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Robotics: Dynamics and Control (ESE 446)
Professor Terry

Final Part 1: SRR/PDR Outline

Problem Statement

The problem involves developing a prototype of a robotic manipulator that will test new baseballs to be used by major league pitchers. Our robotic manipulator will throw a ball from the pitchers mound to home plate, subject to the requirement that the ball is thrown 60-80mph and within the strike box. Our proposed manipulator will be an anthropomorphic arm with 3 revolute joints and an end effector that releases the ball.

This report outlines the requirements of our robotic manipulator, following with the direct kinematics, inverse kinematics, geometric jacobian, and dynamics of the proposed manipulator.

Requirements

1. The robotic manipulator must be positioned at the pitcher's mound and must throw the ball from the mound to home plate (the mound is a distance of 60 feet, 6 inches from home plate and raised 10 inches above the level of home plate).
2. The ball must be thrown within the strike zone, where the strike zone is defined as the area over home plate with an upper limit at the midpoint between the top of the shoulders and the top of the uniform pants of the batter, and a lower limit at the bottom of the knees. The strike zone may vary among players, so the manipulator must take this into account.
3. The ball must be thrown at a speed in the range of 80-100 mph.

Kinematics and Dynamics

Direct Kinematics

DH Parameters of Anthropomorphic Arm

Link	a_i	d_i	α_i	θ_i
1	0	0	$\pi/2$	θ_1^*
2	a_2	0	0	θ_2^*
3	a_3	0	0	θ_3^*

Direct Kinematic Equations

$$r_3^0 = a_2 x_2^0 + a_3 x_3^0$$

$$r_3^0 = a_2 R_2^0 x_2^2 + a_3 R_3^0 x_3^3$$

$$r_3^0 = a_2 R_z(\theta_1^*) R_x\left(\frac{\pi}{2}\right) R_z(\theta_2^*) x_2^2 + a_3 R_z(\theta_1^*) R_x\left(\frac{\pi}{2}\right) R_z(\theta_2^*) R_z(\theta_3^*) x_3^3$$

Inverse Kinematics

$$\theta_1 = Atan2(r_{3y}^0, r_{3x}^0)$$

$$\cos(\theta_3) = \frac{(r_{3x}^0)^2 + (r_{3y}^0)^2 + (r_{2x}^0)^2 - (a_2)^2 - (a_3)^2}{2a_2 a_3}$$

$$\sin(\theta_3) = \pm\sqrt{1 - \cos{(\theta_3)^2}}$$

$$\theta_3 = Atan2(\sin(\theta_3), \cos(\theta_3))$$

$$\sin(\theta_2) = \frac{(a_2 + a_3\cos(\theta_3))\, r_{3z}^0 - a_3 \sin(\theta_3)\sqrt{{r_{3x}^0}^2 + {r_{3y}^0}^2}}{(r_{3x}^0)^2 + (r_{3y}^0)^2 + (r_{3z}^0)^2}$$

$$\cos(\theta_2) = \frac{(a_2 + a_3\cos(\theta_3))\sqrt{{r_{3x}^0}^2 + {r_{3y}^0}^2} + a_3 \sin(\theta_3)\, r_{3z}^0}{(r_{3x}^0)^2 + (r_{3y}^0)^2 + (r_{3z}^0)^2}$$

$$\theta_2 = Atan2(\sin(\theta_2), \cos(\theta_2))$$

Geometric Jacobian

$$r_{0,3}^0 = r_{1,3}^0 = a_2 R_z(\theta_1^*) R_x\left(\frac{\pi}{2}\right) R_z(\theta_2^*) x_2^2 + a_3 R_z(\theta_1^*) R_x\left(\frac{\pi}{2}\right) R_z(\theta_2^*) R_z(\theta_3^*) x_3^3$$

$$r_{2,3}^0 = a_3 R_z(\theta_1^*) R_x\left(\frac{\pi}{2}\right) R_z(\theta_2^*) R_z(\theta_3^*) x_3^3$$

$$R_2^0 = R_z(\theta_1^*) R_x\left(\frac{\pi}{2}\right) R_z(\theta_2^*) =$$

$$= \begin{bmatrix} c(\theta_1^*) & -s(\theta_1^*) & 0 \\ s(\theta_1^*) & c(\theta_1^*) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} c(\theta_2^*) & -s(\theta_2^*) & 0 \\ s(\theta_2^*) & c(\theta_2^*) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c(\theta_1^*)c(\theta_2^*) & -c(\theta_1^*)s(\theta_2^*) & s(\theta_1^*) \\ s(\theta_1^*)c(\theta_2^*) & -s(\theta_1^*)s(\theta_2^*) & -c(\theta_1^*) \\ s(\theta_2^*) & c(\theta_2^*) & 0 \end{bmatrix}$$

$$R_3^0 = R_z(\theta_1^*)R_x\left(\frac{\pi}{2}\right)R_z(\theta_2^*)R_z(\theta_3^*) =$$

$$= \begin{bmatrix} c(\theta_1^*)c(\theta_2^*) & -c(\theta_1^*)s(\theta_2^*) & s(\theta_1^*) \\ s(\theta_1^*)c(\theta_2^*) & -s(\theta_1^*)s(\theta_2^*) & -c(\theta_1^*) \\ s(\theta_2^*) & c(\theta_2^*) & 0 \end{bmatrix} \begin{bmatrix} c(\theta_3^*) & -s(\theta_3^*) & 0 \\ s(\theta_3^*) & c(\theta_3^*) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c(\theta_1^*)c(\theta_2^* + \theta_3^*) & -c(\theta_1^*)s(\theta_2^* + \theta_3^*) & s(\theta_1^*) \\ s(\theta_1^*)c(\theta_2^* + \theta_3^*) & -s(\theta_1^*)s(\theta_2^* + \theta_3^*) & -c(\theta_1^*) \\ s(\theta_2^* + \theta_3^*) & c(\theta_2^* + \theta_3^*) & 0 \end{bmatrix}$$

$$r_{0,3}^0 = r_{1,3}^0 = a_2 \begin{bmatrix} c(\theta_1^*)c(\theta_2^*) \\ s(\theta_1^*)c(\theta_2^*) \\ s(\theta_2^*) \end{bmatrix} + a_3 \begin{bmatrix} c(\theta_1^*)c(\theta_2^* + \theta_3^*) \\ s(\theta_1^*)c(\theta_2^* + \theta_3^*) \\ s(\theta_2^* + \theta_3^*) \end{bmatrix}$$

$$= \begin{bmatrix} a_2 c(\theta_1^*)c(\theta_2^*) + a_3 c(\theta_1^*)c(\theta_2^* + \theta_3^*) \\ a_2 s(\theta_1^*)c(\theta_2^*) + a_3 s(\theta_1^*)c(\theta_2^* + \theta_3^*) \\ a_2 s(\theta_2^*) + a_3 s(\theta_2^* + \theta_3^*) \end{bmatrix}$$

$$r_{2,3}^0 = a_3 \begin{bmatrix} c(\theta_1^*)c(\theta_2^* + \theta_3^*) \\ s(\theta_1^*)c(\theta_2^* + \theta_3^*) \\ s(\theta_2^* + \theta_3^*) \end{bmatrix}$$

$$= \begin{bmatrix} a_3 c(\theta_1^*)c(\theta_2^* + \theta_3^*) \\ a_3 s(\theta_1^*)c(\theta_2^* + \theta_3^*) \\ a_3 s(\theta_2^* + \theta_3^*) \end{bmatrix}$$

$$J_{P_1} = z_0^0 \times r_{0,3}^0$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_2 c(\theta_1^*) c(\theta_2^*) + a_3 c(\theta_1^*) c(\theta_2^* + \theta_3^*) \\ a_2 s(\theta_1^*) c(\theta_2^*) + a_3 s(\theta_1^*) c(\theta_2^* + \theta_3^*) \\ a_2 s(\theta_2^*) + a_3 s(\theta_2^* + \theta_3^*) \end{bmatrix}$$

$$= \begin{bmatrix} -s(\theta_1^*)(a_3 c(\theta_2^* + \theta_3^*) + a_2 c(\theta_2^*)) \\ c(\theta_1^*)(a_3 c(\theta_2^* + \theta_3^*) + a_2 c(\theta_2^*)) \\ 0 \end{bmatrix}$$

$$J_{O_1} = z_0^0 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J_{P_2} = z_1^0 \times r_{1,3}^0$$

$$= \begin{bmatrix} s(\theta_1^*) \\ -c(\theta_1^*) \\ 0 \end{bmatrix} \times \begin{bmatrix} a_2 c(\theta_1^*) c(\theta_2^*) + a_3 c(\theta_1^*) c(\theta_2^* + \theta_3^*) \\ a_2 s(\theta_1^*) c(\theta_2^*) + a_3 s(\theta_1^*) c(\theta_2^* + \theta_3^*) \\ a_2 s(\theta_2^*) + a_3 s(\theta_2^* + \theta_3^*) \end{bmatrix}$$

$$= \begin{bmatrix} -c(\theta_1^*)(a_3 s(\theta_2^* + \theta_3^*) + a_2 s(\theta_2^*)) \\ -s(\theta_1^*)(a_3 s(\theta_2^* + \theta_3^*) + a_2 s(\theta_2^*)) \\ a_3 c(\theta_2^* + \theta_3^*) + a_2 c(\theta_2^*) \end{bmatrix}$$

$$J_{O_2} = z_1^0 = \begin{bmatrix} s(\theta_1^*) \\ -c(\theta_1^*) \\ 0 \end{bmatrix}$$

$$J_{P_3} = z_2^0 \times r_{2,3}^0$$

$$= \begin{bmatrix} s(\theta_1^*) \\ -c(\theta_1^*) \\ 0 \end{bmatrix} \times \begin{bmatrix} a_3 c(\theta_1^*) c(\theta_2^* + \theta_3^*) \\ a_3 s(\theta_1^*) c(\theta_2^* + \theta_3^*) \\ a_3 s(\theta_2^* + \theta_3^*) \end{bmatrix}$$

$$= \begin{bmatrix} -a_3 s(\theta_2^* + \theta_3^*) c(\theta_1^*) \\ -a_3 s(\theta_2^* + \theta_3^*) s(\theta_1^*) \\ a_3 c(\theta_2^* + \theta_3^*) \end{bmatrix}$$

$$J_{O_3} = z_2^0 = \begin{bmatrix} s(\theta_1^*) \\ -c(\theta_1^*) \\ 0 \end{bmatrix}$$

$$J_G = \begin{bmatrix} J_{P_1} & J_{P_2} & J_{P_3} \\ J_{O_1} & J_{O_2} & J_{O_3} \end{bmatrix}$$

$$= \begin{bmatrix} -s(\theta_1^*)(a_3c(\theta_2^*+\theta_3^*)+a_2c(\theta_2^*)) & -c(\theta_1^*)(a_3s(\theta_2^*+\theta_3^*)+a_2s(\theta_2^*)) & -a_3s(\theta_2^*+\theta_3^*)c(\theta_1^*) \\ c(\theta_1^*)(a_3c(\theta_2^*+\theta_3^*)+a_2c(\theta_2^*)) & -s(\theta_1^*)(a_3s(\theta_2^*+\theta_3^*)+a_2s(\theta_2^*)) & -a_3s(\theta_2^*+\theta_3^*)s(\theta_1^*) \\ 0 & a_3c(\theta_2^*+\theta_3^*)+a_2c(\theta_2^*) & a_3c(\theta_2^*+\theta_3^*) \\ 0 & s(\theta_1^*) & s(\theta_1^*) \\ 0 & -c(\theta_1^*) & -c(\theta_1^*) \\ 1 & 0 & 0 \end{bmatrix}$$

Dynamics

Jacobian of the links and motors (determined from examining Jacobian of the end effector)

$$J_G^{m_1} = \begin{bmatrix} J_{p_1}^{m_1} & J_{p_2}^{m_1} & J_{p_3}^{m_1} \\ J_{o_1}^{m_1} & J_{o_2}^{m_1} & J_{o_3}^{m_1} \end{bmatrix}$$

$$J_G^{m_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ k_{r_1} & 0 & 0 \end{bmatrix}$$

$$J_G^{m_2} = \begin{bmatrix} J_{p_1}^{m_2} & J_{p_2}^{m_2} & J_{p_3}^{m_2} \\ J_{o_1}^{m_2} & J_{o_2}^{m_2} & J_{o_3}^{m_2} \end{bmatrix}$$

$$J_G^{m_2} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & k_{r_2}s_1 & 0 \\ 0 & -k_{r_2}c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$J_G^{m_3} = \begin{bmatrix} J_{p_1}^{m_3} & J_{p_2}^{m_3} & J_{p_3}^{m_3} \\ J_{o_1}^{m_3} & J_{o_2}^{m_3} & J_{o_3}^{m_3} \end{bmatrix}$$

$$J_G^{m_3} = \begin{bmatrix} -a_2c_2s_1 & -a_2c_1s_2 & 0 \\ a_2c_1c_2 & -a_2s_1s_2 & 0 \\ 0 & a_2c_2 & 0 \\ 0 & s_1 & k_{r_3}s_1 \\ 0 & -c_1 & -k_{r_3}c_1 \\ 1 & 0 & 0 \end{bmatrix}$$

$$J_G^{l_1} = \begin{bmatrix} J_{p_1}^{l_1} & J_{p_2}^{l_1} & J_{p_3}^{l_1} \\ J_{o_1}^{l_1} & J_{o_2}^{l_1} & J_{o_3}^{l_1} \end{bmatrix}$$

$$J_G^{l_1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$J_G^{l_2} = \begin{bmatrix} J_{p_1}^{l_2} & J_{p_2}^{l_2} & J_{p_3}^{l_2} \\ J_{o_1}^{l_2} & J_{o_2}^{l_2} & J_{o_3}^{l_2} \end{bmatrix}$$

$$J_G^{l_2} = \begin{bmatrix} -l_2 c_2 s_1 & -l_2 c_1 s_2 & 0 \\ l_2 c_1 c_2 & -l_2 s_1 s_2 & 0 \\ 0 & l_2 c_2 & 0 \\ 0 & s_1 & 0 \\ 0 & -c_1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

$$J_G^{l_3} = \begin{bmatrix} J_{p_1}^{l_3} & J_{p_2}^{l_3} & J_{p_3}^{l_3} \\ J_{o_1}^{l_3} & J_{o_2}^{l_3} & J_{o_3}^{l_3} \end{bmatrix}$$

$$J_G^{l_3} = \begin{bmatrix} s_1(l_3 c_{23} + a_2 c_2) & -c_1(l_3 s_{23} + a_2 s_2) & -l_3 s_{23} c_1 \\ c_1(l_3 c_{23} + a_2 c_2) & -s_1(l_3 s_{23} + a_2 s_2) & -l_3 s_{23} s_1 \\ 0 & l_3 c_{23} + a_2 c_2 & l_3 c_{23} \\ 0 & s_1 & s_1 \\ 0 & -c_1 & -c_1 \\ 1 & 0 & 0 \end{bmatrix}$$

B matrix

$$B(q) = \sum_{i=1}^{3} m_{l_i} J_p^{l_i T} J_p^{l_i} + J_{o_i}^{l_i T} R_i^0 I_{l_i}^i R_i^{0T} J_{o_i}^{l_i} + m_{m_i} J_p^{m_i T} J_p^{m_i} + J_{o_i}^{m_i T} R_{m_i}^{0T} I_{m_i}^{m_i} R_{m_i}^{0T} J_{o_i}^{m_i}$$

$$\begin{bmatrix} I_{l_1} + I_{l_2} + I_{m_2} + l_2^2 m_{l_2} + I_{l_3} c_{23}^2 + I_{l_3} s_{23}^2 + I_{m_3} c_2^2 + I_{m_3} s_2^2 + m_{l_3} c_1^2 (l_3 c_{23} + a_2 c_2)^2 - l_2^2 m_{l_2} s_2^2 + m_{l_3} s_1^2 (l_3 c_{23} + a_2 c_2)^2 + a_2^2 m_{m_3} c_1^2 c_2^2 \\ -a_2 m_{m_3} c_1 c_2 s_2 s_{1-2} \\ 0 \end{bmatrix}$$

The rest of the output from matlab (matrix too large to fit legibly on page):

$$\begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{21} & B_{22} & B_{23} \\ B_{31} & B_{32} & B_{33} \end{bmatrix}$$

$$B_{12} = -a2\text{^}2 * mm3 * cos(theta1) * cos(theta2) * sin(theta2) * (sin(theta1) - sin(theta2))$$

B_{13}= 0

B_{22}= Il2 + Il3 + Im3 + Im2*kr2^2 + a2^2*ml3 + a2^2*mm3 + l2^2*ml2 + l3^2*ml3 + 2*a2*l3*ml3*cos(theta3)

B_{23}= ml3*l3^2 + a2*ml3*cos(theta3)*l3 + Im3*kr3

B_{32}= ml3*l3^2 + a2*ml3*cos(theta3)*l3 + Im3*kr3

B_{33}= Im3*kr3^2 + ml3*l3^2

C matrix

$$C(q,\dot{q}) = c_{ij} = \sum_{i=1}^{3} c_{ijk}\dot{q}_k , \quad c_{ijk} = \frac{1}{2}\left(\frac{\partial b_{ij}}{\partial q_k} + \frac{\partial b_{ik}}{\partial q_j} - \frac{\partial b_{jk}}{\partial q_i}\right)$$

$$\begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

Output from Matlab (matrix too large to fit legibly on page):

C_{11}= (a2^2*mm3*sin(4*theta2))/4 - l3^2*ml3*sin(2*theta2 + 2*theta3) - (a2^2*ml3*sin(2*theta2))/2 - (a2^2*mm3*sin(2*theta1))/4 - (a2^2*mm3*sin(2*theta2))/4 - (a2^2*mm3*sin(2*theta1 + 2*theta2))/4 - (l2^2*ml2*sin(2*theta2))/2 - (a2*l3*ml3*sin(theta3))/2 - (3*a2*l3*ml3*sin(2*theta2 + theta3))/2

C_{12}= (a2^2*mm3*sin(4*theta2))/4 - (a2^2*mm3*sin(2*theta2))/4 - (a2^2*ml3*sin(2*theta2))/2 - (l2^2*ml2*sin(2*theta2))/2 - (a2^2*mm3*cos(2*theta1)*sin(2*theta2))/4 - (a2^2*mm3*cos(2*theta2)*sin(2*theta1))/2 - (l3^2*ml3*cos(2*theta2)*sin(2*theta3))/2 - (l3^2*ml3*cos(2*theta3)*sin(2*theta2))/2 - (a2^2*mm3*cos(theta1)*sin(theta2))/4 +

(3*a2^2*mm3*sin(3*theta2)*cos(theta1))/4 - a2*l3*ml3*cos(2*theta2)*sin(theta3) - a2*l3*ml3*sin(2*theta2)*cos(theta3)

C_{13}= -(l3*ml3*(l3*sin(2*theta2 + 2*theta3) + a2*sin(theta3) + a2*sin(2*theta2 + theta3)))/2

C_{21}= ml3*cos(theta1)^2*(l3*cos(theta2 + theta3) + a2*cos(theta2))*(l3*sin(theta2 + theta3) + a2*sin(theta2)) + ml3*sin(theta1)^2*(l3*cos(theta2 + theta3) + a2*cos(theta2))*(l3*sin(theta2 + theta3) + a2*sin(theta2)) + l2^2*ml2*cos(theta2)*sin(theta2) + a2^2*mm3*cos(theta2)*sin(theta2)^3 - a2^2*mm3*cos(theta2)^3*sin(theta2) + a2^2*mm3*cos(theta2)*sin(theta1)*sin(theta2)*(sin(theta1) - sin(theta2))

C_{22}= -a2*l3*ml3*sin(theta3)

C_{23}= -2*a2*l3*ml3*sin(theta3)

C_{31}= (l3*ml3*(l3*sin(2*theta2 + 2*theta3) + a2*sin(theta3) + a2*sin(2*theta2 + theta3)))/2

C_{32}= a2*l3*ml3*sin(theta3)

C_{33}= 0

Gravity Vector

$$g(q) = -\sum_{j=1}^{3} m_{l_j} g_0^T J_{p_i}^{l_i}(q) + \ m_{m_j} g_0^T J_{p_i}^{m_i}(q)$$

$$G_0 = [0\ 0 - g]^T$$

$$g(q) = \begin{vmatrix} 0 \\ g m_{l_3}(l_3 c_{23} + a_2 c_2) + a_2 g m_{m_3} c_2 + g l_2 m_{l_2} c_2 \\ g l_3 m_{l_3} c_{23} \end{vmatrix}$$