Precision studies in traps: Measurement of fundamental constants and tests of fundamental theories

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Abstract

Experiments on single atomic particles confined in Penning ion traps have contributed significantly to the improvements of fundamental constants and to tests of the theory of Quantum Electrodynamics for free and bound electrons. The most precise value of the fine structure constant as well as the electron mass have been derived from trap experiments. Numerous atomic masses of interest for fundamental questions have been determined with precisions of 10^{-9} or below. Further progress is envisaged in the near future.
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1. Introduction

The contribution of ion traps for testing a fundamental theory and for determination of fundamental constants started by the celebrated $g-2$ experiment by Dehmelt and coworkers who measured the magnetic moment of the free electron to 12 significant digits [1] using a single electron confined in a Penning trap – even 15 years later still one of the most outstanding experiments in atomic physics. The deviation of the g factor from the Dirac value $g = 2$ is described by the theory of Quantum Electrodynamics (QED). In the follow-ing years the calculation of the corresponding Feynman diagrams have been performed to a similar accuracy [2] and the agreement between the experimental and theoretical value has been considered as the most precise test of QED at low energies. The theoretical result has been obtained by perturbation theory which results in a series expansion with the fine structure constant α as expansion parameter. For the numerical comparison between experiment and theory, a value for α from non-QED experiments such as the quantum Hall effect or the ac-Josephson effect was needed. Agreement between experiments and calculations based on QED was found with no exception in all kinds of experiments and it was realized that it is a powerful concept to describe electromagnetic processes and considered as a valid theory ("QED

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is the jewel of physics, our proudest possession",
R. Feynman). Consequently the $g - 2$ experiment
and the corresponding calculation was no longer
regarded as test of QED but was used to extract a
value for α. In fact the most recent compilation of
fundamental constants by the CODATA [3] group
in year 2000 lists a value for α which is mainly
based on the electron g factor.

2. g factor of the bound electron

When we consider the g factor of the single
electron bound in hydrogen-like ions additional
deviations from the Dirac value of $g = 2$ occur.
The binding to the nucleus requires the solution of
the Dirac equation. For the ground state of a hy-
drogenic ion of nuclear charge Z this leads to a
value

$$g = 2/3\sqrt{1 + 2(1 - (Z\alpha)^2)}. \tag{1}$$

The binding changes also the QED part, leading to
many more Feynman diagrams which have to be
considered. The evaluation of these diagrams re-
quires enormous mathematical skills. Moreover
the perturbative approach as employed for the free
electron gives no longer reliable results. The
bound-state QED series expansion still is an ex-
pansion in α. The effect of binding can be consid-
ered by a second expansion series in the parameter
$Z\alpha$. For large values of Z this is no longer small
compared to 1. Consequently a non-perturbative
approach for this series has to be taken which
considers all orders in $Z\alpha$ simultaneously. Beier
et al. [4] and Shabaev and Yerokhin [5] have de-
veloped these methods and obtained numerical
values for the Feynman diagrams of the order
(α/π). Higher orders are not yet calculated but
their size can be reliably estimated.

Finally nuclear size effects and nuclear recoil
have an influence on the g factor of the bound
electron which particularly for high values of Z
become very significant. A careful account of the
present status of calculations is given by Yerokhin
et al. at this conference [6]. Fig. 1 summarizes the
theoretical contributions to the bound electron's g
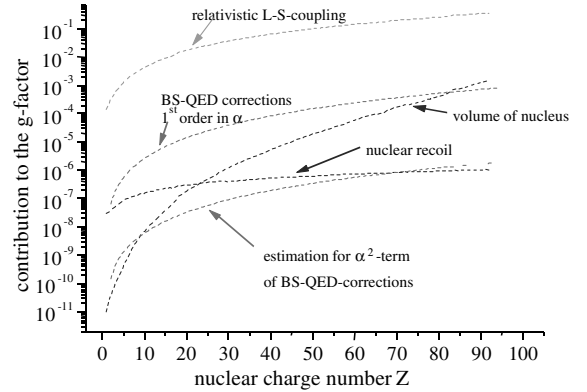factor for different values of the nuclear charge Z.

Fig. 1. Binding, QED, and nuclear corrections to the electron's
g factor bound in hydrogenic ions.

3. Bound state g factor measurements

We have performed measurements of the bound
electron g factor in two hydrogen-like systems:
$^{12}C^{5+}$ and $^{16}O^{7+}$. The experiments use single ions
confined in a Penning trap. The trap consists of a
stack of cylindrical electrodes of 7 mm inner dia-
meter. Three of these electrodes, a ring and two
end electrodes, create a potential minimum in the
axial direction by application of a static voltage
between them. This serves for axial confinement of
an ion. Radial confinement is achieved by the su-
perposition of a magnetic field parallel to the trap
axis. Between the ring and the end electrodes we
placed two correction electrodes. They can be used
to change the potential near the trap center in such
a way that it depends on the square of the coor-
dinates, forming a quadrupole potential. The whole
system including the attached electronics is in
thermal contact to a liquid He bath. The vacuum
in our system is estimated to be better than 10^{-16}
mbar. This number is derived from observed
storage of a cloud of about 30 C^{5+} ions for a
couple of weeks. Not a single charge exchange to
C^{4+} was observed during this time which leads to
an upper limit for the rest-gas density using the
known cross sections for charge exchange colli-
sions. In fact, once a single particle is trapped it
remains in our trap for several months.

The equations of motion for a single ion in such
a potential can be solved analytically and lead to a

superposition of three harmonic oscillations with the frequencies:

$$\omega_- = \frac{\omega_c}{2} - \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}}, \tag{2}$$

$$\omega_+ = \frac{\omega_c}{2} + \sqrt{\frac{\omega_c^2}{4} - \frac{\omega_z^2}{2}}, \tag{3}$$

$$\omega_z = \sqrt{\frac{eU}{Md^2}}. \tag{4}$$

From a measurement of these frequencies the free ion's cyclotron frequency $\omega_c = (q/M)B$ can be determined using the relation:

$$\omega_c^2 = \omega_+^2 + \omega_z^2 + \omega_-^2. \tag{5}$$

Our actual trap consists of two geometrically identical trap arrangements separated by 2.7 cm. The difference between them is that the ring electrode of one trap is made of ferromagnetic nickel while all other electrodes are out of gold-plated copper. The Ni ring distorts the magnetic field in a bottle-like manner. This feature is used as explained below for the determination of the spin direction of the bound electron. A sketch of our trap is shown in Fig. 2.
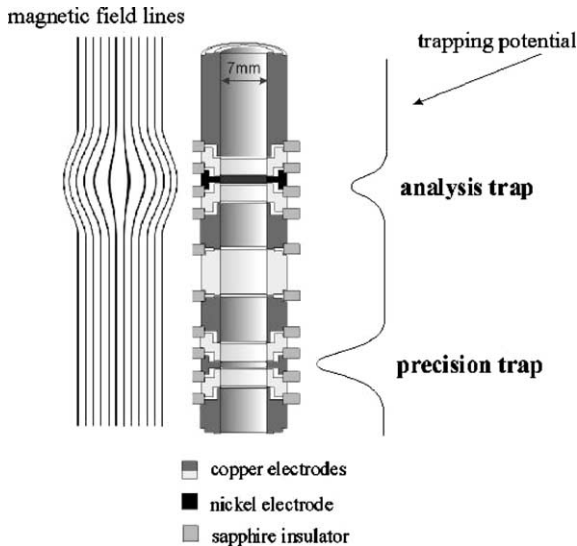
Attached to the electrodes which are split into different segments are electronic resonance circuits tuned to the ion's axial or cyclotron oscillation frequencies. An ion oscillating in the trap induces image currents in the electrodes resulting in a detectable signal in the circuit. When an ion oscillation is continuously in resonance with the circuits the thermal power is dissipated to the environment and the ion reaches the same temperature as the circuit. Keeping the circuits as well as the trap in contact to a liquid helium bath we arrive at an ion temperature of 4 K in a time depending on the quality factor Q of the circuits. Fig. 3 shows an example of resistive cooling of the axial mode of a single O^{7+} ion with a time constant of 132 ms by a superconducting circuit of $Q = 2000$.

We create ions by electron bombardment of a carbon-covered surface which also contains impurities like oxygen and others. Ions of many charges states of different species are formed and trapped in one of the traps. Unwanted ions are removed by excitation of their mass and charge-state dependent axial oscillation frequencies. A single ion of the species under investigation is prepared by carefully lowering the trap potential to boil off the ions until a single particle is left.

The ion's oscillation frequencies are also measured by the induced currents in the trap electrodes: Fig. 4 shows a Fourier transform of the electronic signal induced in the split ring electrode. It peaks at the frequency ω_+. The fractional full

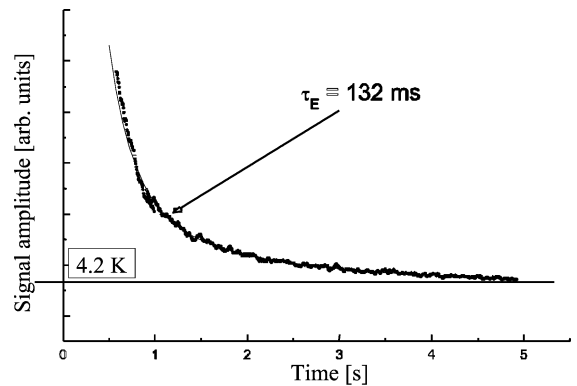Fig. 2. Electrode structure in double Penning trap.

Fig. 3. Resistive cooling of the axial oscillation of a single O^{7+} ion. The exponential decay constant of 132 ms is determined by the Q-value of the cooling circuit.
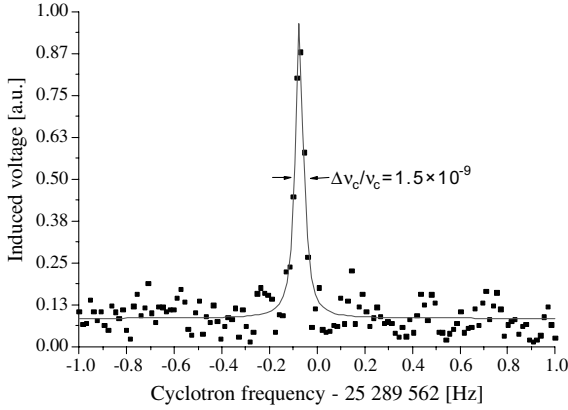
Fig. 4. Fourier transform of the induced voltage from a single O^{7+} ion in a circuit attached to the split ring electrode.

width at half maximum of the curve is of the order of 10^{-9} and its center can be determined to below 10^{-10}. An example for the axial frequency is shown in Fig. 5. Here the noise power in the detection circuit is reduced at the ion's oscillation frequency since at this frequency the induced noise from the residual thermal ion oscillation adds to the noise of the tank circuit with 180° phase difference. The axial frequency at about 369 kHz can be determined with a precision of less than 200 mHz.

In order to determine the g factor of the bound electron, we induce magnetic dipole transitions between the two spin states up and down in the
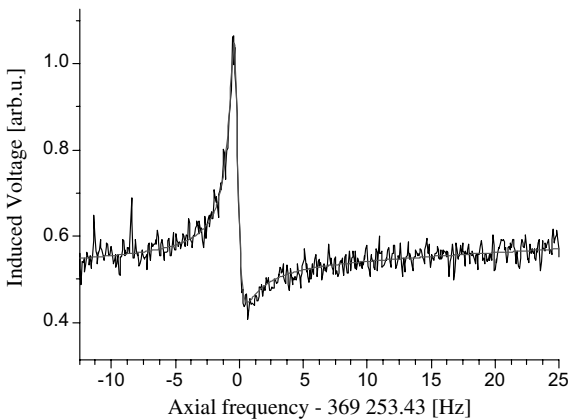

Fig. 5. Axial resonance of a single O^{7+} ion monitored by the induced voltage in a tank circuit attached to an electrode. Change in the resonance of more than 100 mHz can be detected.

magnetic field. The transition frequency ω_L (Larmor frequency) is related to the g factor by

$$\hbar\omega_L = \frac{e}{2mc}B. \tag{6}$$

In our field of 3.8 T this frequency is at 104 GHz. The magnetic field is calibrated by the ion's cyclotron frequency $\omega_c = (q/M)B$ derived using Eq. (5). The g factor follows:

$$g = 2\frac{\omega_L}{\omega_c}\frac{m}{M}\frac{q}{e}. \tag{7}$$

So we have to measure the ratio of Larmor to cyclotron frequency and take the ratio of the electron's to the ion's mass from the CODATA compilation of masses. While the measurement of the cyclotron frequency has been described above we use the "continuous Stern–Gerlach effect" for the detection of the spin direction [7,8]: The inhomogeneous magnetic field in one of our traps ("analysis trap") can be expanded in a power series along the z axis. The odd terms in the expansion vanish because of mirror symmetry: $B \approx B_0 + b_2z^2$. The total force in axial direction which an ion experiences is derived from the sum of the electric quadrupole potential $\Phi_{el} \sim z^2$, and the magnetic potential $\Phi_{magn} = \mu B = \mu(B_0 + b_2z^2)$. Because both parts depend on the square of the axial coordinate the potential remains harmonic. Its frequency, however, depends on the orientation of the magnetic moment μ to the B-field direction. The difference in axial oscillation frequency for the two spin directions is $\delta v_z = b_2\mu/4\pi^2mv_z$. For our geometry this amounts to 0.75 and 0.48 Hz for $^{12}C^{5+}$ and $^{16}O^{7+}$, respectively. Fig. 6 demonstrates for the case of O^{7+} that two distinct different axial frequencies with the expected difference can be observed when the ion is irradiated continuously with microwaves at the Larmor frequency. At the chosen amplitude of this microwave field in Fig. 6 a spin flip occurs typically every few minutes.

A measurement of the Larmor and cyclotron frequency in the analysis trap would be limited in precision by the inhomogeneity of the magnetic field. In order to obtain a higher precision we make use of our double trap structure: After a determination of the spin direction in the analysis trap the ion is transferred by adiabatic changes of
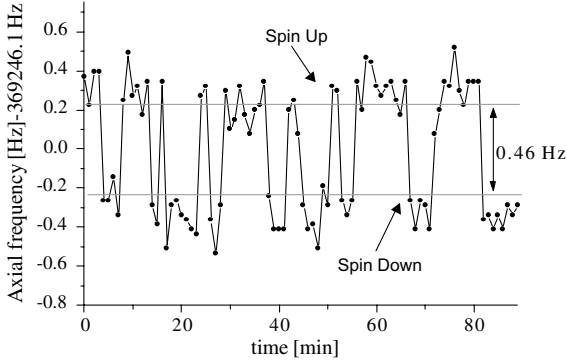
Fig. 6. Center frequency of the axial resonance in the analysis trap when the ion is continuously irradiated by microwaves at the Larmor precession frequency. Two distinct center frequencies of 0.4 Hz difference can be distinguished which refer to the two directions of the bound electron's spin. A spin flip occurs every few minutes.

Fig. 7. Spin flip probability in the precision trap for various values of the ratio v_L/v_C.

the voltages at the trap electrodes to the second trap where the magnetic field is homogeneous ("precision trap"). Here spin flips are induced by the microwave field and simultaneously the cyclotron frequency is measured. After a few minutes the ion is transferred back to the analysis trap and the spin direction is determined in order to decide whether its direction has been changed or not. In addition the axial and cyclotron mode are cooled repetitively. More details can be found in [9]. The procedure for a complete cycle as described above takes about 2 h. We obtain a typical rate of 2–4 spin flips per day depending on the microwave frequency and the performance of the components, particularly the stability of the trap voltage sources. A variation of the trapping voltage (1 V) by a few microvolts could mimic a spin transition by the corresponding change in the axial frequency. For a given microwave frequency and amplitude we obtain a spin flip probability defined as the ratio between successful changes of the spin direction to the total number of attempts. If we vary the frequency of the microwave field we obtain a resonance curve as shown in Fig. 7. Here we plotted directly the spin flip probability versus the ratio ω_L/ω_c. The simultaneous measurement of both frequencies makes sure that temporal fluctuations of the magnetic field strength cancel to first order. The example shown in Fig. 7 contains about
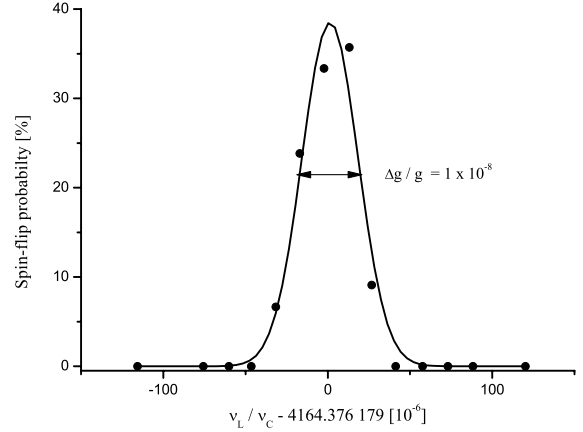
100 spin flips and is the result of about two months of measuring time. The resonance width is of the order of 10^{-8}. Multiplying the center-frequency ratio with the electron/ion-mass ratio and the ion's charge state gives the g factor. The results for $^{12}C^{5+}$ [9] and $^{16}O^{7+}$ are

$$g(C^{5+})_{exp} = 2.001\,041\,596\,3\,(10)\,(44),$$

$$g(O^{7+})_{exp} = 2.000\,047\,026\,0\,(15)\,(44).$$

The first error margin is the sum of our statistical and systematical errors and the second comes from the uncertainty of the electron mass of the CODATA tables, based on a measurement of the electron cyclotron frequency in a Penning trap by Van Dyck and coworkers [10]. Our results can be compared with the most recent theoretical calculations by Yerokhin et al. [6,11]:

$$g(C^{5+})_{theo} = 2.001\,041\,590\,1\,(3),$$

$$g(O^{7+})_{theo} = 2.000\,047\,020\,2\,(6).$$

They are in good agreement within the error bars. Thus the experimental result can be considered as a good test of the bound-state QED calculations.

4. Electron mass determination

As seen from the experimental values for both investigated ions the largest contribution for the

error budget comes from the uncertainty of the electron mass. Similarly as in the initially mentioned case of the free electron's g factor we can take a different point of view: Assuming the correctness of the theoretical values we can derive a value for the electron mass by rewriting Eq. (7):

$$m = \frac{g_{theo}}{2} \frac{\omega_c}{\omega_L} \frac{e}{q} M_{ion}. \qquad (8)$$

Taking into account the mass of the missing electrons and their sufficiently known binding energies we arrive at values for the electron mass in atomic units from measurements on C^{5+} and O^{7+}:

$$m(C^{5+}) = 0.000\,548\,579\,909\,3\,(3),$$

$$m(O^{7+}) = 0.000\,548\,579\,909\,4\,(4).$$

These values agree with the CODATA value [3]

$$m(COD) = 0.000\,548\,579\,911\,0\,(12)$$

but the error bar has been reduced by a factor of 4. It should be stressed that both values from the bound-electron measurements cannot be regarded as independent since a common systematic error could be present in the experimental or the theoretical value for the g factor. More details of the electron mass determination can be found in [12].

5. Mass determinations

Resonant excitations of the cyclotron oscillation of a trapped ion such as shown in Fig. 4 form the basis of precision mass spectrometry which in many cases is linked to fundamental constants. The ratio of cyclotron frequencies of two different ions in the same magnetic field is the inverse of their mass ratio. If carbon as the atomic mass standard is taken as a reference one obtains directly the mass of an ion in atomic units, from which the mass of the neutral atom can be derived. Ideally the masses are compared under identical conditions: Two ions having the same energy should be at the same time and the same place in a perfect trap. Different approaches have been taken to come close to this ideal situation: Making the trap potential as harmonic as possible by correction electrodes, minimizing the temporal fluctuations of the magnetic field of superconducting coils by pressure and temperature stabilization, cooling the ions to the same low temperature, and switching between two ions as fast as possible. These attempts undertaken by different groups have led in the past years to mass comparisons on the level of a part in 10^{-9} or below and even uncertainties as low as 10^{-11} are envisaged [13]. A number of masses have been determined which are directly related to fundamental constants. They are listed in Table 1. Apart from the obviously fundamental values of the electron and proton mass, the ^{3}He mass is of importance for neutrino mass determination [14], the mass of ^{28}Si is required for an attempted new definition of the kilogram based on silicon crystals [15], the ^{133}Cs mass has to be known accurately for a determination of α from photon recoil experiments [16],

Table 1
Examples of precision masses of interest for fundamental constants and interactions determined in Penning traps

Particle	Mass (atomic unit)	Relative uncertainties $[10^{-9}]$	Reference
Electron	0.000 548 579 911 1 (12)	2	D.L. Farnham et al., Phys. Rev. Lett. 75 (1995) 3598
Proton	1.007 825 032 14 (35)	0.35	D.L. Farnham et al., Phys. Rev. Lett. 75 (1995) 3598
^{3}Helium	3.016 029 323 5 (28)	1	T. Fritioff et al., Eur. Phys. J. D 15 (2001) 141
^{28}Silicon	27.976 926 57 (30)	10	P. De Bièvre et al., IEEE Trans. Instr. Meas. 50 (2001) 593
^{133}Cesium	132.905 451 931 (27)	0.2	M.P. Bradley et al., Phys. Rev. Lett. 83 (1999) 4510
$M(\bar{p})/M(p)$	0.999 999 999 91 (9)	0.1	G. Gabrielse et al., Phys. Rev. Lett. 82 (1999) 3198

and finally the mass comparison of the proton and antiproton can be regarded as test of CPT symmetry in hadronic systems [17].

6. Future possibilities

Presently we are working on improvements in the experimental determination of g by testing different schemes to determine the required frequency ratio ω_L/ω_c. Preliminary results have been already obtained [18] and we consider as possible a reduction of the experimental error bar by a factor of 3. Then the experimental error would match the present theoretical uncertainty.

The measurement of the g factor of the bound electron in hydrogen-like systems may also serve for a new determination of α. The largest deviation of g from the free particles value arises from the binding and is described in Eq. (1). Assuming the QED correction as small and known we can determine α from the difference of g for the free and the bound electron. The uncertainty in α is linked to the uncertainty in g by

$$\delta\alpha/\alpha \approx (1/Z\alpha)\delta g/g. \tag{9}$$

In order to minimize the uncertainty of α it would be desirable to measure g at nuclear charges as high as possible. Then, however, the QED part and also the nuclear correction are no longer small and may not be known well enough. As a good compromise between the conflicting requirements we consider a system like Ca^{19+} for which the QED contributions have been calculated [5]. Assuming the same experimental and theoretical uncertainty as we have obtained so far for C^{5+} and O^{7+} we would arrive at an uncertainty for α of about 10^{-8}. This is comparable to the precision of non-QED determinations of α. A more detailed consideration of uncertainties in α from a g factor measurement can be found by the contribution of Th. Beier et al. in this volume [19].

7. Summary

Ion traps have made a significant contribution to the determination of fundamental constants: At present the most precise value for the fine structure constant and for the mass of the electron in atomic units have been derived from ion-trap measurements. These derivations, however, require the validity of the underlying theory of QED. This validity is generally assumed since QED has been tested in numerous experiments and so far without exception no significant discrepancy between experiment and theory has been found. Precision measurements then test the correctness of the actual QED calculation and have in fact in some cases led to the discovery of computational errors in the complex calculations.

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