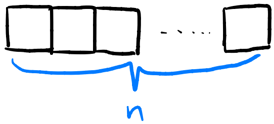
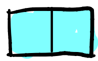
Grace &
Lucy
3-26

Recurrence Example

How many ways are there to perfectly cover $1 \times n$ board



with only monominoes 

and dominoes 

such that there are no two consecutive dominoes?
Find a recurrence.

Setup Let h_n be the number of ways to perfectly cover a $1 \times n$ board. $h_n = \mathcal{N}^{\underbrace{\square\square\square\cdots\square}_{n}}$

Base cases $h_1 = \mathcal{N}^{\square} = |\{\square\}| = 1$

$h_2 = \mathcal{N}^{\square\square} = |\{\square\square, \square\square\}| = 2$

$h_3 = \mathcal{N}^{\square\square\square} = |\{\square\square\square, \square\square\square, \square\square\square\}| = 3$

$h_4 = \mathcal{N}^{\square\square\square\square} = h_3 + h_1 = 4$

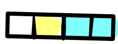
Recurrence

$$h_n = \mathcal{N}^{\underbrace{\square\square\square\cdots\square}_{n}} = h_{n-1} + h_{n-3}, \quad n \geq 3, \quad \text{with } h_0 = 1, \ h_1 = 1, \ h_2 = 2$$

$h_3 = h_2 + h_0$

$h_0 = 1$
$h_1 = 1$
$h_2 = 2$
$h_3 = 1 + 2$

What is h_0?

h_4  

$h_3 + h_1$

end with monomino | end with domino

Footnote Go over $h_0 = 1$ for the same reason that $0! = 1$
Firstly there is 1 way to choose 0 objects.
But this isn't arbitrary; math breaks otherwise.

Why do we want to use h_0? It's simple to compute. Busy Beaver example

Why is $0! == 1$?

By intuition Think about permutations. $5! = 120$ means 120 ways to arrange 5 objects (order matters).

$$\underset{5}{\underline{A}}\ \underset{4}{\underline{B}}\ \underset{3}{\underline{C}}\ \underset{2}{\underline{D}}\ \underset{1}{\underline{E}} = 120 \text{ arrangements}$$

By math
$$n! = n(n-1)!$$
$$1! = 1(0)!$$
$$1 = 0!$$

The simplest proof
$$5! = 5(4!)$$
$$5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

By cos(x) $\cos(x) = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)!} x^{2n}$

A bit harder but a more illustrative example

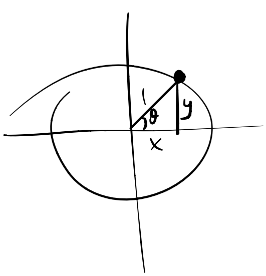
$$= \frac{x^0}{0!} - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$$= 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \cdots$$

$\frac{1}{0!} = 1$ if we define $0!$ to be 1. Otherwise, division by 0 or the series just doesn't work.

Does this infinite series (sum of sequence) work for cos(0)=1?
That is, let x=0; does the series equal 1 when x=0?

✓ $\cos(0) = 1$ from the infinite sum



$$\cos\theta = \frac{x}{1} = x$$
$$\sin\theta = \frac{y}{1} = y$$

Look at sine and cosine graph animation

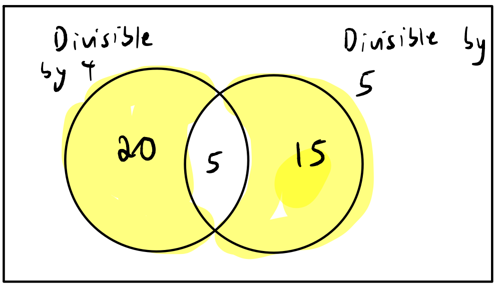
Inclusion-exclusion example

How many numbers between 1 and 100, inclusive, are divisible by 4 or 5 but not both?

25 numbers divisible by 4
20 numbers divisible by 5
5 numbers divisible by 4 and 5

Method 1

Venn diagram



35 ✓

Method 2

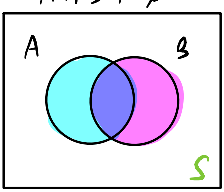
Arithmetic only

Let A = "divisible by 4", $|A| = 25$
B = "divisible by 5", $|B| = 20$

$|A| + |B| - 2|A \cap B| = 25 + 20 - 2(5)$

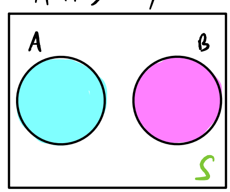
Inclusion-exclusion principle

not disjoint
$A \cap B \neq \emptyset$



$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

disjoint
$A \cap B = \emptyset$

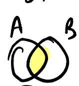


$P(A \cup B) = P(A) + P(B)$

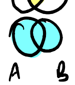
$\cap$ = intersection, "and"
$\cup$ = union, "or"

$\emptyset$ = empty set

$A \cap B$ =



$A \cup B$ =



Grace & Lucy

3-26-22 English D03 2020 December

4) reflexive pronoun

pronoun that ends with "self" himself herself itself themselves

reflexive use when subject == object

$\underbrace{\text{He}}_{\text{subject}}$ did this to $\underbrace{\text{himself}}_{\text{object}}$.

or emphasizes

She $\underbrace{\text{herself}}_{\text{emphasizes}}$ was an award-winning journalist.

21) I take measurements, draft patterns, and cut cloth.

This is grammatically correct.

Default to using commas to separate ideas; use semicolons as well when that becomes confusing.

23) Transition word questions: Always read before & after

40) $\frac{b-a}{b}$

Math 65E 2007 December

54) (1)   Discrete math

Horizontal line test

Function is one-to-one if no horizontal line intersects graph twice

Vertical line test

Not a function if any vertical line intersects more than once

59) D

$nPr = \frac{n!}{(n-r)!}$

$\frac{n!}{(n-2)!} = nP_2$

"How many arrangements of groups of 2 (r) from n objects"

$\frac{n!}{(n-2)!} = n \cdot (n-1)$

$n(n-1) = 30$

$\parallel$

$6 \cdot 5 = 30$

$\frac{n!}{(n-2)!} = \frac{n \cdot (n-1) \cdot \cancel{(n-2) \cdots 1}}{\cancel{(n-2) \cdots 1}}$

$(n-1)! = 5! = 120$

$10P2$

$= \frac{10!}{8!}$

$= 10 \cdot 9$

"10 digits 0-9, how many 2 digit passcodes (no replacement) can you make"

$\frac{0}{10} \cdot \frac{1}{9} = 90$