of calculating it

Inscribe regular hexagon in unit circle ($r = 1$)



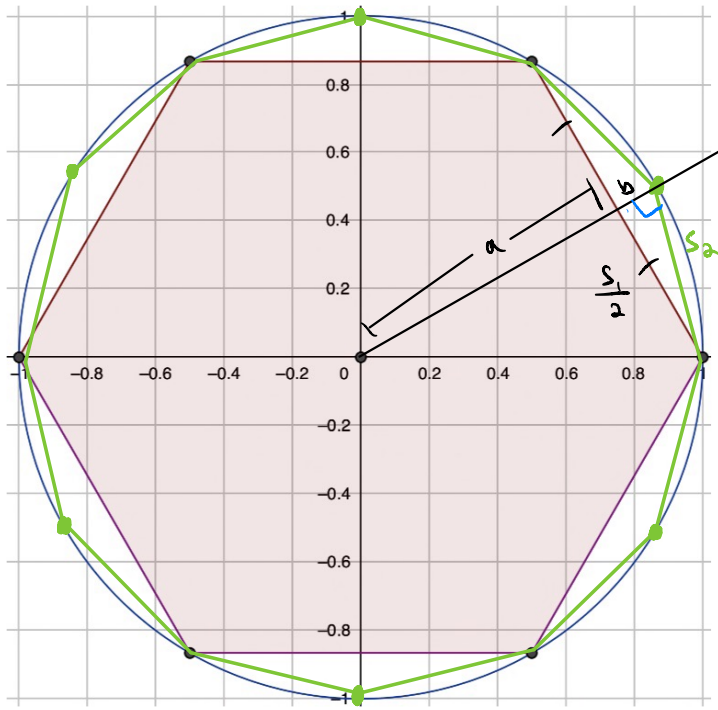
$n = 6$, hexagon

Side lengths are 1



$n = 12$

How do we get new side length, s_2? Old side length, s_1, is 1.



a right angle

Order:

1. a
2. $s_1/2$
3. b

$$a^2 + \left(\frac{s_1}{2}\right)^2 = 1^2$$

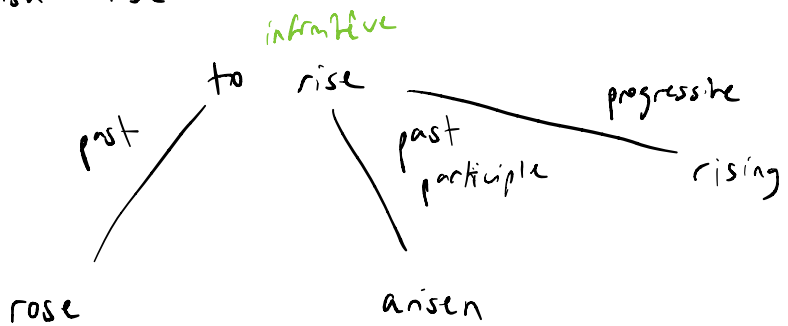
$$a = \sqrt{1 - \left(\frac{s_1}{2}\right)^2}$$

$$b = 1 - a$$

$$s_2 = \sqrt{b^2 + \left(\frac{s_1}{2}\right)^2}$$

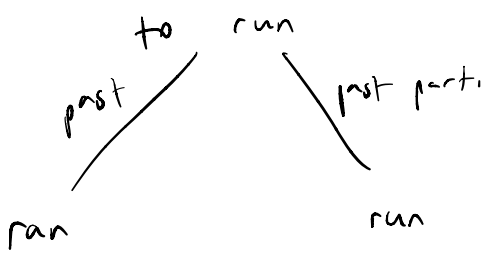
Question: Would you choose this algorithm to calculate π or the $1 - \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \cdots$ method?

English 73C



Past perfect tense { has, had, have } + <past participle>

I have arisen ✓
She had arisen ✓



I had ran. ⊘

I had run ... ✓

She had run a marathon. ✓