Electrical noise and the measurement of absolute temperature, Boltzmann's constant and Avogadro's number

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Electrical noise and the measurement of absolute temperature, Boltzmann's constant and Avogadro's number

T J Ericson

Boltzmann's constant can be measured using comparatively inexpensive electronic amplifiers. The apparatus described in this article is also capable of measuring absolute temperature and Avogadro's number.

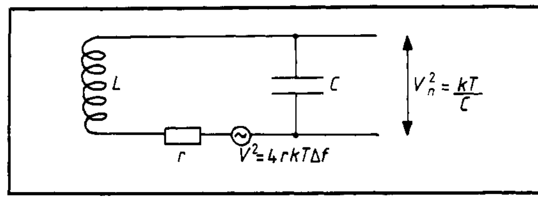

Figure 1 A simple *LC* circuit

Theory

The basic principle involved in this experiment is the measurement of the electrical energy in an *LC* circuit. The simple *LC* (as in figure 1) can be considered to store energy in magnetic or electric fields. The *LC* circuit can therefore be considered to have two degrees of freedom so that the average energy of $\frac{1}{2}CV_n^2$ in the capacitor is equal to $\frac{1}{2}kT$ where T is the absolute temperature of the circuit. Therefore $\frac{1}{2}CV_n^2 = \frac{1}{2}kT$ and so $V_n = (kT/C)^{1/2}$. Consequently by measuring the RMS voltage V_n across C either k can be calculated assuming that C and T are known or T can be measured assuming k and C are known. (The resistor r in the circuit represents the mechanism by which energy is exchanged between the circuit and the external world).

The simplicity of this idea is the consequence of each electromagnetic mode having $\frac{1}{2}kT$ in each degree of freedom. This means that the noise voltage across C is independent of r. The idea that noise in an electric circuit is independent of r may appear strange to some electrical engineers whose standard working formula is the Nyquist equation, $V^2 = 4rkT\Delta f$. The two viewpoints are easily seen to be equivalent when it is remembered that the Nyquist derivation of the noise produced by a resistor r is a consequence of assuming that each mode on a transmission line has an average energy of $\frac{1}{2}kT$ for each degree of freedom. It is also possible by assuming Nyquist's equation to show that $V_n = (kT/C)^{1/2}$. If r is considered a generator of noise voltage $V = (4rkT\Delta f)^{1/2}$ then the voltage across C in the bandwidth Δf is $Q(4rkT\Delta f)^{1/2}$, where Q is the quality factor of the circuit. But the integrated bandwidth of the circuit is $\pi f_0/2Q$, where $f_0 = \omega_0/2\pi$ = resonant frequency of the circuit. Therefore voltage across $C = V_m = Q(4rkTf_0\pi/2Q)^{1/2}$.

Now $Q = L\omega_0/r = 1/C\omega_0 r$. Therefore RMS noise voltage (V_n) across C is

$$\frac{1}{C\omega_0 r}\left(\frac{4rkTf_0C\omega_0 r\pi}{2}\right)^{1/2} = \left(\frac{kT}{C}\right)^{1/2}.$$

Problems

The first problem associated with the measurement of V_n is the requirement that the input impedance of the amplifier should be large compared with the parallel impedance of the *LC* circuit. Also, the amplifier should ideally produce a noise small compared with V_n.

Fortunately circuits using JFETS can reasonably satisfy both these requirements. In order that noise

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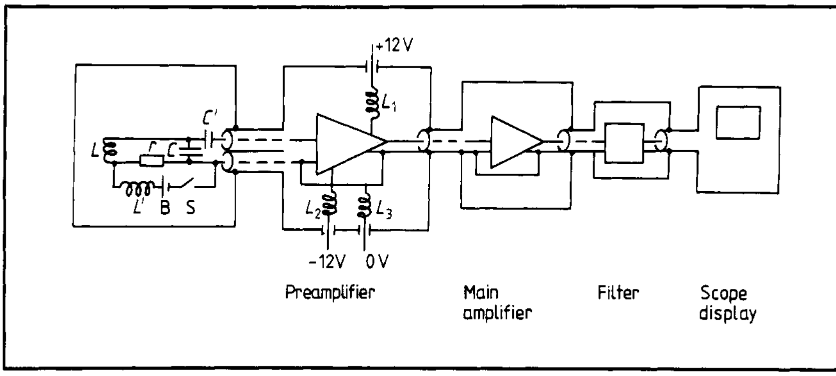


Figure 2 The experimental layout with screening and earthing arrangements

signals can after amplification be displayed on standard laboratory scopes, a centre frequency f_0 of 4 MHz is used, with $C = 40$ pF and $L = 40\,\mu$H.

If r is allowed to have a maximum value of 20 ohm then the band width of the LC circuit is 0.1 MHz. In this apparatus the amplifier is designed to have bandwidth of 0.8 MHz so that nearly all the noise from the LC circuit is displayed on the scope. Figure 2 illustrates the experimental layout together with the screening and earthing arrangements.

Practical details

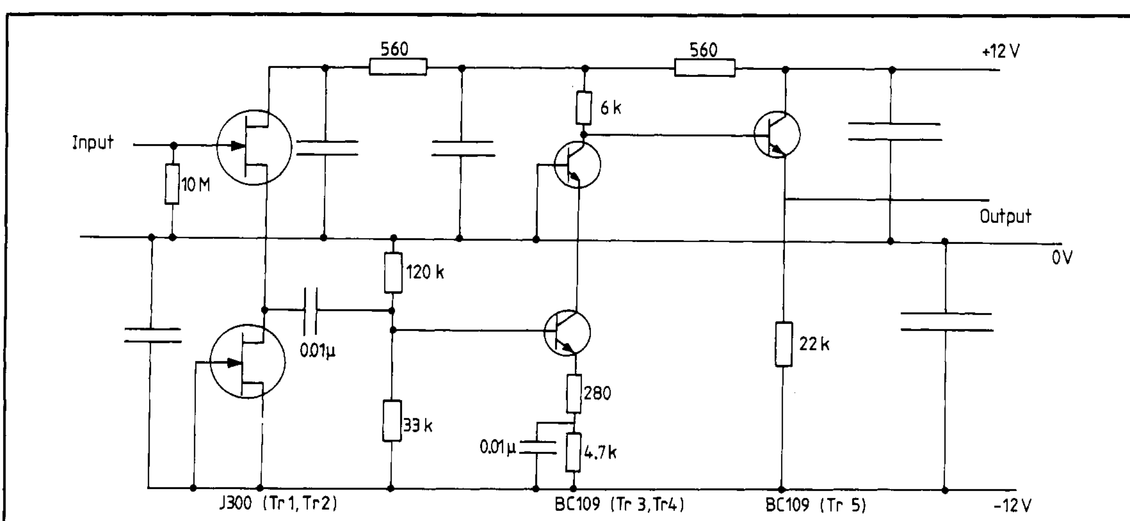
LC circuit (figures 1 and 2) Twenty turns of 22 swg copper enamel coated wire wound on grade B10 toroid FX3853 give an inductance of 22 μH. L consists of two of these inductors in series. C consists of a 30 pF beehive capacitor together with a few centimetres of coaxial cable and the input capacitance of the amplifier.

For the purpose of temperature measurement, r is two tungsten filament torch bulbs (2.5 V and 0.2 A) which are heated by four 1.5 V cells when switch S is closed. L' is an inductor to ensure that the noise voltage produced by r is not bypassed by the battery B and consists of 50 turns of 32 swg copper wire on a FX3853 toroid, approximately 150 μH.

The LC circuit together with B L and S, is enclosed by a 22 swg aluminium screen. S is a reed switch operated by a small magnet outside the aluminium screen. C' prevents the preamplifier being incorrectly biassed when S is closed.

Preamplifier The circuit and circuit board layout are shown in figures 3 and 4. The power supplies of 12 V are decoupled by inductors L_1, L_2, L_3 (50 turns of 32 swg wire on a FX3852 toroid, ~ 100 μH) and lead through capacitors C_1, C_2, and C_3 (10 000 pF lead-through capacitors, RS 126–001). The earth plane of the copper clad board is electrically con-

Figure 3 Circuit for preamplifier (all unmarked capacitors 0.1 μF)



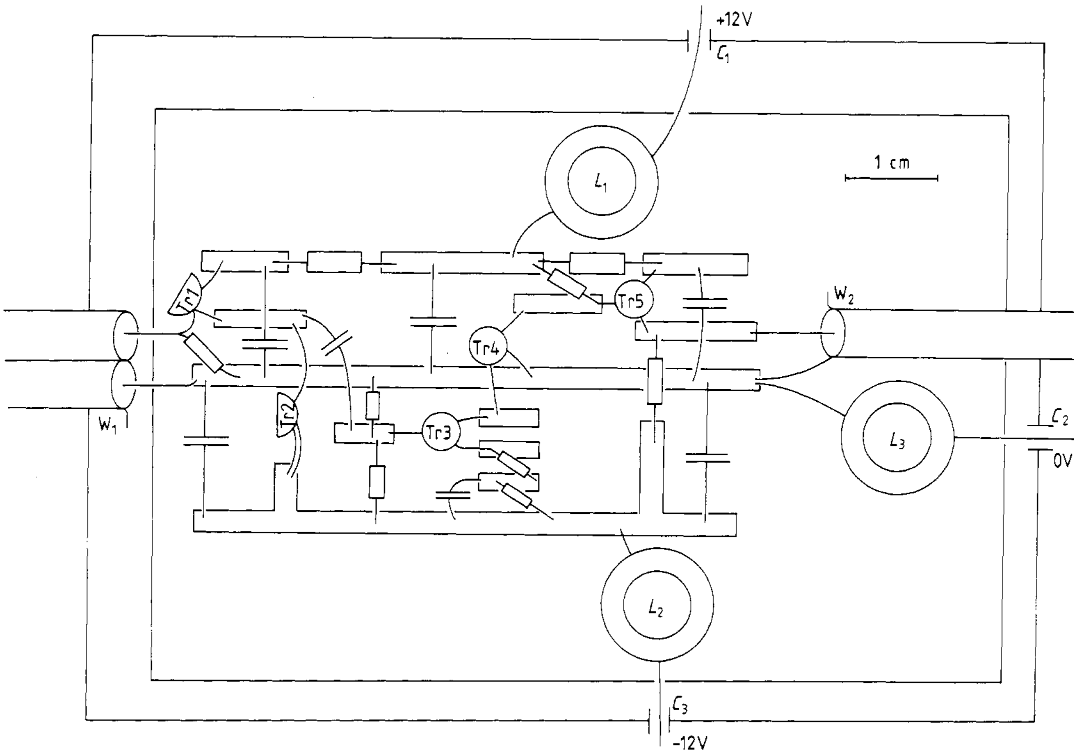


Figure 4 Circuit board for preamplifier

Figure 5 Bandpass filter with component values

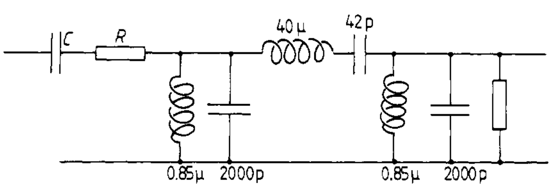
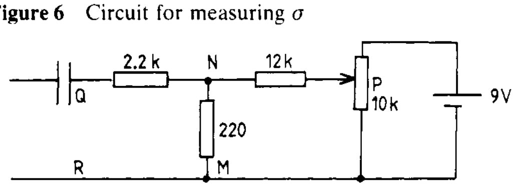


Figure 6 Circuit for measuring σ



nected (by links W_1 and W_2) to the screens of the coaxial cables. All three coaxial cable screens are electrically connected to the cast aluminium box. The circuit is finally connected to a single earth at the scope *via* the main amplifier and filter (see figure 2).

Main amplifier The MC1733 provides sufficient gain with pins G1A and G1B connected together. As this chip has a band width of 40 MHz, care has to be taken to avoid self oscillations. Consequently a copper earth plane was found necessary and interconnections were kept as short as possible. The power supplies are ± 6 V. They are decoupled in the same way as the power supplies for the preamplifier.

The filter This was designed as a Butterworth bandpass filter having a lower 3 dB frequency of 3.5 MHz and an upper 3 dB frequency of 4.3 MHz with a source and load of 100 Ω. Component values are shown in figure 5. C is 0.1 µF and R is adjusted such that R + output resistance of MC1733 = 100 Ω, the output resistance being of the order of 20 Ω from the manufacturer's data sheet.

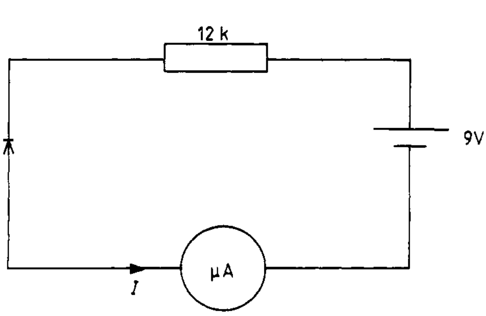


Figure 7 Circuit for measuring σ
The photodiode is RS 305–462

Display of noise signal and measurement of σ The probability density function for the narrow band random noise obeys the equation

$$p_r = \text{const}\exp(-V^2/2\sigma^2).$$

σ, the standard deviation, can be measured by displaying the noise signal on the vertical Y axis of a scope using a sinusoidal signal of say 50 kHz as X sweep. The noise is sampled between V and $V+\delta V$ by measuring the light output from the oscilloscope screen using a photodiode. The transparent plastic protective front cover of the photodiode is masked with aluminium foil except for a horizontal 1 mm slit. The X sweep is adjusted to a few millimetres. The circuit arrangements for measuring σ are shown in figures 6 and 7. QR is attached to the Y input of the oscillator and NM to a digital voltmeter of high input impedance.

In order to measure σ the potentiometer P is adjusted until the current output I of the photodiode is at a maximum I_0. P is then adjusted until $I=0.607\ I_0$ and the digital voltmeter reading V is measured. There are two readings of V for which $I=0.607\ I_0$ (figure 8a). The difference between the readings is 2σ. The experimental arrangement for measuring σ is shown in figure 8b.

Measurement of absolute temperature and Boltzmann's constant

The temperature of a resistor (in this case a tungsten filament) can be determined by measuring σ for three different conditions. Let σ_n^2 = variance with input of amplifier shorted; σ_1^2 = variance with r at room temperature (T_1 K); and σ_2^2 = variance with r hot (TK). Now $\sigma_1^2=(\sigma_{T_1})^2+(\sigma_n)^2$ and $\sigma_2^2=(\sigma_T)^2+(\sigma_n)^2$, where σ_{T_1} = RMS output if the amplifier were noiseless and r at room temperature T_1, and σ_T = RMS output if amplifier were noiseless and r at T. But $(\sigma_T)^2/(\sigma_{T_1})^2=T/T_1$, therefore T can be calculated. The temperature calculated was found to agree within about 3% with the temperature estimated for the change in electrical resistance of the filament.

Boltzmann's constant can also be determined if C (figure 1) is measured. Note that C is the total capacitance in parallel with L, therefore C includes the input capacitance of the amplifier. C can be measured by inserting a known capacitance of, say, 22 pF in parallel with C and measuring the new resonance frequency f_1. If f_0 was the original resonant frequency then $L(C+22)/LC=f_0^2/f_1^2$. Hence C can be calculated. Now, as $\sigma_T=V_n\times$ voltage gain of amplifier, then $C(\sigma_{T_1})^2=k\times T_1\times$ (voltage gain of amplifier)^2. Therefore k can be calculated if the voltage gain of the amplifier is measured. The gain must be measured over the linear region of the amplifier so that it is important that the output of the main amplifier does not exceed 1 V peak to peak. A convenient circuit for measuring gain is shown in figure 9.

Experimental errors In principle the noise output is independent of the value of r and only depends on

Figure 8 **a** Graph of probability density function. **b** Experimental arrangement for measuring σ. The photodiode is attached to the screen. The noise signal is displayed on the scope and is moved vertically by a voltage V controlled by a potentiometer P. The noise is sampled by the light output of the signal on the screen producing a current I through the photodiode

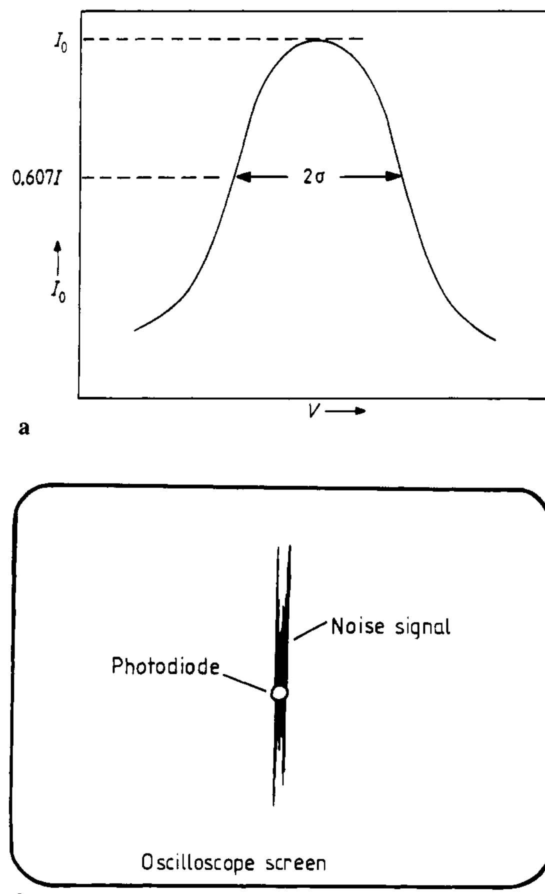
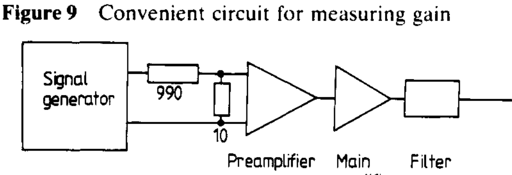


Figure 9 Convenient circuit for measuring gain



the temperature of r. Therefore the value of V_n should be independent of r. In practice, however, V_n decreases by 2% as r increases from 2 Ω to 20 Ω. The probable causes of this error have not been fully investigated. Further analysis and measurement are proceeding to reduce this 2% error. However, for the purposes of the measurements envisaged at present this error is acceptable.

For the accurate measurement of noise it is important that the voltage gain of the amplifiers is linear for the voltages used. The main amplifier MC1733 is sufficiently linear not to produce significant errors for the range of output voltage used.

Conclusion

The apparatus described is capable of measuring absolute temperatures of a tungsten filament bulb up to normal running temperatures. It is also capable of measuring Boltzmann's constant (and consequently Avogadro's number as Avogadro's number $\times k$ = universal gas constant) to an accuracy of a few percent. The display of narrow band noise whose amplitude and bandwidth can be adjusted is also educationally useful.

As Boltzmann's constant is not easily and quickly measured by Brownian motion, (i.e. mechanical noise techniques), electrical noise techniques as described are convenient to demonstrate how the concept of temperature is related to the micro- and macroscopic world.

References

Earls J A 1966 'Undergraduate experiment on thermal and shot noise' *Am. J. Phys.* **34** 575–9

Nyquist H 1928 *Phys. Rev.* **32** 110–3

Pepper M G and Brown J B 1978 'Absolute high-temperature Johnson noise thermometry' *J. Phys. E: Sci. Instrum.* **12** 31–4

Siliconix Small Signal FET Design Catalog 1983 (Siliconix Inc.)

von Thune P C 1976 *Noise Thermometer* US Pat. No. 3937 086, 10 Feb

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'Notes on experiments' enables teachers at both school and tertiary level to share their ideas with other readers. *Physics Education* welcomes submissions from readers who know of some simple improvement to a commercially made piece of apparatus, or who have designed a new gadget or improved a standard experiment. In particular the Editor would welcome brief descriptions of experiments devised or procedures evolved during the course or project work or investigation undertaken by students; such submissions should be made under the joint name of the teacher and the student.

INVESTIGATION OF BOX RESONANCES USING A MICRO

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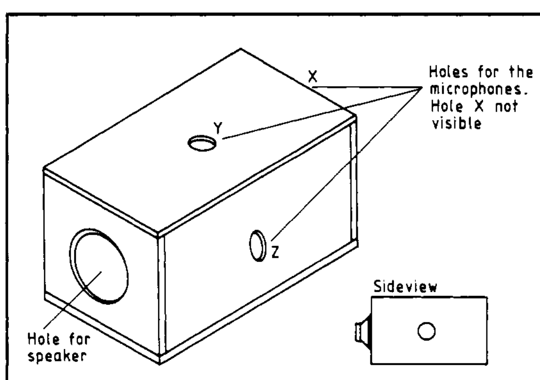
The purpose of this investigation was to measure the resonant frequencies in a box (figure 1) and to compare them with the predicted values. In order to make a detailed analysis it is necessary to take amplitude readings over a wide range of frequencies. A method was developed using a BBC micro with a Unilab interface to control the frequency of

Figure 1 Schematic diagram of the box used. The speaker was mounted externally for simplicity



† Christopher Briggs was in the sixth form when he carried out this project. He is now a student at the University of Bradford. *Physics Education* welcomes contributions from school students.

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