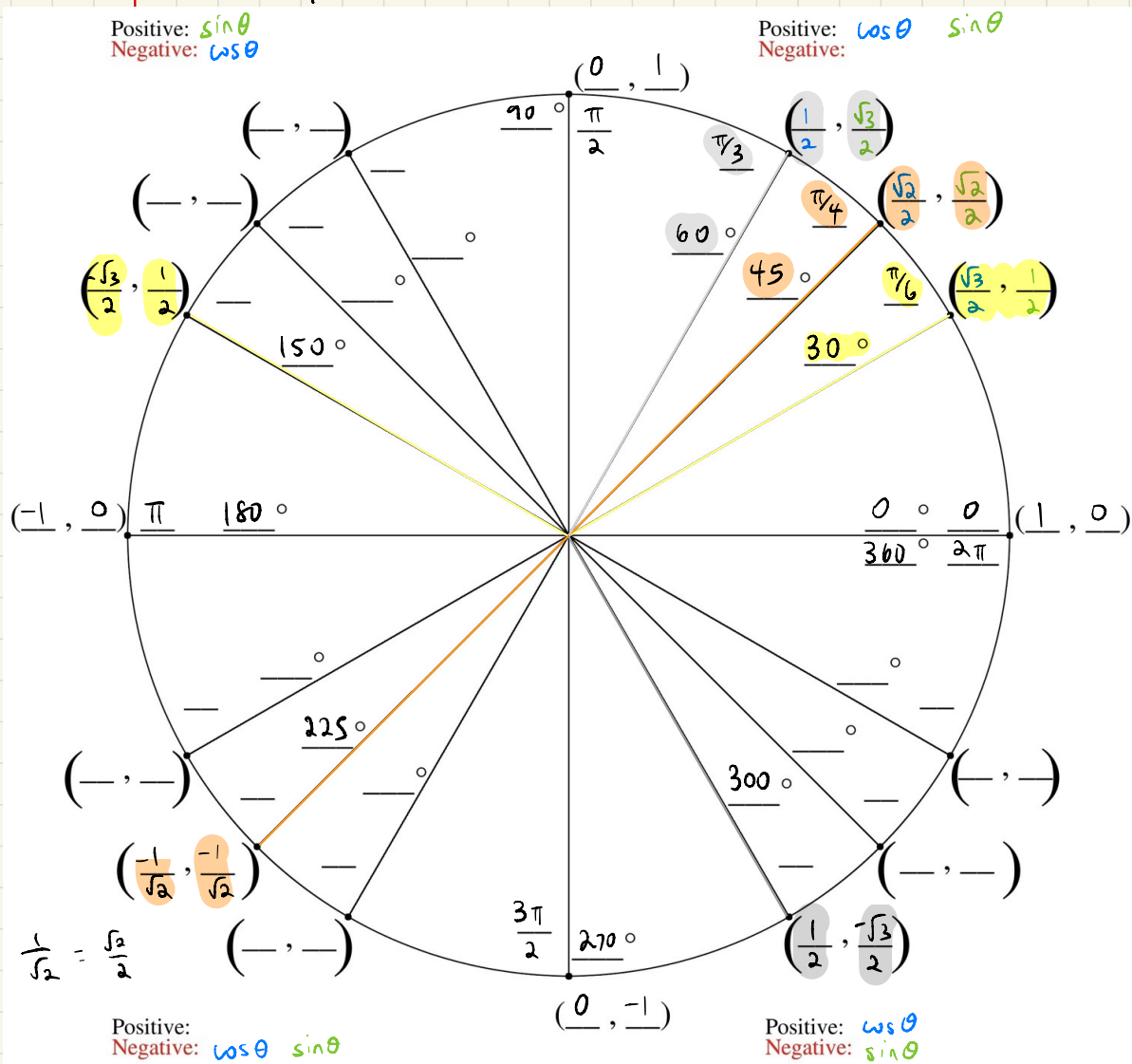
[redacted]

17 April 2022

Extra topics: radian, ellipse, unit circle, contrapositive

Recall $\cos\theta = \frac{adj}{hyp} = \frac{x}{1} = x$ if $r = 1$ (unit circle)

Similarly, $\sin\theta = \frac{opp}{hyp} = \frac{y}{1} = y$ if $r = 1$



$\cos(150°) = -\frac{\sqrt{3}}{2}$ $\sin(150°) = \frac{1}{2}$

$\cos(225°) = -\frac{1}{\sqrt{2}}$ $\sin(225°) = -\frac{1}{\sqrt{2}}$

$\cos(300°) = \frac{1}{2}$ $\sin(300°) = -\frac{\sqrt{3}}{2}$

710 G Math

60) If it is raining, then the parade is cancelled <-> If the parade is not cancelled, it is not raining

By intuition If parade wasn't cancelled, must not have rained.

By math Let p = "It is raining", q = "Parade is cancelled"

Symbols

symbol	meaning
¬	not
∧	and
∨	or
→	implies

p	q	p ∧ q	p ∨ q	p → q	¬p ∨ q
F	F	F	F	T	T
F	T	F	T	T	T
T	F	F	T	F	F
T	T	T	T	T	T

p	¬p
F	T
T	F

q	¬q
F	T
T	F

Law $p \rightarrow q \equiv \neg p \vee q$ Why? By truth table values of $p \rightarrow q$ are the same as the values of $\neg p \vee q$, hence they are logically equivalent.

By intuition p = Duke won
q = I pay ym $20

Proposition If Duke won, I pay ym $20.

If Duke didn't win (¬p), my proposition is true if I pay ym or not. So we use an or (∨).

Else, if Duke won, my proposition is true only if I pay ym $20 (q).

So we or (∨) ¬p, q.

Summary If $p \rightarrow q$, $\neg q \rightarrow \neg p$ ✓ (contrapositive) but $q \rightarrow p$ ✗ (converse) $\neg p \rightarrow \neg q$ ✗

Proof of $p \to q \equiv \neg q \to \neg p$.

Assumptions $p \to q \equiv \neg p \vee q$ proven on previous page
$p \vee q \equiv q \vee p$ proof by truth table & intuition

Begin $p \to q$

Contrapositive $\neg p \vee q$

Commutative $q \vee \neg p$

Contrapositive $\neg q \to \neg p$

[redacted]

[redacted]

4-17-22 Math 2020 June C02

46) practice Law of Sines and Law of Cosines

I promise they aren't hard, just plugging in numbers into a formula – it'll take ~15 min.

Law of Sines practice https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:trig/x9e81a4f98389efdf:law-of-sines/e/law_of_sines

If the link doesn't work, Google it

Law of Cosines practice https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:trig/x9e81a4f98389efdf:law-of-cosines/e/law_of_cos

55) tricky; always confirm

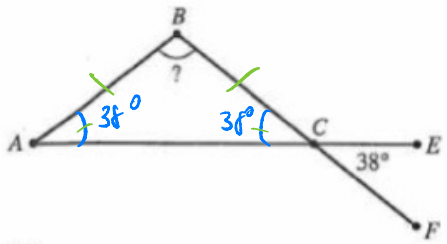
56) fastest way to do complex number problems?

47) ellipse

https://www.khanacademy.org/math/precalculus/x9e81a4f98389efdf:conics/x9e81a4f98389efdf:ellipse-center-radii/v/conic-sections-intro-to-ellipses

5)

5. In the figure below, $\overline{AB}$ is congruent to $\overline{BC}$, and $\overline{AE}$ intersects $\overline{BF}$ at C. What is the measure of $\angle B$?



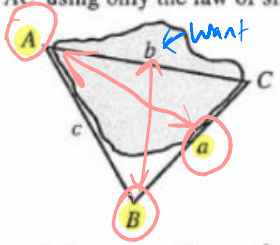
A. 14°
B. 38°
C. 76°
D. 104°
E. 142°

$180 - 2(38) = 104$

46)

46. A surveyor needs to find the length from point A to point C across a lake as shown in the figure below. The measurements of which of the following angles and side lengths are sufficient for the surveyor to determine the length of $\overline{AC}$ using only the law of sines?

Have



(Note: The law of sines says $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$.)

	angle	side length
F.	A, B	a
G.	A	c
H.	B	a, c
J.	B	c
K.	C	a

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{a \sin B}{\sin A} = b$$

59) original 100

× .9 → sale 90

× .7 → clearance 63

.63

$100 ? 10% off + 30% off = 40% off

≠ $60 ✗

✓

D 37%

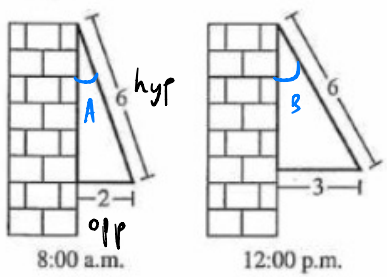
50) Misread

56) Check Desmos

$\sin(\theta) = \frac{\text{opp}}{\text{hyp}}$

55)

55. A 6-foot awning that extended 2 feet horizontally from a vertical building at 8:00 a.m. was adjusted to extend 3 feet horizontally from the building at 12:00 p.m., as shown below. Which of the following expressions equals the positive difference in the measures of the angle between the awning and the building at 8:00 a.m. and at 12:00 p.m. ?



8:00 a.m. 12:00 p.m.

A. $\cos^{-1}\left(\frac{3-2}{6}\right)$

B. $\sin^{-1}\left(\frac{3-2}{6}\right)$

C. $\tan^{-1}\left(\frac{3-2}{6}\right)$

D. $\cos^{-1}\left(\frac{3}{6}\right) - \cos^{-1}\left(\frac{2}{6}\right)$

E. $\sin^{-1}\left(\frac{3}{6}\right) - \sin^{-1}\left(\frac{2}{6}\right)$

B − A (positive difference)

A − B would be negative

$B = \sin^{-1}(3/6)$

$A = \sin^{-1}(2/6)$

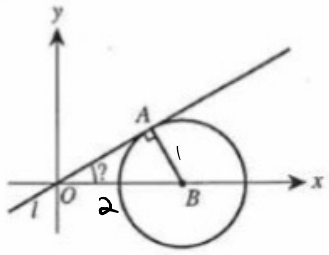
	input	output
sin	θ	ratio
$\sin^{-1}$	ratio	θ
cos	θ	ratio
$\cos^{-1}$	ratio	θ

ratio [−1, 1]

$\sin^{-1}(2)$ = error (solvable with complex numbers)

24. Graphed in the standard (x,y) coordinate plane below is line l and the circle with equation $(x-2)^2+y^2=1$. Line l passes through $O(0,0)$ and is tangent to the circle at A, and B is the center of the circle. What is the measure of $\angle AOB$?

F. 15°
G. 22.5°
H. 30°
J. 45°
K. 60°



$$\sin(\theta) = \frac{1}{2}$$

$$\sin^{-1}(\sin(\theta)) = \sin^{-1}(1/2)$$

$$\theta = \sin^{-1}(1/2)$$