What's the Point of Optimal Transport If We Can't Have Tea, Earl Grey, Hot?

Or: Why the Universe Might Be Playing With Its Food

Jesse Paul

The Inchworm and the Replicator

My advisor and I once spent half an hour debugging a finite element code that was supposed to solve the Monge-Ampère equation. The solver kept crashing, throwing increasingly bizarre error messages, until finally it succeeded—and produced what looked exactly like the transport map for rearranging atoms into a teacup. We stared at it for a while. Then my advisor said, "You know what's funny? We're sitting here with the mathematical blueprint for a Star Trek replicator, and everyone thinks we're just doing abstract mathematics."

But here's what's really funny: Why do we assume that rearranging matter should be hard?

I mean, think about it. Every spring, cherry trees take dirt, water, and air and rearrange them into flowers. No one gave them a PhD in optimal transport theory. Slime molds solve shortest path problems that would make a computer scientist weep. And somewhere right now, in your kitchen, yeast is turning sugar into bread—executing a massively parallel optimization algorithm that would cost millions to simulate.

So the real question isn't "How can we build a replicator?" The real question is: Why does the existence of a machine that rearranges matter for the sheer pleasure of creating strike us as impossible? What does it tell us about ourselves that we instinctively assume the universe prefers things to stay put?

The Universe is Already Playing This Game

Here's something they don't tell you in graduate school: the Monge-Ampère equation—this horrifically nonlinear PDE that makes grown mathematicians cry—might just be the universe having fun.

Think about it. In 1781, Gaspard Monge posed his famous problem: what's the best way to move a pile of dirt to fill a hole? But why frame it as "optimal"? Why not "most playful"? After all, when a child builds sandcastles, she's solving the same transport problem. She's just having more fun with it than Monge did.

The mathematics is undeniable: when you want to transform one distribution of matter into another—whether it's dirt into embankments or atoms into Earl Grey tea—you need to solve for a transport map. This map is governed by the Monge-Ampère equation. In the smooth, classical world, this is already hard enough. But reality isn't smooth. Reality is made of discrete atoms, quantum jumps, discontinuous phase transitions.

This is where the story gets interesting.

When Reality Breaks Your Beautiful Theory (And Why That's Hilarious)

So you have this elegant 18th-century mathematics, and you want to apply it to actual atoms. There's just one problem: atoms are not a continuous fluid. They're quantum mechanical entities, sure, with their probability clouds and wave functions. But when you're tracking where matter actually ends up—during crystallization, phase transitions, self-assembly—you often get these sharp, concentrated peaks. Nucleation sites that act like Dirac deltas. Defect centers that are mathematical singularities. The quantum fuzziness collapses into very specific "here" and "not there" configurations.

Try to take a derivative of a nucleation event and watch your computer have an existential crisis.

The classical mathematicians would throw up their hands here. "It can't be done!" they'd say. "The mathematics doesn't work for discrete particles!" And they'd go back to their smooth manifolds and their differentiable functions, leaving the real world to the engineers.

But here's where it gets subversive. Some mathematical anarchists—let's call them that—decided that if reality didn't fit the mathematics, maybe we needed new mathematics. Enter Alexander Alexandrov, who basically said: "What if instead of thinking about derivatives, we think about this geometrically?"

Alexandrov's solution is genuinely radical. Instead of requiring smoothness, he redefined what we mean by a solution. He created a "weak" framework—weak not as in feeble, but as in flexible, playful, accommodating. It's mathematics that bends rather than breaks. Mathematics that says, "Oh, you're a bunch of discrete atoms that don't have derivatives?

That's cool, let me work with what you've got."

This is the mathematical equivalent of that friend who can make dinner out of whatever random ingredients you have in your fridge. It's improvisational. It's jazz.

The Replicator as an Anti-Authoritarian Technology

Now here's where things get properly Graeberian.

The Star Trek replicator isn't just a neat gadget. It's arguably the most radical technology ever imagined. Not because it's high-tech, but because it fundamentally undermines every power structure based on scarcity.

Can't afford food? Replicate it. Need medicine? Replicate it. Want that thing that only rich people have? Replicate it.

The mathematics we're developing—these weak solutions to the Monge-Ampère equation—are the blueprint for the ultimate democratization of matter itself. And the beautiful irony? We're developing this mathematics not to build replicators, but because we're curious. Because it's fun. Because watching transport maps emerge from nonlinear PDEs is genuinely delightful if you're the kind of person who finds that sort of thing delightful.

Building the Engine: Or, Why Not Let the Universe Do the Work?

The technical papers all imagine replicators as these massive engineering projects. Armies of nanobots! Precisely controlled force fields! Computational requirements that would melt a supercomputer!

But again, why do we assume it has to be hard?

The same Alexandrov solution that tells particles where to go could be interpreted not as instructions for machines, but as a potential field that particles *want* to follow. We already know particles love rolling down potential gradients—that's literally what they do all day long. So maybe instead of forcing atoms into place, we create an energy landscape where the teacup configuration is simply the most fun place to be.

This isn't as crazy as it sounds. Liquid crystals already do this. Colloids self-assemble into complex patterns just because that's where the energy landscape leads them. They're not being forced. They're playing.

The Play Principle of Matter Reconfiguration

What if—and hear me out here—the reason we haven't built replicators yet isn't because they're too hard, but because we're thinking about them wrong?

We approach it as an optimization problem: minimize energy, maximize efficiency, optimize transport. But what if matter, like that inchworm my advisor and I once watched while our code compiled, just wants to explore every possible configuration? What if the Monge-Ampère equation isn't describing the "optimal" path but the most playful one?

There's something deeply suspicious about the fact that the same mathematics that governs optimal transport also shows up in soap bubbles finding their shapes, crystals growing in solution, neurons finding their targets during brain development, or markets reaching equilibrium (when they're not being manipulated by capitalists).

It's almost as if the universe has a favorite game, and that game is "rearrange yourself in interesting ways." The Monge-Ampère equation isn't a law imposed from above; it's the score the universe is improvising from.

Why This Matters (Beyond Getting Your Tea)

Look, I'm not actually saying we should build Star Trek replicators (though we absolutely should). I'm saying that the mathematics we're developing to understand matter reconfiguration reveals something profound about reality itself.

The fact that we need "weak" solutions—solutions that embrace discontinuity, that work with discreteness rather than against it, that find pattern in apparent chaos—tells us that the universe itself is more flexible than our authoritarian mathematics traditionally assumed.

Every time we solve the Monge-Ampère equation for discrete particles, we're proving that:

1. Reality doesn't follow the rules we think it should
2. That's actually fine
3. The workarounds are more interesting than the original rules
4. Maybe the universe is just making it up as it goes along

So here I am, writing my dissertation on weak solutions to fully nonlinear PDEs, and people ask me, "What's the point?"

The point is that I'm literally writing the mathematical manual for matter replicators. The point is that the universe is already a replicator—it's been rearranging matter into increasingly amusing configurations for 14 billion years. The point is that a cup of Earl Grey tea is just atoms playing a very specific game, and we're finally learning the rules.

But mostly, the point is this: if we can't use mathematics to imagine revolutionary technologies that could change society by making scarcity obsolete, then what's the point of doing mathematics at all?

The replicator isn't science fiction. It's atoms playing around. The Monge-Ampère equation isn't abstract mathematics. It's the universe's playbook. And weak solutions aren't mathematical compromise—they're mathematical jazz, improvisation, the recognition that reality is more interesting than our theories.

My advisor was right to stare at that transport map. We weren't just debugging code. We were catching a glimpse of the universe at play, rearranging itself for the sheer joy of rearrangement, solving optimal transport problems not because it has to, but because—apparently—that's what it does for fun.