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# The Cross-Section of Volatility and Expected Returns

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## ABSTRACT

We examine the pricing of aggregate volatility risk in the cross-section of stock returns. Consistent with theory, we find that stocks with high sensitivities to innovations in aggregate volatility have low average returns. Stocks with high idiosyncratic volatility relative to the Fama and French (1993, *Journal of Financial Economics* 25, 2349) model have abysmally low average returns. This phenomenon cannot be explained by exposure to aggregate volatility risk. Size, book-to-market, momentum, and liquidity effects cannot account for either the low average returns earned by stocks with high exposure to systematic volatility risk or for the low average returns of stocks with high idiosyncratic volatility.

IT IS WELL KNOWN THAT THE VOLATILITY OF STOCK RETURNS varies over time. While considerable research has examined the time-series relation between the volatility of the market and the expected return on the market (see, among others, Campbell and Hentschel (1992) and Glosten, Jagannathan, and Runkle (1993)), the question of how aggregate volatility affects the cross-section of expected stock returns has received less attention. Time-varying market volatility induces changes in the investment opportunity set by changing the expectation of future market returns, or by changing the risk-return trade-off. If the volatility of the market return is a systematic risk factor, the arbitrage pricing theory or a factor model predicts that aggregate volatility should also be priced in the cross-section of stocks. Hence, stocks with different sensitivities to innovations in aggregate volatility should have different expected returns.

The first goal of this paper is to provide a systematic investigation of how the stochastic volatility of the market is priced in the cross-section of expected stock returns. We want to both determine whether the volatility of the market

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is a priced risk factor and estimate the price of aggregate volatility risk. Many option studies have estimated a negative price of risk for market volatility using options on an aggregate market index or options on individual stocks.[1] Using the cross-section of stock returns, rather than options on the market, allows us to create portfolios of stocks that have different sensitivities to innovations in market volatility. If the price of aggregate volatility risk is negative, stocks with large, positive sensitivities to volatility risk should have low average returns. Using the cross-section of stock returns also allows us to easily control for a battery of cross-sectional effects, such as the size and value factors of Fama and French (1993), the momentum effect of Jegadeesh and Titman (1993), and the effect of liquidity risk documented by Pástor and Stambaugh (2003). Option pricing studies do not control for these cross-sectional risk factors.

We find that innovations in aggregate volatility carry a statistically significant negative price of risk of approximately $-1\%$ per annum. Economic theory provides several reasons why the price of risk of innovations in market volatility should be negative. For example, Campbell (1993, 1996) and Chen (2002) show that investors want to hedge against changes in market volatility, because increasing volatility represents a deterioration in investment opportunities. Risk-averse agents demand stocks that hedge against this risk. Periods of high volatility also tend to coincide with downward market movements (see French, Schwert, and Stambaugh (1987) and Campbell and Hentschel (1992)). As Bakshi and Kapadia (2003) comment, assets with high sensitivities to market volatility risk provide hedges against market downside risk. The higher demand for assets with high systematic volatility loadings increases their price and lowers their average return. Finally, stocks that do badly when volatility increases tend to have negatively skewed returns over intermediate horizons, while stocks that do well when volatility rises tend to have positively skewed returns. If investors have preferences over coskewness (see Harvey and Siddique (2000)), stocks that have high sensitivities to innovations in market volatility are attractive and have low returns.[2]

The second goal of the paper is to examine the cross-sectional relationship between idiosyncratic volatility and expected returns, where idiosyncratic volatility is defined relative to the standard Fama and French (1993) model.[3] If the Fama–French model is correct, forming portfolios by sorting on idiosyncratic volatility will obviously provide no difference in average returns. Nevertheless, if the Fama–French model is false, sorting in this way potentially provides a set

[1] See, among others, Jackwerth and Rubinstein (1996), Bakshi, Cao and Chen (2000), Chernov and Ghysels (2000), Burashi and Jackwerth (2001), Coval and Shumway (2001), Benzoni (2002), Pan (2002), Bakshi and Kapadia (2003), Eraker, Johannes and Polson (2003), Jones (2003), and Carr and Wu (2003).

[2] Bates (2001) and Vayanos (2004) provide recent structural models whose reduced form factor structures have a negative risk premium for volatility risk.

[3] Recent studies examining total or idiosyncratic volatility focus on the average level of firm-level volatility. For example, Campbell et al. (2001) and Xu and Malkiel (2003) document that idiosyncratic volatility has increased over time. Brown and Ferreira (2003) and Goyal and Santa-Clara (2003) argue that idiosyncratic volatility has positive predictive power for excess market returns, but this is disputed by Bali et al. (2004).

of assets that may have different exposures to aggregate volatility and hence different average returns. Our logic is the following. If aggregate volatility is a risk factor that is orthogonal to existing risk factors, the sensitivity of stocks to aggregate volatility times the movement in aggregate volatility will show up in the residuals of the Fama–French model. Firms with greater sensitivities to aggregate volatility should therefore have larger idiosyncratic volatilities relative to the Fama–French model, everything else being equal. Differences in the volatilities of firms' true idiosyncratic errors, which are not priced, will make this relation noisy. We should be able to average out this noise by constructing portfolios of stocks to reveal that larger idiosyncratic volatilities relative to the Fama–French model correspond to greater sensitivities to movements in aggregate volatility and thus different average returns, if aggregate volatility risk is priced.

While high exposure to aggregate volatility risk tends to produce low expected returns, some economic theories suggest that idiosyncratic volatility should be positively related to expected returns. If investors demand compensation for not being able to diversify risk (see Malkiel and Xu (2002) and Jones and Rhodes-Kropf (2003)), then agents will demand a premium for holding stocks with high idiosyncratic volatility. Merton (1987) suggests that in an information-segmented market, firms with larger firm-specific variances require higher average returns to compensate investors for holding imperfectly diversified portfolios. Some behavioral models, like Barberis and Huang (2001), also predict that higher idiosyncratic volatility stocks should earn higher expected returns. Our results are directly opposite to these theories. We find that stocks with high idiosyncratic volatility have low average returns. There is a strongly significant difference of $-1.06\%$ per month between the average returns of the quintile portfolio with the highest idiosyncratic volatility stocks and the quintile portfolio with the lowest idiosyncratic volatility stocks.

In contrast to our results, earlier researchers either find a significantly positive relation between idiosyncratic volatility and average returns, or they fail to find any statistically significant relation between idiosyncratic volatility and average returns. For example, Lintner (1965) shows that idiosyncratic volatility carries a positive coefficient in cross-sectional regressions. Lehmann (1990) also finds a statistically significant, positive coefficient on idiosyncratic volatility over his full sample period. Similarly, Tinic and West (1986) and Malkiel and Xu (2002) unambiguously find that portfolios with higher idiosyncratic volatility have higher average returns, but they do not report any significance levels for their idiosyncratic volatility premiums. On the other hand, Longstaff (1989) finds that a cross-sectional regression coefficient on total variance for size-sorted portfolios carries an insignificant negative sign.

The difference between our results and the results of past studies is that the past literature either does not examine idiosyncratic volatility at the firm level, or does not directly sort stocks into portfolios ranked on this measure of interest. For example, Tinic and West (1986) work only with 20 portfolios sorted on market beta, while Malkiel and Xu (2002) work only with 100 portfolios sorted on market beta and size. Malkiel and Xu (2002) only use the idiosyncratic volatility of one of the 100 beta/size portfolios to which a stock belongs to proxy for that stock's idiosyncratic risk and, thus, do not examine firm-level idiosyncratic volatility. Hence, by not directly computing differences in average returns between stocks with low and high idiosyncratic volatilities, previous studies miss the strong negative relation between idiosyncratic volatility and average returns that we find.

The low average returns to stocks with high idiosyncratic volatilities could arise because stocks with high idiosyncratic volatilities may have high exposure to aggregate volatility risk, which lowers their average returns. We investigate this conjecture and find that this is not a complete explanation. Our idiosyncratic volatility results are also robust to controlling for value, size, liquidity, volume, dispersion of analysts' forecasts, and momentum effects. We find the effect robust to different formation periods for computing idiosyncratic volatility and for different holding periods. The effect also persists in bull and bear markets, recessions and expansions, and volatile and stable periods. Hence, our results on idiosyncratic volatility represent a substantive puzzle.

The rest of this paper is organized as follows. In Section I, we examine how aggregate volatility is priced in the cross-section of stock returns. Section II documents that firms with high idiosyncratic volatility have very low average returns. Finally, Section III concludes.

## I. Pricing Systematic Volatility in the Cross-Section

### *A. Theoretical Motivation*

When investment opportunities vary over time, the multifactor models of Merton (1973) and Ross (1976) show that risk premia are associated with the conditional covariances between asset returns and innovations in state variables that describe the time-variation of the investment opportunities. Campbell's (1993, 1996) version of the Intertemporal Capital Asset Pricing Model (I-CAPM) shows that investors care about risks both from the market return and from changes in forecasts of future market returns. When the representative agent is more risk averse than log utility, assets that covary positively with good news about future expected returns on the market have higher average returns. These assets command a risk premium because they reduce a consumer's ability to hedge against a deterioration in investment opportunities. The intuition from Campbell's model is that risk-averse investors want to hedge against changes in aggregate volatility because volatility positively affects future expected market returns, as in Merton (1973).

However, in Campbell's setup, there is no direct role for fluctuations in market volatility to affect the expected returns of assets because Campbell's model is premised on homoskedasticity. Chen (2002) extends Campbell's model to a heteroskedastic environment which allows for both time-varying covariances and stochastic market volatility. Chen shows that risk-averse investors also want to directly hedge against changes in future market volatility. In Chen's model, an asset's expected return depends on risk from the market return, changes in forecasts of future market returns, and changes in forecasts of future market volatilities. For an investor more risk averse than log utility, Chen shows that an asset that has a positive covariance between its return and a variable that positively forecasts future market volatilities causes that asset to have a lower expected return. This effect arises because risk-averse investors reduce current consumption to increase precautionary savings in the presence of increased uncertainty about market returns.

Motivated by these multifactor models, we study how exposure to market volatility risk is priced in the cross-section of stock returns. A true conditional multifactor representation of expected returns in the cross-section would take the following form:

$$r_{t+1}^{i} = a_t^i + \beta_{m,t}^{i}\left(r_{t+1}^{m} - \gamma_{m,t}\right) + \beta_{v,t}^{i}(v_{t+1} - \gamma_{v,t}) + \sum_{k=1}^{K} \beta_{k,t}^{i}(f_{k,t+1} - \gamma_{k,t}), \quad (1)$$

where $r_{t+1}^{i}$ is the excess return on stock $i$, $\beta_{m,t}^{i}$ is the loading on the excess market return, $\beta_{v,t}^{i}$ is the asset's sensitivity to volatility risk, and the $\beta_{k,t}^{i}$ coefficients for $k = 1, \ldots, K$ represent loadings on other risk factors. In the full conditional setting in equation (1), factor loadings, conditional means of factors, and factor premiums potentially vary over time. The model in equation (1) is written in terms of factor innovations, so $r_{t+1}^{m} - \gamma_{m,t}$ represents the innovation in the market return, $v_{t+1} - \gamma_{v,t}$ represents the innovation in the factor reflecting aggregate volatility risk, and innovations to the other factors are represented by $f_{k,t+1} - \gamma_{k,t}$. The conditional mean of the market and aggregate volatility are denoted by $\gamma_{m,t}$ and $\gamma_{v,t}$, respectively, while the conditional means of the other factors are denoted by $\gamma_{k,t}$. In equilibrium, the conditional mean of stock $i$ is given by

$$a_t^i = E_t\left(r_{t+1}^{i}\right) = \beta_{m,t}^{i}\lambda_{m,t} + \beta_{v,t}^{i}\lambda_{v,t} + \sum_{k=1}^{K} \beta_{k,t}^{i}\lambda_{k,t}, \quad (2)$$

where $\lambda_{m,t}$ is the price of risk of the market factor, $\lambda_{v,t}$ is the price of aggregate volatility risk, and the $\lambda_{k,t}$ are the prices of risk of the other factors. Note that only if a factor is traded is the conditional mean of a factor equal to its conditional price of risk.

The main prediction from the factor model setting of equation (1) that we examine is that stocks with different loadings on aggregate volatility risk have different average returns.[4] However, the true model in equation (1) is infeasible

[4] While an I-CAPM implies joint time-series as well as cross-sectional predictability, we do not examine time-series predictability of asset returns by systematic volatility. Time-varying volatility risk generates intertemporal hedging demands in partial equilibrium asset allocation problems. In a partial equilibrium setting, Liu (2001) and Chacko and Viceira (2003) examine how volatility risk affects the portfolio allocation of stocks and risk-free assets, while Liu and Pan (2003) show how investors can optimally exploit the variation in volatility with options. Guo and Whitelaw (2003) examine the intertemporal components of time-varying systematic volatility in a Campbell (1993, 1996) equilibrium I-CAPM.

to examine because the true set of factors is unknown and the true conditional factor loadings are unobservable. Hence, we do not attempt to directly use equation (1) in our empirical work. Instead, we simplify the full model of equation (1), which we now detail.

## B. The Empirical Framework

To investigate how aggregate volatility risk is priced in the cross-section of equity returns we make the following simplifying assumptions to the full specification in equation (1). First, we use observable proxies for the market factor and the factor representing aggregate volatility risk. We use the CRSP value-weighted market index to proxy for the market factor. To proxy innovations in aggregate volatility, $(v_{t+1} - \gamma_{v,t})$, we use changes in the *VIX* index from the Chicago Board Options Exchange (CBOE).[5] Second, we reduce the number of factors in equation (1) to just the market factor and the proxy for aggregate volatility risk. Finally, to capture the conditional nature of the true model, we use short intervals—1 month of daily data—to take into account possible time variation of the factor loadings. We discuss each of these simplifications in turn.

### B.1. Innovations in the VIX Index

The *VIX* index is constructed so that it represents the implied volatility of a synthetic at-the-money option contract on the S&P100 index that has a maturity of 1 month. It is constructed from eight S&P100 index puts and calls and takes into account the American features of the option contracts, discrete cash dividends, and microstructure frictions such as bid–ask spreads (see Whaley (2000) for further details).[6] Figure 1 plots the *VIX* index from January 1986 to December 2000. The mean level of the daily *VIX* series is 20.5%, and its standard deviation is 7.85%.

Because the *VIX* index is highly serially correlated with a first-order autocorrelation of 0.94, we measure daily innovations in aggregate volatility by using daily changes in *VIX*, which we denote as $\Delta VIX$. Daily first differences in *VIX* have an effective mean of zero (less than 0.0001), a standard deviation of

[5] In previous versions of this paper, we also consider: Sample volatility, following French et al. (1987); a range-based estimate, following Alizadeh, Brandt, and Diebold (2002); and a high-frequency estimator of volatility from Andersen, Bollerslev, and Diebold (2003). Using these measures to proxy for innovations in aggregate volatility produces little spread in cross-sectional average returns. These tables are available upon request.

[6] On September 22, 2003, the CBOE implemented a new formula and methodology to construct its volatility index. The new index is based on the S&P500 (rather than the S&P100) and takes into account a broader range of strike prices rather than using only at-the-money option contracts. The CBOE now uses *VIX* to refer to this new index. We use the old index (denoted by the ticker *VXO*). We do not use the new index because it has been constructed by backfilling only to 1990, whereas the *VXO* is available in real time from 1986. The CBOE continues to make both volatility indices available. The correlation between the new and the old CBOE volatility series is 98% from 1990 to 2000, but the series that we use has a slightly broader range than the new CBOE volatility series.

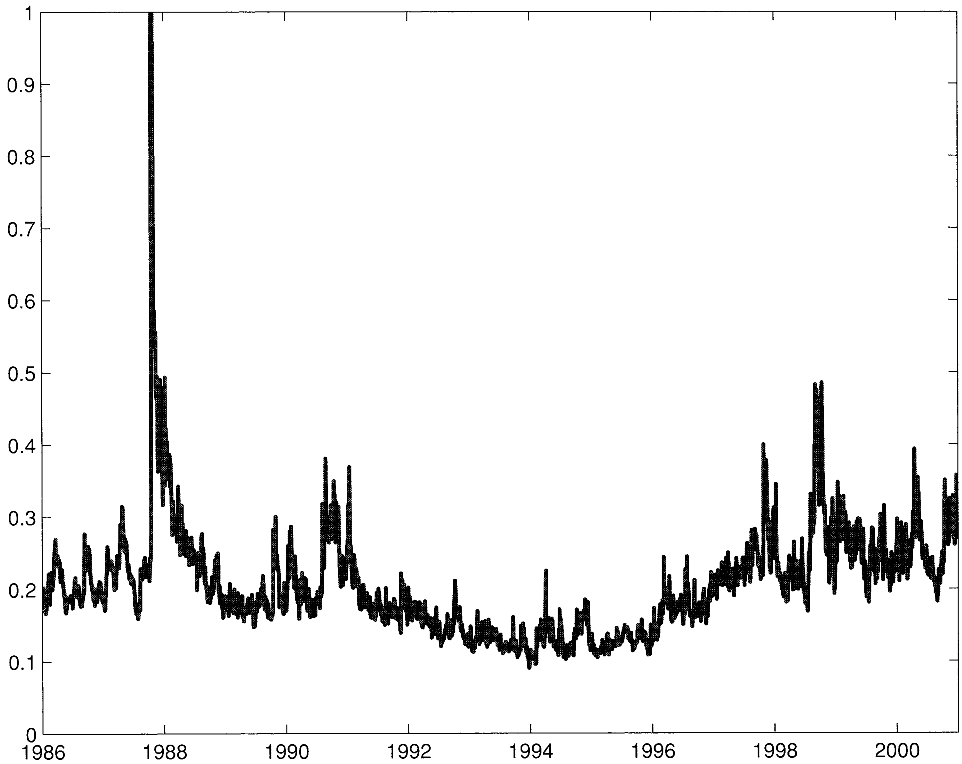


**Figure 1. Plot of *VIX*.** The figure shows the *VIX* index plotted at a daily frequency. The sample period is January 1986 to December 2000.

2.65%, and negligible serial correlation (the first-order autocorrelation of $\Delta VIX$ is $-0.0001$). As part of our robustness checks in Section I.C, we also measure innovations in *VIX* by specifying a stationary time-series model for the conditional mean of *VIX* and find our results to be similar to those using simple first differences. While $\Delta VIX$ appears to be an ideal proxy for innovations in volatility risk because the *VIX* index is representative of traded option securities whose prices directly reflect volatility risk, there are two main caveats with respect to using *VIX* to represent observable market volatility.

The first concern is that the *VIX* index is the implied volatility from the Black–Scholes (1973) model, and we know that the Black–Scholes model is an approximation. If the true stochastic environment is characterized by stochastic volatility and jumps, $\Delta VIX$ will reflect total quadratic variation in both diffusion and jump components (see, for example, Pan (2002)). Although Bates (2000) argues that implied volatilities computed taking into account jump risk are very close to original Black–Scholes implied volatilities, jump risk may be priced differently from volatility risk. Our analysis does not separate jump risk from diffusion risk, so our aggregate volatility risk may include jump risk components.

A more serious reservation about the *VIX* index is that *VIX* combines both stochastic volatility and the stochastic volatility risk premium. Only if the risk premium is zero or constant would $\Delta VIX$ be a pure proxy for the innovation in aggregate volatility. Decomposing $\Delta VIX$ into the true innovation in volatility and the volatility risk premium can only be done by writing down a formal model. The form of the risk premium depends on the parameterization of the price of volatility risk, the number of factors, and the evolution of those factors. Each different model specification implies a different risk premium. For example, many stochastic volatility option pricing models assume that the volatility risk premium can be parameterized as a linear function of volatility (see, for example, Chernov and Ghysels (2000), Benzoni (2002), and Jones (2003)). This may or may not be a good approximation to the true price of risk. Rather than imposing a structural form, we use an unadulterated $\Delta VIX$ series. An advantage of this approach is that our analysis is simple to replicate.

### *B.2. The Pre-Formation Regression*

Our goal is to test whether stocks with different sensitivities to aggregate volatility innovations (proxied by $\Delta VIX$) have different average returns. To measure the sensitivity to aggregate volatility innovations, we reduce the number of factors in the full specification in equation (1) to two, namely, the market factor and $\Delta VIX$. A two-factor pricing kernel with the market return and stochastic volatility as factors is also the standard setup commonly assumed by many stochastic option pricing studies (see, for example, Heston (1993)). Hence, the empirical model that we examine is

$$r_t^i = \beta_0 + \beta_{MKT}^i MKT_t + \beta_{\Delta VIX}^i \Delta VIX_t + \varepsilon_t^i, \tag{3}$$

where *MKT* is the market excess return, $\Delta VIX$ is the instrument we use for innovations in the aggregate volatility factor, and $\beta_{MKT}^i$ and $\beta_{\Delta VIX}^i$ are loadings on market risk and aggregate volatility risk, respectively.

Previous empirical studies suggest that there are other cross-sectional factors that have explanatory power for the cross-section of returns, such as the size and value factors of the Fama and French (1993) three-factor model (hereafter FF-3). We do not directly model these effects in equation (3), because controlling for other factors in constructing portfolios based on equation (3) may add a lot of noise. Although we keep the number of regressors in our pre-formation portfolio regressions to a minimum, we are careful to ensure that we control for the FF-3 factors and other cross-sectional factors in assessing how volatility risk is priced using post-formation regression tests.

We construct a set of assets that are sufficiently disperse in exposure to aggregate volatility innovations by sorting firms on $\Delta VIX$ loadings over the past month using the regression (3) with daily data. We run the regression for all stocks on AMEX, NASDAQ, and the NYSE, with more than 17 daily observations. In a setting in which coefficients potentially vary over time, a 1-month window with daily data is a natural compromise between estimating coefficients with a reasonable degree of precision and pinning down conditional coefficients in an environment with time-varying factor loadings. Pástor and Stambaugh (2003), among others, also use daily data with a 1-month window in similar settings. At the end of each month, we sort stocks into quintiles, based on the value of the realized $\beta_{\Delta VIX}$ coefficients over the past month. Firms in quintile 1 have the lowest coefficients, while firms in quintile 5 have the highest $\beta_{\Delta VIX}$ loadings. Within each quintile portfolio, we value weight the stocks. We link the returns across time to form one series of post-ranking returns for each quintile portfolio.

Table I reports various summary statistics for quintile portfolios sorted by past $\beta_{\Delta VIX}$ over the previous month using equation (3). The first two columns report the mean and standard deviation of monthly total, not excess, simple returns. In the first column under the heading "Factor Loadings," we report the pre-formation $\beta_{\Delta VIX}$ coefficients, which are computed at the beginning of each month for each portfolio and are value weighted. The column reports the time-series average of the pre-formation $\beta_{\Delta VIX}$ loadings across the whole sample. By construction, since the portfolios are formed by ranking on past $\beta_{\Delta VIX}$, the pre-formation $\beta_{\Delta VIX}$ loadings monotonically increase from $-2.09$ for portfolio 1 to 2.18 for portfolio 5.

The columns labeled "CAPM Alpha" and "FF-3 Alpha" report the time-series alphas of these portfolios relative to the CAPM and to the FF-3 model, respectively. Consistent with the negative price of systematic volatility risk found by the option pricing studies, we see lower average raw returns, CAPM alphas, and FF-3 alphas with higher past loadings of $\beta_{\Delta VIX}$. All the differences between quintile portfolios 5 and 1 are significant at the 1% level, and a joint test for the alphas equal to zero rejects at the 5% level for both the CAPM and the FF-3 model. In particular, the 5-1 spread in average returns between the quintile portfolios with the highest and lowest $\beta_{\Delta VIX}$ coefficients is $-1.04\%$ per month. Controlling for the *MKT* factor exacerbates the 5-1 spread to $-1.15\%$ per month, while controlling for the FF-3 model decreases the 5-1 spread to $-0.83\%$ per month.

### *B.3. Requirements for a Factor Risk Explanation*

While the differences in average returns and alphas corresponding to different $\beta_{\Delta VIX}$ loadings are very impressive, we cannot yet claim that these differences are due to systematic volatility risk. We examine the premium for aggregate volatility within the framework of an unconditional factor model. There are two requirements that must hold in order to make a case for a factor risk-based explanation. First, a factor model implies that there should be contemporaneous patterns between factor loadings and average returns. For example, in a standard CAPM, stocks that covary strongly with the market factor should, on average, earn high returns over the same period. To test a factor model, Black, Jensen, and Scholes (1972), Fama and French (1992, 1993), Jagannathan and Wang (1996), and Pástor and Stambaugh (2003), among others, all form portfolios using various pre-formation criteria, but examine

**Table I**

**Portfolios Sorted by Exposure to Aggregate Volatility Shocks**

We form value-weighted quintile portfolios every month by regressing excess individual stock returns on $\Delta VIX$, controlling for the *MKT* factor as in equation (3), using daily data over the previous month. Stocks are sorted into quintiles based on the coefficient $\beta_{\Delta VIX}$ from lowest (quintile 1) to highest (quintile 5). The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The row "5-1" refers to the difference in monthly returns between portfolio 5 and portfolio 1. The Alpha columns report Jensen's alpha with respect to the CAPM or the Fama–French (1993) three-factor model. The pre-formation betas refer to the value-weighted $\beta_{\Delta VIX}$ or $\beta_{FVIX}$ within each quintile portfolio at the start of the month. We report the pre-formation $\beta_{\Delta VIX}$ and $\beta_{FVIX}$ averaged across the whole sample. The second to last column reports the $\beta_{\Delta VIX}$ loading computed over the next month with daily data. The column reports the next month $\beta_{\Delta VIX}$ loadings averaged across months. The last column reports ex post $\beta_{FVIX}$ factor loadings over the whole sample, where *FVIX* is the factor mimicking aggregate volatility risk. To correspond with the Fama–French alphas, we compute the ex post betas by running a four-factor regression with the three Fama–French factors together with the *FVIX* factor that mimics aggregate volatility risk, following the regression in equation (6). The row labeled "Joint test $p$-value" reports a Gibbons, Ross and Shanken (1989) test for the alphas equal to zero, and a robust joint test that the factor loadings are equal to zero. Robust Newey–West (1987) $t$-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

| | | | | | | | | Factor Loadings | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Rank | Mean | Std. Dev. | % Mkt Share | Size | B/M | CAPM Alpha | FF-3 Alpha | Pre-Formation $\beta_{\Delta VIX}$ | Pre-Formation $\beta_{FVIX}$ | Next Month Post-Formation $\beta_{\Delta VIX}$ | Full Sample Post-Formation $\beta_{FVIX}$ |
| 1 | 1.64 | 5.53 | 9.4% | 3.70 | 0.89 | 0.27 | 0.30 | −2.09 | −2.00 | −0.033 | −5.06 |
| | | | | | | [1.66] | [1.77] | | | | [−4.06] |
| 2 | 1.39 | 4.43 | 28.7% | 4.77 | 0.73 | 0.18 | 0.09 | −0.46 | −0.42 | −0.014 | −2.72 |
| | | | | | | [1.82] | [1.18] | | | | [−2.64] |
| 3 | 1.36 | 4.40 | 30.4% | 4.77 | 0.76 | 0.13 | 0.08 | 0.03 | 0.08 | 0.005 | −1.55 |
| | | | | | | [1.32] | [1.00] | | | | [−2.86] |
| 4 | 1.21 | 4.79 | 24.0% | 4.76 | 0.73 | −0.08 | −0.06 | 0.54 | 0.62 | 0.015 | 3.62 |
| | | | | | | [−0.87] | [−0.65] | | | | [4.53] |
| 5 | 0.60 | 6.55 | 7.4% | 3.73 | 0.89 | −0.88 | −0.53 | 2.18 | 2.31 | 0.018 | 8.07 |
| | | | | | | [−3.42] | [−2.88] | | | | [5.32] |
| 5-1 | −1.04 | | | | | −1.15 | −0.83 | | | | |
| | [−3.90] | | | | | [−3.54] | [−2.93] | | | | |
| Joint test $p$-value | | | | | | 0.01 | 0.03 | | | | 0.00 |

post-ranking factor loadings that are computed over the full sample period. While the $\beta_{\Delta VIX}$ loadings show very strong patterns of future returns, they represent past covariation with innovations in market volatility. We must show that the portfolios in Table I also exhibit high loadings with volatility risk over the same period used to compute the alphas.

To construct our portfolios, we take $\Delta VIX$ to proxy for the innovation in aggregate volatility at a daily frequency. However, at the standard monthly frequency, which is the frequency of the ex post returns for the alphas reported in Table I, using the change in *VIX* is a poor approximation for innovations in aggregate volatility. This is because at lower frequencies, the effect of the conditional mean of *VIX* plays an important role in determining the unanticipated change in *VIX*. In contrast, the high persistence of the *VIX* series at a daily frequency means that the first difference of *VIX* is a suitable proxy for the innovation in aggregate volatility. Hence, we should not measure ex post exposure to aggregate volatility risk by looking at how the portfolios in Table I correlate ex post with monthly changes in *VIX*.

To measure ex post exposure to aggregate volatility risk at a monthly frequency, we follow Breeden, Gibbons, and Litzenberger (1989) and construct an ex post factor that mimics aggregate volatility risk. We term this mimicking factor *FVIX*. We construct the tracking portfolio so that it is the portfolio of asset returns maximally correlated with realized innovations in volatility using a set of basis assets. This allows us to examine the contemporaneous relationship between factor loadings and average returns. The major advantage of using *FVIX* to measure aggregate volatility risk is that we can construct a good approximation for innovations in market volatility at any frequency. In particular, the factor mimicking aggregate volatility innovations allows us to proxy aggregate volatility risk at the monthly frequency by simply cumulating daily returns over the month on the underlying base assets used to construct the mimicking factor. This is a much simpler method for measuring aggregate volatility innovations at different frequencies, rather than specifying different, and unknown, conditional means for *VIX* that depend on different sampling frequencies. After constructing the mimicking aggregate volatility factor, we confirm that it is high exposure to aggregate volatility risk that is behind the low average returns to past $\beta_{\Delta VIX}$ loadings.

However, just showing that there is a relation between ex post aggregate volatility risk exposure and average returns does not rule out the explanation that the volatility risk exposure is due to known determinants of expected returns in the cross-section. Hence, our second condition for a risk-based explanation is that the aggregate volatility risk exposure is robust to controlling for various stock characteristics and other factor loadings. Several of these cross-sectional effects may be at play in the results of Table I. For example, quintile portfolios 1 and 5 have smaller stocks, and stocks with higher book-to-market ratios, and these are the portfolios with the most extreme returns. Periods of very high volatility also tend to coincide with periods of market illiquidity (see, among others, Jones (2003) and Pástor and Stambaugh (2003)). In Section I.C, we control for size, book-to-market, and momentum effects, and also specifically disentangle the exposure to liquidity risk from the exposure to systematic volatility risk.

### *B.4. A Factor Mimicking Aggregate Volatility Risk*

Following Breeden et al. (1989) and Lamont (2001), we create the mimicking factor *FVIX* to track innovations in *VIX* by estimating the coefficient $b$ in the following regression:

$$\Delta VIX_t = c + b'X_t + u_t, \tag{4}$$

where $X_t$ represents the returns on the base assets. Since the base assets are excess returns, the coefficient $b$ has the interpretation of weights in a zero-cost portfolio. The return on the portfolio, $b'X_t$, is the factor *FVIX* that mimics innovations in market volatility. We use the quintile portfolios sorted on past $\beta_{\Delta VIX}$ in Table I as the base assets $X_t$. These base assets are, by construction, a set of assets that have different sensitivities to past daily innovations in *VIX*.[7] We run the regression in equation (4) at a daily frequency every month and use the estimates of $b$ to construct the mimicking factor for aggregate volatility risk over the same month.

An alternative way to construct a factor that mimics volatility risk is to directly construct a traded asset that reflects only volatility risk. One way to do this is to consider option returns. Coval and Shumway (2001) construct market-neutral straddle positions using options on the aggregate market (S&P100 options). This strategy provides exposure to aggregate volatility risk. Coval and Shumway approximate daily at-the-money straddle returns by taking a weighted average of zero-beta straddle positions, with strikes immediately above and below each day's opening level of the S&P100. They cumulate these daily returns each month to form a monthly return, which we denote as *STR*.[8] In Section I.D, we investigate the robustness of our results to using *STR* in place of *FVIX* when we estimate the cross-sectional aggregate volatility price of risk.

Once we construct *FVIX*, then the multifactor model (3) holds, except we can substitute the (unobserved) innovation in volatility with the tracking portfolio that proxies for market volatility risk (see Breeden (1979)). Hence, we can write the model in equation (3) as the following cross-sectional regression:

$$r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{FVIX}^i FVIX_t + \varepsilon_t^i, \tag{5}$$

where *MKT* is the market excess return, *FVIX* is the mimicking aggregate volatility factor, and $\beta_{MKT}^i$ and $\beta_{FVIX}^i$ are factor loadings on market risk and aggregate volatility risk, respectively.

[7] Our results are unaffected if we use the six Fama–French (1993) $3 \times 2$ portfolios sorted on size and book-to-market as the base assets. These results are available upon request.

[8] The *STR* returns are available from January 1986 to December 1995, because it is constructed from the Berkeley Option Database, which has reliable data only from the late 1980s and ends in 1995.

To test a factor risk model like equation (5), we must show contemporaneous patterns between factor loadings and average returns. That is, if the price of risk of aggregate volatility is negative, then stocks with high covariation with *FVIX* should have low returns, on average, over the same period used to compute the $\beta_{FVIX}$ factor loadings and the average returns. By construction, *FVIX* allows us to examine the contemporaneous relationship between factor loadings and average returns and it is the factor that is ex post most highly correlated with innovations in aggregate volatility. However, while *FVIX* is the right factor to test a risk story, *FVIX* itself is not an investable portfolio because it is formed with future information. Nevertheless, *FVIX* can be used as guidance for tradeable strategies that would hedge market volatility risk using the cross-section of stocks.

In the second column under the heading "Factor Loadings" of Table I, we report the pre-formation $\beta_{FVIX}$ loadings that correspond to each of the portfolios sorted on past $\beta_{\Delta VIX}$ loadings. The pre-formation $\beta_{FVIX}$ loadings are computed by running the regression (5) over daily returns over the past month. The pre-formation *FVIX* loadings are very similar to the pre-formation $\Delta VIX$ loadings for the portfolios sorted on past $\beta_{\Delta VIX}$ loadings. For example, the pre-formation $\beta_{FVIX}(\beta_{\Delta VIX})$ loading for quintile 1 is $-2.00$ ($-2.09$), while the pre-formation $\beta_{FVIX}(\beta_{\Delta VIX})$ loading for quintile 5 is 2.31 (2.18).

### *B.5. Post-Formation Factor Loadings*

In the next-to-last column of Table I, we report post-formation $\beta_{\Delta VIX}$ loadings over the next month, which we compute as follows. After the quintile portfolios are formed at time $t$, we calculate daily returns of each of the quintile portfolios over the next month, from $t$ to $t + 1$. For each portfolio, we compute the ex post $\beta_{\Delta VIX}$ loadings by running the same regression (3) that is used to form the portfolios using daily data over the next month ($t$ to $t + 1$). We report the next-month $\beta_{\Delta VIX}$ loadings averaged across time. The next-month post-formation $\beta_{\Delta VIX}$ loadings range from $-0.033$ for portfolio 1 to 0.018 for portfolio 5. Hence, although the ex post $\beta_{\Delta VIX}$ loadings over the next month are monotonically increasing, the spread is disappointingly very small.

Finding large spreads in the next-month post-formation $\beta_{\Delta VIX}$ loadings is a very stringent requirement and one that would be done in direct tests of a conditional factor model such as equation (1). Our goal is more modest. We examine the premium for aggregate volatility using an unconditional factor model approach, which requires that average returns be related to the unconditional covariation between returns and aggregate volatility risk. As Hansen and Richard (1987) note, an unconditional factor model implies the existence of a conditional factor model. However, to form precise estimates of the conditional factor loadings in a full conditional setting like equation (1) requires knowledge of the instruments driving the time variation in the betas, as well as specification of the complete set of factors.

The ex post $\beta_{\Delta VIX}$ loadings over the next month are computed using, on average, only 22 daily observations each month. In contrast, the CAPM and FF-3 alphas are computed using regressions measuring unconditional factor exposure over the full sample (180 monthly observations) of post-ranking returns. To demonstrate that exposure to volatility innovations may explain some of the large CAPM and FF-3 alphas, we must show that the quintile portfolios exhibit different post-ranking spreads in aggregate volatility risk sensitivities over the entire sample at the same monthly frequency for which the post-ranking returns are constructed. Averaging a series of ex post conditional 1-month covariances does not provide an estimate of the unconditional covariation between the portfolio returns and aggregate volatility risk.

To examine ex post factor exposure to aggregate volatility risk consistent with a factor model approach, we compute post-ranking *FVIX* betas over the full sample.[9] In particular, since the FF-3 alpha controls for market, size, and value factors, we compute ex post *FVIX* factor loadings also controlling for these factors in a four-factor post-formation regression,

$$r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \beta_{FVIX}^i FVIX_t + \varepsilon_t^i, \tag{6}$$

where the first three factors *MKT, SMB*, and *HML* constitute the FF-3 model's market, size, and value factors. To compute the ex post $\beta_{FVIX}$ loadings, we run equation (6) using monthly frequency data over the whole sample, where the portfolios on the left-hand side of equation (6) are the quintile portfolios in Table I that are sorted on past loadings of $\beta_{\Delta VIX}$ using equation (3).

The last column of Table I shows that the portfolios sorted on past $\beta_{\Delta VIX}$ exhibit strong patterns of post-formation factor loadings on the volatility risk factor *FVIX*. The ex post $\beta_{FVIX}$ factor loadings monotonically increase from $-5.06$ for portfolio 1 to 8.07 for portfolio 5. We strongly reject the hypothesis that the ex post $\beta_{FVIX}$ loadings are equal to zero, with a $p$-value less than 0.001. Thus, sorting stocks on past $\beta_{\Delta VIX}$ provides strong, significant spreads in ex post aggregate volatility risk sensitivities.[10]

### B.6. Characterizing the Behavior of FVIX

Table II reports correlations among the *FVIX* factor, $\Delta VIX$, and *STR*, as well as correlations of these variables with other cross-sectional factors. We denote the daily first difference in *VIX* as $\Delta VIX$, and use $\Delta_m VIX$ to represent the monthly first difference in the *VIX* index. The mimicking volatility factor is highly contemporaneously correlated with changes in volatility at a daily

[9] The pre-formation betas and the post-formation betas are computed using different criteria ($\Delta VIX$ and *FVIX*, respectively). However, Table I shows that the pre-formation $\beta_{FVIX}$ loadings are almost identical to the pre-formation $\beta_{\Delta VIX}$ loadings.

[10] When we compute ex post betas using the monthly change in *VIX*, $\Delta_m VIX$, using a four-factor model similar to equation (6) (except using $\Delta_m VIX$ in place of *FVIX*), there is less dispersion in the post-formation $\Delta_m VIX$ betas, ranging from $-2.46$ for portfolio 1 to 0.76 to portfolio 5, compared to the ex post $\beta_{FVIX}$ loadings.

**Table II**
## Factor Correlations

The table reports correlations of first differences in *VIX*, *FVIX*, and *STR* with various factors. The variable $\Delta VIX$ ($\Delta_m VIX$) represents the daily (monthly) change in the *VIX* index, and *FVIX* is the mimicking aggregate volatility risk factor. The factor *STR* is constructed by Coval and Shumway (2001) from the returns of zero-beta straddle positions. The factors *MKT, SMB, HML* are the Fama and French (1993) factors, the momentum factor *UMD* is constructed by Kenneth French, and *LIQ* is the Pástor and Stambaugh (2003) liquidity factor. The sample period is January 1986 to December 2000, except for correlations involving *STR*, which are computed over the sample period January 1986 to December 1995.

Panel A: Daily Correlation

| | $\Delta VIX$ |
|---|---|
| *FVIX* | 0.91 |

Panel B: Monthly Correlations

| | *FVIX* | $\Delta_m VIX$ | *MKT* | *SMB* | *HML* | *UMD* | *LIQ* |
|---|---|---|---|---|---|---|---|
| $\Delta_m VIX$ | 0.70 | 1.00 | −0.58 | −0.18 | 0.22 | −0.11 | −0.33 |
| *FVIX* | 1.00 | 0.70 | −0.66 | −0.14 | 0.26 | −0.25 | −0.40 |
| *STR* | 0.75 | 0.83 | −0.39 | −0.39 | 0.08 | −0.26 | −0.59 |

frequency, with a correlation of 0.91. At the monthly frequency, the correlation between *FVIX* and $\Delta_m VIX$ is lower, at 0.70. The factors *FVIX* and *STR* have a high correlation of 0.83, which indicates that *FVIX*, formed from stock returns, behaves like the *STR* factor constructed from option returns. Hence, *FVIX* captures option-like behavior in the cross-section of stocks. The factor *FVIX* is negatively contemporaneously correlated with the market return (−0.66), reflecting the fact that when volatility increases, market returns are low. The correlations of *FVIX* with *SMB* and *HML* are −0.14 and 0.26, respectively. The correlation between *FVIX* and *UMD*, a factor capturing momentum returns, is also low at −0.25.

In contrast, there is a strong negative correlation between *FVIX* and the Pástor and Stambaugh (2003) liquidity factor, *LIQ*, at −0.40. The *LIQ* factor decreases in times of low liquidity, which tend to also be periods of high volatility. One example of a period of low liquidity with high volatility is the 1987 crash (see, among others, Jones (2003) and Pástor and Stambaugh (2003)). However, the correlation between *FVIX* and *LIQ* is far from −1, indicating that volatility risk and liquidity risk may be separate effects, and may be separately priced. In the next section, we conduct a series of robustness checks designed to disentangle the effects of aggregate volatility risk from other factors, including liquidity risk.

### *C. Robustness*

In this section, we conduct a series of robustness checks in which we specify different models for the conditional mean of *VIX*, we use windows of different estimation periods to form the $\beta_{\Delta VIX}$ portfolios, and we control for potential cross-sectional pricing effects due to book-to-market, size, liquidity, volume, and momentum factor loadings or characteristics.

### C.1. Robustness to Different Conditional Means of VIX

We first investigate the robustness of our results to the method measuring innovations in *VIX*. We use the change in *VIX* at a daily frequency to measure the innovation in volatility because *VIX* is a highly serially correlated series. However, *VIX* appears to be a stationary series, and using $\Delta VIX$ as the innovation in *VIX* may slightly over-difference. Our finding of low average returns on stocks with high $\beta_{FVIX}$ is robust to measuring volatility innovations by specifying various models for the conditional mean of *VIX*. If we fit an AR(1) model to *VIX* and measure innovations relative to the AR(1) specification, we find that the results of Table I are almost unchanged. Specifically, the mean return of the difference between the first and fifth $\beta_{\Delta VIX}$ portfolios is $-1.08\%$ per month, and the FF-3 alpha of the 5-1 difference is $-0.90\%$, both highly statistically significant. Using an optimal BIC choice for the number of autoregressive lags, which is 11, produces a similar result. In this case, the mean of the 5-1 difference is $-0.81\%$ and the 5-1 FF-3 alpha is $-0.66\%$; both differences are significant at the 5% level.[11]

### C.2. Robustness to the Portfolio Formation Window

In this subsection, we investigate the robustness of our results to the amount of data used to estimate the pre-formation factor loadings $\beta_{\Delta VIX}$. In Table I, we use a formation period of 1 month, and we emphasize that this window is chosen a priori without pre-tests. The results in Table I become weaker if we extend the formation period of the portfolios. Although the point estimates of the $\beta_{\Delta VIX}$ portfolios have the same qualitative patterns as Table I, statistical significance drops. For example, if we use the past 3 months of daily data on $\Delta VIX$ to compute volatility betas, the mean return of the 5th quintile portfolio with the highest past $\beta_{\Delta VIX}$ stocks is 0.79%, compared with 0.60% with a 1-month formation period. Using a 3-month formation period, the FF-3 alpha on the 5th quintile portfolio decreases in magnitude to $-0.37\%$, with a robust $t$-statistic of $-1.62$, compared to $-0.53\%$, with a $t$-statistic of $-2.88$, with a 1-month formation period from Table I. If we use the past 12 months of *VIX* innovations, the 5th quintile portfolio mean increases to 0.97%, while the FF-3 alpha decreases in magnitude to $-0.24\%$ with a $t$-statistic of $-1.04$.

The weakening of the $\beta_{\Delta VIX}$ effect as the formation periods increase is due to the time variation of the sensitivities to aggregate market innovations. The turnover in the monthly $\beta_{\Delta VIX}$ portfolios is high (above 70%) and using longer

[11] In these exercises, we estimate the AR coefficients only using all data up to time $t$ to compute the innovation for $t+1$, so that no forward-looking information is used. We initially estimate the AR models using 1 year of daily data. However, the optimal BIC lag length is chosen using the whole sample.

formation periods causes less turnover; however, using more data provides less precise conditional estimates. The longer the formation window, the less these conditional estimates are relevant at time $t$, and the lower the spread in the pre-formation $\beta_{\Delta VIX}$ loadings. By using only information over the past month, we obtain an estimate of the conditional factor loading much closer to time $t$.

### *C.3. Robustness to Book-to-Market and Size Characteristics*

Small growth firms are typically firms with option values that would be expected to do well when aggregate volatility increases. The portfolio of small growth firms is also one of the Fama–French (1993) 25 portfolios sorted on size and book-to-market that is hardest to price by standard factor models (see, for example, Hodrick and Zhang (2001)). Could the portfolio of stocks with high aggregate volatility exposure have a disproportionately large number of small growth stocks?

Investigating this conjecture produces mixed results. If we exclude only the portfolio among the 25 Fama–French portfolios with the smallest growth firms and repeat the quintile portfolio sorts in Table I, we find that the 5-1 mean difference in returns is reduced in magnitude from $-1.04\%$ for all firms to $-0.63\%$ per month, with a $t$-statistic of $-3.30$. Excluding small growth firms produces a FF-3 alpha of $-0.44\%$ per month for the zero-cost portfolio that goes long portfolio 5 and short portfolio 1, which is no longer significant at the 5% level ($t$-statistic is $-1.79$), compared to the value of $-0.83\%$ per month with all firms. These results suggest that small growth stocks may play a role in the $\beta_{\Delta VIX}$ quintile sorts of Table I.

However, a more thorough characteristic-matching procedure suggests that size or value characteristics do not completely drive the results. Table III reports mean returns of the $\beta_{\Delta VIX}$ portfolios characteristic matched by size and book-to-market ratios, following the method proposed by Daniel et al. (1997). Every month, each stock is matched with one of the Fama–French 25 size and book-to-market portfolios according to its size and book-to-market characteristics. The table reports value-weighted simple returns in excess of the characteristic-matched returns. Table III shows that characteristic controls for size and book-to-market decrease the magnitude of the raw 5-1 mean return difference of $-1.04\%$ in Table I to $-0.90\%$. If we exclude firms that are members of the smallest growth portfolio of the Fama–French 25 size-value portfolios, the magnitude of the mean 5-1 difference decreases to $-0.64\%$ per month. However, the characteristic-controlled differences are still highly significant. Hence, the low returns to high past $\beta_{\Delta VIX}$ stocks are not completely driven by a disproportionate concentration among small growth stocks.

### *C.4. Robustness to Liquidity Effects*

Pástor and Stambaugh (2003) demonstrate that stocks with high liquidity betas have high average returns. In order for liquidity to be an explanation behind the spreads in average returns of the $\beta_{\Delta VIX}$ portfolios, high $\beta_{\Delta VIX}$ stocks

**Table III**

**Characteristic Controls for Portfolios Sorted on $\beta_{\Delta VIX}$**

The table reports the means and standard deviations of the excess returns on the $\beta_{\Delta VIX}$ quintile portfolios characteristic matched by size and book-to-market ratios. Each month, each stock is matched with one of the Fama and French (1993) 25 size and book-to-market portfolios according to its size and book-to-market characteristics. The table reports value-weighted simple returns in excess of the characteristic-matched returns. The columns labeled "Excluding Small, Growth Firms" exclude the Fama–French portfolio containing the smallest stocks and the firms with the lowest book-to-market ratios. The row "5-1" refers to the difference in monthly returns between portfolio 5 and portfolio 1. The $p$-values of joint tests for all alphas equal to zero are less than 1% for the panel of all firms and for the panel excluding small, growth firms. Robust Newey–West (1987) $t$-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

| | All Firms | | Excluding Small, Growth Firms | |
|---|---|---|---|---|
| Rank | Mean | Std. Dev. | Mean | Std. Dev. |
| 1 | 0.32 | 2.11 | 0.36 | 1.90 |
| 2 | 0.04 | 1.25 | 0.02 | 0.94 |
| 3 | 0.04 | 0.94 | 0.05 | 0.89 |
| 4 | −0.11 | 1.04 | −0.10 | 1.02 |
| 5 | −0.58 | 3.39 | −0.29 | 2.17 |
| 5-1 | −0.90 | | −0.64 | |
| | [−3.59] | | [−3.75] | |

must have low liquidity betas. To check that the spread in average returns on the $\beta_{\Delta VIX}$ portfolios is not due to liquidity effects, we first sort stocks into five quintiles based on their historical Pástor–Stambaugh liquidity betas. Then, within each quintile, we sort stocks into five quintiles based on their past $\beta_{\Delta VIX}$ coefficient loadings. These portfolios are rebalanced monthly and are value weighted. After forming the $5 \times 5$ liquidity beta and $\beta_{\Delta VIX}$ portfolios, we average the returns of each $\beta_{\Delta VIX}$ quintile over the five liquidity beta portfolios. Thus, these quintile $\beta_{\Delta VIX}$ portfolios control for differences in liquidity.

We report the results of the Pástor–Stambaugh liquidity control in Panel A of Table IV, which shows that controlling for liquidity reduces the magnitude of the 5-1 difference in average returns from $-1.04\%$ per month in Table I to $-0.68\%$ per month. However, after controlling for liquidity, we still observe the monotonically decreasing pattern of average returns of the $\beta_{\Delta VIX}$ quintile portfolios. We also find that controlling for liquidity, the FF-3 alpha for the 5-1 portfolio remains significantly negative at $-0.55\%$ per month. Hence, liquidity effects cannot account for the spread in returns resulting from sensitivity to aggregate volatility risk.

Table IV also reports post-formation $\beta_{FVIX}$ loadings. Similar to the post-formation $\beta_{FVIX}$ loadings in Table I, we compute the post-formation $\beta_{FVIX}$ coefficients using a monthly frequency regression with the four-factor model in equation (6) to be comparable to the FF-3 alphas over the same sample period. Both the pre-formation $\beta_{\Delta VIX}$ and post-formation $\beta_{FVIX}$ loadings increase from

**Table IV**

**Portfolios Sorted on $\beta_{\Delta VIX}$ Controlling for Liquidity, Volume and Momentum**

In Panel A, we first sort stocks into five quintiles based on their historical liquidity beta, following Pástor and Stambaugh (2003). Then, within each quintile, we sort stocks based on their $\beta_{\Delta VIX}$ loadings into five portfolios. All portfolios are rebalanced monthly and are value weighted. The five portfolios sorted on $\beta_{\Delta VIX}$ are then averaged over each of the five liquidity beta portfolios. Hence, they are $\beta_{\Delta VIX}$ quintile portfolios controlling for liquidity. In Panels B and C, the same approach is used except we control for average trading volume (in dollars) over the past month and past 12-month returns, respectively. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, simple returns. The table also reports alphas from CAPM and Fama–French (1993) regressions. The row "5-1" refers to the difference in monthly returns between portfolio 5 and portfolio 1. The pre-formation betas refer to the value-weighted $\beta_{\Delta VIX}$ within each quintile portfolio at the start of the month. We report the pre-formation $\beta_{\Delta VIX}$ averaged across the whole sample. The last column reports ex post $\beta_{FVIX}$ factor loadings over the whole sample, where *FVIX* is the factor mimicking aggregate volatility risk. To correspond with the Fama–French alphas, we compute the ex post betas by running a four-factor regression with the three Fama–French factors together with the *FVIX* factor, following the regression in equation (6). The row labeled "Joint test $p$-value" reports a Gibbons et al. (1989) test that the alphas equal zero, and a robust joint test that the factor loadings are equal to zero. Robust Newey–West (1987) $t$-statistics are reported in square brackets. The sample period is from January 1986 to December 2000.

| Rank | Mean | Std. Dev. | CAPM Alpha | FF-3 Alpha | Pre-Formation $\beta_{\Delta VIX}$ Loading | Post-Formation $\beta_{FVIX}$ Loading |
|---|---|---|---|---|---|---|
| Panel A: Controlling for Liquidity | | | | | | |
| 1 | 1.57 | 5.47 | 0.21 | 0.19 | −1.89 | −1.87 |
| | | | [1.31] | [1.34] | | [−1.65] |
| 2 | 1.48 | 4.48 | 0.27 | 0.15 | −0.43 | −2.70 |
| | | | [2.25] | [1.68] | | [−2.78] |
| 3 | 1.40 | 4.54 | 0.15 | 0.09 | 0.03 | −1.34 |
| | | | [1.59] | [0.97] | | [−1.90] |
| 4 | 1.30 | 4.74 | 0.02 | −0.02 | 0.49 | 0.49 |
| | | | [0.21] | [−0.17] | | [0.54] |
| 5 | 0.89 | 5.84 | −0.52 | −0.36 | 1.96 | 5.38 |
| | | | [−2.87] | [−2.09] | | [4.26] |
| 5-1 | −0.68 | | −0.73 | −0.55 | | |
| | [−3.04] | | [−2.99] | [−2.15] | | |
| Joint test $p$-value | | | 0.04 | 0.01 | | 0.00 |

*(continued)*

negative to positive from portfolio 1 to 5, consistent with a risk story. In particular, the post-formation $\beta_{FVIX}$ loadings increase from −1.87 for portfolio 1 to 5.38 for portfolio 5. We reject the hypothesis that the ex post $\beta_{FVIX}$ loadings are jointly equal to zero with a $p$-value less than 0.001.

### *C.5. Robustness to Volume Effects*

Panel B of Table IV reports an analogous exercise to that in Panel A except we control for volume rather than liquidity. Gervais, Kaniel, and Mingelgrin (2001) find that stocks with high past trading volume earn higher average

**Table IV**—*Continued*

| Rank | Mean | Std. Dev. | CAPM Alpha | FF-3 Alpha | Pre-Formation $\beta_{\Delta VIX}$ Loading | Post-Formation $\beta_{FVIX}$ Loading |
|---|---|---|---|---|---|---|
| Panel B: Controlling for Volume | | | | | | |
| 1 | 1.10 | 4.73 | −0.11 | −0.13 | −2.08 | −3.12 |
| | | | [−0.58] | [−1.34] | | [−3.17] |
| 2 | 1.18 | 4.01 | 0.08 | −0.08 | −0.47 | −3.39 |
| | | | [0.46] | [−0.92] | | [−4.19] |
| 3 | 1.18 | 3.78 | 0.10 | −0.04 | 0.04 | −2.84 |
| | | | [0.66] | [−0.50] | | [−4.84] |
| 4 | 0.98 | 4.18 | −0.17 | −0.23 | 0.55 | 0.14 |
| | | | [−1.06] | [2.16] | | [0.24] |
| 5 | 0.38 | 5.31 | −0.90 | −0.71 | 2.17 | 4.29 |
| | | | [−3.86] | [−4.84] | | [5.07] |
| 5-1 | −0.72 | | −0.79 | −0.58 | | |
| | [−3.49] | | [−3.22] | [−3.03] | | |
| Joint test $p$-value | | | 0.00 | 0.00 | | 0.00 |
| Panel C: Controlling for Past 12-Month Returns | | | | | | |
| 1 | 1.25 | 5.55 | −0.11 | −0.17 | −2.03 | 0.39 |
| | | | [−0.64] | [−1.08] | | [0.28] |
| 2 | 1.19 | 4.87 | −0.08 | −0.19 | −0.49 | 0.82 |
| | | | [−0.57] | [−1.54] | | [0.68] |
| 3 | 1.28 | 4.76 | 0.02 | −0.08 | 0.03 | 0.97 |
| | | | [0.15] | [−0.73] | | [0.89] |
| 4 | 1.06 | 4.88 | −0.22 | −0.27 | 0.56 | 4.86 |
| | | | [−1.64] | [−2.26] | | [5.50] |
| 5 | 0.36 | 5.87 | −1.05 | −0.90 | 2.11 | 7.17 |
| | | | [−5.01] | [−4.72] | | [5.50] |
| 5-1 | −0.89 | | −0.93 | −0.74 | | |
| | [−4.72] | | [−4.00] | [−3.42] | | |
| Joint test $p$-value | | | 0.00 | 0.00 | | 0.00 |

returns than stocks with low past trading volume. It could be that the low average returns (and alphas) we find for stocks with high $\beta_{FVIX}$ loadings are just stocks with low volume. Panel B shows that this is not the case. In Panel B, we control for volume by first sorting stocks into quintiles based on their trading volume over the past month. We then sort stocks into quintiles based on their $\beta_{FVIX}$ loading and average across the volume quintiles. After controlling for volume, the FF-3 alpha of the 5-1 long–short portfolio remains significant at the 5% level at −0.58% per month. The post-formation $\beta_{FVIX}$ loadings also monotonically increase from portfolio 1 to 5.

### *C.6. Robustness to Momentum Effects*

Our last robustness check controls for the Jegadeesh and Titman (1993) momentum effect in Panel C. Since Jegadeesh and Titman report that stocks with low past returns, or past loser stocks, continue to have low future returns, stocks with high past $\beta_{\Delta VIX}$ loadings may tend to also be loser stocks. Controlling for past 12-month returns reduces the magnitude of the raw $-1.04\%$ per month difference between stocks with low and high $\beta_{FVIX}$ loadings to $-0.89\%$, but the 5-1 difference remains highly significant. The CAPM and FF-3 alphas of the portfolios constructed to control for momentum are also significant at the 1% level. Once again, the post-formation $\beta_{FVIX}$ loadings are monotonically increasing from portfolio 1 to 5. Hence, momentum cannot account for the low average returns to stocks with high sensitivities to aggregate volatility risk.

### *D. The Price of Aggregate Volatility Risk*

Tables III and IV demonstrate that the low average returns to stocks with high past sensitivities to aggregate volatility risk cannot be explained by size, book-to-market, liquidity, volume, or momentum effects. Moreover, Tables III and IV also show strong ex post spreads in the *FVIX* factor. Since this evidence supports the case that aggregate volatility is a priced risk factor in the cross-section of stock returns, the next step is to estimate the cross-sectional price of volatility risk.

To estimate the factor premium $\lambda_{FVIX}$ on the mimicking volatility factor *FVIX*, we first construct a set of test assets whose factor loadings on market volatility risk are sufficiently disperse so that the cross-sectional regressions have reasonable power. We construct 25 investible portfolios sorted by $\beta_{MKT}$ and $\beta_{\Delta VIX}$ as follows. At the end of each month, we sort stocks based on $\beta_{MKT}$, computed by a univariate regression of excess stock returns on excess market returns over the past month using daily data. We compute the $\beta_{\Delta VIX}$ loadings using the bivariate regression (3) also using daily data over the past month. Stocks are ranked first into quintiles based on $\beta_{MKT}$ and then within each $\beta_{MKT}$ quintile into $\beta_{\Delta VIX}$ quintiles.

Jagannathan and Wang (1996) show that a conditional factor model like equation (1) has the form of a multifactor unconditional model, where the original factors enter as well as additional factors associated with the time-varying information set. In estimating an unconditional cross-sectional price of risk for the aggregate volatility factor *FVIX*, we recognize that additional factors may also affect the unconditional expected return of a stock. Hence, in our full specification, we estimate the following cross-sectional regression that includes FF-3, momentum (*UMD*), and liquidity (*LIQ*) factors:

$$\begin{aligned} r_t^i = c &+ \beta_{MKT}^i \lambda_{MKT} + \beta_{FVIX}^i \lambda_{FVIX} + \beta_{SMB}^i \lambda_{SMB} \\ &+ \beta_{HML}^i \lambda_{HML} + \beta_{UMD}^i \lambda_{UMD} + \beta_{LIQ}^i \lambda_{LIQ} + \varepsilon_t^i, \end{aligned} \tag{7}$$

where the $\lambda$s represent unconditional prices of risk of the various factors. To check robustness, we also estimate the cross-sectional price of aggregate volatility risk by using the Coval and Shumway (2001) *STR* factor in place of *FVIX* in equation (7).

We use the $25\beta_{MKT} \times \beta_{\Delta VIX}$ base assets to estimate factor premiums in equation (7) following the two-step procedure of Fama–MacBeth (1973). In the first stage, betas are estimated using the full sample. In the second stage, we use cross-sectional regressions to estimate the factor premia. We are especially interested in ex post factor loadings on the *FVIX* aggregate volatility factor, and the price of risk of *FVIX*. Panel A of Table V reports the results. In addition to the standard Fama and French (1993) factors *MKT*, *SMB*, and *HML*, we include the momentum factor *UMD* and Pástor and Stambaugh's (2003) non-traded liquidity factor, *LIQ*. We estimate the cross-sectional risk premium for *FVIX* together with the Fama–French model in Regression I. In Regression II, we check robustness of our results by using Coval and Shumway's (2001) *STR* option factor. Regressions III and IV also include the additional regressors *UMD* and *LIQ*.

In general, Panel A shows that the premiums of the standard factors (*MKT*, *SMB*, *HML*) are estimated imprecisely with this set of base assets. The premium on *SMB* is consistently estimated to be negative because the size strategy performed poorly from the 1980s onward. The value effect also performed poorly during the late 1990s, which accounts for the negative coefficient on *HML*.

In contrast, the price of volatility risk in Regression I is $-0.08\%$ per month, which is statistically significant at the 1% level. Using the Coval and Shumway (2001) *STR* factor in Regression II, we estimate the cross-sectional price of volatility risk to be $-0.19\%$ per month, which is also statistically significant at the 1% level. These results are consistent with the hypothesis that the cross-section of stock returns reflects exposure to aggregate volatility risk, and the price of market volatility risk is significantly negative.

When we add the *UMD* and *LIQ* factors in Regressions III and IV, the estimates of the *FVIX* coefficient are essentially unchanged. When *UMD* is added, its coefficient is insignificant, while the coefficient on *FVIX* barely moves from the $-0.080$ estimate in Regression I to $-0.082$. The small effect of adding a momentum control on the *FVIX* coefficient is consistent with the low correlation between *FVIX* and *UMD* in Table II and with the results in Table IV showing that controlling for past returns does not remove the low average returns on stocks with high $\beta_{FVIX}$ loadings. In the full specification Regression IV, the *FVIX* coefficient becomes slightly smaller in magnitude at $-0.071$, but the coefficient remains significant at the 5% level with a robust $t$-statistic of $-2.02$. Moreover, *FVIX* is the only factor to carry a relatively large absolute $t$-statistic in the regression, which estimates seven coefficients with only 25 portfolios and 180 time-series observations.

Panel B of Table V reports the first-pass factor loadings on *FVIX* for each of the 25 base assets from Regression I in Panel A. Panel B confirms that the portfolios formed on past $\beta_{\Delta VIX}$ loadings reflect exposure to volatility risk measured by *FVIX* over the full sample. Except for two portfolios (the two lowest $\beta_{MKT}$ portfolios corresponding to the lowest $\beta_{\Delta VIX}$ quintile), all the *FVIX* factor loadings increase monotonically from low to high. Examination of the realized *FVIX* factor loadings demonstrates that the set of base assets, sorted on past $\beta_{\Delta VIX}$ and past $\beta_{MKT}$, provides disperse ex post *FVIX* loadings.

**Table V**

**Estimating the Price of Volatility Risk**

Panel A reports the Fama–MacBeth (1973) factor premiums on 25 portfolios sorted first on $\beta_{MKT}$ and then on $\beta_{\Delta VIX}$. *MKT* is the excess return on the market portfolio, *FVIX* is the mimicking factor for aggregate volatility innovations, *STR* is Coval and Shumway's (2001) zero-beta straddle return, *SMB* and *HML* are the Fama–French (1993) size and value factors, *UMD* is the momentum factor constructed by Kenneth French, and *LIQ* is the aggregate liquidity measure from Pástor and Stambaugh (2003). In Panel B, we report ex post factor loadings on *FVIX*, from the regression specification I (Fama–French model plus *FVIX*). Robust $t$-statistics that account for the errors-in-variables for the first-stage estimation in the factor loadings are reported in square brackets. The sample period is from January 1986 to December 2000, except for the Fama–MacBeth regressions with *STR*, which are from January 1986 to December 1995.

Panel A: Fama–MacBeth (1973) Factor Premiums

| | I | II | III | IV |
|---|---|---|---|---|
| Constant | −0.145 | −0.527 | −0.202 | −0.247 |
| | [−0.23] | [−0.88] | [−0.31] | [−0.36] |
| *MKT* | 0.977 | 1.276 | 1.034 | 1.042 |
| | [1.11] | [1.47] | [1.13] | [1.13] |
| *FVIX* | −0.080 | | −0.082 | −0.071 |
| | [−2.49] | | [−2.39] | [−2.02] |
| *STR* | | −0.194 | | |
| | | [−2.32] | | |
| *SMB* | −0.638 | −0.246 | −0.608 | −0.699 |
| | [−1.24] | [−0.59] | [−1.13] | [−1.25] |
| *HML* | −0.590 | −0.247 | −0.533 | −0.232 |
| | [−0.95] | [−0.40] | [−0.82] | [−0.34] |
| *UMD* | | | 0.827 | 0.612 |
| | | | [0.83] | [0.59] |
| *LIQ* | | | | −0.021 |
| | | | | [−1.00] |
| Adj $R^2$ | 0.67 | 0.56 | 0.65 | 0.79 |

Panel B: Ex Post Factor Loadings on *FVIX*

| Pre-ranking on $\beta_{MKT}$ | Pre-ranking on $\beta_{\Delta VIX}$ | | | | |
|---|---|---|---|---|---|
| | 1 low | 2 | 3 | 4 | 5 |
| Low 1 | −1.57 | −5.89 | −3.83 | −3.35 | −1.03 |
| | [−0.46] | [−3.23] | [−1.93] | [−1.99] | [−0.45] |
| 2 | −3.49 | −4.47 | −4.01 | −2.00 | −0.54 |
| | [−1.67] | [−3.18] | [−3.11] | [−1.66] | [−0.31] |
| 3 | −5.74 | −3.49 | −2.56 | −0.95 | 3.72 |
| | [−3.16] | [−2.84] | [−2.21] | [−0.78] | [2.30] |
| 4 | −5.80 | −1.41 | −0.34 | 3.39 | 6.66 |
| | [−4.13] | [−1.00] | [−0.29] | [2.69] | [3.85] |
| High 5 | −3.69 | −0.57 | 3.52 | 7.81 | 11.70 |
| | [−2.05] | [−0.45] | [1.76] | [3.32] | [3.13] |

From the estimated price of volatility risk of $-0.08\%$ per month in Table V, we revisit Table I to measure how much exposure to aggregate volatility risk accounts for the large spread in the ex post raw returns of $-1.04\%$ per month between the quintile portfolios with the lowest and highest past $\beta_{\Delta VIX}$ coefficients. In Table I, the ex post spread in *FVIX* betas between portfolios 5 and 1 is $8.07 - (-5.06) = 13.13$. The estimate of the price of volatility risk is $-0.08\%$ per month. Hence, the ex post 13.13 spread in the *FVIX* factor loadings accounts for $13.13 \times -0.080 = -1.05\%$ of the difference in average returns, which is almost exactly the same as the ex post $-1.04\%$ per month number for the raw average return difference between quintile 5 and quintile 1. Hence, virtually all of the large difference in average raw returns in the $\beta_{\Delta VIX}$ portfolios can be attributed to exposure to aggregate volatility risk.

### E. A Potential Peso Story?

Despite being statistically significant, the estimates of the price of aggregate volatility risk from Table V are small in magnitude ($-0.08\%$ per month, or approximately $-1\%$ per annum). Given these small estimates, an alternative explanation behind the low returns to high $\beta_{\Delta VIX}$ stocks is a Peso problem. By construction, *FVIX* does well when the *VIX* index jumps upward. The small negative mean of *FVIX* of $-0.08\%$ per month may be due to having observed a smaller number of volatility spikes than the market expected ex ante.

Figure 1 shows that there are two episodes of large volatility spikes in our sample coinciding with large negative moves of the market: October 1987 and August 1998. In 1987, *VIX* volatility jumped from 22% at the beginning of October to 61% at the end of October. At the end of August 1998, the level of *VIX* reached 48%. The mimicking factor *FVIX* returned 134% during October 1987, and 33.6% during August 1998. Since the cross-sectional price of risk of *FVIX* is $-0.08\%$ per month, from Table V, the cumulative return over the 180 months in our sample period is $-14.4\%$. A few more large values could easily change our inference. For example, only one more crash, with an *FVIX* return of the same order of magnitude as the August 1998 episode, would be enough to generate a positive return on the *FVIX* factor. Using a power law distribution for extreme events, following Gabaix et al. (2003), we would expect to see approximately three large market crashes below three standard deviations during this period. Hence, the ex ante probability of having observed another large spike in volatility during our sample is quite likely.

Hence, given our short sample, we cannot rule out a potential Peso story and, thus, we are not extremely confident about the long-run price of risk of aggregate volatility. Nevertheless, if volatility is a systematic factor as asset pricing theory implies, market volatility risk should be reflected in the cross-section of stock returns. The cross-sectional Fama–MacBeth (1973) estimates of the negative price of risk of *FVIX* are consistent with a risk-based story, and our estimates are highly statistically significant with conventional asymptotic distribution theory that is designed to be robust to conditional heteroskedasticity. However, since we cannot convincingly rule out a Peso problem explanation, our $-1\%$ per annum cross-sectional estimate of the price of risk of aggregate volatility must be interpreted with caution.

## II. Pricing Idiosyncratic Volatility in the Cross-Section

The previous section examines how systematic volatility risk affects cross-sectional average returns by focusing on portfolios of stocks sorted by their sensitivities to innovations in aggregate volatility. In this section, we investigate a second set of assets sorted by idiosyncratic volatility defined relative to the FF-3 model. If market volatility risk is a missing component of systematic risk, standard models of systematic risk, such as the CAPM or the FF-3 model, should misprice portfolios sorted by idiosyncratic volatility because these models do not include factor loadings measuring exposure to market volatility risk.

### *A. Estimating Idiosyncratic Volatility*

#### *A.1. Definition of Idiosyncratic Volatility*

Given the failure of the CAPM to explain cross-sectional returns and the ubiquity of the FF-3 model in empirical financial applications, we concentrate on idiosyncratic volatility measured relative to the FF-3 model

$$r_t^i = \alpha^i + \beta_{MKT}^i MKT_t + \beta_{SMB}^i SMB_t + \beta_{HML}^i HML_t + \varepsilon_t^i. \tag{8}$$

We define idiosyncratic risk as $\sqrt{\text{var}(\varepsilon_t^i)}$ in equation (8). When we refer to idiosyncratic volatility, we mean idiosyncratic volatility relative to the FF-3 model. We also consider sorting portfolios on total volatility, without using any control for systematic risk.

#### *A.2. A Trading Strategy*

To examine trading strategies based on idiosyncratic volatility, we describe portfolio formation strategies based on an estimation period of $L$ months, a waiting period of $M$ months, and a holding period of $N$ months. We describe an $L/M/N$ strategy as follows. At month $t$, we compute idiosyncratic volatilities from the regression (8) on daily data over an $L$-month period from month $t - L - M$ to month $t - M$. At time $t$, we construct value-weighted portfolios based on these idiosyncratic volatilities and hold these portfolios for $N$ months. We concentrate most of our analysis on the 1/0/1 strategy, in which we simply sort stocks into quintile portfolios based on their level of idiosyncratic volatility computed using daily returns over the past month, and we hold these value-weighted portfolios for 1 month. The portfolios are rebalanced each month. We also examine the robustness of our results to various choices of $L$, $M$, and $N$.

The construction of the $L/M/N$ portfolios for $L > 1$ and $N > 1$ follows Jegadeesh and Titman (1993), except our portfolios are value weighted. For example, to construct the 12/1/12 quintile portfolios, each month we construct a value-weighted portfolio based on idiosyncratic volatility computed from daily data over the 12 months of returns ending 1 month prior to the formation date. Similarly, we form a value-weighted portfolio based on 12 months of returns ending 2 months prior, 3 months prior, and so on up to 12 months prior. Each of these portfolios is value weighted. We then take the simple average of these 12 portfolios. Hence, each quintile portfolio changes 1/12th of its composition each month, where each 1/12th part of the portfolio consists of a value-weighted portfolio. The first (fifth) quintile portfolio consists of 1/12th of the lowest value-weighted (highest) idiosyncratic stocks from 1 month ago, 1/12th of the value-weighted lowest (highest) idiosyncratic stocks from 2 months ago, etc.

### B. Patterns in Average Returns for Idiosyncratic Volatility

Table VI reports average returns of portfolios sorted on total volatility, with no controls for systematic risk, in Panel A and of portfolios sorted on idiosyncratic volatility in Panel B.[12] We use a 1/0/1 strategy in both cases. Panel A shows that average returns increase from 1.06% per month going from quintile 1 (low total volatility stocks) to 1.22% per month for quintile 3. Then, average returns drop precipitously. Quintile 5, which contains stocks with the highest total volatility, has an average total return of only 0.09% per month. The FF-3 alpha for quintile 5, reported in the last column, is −1.16% per month, which is highly statistically significant. The difference in the FF-3 alphas between portfolio 5 and portfolio 1 is −1.19% per month, with a robust $t$-statistic of −5.92.

We obtain similar patterns in Panel B, where the portfolios are sorted on idiosyncratic volatility. The difference in raw average returns between quintile portfolios 5 and 1 is −1.06% per month. The FF-3 model is clearly unable to price these portfolios since the difference in the FF-3 alphas between portfolio 5 and portfolio 1 is −1.31% per month, with a $t$-statistic of −7.00. The size and book-to-market ratios of the quintile portfolios sorted by idiosyncratic volatility also display distinct patterns. Stocks with low (high) idiosyncratic volatility are generally large (small) stocks with low (high) book-to-market ratios. The risk adjustment of the FF-3 model predicts that quintile 5 stocks should have high, not low, average returns.

The findings in Table VI are provocative, but there are several concerns raised by the anomalously low returns of quintile 5. For example, although quintile 5 contains 20% of the stocks sorted by idiosyncratic volatility, quintile 5 is only a small proportion of the value of the market (only 1.9% on average). Are these patterns repeated if we only consider large stocks, or only stocks traded on the NYSE? The next section examines these questions. We also examine whether the phenomena persist if we control for a large number of cross-sectional effects that the literature has identified either as potential risk factors or anomalies. In particular, we control for size, book-to-market, leverage, liquidity, volume,

[12] If we compute idiosyncratic volatility relative to the CAPM, we obtain almost identical results to Panel B of Table VI. Each quintile portfolio of idiosyncratic volatility relative to the CAPM has a correlation of above 99% with its corresponding quintile counterpart when idiosyncratic volatility is computed relative to the FF-3 model.

**Table VI**

## Portfolios Sorted by Volatility

We form value-weighted quintile portfolios every month by sorting stocks based on total volatility and idiosyncratic volatility relative to the Fama–French (1993) model. Portfolios are formed every month, based on volatility computed using daily data over the previous month. Portfolio 1 (5) is the portfolio of stocks with the lowest (highest) volatilities. The statistics in the columns labeled Mean and Std. Dev. are measured in monthly percentage terms and apply to total, not excess, simple returns. Size reports the average log market capitalization for firms within the portfolio and B/M reports the average book-to-market ratio. The row "5-1" refers to the difference in monthly returns between portfolio 5 and portfolio 1. The Alpha columns report Jensen's alpha with respect to the CAPM or Fama–French (1993) three-factor model. Robust Newey–West (1987) $t$-statistics are reported in square brackets. Robust joint tests for the alphas equal to zero are all less than 1% for all cases. The sample period is July 1963 to December 2000.

| Rank | Mean | Std. Dev. | % Mkt Share | Size | B/M | CAPM Alpha | FF-3 Alpha |
|---|---|---|---|---|---|---|---|
| | | | Panel A: Portfolios Sorted by Total Volatility | | | | |
| 1 | 1.06 | 3.71 | 41.7% | 4.66 | 0.88 | 0.14 | 0.03 |
| | | | | | | [1.84] | [0.53] |
| 2 | 1.15 | 4.48 | 33.7% | 4.70 | 0.81 | 0.13 | 0.08 |
| | | | | | | [2.14] | [1.41] |
| 3 | 1.22 | 5.63 | 15.5% | 4.10 | 0.82 | 0.07 | 0.12 |
| | | | | | | [0.72] | [1.55] |
| 4 | 0.99 | 7.15 | 6.7% | 3.47 | 0.86 | −0.28 | −0.17 |
| | | | | | | [−1.73] | [−1.42] |
| 5 | 0.09 | 8.30 | 2.4% | 2.57 | 1.08 | −1.21 | −1.16 |
| | | | | | | [−5.07] | [−6.85] |
| 5-1 | −0.97 | | | | | −1.35 | −1.19 |
| | [−2.86] | | | | | [−4.62] | [−5.92] |
| | | | Panel B: Portfolios Sorted by Idiosyncratic Volatility Relative to FF-3 | | | | |
| 1 | 1.04 | 3.83 | 53.5% | 4.86 | 0.85 | 0.11 | 0.04 |
| | | | | | | [1.57] | [0.99] |
| 2 | 1.16 | 4.74 | 27.4% | 4.72 | 0.80 | 0.11 | 0.09 |
| | | | | | | [1.98] | [1.51] |
| 3 | 1.20 | 5.85 | 11.9% | 4.07 | 0.82 | 0.04 | 0.08 |
| | | | | | | [0.37] | [1.04] |
| 4 | 0.87 | 7.13 | 5.2% | 3.42 | 0.87 | −0.38 | −0.32 |
| | | | | | | [−2.32] | [−3.15] |
| 5 | −0.02 | 8.16 | 1.9% | 2.52 | 1.10 | −1.27 | −1.27 |
| | | | | | | [−5.09] | [−7.68] |
| 5-1 | −1.06 | | | | | −1.38 | −1.31 |
| | [−3.10] | | | | | [−4.56] | [−7.00] |

turnover, bid–ask spreads, coskewness, dispersion in analysts' forecasts, and momentum effects.

### *C. Controlling for Various Cross-Sectional Effects*

Table VII examines the robustness of our results with the 1/0/1 idiosyncratic volatility portfolio formation strategy to various cross-sectional risk factors. The

**Table VII**

## Alphas of Portfolios Sorted on Idiosyncratic Volatility

The table reports Fama and French (1993) alphas, with robust Newey–West (1987) $t$-statistics in square brackets. All the strategies are 1/0/1 strategies described in Section II.A for idiosyncratic volatility computed relative to FF-3, but control for various effects. The column "5-1" refers to the difference in FF-3 alphas between portfolio 5 and portfolio 1. In the panel labeled "NYSE Stocks Only," we sort stocks into quintile portfolios based on their idiosyncratic volatility, relative to the FF-3 model, using only NYSE stocks. We use daily data over the previous month and rebalance monthly. In the panel labeled "Size Quintiles," each month we first sort stocks into five quintiles on the basis of size. Then, within each size quintile, we sort stocks into five portfolios sorted by idiosyncratic volatility. In the panels controlling for size, liquidity volume, and momentum, we perform a double sort. Each month, we first sort stocks based on the first characteristic (size, book-to-market, leverage, liquidity, volume, turnover, bid–ask spreads, or dispersion of analysts' forecasts) and then, within each quintile we sort stocks based on idiosyncratic volatility relative to the FF-3 model. The five idiosyncratic volatility portfolios are then averaged over each of the five characteristic portfolios. Hence, they represent idiosyncratic volatility quintile portfolios controlling for the characteristic. Liquidity represents the Pástor and Stambaugh (2003) historical liquidity beta, leverage is defined as the ratio of total book value of assets to book value of equity, volume represents average dollar volume over the previous month, turnover represents volume divided by the total number of shares outstanding over the past month, and the bid–ask spread control represents the average daily bid–ask spread over the previous month. The coskewness measure is computed following Harvey and Siddique (2000) and the dispersion of analysts' forecasts is computed by Diether et al. (2002). The sample period is July 1963 to December 2000 for all controls with the exceptions of liquidity (February 1968 to December 2000), the dispersion of analysts' forecasts (February 1983 to December 2000), and the control for aggregate volatility risk (January 1986 to December 2000). All portfolios are value weighted.

| | | Ranking on Idiosyncratic Volatility | | | | | |
|---|---|---|---|---|---|---|---|
| | | 1 Low | 2 | 3 | 4 | 5 High | 5-1 |
| NYSE Stocks Only | | 0.06 | 0.04 | 0.02 | −0.04 | −0.60 | −0.66 |
| | | [1.20] | [0.75] | [0.30] | [−0.40] | [−5.14] | [−4.85] |
| Size Quintiles | Small 1 | 0.11 | 0.26 | 0.31 | 0.06 | −0.43 | −0.55 |
| | | [0.72] | [1.56] | [1.76] | [0.29] | [−1.54] | [−1.84] |
| | 2 | 0.19 | 0.20 | −0.07 | −0.65 | −1.73 | −1.91 |
| | | [1.49] | [1.74] | [−0.67] | [−5.19] | [−8.14] | [−7.69] |
| | 3 | 0.12 | 0.21 | 0.03 | −0.27 | −1.49 | −1.61 |
| | | [1.23] | [2.40] | [0.38] | [−3.36] | [−10.1] | [−7.65] |
| | 4 | 0.03 | 0.22 | 0.17 | −0.03 | −0.82 | −0.86 |
| | | [0.37] | [2.57] | [2.47] | [−0.45] | [−6.61] | [−4.63] |
| | Large 5 | 0.09 | 0.04 | 0.03 | 0.14 | −0.17 | −0.26 |
| | | [1.62] | [0.72] | [0.51] | [1.84] | [−1.40] | [−1.74] |
| Controlling for Size | | 0.11 | 0.18 | 0.09 | −0.15 | −0.93 | −1.04 |
| | | [1.30] | [2.49] | [1.35] | [−1.99] | [−6.81] | [−5.69] |
| Controlling for Book-to-Market | | 0.61 | 0.69 | 0.71 | 0.50 | −0.19 | −0.80 |
| | | [3.02] | [2.80] | [2.49] | [1.47] | [−0.48] | [−2.90] |
| Controlling for Leverage | | 0.11 | 0.11 | 0.08 | −0.24 | −1.12 | −1.23 |
| | | [2.48] | [2.20] | [1.19] | [−2.45] | [−7.81] | [−7.61] |
| Controlling for Liquidity | | 0.08 | 0.09 | −0.01 | −0.16 | −1.01 | −1.08 |
| | | [1.71] | [1.53] | [−0.09] | [−1.62] | [−8.61] | [−7.98] |
| Controlling for Volume | | −0.03 | 0.02 | −0.01 | −0.39 | −1.25 | −1.22 |
| | | [−0.49] | [0.39] | [−0.32] | [−7.11] | [−10.9] | [−8.04] |
| Controlling for Turnover | | 0.11 | 0.03 | −0.11 | −0.49 | −1.34 | −1.46 |
| | | [2.49] | [0.58] | [−1.79] | [−6.27] | [−11.0] | [−10.7] |
| Controlling for Bid–Ask Spreads | | −0.07 | −0.01 | −0.09 | −0.49 | −1.26 | −1.19 |
| | | [−1.21] | [−0.18] | [−1.14] | [−5.36] | [−9.13] | [−6.95] |
| Controlling for Coskewness | | −0.02 | −0.00 | 0.01 | −0.37 | −1.40 | −1.38 |
| | | [−0.32] | [−0.02] | [0.08] | [−2.30] | [−6.07] | [−5.02] |
| Controlling for Dispersion | | 0.12 | −0.07 | 0.11 | 0.01 | −0.27 | −0.39 |
| in Analysts' Forecasts | | [1.57] | [−0.76] | [1.12] | [0.09] | [−1.76] | [−2.09] |

table reports FF-3 alphas, the difference in FF-3 alphas between the quintile portfolios with the highest and lowest idiosyncratic volatilities, together with $t$-statistics to test their statistical significance.[13] All the portfolios formed on idiosyncratic volatility remain value weighted.

### *C.1. Using Only NYSE Stocks*

We examine the interaction of the idiosyncratic volatility effect with firm size in two ways. First, we rank stocks based on idiosyncratic volatility using only NYSE stocks. Excluding NASDAQ and AMEX has little effect on our results. The highest quintile of idiosyncratic volatility stocks has an FF-3 alpha of $-0.60\%$ per month. The 5-1 difference in FF-3 alphas is still large in magnitude, at $-0.66\%$ per month, with a $t$-statistic of $-4.85$. While restricting the universe of stocks to only the NYSE mitigates the concern that the idiosyncratic volatility effect is concentrated among small stocks, it does not completely remove this concern because the NYSE universe still contains small stocks.

### *C.2. Controlling for Size*

Our second examination of the interaction of idiosyncratic volatility and size uses all firms. We control for size by first forming quintile portfolios ranked on market capitalization. Then, within each size quintile, we sort stocks into quintile portfolios ranked on idiosyncratic volatility. Thus, within each size quintile, quintile 5 contains the stocks with the highest idiosyncratic volatility.

The second panel of Table VII shows that in each size quintile, the highest idiosyncratic volatility quintile has a dramatically lower FF-3 alpha than the other quintiles. The effect is not most pronounced among the smallest stocks. Rather, quintiles 2-4 have the largest 5-1 differences in FF-3 alphas, at $-1.91\%$, $-1.61\%$, and $-0.86\%$ per month, respectively. The average market capitalization of quintiles 2-4 is, on average, 21% of the market. The $t$-statistics of these alphas are all above 4.5 in absolute magnitude. In contrast, the 5-1 alphas for the smallest and largest quintiles are actually statistically insignificant at the 5% level. Hence, it is not small stocks that are driving these results.

The row labeled "Controlling for Size" averages across the five size quintiles to produce quintile portfolios with dispersion in idiosyncratic volatility, but which contain all sizes of firms. After controlling for size, the 5-1 difference in FF-3 alphas is still $-1.04\%$ per month. Thus, market capitalization does not explain the low returns to high idiosyncratic volatility stocks.

In the remainder of Table VII, we repeat the explicit double-sort characteristic controls, replacing size with other stock characteristics. We first form portfolios based on a particular characteristic, then we sort on idiosyncratic volatility, and finally we average across the characteristic portfolios to create portfolios

[13] We emphasize that the difference in mean raw returns between quintile 5 and 1 portfolios is very similar to the difference in the FF-3 alphas, but we focus on FF-3 alphas as they control for the standard set of systematic factors.

that have dispersion in idiosyncratic volatility but contain all aspects of the characteristic.

### *C.3. Controlling for Book-to-Market Ratios*

It is generally thought that high book-to-market firms have high average returns. Thus, in order for the book-to-market effect to be an explanation of the idiosyncratic volatility effect, the high idiosyncratic volatility portfolios must be primarily composed of growth stocks that have lower average returns than value stocks. The row labeled "Controlling for Book-to-Market" shows that this is not the case. When we control for book-to-market ratios, stocks with the lowest idiosyncratic volatility have high FF-3 alphas, and the 5-1 difference in FF-3 alphas is −0.80% per month, with a $t$-statistic of −2.90.

### *C.4. Controlling for Leverage*

Leverage increases expected equity returns, holding asset volatility and asset expected returns constant. Asset volatility also prevents firms from increasing leverage. Hence, firms with high idiosyncratic volatility could have high asset volatility but relatively low equity returns because of low leverage. The next line of Table VII shows that leverage cannot be an explanation of the idiosyncratic volatility effect. We measure leverage as the ratio of total book value of assets to book value of equity. After controlling for leverage, the difference between the 5-1 alphas is −1.23% per month, with a $t$-statistic of −7.61.

### *C.5. Controlling for Liquidity Risk*

Pástor and Stambaugh (2003) argue that liquidity is a systematic risk. If liquidity is to explain the idiosyncratic volatility effect, high idiosyncratic volatility stocks must have low liquidity betas, giving them low returns. We check this explanation by using the historical Pástor–Stambaugh liquidity betas to measure exposure to liquidity risk. Controlling for liquidity does not remove the low average returns of high idiosyncratic volatility stocks. The 5-1 difference in FF-3 alphas remains large at −1.08% per month, with a $t$-statistic of −7.98.

### *C.6. Controlling for Volume*

Gervais et al. (2001) find that stocks with higher volume have higher returns. Perhaps stocks with high idiosyncratic volatility are merely stocks with low trading volume? When we control for trading volume over the past month, the 5-1 difference in alphas is −1.22% per month, with a $t$-statistic of −8.04. Hence, the low returns on high idiosyncratic volatility stocks are robust to controlling for volume effects.

### C.7. Controlling for Turnover

Our next control is turnover, measured as trading volume divided by the total number of shares outstanding over the previous month. Turnover is a noisy proxy for liquidity. Table VII shows that the low alphas on high idiosyncratic volatility stocks are robust to controlling for turnover. The 5-1 difference in FF-3 alphas is $-1.19\%$ per month, and it is highly significant with a $t$-statistic of $-8.04$. Examination of the individual turnover quintiles (not reported) indicates that the 5-1 differences in alphas are most pronounced in the quintile portfolio with the highest, not the lowest, turnover.

### C.8. Controlling for Bid-Ask Spreads

An alternative liquidity control is the bid–ask spread, which we measure as the average daily bid–ask spread over the previous month for each stock. In order for bid–ask spreads to be an explanation, high idiosyncratic volatility stocks must have low bid–ask spreads and corresponding low returns. Controlling for bid–ask spreads does little to remove the effect. The FF-3 alpha of the highest idiosyncratic volatility portfolio is $-1.26\%$, while the 5-1 difference in alphas is $-1.19\%$ and remains highly statistically significant with a $t$-statistic of $-6.95$.

### C.9. Controlling for Coskewness Risk

Harvey and Siddique (2000) find that stocks with more negative coskewness have higher returns. Stocks with high idiosyncratic volatility may have positive coskewness, giving them low returns. Computing coskewness following Harvey and Siddique (2000), we find that exposure to coskewness risk is not an explanation. The FF-3 alpha for the 5-1 portfolio is $-1.38\%$ per month, with a $t$-statistic of $-5.02$.

### C.10. Controlling for Dispersion in Analysts' Forecasts

Diether, Malloy, and Scherbina (2002) provide evidence that stocks with higher dispersion in analysts' earnings forecasts have lower average returns than stocks with low dispersion of analysts' forecasts. They argue that dispersion in analysts' forecasts measures differences of opinion among investors. Miller (1977) shows that if there are large differences in stock valuations and short sale constraints, equity prices tend to reflect the view of the more optimistic agents, which leads to low future returns for stocks with large dispersion in analysts' forecasts.

If stocks with high dispersion in analysts' forecasts tend to be more volatile stocks, then we may be finding a similar anomaly to Diether et al. (2002). Over Diether et al.'s sample period, 1983–2000, we test this hypothesis by performing a characteristic control for the dispersion of analysts' forecasts. We take the quintile portfolios of stocks sorted on increasing dispersion of analysts' forecasts (Table VI of Diether et al. (2002, p. 2128)) and within each quintile sort stocks on idiosyncratic volatility. Note that this universe of stocks contains mostly large firms, where the idiosyncratic volatility effect is weaker, because multiple analysts usually do not make forecasts for small firms.

The last two lines of Table VII present the results for averaging the idiosyncratic volatility portfolios across the forecast dispersion quintiles. The 5-1 difference in alphas is still $-0.39\%$ per month, with a robust $t$-statistic of $-2.09$. While the shorter sample period may reduce power, the dispersion of analysts' forecasts reduces the noncontrolled 5-1 alpha considerably. However, dispersion in analysts' forecasts cannot account for all of the low returns to stocks with high idiosyncratic volatility.[14]

### D. A Detailed Look at Momentum

Hong, Lim, and Stein (2000) argue that the momentum effect documented by Jegadeesh and Titman (1993) is asymmetric and has a stronger negative effect on declining stocks than a positive effect on rising stocks. A potential explanation behind the idiosyncratic volatility results is that stocks with very low returns have very high volatility. Of course, stocks that are past winners also have very high volatility, but loser stocks could be overrepresented in the high idiosyncratic volatility quintile.

In Table VIII, we perform a series of robustness tests of the idiosyncratic volatility effect to this possible momentum explanation. In Panel A, we perform $5 \times 5$ characteristic sorts first over past returns, and then over idiosyncratic volatility. We average over the momentum quintiles to produce quintile portfolios sorted by idiosyncratic risk that control for past returns. We control for momentum over the previous 1 month, 6 months, and 12 months. Table VIII shows that momentum is not driving the results. Controlling for returns over the past month does not remove the very low FF-3 alpha of quintile 5 ($-0.59\%$ per month), and the 5-1 difference in alphas is still $-0.66\%$ per month, which is statistically significant at the 1% level. When we control for past 6-month returns, the FF-3 alpha of the 5-1 portfolio increases in magnitude to $-1.10\%$ per month. For past 12-month returns, the 5-1 alpha is even larger in magnitude at $-1.22\%$ per month. All these differences are highly statistically significant.

In Panel B, we closely examine the individual $5 \times 5$ FF-3 alphas of the quintile portfolios sorted on past 12-month returns and idiosyncratic volatility. Note that if we average these portfolios across the past 12-month quintile portfolios, and then compute alphas, we obtain the alphas in the row labeled "Past 12-months" in Panel A of Table VIII. This more detailed view of the

[14] We can also reverse the question and ask if the low average returns of stocks with high dispersion of analysts' forecasts are due to the low returns on stocks with high idiosyncratic volatility by first sorting stocks on idiosyncratic volatility and then by forecast dispersion. Controlling for idiosyncratic volatility, the FF-3 alpha for the quintile portfolio, that is long stocks with the highest forecast dispersion and short stocks in the quintile portfolio with the lowest forecast dispersion, is $-0.36\%$ per month, which is insignificant at the 5% level (the robust $t$-statistic is $-1.47$).

**Table VIII**

**Alphas of Portfolios Sorted on Idiosyncratic Volatility Controlling for Past Returns**

The table reports Fama and French (1993) alphas, with robust Newey–West (1987) $t$-statistics in square brackets. All the strategies are 1/0/1 strategies described in Section II.A, but control for past returns. The column "5-1" refers to the difference in FF-3 alphas between portfolio 5 and portfolio 1. In the first three rows labeled "Past 1-month" to "Past 12-months," we control for the effect of momentum. We first sort all stocks on the basis of past returns, over the appropriate formation period, into quintiles. Then, within each momentum quintile, we sort stocks into five portfolios sorted by idiosyncratic volatility, relative to the FF-3 model. The five idiosyncratic volatility portfolios are then averaged over each of the five characteristic portfolios. Hence, they represent idiosyncratic volatility quintile portfolios controlling for momentum. The second part of the panel lists the FF-3 alphas of idiosyncratic volatility quintile portfolios within each of the past 12-month return quintiles. All portfolios are value weighted. The sample period is July 1963 to December 2000.

| | Ranking on Idiosyncratic Volatility | | | | | |
|---|---|---|---|---|---|---|
| | 1 Low | 2 | 3 | 4 | 5 High | 5-1 |
| | | | Panel A: Controlling for Momentum | | | |
| Past 1 month | 0.07 | 0.08 | 0.09 | −0.05 | −0.59 | −0.66 |
| | [0.43] | [0.94] | [1.26] | [−0.47] | [−3.60] | [−2.71] |
| Past 6 months | −0.01 | −0.12 | −0.28 | −0.45 | −1.11 | −1.10 |
| | [−0.20] | [−1.86] | [−3.60] | [−5.20] | [−9.35] | [−7.18] |
| Past 12 months | 0.01 | −0.05 | −0.28 | −0.64 | −1.21 | −1.22 |
| | [0.15] | [−0.76] | [−3.56] | [−6.95] | [−11.5] | [−9.20] |
| | | | Panel B: Past 12-Month Quintiles | | | |
| Losers 1 | −0.41 | −0.83 | −1.44 | −2.11 | −2.66 | −2.25 |
| | [−1.94] | [−3.90] | [−6.32] | [−9.40] | [−10.6] | [−7.95] |
| 2 | −0.08 | −0.24 | −0.64 | −1.09 | −1.70 | −1.62 |
| | [−0.49] | [−1.58] | [−4.40] | [−6.46] | [−8.90] | [−7.00] |
| 3 | −0.06 | −0.11 | −0.26 | −0.48 | −1.03 | −0.97 |
| | [−0.52] | [−1.16] | [−2.15] | [−3.49] | [−7.93] | [−5.85] |
| 4 | 0.15 | 0.07 | 0.23 | −0.03 | −0.65 | −0.80 |
| | [1.57] | [0.65] | [2.27] | [−0.29] | [−4.76] | [−4.89] |
| Winners 5 | 0.45 | 0.85 | 0.71 | 0.52 | −0.03 | −0.48 |
| | [3.52] | [5.44] | [3.97] | [2.63] | [−0.13] | [−2.01] |

interaction between momentum and idiosyncratic volatility reveals several interesting facts.

First, the low returns to high idiosyncratic volatility are most pronounced for loser stocks. The 5-1 differences in alphas range from −2.25% per month for the loser stocks, to −0.48% per month for the winner stocks. Second, the tendency for the momentum effect to be concentrated more among loser, rather than winner, stocks cannot account for all of the low returns to high idiosyncratic volatility stocks. The idiosyncratic volatility effect appears significantly in every past return quintile. Hence, stocks with high idiosyncratic volatility earn low average returns, no matter whether these stocks are losers or winners.

Finally, the momentum effect itself is also asymmetric across the idiosyncratic volatility quintiles. In the first two idiosyncratic volatility quintiles, the alphas of losers (winners) are roughly symmetrical. For example, for stocks with the lowest idiosyncratic volatilities, the loser (winner) alpha is −0.41% (0.45%). In the second idiosyncratic volatility quintile, the loser (winner) alpha is −0.83% (0.85%). However, as idiosyncratic volatility becomes very high, the momentum effect becomes highly skewed towards extremely low returns on stocks with high idiosyncratic volatility. Hence, one way to improve the returns to a momentum strategy is to short past losers with high idiosyncratic volatility.

### *E. Is It Exposure to Aggregate Volatility Risk?*

A possible explanation for the large negative returns of high idiosyncratic volatility stocks is that stocks with large idiosyncratic volatilities have large exposure to movements in aggregate volatility. We examine this possibility in Table IX. The first row of Panel A reports the results of quintile sorts on idiosyncratic volatility, controlling for $\beta_{\Delta VIX}$. This is done by first sorting on $\beta_{\Delta VIX}$ and then on idiosyncratic volatility, and then averaging across the $\beta_{\Delta VIX}$ quintiles. We motivate using past $\beta_{\Delta VIX}$ as a control for aggregate volatility risk because we have shown that stocks with past high $\beta_{\Delta VIX}$ loadings have high future exposure to the *FVIX*-mimicking volatility factor.

Panel A of Table IX shows that after controlling for aggregate volatility exposure, the 5-1 alpha is −1.19% per month, almost unchanged from the 5-1 quintile idiosyncratic volatility FF-3 alpha of −1.31% in Table VI with no control for systematic volatility exposure. Hence, it seems that $\beta_{\Delta VIX}$ accounts for very little of the low average returns of high idiosyncratic volatility stocks. Panel B of Table IX reports ex post *FVIX* factor loadings of the $5 \times 5\beta_{\Delta VIX}$ and idiosyncratic volatility portfolios, where we compute the post-formation *FVIX* factor loadings using equation (6). We cannot interpret the alphas from this regression, because *FVIX* is not a tradeable factor, but the *FVIX* factor loadings give us a picture of how exposure to aggregate volatility risk may account for the spreads in average returns on the idiosyncratic volatility sorted portfolios.

Panel B shows that in the first three $\beta_{\Delta VIX}$ quintiles, we obtain almost monotonically increasing *FVIX* factor loadings that start with large negative ex post $\beta_{FVIX}$ loadings for low idiosyncratic volatility portfolios and end with large positive ex post $\beta_{FVIX}$ loadings. However, for the two highest past $\beta_{\Delta VIX}$ quintiles, the *FVIX* factor loadings have absolutely no explanatory power. In summary, exposure to aggregate volatility partially explains the puzzling low returns to high idiosyncratic volatility stocks, but only for stocks with very negative and low past loadings to aggregate volatility innovations.

### *F. Robustness to Different Formation and Holding Periods*

If risk cannot explain the low returns to high idiosyncratic volatility stocks, are there other explanations? To help disentangle various stories, Table X reports FF-3 alphas of other $L/M/N$ strategies described in Section II.A. First, we examine possible contemporaneous measurement errors in the 1/0/1

**Table IX**

**The Idiosyncratic Volatility Effect Controlling for Aggregate Volatility Risk**

We control for exposure to aggregate volatility using the $\beta_{\Delta VIX}$ loading at the beginning of the month, computed using daily data over the previous month following equation (3). We first sort all stocks on the basis of $\beta_{\Delta VIX}$ into quintiles. Then, within each $\beta_{\Delta VIX}$ quintile, we sort stocks into five portfolios sorted by idiosyncratic volatility, relative to the FF-3 model. In Panel A, we report FF-3 alphas of these portfolios. We average the five idiosyncratic volatility portfolios over each of the five $\beta_{\Delta VIX}$ portfolios. Hence, these portfolios represent idiosyncratic volatility quintile portfolios controlling for exposure to aggregate volatility risk. The column "5-1" refers to the difference in FF-3 alphas between portfolio 5 and portfolio 1. In Panel B, we report ex post *FVIX* factor loadings from a regression of each of the $25\beta_{\Delta VIX} \times$ idiosyncratic volatility portfolios onto the Fama–French (1993) model augmented with *FVIX* as in equation (6). Robust Newey–West (1987) $t$-statistics are reported in square brackets. All portfolios are value weighted. The sample period is from January 1986 to December 2000.

Panel A: FF-3 Alphas

| | Ranking on Idiosyncratic Volatility | | | | | |
|---|---|---|---|---|---|---|
| | 1 Low | 2 | 3 | 4 | 5 High | 5-1 |
| Controlling for Exposure to Aggregate Volatility Risk | 0.05 | 0.01 | −0.14 | −0.49 | −1.14 | −1.19 |
| | [0.83] | [0.09] | [−1.14] | [−3.08] | [−5.00] | [−4.72] |

Panel B: *FVIX* Factor Loadings

| | | Ranking on Idiosyncratic Volatility | | | | |
|---|---|---|---|---|---|---|
| | | 1 Low | 2 | 3 | 4 | 5 High |
| $\beta_{\Delta VIX}$ Quintiles | Low 1 | −6.40 | −1.98 | −0.55 | 8.80 | 7.51 |
| | | [−3.82] | [−0.78] | [−0.23] | [2.16] | [2.31] |
| | 2 | −2.66 | −3.21 | 0.06 | −3.04 | 5.37 |
| | | [−2.27] | [−2.06] | [0.05] | [−2.00] | [1.80] |
| | 3 | −6.51 | −2.74 | −1.93 | −0.31 | 7.25 |
| | | [−4.50] | [−2.41] | [−1.14] | [−0.29] | [3.37] |
| | 4 | 5.65 | 3.73 | 3.50 | 1.33 | 8.22 |
| | | [2.31] | [2.08] | [2.83] | [0.87] | [3.97] |
| | High 5 | 7.53 | 2.46 | 8.60 | 7.53 | 5.79 |
| | | [5.16] | [1.16] | [3.72] | [2.53] | [1.65] |

strategy by setting $M = 1$. Allowing for a 1-month lag between the measurement of volatility and the formation of the portfolio ensures that the portfolios are formed only with information definitely available at time $t$. The top row of Table X shows that the 5-1 FF-3 alpha on the 1/1/1 strategy is −0.82% per month, with a $t$-statistic of −4.63.

A possible behavioral explanation for our results is that higher idiosyncratic volatility does earn higher returns over longer horizons than 1 month, but short-term overreaction forces returns to be low in the first month. If we hold high idiosyncratic volatility stocks for a long horizon ($N = 12$ months), we might see a positive relation between past idiosyncratic volatility and future average returns. The second row of Table X shows that this is not the case. For the 1/1/12

**Table X**

**Quintile Portfolios of Idiosyncratic Volatility for *L/M/N* Strategies**

The table reports Fama and French (1993) alphas, with robust Newey–West (1987) $t$-statistics in square brackets. The column "5-1" refers to the difference in FF-3 alphas between portfolio 5 and portfolio 1. We rank stocks into quintile portfolios of idiosyncratic volatility, relative to FF-3, using *L/M/N* strategies described in Section II.A. At month $t$, we compute idiosyncratic volatilities from the regression (8) on daily data over an $L$ month period from months $t-L-M$ to month $t-M$. At time $t$, we construct value-weighted portfolios based on these idiosyncratic volatilities and hold these portfolios for $N$ months, following Jegadeesh and Titman (1993), except our portfolios are value weighted. The sample period is July 1963 to December 2000.

| | Ranking on Idiosyncratic Volatility | | | | | |
|---|---|---|---|---|---|---|
| Strategy | 1 low | 2 | 3 | 4 | 5 High | 5-1 |
| 1/1/1 | 0.06 | 0.04 | 0.09 | −0.18 | −0.82 | −0.88 |
| | [1.47] | [0.77] | [1.15] | [−1.78] | [−4.88] | [−4.63] |
| 1/1/12 | 0.03 | 0.02 | −0.02 | −0.17 | −0.64 | −0.67 |
| | [0.91] | [0.43] | [−0.37] | [−1.79] | [−5.27] | [−4.71] |
| 12/1/1 | 0.04 | 0.08 | −0.01 | −0.29 | −1.08 | −1.12 |
| | [1.15] | [1.32] | [−0.08] | [−2.02] | [−5.36] | [−5.13] |
| 12/1/12 | 0.04 | 0.04 | −0.02 | −0.35 | −0.73 | −0.77 |
| | [1.10] | [0.54] | [−0.23] | [−2.80] | [−4.71] | [−4.34] |

strategy, we still see very low FF-3 alphas for quintile 5, and the 5-1 difference in alphas is still −0.67% per month, which is highly significant.

By restricting the formation period to $L = 1$ month, our previous results may just be capturing various short-term events that affect idiosyncratic volatility. For example, the portfolio of stocks with high idiosyncratic volatility may be largely composed of stocks that have just made, or are just about to make, earnings announcements. To ensure that we are not capturing specific short-term corporate events, we extend our formation period to $L = 12$ months. The third row of Table X reports FF-3 alphas for a 12/1/1 strategy. Using one entire year of daily data to compute idiosyncratic volatility does not remove the anomalous high idiosyncratic volatility-low average return pattern: The 5-1 difference in alphas is −1.12% per month. Similarly, the patterns are robust for the 12/1/12 strategy, which has a 5-1 alpha of −0.77% per month.

### *G. Subsample Analysis*

Table XI investigates the robustness of the low returns to stocks with high idiosyncratic volatility over different subsamples. First, the effect is observed in every decade from the 1960s to the 1990s. The largest difference in alphas between portfolio 5 and portfolio 1 occurs during the 1980s, with an FF-3 alpha of −2.23% per month, and we observe the smallest magnitude of the FF-3 alpha of the 5-1 portfolio during the 1970s, during which time the FF-3 alpha is −0.77% per month. In every decade, the effect is highly statistically significant.

A possible explanation for the idiosyncratic volatility effect may be asymmetry of return distributions across business cycles. Volatility is asymmetric

**Table XI**

**The Idiosyncratic Volatility Effect over Different Subsamples**

The table reports Fama and French (1993) alphas, with robust Newey–West (1987) $t$-statistics in square brackets. The column "5-1" refers to the difference in FF-3 alphas between portfolio 5 and portfolio 1. We rank stocks into quintile portfolios of idiosyncratic volatility, relative to FF-3, using the 1/0/1 strategy described in Section II.A and examine robustness over different sample periods. The stable and volatile periods refer to the months with the lowest and highest 20% absolute value of the market return, respectively. The full sample period is July 1963 to December 2000.

| | Ranking on Idiosyncratic Volatility | | | | | |
|---|---|---|---|---|---|---|
| Subperiod | 1 Low | 2 | 3 | 4 | 5 High | 5-1 |
| Jul 1963–Dec 1970 | 0.06 | 0.03 | 0.09 | −0.36 | −0.94 | −1.00 |
| | [1.23] | [0.42] | [0.73] | [−2.18] | [−5.81] | [−5.62] |
| Jan 1971–Dec 1980 | −0.24 | 0.32 | 0.19 | 0.03 | −1.02 | −0.77 |
| | [−2.53] | [3.20] | [1.55] | [0.21] | [−5.80] | [−3.14] |
| Jan 1981–Dec 1990 | 0.15 | 0.08 | −0.16 | −0.66 | −2.08 | −2.23 |
| | [2.14] | [1.07] | [−1.25] | [−4.82] | [−10.1] | [−9.39] |
| Jan 1991–Dec 2000 | 0.16 | −0.01 | 0.14 | −0.48 | −1.39 | −1.55 |
| | [1.34] | [−0.08] | [0.77] | [−2.41] | [−3.31] | [−3.19] |
| NBER Expansions | 0.06 | 0.02 | 0.08 | −0.33 | −1.19 | −1.25 |
| | [1.26] | [0.25] | [1.01] | [−3.18] | [−7.07] | [−6.55] |
| NBER Recessions | −0.10 | 0.64 | −0.01 | −0.34 | −1.88 | −1.79 |
| | [−0.65] | [3.58] | [−0.04] | [−1.32] | [−3.32] | [−2.63] |
| Stable Periods | 0.05 | −0.02 | −0.11 | −0.62 | −1.66 | −1.71 |
| | [0.44] | [−0.25] | [−1.07] | [−4.06] | [−6.56] | [−4.75] |
| Volatile Periods | −0.04 | 0.24 | 0.32 | 0.18 | −0.93 | −0.89 |
| | [−0.29] | [1.69] | [2.32] | [0.55] | [−2.40] | [−2.02] |

(and larger with downward moves), so stocks with high idiosyncratic volatility may have normal average returns during expansionary markets, and their low returns may mainly occur during bear market periods, or recessions. We may have observed too many recessions in the sample relative to what agents expected ex ante. We check this hypothesis by examining the returns of high idiosyncratic volatility stocks conditioning on NBER expansions and recessions. During NBER expansions (recessions), the FF-3 alpha of the 5-1 portfolio is −1.25% (−1.79%). Both the expansion and recession differences in FF-3 alphas are significant at the 1% level. There are more negative returns to high idiosyncratic volatility stocks during recessions, but the fact that the $t$-statistic in NBER expansions is −6.55 shows that the low returns from high idiosyncratic volatility also thrive during expansions.

A final possibility is that the idiosyncratic volatility effect is concentrated during the most volatile periods in the market. To test for this possibility, we compute FF-3 alphas of the difference between quintile portfolios 5 and 1 conditioning on periods with the lowest or highest 20% of absolute moves of the market return. These are ex post periods of low or high market volatility. During stable (volatile) periods, the difference in the FF-3 alphas of the fifth and first quintile portfolios is −1.71% (−0.89%) per month. Both the differences in alphas during stable and volatile periods are significant at the 5% level. The most negative returns of the high idiosyncratic volatility strategy are earned during periods when the market is stable. Hence, the idiosyncratic volatility effect is remarkably robust across different subsamples.

## III. Conclusion

Multifactor models of risk predict that aggregate volatility should be a cross-sectional risk factor. Past research in option pricing has found a negative price of risk for systematic volatility. Consistent with this intuition, we find that stocks with high past exposure to innovations in aggregate market volatility earn low future average returns. We use changes in the *VIX* index constructed by the Chicago Board Options Exchange to proxy for innovations in aggregate volatility.

To find the component of market volatility innovations that is reflected in equity returns, we construct a factor to mimic innovations in market volatility following Breeden et al. (1989) and Lamont (2001). We first form portfolios on the basis of their past sensitivity to first differences in the *VIX* index. Then, we project innovations in *VIX* onto these portfolios to produce a factor that mimics aggregate volatility risk, which we term *FVIX*. This portfolio of basis assets is maximally correlated with the realized aggregate volatility innovations. Portfolios constructed by ranking on past betas to first differences in *VIX* also exhibit strong patterns in post-formation *FVIX* factor loadings. In particular, the ex post increasing patterns in *FVIX* factor loadings correspond to decreasing Fama–French (1993) alphas over the same period that the alphas are computed.

We estimate a cross-sectional price of volatility risk of approximately $-1\%$ per annum, and this estimate is robust to controlling for size, value, momentum, and liquidity effects. Hence, the decreasing average returns to stocks with high past sensitivities to changes in *VIX* is consistent with the cross-section of returns pricing aggregate volatility risk with a negative sign. However, despite the statistical significance of the negative volatility risk premium, its small size and our relatively small sample mean that we cannot rule out a potential Peso problem explanation. Since the *FVIX* portfolio does well during periods of market distress, adding another volatility spike like October 1987 or August 1998 to our sample would change the sign of the price of risk of *FVIX* from negative to positive. Nevertheless, our estimate of a negative price of risk of aggregate volatility is consistent with a multifactor model or Intertemporal CAPM. In these settings, aggregate volatility risk is priced with a negative sign because risk-averse agents reduce current consumption to increase precautionary savings in the presence of higher uncertainty about future market returns. Our results are also consistent with the estimates of a negative price of risk for aggregate volatility estimated by many option pricing studies.

We also examine the returns of a set of test assets that are sorted by idiosyncratic volatility relative to the Fama–French (1993) model. We uncover a very robust result. Stocks with high idiosyncratic volatility have abysmally low average returns. In particular, the quintile portfolio of stocks with the highest idiosyncratic volatility earns total returns of just −0.02% per month in our sample. These low average returns to stocks with high idiosyncratic volatility cannot be explained by exposures to size, book-to-market, leverage, liquidity, volume, turnover, bid-ask spreads, coskewness, or dispersion in analysts' forecasts characteristics. The effect also persists in bull and bear markets, NBER recessions and expansions, volatile and stable periods, and is robust to considering different formation and holding periods as long as 1 year. Although we argue that aggregate volatility is a new cross-sectional, systematic factor, exposure to aggregate volatility risk accounts for very little of the anomalous low returns of stocks with high idiosyncratic volatility. Hence, the cross-sectional expected return patterns found by sorting on idiosyncratic volatility present something of a puzzle.

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