Calculating vertical velocity of APEX floats

The vertical velocity of water can be determined as the difference between the measured vertical velocity of a float and the vertical velocity that the float would have in stationary water, i.e.

$$W_{Water} = W_{Float} - W_0 \tag{1}$$

The vertical motion of the float can be determined form the rate of change of pressure, but the velocity the float would have in stationary water must be modelled. Merckelbach, Smeed & Griffiths (2009), hereafter MSG09, have used this approach to determine vertical water velocity from glider measurements.

If acceleration can be neglected then the vertical velocity of a descending float in motionless water is determined by a balance of forces arising from drag, buoyancy and gravity.

$$\rho C_D A W_0^2 = \left| g(\rho V - M) \right| \tag{2}$$

where ρ is the density of water, C_D is the drag coefficient, A, the cross-sectional area of the float, g acceleration due to gravity, M is the mass of the float and V the volume.

The mass of the float is fixed but the density of the surrounding water varies as does the volume of the float. Three factors determine the volume of the float:

- The piston position is used to control the ascent and descent rates. This is recorded as “piston counts’, k. We assume that the volume change is proportional to k and can be expressed as $\Delta V\left(k - k_0\right)$. Where the float has volume V_0 when $k = k_0$.
- The compression of the float with increasing pressure with compressibility κ.
- Thermal expansion of the float expressed as $\alpha\left(T - T_0\right)$

Defining ρ_0 as the density at which the float is neutrally buoyant when $k = k_0$, $p = 0$ and $T = T_0$ and assuming that the changes in volume are small compared to V_0, equation (1) can be approximated as

$$\frac{W_0^2}{W_F^2} = \left| \frac{\Delta V}{V_0}(k - k_0) + \left(\frac{\rho}{\rho_0} - 1 - \kappa p + \alpha\left(T - T_0\right) \right) \right| \tag{3}$$

where the scale speed, W_F is given by $W_F^2 = \dfrac{C_D A}{g V_0}$,

There are then 7 parameters in equation (3): $W_F, \dfrac{\Delta V}{V_0},\ k_0,\ \kappa,\ \alpha,\ \rho_0, \text{and } T_0$.

To estimate the value of these parameters the data from float deployments was analysed as follows.

The vertical velocity, W, of the float was determined form the rate of change of pressure. From this was calculated the variable

$$W_2 = \operatorname{sign}(W)\, W^2 \tag{4}$$

so that W_2, is positive when the float is rising and negative as it dives. The parameters $a, b, c, \text{ and } d$ were then determined by minimising the residual sum of squares

$$\sum_{Data} \left(W_2^2 - W_M^2\right)^2 \tag{5}$$

where

$$W_M^2 = \alpha + b\,k + c\,\rho + d\,p \tag{6}$$

From these we can determine

$$k_0 = -\frac{a}{b},\ \frac{\Delta V}{V_0} = \frac{b}{\rho_0 c},\ \kappa = \frac{d}{\rho_0 c},\ \text{and } W_f = \mathrm{sqrt}\left(\rho_0 c\right) \tag{7}$$

The robustness of the solution was tested by minimising (5) for a number of different subsets of the data. It was found that the solution for $a \text{ and } b$ was robust but that the solution of $c \text{ and } d$ was dependent up on the data used. This is to be expected since $\rho \text{ and } p$ are closely correlated with both increasing as a function of depth. The temperature term was not included either because this to is closely correlated with $\rho \text{ and } p$. Parameters $c \text{ and } d$ are not expected to vary significantly during the deployment, however, MSG09 showed that the drag coefficient for gliders did increase due to biofouling. The same may be expected to happen for floats. Thus a modified form of (6) was used. First the parameters $\frac{V_0}{\Delta V}$ and κ were determined from the floats design specification and/or the minimisation process using the complete dataset.[1] Then

$$W_M^2 = \alpha + b\left(k + \frac{V_0}{\Delta V}\left(\frac{\rho}{\rho_0} - \kappa p\right)\right) \tag{8}$$

was substituted in (5) for subsections of the data to determine the change in W_f with time.

Float ID	Volume increment $\frac{V_0}{\Delta V}$ x 10^5	Compressibility κ (dbar^{-1}) x 10^6
4976	11.5	2.03
4977	6.42	2.93

Note on calculations

Program xem_vinterp.m changes sign of W on up profiles so that w is always negative for ease of plotting.

[1] At this time I have used minimisation but I expect that the values of these parameters are known by someone.