Lectures in Open Economy Macroeconomics

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Chapter 1

The Current Account in an Endowment Economy

The purpose of this chapter is to build a canonical dynamic, general equilibrium model of the open economy capable of capturing a number of basic empirical regularities associated with the current account and other macroeconomic variables of interest.

1.1 The Model Economy

Consider an economy populated by a large number of infinitely lived households with preferences described by the utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t), \tag{1.1}$$

where c_t denotes consumption of the single, perishable good available in the economy, and U denotes the single-period utility function, which is assumed to be strictly increasing and strictly concave.

The evolution of the debt position of the representative household is given by

$$d_t = (1+r)d_{t-1} + c_t - y_t. \tag{1.2}$$

where d_t denotes the debt position chosen in period t, r denotes the interest rate, assumed to be constant, and y_t is an exogenous and stochastic endowment of goods. The endowment process represents the sole source of uncertainty in this economy. The above constr4aint states that the change in the level of debt, $d_t - d_{t-1}$, has two sources, interest services on previously acquired debt, rd_{t-1}, and excess expenditure over income, $c_t - y_t$. Households are subject to the following borrowing constraint that prevents them from engaging in Ponzi games:

$$\lim_{j\to\infty} E_t \frac{d_{t+j}}{(1+r)^j} \leq 0. \tag{1.3}$$

This limit conditions says that the household's debt position must grow at a rate lower than the interest rate r. The optimal allocation of debt will always feature this constraint holding with strict equality. This is because if the allocation $\{c_t, d_t\}_{t=0}^{\infty}$ satisfies the no-Ponzi-game constraint with strict inequality, then one can choose an alternative allocation $\{c'_t, d'_t\}_{t=0}^{\infty}$ that also satisfies the no-Ponzi-game constraint and satisfies $c'_t \geq c_t$, with $c'_{t'} > c_{t'}$ for at least one date $t' \geq 0$. This alternative allocation is clearly strictly prefer to the original one because the single period utility function is strictly increasing. The household chooses sequences for c_t, and d_t for $t \geq 0$, so as to maximize (1.1) subject to (1.2) and (1.3). The optimality conditions associated with this problem are (1.2), (1.3) holding with equality, and the following Euler condition:

$$U'(c_t) = \beta(1+r)E_t U'(c_{t+1}). \tag{1.4}$$

The interpretation of this expression is simple. If the household sacrifices one unit of consumption in period t and invests it in financial assets, its period-t utility falls by $U'(c_t)$. In period $t+1$ the household receives the unit of goods invested plus interests, $1+r$, yielding $\beta(1+r)U'(c_{t+1})$ utils. At the optimal allocation, the cost and benefit must equal each other.

We make two additional assumptions that greatly facilitates the analysis. First we require that the subjective and pecuniary rates of discount, β and $1/(1+r)$ are equal to each other, that is,

$$\beta(1+r) = 1.$$

This assumption elininates long-run growth in consumption. Second, we assume that the period utility index is quadratic and given by

$$U(c) = -\frac{1}{2}(c - \bar{c})^2, \tag{1.5}$$

with $c < \bar{c}$.[1] This particular functional form makes it possible to obtain a closed-form solution of the model. Under these assumptions, the Euler

[1]You should recognize these as the assumptions of the Hall (1978) permanent income model of consumption.

condition (1.4) collapses to

$$c_t = E_t c_{t+1}, \tag{1.6}$$

which says that consumption follows a random walk.

We will now use the household's sequential budget constraint (1.2) and the no-Ponzi-scheme constriant (1.3) holding with equality—also known as the transversality condition—to derive an intertemporal resource constraint. Begin by expressing the sequential budget constraint in period t as

$$(1+r)d_{t-1} = d_t + y_t - c_t$$

Lead this equation 1 period and use it to get rid of d_t:

$$(1+r)d_{t-1} = y_t - c_t + \frac{y_{t+1} - c_{t+1}}{1+r} + \frac{d_{t+1}}{1+r}$$

Repeat this procedure s times to get

$$(1+r)d_{t-1} = \sum_{j=0}^{s} \frac{y_{t+j} - c_{t+j}}{(1+r)^j} + \frac{d_{t+s}}{(1+r)^s}$$

Apply expectations conditional on information available at time zero and take the limit for $s \to \infty$ using the transversality condition (equation (1.3) holding with equality) to get the following intertemporal resource constraint:

$$(1+r)d_{t-1} = E_t \sum_{j=0}^{\infty} \frac{y_{t+j} - c_{t+j}}{(1+r)^j}.$$

Intuitively, this equation says that the initial net foreign debt position must be equal to the expected present discounted value of current and future differences between output and absorption.

Now use the Euler equation (1.6) to deduce that $E_t c_{t+j} = c_t$. Using this result to get rid of expected future consumption in the above expression and rearrange to obtain

$$(1+r)[rd_{t-1} + c_t] = rE_t \sum_{j=0}^{\infty} \frac{y_{t+j}}{(1+r)^j}. \tag{1.7}$$

To be able to fully characterize the equilibrium in this economy, we assume that the endowment process follows an AR(1) process of the form,

$$y_t = \rho y_{t-1} + \epsilon_t$$

Then, the j-period ahead forecast of output in period t is given by

$$E_t y_{t+j} = \rho^j y_t.$$

Using this expression to eliminate expectations of future income from equation (1.7), we obtain

$$\begin{aligned}(1+r)[rd_{t-1}+c_t] &= y_t r \sum_{j=0}^{\infty} \left(\frac{\rho}{(1+r)}\right)^j \\ &= y_t r \frac{1+r}{1+r-\rho}\end{aligned}$$

Solving for c_t, we obtain

$$c_t = \frac{r}{1+r-\rho} y_t - rd_{t-1}$$

according to this expression, households spend each period the annuity associated with their endowment stream. Part of this income they allocate to consumption and the rest to serve their financial oblitations. Letting $tb_t \equiv y - c_t$ and $ca_t \equiv -rd_{t-1} + tb_t$ denote, respectively, the trade balance and the current account in period t, we have

$$tb_t = rd_{t-1} + \frac{1-\rho}{1+r-\rho} y_t$$

$$ca_t = \frac{1-\rho}{1+r-\rho} y_t$$

1.2 Response of the Current Account to Output Shocks

Assume that $0 < \rho < 1$, so that endowment shocks are positively serially correlated, and consider the response of our model economy to an unanticipated endowment shock. Two polar cases are of particular interest. In the first case, the endowment shock is assumed to be purely transitory, $\rho = 0$. In this case, only a very small fraction, $r/(1+r$, of the increase in output is allocated to current consumption. Most of the endowment increase—a fraction $1/(1+r))$—is saved. The intuition for this result is clear. Because the shock is temporary, households smooth consumption by eating a tiny part of it today and leaving the rest for future consumption. In this case, the current account plays the role of a shock absorber. Households borrow to finance negative shocks and save in response to positive shocks. As a result, the current account is procyclical, improving during expansions and deteriorating during contractions.

Consider the case of highly persistent shocks, $\rho \to 1$. In this case, households allocate all of the increase in output to current consumpiton. Thus, the current account is unchanged. Intuitively, if the shock is permanent, the best response is to adjust to the new level of output by increasing or reducing the standard of living according to whether the shock is positive or negative.

Therefore, in this model, which captures the esence of what has became known as the intertemporal approach to the current account, external borrowing is conducted under the the principle: 'finance temporary shocks, adjust to permanent shocks.'

1.3 Current Account Dynamics With Nonstationary Income Shocks

The main prediction of the model presented in the previous section is the procyclicality of the current account-to-output ratio in economies holding a positive asset position with respect to the rest of the world. As we will see shortly, this prediction is counterfactual. In most countries the current account is countercyclical. That is, periods of economic expansion are typically characterized by current account deficits, and recessions take place in the context of current account surpluses in the external account. Here, we explore a simple modification to the model that gets around its problematic prediction. The modification consists in assuming that the the income process is nonstationary. Istead, we assume stationarity in the growth rate of income. Consider an infinitely lived household with preferences given by

$$\max -\frac{1}{2}E_0 \sum_{0}^{\infty} \beta^t (c_t - \bar{c}_t)^2.$$

Note that the satiation point $\bar{c}_t$, is now time varying. We will say more about this variable shortly. As in the model of the previous section, the household's problem consists in maximazing this utility fucntion subject to the resource constraint

$$d_t = (1 + r)d_{t-1} + c_t - y_t$$

Suppose that the endowment process evolves according to the following law of motion

$$y_t = (1+\mu)(1+\epsilon_t)y_{t-1},$$

where ϵ_t is an i.i.d. shock with mean zero. The parameter $\mu \geq 0$ denotes the mean growth rate of he endowment. The level of income is nonstationary, in the sense that an output shock produces a permanent increase in the level of output. The growth rate of output, given by $\ln(y_t/y_{t-1} \approx \mu + \epsilon_t$, is i.i., with mean μ.

Let us scale all variables in the model by tthe level of income. Dividing the resource constraint through by y_{t-1} we obtain

$$\tilde{d}_t = \frac{1+r}{1+\mu}\frac{\tilde{d}_{t-1}}{1+\epsilon_t} + \tilde{c}_t - 1,$$

where a tilde on a variable denotes the ratio of that varialbe to output. Assume that the subsistence level of consumption, $\bar{c}_t$ satisfies $\bar{c}_t/y_t = \bar{c}$, where $\bar{c}$ is a pocitive constant. Then we can write the utility functon as

$$-\frac{1}{2}E_0\sum_{0}^{\infty}\beta^t(\tilde{c}_t - \bar{c})^2 y_t^2$$

The first-order conditon of the household's problem is

$$(\tilde{c}_t - \bar{c})y_t^2 = \beta\frac{1+r}{1+\mu}E_t(\tilde{c}_{t+1} - \bar{c})\frac{y_{t+1}^2}{1+\epsilon_{t+1}}$$

For convenience, we will assume that the drift parameter is nil. That is,

$$\mu = 0.$$

Then, imposing the parameter restriction $\beta(1+r) = 1$, the above optimality conditions can be written as

$$\tilde{c}_t - \bar{c} = E_t(\tilde{c}_{t+1} - \bar{c})(1+\epsilon_{t+1})$$

Linearizing the resource constraint and this expression around the deterministic steady state, we obtain, respectively

$$\Delta d_t = (\Delta d_{t-1} - \tilde{d}\epsilon_t) + \Delta c_t$$

and

$$\Delta c_t = (1+r)E_t\Delta c_{t+1}.$$

Solving the first of these equations forward yields

$$-(1+r)\Delta d_{t-1} = E_t \sum_{j=0}^{\infty} (1+r)^{-j} [\Delta c_{t+j} - \tilde{d}\epsilon_{t+j}]$$

Using the linearized Euler equation and the fact that the income innovation has mean zero, yields

$$-(1+r)\Delta d_{t-1} = \frac{1+r}{r}\Delta c_t - \tilde{d}\epsilon_t$$

Solving for consumption, we have that

$$\Delta c_t = -r\Delta d_{t-1} + \frac{r\tilde{d}}{1+r}\epsilon_t$$

Letting $tby_t \equiv (y_t - c_t)/y_t \equiv 1 - \tilde{c}_t$ denote the trade balance to output ratio in period t, we have that

$$\Delta tby_t = r\Delta d_{t-1} - \frac{rd}{1+r}\epsilon_t$$

So the trade balance to output ratio deteriorates in response to a positive innovation in output growth. The intuition behind this result is as follows: Because the income shock is permanent, consumption increases roughly by the same amount as income. But if the country is indebted, consumption must be less than, as the economy allocates resources to service its external obligations. Therefore, if consumption and income increase by the same amount, the percent increase in consumption is larger than the percent increase in income. Thus, the trade balance to output ratio falls. An interesting testable implication of the simple model analyzed here is that the trade balance to output ratio should be more countercyclical the more indebted is the country. Moreover, the model predicts that the trade balance to output ratio must behave procyclicaly in countries holding a positive net foreign asset position.

1.4 Empirical Evaluation of the Endowment Economy

A key implication of the model economy we are analyzing is that the trade balance and the current account improve in response to a positive output shock. Here we ask the question of whether this prediction is born by the data. To this end, we consider the predictions of a simple empirical model.

The empirical model we consider is the following VAR system:[2]

$$A \begin{bmatrix} \hat{y}_t \\ \hat{\imath}_t \\ tby_t \\ \hat{R}_t^{us} \\ \hat{R}_t \end{bmatrix} = B \begin{bmatrix} \hat{y}_{t-1} \\ \hat{\imath}_{t-1} \\ tby_{t-1} \\ \hat{R}_{t-1}^{us} \\ \hat{R}_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_t^y \\ \epsilon_t^i \\ \epsilon_t^{tby} \\ \epsilon_t^{rus} \\ \epsilon_t^r \end{bmatrix} \tag{1.8}$$

where y_t denotes real gross domestic output, i_t denotes real gross domestic investment, tby_t denotes the trade balance to output ratio, R_t^{us} denotes the gross real US interest rate, and R_t denotes the gross real (emerging) country interest rate. A hat on top of y_t and i_t denotes log deviations from a log-linear trend. A hat on R_t^{us} and R_t denotes simply the log. We measure R_t^{us} as the 3-month gross Treasury bill rate divided by the average gross US inflation over the past four quarters.[3] We measure R_t as the sum of J. P. Morgan's EMBI+ stripped spread and the US real interest rate. Output, investment, and the trade balance are seasonally adjusted. More details on the data are provided in Uribe and Yue (2003).

We identify our VAR model by imposing the restriction that the matrix A be lower triangular with unit diagonal elements. Because R_t^{us} and R_t appear at the bottom of the system, our identification strategy presupposes that innovations in world interest rates (ϵ_t^{rus}) and innovations in country interest rates (ϵ_t^r) percolate into domestic real variables with a one-period lag. At the same time, the identification scheme implies that real domestic shocks (ϵ_t^y, ϵ_t^i, and ϵ_t^{tby}) affect financial markets contemporaneously. We believe our identification strategy is a natural one, for, conceivably, decisions such as employment and spending on durable consumption goods and investment goods take time to plan and implement. Also, it seems reasonable to assume that financial markets are able to react quickly to news about the state of the business cycle in emerging economies.[4]

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[2]The material in this section draws heavily from Uribe and Yue (2003).

[3]Using a more forward looking measure of inflation expectations to compute the US real interest rate does not significantly alter our main results.

[4]But alternative ways to identify ϵ_t^{rus} and ϵ_t^r are also possible. In Uribe and Yue (2003), we explore an identification scheme that allows for real domestic variables to react contemporaneously to innovations in the US interest rate or the country spread. Under this alternative identification strategy, the point estimate of the impact of a US-interest-rate shock on output and investment is slightly positive. For both variables, the two-standard-error intervals around the impact effect include zero. Because it would be difficult for most

We estimate the VAR system (1.8) equation by equation using an instrumental-variable method for dynamic panel data.[5] The estimation results are shown in table 1.1. The estimated system includes an intercept and country specific fixed effects (not shown in the table). We include a single lag in the VAR because adding longer lags does not improve the fit of the model. In estimating the VAR system, we assume that R_t^{us} follows a simple univariate $AR(1)$ process (i.e., we impose the restriction $A_{4i} = B_{4i} = 0$, for all $i \neq 4$). We adopt this restriction for a number of reasons. First, it is reasonable to assume that disturbances in a particular (small) emerging country will not affect the real interest rate of a large country like the United States. Second, the assumed AR(1) specification for R_t^{us} allows us to use a longer time series for R^{us} in estimating the fourth equation of the VAR system, which delivers a tighter estimate of the autoregressive coefficient $B(4,4)$. (Note that R_t^{us} is the only variable in the VAR system that does not change from country to country.) Lastly, the unrestricted estimate of the R_t^{us} equation features statistically insignificant coefficients on all variables except those associated with the lagged US interest rate (B_{44}) and the contemporaneous trade balance-to-GDP ratio (A_{43}). In addition, the point estimate of A_{43} is small.[6] We suspect that the positive coefficient on tby_t in the R_t^{us} equation is reflective of omitted domestic US variables, particularly variables measuring US aggregate activity. This is because in periods of economic expansion in the United States, the Fed typically tightens monetary policy. At the same time, during expansions the US economy typically runs trade balance deficits, which means that those small countries that export primarily to the United States are likely to run trade surpluses during such periods. These omitted variables would contaminate our estimate of ϵ_t^{rus} insofar as domestic US shocks transmit to emerging market economies through channels other than the US interest rate, such as the terms of trade. Obviously, our estimate

models of the open economy to predict an expansion in output and investment in response to an increase in the world interest rate, we conclude that our maintained identification assumption that real variables do not react contemporaneously to innovations in external financial variables is more plausible than the alternative described here.

[5]Our model is a dynamic panel data model with unbalanced long panels ($T > 30$). The model is estimated using the Anderson and Hsiao's (1981) procedure, with lagged levels serving as instrument variables. Judson and Owen (1999) find that compared to the GMM estimator proposed by Arellano and Bond (1991) or the least square estimator with (country specific) dummy variables, the Anderson-Hsiao estimator produces the lowest estimate bias for dynamic panel models with $T > 30$.

[6]In effect, the point estimate of $A_{4,3}$ is -0.082, which implies that a large increase in the trade balance of 1 percentage point of GDP ($\Delta tby_t = 0.01$) produces an increase in the US interest rate of 0.08 percentage points.

of world interest rate shocks depend crucially on the maintained specification of the fourth equation in the VAR system. But clearly the estimate of the country spread shocks is independent of the particular specification assumed for the fourth equation of the VAR system. Using the unrestricted estimate of the R_t^{us} equation delivers impulse responses to US interest rate shocks that are similar to those implied by the AR(1) specification but with much wider error bands around them.[7] We estimate the $AR(1)$ process for R_t^{us} for the period 1987:Q3 to 2002:Q4. This sample period corresponds to the Greenspan era, which arguably ensures homogeneity in the monetary policy regime in place in the United States.

Figure 1.1 displays the response of the empirical model to a positive output shock of unit size. Output increases on impact and then converges gradually to its long-run level. In turn, the trade balance falls significantly below trend for a couple of quarters and before returning gradually to its pre-shock level. The decline in the trade balance in response to the output shock indicates that, on impact, domestic absorption increases by more than output. This is at odds with the predictions of our theoretical model.

The impulse response of investment, a variable that our endowment theoretical economy does not feature, could be the clue for explaining the coutercyclical behavior of the trade balance. The figure shows that in response to a one percent increase in output, investment spending jumps up by about three percent. This suggests that a promising avenue for explaining the joint beavior of output and the external accounts is introduce investment in the theoretical model. This is precisely what we will accomplish in the next chapter.

1.5 Other Tests of the Model

Hall (1978) was the first to explore the econometric implication of the simple model developed in this chapter. Specifically, Hall tested the prediction that consumption must follow a random walk. Hall's work motivated a large empirical literature devoted to testing the empirical relevance of the model described above. Campbell (1987), in particular, deduced and tested a number of theoretical restrictions on the equilibrium behavior of national savings. In the context of the open economy, Campbell's restrictions are readily expressed in terms of the current account. Here we review these restrictions and their empirical validity.

[7] The results are available from the authors upon request.

We start by presenting an additional representation of the current account that involves expected future changes in income. Noting that the current account in period t, denoted ca_t, is given by $y_t - c_t - rd_{t-1}$ we can write equation (1.7) as

$$-(1+r)ca_t = -y_t + rE_t \sum_{j=1}^{\infty} (1+r)^{-j} y_{t+j}.$$

Defining $\Delta x_{t+1} = x_{t+1} - x_t$, it is simple to show that

$$-y_t + rE_t \sum_{j=1}^{\infty} (1+r)^{-j} y_{t+j} = (1+r) \sum_{j=1}^{\infty} (1+r)^{-j} E_t \Delta y_{t+j}$$

Combining the above two expression we can write the current account as

$$ca_t = -\sum_{j=1}^{\infty} (1+r)^{-j} E_t \Delta y_{t+j} \tag{1.9}$$

Intuitively, this expression states that the country borrows from the rest of the world (runs a current account deficit) income is expected to grow in the future. Similarly, the country chooses to build its net foreign asset position (runs a current account surplus) when income is expected to decline in the future. In this case the country saves for a rainy day.

Consider now an empirical representation of the time series Δy_t and ca_t. Define

$$x_t = \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}$$

Consider estimating a VAR system including x_t:

$$x_t = D x_{t-1} + \epsilon_t$$

Let H_t denote the information contained in the vector x_t. Then, from the above VAR system, we have that the forcst of x_{t+j} given H_t is given by

$$E_t[x_{t+j}|H_t] = D^j x_t$$

It follows that

$$\sum_{j=1}^{\infty} (1+r)^{-j} E_t[\Delta y_{t+j}|H_t] = \begin{bmatrix} 1 & 0 \end{bmatrix} [I - D/(1+r)]^{-1} D/(1+r) \begin{bmatrix} \Delta y_t \\ ca_t \end{bmatrix}$$

Let $F \equiv - \left[\begin{array}{cc} 1 & 0 \end{array} \right] [I - D/(1+r)]^{-1} D/(1+r)$. Now consider running a regression of the left and right hand side of equation (1.9) onto the vector x_t. Since x_t includes ca_t as one element, we obtain that the regression coeffient for the left-hand side regression is the vector $[0 \quad 1]$. The regression coefficients of the right-hand side regression is F. So the model implies the following restriction on the vector F:

$$F = [0 \quad 1].$$

Nason and Rogers (2003) perform an econometric test of this restriction. They estimate the VAR system using Canadian data on the current account and GDP net of investment and government spending. The estimation sample is 1963:Q1 to 1997:Q4. The VAR system that Nason and Rogers estimate includes 4 lags. In computing F, they calibrate r at 3.7 percent per year. Their data strongly rejects the above cross-equation restriction of the model. The Wald statistic associated with null hypothesis that $F = [0] quad 1]$ is 16.1, with an asymptotic p-value of 0.04. This p-value means that if the null hypothesis was true, then the Wald statistic, which reflects the discrepancy of F from $[0 \quad 1]$, would take a value of 16.1 or higher only 4 out of 100 times.

Consider now an additional testable cross-equation restriction on the theoretical model. From equation (1.9) it follows that

$$E_t ca_{t+1} - (1+r) ca_t - E_t \Delta y_{t+1} = 0. \tag{1.10}$$

According to this expression, the variable $ca_{t+1} - (1+r)ca_t - \Delta y_{t+1}$ is unpredictable in period t. In perticular, if one runs a regression of this variable on current and past values of x_t, all coefficients should be equal to zero.[8]

This restriction is not valid in a more general version of the model featuring private demand shocks. Consider, for instance, a variation of the model economy where the bliss point is a random variable. Specifically, replace $\bar{c}$ in equation (1.5) by $\bar{c} + \mu_t$, where $\bar{c}$ is still a constant, and μ_t is an i.i.d. shock with mean zero. In this environment, equation (1.10) becomes

$$E_t ca_{t+1} - (1+r) ca_t - E_t \Delta y_{t+1} = \mu_t$$

Clearly, because in general μ_t is correlated with ca_t, the orthogonality condition stating that $ca_{t+1} - (1+r)ca_t - \Delta y_{t+1}$ be orthogonal to variables

[8]Consider projecting the left- and right-hand sides of this expression on the information set H_t. This projection yields the orthogonality restriction $[0 \quad 1][D-(1+r)I]-[1 \quad 0]D = [0 \quad 0]$.

dated t or earlier, will not hold. Nevertheless, in this case we have that $ca_{t+1} - (1+r)ca_t - \Delta y_{t+1}$ should be unpredictable given information available in peroiod $t-1$ or earlier.[9] Both of the orthogonality conditions discussed here are strongly rejected by the data. Nason and Rogers (2003) find that a test of the hypothesis that all coefficients are zero in a regression of $ca_{t+1} - (1+r)ca_t - \Delta y_{t+1}$ onto current and past values of x_t has a p-value of 0.06. The p-value associated with a regression featuring as regressors past values of x_t is 0.01.

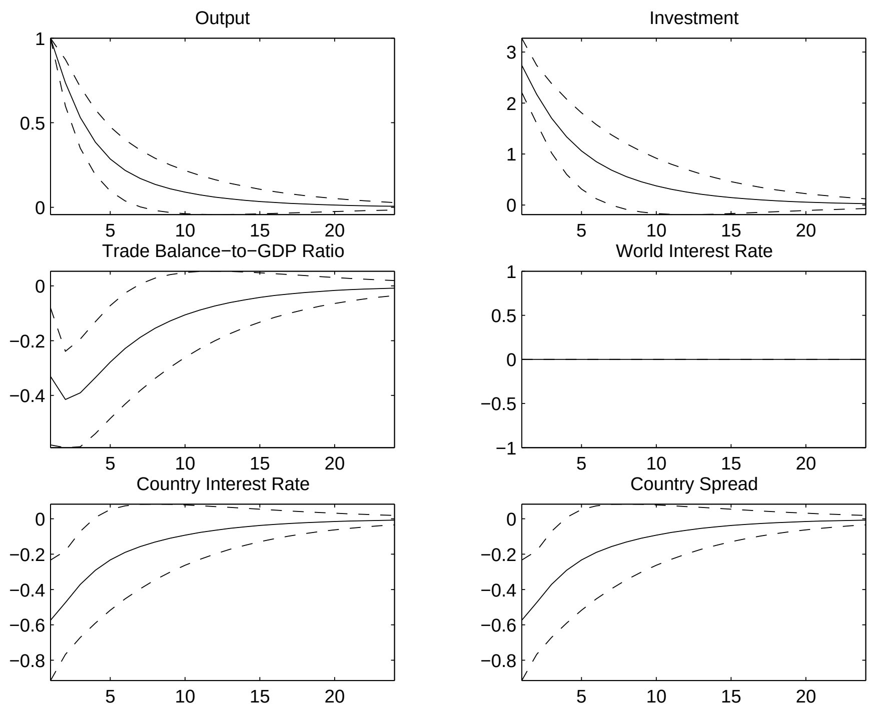
[9]In particular, one can consider projecting the above expressino onto Δy_{t-1} and ca_{t-1}, This yields the orthogonality condition $[0 \quad 1][D-(1+r)I]D - [1 \quad 0]D^2 = [0 \quad 0]$.

Table 1.1: Parameter Estimates of the VAR system

Independent Variable	Dependent Variable $\hat{y}_t$	$\hat{\imath}_t$	tby_t	$\hat{R}^{us}_t$	$\hat{R}_t$
$\hat{y}_t$	–	2.739 (10.28)	0.295 (2.18)	–	−0.791 (−3.72)
$\hat{y}_{t-1}$	.282 (2.28)	−1.425 (−4.03)	−0.032 (−0.25)	–	0.617 (2.89)
$\hat{\imath}_t$	–	–	−0.228 (−6.89)	–	0.114 (1.74)
$\hat{\imath}_{t-1}$	0.162 (4.56)	0.537 (3.64)	0.040 (0.77)	–	−0.122 (−1.72)
tby_t	–	–	–	–	0.288 (1.86)
tby_{t-1}	0.267 (4.45)	−0.308 (−1.30)	0.317 (2.46)	–	−0.190 (−1.29)
$\hat{R}^{us}_t$	–	–	–	–	0.501 (1.55)
$\hat{R}^{us}_{t-1}$	0.0002 (0.00)	−0.269 (−0.47)	−0.063 (−0.28)	.830 (10.89)	0.355 (0.73)
$\hat{R}_{t-1}$	−0.170 (−3.93)	−0.026 (−0.21)	0.191 (3.54)	-	0.635 (4.25)
R^2	0.724	0.842	0.765	0.664	0.619
S.E.	0.018	0.043	0.019	0.007	0.031
No. of obs.	165	165	165	62	160

Notes: t-statistics are shown in parenthesis. The system was estimated equation by equation. All equations except for the $\hat{R}^{us}_t$ equation were estimated using instrumental variables with panel data from Argentina, Brazil, Ecuador, Mexico, Peru, Philippines, and South Africa, over the period 1994:1 to 2001:4. The $\hat{R}^{us}_t$ equation was estimated by OLS over the period 1987:1-2002:4.

Figure 1.1: Impulse Response To An Output Shock



Notes: (1) Solid lines depict point estimates of impulse responses, and broken lines depict two-standard-deviation error bands. (2) The responses of Output and Investment are expressed in percent deviations from their respective log-linear trends. The responses of the Trade Balance-to-GDP ratio, the country interest rate, the US interest rate, and the country spread are expressed in percentage points. The two-standard-error bands are computed using the delta method.

Chapter 2

The Current Account in an Economy with Capital

A theme of the previous chapter was that the simple endowment economy fails to predict the coutercyclicality of the trade balance-to-output ratio in economies maintaining a positive net foreign asset position. In this chapter, we show that allowing for capital accumulation can help resolve this problem.

2.1 No Capital Adjustment Costs

Consider a small open economy populated by a large number of infinitely lived households with preferences described by the utility function

$$\sum_{t=0}^{\infty} \beta^t U(c_t). \tag{2.1}$$

Households seek to maximize this utility function subject to the following three constraints:

$$b_t = (1+r)b_{t-1} + \theta_t F(k_t) - c_t - i_t,$$

$$k_{t+1} = k_t + i_t,$$

and

$$\lim_{t\to\infty} \frac{b_t}{(1+r)^t} \geq 0.$$

The production function F is assumed to be strictly increasing, strictly concave, and to satisfy the Inada conditions. The variable k_t denotes the stock of physical capital, and i_t denotes investment. For the sake of simplicity, we assume that the capital stock does not depreciate. We relax this assumption later on. The variable θ_t denotes an exogenous productivity shock. Here, we wish to characterize the response of the economy to a permanent increase in θ_t.

The Lagrangean associated with the household's problem is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t) + \lambda_t \left[(1+r)b_{t-1} + k_t + \theta_t F(k_t) - c_t - k_{t+t} - b_{t+1} \right] \right\}$$

The first-order conditions corresponding to this problem are

$$U'(c_t) = \lambda_t,$$

$$\lambda_t = \beta(1+r)\lambda_{t+1},$$

$$\lambda_t = \beta\lambda_{t+1}[1 + \theta_{t+1}F'(k_{t+1})],$$

$$b_t = (1+r)b_{t-1} + \theta_t F(k_t) - c_t - k_{t+1} + k_t,$$

and

$$\lim_{t\to\infty} \frac{b_t}{(1+r)^t} = 0.$$

As in the endowment economy example, we assume that $\beta(1+r) = 1$. Then the above optimality conditions can be reduced to the following two equilibrium conditions:

$$r = \theta_{t+1}F'(k_{t+1}); \quad t \geq 0, \tag{2.2}$$

$$c_t = rb_{t-1} + \frac{r}{1+r} \sum_{j=0}^{\infty} \frac{\theta_{t+j}F(k_{t+j}) - k_{t+j+1} + k_{t+j}}{(1+r)^j} \tag{2.3}$$

The first of these equilibrium conditions states that investment is allocated in such a way that the future expected marginal product of capital is equalized to the rate of return on foreign bonds. It follows that k_{t+1} is increasing in the future exprected level of the productivity shock, θ_{t+1}, and decreasing in the opportunity cost of holding physical capital, r. Formally,

$$k_{t+1} = \kappa(\theta_{t+1}, r); \quad \kappa_1 > 0, \kappa_2 < 0.$$

The second equilibrium condition says that consumption equals the interest flow on a broad definition of wealth, which includes not only financial wealth, b_{-1}, but also the present discounted value of the differences between output and investment.

2.1.1 A Permanent Productivity Shock

Suppose now that up until period -1 the technology factor θ was constant and equal to $\bar{\theta}$. Suppose further that in period 0 there is a permanent increase in the technology factor to $\theta' > \bar{\theta}$. That is, $\theta_t = \bar{\theta}$ for $t < 0$ and $\theta_t = \theta'$ for $t \geq 0$. Let $\bar{k}$ denote the pre-shock steady-state level of capital. Equation (2.2) implies that $\bar{k}$ is implicitly given by $r = \bar{\theta}F'(\bar{k})$. Suppose that in period 0 the capital stock is at its pre-shock steady-state level. That is, $k_0 = \bar{k}$. Using again equation (2.2), it is clear that in period 0, and in response to the productivity shock, the capital stock experiences a once-and-for-all increase from $k\bar{L}$ to a level k^*, given by the solution to the equation $r = \theta' F'(,^*)$. Thus, $k_{t+1} = k^*$ for $t \geq 0$. Plugging this path for the capital stock into equation (2.3) and evaluating that equation at $t = 0$ we get

$$c_0 = rb_{-1} + \frac{r}{1+r}\left[\theta' F(\bar{k}) - k^* + \bar{k}\right] + \frac{1}{1+r}\theta' F(k^*)$$

The trade balance, in turn, is given by $tb_t = \theta_t F(k_t) - c_t - i_t$. Theus, we have

$$tb_0 = -rb_{-1} - \frac{1}{1+r}\left[\theta' F(k^*) - \theta' F(\bar{k}) + (k^* - \bar{k})\right]$$

Before period zero, the trade balance is simply equal to $-rb_{-1}$. This implies that in response to the permanent technology shock, the trade balance deteriorates in period zero. In period 1 the trade balance improves. To see this, note that output increases from $\theta' F(k_0)$ to $\theta' F(k^*)$. On the demand side, in period 1 inventment falls to zero, while consumption remains constant. Thus, the trade balance, given by output minus investment minus consumption, necessarily goes up in period 1.

The initial deterioration of the trade balance implied by the model is an important result. For it is in line with two features of the data as described by the impulse respose functions shown in figure1.1: first, in response to a positive output shock (in this case a positive productivity shock), output and investment increase. Second, the trade balance deteriorates. In obtaining a deterioration of the trade balance in response to a positive output shock, the assumption that the technology shock was persistent plays a significant role. For a permanent increase in productivity induces a strong response in domestic absorption. To see this, note first that because the increase in output is expected to persist over time, households' propensity to consume out of current income is high (this is the basic result derived from our study of the endowment economy in chapter 1). Also, because the productivity of capital, θ_t, is expected to stay high in the future, it pays to strongly increase investment spending.

Another important factor in generating a decline in the trade balance in response to a positive prodctivity shock is the assumed absence of capital adjustment costs. Note that in response to the increase in future expected productivity, the entire adjustment in investment occurs in period zero. Indeed, investment falls to zero in period 1 and remains at that level thereafter. In the presence of costs of adjusting the stock of capital, investment spending would be spread over a number of periods, dampening the increase in domestic absorption in the date the shock occurs. We will study the role of adjustment costs more closely shortly.

2.1.2 A Temporary Productivity shock

To stress the importance of persistence in productivity movements in inducing a deterioration of the trade balance in response to a positive output shock, it is worth analyzing the effect of a purely temporary shock. Specifically, suppose that up until period -1 the productivity factor θ_t was constant and equal to $\bar{\theta}$. Suppose also that in period -1 people assigned a zero probability to the event that θ_0 would be different from $\bar{\theta}$. In period 0, however, a zero probability event happens. Namely, $\theta_0 = \theta' > \bar{\theta}$. Furthermore, suppose that everybody correctly expects the productivity shock to be purely temporary. That is, $\theta_t = \bar{\theta}$ for all $t > 0$.

In this case, equation (2.2) implies that the capital stock, and therefore also investment, are unaffected by the productivity shock. That is, $k_t = \bar{k}$ for all $t \geq 0$, where $\bar{k}$ is defined above. This is intuitive. The productivity of capital unexpectedly increases in period zero. As a result, households would like to have more capital in that period. But k_0 is fixed in period zero. Investment in period zero can only increase the future stock of capital. But agents have no incentives to have a higher capital stock in the future, because the its productivity is expected to go back down to its historic level $\bar{\theta}$ right after period zero.

The positive productivity shock in period zero does produce an increase in output in that period, from $\bar{\theta}F(\bar{k})$ to $\theta' F(\bar{k})$. That is

$$y_0 = y_{-1} + (\theta' - \bar{\theta})F(\bar{k}),$$

where $y_{-1} \equiv \bar{\theta}F(\bar{k})$ is the pre-shock steady-state level of output prevailing up until period -1. This output effect induces higher consumption. In effect, using equation (2.3) we have that

$$c_0 = c_{-1} + \frac{r}{1+r}(\theta' - \bar{\theta})F(\bar{k}),$$

where $c_{-1} \equiv -rb_{-1} + \bar{\theta}F(\bar{\theta})$ is the pre-shock steady-state level of consumption prevailing up until period -1. Basically, households invest the entire increase in output in the international financial mnarket and increase consumption by the interest flow associated with that financial investment.

Because the capital stock is unchanged by the shock, we have that investment in physical capital is nil for all $t \geq 0$. Taking this into account and combining the above two expressions we get that the trade balance in period 0 is given by

$$tb_0 - tb_{-1} = (y_0 - y_{-1}) - (c_0 + i_0) + (c_{-1} + i_{-1}) = \frac{1}{1+r}(\theta' - \bar{\theta})F(k_0) > 0.$$

This expression shows that the trade balance improves on impact. The reason for this counterfactual response is simple: 'Frims' have no incentive to invest, as the productivity of capital is short lived, and consumers save most of the increase in income in order to smooth consumption over time.

2.2 Capital Adjustment Costs

Consider now an economy identical to the one described above but in which changes in the stock of capital come at a cost. Capital adjustment costs are typically introduced, in different forms, in small open economy models to dumpen the volatility of investment over the business cycle (see, e.g., Mendoza, 1991; and Schmitt-Grohé, 1998). Suppose that the sequential budget constraint is of the form

$$b_t = (1+r)b_{t-1} + \theta_t F(k_t) - c_t - i_t - \frac{1}{2}\frac{i_t^2}{k_t}$$

Here, capital adjustment costs are given by $i_t^2/(2k_t)$. Adjustment costs are strictly convex. Moreover, both the level of adjustment costs as well as the marginal adjustment cost vanish at the steady-state value of investment, $i_t = 0$. As in the economy without adjustment costs, the law of motion of the capital stock is given by

$$k_{t+1} = k_t + i_t.$$

Also, households are subject to the now familiar no-Ponzi-game constraint

$$\lim_{t \to \infty} \frac{b_t}{(1+r)^t} \geq 0$$

The Lagrangean associated with the problem of maximazing the utility function given by (6.1) subject to the above three constraints is

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \left\{ U(c_t) + \lambda_t \left[(1+r)b_{t-1} + \theta_t F(k_t) - c_t - i_t - \frac{1}{2}\frac{i_t^2}{k_t} - b_{t+1} + q_t(k_t + i_t - k_{t+1}) \right] \right\}$$

The first-order conditions to this problem are [as in the endowment economy example, we assume that $\beta(1+r) = 1$]

$$i_t = (q_t - 1)k_t$$

$$q_t = \frac{\theta_{t+1}F'(k_{t+1}) + \frac{1}{2}(i_{t+1}/k_{t+1})^2 + q_{t+1}}{1+r}$$

$$k_{t+1} = k_t + i_t$$

$$c_t = rb_{t-1} + \frac{r}{1+r}\sum_{j=0}^{\infty}\frac{\theta_{t+j}F(k_{t+j}) - i_{t+j} - \frac{1}{2}(i_{t+j}^2/k_{t+j})}{(1+r)^j}$$

The variable q_t is the ratio of the Lagrange multiplier associated with the capital accumulation equation to that of the sequential budget constraint. This variable represents the relative price of installed capital in terms of investment goods, and is known as Tobin's q. As q_t increases, agents have incentives to allocate goods to the production of capital—i.e., to increase i_t.

2.2.1 Dynamics of the capital stock

Eliminating i_t from the above equations we get the following two first-order, non-linear difference equations in k_t and q_t:

$$k_{t+1} = q_t k_t \tag{2.4}$$

$$q_t = \frac{\theta_{t+1}F'(q_t k_t) + (q_{t+1} - 1)^2/2 + q_{t+1}}{1+r} \tag{2.5}$$

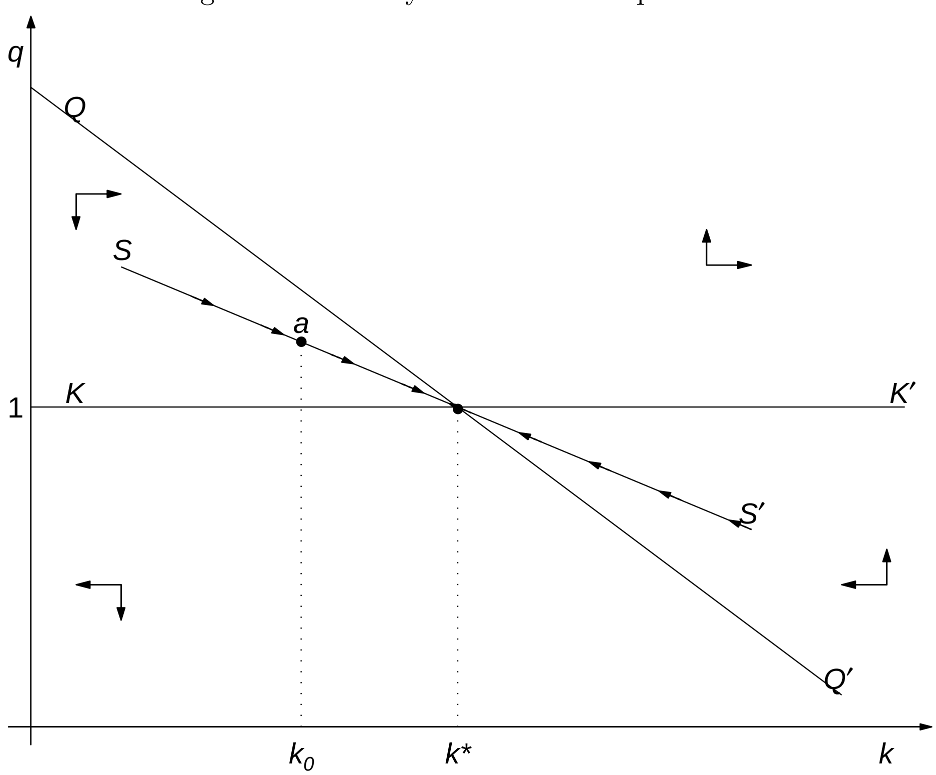
The perfect foresight solution to these equations is depicted in figure 2.1. The horizontal line $\overline{KK'}$ corresponds to the pairs (k_t, q_t) for which $k_{t+1} = k_t$ in equation (2.4). That is,

$$q = 1. \tag{2.6}$$

Above the locus $\overline{KK'}$, the capital stock grows over time and below $\overline{KK'}$ the capital stock declines over time. The locus $\overline{QQ'}$ corresponds to the pairs (k_t, q_t) for which $q_{t+1} = q_t$ in equation (2.5). That is, the paisr satisfying

$$rq = \theta F'(qk) + (q-1)^2/2. \tag{2.7}$$

Figure 2.1: The Dynamics of the Capital Stock



Note that jointly equations (2.6) and (2.7) determine the steady-state value of the capital stock, which we denote by k^*, and the steady-state value of the Tobin's q, 1. For q_t near unity, the locus $\overline{QQ'}$ is downward sloping. Above and to the right of $\overline{QQ'}$ q increases over time and below and to the left of $\overline{QQ'}$ q decreases over time.

The system (2.4)-(2.5) is saddle-path stable. The locus $\overline{SS'}$ represents the converging saddle path. If the initial capital stock is different from its long-run level, both q and k converge monotonically to their steady states along the saddle path.

2.2.2 A Permanent Technology Shock

Suppose now that in period 0 the technology factor θ_t increases permanently from $\bar{\theta}$ to $\theta' > \bar{\theta}$. It is clear from equation (2.6) that the locus $\overline{KK'}$ is not affected by the productivity shock. On the other hand, it is clear from equation (2.7) that the locus $\overline{QQ'}$ shifts up and to the right. It follows that in response to a permanent increase in productivity, the long-run level of capital experiences a permanent increase. The price of capital, q_t, on the other hand, is not affected in the long run.

Consider now the transition to the new steady state. Suppose that the steady-state value of capital prior to the innovation in productivity is k_0 in figure 2.1. Then the new steady-state values of k and q are given by k^* and 1. In the period of the shock, the capital stock does not move. The price of installed capital, q_t, jumps to the new saddle path, point a in the figure. This increase in the price of installed capital induces an increase in investment, which in turn makes capital grow over time. After the initial impact, q_t decreases toward 1. Along this transition, the capital stock increases monotonically towards its new steady-state k^*.

The equilibrium dynamics of output, investment, and capital are quite different in the presence of adjustment costs from those those that arise in the absence thereof. Specifically, in the frictionless envirinment, investment experiences a an increase in the period the productivity shock hits the economy and then vanishes. Output increases in periods 0 and 1 reaching its steady state in period 1. The capital stock, in turn, displays a once-and-for-all increase one period after the shock. Under capital adjustment costs, all of these variables adjust gradually to the unexpected increase in productivity. The more pronounced are adjustment costs, the more sluggish is the response of investment, thereby making it less likely that the trade balance deteriorate in response to a positive technology shock as required for the model to be in line with the data. This observation opens the question of what would the model predict for the behavior of the trade balance in response to output shocks when one introduces a realistic degree of adjustment costs. We address this issue in the next chapter.

Chapter 3

The Current Account in a Real-Business-Cycle Model

In the previous two chapters, we arrived at the conclusion that a model driven by productivity shocks can explain the observed countercyclicality of the current account. We also established that two features of the model are important in making this prediction possible. First, productivity shocks must be sufficiently persistent. Second, capital adjustment costs must not be too strong. In this chapter, we introduce realistic measures of the degrees of persistence and adjustment costs. We also extend the model of the previous chapter by allowing for three new features that add realism to the model. Namely, endogenous labor supply and demand, uncertainty in the technology shock process, and capital depreciation.

3.1 The Model (Model 1)

As in the models we studied in previous chapters, in the present formulation, which follows closely the work of Mendoza (1991) and Schmitt-Grohé and Uribe (2003), we assume that the economy is populated by an infinite number of identical households. These households have preferences described by the following utility function:

$$E_0 \sum_{t=0}^{\infty} \theta_t U(c_t, h_t), \tag{3.1}$$

$$\theta_0 = 1, \tag{3.2}$$

$$\theta_{t+1} = \beta(c_t, h_t)\theta_t \qquad t \geq 0, \tag{3.3}$$

where $\beta_c < 0$, $\beta_h > 0$. This preference specification induces stationary, in the sense that the nonstochastic steady state is independent of the initial state of the economy (namely, independent of the initial level of financial wealth, physical capital, and total factor productivity).

The period-by-period budget constraint of the representative household is given by

$$\frac{d_t}{1+r_t} = d_{t-1} - y_t + c_t + i_t + \Phi(k_{t+1} - k_t), \tag{3.4}$$

where d_t denotes the household's debt position at the end of period t, r_t denotes the interest rate at which domestic residents can borrow in international markets in period t, y_t denotes domestic output, c_t denotes consumption, i_t denotes gross investment, and k_t denotes physical capital. The function $\Phi(\cdot)$ is meant to capture capital adjustment costs and is assumed to satisfy $\Phi(0) = \Phi'(0) = 0$. As pointed out earlier, small open economy models typically include capital adjustment costs to avoid excessive investment volatility in response to variations in the domestic-foreign interest rate differential. The restrictions imposed on Φ ensure that in the nonstochastic steady state adjustment costs are zero and the domestic interest rate equals the marginal product of capital net of depreciation. Output is produced by means of a linearly homogeneous production function that takes capital and labor services as inputs,

$$y_t = A_t F(k_t, h_t), \tag{3.5}$$

where A_t is an exogenous stochastic productivity shock. The stock of capital evolves according to

$$k_{t+1} = i_t + (1-\delta)k_t, \tag{3.6}$$

where $\delta \in (0,1)$ denotes the rate of depreciation of physical capital.

Households choose processes $\{c_t, h_t, y_t, i_t, k_{t+1}, d_t, \theta_{t+1}\}_{t=0}^{\infty}$ so as to maximize the utility function (6.1) subject to (3.2)-(3.6) and a no-Ponzi constraint of the form

$$\lim_{j\to\infty} E_t \frac{d_{t+j}}{\prod_{s=0}^{j}(1+r_s)} \leq 0. \tag{3.7}$$

Letting $\theta_t \eta_t$ and $\theta_t \lambda_t$ denote the Lagrange multipliers on (3.3) and (3.4), the first-order conditions of the household's maximization problem are (3.3)-(3.7) holding with equality and:

$$\lambda_t = \beta(c_t, h_t)(1+r_t)E_t\lambda_{t+1} \tag{3.8}$$

$$\lambda_t = U_c(c_t, h_t) - \eta_t \beta_c(c_t, h_t) \tag{3.9}$$

$$\eta_t = -E_t U(c_{t+1}, h_{t+1}) + E_t \eta_{t+1} \beta(c_{t+1}, h_{t+1}) \tag{3.10}$$

$$-U_h(c_t, h_t) + \eta_t \beta_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t) \tag{3.11}$$

$$\lambda_t[1+\Phi'(k_{t+1}-k_t)] = \beta(c_t, h_t) E_t \lambda_{t+1} \left[A_{t+1} F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})\right] \tag{3.12}$$

These first-order conditions are fairly standard, except for the fact that the marginal utility of consumption is not given simply by $U_c(c_t, h_t)$ but rather by $U_c(c_t, h_t) - \beta_c(c_t, h_t)\eta_t$. The second term in this expression reflects the fact that an increase in current consumption lowers the discount factor ($\beta_c < 0$). In turn, a unit decline in the discount factor reduces utility in period t by η_t. Intuitively, η_t equals the present discounted value of utility from period $t+1$ onward. To see this, iterate the first-order condition (3.10) forward to obtain: $\eta_t = -E_t \sum_{j=1}^{\infty} \left(\frac{\theta_{t+j}}{\theta_{t+1}}\right) U(c_{t+j}, h_{t+j})$. Similarly, the marginal disutility of labor is not simply $U_h(c_t, h_t)$ but instead $U_h(c_t, h_t) - \beta_h(c_t, h_t)\eta_t$.

In this model, the interest rate faced by domestic agents in world financial markets is assumed to be constant and given by

$$r_t = r. \tag{3.13}$$

The law of motion of the productivity shock is given by:

$$\ln A_{t+1} = \rho \ln A_t + \epsilon_{t+1}; \quad \epsilon_{t+1} \sim NIID(0, \sigma_\epsilon^2); \quad t \geq 0. \tag{3.14}$$

A competitive equilibrium is a set of processes $\{d_t, c_t, h_t, y_t, i_t, k_{t+1}, \eta_t, \lambda_t, r_t\}$ satisfying (3.4)-(3.14), given the initial conditions A_0, d_{-1}, and k_0 and the exogenous process $\{\epsilon_t\}$.

We parameterize the model following Mendoza (1991), who uses the following functional forms for preferences and technology:

$$U(c, h) = \frac{\left[c - \omega^{-1} h^\omega\right]^{1-\gamma} - 1}{1 - \gamma}$$

$$\beta(c, h) = \left[1 + c - \omega^{-1} h^\omega\right]^{-\psi_1}$$

$$F(k, h) = k^\alpha h^{1-\alpha}$$

$$\Phi(x) = \frac{\phi}{2} x^2; \quad \phi > 0.$$

The assumed functional forms for the period utility function and the discount factor imply that the marginal rate of substitution between consumption and leisure depends only on labor. In effect, combining equations (3.9) and (3.11) yields

$$h_t^{\omega-1} = A_t F_h(k_t, h_t) \tag{3.15}$$

The right-hand side of this expression is the marginal product of labor, which in equilibrium equals the real wage rate. The left-hand side is the marginal rate of substitution of leisure for consumption. The above expression thus states that the labor supply depends only upon the wage rate and in particular that it is independent of the level of wealth.

We also follow Mendoza (1991) in assigning values to the structural parameters of the model. Mendoza calibrates the model to the Canadian economy. The time unit is meant to be a year. The parameter values are shown on table 3.1. All parameter values are standard in the real-business-cycle

Table 3.1: Calibration of the Samll Open RBC Economy

γ	ω	ψ_1	α	ϕ	r	δ	ρ	σ_ϵ
2	1.455	.11	.32	0.028	0.04	0.1	0.42	0.0129

literature. It is of interest to review the calibration of the parameter ψ_1 defining the elasticity of the discount factor with respect to the composite $c - h^\omega/\omega$. This parameter determines the stationarity of the model and the speed of convergence to the steady state. The value assigned to ψ_1 is set so as to match the average Canadian trade-balance-to-GDP ratio. To see how in steady state this ratio is linked to the value of ψ_1, use equation (3.12) in steady state to get

$$\frac{k}{h} = \left(\frac{\alpha}{r+\delta}\right)^{\frac{1}{1-\alpha}}$$

It follows from this expression that the steady-state capital-labor ratio is independent of the parameter ψ_1. Given the capital-labor ratio, equilibrium condition (3.15) implies that the steady-state value of hours is also independent of ψ_1 and given by

$$h = \left[(1-\alpha)\left(\frac{k}{h}\right)^{\alpha}\right]^{\frac{1}{\omega-1}}.$$

Given the steady-state values of hours and the capital-labor ratio, we can find directly the steady-state values of capital, investment ($i = \delta k$), and output ($y = k^\alpha h^{1-\alpha}$), independently of ψ_1. Now note that in the steady state the trade balance, tb, is given by $y - c - i$. This expression and equilibrium condition (3.8) imply the following steady-state condition relating the trade balance to ψ_1: $[1 + y - i - tb - h^\omega/\omega]^{-\psi_1}(1 + r) = 1$. Using the specific functional form for the discount factor, this expression can be solved for the trade balance-to-output ratio to obtain:

$$\frac{tb}{y} = 1 - \frac{i}{y} - \frac{\left[(1+r)^{1/\psi_1} + \frac{h^\omega}{\omega} - 1\right]}{y}.$$

This expression can be solved for ψ_1 given tb/y, α, r, δ, and ω. All other things constant, the larger is the trade-balance-to-output ratio, the larger is the required value of ψ_1.

Approximating Equilibrium Dynamics

We look for solutions to the equilibrium conditions (3.4)-(3.14) where the vector $x_t \equiv \{d_{t-1}, c_t, h_t, y_t, i_t, k_{t+1}, \eta_t, \lambda_t, r_{t-1}, A_t\}$ fluctuates in a small neighborhood around its nonstochastic steady-state level. Because in any such solution the stock of debt is bounded, we have that the transversality condition $\lim_{j\to\infty} E_t d_{t+j}/(1+r)^j = 0$ is always satisfied. We thus focus on bounded solutions to the system (3.4)-(3.6) and (3.8)-(3.14) of ten equations in ten variables given by the elements of the vector x_t. The system can be written as

$$E_t f(x_{t+1}, x_t) = 0.$$

This expression describes a system of non-linear stochastic difference equations. Closed form solutions for this type of system are not typically available. We therefore must resort to an approximate solution.

There are a number of techniques that have been devised to solve dynamic systems like the one we are studying. The technique we will employ here consists in applying a first-order Taylor expansion (i.e., linearizing) the system of equations around the nonstochastic steady state. The resulting linear system can be readily solved using well-established techniques.

Before linearizing the equilibrium conditions, we introduce a convenient variable transformation. For any variable, say y_t, we define $\widehat{y}_t = \ln(y_t/y)$, where y is the nonstochastic steady-state value of y_t. The transformation is useful because it allows us to interpret movements in any variable as percentage deviations from its steady-state value. To see this, note that for small deviations of y_t from y (or, more precisely, up to first order), it is the case that $\widehat{y}_t \approx (y_t - y)/y$. The appendix displays the log-linearized version of the equilibrium conditions. The linearized version of the equilibrium system can be written as

$$A\widehat{x}_{t+1} = B\widehat{x}_t,$$

where A and B are square matrices conformable with x_t. The ten equations that form the above linearized equilibrium model contain 3 state variables,

$\widehat{k}_t$, $\widehat{d}_{t-1}$, and $\widehat{A}_t$. State variables are variables whose values in each period $t \geq 0$ are either predetermined (determined before t) as is the case with $\widehat{k}_t$ and $\widehat{d}_{t-1}$, or are determined in t but in an exogenous fashion, as is the case with $\widehat{A}_t$. Of these state variables, two are endogenous, $\widehat{k}_t$ and $\widehat{d}_{t-1}$, and one is exogenous, $\widehat{A}_t$. In addition, the model posseses seven co-state variables (variables that are non-predetermined in preiod t), $\hat{c}_t$, $\hat{h}_t$, $\hat{\lambda}_t$, $\hat{\eta}_t$, r_t, $\widehat{i}_t$, and y_t, all of which are endogenous. All the coefficients of the linear system (the elements of A and B) are known functions of the deep structural parameters of the model to which we assigned values in the calibration section. The linearized system has three known initial conditions $\widehat{k}_0$, $\widehat{d}_{-1}$, and $\widehat{A}_0$. To determine the initial value of the remaining seven variables, we impose a terminal condition requiring that at any point in time the system be expected to converge to the nonstochastic sready state. Formally, the terminal condition takes the form

$$\lim_{j \to \infty} |E_t x_{t+j}| = 0.$$

In the next chapter, we will study in detail a technique available to solve linear systems like the one describing the dynamics of our linearized equilibrium conditions. There, we will also describe how to compute second moments and impulse response functions.

3.2 The Model's Performance

Table 3.2 displays some unconditional second moments of interest implied by our model. It should not come as a surprise that the model does very well in replicating the volatility of output, the volatility of investment, and the serial correlation of output. For we picked values for the parameters σ_ϵ, ϕ, and ρ so as to match these three moments. But the model perfroms relatively well along other dimensions. For instance, it correctly implies a volatility ranking featuring investment above output and output above consumption. Also, as in the data, in the model the trade balance is negatively correlated with output. The model overestimates the correlation of hours with output and the correlation of consumption with output. Note in particular that the implied correlation between hours and output is exactly unity. This prediction is due to the assumed functional form for the period utility index. In effect, equilibrium condition (3.15), equating the marginal produc of labor to the marginal rate of substitution between consumption and leisure, can be written as $h_t^\omega = (1-\alpha)y_t$. The log-linearized version of this condition is $\omega\widehat{h}_t = \widehat{y}_t$, which implies that $\widehat{h}_t$ and $\widehat{y}_t$ are perfectly correlated.

Table 3.2: Business-cycle properties: Data and Model

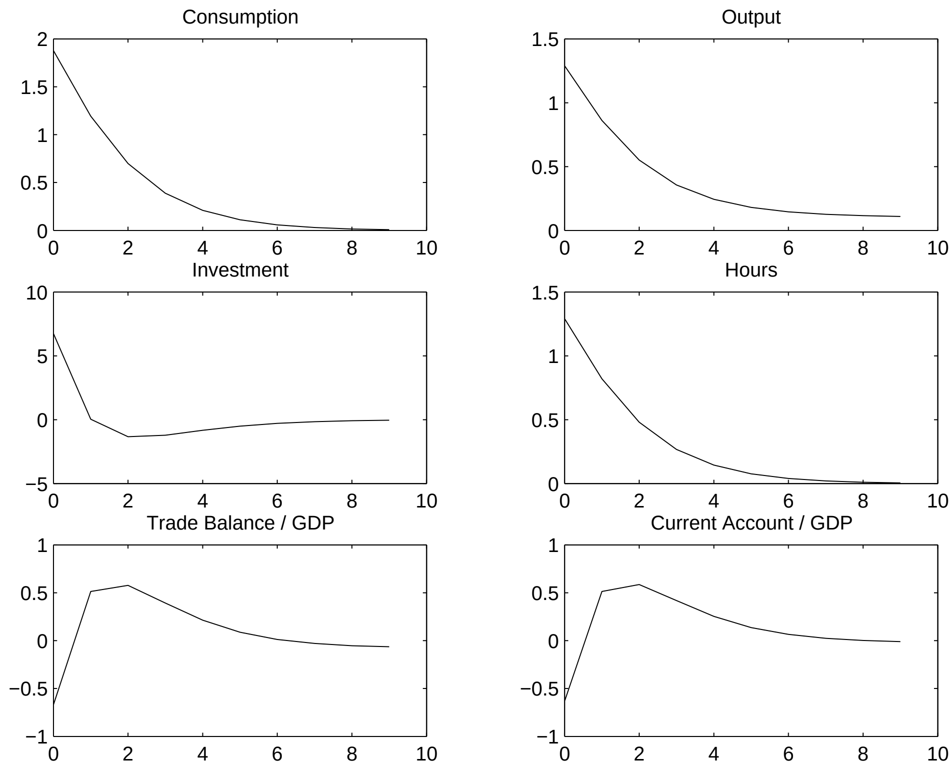
Variable	Canadian Data			Model		
	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}	σ_{x_t}	$\rho_{x_t,x_{t-1}}$	ρ_{x_t,GDP_t}
y	2.8	0.61	1	3.1	0.61	1
c	2.5	0.7	0.59	2.3	0.7	0.94
i	9.8	0.31	0.64	9.1	0.07	0.66
h	2	0.54	0.8	2.1	0.61	1
$\frac{tb}{y}$	1.9	0.66	-0.13	1.5	0.33	-0.012
$\frac{ca}{y}$				1.5	0.3	0.026

Note. The first three columns were taken from Mendoza (1991). σ_{x_t} is measured in percent.

Figure 3.1. displays the impulse response functions of a number of variables of interest to a technology shock of size 1 in period 0. The model predicts an expansion in output, consumption, investment, and hours. The increase in domestic absorption is larger than the increase in output, which results in a deterioration of the trade balance. This latter implication is in line with the predictions of the empirical VAR model studied in a previous chapter (see figure 1.1).

In previous chapters, we deduced that the negative response of the trade balance to a positive technology shock was not a general implication of the neoclassical model. In particular, two conditions must be met for the model to generate a deterioration in the external accounts in response to an improvement in total factor productivity. First, capital adjustment costs must not be too stringent. Second, the productivity shock must be sufficiently persistent. To illustrate this conclusion, figure 3.2 displays the impulse response function of the trade balance-to-GDP ratio to a technology shock of unit size in period 0 under three alternative parameter specifications. The solid line reproduces the benchmark case for comparison. The broken line depicts an economy where the persistence of the productivity shock is half as large as in the benchmark economy ($\rho = 0.21$). Here, because the productivity shock is expected to die out quickly, the response of investment to the positive technology shock is relatively weak. In addition, because the increase in output is more temporary in this environment, consumption smoothing induces households to save most of increase in income. As a re-

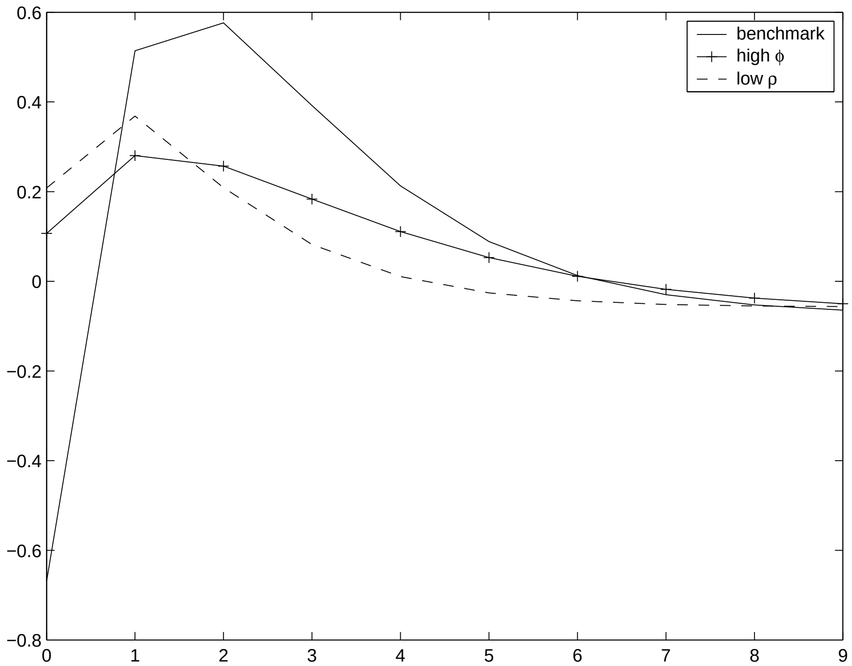
Figure 3.1: Small Open Economy RBC Model: Impulse Responses to a Positive Technology Shock



sult, the expansion in aggregate domestic absorption is modest. Because the size of the productivity shock is the same as in the benchmark economy, the initial response of output and hours is identical in both economies (recall that, by equation (3.15), h_t depends only on k_t and A_t). The end result is an initial improvement in the trade balance.

The crossed line depicts the case of high capital adjustment costs. Here the parameter ϕ equals 0.084, a value three times as large as in the benchmark case. In this environment, high adjustment costs discourage firms from increasing investment spending by as much as they would in the benchmark economy in response to the increase in productivity. As a result, the response of aggregate domestic demand is weaker, which leads to a higher trade balance.

Figure 3.2: Small Open Economy RBC Model: Response of the Trade Balance To a Positive Technology Shock Under Alternative Parameterizations



3.3 Alternative Ways To Close The Small Open Economy Model

Computing business-cycle dynamics in the standard small open economy model is problematic. In this model, domestic residents have only access to a risk-free bond whose rate of return is exogenously determined abroad. As a consequence, the deterministic steady state of the model depends on initial conditions. In particular, the nonstochastic steady state depends upon the country's initial net foreign asset position.[1] Put differently, transient shocks have long-run effects on the state of the economy. That is, the equilibrium dynamics possesses a random walk component. The random walk property of the dynamics implies that the unconditional variance of variables such as asset holdings and consumption is infinite. Thus, endogenous

[1]If the real rate of return on the foreign bond exceeds (is less than) the subjective rate of discount, the model displays perpetual positive (negative) growth. It is standard to eliminate this source of dynamics by assuming that the subjective discount rate equals the (average) real interest rate.

variables in general wonder around an infinitely large region in response to bounded shocks. This introduces serious computational difficulties because all available techniques are valid locally around a given stationary path.

To resolve this problem, researchers resort to a number of modifications to the standard model that have no other purpose than to induce stationarity of the equilibrium dynamics. Obviously, because these modifications basically remove the built-in random walk property of the canonical model, they all necessarily alter the low-frequency properties of the model. The focus of this section is to assess the extent to which these stationarity-inducing techniques affect the equilibrium dynamics at business-cycle frequencies.[2]

We compare the business-cycle properties of five variations of the small open economy model. Earlier in this chapter, we considered a model with an endogenous discount factor (Uzawa, 1968 type preferences). More recent papers using this type of preferences in the context of open economy models include Obstfeld (1990), Mendoza (1991), Schmitt-Grohé (1998), and Uribe (1997). In this model, the subjective discount factor, typically denoted by β, is assumed to be decreasing in consumption. Agents become more impatient the more they consume. The reason why this modification makes the steady state independent of initial conditions becomes clear from inspection of the Euler equation $\lambda_t = \beta(c_t)(1+r)\lambda_{t+1}$. Here, λ_t denotes the marginal utility of wealth, and r denotes the world interest rate. In the steady state, this equation reduces to $\beta(c)(1+r) = 1$, which pins down the steady-state level of consumption solely as a function of r and the parameters defining the function $\beta(\cdot)$. Kim and Kose (2001) compare the business-cycle implications of this model to those implied by a model with a constant discount factor. They find that both models feature similar comovements of macroeconomic aggregates.

Here, we will also consider a simplified specification of Uzawa preferences where the discount factor is assumed to be a function of aggregate per capita consumption rather than individual consumption. This specification is arguably no more arbitrary than the original Uzawa specification and has a number of advantages. First, it also induces stationarity since the above Euler equation still holds. Second, the modified Uzawa preferences result in a model that is computationally much simpler than the standard Uzawa model, for it contains one less Euler equation and one less Lagrange multiplier. Finally, the quantitative predictions of the modified Uzawa model are not significantly different from those of the original model.

In subsection 3.3.2 we study a model with a debt-elastic interest-rate pre-

[2]The material in this section is taken from Schmitt-Grohé and Uribe (2003).

mium. This stationarity inducing technique has been recently used, among others, by Senhadji (1994), Mendoza and Uribe (2000), and Schmitt-Grohé and Uribe (2001). In this model, domestic agents are assumed to face an interest rate that is increasing in the country's net foreign debt. To see why this device induces stationarity, let $p(d_t)$ denote the premium over the world interest rate paid by domestic residents, and d_t the stock of foreign debt. Then in the steady state the Euler equation implies that $\beta[1+r+p(d)] = 1$. This expression determines the steady-state net foreign asset position as a function of r and the parameters that define the premium function $p(\cdot)$ only.

Subsection 3.3.3 features a model with convex portfolio adjustment costs. This way of ensuring stationarity has recently been used by Neumeyer and Perri (2001). In this model, the cost of increasing asset holdings by one unit is greater than one because it includes the marginal cost of adjusting the size of the portfolio. The Euler equation thus becomes $\lambda_t[1 + \psi'(d_t)] = \beta(1 + r)\lambda_{t+1}$, where $\psi(\cdot)$ is the portfolio adjustment cost. In the steady state, this expression simplifies to $1 + \psi'(d) = \beta(1 + r)$, which implies a steady-state level of foreign debt that depends only on parameters of the model.

The models discussed thus far all feature incomplete asset markets. Subsection 3.3.4 presents a model of a small open economy with complete asset markets. Under complete asset markets, the marginal utility of consumption is proportional across countries. So one equilibrium condition states that $U_c(c_t) = \alpha U^*(c_t^*)$, where U denotes the period utility function and stars are used to denote foreign variables. Because the domestic economy is small, c_t^* is determined exogenously. Thus, stationarity of c_t^* implies stationarity of c_t.

For the purpose of comparison, in subsection 3.3.5 we also study the dynamics of the standard small open economy model without any type of stationarity-inducing features, such as the economy analyzed in Correia et al. (1995). In this economy, the equilibrium levels of consumption and net foreign assets display a unit root. As a result unconditional second moments are not well defined. For this reason, we limit the numerical characterization of this model to impulse response functions.

All models are calibrated in such a way that they predict identical steady states. The functional forms of preferences and technologies are also identical across models. The basic calibration and parameterization is as in the previous section. The business-cycle implications of the alternative models are measured by second moments and impulse responses. The central result of this section is that all models deliver virtually identical dynamics at business-cycle frequencies. The complete-asset-market model induces some-what smoother consumption dynamics but similar implications for hours and investment.

3.3.1 Endogenous Discount Factor Without Internalization (Model 1a)

Consider an alternative formulation of the endogenous discount factor model where domestic agents do not internalize the fact that their discount factor depends on their own levels of consumption and effort. Alternatively, suppose that the discount factor depends not upon the agent's own consumption and effort, but rather on the average per capita levels of these variables. Formally, preferences are described by (6.1), (3.2), and

$$\theta_{t+1} = \beta(\tilde{c}_t, \tilde{h}_t)\theta_t \qquad t \geq 0, \tag{3.16}$$

where $\tilde{c}_t$ and $\tilde{h}_t$ denote average per capital consumption and hours, which the individual household takes as given.

The first-order conditions of the household's maximization problem are (3.2), (3.4)-(3.7), (3.16) holding with equality and:

$$\lambda_t = \beta(\tilde{c}_t, \tilde{h}_t)(1 + r_t)E_t\lambda_{t+1} \tag{3.17}$$

$$\lambda_t = U_c(c_t, h_t) \tag{3.18}$$

$$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t) \tag{3.19}$$

$$\lambda_t[1+\Phi'(k_{t+1}-k_t)] = \beta(\tilde{c}_t, \tilde{h}_t)E_t\lambda_{t+1}\left[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})\right] \tag{3.20}$$

In equilibrium, individual and average per capita levels of consumption and effort are identical. That is,

$$c_t = \tilde{c}_t \tag{3.21}$$

and

$$h_t = \tilde{h}_t. \tag{3.22}$$

A competitive equilibrium is a set of processes $\{d_t,\ c_t, h_t,\ \tilde{c}_t, \tilde{h}_t, y_t,\ i_t,\ k_{t+1},\ \lambda_t, r_t, A_t\}$ satisfying (3.4)-(3.7), (3.13), (3.14), (3.17)-(3.22) all holding with equality, given A_0, d_{-1}, and k_0 and the stochastic process $\{\epsilon_t\}$. Note that the equilibrium conditions include one Euler equation less, equation (3.10), and one variable less, η_t, than the standard endogenous-discount-factor model of subsection 3.1. This feature facilitates the computation of the equilibrium dynamics using perturbation methods.[3]

We evaluate the model using the same functional forms and parameter values as in Model 1.

[3] It is remarkable that the degree of computational complexity is reversed when the

3.3.2 Debt Elastic Interest Rate (Model 2)

In Model 2, stationarity is induced by assuming that the interest rate faced by domestic agents, r_t, is increasing in the aggregate level of foreign debt, which we denote by $\tilde{d}_t$. Specifically, r_t is given by

$$r_t = r + p(\tilde{d}_t), \tag{3.23}$$

where r denotes the world interest rate and $p(\cdot)$ is a country-specific interest rate premium. The function $p(\cdot)$ is assumed to be strictly increasing.

Preferences are given by equation (6.1). Unlike in the previous model, preferences are assumed to display a constant subjective rate of discount. Formally,

$$\theta_t = \beta^t,$$

where $\beta \in (0,1)$ is a constant parameter.

The representative agent's first-order conditions are (3.4)-(3.7) holding with equality and

$$\lambda_t = \beta(1+r_t)E_t\lambda_{t+1} \tag{3.24}$$

$$U_c(c_t, h_t) = \lambda_t, \tag{3.25}$$

$$-U_h(c_t, h_t) = \lambda_t A_t F_h(k_t, h_t). \tag{3.26}$$

$$\lambda_t[1+\Phi'(k_{t+1}-k_t)] = \beta E_t\lambda_{t+1}\left[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2}-k_{t+1})\right]. \tag{3.27}$$

Because agents are assumed to be identical, in equilibrium aggregate per capita debt equals individual debt, that is,

$$\tilde{d}_t = d_t. \tag{3.28}$$

A competitive equilibrium is a set of processes $\{d_t, \tilde{d}_{t+1}, c_t, h_t, y_t, i_t, k_{t+1}, r_t, \lambda_t\}_{t=0}^{\infty}$ satisfying (3.4)-(3.7), and (3.23)-(3.28) all holding with equality, given (3.14), A_0, d_{-1}, and k_0.

We adopt the same forms for the functions U, F, and Φ as in Model 1. We use the following functional form for the risk premium:

$$p(d) = \psi_2\left(e^{d-\bar{d}} - 1\right),$$

computational technique consists in iterating a Bellman equation over a discretized state space. The economy with a noninternalized discount factor features an externality, which which complicates significantly the task of computing the equilibrium by value function iterations. A similar comment applies to the computation of equilibrium in a model with an interest-rate premium that depends on the aggreage level of external debt to be discussed in the next subsection.

where ψ_2 and $\bar{d}$ are constant parameters.

We calibrate the parameters γ, ω, α, ϕ, r, δ, ρ, and σ_ϵ using the values shown in table 3.1. We set the subjective discount factor equal to the world interest rate; that is,

$$\beta = \frac{1}{1+r}.$$

The parameter $\bar{d}$ equals the steady-state level of foreign debt. To see this, note that in steady state, the equilibrium conditions (3.23) and (3.24) together with the assumed form of the interest-rate premium imply that $1 = \beta\left[1 + r + \psi_2\left(e^{d-\bar{d}} - 1\right)\right]$. The fact that $\beta(1+r) = 1$ then implies that $d = \bar{d}$. If follows that in the steady state the interest rate premium is nil. We set $\bar{d}$ so that the steady-state level of foreign debt equals the one implied by Model 1. Finally, we set the parameter ψ_2 so as to ensure that this model and Model 1 generate the same volatility in the current-account-to-GDP ratio. The resulting values of β, $\bar{d}$, and ψ_2 are given in table 2.

Table 3.3: Model 2: Calibration of Parameters Not Shared With Model 1

β	$\bar{d}$	ψ_2
0.96	0.7442	0.000742

3.3.3 Portfolio Adjustment Costs (Model 3)

In this model, stationarity is induced by assuming that agents face convex costs of holding assets in quantities different from some long-run level. Preferences and technology are as in Model 2. In contrast to what is assumed in Model 2, here the interest rate at which domestic households can borrow from the rest of the world is constant and equal to the world interest, that is, equation (3.13) holds. The sequential budget constraint of the household is given by

$$d_t = (1 + r_{t-1})d_{t-1} - y_t + c_t + i_t + \Phi(k_{t+1} - k_t) + \frac{\psi_3}{2}(d_t - \bar{d})^2, \qquad (3.29)$$

where ψ_3 and $\bar{d}$ are constant parameters defining the portfolio adjustment cost function. The first-order conditions associated with the household's maximization problem are (3.5)-(3.7), (3.25)-(3.27), (3.29) holding with equality and

$$\lambda_t[1 - \psi_3(d_t - \bar{d})] = \beta(1 + r_t)E_t\lambda_{t+1} \qquad (3.30)$$

This optimality condition states that if the household chooses to borrow an additional unit, then current consumption increases by one unit minus the marginal portfolio adjustment cost $\psi_3(d_t - \bar{d})$. The value of this increase in consumption in terms of utility is given by the left-hand side of the above equation. Next period, the household must repay the additional unit of debt plus interest. The value of this repayment in terms of today's utility is given by the right-hand side. At the optimum, the marginal benefit of a unit debt increase must equal its marginal cost.

A competitive equilibrium is a set of processes $\{d_t, c_t, h_t, y_t, i_t, k_{t+1}, r_t, \lambda_t\}_{t=0}^{\infty}$ satisfying (3.5)-(3.7), (3.13), (3.25)-(3.27), (3.29), and (3.30) all holding with equality, given (3.14), A_0, d_{-1}, and k_0.

Preferences and technology are parameterized as in Model 2. The parameters γ, ω, α, ϕ, r, δ, ρ, and σ_ϵ take the values displayed in table 3.1. As in model 2, the subjective discount factor is assumed to satisfy $\beta(1+r) = 1$. This assumption and equation (3.30) imply that the parameter $\bar{d}$ determines the steady-state level of foreign debt ($d = \bar{d}$). We calibrate $\bar{d}$ so that the steady-state level of foreign debt equals the one implied by models 1, 1a, and 2 (see table 3.3). Finally, we assign the value 0.00074 to ψ_3, which ensures that this model and model 1 generate the same volatility in the current-account-to-GDP ratio. This parameter value is almost identical to that assigned to ψ_2 in model 2. This is because the log-linearized versions of models 2 and 3 are almost identical. Indeed, the models share all equilibrium conditions but the resource constraint (equations (3.4) and (3.29)), the Euler equations associated with the optimal choice of foreign bonds (equations (3.24) and (3.30)), and the interest rate faced by domestic households (equations (3.13) and (3.23)). The log-linearized versions of the resource constraints are the same in both models. The log-linear approximation to the domestic interest rate is given by $\widehat{1+r_t} = \psi_2 d(1+r)^{-1}\widehat{d}_t$ in Model 2 and by $\widehat{1+r_t} = 0$ in Model 3. In turn, the log-linearized versions of the Euler equation for debt are $\widehat{\lambda}_t = \psi_2 d(1+r)^{-1}\widehat{d}_t + E_t\widehat{\lambda}_{t+1}$ in model 2 and $\widehat{\lambda}_t = \psi_3 d\widehat{d}_t + E_t\widehat{\lambda}_{t+1}$ in Model 3. It follows that for small values of ψ_2 and ψ_3 satisfying $\psi_2 = (1+r)\psi_3$ models 2 and 3 will imply similar dynamics.

3.3.4 Complete Asset Markets (Model4)

All model economies considered thus far feature incomplete asset markets. In those models agents have access to a single financial asset that pays a risk-free real rate of return. In the model studied in this subsection, agents have access to a complete array of state-contingent claims. This assumption per se induces stationarity in the equilibrium dynamics.

Preferences and technology are as in model 2. The period-by-period budget constraint of the household is given by

$$E_t r_{t+1} b_{t+1} = b_t + y_t - c_t - i_t - \Phi(k_{t+1} - k_t), \tag{3.31}$$

where r_{t+1} is a stochastic discount factor such that the period-t price of a random payment b_{t+1} in period $t+1$ is given by $E_t r_{t+1} b_{t+1}$. Note that because $E_t r_{t+1}$ is the price in period t of an asset that pays 1 unit of good in every state of period $t+1$, it follows that $1/[E_t r_{t+1}]$ denotes the risk-free real interest rate in period t. Households are assumed to be subject to a no-Ponzi-game constraint of the form

$$\lim_{j\to\infty} E_t q_{t+j} b_{t+j} \geq 0, \tag{3.32}$$

at all dates and under all contingencies. The variable q_t represents the stochatic discount factor between periods 0 and t, such that the period-0 price of a random payment b_t in period t is given by $E_0 q_t b_t$. We have that q_t satisfies

$$q_t = r_1 r_2 \dots r_t,$$

with $q_0 \equiv 1$. The first-order conditions associated with the household's maximization problem are (3.5), (3.6), (3.25)-(3.27), (3.31), and (3.32) holding with equality and

$$\lambda_t r_{t+1} = \beta \lambda_{t+1}. \tag{3.33}$$

A difference between this expression and the Euler equations that arise in the models with incomplete asset markets studied in previous sections is that under complete markets in each period t there is one first-order condition for each possible state in period $t+1$, whereas under incomplete markets the above Euler equation holds only in expectations.

In the rest of the world, agents have access to the same array of financial assets as in the domestic economy. Consequently, one first-order condition of the foreign household is an equation similar to (3.33). Letting starred letters denote foreign variables or functions, we have

$$\lambda_t^* r_{t+1} = \beta \lambda_{t+1}^*. \tag{3.34}$$

Note that we are assuming that domestic and foreign households share the same subjective discount factor. Combining the domestic and foreign Euler equations—equations (3.33) and (3.34)—yields

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\lambda_{t+1}^*}{\lambda_t^*}.$$

This expression holds at all dates and under all contingencies. This means that the domestic marginal utility of consumption is proportional to its foreign counterpart. Formally,

$$\lambda_t = \xi \lambda_t^*,$$

where ξ is a constant parameter determining differences in wealth across countries. We assume that the domestic economy is small. This means that λ_t^* must be taken as an exogenous variable. Because we are interested only in the effects of domestic productivity shocks, we assume that λ_t^* is constant and equal to λ^*, where λ^* is a parameter. The above equilibrium condition then becomes

$$\lambda_t = \psi_4, \tag{3.35}$$

where $\psi_4 \equiv \xi \lambda^*$ is a constant parameter.

A competitive equilibrium is a set of processes $\{c_t, h_t, y_t, i_t, k_{t+1}, \lambda_t\}_{t=0}^{\infty}$ satisfying (3.5), (3.6), (3.25)-(3.27), and (3.35), given (3.14), A_0, and k_0.

The functions U, F, and Φ are parameterized as in the previous models. The parameters γ, β, ω, α, ϕ, δ, ρ, and σ_ϵ take the values displayed in tables 3.1 and 3.3. The parameter ψ_4 is set so as to ensure that the steady-state level of consumption is the same in this model as in models 1 to 3.

3.3.5 The Nonstationary Case (Model 5)

For comparison with the models considered thus far, in this section we describe a version of the small open economy model that displays no stationarity. In this model (a) The discount factor is constant; (b) the interest rate at which domestic agents borrow from the rest of the world is constant (and equal to the subjective discount factor); (c) agents face no frictions in adjusting the size of their portfolios; and (d) markets are incomplete in the sense that domestic households have only access to a single risk-free international bond. This specification of the model induces a random walk component in the equilibrium marginal utility of consumption and the net foreign asset position.

A competitive equilibrium in the nonstationary model is a set of processes $\{d_t,\ c_t,\ h_t,\ y_t,\ i_t,\ k_{t+1},\ r_t,\ \lambda_t\}_{t=0}^{\infty}$ satisfying (3.4)-(3.7), (3.13), and (3.24)-(3.27) all holding with equality, given (3.14), A_0, d_{-1}, and k_0. We calibrate the model using the parameter values displayed in tables 3.1 and 3.3.

3.3.6 Quantitative Results

Table 3.4 displays a number of unconditional second moments of interest observed in the data and implied by models 1 - 4.[4] For all models, we compute the equilibrium dynamics by solving a log-linear approximation to the set of equilibrium conditions. The appendix shows the log-linear version of model 1. The Matlab computer code used to compute the unconditional second moments and impulse response functions for all models presented in this section is available at www.econ.duke.edu/∼uribe.

Although the focus of this section is not to assess the models' abilities to match the data, as a reference we include in the first column of the table the observed second moments using Canadian data. As pointed out by Mendoza (1991), the small open real business cycle model captures a number of features of business cycles in Canada. Specifically, as in the data, all four models predict the following ranking of volatilities, in ascending order, consumption, output, and investment. The models also correctly predict that the components of aggregate demand and hours are procyclical, and that the correlation of the trade balance with GDP is close to zero.[5] The models overestimate the procyclicality of labor. In the data the correlation between hours and output is 0.8, whereas the models imply a perfect correlation. As pointed out earlier, this implication of the models is driven by the assumed preference specification.

The main result of this section is that regardless of how stationarity is induced, the model's predictions regarding second moments are virtually identical. This result is evident from table 3.4. The only noticeable difference arises in model 4, the complete markets case, which as expected predicts less volatile consumption. The low volatility of consumption in the complete markets model introduces an additional difference between the predictions of this model and models 1-3. Because consumption is smoother in model 4, its role in determining the cyclicality of the trade balance is smaller. As a result, model 4 predicts that the correlation between output and the trade balance is positive, whereas models 1-3 imply that it is negative.

Figure 3.3 demonstrates that models 1-5 also imply virtually identical impulse response functions to a technology shock. Each panel shows the impulse response of a particular variable in the six models. For all variables

[4]Model 5 is nonstationary, and therefore does not have well defined unconditional second moments.

[5]Indeed, models 1-3 predict correctly that the trade balance is countercyclical. The correlation is so close to zero, however, that its sign does depend on the solution method employed. Mendoza, for example, approximates the solution to model 1 by discretizing the state space and finds a small but positive correlation.

but consumption and the trade-balance-to-GDP ratio, the impulse response functions are so similar that to the naked eye the graph appears to show just a single line. Again, the only small but noticeable difference is given by the responses of consumption and the trade-balance-to-GDP ratio in the complete markets model. In response to a positive technology shock, consumption increases less when markets are complete than when markets are incomplete. This in turn, leads to a smaller decline in the trade balance in the period in which the technology shock occurs.

3.4 Appendix

3.4.1 Log-Linearization of the Equilibrium Conditions

Let $\widehat{x}_t \equiv \log(x_t/\bar{x})$ denote the log-deviation of x_t from its steady-state value. Then, taking a first-order log-linearization of the model we get:

$$s_{tb}\widehat{d}_t = s_{tb}\frac{r}{1+r}\widehat{r}_{t-1} + s_{tb}(1+r)\widehat{d}_{t-1} - r[\widehat{y}_t - s_c\widehat{c}_t - s_i\widehat{i}_t]$$

$$\widehat{y}_t = \widehat{A}_t + \alpha\widehat{k}_t + (1-\alpha)\widehat{h}_t$$

$$\widehat{k}_{t+1} = (1-\delta)\widehat{k}_t + \delta\widehat{i}_t$$

$$\widehat{\lambda}_t = \frac{r}{1+r}\widehat{r}_t + \epsilon_{\beta c}\widehat{c}_t + \epsilon_{\beta h}\widehat{h}_t + E_t\widehat{\lambda}_{t+1}$$

$$\widehat{\lambda}_t = \frac{(1-\beta)\epsilon_c}{(1-\beta)\epsilon_c - \beta\epsilon_{\beta c}}[\epsilon_{cc}\widehat{c}_t + \epsilon_{ch}\widehat{h}_t] - \frac{\beta\epsilon_{\beta c}}{(1-\beta)\epsilon_c - \beta\epsilon_{\beta c}}[\widehat{\eta}_t + \epsilon_{\beta cc}\widehat{c}_t + \epsilon_{\beta ch}\widehat{h}_t]$$

$$\widehat{\eta}_t = (1-\beta)[\epsilon_c E_t\widehat{c}_{t+1} + \epsilon_h E_t\widehat{h}_{t+1}] + \beta[E_t\widehat{\eta}_{t+1} + \epsilon_{\beta c}\widehat{c}_t + \epsilon_{\beta h}\widehat{h}_t]$$

$$\frac{(1-\beta)\epsilon_h}{(1-\beta)\epsilon_h + \beta\epsilon_{\beta h}}[\epsilon_{hc}\widehat{c}_t + \epsilon_{hh}\widehat{h}_t] + \frac{\beta\epsilon_{\beta h}}{(1-\beta)\epsilon_h + \beta\epsilon_{\beta h}}[\widehat{\eta}_t + \epsilon_{\beta hc}\widehat{c}_t + \epsilon_{\beta hh}\widehat{h}_t] = \widehat{\lambda}_t + \widehat{A}_t + \alpha\widehat{k}_t - \alpha\widehat{h}_t$$

$$\begin{aligned}\widehat{\lambda}_t + \phi k\widehat{k}_{t+1} - \phi k\widehat{k}_t \quad = \quad & \epsilon_{\beta c}\widehat{c}_t + \epsilon_{\beta h}\widehat{h}_t + E_t\widehat{\lambda}_{t+1} + \beta(\beta^{-1} + \delta - 1)[E_t\widehat{A}_{t+1} \\ & +(1-\alpha)E_t\widehat{h}_{t+1} - (1-\alpha)\widehat{k}_{t+1} + \beta\phi k E_t\widehat{k}_{t+2} - \beta\phi k\widehat{k}_{t+1}\end{aligned}$$

$$\widehat{r}_t = 0$$

$$\hat{A}_t = \rho\hat{A}_{t-1} + \epsilon_t,$$

where $\epsilon_{\beta c} \equiv c\beta_c/\beta$, $\epsilon_{\beta h} \equiv h\beta_h/\beta$, $\epsilon_{\beta cc} \equiv c\beta_{cc}/\beta_c$, $\epsilon_{\beta ch} \equiv h\beta_{ch}/\beta_c$, $\epsilon_c \equiv cU_c/U$, $\epsilon_{cc} = cU_{cc}/U_c$, $\epsilon_{ch} = hU_{ch}/U_c$, $s_{tb} \equiv tb/y$, $s_c \equiv c/y$, $s_i = i/y$. In the log-linearization we are using the particular forms assumed for the production function and the capital adjustment cost function.

3.5 Exercise

Real Business Cycles in a Small Open Economy with an internalized debt-elastic interest-rate premium

Consider a variation of the small open economy RBC model with a debt elastic interest rate premium studied in Schmitt-Grohé and Uribe (SGU) (*JIE*, 2003, section 3, model 2) that allows agents to internalize the dependence of the interest rate premium on the level of debt. . Specifically, suppose that the function $p(\cdot)$ depends on the individual debt position, d_t, rather than on the aggregate per capita level of debt, $\tilde{d}_t$.

1. Derive the model's equilibrium conditions.

2. Use the same forms for the functions U, F, Φ, and p as in SGU. Calibrate the parameters γ, ω, α, ϕ, r, δ, ρ, σ_ϵ, β, $\bar{d}$, and ψ_2 using tables 1 and 2 in SGU. Calculate the model's nonstochastic steady state and compare it to that of model 2 in SGU.

3. Compute the unconditonal standard deviation, serial correlation, and correlation with output of output, consumption, investment, hours, the trade balance-to-output ratio, and the current account-to-output ratio implied by the model. Compare these statistics to those associated with model 2 in SGU (reported in their table 3).

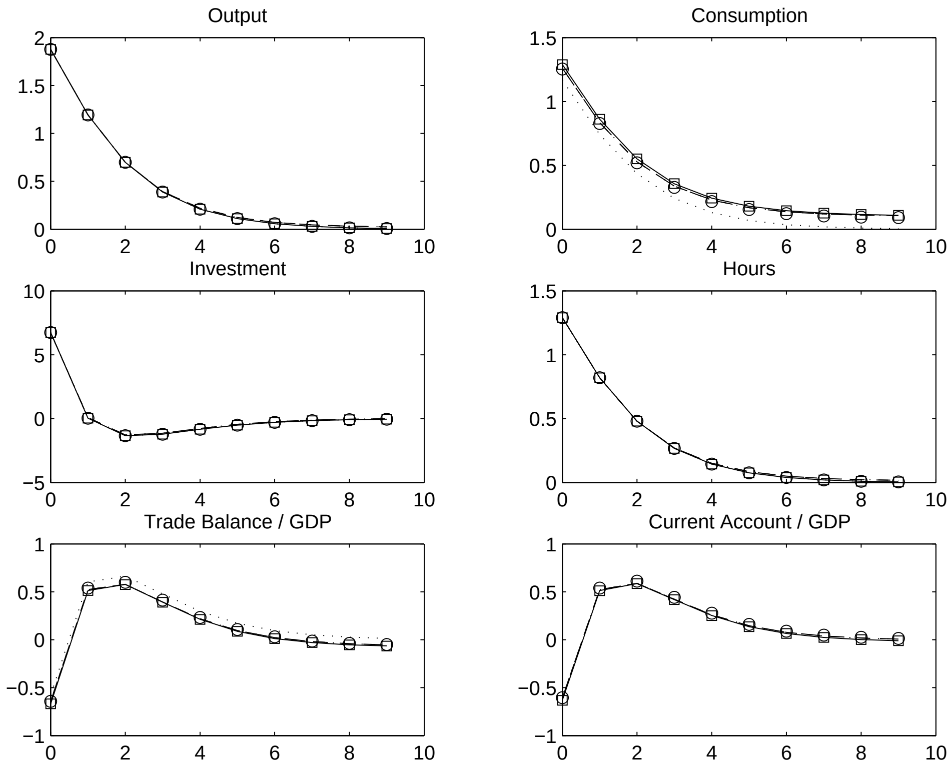
4. Compute the impulse response functions of output, consumption, investment, hours, the trade balance-to-output ratio, and the current account-to-output ratio implied by the model. Compare these impulse responses to those associated with model 2 in SGU (shown in their figure 1).

Table 3.4: Observed and Implied Second Moments

	Data	Model 1	Model 1a	Model 2	Model 3	Model 4
<u>Volatilities</u>:						
std(y_t)	2.8	3.1	3.1	3.1	3.1	3.1
std(c_t)	2.5	2.3	2.3	2.7	2.7	1.9
std(i_t)	9.8	9.1	9.1	9	9	9.1
std(h_t)	2	2.1	2.1	2.1	2.1	2.1
std($\frac{tb_t}{y_t}$)	1.9	1.5	1.5	1.8	1.8	1.6
std($\frac{ca_t}{y_t}$)		1.5	1.5	1.5	1.5	
<u>Serial Correlations</u>:						
corr(y_t, y_{t-1})	0.61	0.61	0.61	0.62	0.62	0.61
corr(c_t, c_{t-1})	0.7	0.7	0.7	0.78	0.78	0.61
corr(i_t, i_{t-1})	0.31	0.07	0.07	0.069	0.069	0.07
corr(h_t, h_{t-1})	0.54	0.61	0.61	0.62	0.62	0.61
corr($\frac{tb_t}{y_t}, \frac{tb_{t-1}}{y_{t-1}}$)	0.66	0.33	0.32	0.51	0.5	0.39
corr($\frac{ca_t}{y_t}, \frac{ca_{t-1}}{y_{t-1}}$)		0.3	0.3	0.32	0.32	
<u>Correlations with Output</u>:						
corr(c_t, y_t)	0.59	0.94	0.94	0.84	0.85	1
corr(i_t, y_t)	0.64	0.66	0.66	0.67	0.67	0.66
corr(h_t, y_t)	0.8	1	1	1	1	1
corr($\frac{tb_t}{y_t}, y_t$)	-0.13	-0.012	-0.013	-0.044	-0.043	0.13
corr($\frac{ca_t}{y_t}, y_t$)		0.026	0.025	0.05	0.051	

Note. The first column was taken from Mendoza (1991). Standard deviations are measured in percent per year.

Figure 3.3: Impulse Response to a Unit Technology Shock in Models 1 - 5



Note. Solid line: Endogenous discount factor model; Squares: Endogenous discount factor model without internalization; Dashed line: Debt-elastic interest rate model; Dash-dotted line: Portfolio adjustment cost model; Dotted line: complete asset markets model; Circles: Model without stationarity inducing elements.

Chapter 4

Solving Dynamic General Equilibrium Models By Linear Approximation

The equilibrium conditions of the simple real business cycle model we studied in the previous chapter takes the form of a nonlinear stochastic vector difference equation. Reduced forms of this sort are common in Macroeconomics. A problem that one must face is that, in general, it is impossible to solve such systems. But fortunately one can obtain good approximations to the true solution in relatively easy ways. In the previous chapter, we introduced one particular strategy, consisting in linearizing the equilbrium conditions around the nonstochastic steady state. Here we explain in detail how to solve the resulting system of linear stochastic difference equations. In addition, we show how to use the solution to compute second moments and impuse response functions.

Consider a vector x_t of order $n \times 1$ whose law of motion is of the form

$$AE_t x_{t+1} = Bx_t + C \tag{4.1}$$

It is convenient to partition x_t into two components

$$x_t \equiv \begin{bmatrix} s_t \\ c_t \end{bmatrix}.$$

The vector s_t is of order $n_s \times 1$ and contains either endogenous predetermined variables that cannot adjust contemporaneously to news, or exogenous variables. The vector c_t is of order $n_c \times 1$ and contains endogenous variables that can adjust contemporaneously to news. The vectors s_t and c_t are also referred to as the state and the control of the system, respectively. Using the RBC model of the previous chapter as an example, we have that the capital stock and the net debt position of the economy are endogenous state variables, the productivity shock is an exogenous state variable, and consumption and hours are two of the control variables of the system. Of course, we have that $n_s + n_c = n$. We further partitiion the state vector into the endogenous component, s_t^{en}, and the exogenous component, s_t^{ex},

$$s_t \equiv \begin{bmatrix} s_t^{en} \\ s_t^{ex} \end{bmatrix},$$

where s_t^{en} is of order $n_{en} \times 1$ and s_t^{ex} is of order $n_{ex} \times 1$, with $n_{en} + n_{ex} = n_s$.

The initial conditions of the system is

$$s_0 = s,$$

where s is an $n_s \times 1$ vector of constants. There are no initial conditions for the control variables. Instead, we impose terminal conditions, by requiring that at each point the variables of the system be expected not to explode. Formally,

$$\lim_{j \to \infty} |E_t x_{t+j}| < \infty$$

The source of uncertainty in the system is a vector μ_t of innovations to the state of the economy. That is,

$$\mu_{t+1} \equiv s_{t+1} - E_t s_{t+1},$$

The vector μ_t is given by

$$\mu_t = \begin{bmatrix} \emptyset \\ \epsilon_t \end{bmatrix},$$

where ϵ_t is of order $n_{ex} \times 1$ and denotes the innovation to the exogenous state vector and is i.i.d. with mean $\emptyset$ and variance/covariance matrix Σ_ϵ.

The $n \times n$ matrix A may be singular, reflecting the presence of equations involving only elements of x_t (i.e., involving no elements of $E_t x_{t+1}$). One should not interpret the singularity of A as indicating multiple solutions or no solution to the system.To see this, consider the following example:

$$x_{t+1} = a x_t + y_t$$

$$y_t = b x_t$$

with x_t a predetermined state variable and y_t a non-predetermined control variable and $|a+b| < 1$. If one writes the system in the form given by (4.1)

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} a & 1 \\ b & -1 \end{bmatrix}$$

Clearly, A is not invertible. However, the system has a unique solution given by

$$x_t = (a+b)^t x_0; \quad t \geq 0.$$

We look for the recursive representation of the solution to the system (4.1), which is of the form:

$$c_t = \Pi s_t + L$$

$$E_t s_{t+1} = \Phi s_t + N$$

The nonstochastic steady state (4.1) is given by a couple of vectors $\bar{c}$ and $\bar{s}$ satisfying

$$\begin{bmatrix} \bar{s} \\ \bar{c} \end{bmatrix} \equiv \bar{x} = (A-B)^{-1} C$$

Define $\hat{x}_t = x_t - \bar{x}$ as the deviation of x_t from its steady-state value. Then we can concentrate in solving the following homogeneous system in $\hat{x}_t$:

$$A E_t \hat{x}_{t+1} = B \hat{x}_t$$

Next, we show how to compute the matrices Φ, Π, L, and N.

4.1 The Schur Decomposition Method

To solve the above system, we use the generalized Schur decomposition of the matrices A and B.[1] The generalized Schur decomposition of A and B is given by upper triangular matrices a and b and orthonormal matrices q and z satisfying:[2]

$$qAz = a$$

and

$$qBz = b.$$

Let

$$y_t \equiv z' \hat{x}_t.$$

[1] More formal descriptions of the method can be found in Klein (2000) and Sims (1996).

[2] Recall that a matrix a is said to be upper triangular if elements $a_{ij} = 0$ for $i > j$. A matrix z is orthonormal if $z'z = zz' = I$.

Then we have that

$$aE_t y_{t+1} = b y_t$$

Now partition a, b, z, and y_t as

$$a = \begin{bmatrix} a_{11} & a_{12} \\ 0 & a_{22} \end{bmatrix}, \; b = \begin{bmatrix} b_{11} & b_{12} \\ 0 & b_{22} \end{bmatrix}; \; z = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}; \; y_t = \begin{bmatrix} y_t^1 \\ y_t^2 \end{bmatrix},$$

where a_{22} and b_{22} are of order $n_c \times n_c$, z_{12} is of order $n_s \times n_c$, and y_t^2 is of order $n_c \times 1$. Then we have that

$$a_{22} E_t y_{t+1}^2 = b_{22} y_t^2,$$

or

$$b_{22}^{-1} a_{22} E_t y_{t+1}^2 = y_t^2.$$

Assume, without loss of generality, that the ratios abs(a_{ii}/b_{ii}) are decreasing in i. Suppose further that the number of ratios less than unity is exactly equal to the number of control variables, n_c, and that the number of ratios greater than one is equal to the number of state variables, n_s. By construction, the eigenvalues of $b_{22}^{-1} a_{22}$ are all less than unity in modulus.[3] Thus, the requirement $\lim_{j\to\infty} |E_t y_{t+j}^2| < \infty$ is satisfied only if $y_t^2 = 0$. In turn, by the definition of y_t^2, this restriction implies that

$$z_{12}' \hat{s}_t + z_{22}' \hat{c}_t = 0,$$

or

$$\hat{c}_t = -{z_{22}'}^{-1} z_{12}' \hat{s}_t;$$

so that

$$\Pi = -{z_{22}'}^{-1} z_{12}'.$$

The fact that $y_t^2 = 0$ also implies that

$$a_{11} y_{t+1}^1 = b_{11} y_t^1,$$

or

$$y_{t+1}^1 = {a_{11}}^{-1} b_{11} y_t^1$$

Now

$$y_t^1 = z_{11}' \hat{s}_t + z_{21}' \hat{c}_t$$

[3]Here we are applying a number of properties of upper triangular matrices. Namely, (a) The inverse of a nonsingular upper triangular matrix is upper triangular. (b) the product of two upper triangular matrices is upper triangular. (c) The eigenvalues of an upper triangular matrix are the elements of its main diagonal.

so

$$y_t^1 = [z'_{11} - z'_{21}{z'_{22}}^{-1}z'_{12}]\hat{s}_t.$$

Combining this expression with the equation describing the evolution of $\hat{y}_t$ shown two lines above, we get

$$\hat{s}_{t+1} = [z'_{11} - z'_{21}{z'_{22}}^{-1}z'_{12}]^{-1}{a_{11}}^{-1}b_{11}[z'_{11} - z'_{21}{z'_{22}}^{-1}z'_{12}]\hat{s}_t;$$

so that

$$\Phi = [z'_{11} - z'_{21}{z'_{22}}^{-1}z'_{12}]^{-1}{a_{11}}^{-1}b_{11}[z'_{11} - z'_{21}{z'_{22}}^{-1}z'_{12}].$$

We can simplify this expression for Π by using the following restrictions:

$$I = z'z = \begin{bmatrix} z'_{11}z_{11} + z'_{21}z_{21} & z'_{11}z_{12} + z'_{21}z_{22} \\ z'_{12}z_{11} + z'_{22}z_{21} & z'_{12}z_{12} + z'_{22}z_{22} \end{bmatrix}$$

to write:[4]

$$\Pi = z_{21}z_{11}^{-1}$$

$$\Phi = z_{11}a_{11}^{-1}b_{11}z_{11}^{-1}.$$

Finally, by the definitions of $\hat{c}_t$ and $\hat{s}_t$, it follows that

$$L = \bar{c} - \Pi\bar{s}$$

and

$$N = \bar{s} - \Phi\bar{s}$$

4.1.1 Matlab Code

Paul Klein of the University of Western Ontario has written Matlab programs that compute the matrices Φ and Π given A, B, and n_s. The set of programs consists of three files, solab.m, qzswitch.m, and qzdiv.m and is available on Klein's website, http://www.ssc.uwo.ca/economics/faculty/klein/. The programs qzswitch.m and qzdiv.m were created by Chris Sims of Princeton University.

[4] To obtain K use the element $(2,1)$ of $z'z$. To obtain the reduced expression for Φ, use element $(2,1)$ of $z'z$ to get $z'_{12}z_{11} = -z'_{22}z_{21}$. Pre multiply by ${z'_{22}}^{-1}$ and post multiply by z_{11}^{-1} to get ${z'_{22}}^{-1}z'_{12} = -z_{21}{z_{11}}^{-1}$. Use this expression to eliminate ${z'_{22}}^{-1}z'_{12}$ From the square bracket in the expression for Φ. Then this square bracket becomes $[z'_{11} + z'_{21}z_{21}{z_{11}}^{-1}]$. Now use element $(1,1)$ of $z'z$ to write $z'_{21}z_{21} = I - z'_{11}z_{11}$. Using this equation to eliminate $z'_{21}z_{21}$ from the expression in square brackets, we get $[z'_{11} + (I - z'_{11}z_{11}){z_{11}}^{-1}]$, which is simply z_{11}^{-1}.

4.2 Computing Second Moments

Start with the equilibrium law of motion of the deviation of the state vector with respect to its steady-state value, which is given by

$$\hat{s}_{t+1} = \Phi \hat{s}_t + \mu_{t+1}, \tag{4.2}$$

Let

$$\Sigma_s \equiv E \hat{s}_t \hat{s}_t'$$

denote the unconditional variance/covariance matrix of $\hat{s}_t$ and

$$\Sigma_\mu \equiv \begin{bmatrix} \emptyset & \emptyset \\ \emptyset & \Sigma_\epsilon \end{bmatrix}$$

the variance/covariance matrix of μ_t. Then we have that

$$\Sigma_s = \Phi \Sigma_s \Phi' + \Sigma_\mu.$$

We will describe three numerical methods to compute Σ_s.

4.2.1 Method 1

Suppose Φ can be decomposed as $\Phi = R \Gamma R^{-1}$, where Γ is a diagonal matrix. Then letting

$$\tilde{s}_t = R^{-1} \hat{s}_t,$$

we can write

$$\tilde{s}_{t+1} = \Gamma \tilde{s}_t + R^{-1} \mu_{t+1}.$$

Now defining $\Sigma_{\tilde{s}} = E \tilde{s}_t \tilde{s}_t$ and $\Sigma_{\tilde{\mu}} = R^{-1} \Sigma_\mu R^{-1\prime}$, we have

$$\Sigma_{\tilde{s}} = \Gamma \Sigma_{\tilde{s}} \Gamma' + \Sigma_{\tilde{\mu}}$$

It follows that

$$\Sigma_{\tilde{s},ij} = \frac{\Sigma_{\tilde{\mu},ij}}{1 - \Gamma_i \Gamma_j} \quad i, j = 1, \dots n_s,$$

where $\Sigma_{\tilde{s},ij}$ denotes the element of $\Sigma_{\tilde{s}}$ located in the ith row and the jth column (a similar notation applies for $\Sigma_{\tilde{\mu},ij}$) and Γ_i denotes the ith diagonal element of Γ. Now use the fact that $\hat{s}_t = R \tilde{s}_t$ to obtain

$$\Sigma_s = R \Sigma_{\tilde{s}} R'.$$

4.2.2 Method 2

Apply the following iterative method:

$$\Sigma_{s,t+1} = \Phi \Sigma_{s,t} \Phi' + \Sigma_{\mu}$$

$$\Sigma_{s,0} = I$$

Stop when $\Sigma_{s,t}$ does not change much as t increases.

4.2.3 Method 3

The following procedure, called doubling algorithm, is a faster version of the one described above.

$$\Sigma_{s,t+1} = \Phi_t \Sigma_{s,t} \Phi_t' + \Sigma_{\mu,t}$$

$$\Phi_{t+1} = \Phi_t \Phi_t$$

$$\Sigma_{\mu,t+1} = \Phi_t \Sigma_{\mu,t} \Phi_t' + \Sigma_{\mu,t}$$

$$\Sigma_{s,0} = I$$

$$\Phi_0 = \Phi$$

$$\Sigma_{\mu,0} = \Sigma_{\mu}$$

4.2.4 Other second moments

Once the variance/covariance matrix of the state vector has been computed, it is easy to find other second moments of interest. Consider for instance the variance/covariance matrix $E\hat{s}_t\hat{s}'_{t-j}$ for $j > 0$.

$$\begin{aligned} E\hat{s}_t\hat{s}'_{t-j} &= E[\Phi^j \hat{s}_{t-j} + \sum_{h=0}^{j-1} \Phi^h \mu_{t-h}]\hat{s}'_{t-jl} \\ &= \Phi^j E\hat{s}_{t-1}\hat{s}'_{t-j} \\ &= \Phi^j \Sigma_s \end{aligned}$$

Similarly, consider the variance covariance matrix of linear combinations of the state vector s_t. For instance, the co-state, or control vector c_t is given by $c_t = \Pi s_t$. Then

$$\begin{aligned} E\hat{c}_t\hat{c}_t' &= \Pi[E\hat{s}_t\hat{s}_t']\Pi' \\ &= \Pi \Sigma_s \Pi'. \end{aligned}$$

and, more generally,

$$
\begin{aligned}
E\hat{c}_t\hat{c}'_{t-j} &= \Pi[E\hat{s}_t\hat{s}'_{t-j}]\Pi' \\
&= \Pi\Phi^j\Sigma_s\Pi',
\end{aligned}
$$

for $j \geq 0$.

4.3 Impulse Response Functions

The impulse response function traces the expected behavior of the system from period 0 on given information available in period 0, in a situation where in period 0 all but one (or some) element(s) of the state vector take their steady-state values. Let the initial state of the system be s, that is, $s_0 = s$. Using the law of motion $E_t\hat{s}_{t+1} = \Phi\hat{s}_t$ for the state vector and the law of iterated expectations we get that the impulse response of the state vector in period t is given by

$$IR(\hat{s}_t) \equiv E_0\hat{s}_t = \Pi^t s; \quad t \geq 0.$$

The response of the vector of controls $\hat{c}_t$ is given by

$$IR(\hat{c}_t) = \Pi\Phi^t s.$$

4.4 Higher Order Approximations

In this chapter, we focused on a first-order approximation to the solution of a nonlinear system of stochastic difference equations of the form $E_t f(x_{t+1}, x_t) = 0$.. But higher order approximations are relatively easy to obtain. Indeed, there is a sense in which higher order approximations are simpler than the first order approximation. Namely, obtaining a higher-order approximation to the solution of the non-linear system is a sequential procedure. Specifically, the coefficients of the ith term of the jth-order approximation are given by the coefficients of the ith term of the ith order approximation, for $j > 1$ and $i < j$. So if the first-order approximation to the solution is available, then obtaining the second order approximation requires only to compute the coefficients of the quadratic terms, since the coefficients of the linear terms are those of the first order approximation. More importantly, obtaining the coefficients of the ith order terms of the approximate solution given all lower-order coefficients involves solving a *linear* system of equations.

Schmitt-Grohé and Uribe (2004) describe in detail how to obtain a seond-order approximation to the solution of the nonlinear system and provide MATLAB code that implements the approximation.

Chapter 5

Business Cycles In Emerging Economies (In Progress)

In discussions of business cycles in small open economies, a critical distinction is between developed and emerting economies. The group of emerging economies is composed by middle income countries with relatively small levels of market capitalization tradable internationally,

5.1 Some Empirical Regularities

A striking difference between developed and emerging economies is that observed business cycles in emerging countries are twice as volatile than in developed countries. Table 5.1 illustrates this contrast by displaying key business-cycle properties in Argentina and and Canada. The volatility of detrended output is 4.6 in Argentina but only 2.8 in Canada. Another remarkable difference between developing and developed countries suggested by the table is that the trade balance-to-output ratio is much more countercylical in emerging countries than in develped countries. Periods of economic boom (contraction) are characterized by relatively larger trade deficits (surpluses) in emerging countries than in develped countries. A third difference between developed and devellping economies that one may extrapolate from the table is that in the latter group consumption appears to be more volatile than output at business-cycle frequences, whereas the reverse is the case in the former set of countries. Two additional differences between the business cycle in Argentina and Canada are that in Argentina, the correlation of the components of aggregate demand with GDP are twice as high as in Canada, and that in Argentina hours and productivity are less correlated with GDP

Table 5.1: Business Cycles in Argentina and Canada

Variable	σ_x	$corr(x_t, x_{t-1})$	$corr(x_t, GDP_t)$
GDP			
Argentina	4.6	0.79	1
Canada	2.8	0.61	1
Consumption			
Argentina	5.4		0.96
Canada	2.5	0.70	0.59
Investment			
Argentina	13.3		0.94
Canada	9.8	0.31	0.64
TB/GDP			
Argentina	2.3		-0.84
Canada	1.9	0.66	-0.13
Hours			
Argentina	4.1		0.76
Canada	2.0	0.54	0.80
Productivity			
Argentina	3.0		0.48
Canada	1.7	0.37	0.70

Source: Mendoza (1991), Kydland and Zarazaga (1997). For Argentina, data on hours and productivity are limited to the manufacturing sector.

than in Canada.

One dimension along which business cycles in Argentina and Canada are similar is the procyclicality of consumption, investment, hours,and productivity. In both types of country, these variables move in tandem with ouput.

The differences between the business cycles of Argentina and Canada turn out to hold much more generally. In effect, table 5.2, taken from Aguiar and Gopinath (2004) displays average business cycle facts in developed and emerging economies. The table averages second moments of detrended data for a group of 13 emerging countries and a group of 13 small developed countries (the list of countries appears at the foot of the table).

Table 5.2: Business Cycles: Emerging Vs. Developed Economies

Moment	Emerging Countries	Developed Countries
σ_y	2.02	1.04
$\sigma_{\Delta y}$	1.87	0.95
ρ_y	0.86	0.9
$\rho_{\Delta y}$	0.23	0.09
σ_c/σ_y	1.32	0.94
σ_i/σ_y	3.96	3.42
$\sigma_{tb/y}$	2.09	0.71
$\rho_{tb/y,y}$	-0.58	-0.26
$\rho_{c,y}$	0.74	0.69
$\rho_{i,y}$	0.87	0.75

Note: Average values of moments for 13 emerging countries and 13 developed countries. Emerging countries: Argentina, Brazil, Ecuador, Israel, Korea, Malaysia, Mexico, Peru, Philippines, Slovak Republic, South Africa, Thailand, and Turkey. Developed Countires: Australia, Austria, Belgium, Canada, Denmark, Finland, Netherlands, New Zeland, Norway, Portugal, Spain, Sweden, Switzerland. Data are detrended using a band-pass filter including frequencies between 6 and 32 quarters with 12 leads and lags.

Source: Aguiar and Gopinath (2004).

For all countries, the time series are at least 40 quarters long. The data is detrended using a band-pass filter that leaves out all frequencies above 32 quarters and below 6 quarters. The table concurs with the conclusions drawn from the comparison of cusiness cycles in Argentina and Canada. In particular, emerging countires are significantly more volatile and display a much more countercyclical trade-balance share, than developed countries. Also, consumption is more volatile than output in emerging countries but less volatile than output in developed countries.

5.2 Trends and Cycles

Aguiar and Gopinath (2004) argue that key differences between the business cycle in developed and emerging countries can be explained by the fact that in emerging countries permanent productivity shocks are relatively more important than transitory productivity shocks vis-a-vis developed countries.

It is easy to imagine how this hypothesis can explain the higher volatility of output in emerging countries. But how can difference in the relative importance of permanent and transitory shocks explain the observed stronger countercyclicality of the trade balance in emerging countries? Suppose first that the small open economy is hit by a positive transitory shock to productivity. Because the improvement in productivity is expected to revert to its mean, agents expect output to be higher in the short run than in the long run. As a rexult, in this case the general tendency is to save part of the increase in output for consumption in the future. This tendency to save dampens the response of consumption creating a tendency for the trade balance to be procyclical. Consider now the case of a permanent shock that takes the form of a mean reverting increase in the growth rate of productivity.In this case, agents expect the level of output and productivity to be higher in the future than in the present. As a result, households have an incentive to borrow agains future income to expand present consumption, and firms have an incentive to invest in physical capital to take advantate of higher future higher productivity. In this situation, aggregate has a tendency to expand more vigorously than output, inducing a deterioration in the trade balance. This explanation also provides intuition for why consumption should be expected to be more volatile the larger the importance of permanent shocks relative to transitory disturbances.

5.2.1 A Small Open Economy With Nonstationary Shocks

Aguiar and Gopinath consider a version of the model with a debt-elastic interest-rate premium presented in chapter 3 (section 3.3.2) augmented with nonstationary shocks á la King, Plosser, and Rebelo (1988b). Spedivically, suppose that the production technology takes the form

$$Y_t = a_t K_t^{\alpha} (X_t h_t)^{1-\alpha}, \tag{5.1}$$

where Y_t denotes ouput in period t, K_t denotes capital in period t, h_t denotes hours worked in period t, and a_t and X_t represent productivity shocks. We use upper case letters to denote variables that will contain a trend in equilibrium, and lower case letters to denote varibles that do not contain a trend in equilibrium.

The productivity shock a_t is assumed to follow first-order autorregressive process in logs. That is,

$$\ln a_{t+1} = \rho_a \ln a_t + \epsilon^a_{t+1}; .$$

The productivity shock X_t is assumed to be nonstationary. Specifically, we assume that the growth rate of X_t,

$$g_t \equiv \frac{X_t}{X_{t-1}},$$

follows a first-order autorregressive process of the form

$$\ln(g_{t+1}/g) = \rho_g \ln(g_t/g) + \epsilon^a_{t+1};.$$

The innovations ϵ^a_t and ϵ^g_t are assumed to be mutually independent, i.i.d., and to have mean zero and standard deviations σ_a and σ_g, respectively. The parameter g is a measure of the average gross growth rate of the productivity factor X_t.

Household face the following period-by-period budget constraint

$$\frac{D_t}{1+r_t} = D_{t-1} - Y_t + C_t + I_t + \frac{\phi}{2X_t}(K_{t+1} - gK_t)^2. \tag{5.2}$$

The nonstationary shock X_t appears in the adjustment const function to ensure that adjustment costs grow at the same rate as the rest of the economy in the deterministic path. Similarly, to avoid a trend in hours worked, we modify the specification of the interest-rate premium and the utility function as follows:

$$r_t = r + \psi_2 \left(e^{\tilde{D}_t/X_t - \bar{d}} - 1 \right),$$

and

$$\sum_{t=0}^{\infty} \beta^t \frac{\left[C_t - \omega^{-1} X_t h_t^{\omega}\right]^{1-\gamma} - 1}{1-\gamma}. \tag{5.3}$$

The variable $\tilde{D}_t$ denotes the aggregate level of external debt per capita, which the household takes as exogenous. In equilibrium, we have that $\tilde{D}_t = D_t$.

Households are subject to a no-Ponzi-scheme constraint of the form $\lim_{j\to\infty} E_t \frac{D_{t+j}}{\prod_{s=0}^{j}(1+r_s)} \leq 0$.

The household seeks to maximize the utility fucntion (5.3) subject to (5.1), (5.2) and the no-Ponzi-game constraint, taking as given the processes a_t, X_t, and r_t and the initial conditions K_0 and D_{-1}.

5.2.2 Equilibrium

Define $y_t = Y_t/X_t$, $c_t = C_t/X_t$, $d_t = D_t/X_t$, and $k_t = K_t/X_{t-1}$. Then, a stationary competitive equilibrium is give by a stationary solution to the following equations:

$$[c_t - \omega^{-1} h_t^{\omega}]^{-\gamma} = \lambda_t$$

$$h_t^{\omega-1} = (1-\alpha) a_t \left(\frac{k_t}{g_t h_t}\right)^{\alpha}$$

$$\lambda_t = \beta \left[1 + r + \psi_2 \left(e^{d_t - \bar{d}} - 1\right)\right] E_t g_{t+1}^{-\gamma} \lambda_{t+1}$$

$$\lambda_t [1 + \phi(k_{t+1} - \frac{g}{g_t} k_t)] = \beta E_t g_{t+1}^{-\gamma} \lambda_{t+1} \left[1 - \delta + \alpha a_{t+1} \left(\frac{k_{t+1}}{g_{t+1} h_{t+1}}\right)^{\alpha-1} + \phi g \left(k_{t+2} - \frac{g}{g_{t+1}} k_{t+1}\right)\right]$$

$$\frac{d_t}{1 + r_t} = \frac{d_{t-1}}{g_t} - a_t \left(\frac{k_t}{g_t}\right)^{\alpha} h_t^{1-\alpha} + c_t + i_t + \frac{\phi}{2} \left(k_{t+1} - \frac{g}{g_t} k_t\right)^2$$

$$r_t = r + \psi_2 \left(e^{d_t - \bar{d}} - 1\right),$$

This zaga will be continued]

Chapter 6

Exchange-Rate-Based Inflation Stabilization

The main conclusion from

For many emerging markets, business fluctuations are marked by balance-of-payment crises. That is, sharp reallocations of domestic agents' portfolios against domestic assets and in favor of foreign assets. Often, the domestic asset household

1. Main elements of the model:

   (a) Continuous-time model of a small, open, endowment economy

   (b) Traded and nontraded goods

   (c) Money is motivated by a cash-in-advance constraint

   (d) The government sets the devaluation rare at a low level for T periods and then is expected to raise it to a higher permaennt level.

2. Results:

   (a) Consumption increases on impact and remains constant at this higher level until time T, when it falls to its long-run level (which is lower than the pre-stabilizaiton level).

   (b) The real exchange rate (the relative price of nontradables in terms of tradables) jumps up (appreciates) on impact, remains constant until period T, and then falls to its long-run level, (which is lower than its pre-stabilization level).

(c) The trade balance deteriorates on impact, remains constant until period T, and then jumps up to a level hgher than the one prevailing before the announcement of the stabilization program.

(d) In period T, there is a BOP crisis, as agents reduce their money holdings in a discrete fashion in exchange for foreign bonds (thus, the central bank loses reserves in exactly the amount in which real balances fall)

3. Intuition: In this model, the rate of devaluation acts as a tax on consumption. Thus, a temporary decline in the devaluation rate induces intertemporal substitution in favor of current consumption and against future consumption.

Households

Households maximize

$$\int_0^\infty e^{-\rho t}[u(c_t^T) + v(c_t^N)] \tag{6.1}$$

subject to

$$W_t = E_t b_t^p + M_t$$
$$\dot{W}_t = rE_t b_t^p + \dot{E}_t b_t^p + P_t^T[(y^T - c_t^T) + p_t(y^N - c_t^N)] - P_t^T \tau_t$$
$$M_t \geq \alpha P_t^T (c_t^T + p_t c_t^N)$$

where W denotes nominal financial wealth, M denotes nominal money balances, b denotes holdings of a foreign-currency-denominated bond paying the constant interest rate r in foreign currency, E denotes the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency, c^T and c^N denote consumption of tradables and nontradables, respectively, y^T and y^N denote the endowments of tradables and nontradables, respectively, P_t^T denotes the domestic curency price of tradbles, and p denotes the real exchange rate, defined as the price of nontradables in terms of tradables. The foreign currency price of tradables is assumed to be equal to one and the law of one price is assumed to hold for tradables. Thus, $P_t^T = P^* E_t = E_t$. Finally, τ_t denote real lump-sum taxes measured in terms of tradables.

Let $\epsilon_t \equiv \dot{E}_t / E_t$ denote the rate of devaluation. Use the first two constraints to eliminate b_t^p,

$$\frac{\dot{W}_t}{E_t} - \frac{\dot{E}_t}{E_t}\frac{W_t}{E_t} = r\frac{W_t}{E_t} - (r + \dot{E}_t/E_t)\frac{M_t}{E_t} + y^T - c_t^T + p_t(y^N - c_t^N) - \tau_t$$

Let $w_t \equiv W_t/E_t$ and $m_t = M_t/E_t$ denote real financial wealth and real balances measured in terms of tradables. Then the above expresion can be written as

$$\dot{w}_t = rw_t - (r + \epsilon_t)m_t + y^T - c_t^T + p_t(y^N - c_t^N) - \tau_t$$

In addition, households are subject to a no-Ponzi-game constraint of the form:

$$\lim_{t \to \infty} e^{-rt} w_t \geq 0$$

The above two constraints are equivalent to the following intertemporal budget constraint

$$w_0 \geq \int_0^\infty e^{-rt}\left[(r + \epsilon_t)m_t - y^T + c_t^T - p_t(y^N - c_t^N) + \tau_t\right] dt$$

Under perfect foresight and free capital mobility, the domestic nominal interest rate, i_t, must satisfy the uncovered interst parity condition, that is,

$$i_t = r + \epsilon_t$$

To the extent that the nominal interes rate is positive, the cash-in-advance will hold with strict equality. Assuming that this is the case at all times (something that we will have to check when we characterize the equilibrium), we can use the cash-in-advance constraint to eliminate m_t from the intertemporal resource constraint

$$w_0 \geq \int_0^\infty e^{-rt} \left[(1+\alpha i_t)(c_t^T + p_t c_t^N) + \tau_t - y^T - p_t y^N\right] \tag{6.2}$$

Households maximize (6.1) subject to (6.2). The first-order conditions are (6.2) holidng with equality and

$$u'(c_t^T) = \lambda(1+\alpha i_t) \tag{6.3}$$

$$\frac{v'(c_t^N)}{u'(c_t^T)} = p_t \tag{6.4}$$

$$w_0 = \int_0^\infty e^{-rt} \left[(1+\alpha i_t)(c_t^T + p_t c_t^N) - y^T - p_t y^N - \tau_t\right] \tag{6.5}$$

The consolidated government

The government's flow budget constraint is:

$$\dot{b}_t^g = r b_t^g + \tau_t + \frac{\dot{M}_t}{E_t}$$

Using the fact that $\dot{m}_t = \dot{M}_t/E_t - \epsilon_t m_t$ we have

$$\dot{b}_t^g - \dot{m}_t = r b_t^g + \tau_t + \epsilon_t m_t$$

or, adding and subtracting $r m_t$ on the right-hand side

$$\dot{b}_t^g - \dot{m}_t = r(b_t^g - m_t^g) + \tau_t + i_t m_t$$

We will assume that the government is subject to a no-Ponzi-game constraint whereby the limit of the present discounted value of the real value of its total liabilities, $m_t - b_t^g m_t$, must be zero, that is

$$\lim_{t\to\infty} e^{-rt}(m_t - b_t^g) = 0$$

The above two expressions imply that

$$m_0 - b_0^g = \int_0^\infty [i_t m_t + \tau_t] dt \tag{6.6}$$

Exchange rate policy

$$\epsilon_t = \begin{cases} \epsilon^L & t < T \\ \epsilon^H > \epsilon^L & t \geq T \end{cases} \tag{6.7}$$

6.0.3 Market-clearing conditions

$$c_t^N = y^N \tag{6.8}$$

Combining equations (6.5) and (6.6) and taking into account that $w_0 = m_0 + b_0^p$ yields

$$b_0 = \int_0^\infty e^{-rt} \left[c_t^T - y^T \right] \tag{6.9}$$

where $b_0 \equiv b_0^p + b_0^g$.

Chapter 7

Models of Balance of Payments Crises

A balance of payments (BOP) crisis is a situation in which the private sector changes the composition of its asset portfolio away from domestic currency at a point in which the monetary authority is maintaining a fixed exchange rate policy. Because at an aggregate level the only way in which the private sector as a whole can get rid of part of its money holdings is by running to the central bank.

pegging the and in favor of foreign assets. Suppose the government is pegging the exchange rate. Under this policy regime, the central bank must stand ready to exchange domestic currency (pesos, say) for foreign currency (dollars) and vice versa at a fixed rate. Suppose for some reason private agents decide in unison to get rid of a significant part of their peso holdings and acquire dollars.

7.1 The Krugman Model

In a celebrated paper, Krugman (1979) develops a model of Balance of Payments Crises that features fiscal deficits at the center of the problem.

The demand for money is given by

$$m_t = L(i_t, y); \quad L_1 < 0$$

where $m_t = M_t/P_t$ denotes real balances, i_t denotes the nominal interestr rate and y denotes a constant level of output (or aggregate spending).

7.1.1 PPP and Uncovered Interest Parity Condition

We assume that there is a single good and that PPP holds. We further assume that the foreign currency price of the good is constant and normalized to unity. Thus, we have that

$$P_t = E_t,$$

where P_t is the domestic nominal price level and E_t is the nominal exchange rate, defined as the price of foreign currency in terms of domestic currency.

The model is in perfect foresight and free capital mobility is assumed. Thus, the nominal interest rate satisfies the uncovered interest parity condition

$$i_t = r + \epsilon_t,$$

where r denotes the world dollar interest rate and $\epsilon_t = \dot{E}_t/E_t$ denotes the devaluation rate of the domestic currency. For simplicity, assume that the world interest rate is zero:[1]

$$r = 0.$$

7.1.2 The government

The government holds reserves in the form of foreign-currency-denominated bonds, which we denote by k, and prints domestic currency. In addition, the government runs a constant primary deficit of $g > 0$. The flow budget constraint of the government is given by

$$\dot{k}_t = \epsilon_t m_t + \dot{m}_t - g.$$

According to this expression, the only source of revenue of the governenment is seignorage, $\epsilon_t m_t + \dot{m}_t$.

When the government pegs the exchange rate, discrete changes in the supply of money are reflected one-by-one in changes in government reserves, that is,

$$\Delta k_t = \Delta m_t.$$

7.1.3 The Monetary/Fiscal Regime

In period zero the government announces a curency peg. The government is committed to maintaining the peg as long as foreign reserves are above a certian floor, which we denote by $\underline{k}$. As soon as foreign reserves fall below this threshold, the government abondones the peg and begins to print

[1] Modifying the model to allow for interest-bearing bonds is straightforward.

enough money to finance the deficit. As indicated, fiscal policy is characterized by a constant flow of primary deficit throughout.

7.1.4 A Balance-of-Payments (BOP) Crisis

While the peg is in place, we have that $\epsilon_t = 0$. At the same time, as long as the peg is maintained, the demand for money is constant and equal to $L(0, y)$. Thus, we have that while exchange-rate program is alive, $\dot{m}_t = 0$. Then, combining the demand for money and the flow budget constraint of the government we have that

$$\dot{k}_t = -g.$$

Note that by pegging the exchange rate, the government relinquishes its sole source of revenue, seignorage. According to this expression, the stock of reserves is falling at the rate of g per period. The higher the fiscal deficit, the faster the rate at which reserves fall. This expression also shows that the peg cannot last forever. For in that case reserves would fall below the threshold $\underline{k}$ in finite time.

Let T denote the time at which the peg is abandoned. Then, clearly, $k_T = \underline{k}$. One important element of the Krugman model is that the *level* of the nominal exchange rate cannot jump in period T. A perfectly anticipated discrete change in the nominal exchange rate would induce a massive run against the domestic currency an instant before the crisis. Such run would would in turn anticipate the crisis. This logic lead to a crisis at a point early enough to imply no jump in E_t.

Let's first find the devaluation rate that allows the government to collect enough revenue to finance the deficit. This devaluation rate, which we denote by $\bar{\epsilon}$, must satisfy

$$\bar{\epsilon} L(\bar{\epsilon}, y) = g.$$

This is a Laffer curve type relation, which in general has 2 or no solution. We assume it hase 2 and take the one on the left side of the Laffer curve. Then, in the neighborhood of the solution we have that the higher the inflation tax, ϵ, the higher the inflation tax revenue. Clearly, $\bar{\epsilon} > 0$. This implies that at time T there is a discrete decline in the demand for real balances from $L(0, y)$ to $L(\bar{\epsilon}, y)$. The corresponding change in foreign reserves is given by

$$\Delta k_T = L(\bar{\epsilon}, y) - L(0, y) < 0.$$

This means that at $t = T$ the government loses a discrete amount of foreign reserves. This is what Krugman calls a BOP crisis. Because the opportunity cost of holding money, ϵ_t, jumps up at T, the public gets rid of part of their real balances, accelerating the drainage of foreign reserves of the cental bank.

7.1.5 Computing T

The crisis takes palce when foreign reserves, k_t, hit the lower bound $\underline{k}$. Thus, T must satisfy

$$-(k_0 - \underline{k}) = \int_0^T \dot{k}dt + \Delta k_T,$$

where k_0 denotes the initial (post annoucement of the peg) stock of foreign reserves held by the cental bank. This expression can be written as

$$-(k_0 - \underline{k}) = -\int_0^T g\,dt + L(\bar{\epsilon}, y) - L(0, y).$$

or

$$-(k_0 - \underline{k}) = -gT + L(\bar{\epsilon}, y) - L(0, y).$$

Solving for T we have

$$T = \frac{(k_0 - \underline{k}) - [L(0, y) - L(\bar{\epsilon}, y)]}{g}$$

We note the following intuitive implications of this expression:

- The higher the fiscal deficit, g, the sooner the BOP crises takes place.
- The higher the initial stock of reserves and the lower the lower bound of reserves, the the later the crises occurs.
- The higher the interest elasticity of money demand, the sooner the BOP crises occurs.

A numerical example

Suppose the iinitial stock of foreign reserves is 10%, the lower bound on reserves is 0, the fiscal deficit is 2% of GDP, and the money demand function is given by $y(a - bi)$, with $a = 0.2$ and $b = 0.25$.

Then $\bar{\epsilon}$ is given by

$$\bar{\epsilon}(0.2 - 0.25\bar{\epsilon}) = 0.02$$

Solving this quadratic polynomial for the smallest root (for we want to be on the upward-sloping side of the Laffer curve) yields

$$\bar{\epsilon} = 11\%.$$

Then T is given by

$$
\begin{aligned}
T &= \frac{(k_0 - \underline{k}) - yb(\bar{\epsilon}}{g} \\
&= \frac{0.1 - 0.25 \cdot 0.11}{0.02} \\
&= 3.6 \text{ years.}
\end{aligned}
$$

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Economics 731
International Monetary
Theory and Policy

1. Consider the simple Krugman model discussed in class, but now assume that the world interest rate, r, is positive. Provide an implicit expression for the post collapse rate of devaluation $\bar{\epsilon}$. Derive an expression for the time of the crisis, T.

References

Aguiar, Mark, and Gita Gopinath "Emerging Market Business Cycles: The Cycle is the Trend," NBER working paper No. 10734, August 2004.

Arellano, M. and S. Bond, "Some Tests of Specification for Panel Data: Monte Carlo Evidence and An Application to Employment Equations," *Review of Economic Studies*, 58, (1991): 277-297.

Anderson, T. H. and Hsiao, C., "Estimation of Dynamic Models With Error Components," *Journal of the American Statistical Association*, 76, (1981): 598-606.

Campbell, John Y., "Does Saving Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis," *Econometrica*, 55, (November 1987): pp. 1249-1273.

Correia, Isabel, João C.Neves, and Sergio Rebelo, "Business Cycles in a Small Open Economy," *European Economic Review*, 39, (1995): 1089-1113.

Hall, Robert, "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy*, 86, (1978): 971-987.

Judson, Ruth A. and Ann L. Owen, "Estimating Dynamic Panel Data Models," *A Guide For Macroeconomists*, Economic Letters, (65): 1999, pp. 9-15.

Kim, Sunghyun Henry and Ayhan Kose "Dynamics of Open Economy Business Cycle Models: Understanding the Role of the Discount Factor," Tufts University, 2001, forthcoming, Macroeconomic Dynamics.

Klein, Paul, "Using the Generalized Schur Form to Solve a Multivariate Linear Rational Expectations Model," *Journal of Economic Dynamics and Control*, 24, (2000): 1405-1423.

Krugman, Paul R., "A Model of Balance of Payments Crises," *Journal of Money, Credit and Banking*, 11, (1979): 311-25.

Kydland, Finn E., and Carlos E. J. M. Zarazaga, "Is the Business Cycle of Argentina Different?," *Economic Review*, Federal Reserve Bank of

Dallas, (1997): 21-36.

Mendoza, Enrique, "Real Business cycles in a small-open economy," *American Economic Review*, 81, (1991): 797-818.

Mendoza, Enrique and Martín Uribe, "Devaluation Risk and the Business-Cycle Implications of Exchange-Rate Management," *Carnegie-Rochester Conference Series on Public Policy*, 53, (December 2000): 239-96.

Nason, James M. and John H. Rogers "The Present-Value Model of the Current Account Has Been Rejected: Round Up the Usual Suspects," Board of Governors of the Federal Reserve System, International Finance Discussion Paper No. 760, February 2003.

Neumeyer, Pablo A. and Fabrizio Perri "Business Cycles in Emerging Markets:The Role of Interest Rates," manuscript, New York University, November 2001.

Obstfeld, Maurice, "Intertemporal dependence, impatience, and dynamics," *Journal of Monetary Economics*, 26, (1990): 45-75.

Obstfeld, Maurice, "Rational and Self-Fulfilling Balance of Payments Crises," *American Economic Review*, 76, (1986): 72-81.

Obstfeld, Maurice, "The Logic of Currency Crises," *Cahiers Economiques et Monetaires, Bank of France*, 43, (1994): 189-213.

Schmitt-Grohé, Stephanie, "The international transmission of economic fluctuations: Effects of U.S. business cycles on the Canadian economy," *Journal of International Economics*, 44, (April 1998): 257-287.

Schmitt-Grohé, Stephanie and Martín Uribe, "Stabilization Policy and the Costs of Dollarization," *Journal of Money, Credit, and Banking*, 33, (May 2001): 482-509.

Schmitt-Grohé, Stephanie and Martín Uribe, "Closing Small Open Economy Models," *Journal of International Economics*, 61, (October 2003): pp. 163-185 .

Schmitt-Grohé, Stephanie and Martín Uribe, "Solving Dynamic General Equilibrium Models Using a Second-Order Approximation to the Policy Function," *Journal of Economic Dynamics and Control*, vol. 28, (January 2004): pp. 755-775.

Senhadji, Abdelhak S. "Adjustment of a Small Open Economy to External Shocks," Dissertation, University of Pennsylvania, 1994.

Sims, Christopher "Solving Linear Rational Expectations Models," mimeo, Yale University/ 1996.

Uribe, Martín, "Exchange rate based inflation stabilization: the initial real effects of credible plans," *Journal of Monetary Economics*, 39, (1997): 197-221.

Uribe, Martín and Z. Vivian Yue "Country Spreads and Emerging Countries: Who Drives Whom?," NBER working paper No. 10018, October 2003.

Uzawa, H., 1968, Time preference, the consumption function and optimum asset holdings, in: Wolfe, J.N. (Ed.), Value, Capital and Growth: Papers in Honor of Sir John Hicks, (Edinburgh, The University of Edinburgh Press) pp. 485-504.