



Figure 1: Part of the function  $f(x) = x^2$ .

The slope of the line through the points  $A$  and  $B$  is:

$$\text{slope}_{AB} = \frac{\Delta y}{\Delta x} = \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad (1)$$

We want to determine the slope in point  $A$ . Therefore, we move point  $B$  to point  $A$ . This means that  $\Delta x$  will slowly become 0. This way, we get a *limit*.

$$\text{slope}_A = \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \quad (2)$$

Note that this limit will give us a *function*.

### Example

Find the slope in point  $A(1, 1)$  for the function  $f(x) = x^2$ .

First we calculate the limit function:

$$\begin{aligned} \text{slope}_A &= \lim_{\Delta x \rightarrow 0} \frac{(x + \Delta x)^2 - x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{x^2 + 2x\Delta x + (\Delta x)^2 - x^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} \frac{2x\Delta x + (\Delta x)^2}{\Delta x} \\ &= \lim_{\Delta x \rightarrow 0} 2x + \Delta x \\ &= 2x \end{aligned} \quad (3)$$

The slope in  $A(1, 1)$  is 2.