Calculating optical depth from absorption coefficient

1. **Between 691.5 cm^{-1} and 692.5 cm^{-1}, the HITRAN 2016 database contains the following parameters for lines of the relevant gases with line strengths above 10^{-21} cm/molecule:**

23	2	691.558689	1.144E-21	8.111E-01	0.1213	1.053	43.1975	0.75
43	1	691.627250	1.066E-21	5.078E-02	0.1000	0.200	205.8231	0.75
11	1	691.767164	4.587E-21	1.024E+01	0.0976	0.396	1022.9201	0.72
2	1	691.972420	9.101E-20	9.153E-01	0.0687	0.090	362.7883	0.75
3	1	692.034820	1.140E-21	9.259E-02	0.0746	0.103	191.7092	0.78
43	1	692.125070	1.132E-21	5.151E-02	0.1000	0.200	184.4532	0.75
2	1	692.129005	4.052E-21	9.650E-01	0.0688	0.091	1007.1335	0.75
11	1	692.312700	1.013E-20	1.153E+01	0.0931	0.302	1026.6435	0.72
26	1	692.318440	1.708E-20	3.312E+00	0.0787	0.147	912.6394	0.75
2	1	692.400097	3.727E-21	9.665E-01	0.0687	0.090	1031.1292	0.75
3	1	692.468700	1.212E-21	9.278E-02	0.0755	0.103	158.1653	0.78

Compute the optical depth at a wavenumber of 691.97242 cm^{-1} for a 1 km thick layer at a pressure of $p = 102$ mb and temperature of $T = 217$ K. The volume mixing ratios of the radiatively active gases are q_{CO_2} = 3.70 × 10^{-4}.

a) The number of air molecules per cm^2 in this layer is:

$$u_{air} = \frac{\Delta z N_A p}{R^* T} = \frac{(1000\ \mathrm{m})(6.022\times 10^{23}\ molec\ \mathrm{mol}^{-1})(10200\ \mathrm{kg\ m^{-1}\ s^{-2}})}{(8.314\ \mathrm{kg\ m^2\ s^{-2}\ mol^{-1}\ K^{-1}})(217\ \mathrm{K})}$$

$$= 3.4\times 10^{27}\ molec\ \mathrm{m}^{-2} = 3.4\times 10^{23}\ molec\ \mathrm{cm}^{-2}$$

Where N_A is Avogadro constant and R^* is ideal gas constant.

b) Calculate the absorption coefficient.

The monochromatic absorption coefficient from the program is

$$k_\nu = 2.024\times 10^{-18}\ \mathrm{cm^2\ molec^{-1}}$$

c) Calculate the optical depth of the layer.

The optical depth of the layer at this wavenumber is the absorber amount of CO_2 times the absorption coefficient:

The number of air molecules per cm^2 in this layer is

$$\tau = k_\nu u_{\mathrm{CO_2}} = k_\nu q_{\mathrm{CO_2}} u_{\mathrm{air}}$$

$$= \left(2.024 \times 10^{-18}\ \mathrm{cm^2\ molec^{-1}}\right)\left(3.7 \times 10^{-4}\right)\left(3.4 \times 10^{23}\ \mathrm{molec\ cm^{-2}}\right) = 255$$