The formal definition of the unilateral or one-sided Laplace integral is:

$$\mathcal{L}\{f(t)\} = F(S) = \int_0^\infty f(t)\, \mathrm{e}^{-st}\, \mathrm{d}t$$

In case $f(0)$ does not exists, one can use

$$\mathcal{L}\{f(t)\} = F(s) = \lim_{\varepsilon \downarrow 0} \int_\varepsilon^\infty f(t)\, \mathrm{e}^{-st}\, \mathrm{d}t$$

Linearity:

$$\mathcal{L}\{af(t) + bg(t)\} = aF(s) + bF(s)$$

Convolution:

$$\mathcal{L}\{f(t) * g(t)\} = \int_0^\infty f(\tau)\, g(t-\tau)\, \mathrm{d}\tau = F(s) \cdot G(s)$$

Initial value theorem:

$$f(0^+) = \lim_{s \to \infty} sF(s)$$

Final value theorem:

$$f(\infty) = \lim_{s \to 0} sF(s)$$

if all poles of $sF(s)$ are in the left half-plane.

Table 1: Laplace transforms.

Function	$f(t)$	$F(s)$	Convergence
Unit impulse	$\delta(t)$	1	For all s
Delayed unit impulse	$\delta(t-\tau)$	$\mathrm{e}^{-\tau s}$	$\Re(s)>0$
Unit step (Heaviside)	$H(t)$	$\frac{1}{s}$	$\Re(s)>0$
Delayed unit step (Heaviside)	$H(t-\tau)$	$\frac{1}{s}\mathrm{e}^{-\tau s}$	$\Re(s)>0$
Ramp	t	$\frac{1}{s^2}$	$\Re(s)>0$
Power (n integer)	t^n	$\frac{n!}{s^{n+1}}$	$\Re(s)>0, n>-1$
Exponential decay	e^{-at}	$\frac{1}{s+a}$	$\Re(s)>-a$
Exponential approach	$1-\mathrm{e}^{-at}$	$\frac{1}{s(s+a)}$	$\Re(s)>-a$
Sine	$\sin(\omega t)$	$\frac{\omega}{s^2+\omega^2}$	$\Re(s)>0$
Cosine	$\cos(\omega t)$	$\frac{s}{s^2+\omega^2}$	$\Re(s)>0$
Hyperbolic sine	$\sinh(\alpha t)$	$\frac{\alpha}{s^2-\alpha^2}$	$\Re(s)>\lvert\alpha\rvert$
Hyperbolic cosine	$\cos(\alpha t)$	$\frac{s}{s^2-\alpha^2}$	$\Re(s)>\lvert\alpha\rvert$
Exponential decay sine	$\mathrm{e}^{-at}\sin(\omega t)$	$\frac{\omega}{(s+a)^2+\omega^2}$	$\Re(s)>-a$
Exponential decay cosine	$\mathrm{e}^{-at}\cos(\omega t)$	$\frac{s+a}{(s+a)^2+\omega^2}$	$\Re(s)>-a$
nth root	$\sqrt[n]{t}$	$\frac{1}{s^{\frac{1}{n}+1}}\Gamma\left(\frac{1}{n}+1\right)$	$\Re(s)>0$
Natural logarithm	$\ln t$	$\frac{-\ln(s)-\gamma}{s}$	$\Re(s)>0$