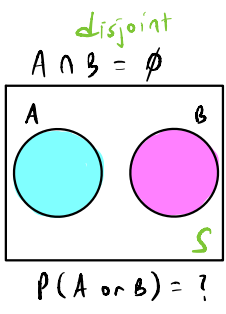
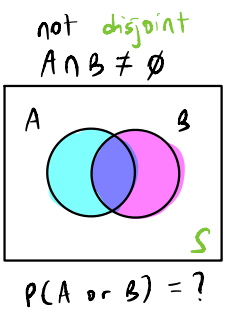


Angela &
 Andrey
 3-20

Inclusion -
 Exclusion
 Principle



\emptyset = empty set, $\{ \}$, no elements inside

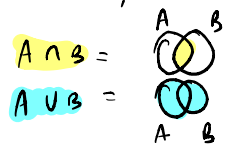
S = sample space, all possible outcomes

A = event A

B = event B

\cap = intersection, "and"

\cup = union, "or"



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = P(A) + P(B)$
 $P(A \cap B) = 0$

Example
 (just ACT
 question we
 did!)

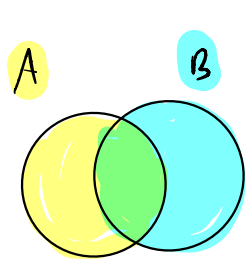
You have 10 cards. The first 5 are labeled A, and the last 5 are labeled B. Also, the first 2 are red, the next 2 are green, the next 2 are blue, the next 2 are orange, and the last 2 are pink.

$P(A) = \frac{1}{2}$
 $P(\text{red}) = \frac{1}{5}$

$P(A \text{ and red}) = \frac{1}{10}$

If you draw randomly, what is the probability that you draw a card labeled A or a red card?

$\frac{6}{10} = P(A) + P(\text{red}) - P(A \text{ and red})$
 $\frac{5}{10} + \frac{2}{10} - \frac{1}{10}$



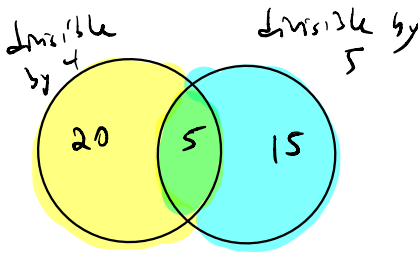
A or B

How many numbers between 1 and 100, inclusive, are divisible by 4 or 5 but not both?

25 numbers
divisible by 4

20 numbers
divisible by 5

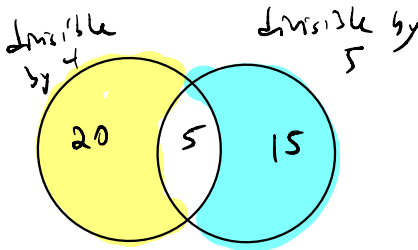
5 numbers
divisible by
4 and 5



$$25 + 20 - 2 \times 5 = 35$$

Or $20 + 15 = 35$

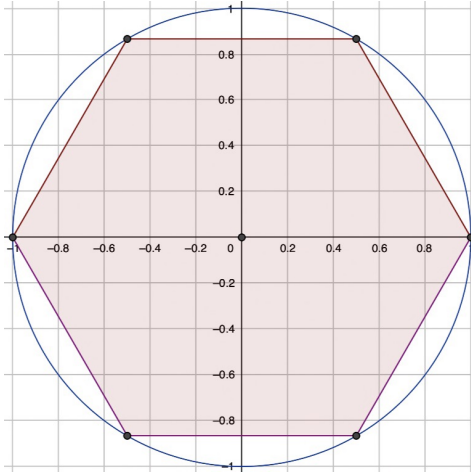
(*) Subtract the intersection ^{2x5} twice because we want



but $25 + 20$ actually adds the intersection of 5 numbers ^{twice}.

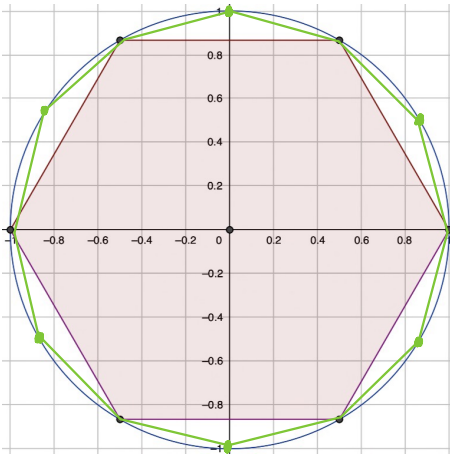
π day! Here's the Archimedean method of calculating it

Inscribe regular hexagon in unit circle ($r=1$)



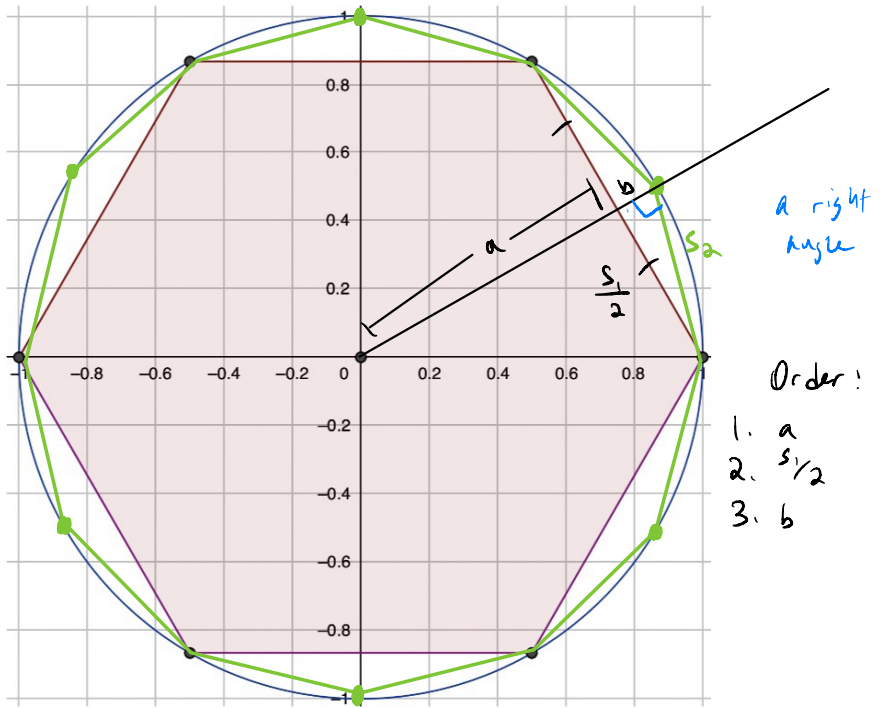
$n=6$, hexagon

Side lengths are 1



$n=12$

How do we get new side length, s_2 ? Old side length, s_1 , is 1.



Order!
 1. a
 2. $s_1/2$
 3. b

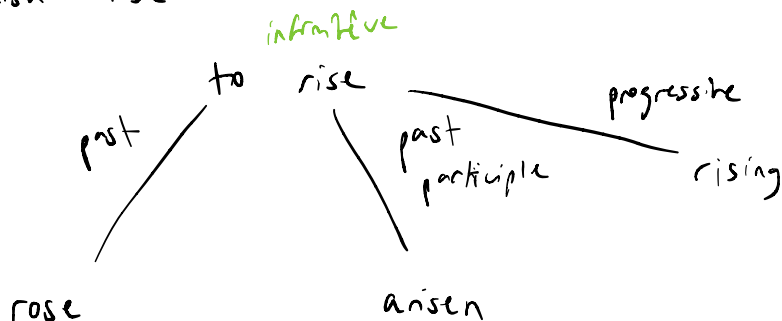
$$a^2 + \left(\frac{s_1}{2}\right)^2 = 1^2$$

$$a = \sqrt{1 - \left(\frac{s_1}{2}\right)^2}$$

$$b = 1 - a$$

$$s_2 = \sqrt{b^2 + \left(\frac{s_1}{2}\right)^2}$$

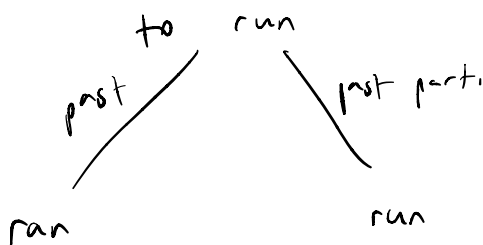
Question: Would you choose this algorithm to calculate π or the $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots$ method?



Past perfect tense { has, had, have } + <past participle>

I have arisen ✓

She had arisen ✓



I had ran. ✗

I had run ... ✓

She had run
a marathon. ✓