IT 327 – SUPPLEMENTARY MATERIAL CHAPTER 1

DC Electric Circuits

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Chapter 1: DC Electric Circuits

Preview

All electronic circuits consist of elements connected in such a way that they form a complete loop, or circuit. Figure 1-2 is an example of such a circuit, and includes a voltage rise (E1) and a load (R1). To understand the operation of an electronic circuit, we need to understand the terms voltage, current, and resistance, and the basic law that relates these three terms: Ohm's law.

All electronic circuits consist of elements connected either in series or in parallel. Some circuits contain combinations of these connections (series-parallel), but such circuits will not be considered in this text.

1-1 Voltage, Current and Resistance

Voltage (named after Alessandro Volta) is the force that causes electrons to want to move, and is thus often also referred to as electro-motive force. Voltage can be produced by batteries (through electrochemical reactions), by rubbing (as with static electricity), and by generators (wires and magnets moving with respect to each other).

In the Bohr model of the atom, the outer shell of electrons (the valence shell) are the only electrons that participate in what we know as electrical current. Current is the flow of electrons, and when 6.241×10^{18} electrons (1 Coulomb) pass a given point in 1 second, we call that 1 Ampere (or Amps) of current (named after André-Marie Ampère). In typical electronic circuits, currents range from fA to mA; in typical motor and other power circuits, currents range from mA to hundreds of Amps.

Any time a voltage is applied to a material, some current flows. The amount of current that flows is limited by the resistivity offered by the material. Resistivity is a property that varies greatly in common materials, as shown in Table 1-1. Resistance is a specific instantiation of a material.

The equation that relates these three electrical parameters is Ohm's Law, first identified by Georg Ohm in 1827:

$$I = \frac{E}{R} \qquad \textit{Ohm's Law} \qquad \mathbf{1.1}$$

To better become familiar with the terms used in this and subsequent chapters, these terms are summarized in Table 1-2.

In most ways, electrical current behaves like the flow of water, so it is useful to think of it this way. For water to flow, there must be some force (pressure) to make it want to move; in electricity, this force is Voltage, or electro-motive force (EMF). There must also be some channel in which the water can flow, typically a pipe. In electricity, this is a wire. And finally, there is always opposition to flow, known as head in pressurized water systems, and known as resistance in electricity.

For example, if we wish to have 150 mA flow through a material, we have two options: either adjust the voltage until that much current flows, or adjust the

Material	Resistivity (Ω-cm)
Silver	1.59×10^{-6}
Copper	1.7×10^{-6}
Gold	2.4×10^{-6}
Aluminum	2.8×10^{-6}
Tungsten	5.6×10^{-6}
Nichrome	1.5×10^{-4}
Carbon (amorphous)	3.5×10^{-3}
Germanium	4.6×10^{1}
Sea water	2.0×10^{1}
Drinking water	2.0×10^{3} to 2.0×10^{5}
Silicon	6.4×10^{4}
Polyethylene	5.0×10^{10}
Glass	1.0×10^{12} to 1.0×10^{16}
Carbon (diamond)	1.0×10^{14}
Hard Rubber	1.0×10^{15}
Air	1.3×10^{18} to 3.3×10^{18}
Quartz	7.4×10^{19}
PET	1.0×10^{23}
Teflon	1.0×10^{25} to 1.0×10^{27}

Table 1-1: Resistivities of common materials. Note that the range is HUGE: from 10^{-6} to 10^{27}, or about 33 orders of magnitude!

Term	Units	Unit Abbrev	Symbol	Meaning
Voltage	Volts	V	E	ElectroMotive Force (EMF)
Current	Amperes	A	I	Flow of electrons
Resistance	Ohms	Ω	R	Opposition to electron flow
Power	Watts	W	P	Energy/unit time; Joules/sec
Frequency	Hertz	Hz	f	Cycles/sec
Capacitance	Farads	F	C	Opposes changes in voltage
Inductance	Henries	H	L	Opposes changes in current
Reactance	Ohms	Ω	X	Opposition to changes in E or I
Impedance	Ohms	Ω	Z	Vector sum of total reactance and resistance in a circuit

Table 1-2: Summary of terms to be used in Chapters 1 and 2 of this text.

resistance of the material until that much current flows. In the case of a fixed resistance (200 Ω, for instance), the voltage necessary to cause 150 mA of current to flow would be:

E = IR = (150 mA)(200 Ω) = ***30 Volts***

If we wish to use a resistor that, with 10 Volts applied, gives us 150 mA of current, we would need a resistance of:

R = E/I = (10 V)/(150 mA) = ***66.67 Ohms***

Thus, Ohm's Law allows us to predict very consistently the current, voltage, or the resistance of a given circuit, if two of them are already known.

The significance of Ohm's Law is not immediately apparent, so a word should be said about it. Electricity has been known about for many centuries. Indeed, it was the ancient Greeks who gave it the name we use today. Their experiments showed that electrical effects could be produced by rubbing different items on amber, the Greek word for which is *electron*. It has been most often thought of as a fluid, and so many terms used with electricity today come from that concept, including current and voltage. How current and voltage are related is what many experimenters worked to determine, and it was Georg Ohm, in about 1827, who formalized this relationship experimentally. It is a fundamental and very powerful tool for analyzing nearly all circuits, and will be used repeatedly in this text.

Another equation that is essential in understanding electrical circuits is the Power Formula:

$$P = IE \qquad \textit{Power Formula} \qquad 1.2$$

This equation allows us to relate the voltage and current in a circuit to the total power being used by that circuit. It also tells us that if NO current flows (I = 0), there is NO power dissipation. Likewise, if there is no voltage drop (E = 0), there also is NO power dissipation.

Since Ohm's Law and the Power Formula both use I and E, we can combine these equations and solve for different variables, giving 12 versions of these formulas, as shown in Figure 1-1. The versions that will be of the most significance in this text are:

$P = E^2/R$ $P = I^2R$ $P = IE$

$R = E/I$ $E = P/I$ $I = P/E$

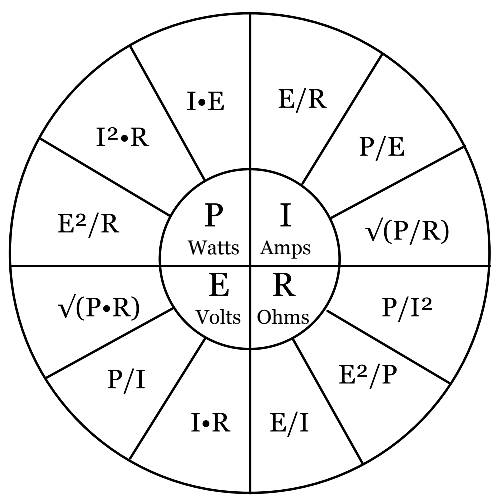

Figure 1-1: The twelve forms of the combination of Ohm's Law and the Power Formula.

1-2 Electrical Circuits

For the purposes of this text, an electrical circuit consists of a voltage rise or source and at least one load. Figure 1-2 is an example of a simple electrical circuit consisting of these two elements.

This is called a *circuit* because the flow of current goes all the way around and back to the source, making it a complete circuit. Incomplete circuits are *open* circuits (Figure 1-3). Circuits with no load and only the voltage source are *short* circuits (Figure 1-4). All useful circuits are composed of at least one voltage source (either DC or AC) and at least one load (represented by the resistor).

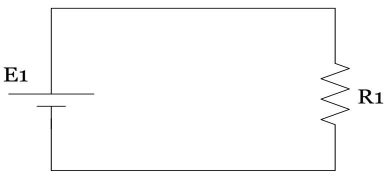

Figure 1-2: A simple circuit with a voltage source and a resistor.

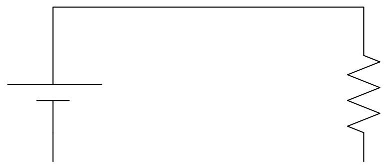

Figure 1-3: An example of an *open* circuit.

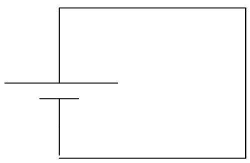

Figure 1-4: An example of a *short* circuit.

Voltage which does not change over time is known as DC (Direct Current) voltage. Voltage which does change over time is known as AC (Alternating Current) voltage. Examples of DC include batteries, power supplies, and solar panels. Examples of AC include the 120 Volts, 60 Hz power common in the USA, automotive alternators, and most portable generators.

1-3 Series Circuits: Voltage Drops

There are only two ways of connecting multiple elements in circuits: in series, or in parallel. Some very complex circuits may use a combination of these, but in this text we will confine ourselves to only these two, and will not analyze any circuits with a combination of these. Thus, we will analyze *series circuits*, and then *parallel circuits*.

The characteristics of each type of circuit are completely different. And since series circuits are generally easier to comprehend, we will cover them first.

Figure 1-4 is an example of a series circuit with one voltage source and two loads, both connected in series. Such a circuit has the following inherent characteristics:

1. All elements in the circuit have the same current.
2. Each resistor drops only a portion of the total voltage.
3. Each resistor dissipates only a portion of the total power.
4. If any element in the circuit is removed, ALL the current stops flowing, and NO power is then dissipated. This is an open circuit.
5. Each resistor drops a voltage directly proportional to its respective value. This is known as the *voltage divider rule*.
6. Each resistor dissipates an amount of power directly proportional to its respective value. This is a direct result of characteristic #5 above.

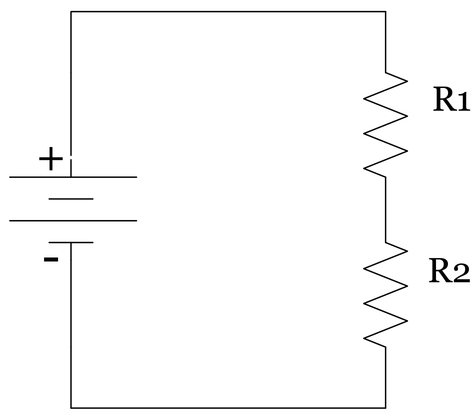

Figure 1-4: An example of a simple series circuit with one voltage source and two loads, connected in series.

The voltage divider rule is very useful, and is formalized as follows:

$$V_{R_n} = V_T * \frac{R_n}{R_T} \qquad \textbf{\textit{Voltage Divider Rule}} \qquad \mathbf{1.3}$$

An example analysis of a simple series circuit will be very helpful. We will use the example shown in Figure 1-5. To analyze a circuit means to calculate all the values not directly given. This circuit gives us the value of the applied voltage (12 Volts), of R1 (2 kΩ), and of R2 (4 kΩ). Table 1-3 shows all the other values remaining to be calculated.

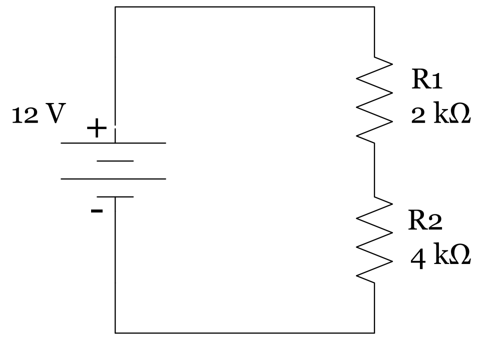

Figure 1-5: A simple series circuit to be analyzed.

The first step is to calculate the total resistance. In series circuits, this is found by simply adding together all of the individual resistances:

$R_{Total} = R_1 + R_2 = 2\ k\Omega + 4\ k\Omega = \mathbf{6\ k\Omega}$

We next find the total current, using Ohm's Law:

$I_{Total} = E_{Total} / R_{Total} = 12\ V / 6\ k\Omega = \mathbf{2\ mA}$

Next we calculate the voltage drops across each resistor. This can be done using either Ohm's Law or the Voltage Divider Rule, so we will do one of each:

$V_{R1} = (I_{R1})(R_1) = (2\ mA)(2\ k\Omega) = \mathbf{4\ V}$

$V_{R2} = V_{Total} * (R_2/R_{Total}) = 12\ V * (4\ k\Omega / 6\ k\Omega)$

$= 12\ V * .667 = \mathbf{8\ V}$

	Voltage	Resistance	Current	Power
Total	**12 V**			
R1		**2 kΩ**		
R2		**4 kΩ**		

Table 1-3: Values given for Figure 1.5. Note that only 3 values are given, leaving 9 values to be calculated.

Characteristic #1 of series circuits says that all elements in the circuit have the same current, so we can find all the missing currents by simply putting in the value calculated for the total current:

$I_{R1} = I_{R2} = I_{Total} = \mathbf{2\ mA}$

This leaves only the powers to be calculated, which can be done by using any of the equations given for power on the page 3, so let's use one of each:

$P_{Total} = I_{Total} * E_{Total} = 2\ mA * 12\ V = \mathbf{24\ mW}$

$P_{R1} = I_{R1}^2 * R_1 = (2\ mA)^2 * 2\ k\Omega = \mathbf{8\ mW}$

$P_{R2} = E_{R2}^2 / R_2 = (8\ V)^2 / 4\ k\Omega = \mathbf{16\ mW}$

With all the values now calculated, we can fill in Table 1-3, creating a completely filled-in table as shown in Table 1-4.

	Voltage	Resistance	Current	Power
Total	**12 V**	6 kΩ	2 mA	24 mW
R1	4 V	**2 kΩ**	2 mA	8 mW
R2	8 V	**4 kΩ**	2 mA	16 mw

Table 1-4: All the values for Figure 1-5.

Table 1-4 allows us to see examples of some of the characteristics described for series circuits on page 4. Characteristic #2 states that each resistor drops only a portion of the total voltage. Together with characteristic #5 (the Voltage Divider Rule), we would expect R_2 to drop twice as much voltage as R_1, since it is twice as great in value; Table 1-4 shows that in the Voltage column: $V_{R1} = 4$ V, while $V_{R2} = 8$ V.

We also note that $V_{R1} + V_{R2} = V_{Total}$ (4 V + 8 V = 12 V). This is known as Kirchoff's Voltage Law, and is formalized as follows for series circuits:

$$\sum E = \sum V \qquad \textbf{\textit{Kirchoff's Voltage Law}} \qquad \mathbf{1.4}$$

In words, this would be stated: *The sum of the voltage rises must equal the sum of the voltage drops.*

Also in Table 1-4 we see characteristic #3, together with #6 exemplified: R_2 dissipates twice as much power as R_1, since it is twice as great in value and they both have the same current.

This brings up two things that should now be formalized. Generally, E means a voltage *rise*, while V means a voltage *drop*. Although this definition is not always followed in the literature, we will try to maintain this distinction for purpose of clarity. The other point is that in all electric circuits, all the current that leaves the voltage source returns to that source – none is lost. But none of the voltage remains. In Figure 1-5, we start with 12 Volts, but R_1 drops 4 Volts, and then R_2 drops 8 Volts, leaving none. Formalized as Kirchoff's Voltage Law, it simply points out that voltage drops are an integral part of electrical circuits, and we see this particularly well in series circuits.

One more example should help this section on series circuits. This time we will use four series resistors, and we will also look at the intermediate voltage *nodes*. A node is defined as a point in a circuit with a common voltage; while that is a bit of an abstract concept initially, Figure 1-6 will help clarify this.

The circuit depicted in Figure 1-6 has one voltage source, four resistors, and five nodes. All circuits have at least two nodes: V_{Total} and V_0, which are always the total voltage (or just the applied voltage) and ground (or the reference node). In this circuit, there are 3 additional nodes, labeled V_1, V_2 and V_3.

Using the same analysis process as before, we can fill out the table for all the parameters in the circuit, as shown in Table 1-5.

$R_{Total} = R_1 + R_2 + R_3 + R_4 = 25\ \Omega + 50\ \Omega + 75\ \Omega + 100\ \Omega =$ **$250\ \Omega$**

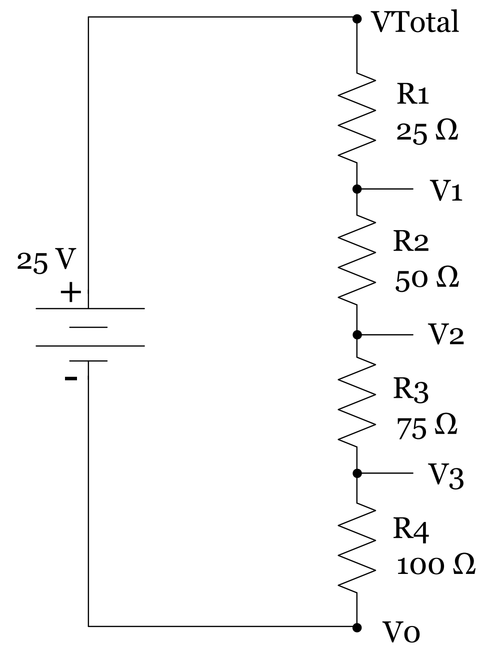

Figure 1-6: A series circuit with one voltage source, four resistors, and five nodes (VTotal, V0, V1, V2, V3).

	Voltage	Resistance	Current	Power
Total	**25 V**	250 Ω	100 mA	2.5 W
R1	2.5 V	**25 Ω**	100 mA	0.25 W
R2	5.0 V	**50 Ω**	100 mA	0.5 W
R3	7.5 V	**75 Ω**	100 mA	0.75 W
R4	10.0 V	**100 Ω**	100 mA	1.0 W

Table 1-5: All the values for Figure 1-6.

$I_{Total} = E_{Total} / R_{Total} = 25\text{ V} / 250\ \Omega =$ **100 mA**
$V_{R1} = (I_{R1})(R_1) = (100\text{ mA})(25\ \Omega) =$ **2.5 V**
$V_{R2} = (I_{R2})(R_2) = (100\text{ mA})(50\ \Omega) =$ **5.0 V**
$V_{R3} = (I_{R3})(R_3) = (100\text{ mA})(75\ \Omega) =$ **7.5 V**
$V_{R4} = (I_{R4})(R_4) = (100\text{ mA})(100\ \Omega) =$ **10.0 V**

$P_{Total} = I_{Total} * E_{Total} = 100\text{ mA} * 25\text{ V} =$ **2.5 W**
$P_{R1} = I_{R1} * V_{R1} = 100\text{ mA} * 2.5\text{ V} =$ **0.25 W**
$P_{R2} = I_{R2} * V_{R2} = 100\text{ mA} * 5.0\text{ V} =$ **0.5 W**
$P_{R3} = I_{R3} * V_{R3} = 100\text{ mA} * 7.5\text{ V} =$ **0.75 W**
$P_{R4} = I_{R4} * V_{R4} = 100\text{ mA} * 10.0\text{ V} =$ **1.0 W**

Additionally, we can also calculate the voltage at each of the nodes. Starting with the given values, we know that $V_{Total} = 25$ V, and that $V_0 = 0$ V. And then, knowing that each resistor drops a portion of the total voltage, and using these voltage drops in Table 1-5, we can calculate V1 as:

$V_1 = V_{Total} – V_{R1} = 25\text{ V} – 2.5\text{ V} =$ **22.5 V**

We also can calculate V1 by going from the bottom up, as:

$V_1 = V_0 + V_{R4} + V_{R3} + V_{R2} = 0\text{ V} + 10.0\text{ V} + 7.5\text{ V} + 5.0\text{ V} =$ **22.5 V**

Similarly, we can calculate V2 and V3 by either starting at the top and going down, or by starting at the bottom and going up, whichever is easier, as:

$V_2 = V_{Total} – V_{R1} – V_{R2} = 25\text{ V} – 2.5\text{ V} – 5.0\text{ V} =$ **17.5 V**
$V_2 = V_0 + V_{R4} + V_{R3} = 10.0\text{ V} + 7.5\text{ V} =$ **17.5 V**
$V_3 = V_{Total} – V_{R1} – V_{R2} – V_{R3} = 25\text{ V} – 2.5\text{ V} – 5.0\text{ V} – 7.5\text{ V} =$ **10.0 V**
$V_3 = V_0 + V_{R4} = 0\text{ V} + 10.0\text{ V} =$ **10.0 V**

In this example, it is very easy to see Kirchoff's Voltage Law (the sum of the voltage rises must equal the sum of the voltage drops):

25 V = 2.5 V + 5.0 V + 7.5 V + 10.0 V

Because this law is ALWAYS true in series circuits, it can be used to verify calculations.

1-4 Series Circuits: Voltage Rises

As mentioned before, the most common types of DC voltage sources (or *rises*) are batteries and power supplies (which take AC and convert it to DC). And particularly with batteries, it is very common to connect them together in such a way that more voltage is obtained.

There is another parameter that describes batteries with which we need to become familiar, and that is their *ampacity* – their capacity to deliver current. As always, the actual amount of current that a battery or power supply actually delivers is a function of Ohm's Law – it is limited by the resistance of the circuit. But if we were to provide an optimal circuit (one with NO resistance, or $R_{Total} = 0\ \Omega$), we would still not obtain an infinite current, as no battery or power supply can provide that. The reason is that all voltage sources have an *internal* resistance that is an integral part of their physical construction – it is not desirable, but it is inevitable. This internal resistance is modeled as shown in Figure 1-7, where we have a voltage source together with an internal resistance, surrounded by a dotted line, indicating that these two elements are physically inseparable.

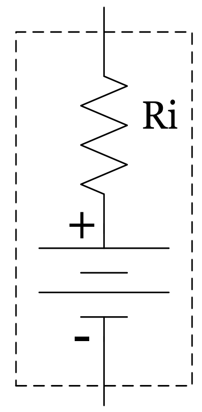

Figure 1-7: A voltage source (or rise), shown with its inherent internal resistance.

The *ampacity* of a voltage source is the maximum amount of current that it can deliver. In the case of a D-cell type battery, this value is typically about 2.0 A, meaning that if it is an alkaline battery (1.5 V), its internal resistance is:

$R = E / I = 1.5\text{ V} / 2.0\text{ A} = \mathbf{0.75\ \Omega}$

If two such batteries are connected in series, our voltage becomes:

$V_{Total} = V_1 + V_2 = 1.5\text{ V} + 1.5\text{ V} = \mathbf{3.0\ V}$

Our total resistance becomes:

$R_{Total} = R_{iV1} + R_{iV2} = 0.75\ \Omega + 0.75\ \Omega = \mathbf{1.5\ \Omega}$

And our ampacity becomes:

$I_{Total} = V_{Total} / R_{Total} = 3.0\text{ V} / 1.5\ \Omega = \mathbf{2.0\ A}$

Which we note is the same as before. So, by connecting two batteries in series, we have doubled our voltage, but we have not changed our ampacity. This deserves another example, one which should not be actually tried but which illustrates what happens when batteries are connected in series.

What if we were to series connect two different types of batteries, such as:

V_1 = alkaline battery = 1.5 V @ 2.0 A; $R_i = 0.75\ \Omega$

V_2 = NiCd battery = 1.2 V @ 1.5 A; $R_i = 0.8\ \Omega$

Connecting them together in series gives:

$V_{Total} = V_1 + V_2 = 1.5\text{ V} + 1.2\text{ V} = \mathbf{2.7\ V}$

Our total resistance becomes:

$R_{Total} = R_{iV1} + R_{iV2} = 0.75\ \Omega + 0.8\ \Omega = \mathbf{1.55\ \Omega}$

And our ampacity becomes the lesser of the two ampacities, so: = **1.5A**

So we note that, while we have increased our voltage, we have not kept the ampacity of V_1, but rather that of the lesser ampacity of V_2. Thus, the voltage source with the higher internal resistance becomes the major limit to our ampacity.

1-5 Parallel Circuits: Voltage Drops

Figure 1-8 is an example of a parallel circuit with one voltage source and two loads, both connected in parallel. Such a circuit has the following inherent characteristics:

1. All elements in the circuit have the same voltage.
2. Each resistor drops all of the total voltage.

3. Each resistor dissipates only a portion of the total power.
4. If any element in the circuit is removed, only the current in that branch stops flowing; the other branches are completely unaffected.
5. The current each resistor allows to flow is inversely proportional to its respective value. This is known as the *current divider rule*.
6. Each resistor dissipates an amount of power inversely proportional to its respective value. This is a direct result of characteristic #5 above.

The current divider rule (or formula) is very useful, and is formalized as follows:

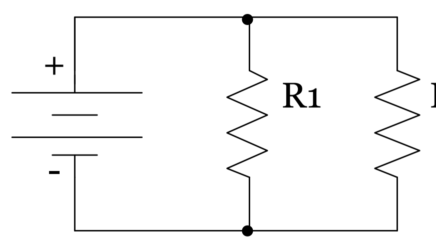

Figure 1-8: An example of a simple parallel circuit, with one voltage source and two loads connected in parallel.

$$I_{R_n} = I_T * \frac{R_T}{R_n} \qquad \textit{Current Divider Formula} \qquad 1.5$$

A careful comparison of the above characteristics to those listed on page 4 for series circuits shows that everything is reversed, with the exception of characteristic #3.

Parallel circuits are the most common kind, in our everyday experience. Specifically, in our home we may have several loads (represented by resistors) connected to the voltage source (the AC power outlet). Regardless of whether they are a blender, a refrigerator, a stereo, or a computer, all of our loads drop the full 120 V. If we disconnect one of our loads, it has no effect on any other load. This is true for all parallel circuits.

As with series circuits, the best way to understand them is to analyze them. We will use the example shown in Figure 1-9. As before, this means to calculate all the values not directly given. This circuit gives us the value of the applied voltage (20 Volts), of R1 (35 Ω), and of R2 (also 35 Ω). Table 1-6 shows all the other values remaining to be calculated.

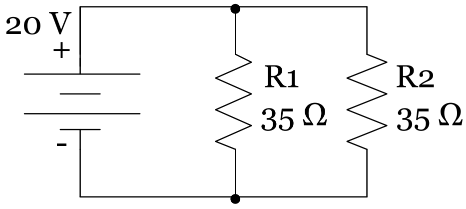

Figure 1-9: A simple parallel circuit to be analyzed.

The first step is to calculate the total resistance. In parallel circuits, this is found by adding the inverses of each resistance as follows:

$R_{Total} = 1 / (1/R_1 + 1/R_2) = 1 / (1/35\ \Omega + 1/35\ \Omega = \mathbf{17.5\ \Omega}$

We next find the total current, using Ohm's Law:

$I_{Total} = E_{Total} / R_{Total} = 20\ V / 17.5\ \Omega = \mathbf{1.143\ A}$

	Voltage	Resistance	Current	Power
Total	**20 V**			
R1		**35 Ω**		
R2		**35 Ω**		

Table 1-6: Values given for Figure 1-9. Note that only 3 values are given, leaving 9 values to be calculated.

Since this is a parallel circuit, characteristics #1 and #2 apply, so both resistors drop 20V. Thus, the current for each resistor can be found by Ohm's Law:

$I_{R1} = V_{R1} / R_1 = 20\ V / 35\ \Omega = \mathbf{0.5714\ A}$

$I_{R2} = V_{R2} / R_2 = 20\ V / 35\ \Omega = \mathbf{0.5714\ A}$

And since the total current is simply the sum of all the branch currents, we verify the total current calculated above by adding I_{R1} and I_{R2} together:

$I_{Total} = I_{R1} + I_{R2} = 0.571\ A + 0.571\ A = \mathbf{1.143\ A}$

And finally, we can find the power dissipations using any of the power equations, as follows:

$P_{Total} = I_{Total} * E_{Total} = 1.143\ A * 20\ V = \mathbf{22.86\ W}$

$P_{R1} = I_{R1}^2 * R_1 = (0.5714\ A)^2 * 35\ \Omega = \mathbf{11.43\ W}$

$P_{R2} = E_{R2}^2 / R_2 = (20\ V)^2 / 35\ \Omega = \mathbf{11.43\ W}$

Table 1-7 gives all the values, both provided and calculated, for Figure 1-9.

	Voltage	Resistance	Current	Power
Total	**20 V**	17.5 Ω	1.143 A	22.86 W
R1	20 V	**35 Ω**	0.5714 A	11.43 W
R2	20 V	**35 Ω**	0.5714 A	11.43 W

Table 1-7: All the values for Figure 1-9.

Table 1-7 allows us to see examples of some of the characteristics described for parallel circuits on pages 7 & 8. Characteristics #1 and #2 state that each resistor drops the same voltage, and that voltage is the total voltage. Together with characteristic #5 (the Current Divider Rule), we would expect R_1 and R_2 to have exactly half of the total current, since they are equal in value; Table 1-7 shows this to be true.

We also note that $I_{R1} + I_{R2} = I_{Total}$ (0.5714 A + 0.5714 A = 1.143 A). This is known as Kirchoff's Current Law, and is formalized as follows for parallel circuits:

$$\sum I_{in} = \sum I_{out} \qquad \textit{Kirchoff's Current Law} \qquad \mathbf{1.6}$$

In words, this would be stated: *The sum of the currents entering a node must equal the sum of the currents leaving a node.* For example, we note in Table 1-7 that the sum of the currents entering the 20 V node (1.143 A) is equal to the sum of the currents leaving that node ($I_{R1} + I_{R2}$).

Also in Table 1-7 we see characteristic #3, together with #6 exemplified: R_1 dissipates the same power as R_2, since they are of equal value. We also see that the power of R_1 plus the power of R_2 is equal to the total power.

The circuit of Figure 1-9 is quite simple, so a more analysis of a more complex circuit should be undertaken to further illustrate these characteristics. We will use the circuit of Figure 1-10 for this analysis.

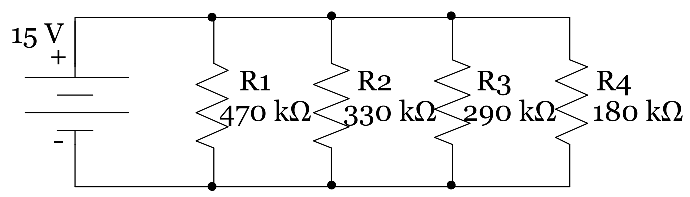

Figure 1-10: A more complex parallel circuit with one voltage source and four loads.

As usual, we will fill out a table for all the values of this circuit, both given and calculated. We will begin by finding the total resistance, followed by the total current:

$R_{Total} = 1 / (1/R_1 + 1/R_2 + 1/R_3 + 1/R_4)$

$= 1 / (1/470\text{ k}\Omega + 1/330\text{ k}\Omega + 1/290\text{ k}\Omega + 1/180\text{ k}\Omega)$

$=$ **70.61 kΩ**

$I_{Total} = E_{Total} / R_{Total} = 15\text{ V} / 70.61\text{ k}\Omega$

$=$ **212.4 μA**

The individual branch currents can be found using Ohm's Law for each branch, since the resistances and voltage drops are all known:

$I_{R1} = E_{R1} / R_1 = 15\text{ V} / 470\text{ k}\Omega =$ **31.91 μA**

$I_{R2} = E_{R2} / R_2 = 15\text{ V} / 330\text{ k}\Omega =$ **45.45 μA**

$I_{R3} = E_{R3} / R_3 = 15\text{ V} / 290\text{ k}\Omega =$ **51.72 μA**

$I_{R4} = E_{R4} / R_4 = 15\text{ V} / 180\text{ k}\Omega =$ **83.33 μA**

	Voltage	Resistance	Current	Power
Total	**15 V**	70.61 kΩ	212.4 μA	3.186 mW
R1	15 V	**470 kΩ**	31.91 μA	478.7 μW
R2	15 V	**330 kΩ**	45.45 μA	681.8 μW
R3	15 V	**290 kΩ**	51.72 μA	775.8 μW
R4	15 V	**180 kΩ**	83.33 μA	1.25 mW

Table 1-8: All the values for Figure 1-10.

Similarly, the branch currents could also be found using the Current Divider Formula (Equation 1.5) as follows:

$I_{R1} = I_{Total} * (R_{Total} / R_1) = 212.4\ \mu\text{A} * (70.61\text{ k}\Omega / 470\text{ k}\Omega) =$ **31.91 μA**

$I_{R2} = I_{Total} * (R_{Total} / R_2) = 212.4\ \mu\text{A} * (70.61\text{ k}\Omega / 330\text{ k}\Omega) =$ **45.45 μA**

$I_{R3} = I_{Total} * (R_{Total} / R_3) = 212.4\ \mu\text{A} * (70.61\text{ k}\Omega / 290\text{ k}\Omega) =$ **51.72 μA**

$I_{R4} = I_{Total} * (R_{Total} / R_4) = 212.4\ \mu\text{A} * (70.61\text{ k}\Omega / 180\text{ k}\Omega) =$ **83.33 μA**

Either way we calculate the branch currents – using Ohm's Law or the Current Divider Formula – the results are the same.

And finally, all the power dissipations can be found, using any of the power formulas since we now know all the voltage drops, currents, and resistances for all the branches. We will use the P = IE formula:

$P_{Total} = I_{Total} * E_{Total}$ = 212.4 μA * 15 V = **3.186 mW**
$P_{R1} = I_{R1} * E_{R1}$ = 31.91 μA * 15 V = **478.7 μW**
$P_{R2} = I_{R2} * E_{R2}$ = 45.45 μA * 15 V = **681.8 μW**
$P_{R3} = I_{R3} * E_{R3}$ = 51.72 μA * 15 V = **775.8 μW**
$P_{R4} = I_{R4} * E_{R4}$ = 83.33 μA * 15 V = **1.25 mW**

Table 1-8, even more than Table 1-7, allows us to see the characteristics of parallel circuits described on pages 7-8. For characteristics #1 and #2, we see that all 4 resistors drop the same voltage – the total voltage of 15 Volts. We see that each resistor dissipates only a portion of the total power of 3.186 mW (characteristic #3), and that this portion of the total power dissipation is an inverse function of the value of the resistance (characteristic #6) – the power dissipation of R1 (the largest resistance) is only 478.7 μW, which is the smallest of all the power dissipations. We also see this inverse relationship in the current through each resistor (Kirchoff's Current Law), in that the largest resistance (R1) has the smallest current (31.91 μA).

We should also note that most of the characteristics of parallel circuits (#1, 2, 4, 5, 6) are opposite those of series circuits.

Conductance

Often in considering parallel circuits, they are analyzed by using the concept of conductance, which is how readily a certain device conducts electricity. Conductance is simply the reciprocal of resistance; its symbol is G, and its units are Siemens (S). Therefore:

$R = 1/G$ $\qquad$ $G = 1/R$

For example, analyzing the circuit of Figure 1-10 would give the following conductances:

$G_1 = 1/R_1$ = 1/470 kΩ = 2.13 μS
$G_2 = 1/R_2$ = 1/330 kΩ = 3.03 μS
$G_3 = 1/R_3$ = 1/290 kΩ = 3.45 μS
$G_4 = 1/R_4$ = 1/180 kΩ = 5.56 μS
$G_T = G_1 + G_2 + G_3 + G_4$ = 14.17 μS

And then by simply inverting the total conductance, G_T, we find the total resistance:

$R_T = 1/G_T$ = 1/14.17 μS = 70.6 kΩ, which is the same answer found by the previous method.

1-6 Parallel Circuits: Voltage Rises

For this section, one of the most important characteristics of parallel circuits is #1 (from p. 7): all elements in the circuit have the same voltage. In the example of Figure 1-11, we have two batteries, both at 1.5 Volts, but at different ampacities.

Since connecting voltage sources in *parallel* does not increase the voltage, there must be something it does increase – and that is the ampacity. The parallel combination in Figure 1-11 gives 1.5 V, at an ampacity of 4.5 Amps. Such a connection would be useful where one battery is too weak – which is exactly what is done when one good battery in an automobile is used to "jump" another car whose battery is inadequate. Both batteries have the same voltage, and when connected in parallel, the ampacities add together to boost the current available to the starter motor.

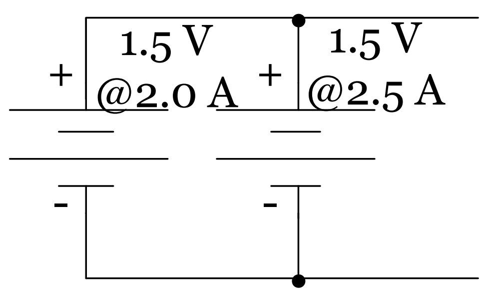

Figure 1-11: Two voltage rises (sources) in parallel.

A word should be said about what would happen if one were to connect in parallel two batteries of *different* voltages – for instance an alkaline battery at 1.5 V and a NiCd battery at 1.2 V. Once they are connected together, they are *forced* to have the same voltage, and since the 1.2 V battery cannot be raised to 1.5 V, the 1.5-V battery is drug down to 1.2 V by drawing current from it and through the 1.2-Volt battery. Such a connection quickly discharges the 1.5-Volt battery and heats up both batteries with the wasted current. Such heating can occur very quickly and can permanently damage or destroy both batteries. So, the bottom line is – never connect in parallel any voltage sources which are NOT the same voltage.

1-7 Summary

Chapter 1 on DC electric circuits can be summarized with the following main points:

1. An electric circuit has a voltage source, a load, and current which flows from the output of the voltage source back to the other terminal of the voltage source.
2. Voltage (*electromotive force*) is the force that causes electrons to move.
3. Current is the flow of electrons. One Amp = 6.241×10^{18} (1 Coulomb) per second.
4. The amount of current that flows is directly proportional to the voltage, and inversely proportional to the resistance. This is Ohm's Law (Equation 1.1).
5. Power in an electric circuit is the product of the current (I) and the voltage (E). This is the Power Formula (Equation 1.2).
6. The only useful circuit is a complete circuit; open and short circuits are not useful.
7. Electrical components can only be connected in two ways: in series and in parallel.
8. It is essential to understand the characteristics of series circuits, which are:
    a. All elements in the circuit have the same current.
    b. Each resistor drops only a portion of the total voltage.
    c. Each resistor dissipates only a portion of the total power.
    d. If any element in the circuit is removed, ALL the current stops flowing, and NO power is then dissipated. This is an open circuit.
    e. Each resistor drops a voltage directly proportional to its respective value. This is known as the Voltage Divider Rule.
    f. Each resistor dissipates an amount of power directly proportional to its respective value. This is a direct result of characteristic #e above.
9. In all complete electrical circuits, the sum of the voltage rises must equal the sum of the voltage drops. This is known as Kirchoff's Voltage Law (Equation 1.4).
10. A node in an electrical circuit is a point with a common voltage. All electrical circuits have at least two nodes: the source voltage and the reference node (usually ground).
11. Connecting voltage sources in series <u>adds</u> the voltages, but the ampacity becomes the <u>smallest</u> of the individual ampacities.
12. It is essential to understand the characteristics of parallel circuits, which are:
    a. All elements in the circuit have the same voltage.
    b. Each resistor drops all of the total voltage.
    c. Each resistor dissipates only a portion of the total power.
    d. If any element in the circuit is removed, only the current in that branch stops flowing; the other branches are completely unaffected.
    e. The current each resistor allows to flow is inversely proportional to its respective value. This is known as the current divider rule.
    f. Each resistor dissipates an amount of power inversely proportional to its respective value. This is a direct result of characteristic #e above.
13. All but one of the characteristics of series and parallel circuits are opposite each other.
14. In all complete electrical circuits, the sum of the currents entering a node must equal the sum of the currents leaving that node. This is known as Kirchoff's Current Law (Equation 1.6).
15. Connecting voltage sources in parallel <u>adds</u> the ampacities, but the voltage remains unchanged.

Problems

1. How widely does resistivity range, between silver and Teflon? (5 pts)
2. What are the units of electromotive force? (5 pts)
3. What is electrical current? (5 pts)
4. What is a short circuit? (5 pts)
5. What is an open circuit? (5 pts)
6. A circuit as in Figure 1P-1 has E = 14 Volts and I = 3.5 Amps. Find R and P. (10 pts)
7. A circuit as in Figure 1P-1 has E = 25 Volts and R = 860 kΩ. Find I. (5 pts)
8. A circuit as in Figure 1P-1 has I = 35 μA and R = 33 kΩ. Find E. (5 pts)
9. A circuit as in Figure 1P-1 has E = 18 Volts and R = 370 Ω. Find P. (5 pts)
10. A circuit as in Figure 1P-1 has I = 15 Amps and R = 0.25 Ω. Find P. (5 pts)
11. Find all the missing values for the circuit in Figure 1P-2. (70 pts)
12. Find all the missing values for the circuit in Figure 1P-3. (75 pts)

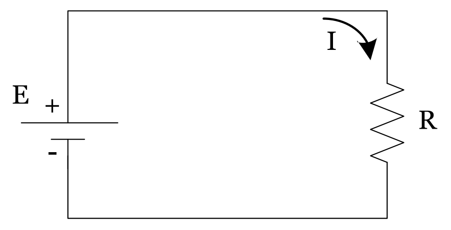

Figure 1P-1: A simple series circuit.

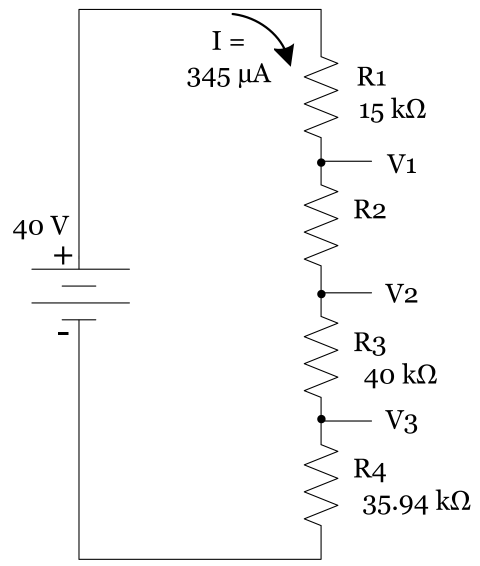

Figure 1P-2: A 4-load series circuit.

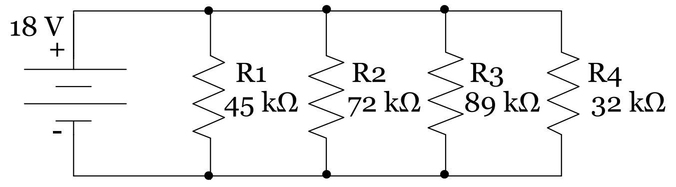

Figure 1P-3: A 4-load parallel circuit.

Answers to Numerical Problems

6. **4.0 Ω; 49 W**
7. **29.07 µA**
8. **1.155 V**
9. **875.7 mW**
10. **56.25 W**
11.

	Voltage	Resistance	Current	Power
Total	40 V	**115.94 kΩ**	345 µA	**13.8 mW**
R1	**5.175 V**	15 kΩ	345 µA	**1.7854 mW**
R2	**8.625 V**	**25 kΩ**	345 µA	**2.9756 mW**
R3	**13.8 V**	40 kΩ	345 µA	**4.761 mW**
R4	**12.4 V**	35.94 kΩ	345 µA	**4.278 mW**
V1	**34.825 V**			
V2	**26.2 V**			
V3	**12.4 V**			

12.

	Voltage	Resistance	Current	Power
Total	18 V	**12.723 kΩ**	**1.4148 mA**	**25.467 mW**
R1	**18 V**	45 kΩ	**400 µA**	**7.2 mW**
R2	**18 V**	72 kΩ	**250 µA**	**4.5 mW**
R3	**18 V**	89 kΩ	**202.25 µA**	**3.641 mW**
R4	**18 V**	32 kΩ	**562.5 µA**	**10.125 mW**