Noise in Parallel MRI

How to determine whether single-coil assumptions still hold (they don't)

Jesse Knight, Alan R Moody, April Khademi
*Image Analysis in Medicine Laboratory (IAMLAB), School of Engineering, University of Guelph **Department of Medical Imaging, University of Toronto

Background

Abstract

- Existing image processing tools require assumptions about MRI noise [1]–[3]:

1. Stationary PDF
2. Gaussian (or other known) PDF
3. Spatially Uncorrelated

- Assumptions may not hold for Parallel MRI, rendering existing tools invalid
- Need an exploratory analysis framework to test these assumptions

Parallel MRI

- Conventional MRI is notoriously slow (vs. CT)
- Parallel MRI promises $2\times$ to $4\times$ reduced scan time
- Subsample Fourier space using multiple spatially-sensitive readout coils [1]
- Using coil sensitivity profiles, the subsampled data can be [4]:
  - Unwrapped in the spatial domain (e.g. SENSE, Figure 1)
  - Interpolated in Fourier space (e.g. GRAPPA)
- PMRI for all future scanning protocols: thoracic, fMRI, any clinical

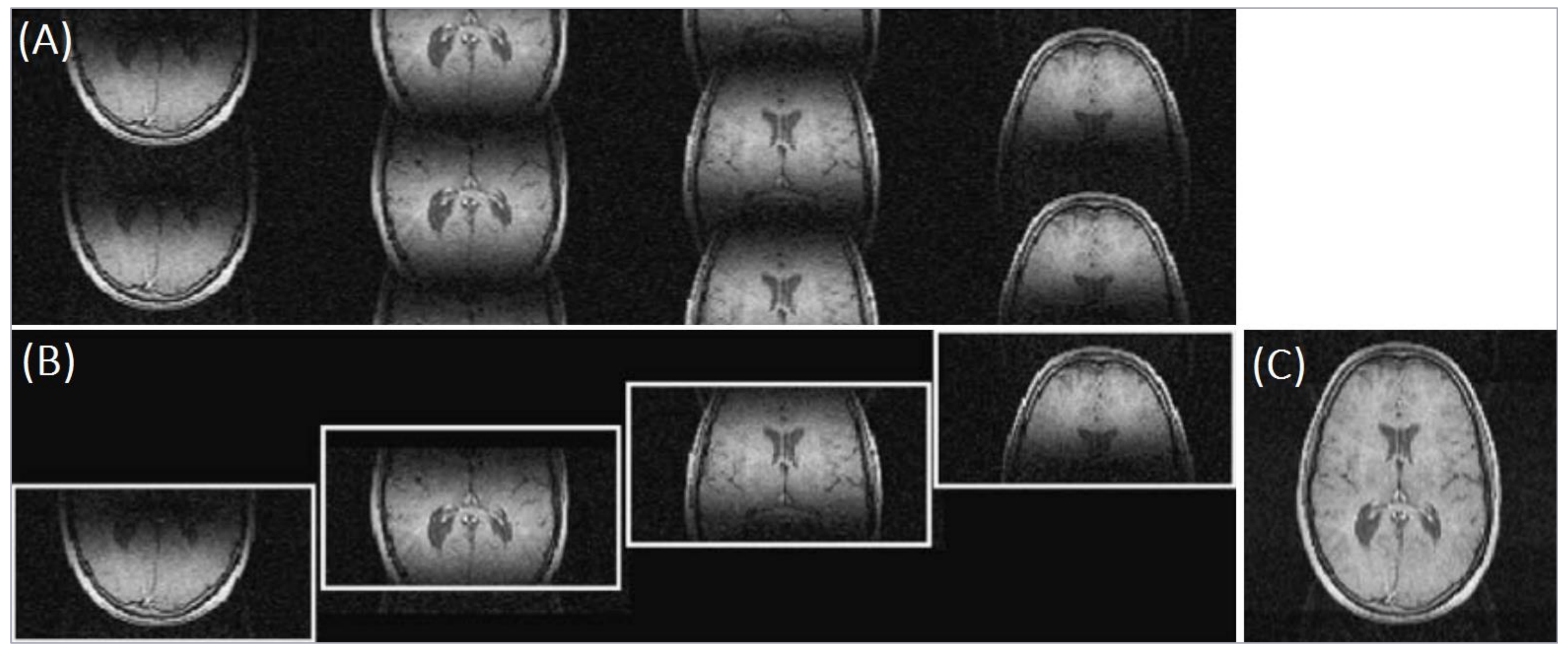


Figure 1: PRMI reconstruction (A) original spatial aliasing due to Fourier space subsampling (B) region selection based on coil sensitivity profiles (C) reconstructed image (adapted from [4])

Single Coil Noise Assumptions

- Conventional MRI noise distributions are modelled as additive field [1], [2]:

$$y(x_1,x_2) = f(x_1,x_2) + n(x_1,x_2), \quad \begin{cases} y(x_1,x_2): \textit{observed image} \\ f(x_1,x_2): \textit{ideal signal} \\ n(x_1,x_2): \textit{noise image} \end{cases}$$

- 3 single coil noise assumptions: 1. Stationary; 2. Known PDF; 3. Spatially Uncorrelated
- These assumptions incorporated into popular analysis tools:
  - Segmentation & coregistration [3]
  - Gaussian Mixture Models
  - Synthetic image volumes (*BrainWeb*)
- PMRI noise characteristics are different [2]: do these assumptions still hold?

Exploratory Noise Analysis

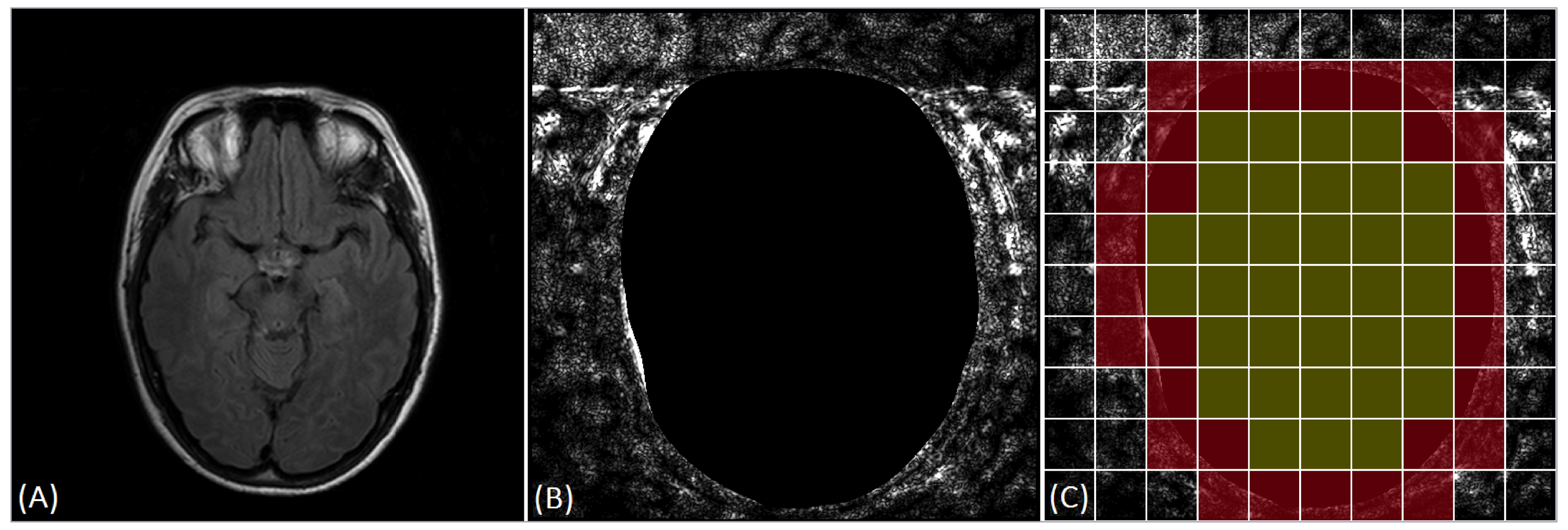
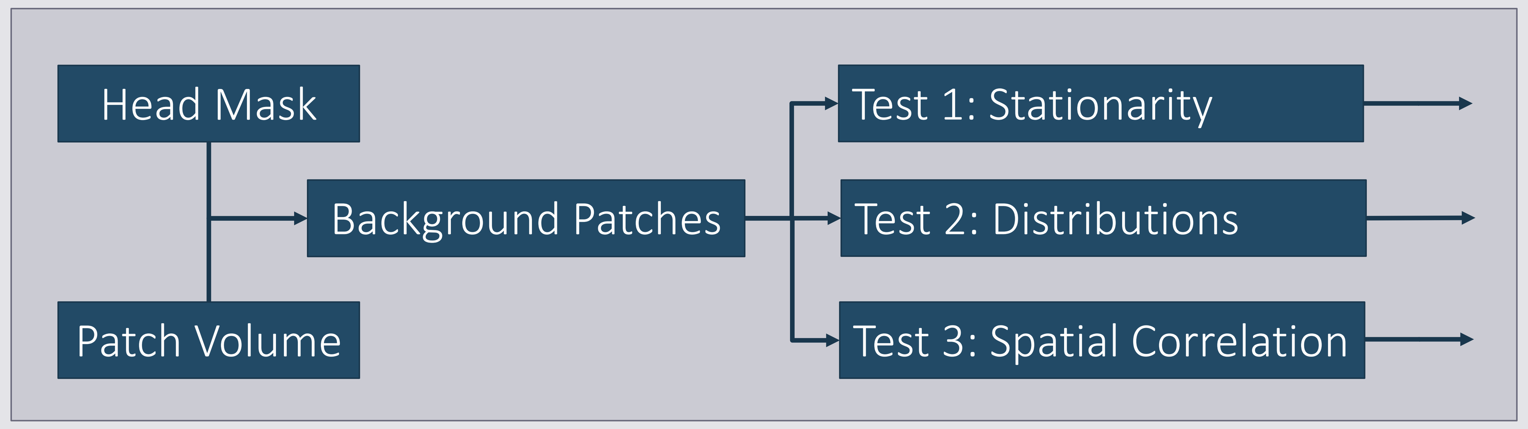


Figure 2: Patching pipeline (A) original PMRI FLAIR image (B) background signal at 100 x contrast scale (C) patched background signal: uncoloured patches are completely background and used for testing; yellow are completely head; red are a mixture.

Noise Analysis Pipeline



Test 1: Stationarity

- **Question**: Is the noise distribution consistent across the image plane?
- Test: 2-Sample Kolmogorov-Smirnov (KS) Test comparing patch distributions
- H_0: Data in both patches come from the same distribution

Test statistic $\Delta_{1,2}$ of the KS Test

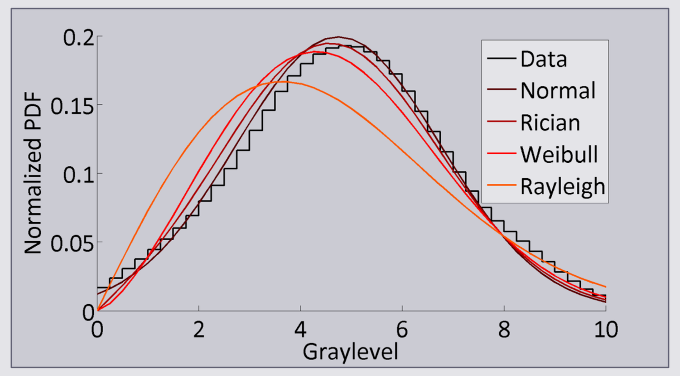
$$\Delta = \sup_Z |\,\mathrm{CDF}_1(Z) - \mathrm{CDF}_2(Z)\,|$$

$$\Delta_{1,2} > c(\alpha)\sqrt{\frac{N_1+N_2}{N_1 N_2}}$$

Test 2: Distributions

- **Question**: Do the noise data follow a known distribution, e.g. Gaussian, Rician, Rayleigh…
- Test: 2-Sample KS Test comparing patch data to EM-optimally fit distribution
- H_0: patch data comes from the distribution

Fitting distributions to patch data



Test 3: Spatial Correlation

- **Question**: Are the data in a given patch spatially correlated?
- Test: 2D Spatial Correlation (2DSC) Test: compare observed M_2 statistic with M_2 from random permutations of same data
- H_0: patch data are spatially uncorrelated

Test statistic M_2 of the 2DSC Test

$$W_{i,j} = \|s_i - s_j\|$$

$$U_{i,j} = |Z(s_i) - Z(s_j)|$$

$$M_2 = \sum_{i=1}^{N}\sum_{j=1}^{N} W_{i,j} U_{i,j}$$

Results

Rejecting Single Coil Assumptions

- Test database: 12 SENSE-reconstructed FLAIR volumes:
- Volume size: $560\times560\times50$ / Patch size: $56\times56\times1$ = 5000 patches per volume
  - **Test 1:** 96 % Nonstationary PDF
  - **Test 2:** 90 % No Standard Data Distribution
  - **Test 3:** 88 % Spatially Correlated
- Computational expense: infeasible to test all patches; random sampling sufficient

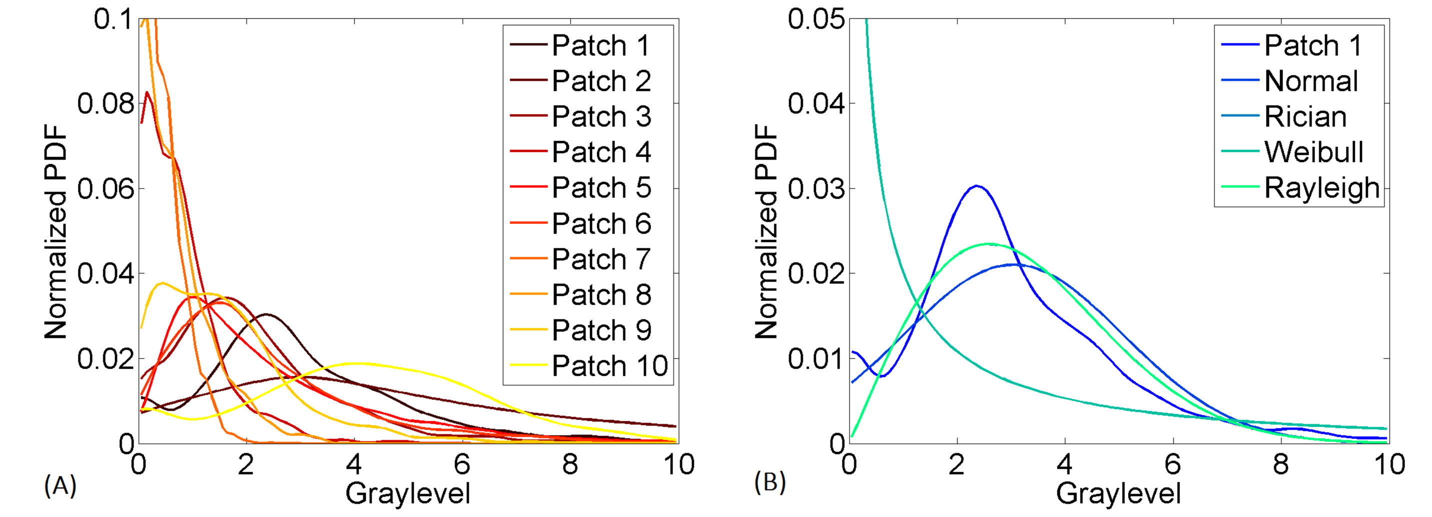


Figure 3. Sample patch probability distribution functions (a) from 10 randomly selected patches and (b) from a single patch, and the corresponding best fit distributions.

Table 1: Test results for 12 image volumes. Shown are the percentages (%) of patch tests for which the hypotheses of single-coil could not be rejected at $\alpha = 0.05$. Null Hypotheses are Stationarity: Test 1; PDF Match: Test 2; Spatial Uncorrelation: Test 3. *1000 patches tested only

Volume No.		1	2	3	4	5	6	7	8	9	10	11	12	Mean
# Tested Patches		2050	2498	2409	2408	2500	1917	1871	2092	2093	2113	2459	2468	2240
% Stationary		3.8	5.2	6.7	2.8	1.7	4.2	5.6	1.4	8.5	5.5	4	2.7	4.34
% PDF Match	Gaussian	0	0	0.2	0	0	0	1.2	0	0.2	0.1	0	0	0.14
	Rician	0.6	0.2	0.3	0.6	0.7	0.1	1.3	0.5	0.8	0.5	0.3	0.2	0.51
	Rayleigh	0.3	0.1	0	0.4	0.3	0.1	1	0.4	0.5	0.2	0.2	0.2	0.31
	Weibull	11.3	7.5	5.6	12.3	14.8	9.4	10	18.1	6.6	7.7	10.7	11.2	10.4
% Spatially Uncorrelated*		11.3	12.9	14.2	10.3	9.4	13.4	11.6	9.7	17.7	13.4	12	10.9	12.23

Future Work

- Investigate implementation parameters:
  - KS Test alternatives for discrete distributions (e.g. uint8 volumes)
  - Quantify impacts of patch size
- Apply testing framework to compare and contrast:
  - PMRI Reconstruction methods (synthetic vs. SENSE vs. GRAPPA)
  - Scanner manufacturers (GE vs. Siemens vs. Philips)
- Develop new model-free image analysis tools for when single-coil assumptions fail

References

[1] P. Thunberg and P. Zetterberg, "Noise distribution in SENSE- and GRAPPA-reconstructed images: a computer simulation study.," *Magn. Reson. Imaging*, vol. 25, no. 7, pp. 1089–1094, Sep. 2007.
[2] O. Dietrich, J. G. Raya, S. B. Reeder, M. Ingrisch, M. F. Reiser, and S. O. Schoenberg, "Influence of multichannel combination, … on MRI noise characteristics.," *Magn. Reson. Imaging*, vol. 26, no. 6, pp. 754–762, Jul. 2008.
[3] J. Ashburner and K. J. Friston, "Unified segmentation.," *Neuroimage*, vol. 26, no. 3, pp. 839–851, Jul. 2005.
[4] M. Blaimer, F. Breuer, M. Mueller, R. M. Heidemann, M. A. Griswold, and P. M. Jakob, "SMASH, SENSE, PILS, GRAPPA: how to choose the optimal method.," *Top. Magn. Reson. Imaging*, vol. 15, no. 4, pp. 223–36, Aug. 2004.

Acknowledgements

NSERC CRSNG
University of Guelph
Sunnybrook Research Institute
Medical Imaging University of Toronto