$$p(t) = u(t) \cdot i(t) = \hat{u}\sin(\omega t) \cdot \hat{\imath}\sin(\omega t)$$
$$= \hat{u}\hat{\imath}\sin^2(\omega t)$$

$$P_{gem} = \frac{1}{T}\int_0^T u(t) \cdot i(t)\,\mathrm{d}t \tag{1a}$$

$$= \frac{1}{T}\int_0^T \hat{u}\sin(\omega t) \cdot \hat{\imath}\sin(\omega t)\,\mathrm{d}t \tag{1b}$$

$$= \frac{1}{T}\int_0^T \hat{u}\hat{\imath}\sin^2(\omega t)\,\mathrm{d}t \tag{1c}$$

$$= \hat{u}\hat{\imath}\frac{1}{T}\int_0^T \sin^2(\omega t)\,\mathrm{d}t \tag{1d}$$

$$= \hat{u}\hat{\imath}\frac{1}{T}\int_0^T \left(\frac{1}{2} - \frac{1}{2}\cos(2\omega t)\right)\,\mathrm{d}t \tag{1e}$$

$$= \frac{\hat{u}\hat{\imath}}{2T}\left(\int_0^T 1\,\mathrm{d}t - \int_0^T \cos(2\omega t)\,\mathrm{d}t\right) \tag{1f}$$

$$= \frac{\hat{u}\hat{\imath}}{2T}\left(T - 0\right) \tag{1g}$$

$$= \frac{1}{2}\hat{u}\hat{\imath} \tag{1h}$$