Choice Deferral and Search Fatigue*

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Abstract

When gathering information to make decisions, individuals often have to delay making a decision because the process of gathering information is interrupted, and the individual is not yet ready to make a decision. The paper considers a model of choice deferral based on time-varying search costs, potentially based on search fatigue, in which individuals have to strategically decide whether to defer choice, taking into account the current available information, and when they will have again a chance to gain further information. We find that individuals are more likely to defer choice when the amount of information gained when gathering information is greater, when there is an opportunity to gather information again sooner, when the individuals discount less the future, and when the likelihood of having search fatigue is lower. We also consider the case in which individuals incur costs of re-starting a process of information gathering, and when the individual has greater information about the extent of search fatigue.

1. Introduction

When gathering information to make decisions, individuals have often to delay making a decision because the process of gathering information is interrupted, and the individual is not yet ready to make a decision. This possible choice deferral can be based on time-varying search costs, potentially based on search fatigue, and individuals have to strategically decide whether to defer choice, taking into account the current available information, and when they will have again a chance to gain further information. Choice deferrals occur often in the health, food, financial, entertainment, and general consumption domains.

For example, when a consumer searches online for information to decide whether to purchase a certain product, the consumer may at some point have to interrupt the search process without yet sufficiently diagnostic information, and at that point has to decide whether to purchase the product right away, or delay purchase until a future data when the consumer is again able to gather information on the product. The interruption could be based on search fatigue increasing the search costs, or because the opportunity costs of searching for information increased temporarily. This could be, for example, because the consumer starts gathering information before dinner, and then the dinner time arrives and the consumer can choose to either purchase the product then, or delay choice when the consumer has again a chance to look for information after dinner. Interruptions can also occur in offline shopping, when after shopping for a while the consumer may have to leave at some point, without having made a decision.

Another possibility could be a manager having to make a strategic decision, for example, whether to launch a new product. The manager could be gathering information about whether to launch the product, and then have that process be interrupted with another managerial activity that may last some period of time. The manager then has to decide whether to launch the product right away, or wait until the manager has more time to analyze the potential success of the product launch.

In order to consider the possibility of choice deferral, we formulate a model in which an individual can be either in a state of low search or high search costs, and move across states at some hazard rate. In the consumer setting, these hazard rates could be relatively high for online shopping, but be lower for offline shopping, especially if it is a store that is of difficult access. The individual gathers information when she is in the low search costs, and prefers not to gather information when she is in the high search costs state. To simplify the analysis we consider that the individual has zero search costs in the low search costs state, and very high search costs in the high search costs state.

In order to obtain strategic effects at the time when search costs increase, we consider that the individual either discounts the future or has a fixed cost of re-starting the search process. Either of these possibilities leads the individual, at the time when the search costs increase, to potentially decide not to defer choice, even though the individual did not make a choice up to that point. If there is neither discounting nor fixed costs of re-starting the search, individuals would just defer choice automatically when the search costs increase.

When gathering information (that is, in the low search costs state), the individual makes a choice if the individual obtains sufficiently diagnostic information. When the search costs increase, the consumer may decide to make a choice then, because, even though the individual did not receive sufficient positive information, the current evaluation is close enough such that it is better to make the choice now, than wait for the search costs to come down again.

We find that individuals are more likely to defer choice when the amount of information gained when gathering information is greater, when there is an opportunity to gather information again sooner, when the individuals discounts less the future, and when the likelihood of having search fatigue is lower. If information gained when gathering information is greater, there is a greater payoff of deferring choice, and waiting for more information to arrive. When the opportunity to gather information again occurs sooner, or individuals discount less the future, the benefits of having greater information becomes in expectation more valuable when evaluated at the time when the deferral decision is made, and individuals defer more choice. When the likelihood of having search fatigue is greater, the individuals know that when they will have again a change to gather information, they may get tired quickly, therefore leading to a lower payoff of deferring choice. Seen the other way, the individuals do more choice deferral when the likelihood of search fatigue is lower.

We also consider the case in which individual incur costs of re-starting a process of information gathering.[1] In that case, we do not need discounting for there to be strategic effects at the time when the choice deferral decision is made. This case could be important empirically as the extent of time discounting between different opportunities to gather information may be relatively small (for example, weeks), which would lead by itself to small strategic effects at the time when the choice deferral decision is made. The existence of costs of re-starting the information gathering phase can be see as a possibility, and lead to significant strategic

[1]See Byrne and de Roos (2022) on evidence on the existence of start-up search costs.

effects at the time when the choice deferral decision is made.

As the individual does more search, the individual may learn that she is getting more tired of search. In that case we find, that the individual requires a less diagnostic information for the individual to decide to make a choice before having to decide of whether to defer choice. The extent of less information is decreasing in the discount rate and in the likelihood of returning to a fully-rested search mode, and increasing in the rate at which the individual gets fatigued from search, and in the amount of information gained during search.

A greater discount rate makes the individual less willing to wait for further information, and this leads the decrease in requirements to make the choice to be also less accentuated when the individual gets tired from search. A greater speed with which the individual returns to the fully-rested search mode makes the individual to be less strategic on reducing the requirements to make a choice when she has more search fatigue. A greater rate of search fatigue makes an individual to be more concerned about not being able to do further search, and therefore the individual reduces further the requirements to make a choice when there is greater search fatigue. If there is a greater amount of information gained during search, the individual has greater requirements to make a choice, and this also leads to a further reduction of requirements to make a choice when the individual gets tired from search.

We also derive the firm's optimal pricing strategy given the DM's choice deferral behaviors. If the initial expected value of adopting the alternative is low, the firm sets a price such that the consumer does not adopt immediately in both search mode and no-search mode. In such a case, we find that the optimal price should be higher when the amount of information gained when gathering information is greater, when there is an opportunity to gather information again sooner, when the individuals discounts less the future, and when the likelihood of having search fatigue is lower. These results show how firms should use data on consumer browsing sessions to determine price, and provide managerial implications on how price should change following other interventions to reduce search fatigue or restart consumer search sooner, such as redesigning user interface, ad re-targeting and push notifications. We also find that the comparatives statics can be reversed if the initial expected value of adopting the alternative is sufficiently high.

There is substantial work documenting the existence of choice deferrals by individuals, because of the inability to make a decision (see Anderson 2003, Chernev, Böckenholt, and Goodman 2015, Scheibehenne, Greifeneder, and Todd 2010, for reviews). This work has characterized the causes for choice deferral, and its consequences. For example, this work has investigated the role of dominance relations, option desirability, attribute commonality, and attribute alignability on choice deferral (e.g., Chernev and Hamilton 2009, Dhar 1997, Gourville and Soman 2005, Tversky and Shafir 1982), and that the option of choice deferral may affect individual choices and affect behavioral effects (e.g., Dhar and Simonson 2003). Bhatia and Mullet (2016) consider a sequential learning model with the possibility of choice deferral which provides an explanation for several of the behavioral effects obtained. A significant explanation for not choosing has been choice overload, the existence of too many options may deter choice, which can also been seen as deferral of choice. Examples of work providing explanations for this effect of choice overload include Kamenica (2008), Kuksov and Villas-Boas (2010), Villas-Boas (2009). Here, the existence of multiple alternatives is not going to play any role, and the decision of choice deferral comes from the difficulty of the decision being made, and from time varying search costs (or, alternatively, time-varying information gained). There is also work showing evidence of search gaps and search fatigue when consumers search across multiple alternatives (e.g., Ursu, Zhang, and Honka 2022).

In relation to the existing literature, a significant innovation of this paper is to formally consider the next choice opportunities once choice is deferred. That is, while in the existing literature choice deferral is considered as no choice forever, here we formally consider the possibility of future choice, once the individuals have again a chance of searching for information.

The remainder of the paper is organized as follows. The next section introduces a basic model of choice deferral with discounting. Section 3 presents the analysis and results, and Section 4 considers the effect of search fatigue. Section 5 considers the case in which there are start-up search costs. Section 6 discusses optimal pricing. Section 7 concludes. The Appendix collects the proofs of the the results.

2. The Model

Consider the following simple model of choice deferrals. A decision-maker (DM) is gradually collecting information about whether to take an alternative. Suppose time is continuous. The DM can be either in a “search” mode or in a “no-search” mode. Whether the DM is in the “search” or in the “no-search” mode is exogenous, and tries to capture the idea that the DM sometimes has low costs of learning information about the alternative, and other times the DM has high costs of learning information about the alternative.

If the DM is in the search mode, the DM moves to the no-search mode with constant hazard rate λ. If the DM is in the no-search mode, the DM moves to the search mode with constant hazard rate β.

This set-up captures the idea that the DM has sometimes the ability to search for information, and other times cannot search for information. When the DM is in the search mode, the DM uptates the DM's beliefs about the alternative, and can choose to adopt the alternative at any time. When the DM switches from the search mode to the no-search mode, the DM may want to lower the threshold to adopt the alternative, as the DM will be for some period of time without learning new information about the alternative. At that instance if the DM's beliefs about the alternative are not sufficiently high the DM will choose to defer choice until the DM is again in the choice mode. This can also be interpreted as search fatigue, as the DM suddenly has a high cost of search for information, stops getting information on the alternative, and decides to delay making a choice until the DM has again a chance to learn more information about the alternative (the DM gets sufficiently rested such that the DM returns to the search mode).

The existence of constant hazard of moving between the search and no-search mode allows the problem to be stationary so that the threshold of whether to adopt the alternative is constant over time. If the hazard rates of moving between search modes are not constant, then the threshold of whether to adopt the alternative would also not be constant leading to significant complications in the analysis (it could still be characterized numerically, but analytic results would be difficult to obtain).

Note that this set-up can be interpreted as search fatigue leading to the DM stopping to search with the DM being endowed with a search fatigue limit when starting a search process, but not knowing when that search fatigue limit is. With a constant hazard rate the process is memoryless, and therefore, from the point of view of the DM, she gets search fatigue with the same likelihood, independently of how long the DM has been searching. If the DM understands over time that she is getting fatigued from search we would expect that the hazard rate of moving from the search mode to the no-search mode to be increasing in the length of time that the DM has been in the search mode. This would lead the threshold to adopt the alternative to vary over time (in fact, to decrease with the length of time in the search mode), which would be more complicated to analyze. This case is considered in Section 4 when search fatigue occurs over two stages.

Note that, similarly, we could expect that the hazard rate of moving from the no-search mode to the search mode to be increasing in the length of time spent in the no-search mode, because a greater rest from search should lead to a greater likelihood of being ready to search for information again. This possibility would not affect the results presented here, as the

DM would prefer to continue waiting until the switch to the search mode, as that switch is expected to be sooner.

At a moment in time the DM has some expected value of the payoff of the alternative, which we denote by x. When the DM is in the search mode x evolves as a Brownian motion with a constant variance σ^2. This can be interpreted as the DM learning over time about equally important and independent attributes, and there being an infinite number of attributes. When the DM is in the no-search mode the expected value x stays fixed (as no information is gained).

If learning is done with signals about the overall value of the product, or if attributes have unequal importance with the DM checking first the most important attributes, or if there is non-zero correlation between the attributes, then we would have σ^2 to be decreasing over time, leading again to a threshold to adopt the alternative that is varying (decreasing) over time, which is a more complicated case to consider. The case presented here can be seen as the extreme case in the amount of information learned over time is constant, in contrast to the other extreme case in which everything about the alternative is learned at the first time. The real-world would be somewhere between these two extreme cases.

The payoff of not adopting the alternative is set at zero. The DM discounts the future at a continuous-time discount rate r and does not incur any on-going search costs when learning information. The optimal search behavior of the DM would be to adopt the alternative, when in the search mode, if the expected payoff of the alternative x reaches a threshold $\overline{x}$. When in the no-search mode, the DM would adopt the alternative if the expected payoff of the alternative is above some threshold $\widetilde{x}$. We note that $\widetilde{x} < \overline{x}$. Furthermore, note that at the instant at which the DM moves from the search mode to the no-search mode, if $x \in [\widetilde{x}, \overline{x})$, the DM chooses to adopt the alternative immediately because of the delay of getting any additional information. If at that instance $x < \widetilde{x}$ the DM decides not to adopt the alternative then, and waits until the DM switches again to the search mode and gain further information then. This is the case in which the DM defers choice. Note that this means that there is a positive mass probability of the DM adopting the alternative at an instant when the DM moves from the search to the no-search mode. The next section will study how to obtain the thresholds $\overline{x}$ and $\widetilde{x}$, and their properties.

Note that discounting is crucial for the problem as presented. If there is no discounting no DM would adopt the alternative when the DM moves from the search to the no-search mode, and there is no benefit of strategic choice deferral (there is just automatic choice deferral). One alternative to discounting is to have start-up search costs, each time the search mode starts, and that case is considered in Section 5.

This basic model assumes that that there are not on-going search costs. If there are on-going search costs when learning for information, then the DM would also have a lower threshold for when to never engage in the search process again if the expected payoff of adopting the alternative is low enough. We do not consider this case in the base model to simplify the analysis, as this case is not essential to obtain the choice deferral effects. The on-going search costs are considered in Section 5.

The assumption that the payoff of not adopting the alternative is zero is not without loss of generality. In fact, if this payoff is positive the DM has to consider the trade-off of losing the discounted payoff of not adopting the alternative versus continuing to search for further information on the possible alternative. This would lead again to the existence of a lower threshold for when to never engage in the search process again and take the no adoption option, if the expected payoff of adopting the alternative is low enough. We again do not consider this possibility to simplify the analysis, as this possibility is not essential to obtain the choice deferral effects.

3. Analysis

In order to consider the optimal decisions of the DM, we have to consider the expected present discounted value of the DM under the optimal decisions, depending on the state in which the DM is. Let $V(x)$ be the expected discounted payoff for the DM if the DM is in the search mode, and $W(x)$ be the expected payoff for the DM if the DM is in the no-search mode.

The Bellman equation for $V(x)$ for $x < \widetilde{x}$ can be written as

$$V(x) = (1 - \lambda\, dt)e^{-r\, dt} EV(x + dx) + \lambda\, dt W(x). \tag{1}$$

(Note that we could have $e^{-r\, dt} EW(x+dx)$ instead of $W(x)$ in (1) and the subsequent analysis would not change, as the second order terms in dt disappear.) The Bellman equation for $V(x)$ for $x \in (\widetilde{x}, \overline{x})$ can be written as

$$V(x) = (1 - \lambda\, dt)e^{-r\, dt} EV(x + dx) + \lambda\, dt x. \tag{2}$$

Finally, the Bellman equation for $W(x)$ can be obtained to be

$$W(x) = \beta\, dt V(x) + (1 - \beta\, dt)e^{-r\, dt} W(x), \tag{3}$$

from which one can obtain $W(x) = \frac{\beta}{r+\beta} V(x)$. Substituting into (1), and using Itô's Lemma, we can obtain the second order differential equation in $V(x)$ for $x < \widetilde{x}$ as

$$r\frac{r+\beta+\lambda}{r+\beta} V(x) = \frac{\sigma^2}{2} V''(x). \tag{4}$$

Given that $\lim_{x \to -\infty} V(x) = 0$, as the expected payoff of the DM has to approach zero if the expected payoff of the alternative approaches negative infinity, we have that the solution to (4) satisfies

$$V(x) = A_1 e^{\mu x} \tag{5}$$

where $\mu = \sqrt{\frac{2r}{\sigma^2}\frac{r+\beta+\lambda}{r+\beta}}$ and A_1 is a constant to be determined.

Similarly, applying Itô's Lemma to (2), we can obtain the second order differential equation in $V(x)$ for $x \in (\widetilde{x}, \overline{x})$ as

$$V(x) = A_2 e^{\widetilde{\mu} x} + A_3 e^{-\widetilde{\mu} x} + \frac{\lambda}{r+\lambda} x, \tag{6}$$

where $\widetilde{\mu} = \sqrt{\frac{2(r+\lambda)}{\sigma^2}}$ and A_2 and A_3 are constants to be determined.

Using value matching and smooth pasting of $V(x)$ at $\widetilde{x}$ and $\overline{x}$, $V(\widetilde{x}^-) = V(\widetilde{x}^+), V'(\widetilde{x}^-) = V'(\widetilde{x}^+)$, $V(\overline{x}) = \overline{x}$, and $V'(\overline{x}) = 1$, and $W(\widetilde{x}) = \widetilde{x}$, we obtain a system of five equations (presented in the Appendix) to obtain $\widetilde{x}, \overline{x}, A_1, A_2$, and A_3.

Using $\delta = \overline{x} - \widetilde{x}$ we can obtain (see Appendix)

$$\beta(D-1)\left\{\mu(r+\lambda)\left[1 + D - \frac{\delta\widetilde{\mu}}{D-1}(1+D^2)\right] + \widetilde{\mu}(r-\lambda)(D-1) - \delta\widetilde{\mu}^2 r(1+D)\right\} + \\ (r+\lambda)[r(D^2-1)(\mu - \widetilde{\mu}^2\delta) + r\widetilde{\mu}(1+D^2)(1-\mu\delta) + 2\widetilde{\mu}\lambda D] = 0 \tag{7}$$

which determines δ, where $D = e^{\widetilde{\mu}\delta}$. We can then also obtain $\widetilde{x}$ as a function of δ as

$$\widetilde{x} = \beta \frac{r + r\widetilde{\mu}\delta + \lambda D}{D[\widetilde{\mu} r(r+\beta+\lambda) + \mu(r+\beta)(r+\lambda)] - \widetilde{\mu}\beta r}. \tag{8}$$

Figure 1 illustrates a sample path in which the individual makes the decision to take the alternative during the search mode after several choice deferrals. Figure 2 illustrates a sample path in which the individual makes the decision to take the alternative when switching from the search to the no-search mode after several choice deferrals.

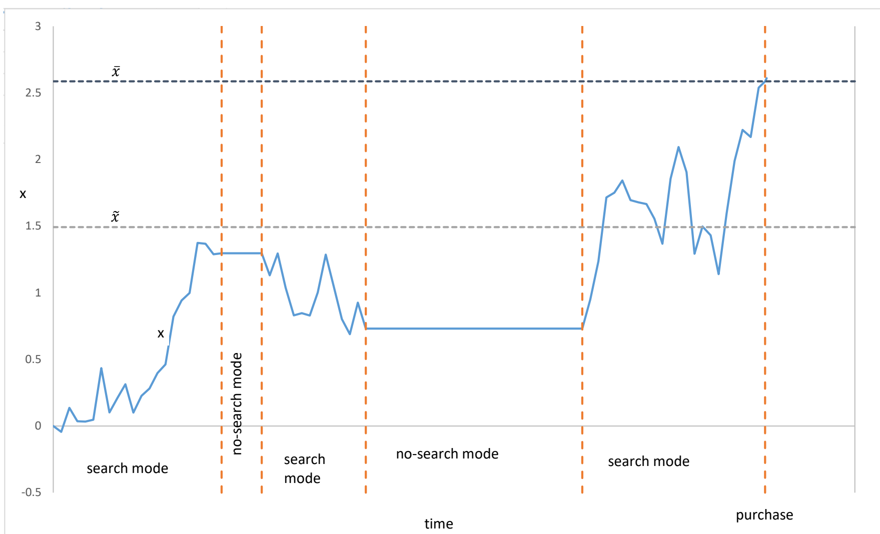

Figure 1: Example of sample path of individual expected payoff when making decision during the search mode with $x_0 = 0, r = .05, \lambda = \beta = .5$, and $\sigma^2 = 1$. For these parameter values we have $\overline{x} \approx 2.59$ and $\widetilde{x} = 1.49$.

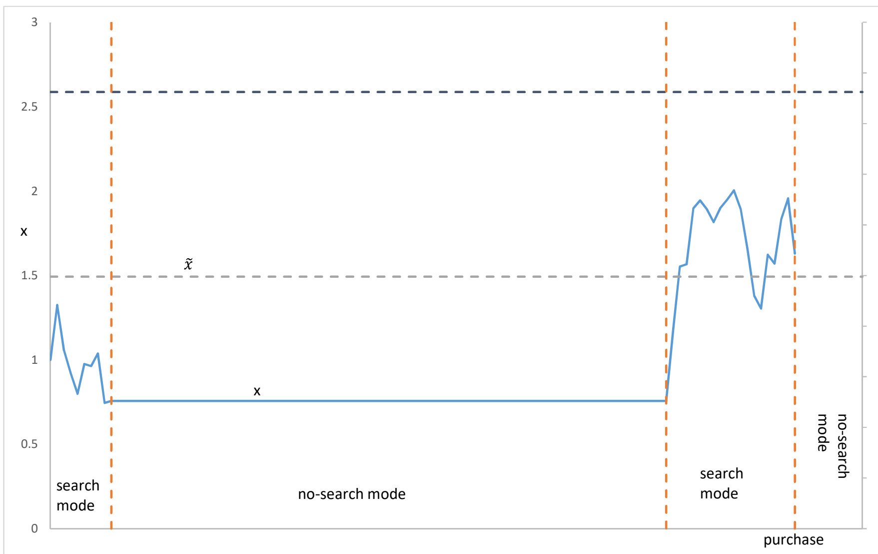

Figure 2: Example of sample path of individual expected payoff when making decision when moving search to no-search mode with $x_0 = 1, r = .05, \lambda = \beta = .5$, and $\sigma^2 = 1$. For these parameter values we have $\overline{x} \approx 2.59$ and $\widetilde{x} = 1.49$.

Given the complexity of (7) it is difficult to analyze properties of $\overline{x}$ and $\widetilde{x}$ for the general case. It is however possible to consider some particular cases, and we focus here in the cases in which $\beta \to 0$, the case in which once the DM leaves the search mode the DM never comes back to that mode, and in which $\beta \to \infty$, the case in which DM returns infinitely quickly to the search mode, once the DM goes to the no-search mode. We then present numerical illustrations of the general case.

Case of $\beta \to 0$:

In the case of $\beta \to 0$ we have that $\widetilde{x} \to 0$, such that when the search mode ends the DM adopts the alternative as long as $x \geq 0$. We can also then obtain that $\overline{x}$ in the limit solves

$$e^{\mu\overline{x}}(1-\mu\overline{x}) + \frac{\lambda}{r} = 0. \tag{9}$$

From this we can obtain that $\overline{x} > 1/\mu$ and that at the limit $\overline{x}$ is increasing in σ^2 and decreasing in λ and r.

For this case, we can obtain that $\widetilde{x}/\beta \to \frac{1}{r(\mu+\widetilde{\mu})}$, which shows that for β small, $\widetilde{x}$ is increasing in both β and σ^2, and decreasing in the discount rate r, and the hazard rate of moving to the no-search mode, λ.

We collect these results in the following proposition.

Proposition 1. *Suppose that $\beta \to 0$. Then the purchase threshold in the search mode, $\overline{x}$ is increasing in σ^2 and decreasing in both λ and r, and the purchase threshold in the no-search mode, $\widetilde{x}$, is increasing in both β and σ^2, and decreasing in both λ and r.*

As the information gained in the search mode, σ^2, is greater the DM gains more from search, and chooses to search more, which results in both purchase thresholds to increase. When the discount rate increases, the present value of delaying purchase is reduced, and therefore the DM searches less, which means that both purchase thresholds fall. Similarly, when the likelihood of moving from the search mode to the no-search mode increases, the likelihood of being able to continue to search decreases, and therefore the DM prefers to make the adoption decision sooner, which means that both purchase thresholds fall. The extent of choice deferral can be seen as decreasing in δ, and, therefore, is decreasing in σ^2 and increasing in the λ and r. As the amount of information gained during the search mode, σ^2, increases the DM raises the purchase threshold when in the search mode, $\overline{x}$, but then accepts almost any outcome above zero when moving to the no-search mode, which means that the extent of choice deferral is lower. Similarly, as the discount rate, r, or the rate at which the DM moves from the search to the no-search mode, λ, increases, the DM decreases the purchase threshold when in the search mode, $\overline{x}$, and therefore the extent of choice deferral increases.

Case of $\beta \to \infty$:

In the case of $\beta \to \infty$ we have that $\delta \to 0$ and $\overline{x}, \widetilde{x} \to \sqrt{\frac{\sigma^2}{2r}}$. This shows that, as one may expect, when the DM is more likely to come back to the search mode, the DM is more demanding on the expected payoff of the alternative to decide to adopt it (in comparison to the case of $\beta \to 0$).

In this case of $\beta \to \infty$ it is also interesting to see the rate at which δ converges to zero, and the rate at which $\overline{x}$ and $\widetilde{x}$ converge to $\sqrt{\frac{\sigma^2}{2r}}$.

To see this note that as $\beta \to \infty$ we can obtain from (7) that

$$\beta(D-1)^2 \to 2(r+\lambda) \tag{10}$$

from which we can obtain that

$$\delta\sqrt{\beta} \to 2\frac{r+\lambda}{\sigma}, \tag{11}$$

which shows that the difference in the thresholds between not deferring choice and deferring choice, δ, is decreasing on what is learned during the search mode, σ^2, increasing in the rate at which the search mode is interrupted, λ, and in the discount rate, r, and decreasing in the rate at which the DM returns to the search mode from the no-search mode, β. The effect of the amount of learning is interesting. As there is a greater amount of learning, there is a greater difference between the search and the no-search state, and therefore the DM is more likely to choose to defer choice. As the discount rate is greater, the benefit of deferring choice is lower, and therefore the DM has a lower preference to defer choice, a greater δ. As the rate at which the search mode is interrupted is greater, the benefit of deferring choice is also lower (as when the search state occurs, it will be not as long lived), and the DM has again a lower preference to defer choice, a greater δ.

Note that the comparative statics on δ (and therefore extent of choice deferral) of σ^2, r, and λ are the opposite from the case of $\beta \to 0$. When $\beta \to 0$, the chances of returning to the search mode are very low, leading the effect of σ^2, r, and λ on choice deferral to be mostly through their effect on the purchase threshold in the search mode, $\overline{x}$, which is increasing in σ^2 and decreasing in r and λ. On the other hand, when $\beta \to \infty$ the changes of returning to the search mode are very high, and therefore the effect of these variables on choice deferral can be seen as mostly through $\widetilde{x}$, leading to the opposite effects on choice deferral.

With this result on δ we can now turn to the question of the levels of the thresholds $\overline{x}$ and $\widetilde{x}$. To see this we can obtain from (8), taking the limit when $\beta \to \infty$,

$$\widetilde{x} \;\to\; \frac{1}{\mu} - \delta\frac{r}{r+\lambda} \tag{12}$$

$$\overline{x} \;\to\; \frac{1}{\mu} + \delta\frac{\lambda}{r+\lambda}, \tag{13}$$

from which we can obtain that $\widetilde{x}$ is lower than $1/\mu$ and that $\overline{x}$ is greater than $1/\mu$ for β large. We can also obtain that both $\widetilde{x}$ and $\overline{x}$, when β is large, are increasing in σ^2 and decreasing in r and λ. We summarize these results in the following proposition.

Proposition 2. *Suppose that $\beta \to \infty$. Then the purchase threshold in both the search mode and in the no-search mode, $\overline{x}$ and $\widetilde{x}$, are increasing in σ^2 and decreasing in both λ and r.*

General Case

We illustrate the results for the general case in Figures 3-6.

Figure 3 illustrates how the purchase thresholds $\overline{x}$ and $\widetilde{x}$ increase with the rate at which the individual switches from the no-search mode to the search mode, β, and that the difference $\overline{x} - \widetilde{x}$, which is inversely related to the extent of choice deferral, decreases with β.

Figure 4 illustrates how the purchase thresholds $\overline{x}$ and $\widetilde{x}$ decrease with the discount rate r, for a case of β low ($\beta = .1$), and a case of β high ($\beta = 5$). The figure also illustrates how the difference $\overline{x} - \widetilde{x}$ decreases in r for β low, but increases in r for β high. As discussed above, when β is low, a higher discount rate has a greater effect on the purchase threshold in the search mode, $\overline{x}$, which leads to a decrease in the difference $\overline{x} - \widetilde{x}$, which means an increase in choice deferral. When β is high, an increase in the discount rate has the effect of decreasing choice deferral, which means that the difference $\overline{x} - \widetilde{x}$ is greater.

Figure 5 illustrates how the purchase thresholds $\overline{x}$ and $\widetilde{x}$ decrease with the rate at which the the individual switches from the search mode to the no-search mode, λ, for a case of β low ($\beta = .1$), and a case of β high ($\beta = 5$). The figure illustrates how the difference $\overline{x} - \widetilde{x}$ decreases in λ for β low, but can increase in λ for β high. The rationale is similar to the one regarding the effect of the discount rate discussed above.

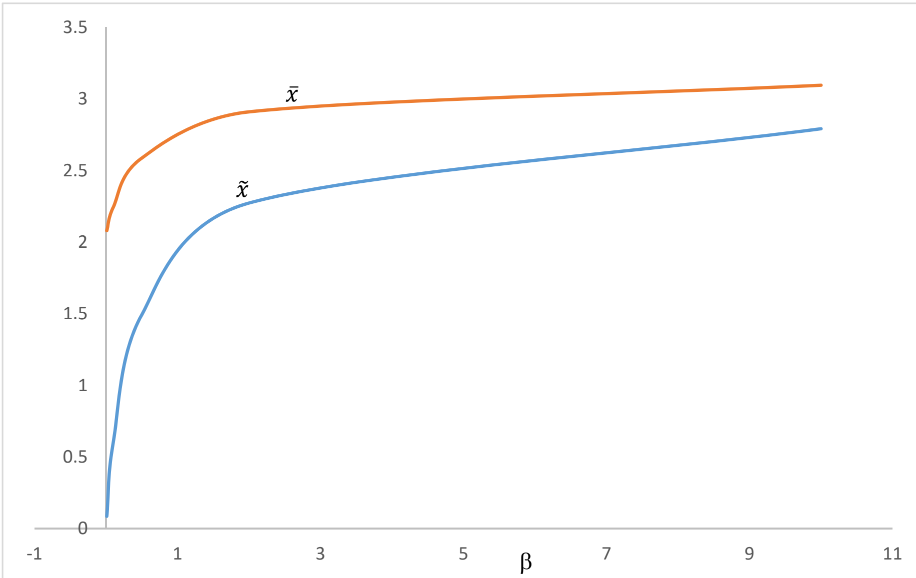

Figure 3: Example of the purchase thresholds $\overline{x}$ and $\widetilde{x}$ as a function of β for $r = .05, \lambda = .5$, and $\sigma^2 = 1$.

Figure 6 illustrates how the purchase thresholds $\overline{x}$ and $\widetilde{x}$ increase with the amount of information learned in the search mode, σ^2, for a case of β low ($\beta = .1$), and a case of β high ($\beta = 5$). The figure illustrates how the difference $\overline{x} - \widetilde{x}$ also increases in σ^2, showing how the extent of choice deferral decreases in σ^2.

4. Two Search Modes

We now consider a set-up in which the DM can become aware of being tired from search over time. We consider this possibility with the existence of two search modes, search mode 1 and search mode 2. The DM moves from search mode 1 to search mode 2 at hazard rate λ, and then from search mode 2 to the no-search mode at also the hazard rate λ. Once in the no-search mode the DM can move to search mode 1 at hazard rate β. This captures the idea of the DM being aware of search fatigue, by the DM realizing that once in search mode 2, the DM is aware that the switch to no-search mode will come sooner than in search mode 1.

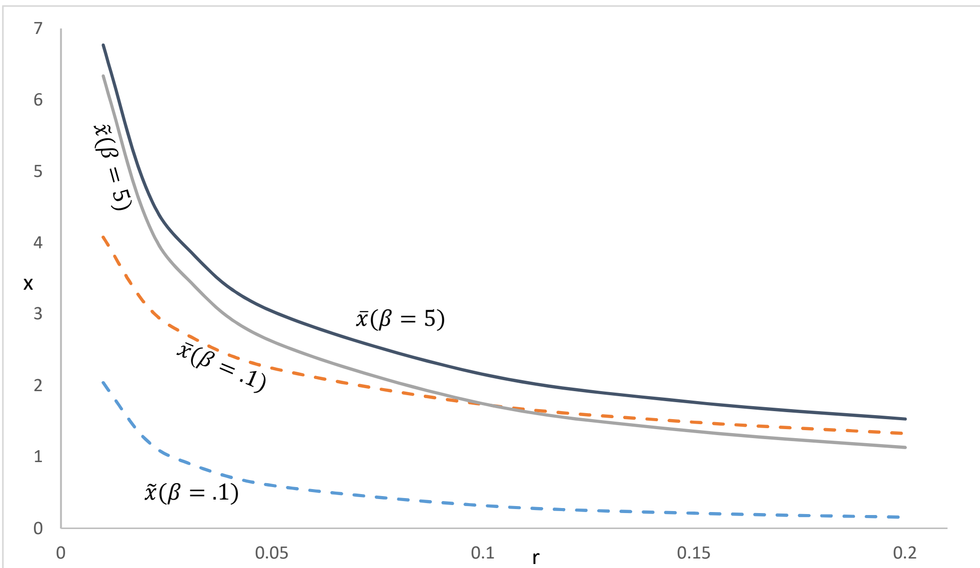

Figure 4: Example of the purchase thresholds $\overline{x}$ and $\widetilde{x}$ as a function of r for $\lambda = .5$, $\sigma^2 = 1$, and $\beta = .1, 5$.

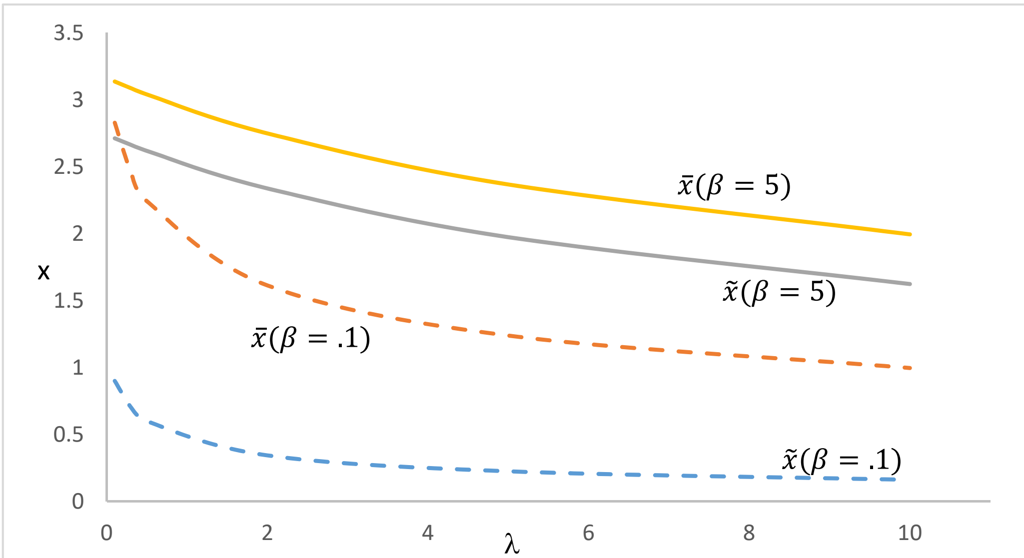

Figure 5: Example of the purchase thresholds $\overline{x}$ and $\widetilde{x}$ as a function of λ for $r = .05$, $\sigma^2 = 1$, and $\beta = .1, 5$.

In the construction of optimal decision-making we are looking for three thresholds, $\overline{x}, \underline{x}$, and $\widetilde{x}$, with $\widetilde{x} < \underline{x} < \overline{x}$, such that in search mode 1 the DM adopts the alternative if $x \geq \overline{x}$, in search mode 2 the DM adopts the alternative if $x \geq \underline{x}$, and in the no-search mode the DM adopts the alternative if $x \geq \widetilde{x}$. Let $V_1(x)$ be the expected payoff in search mode 1, $V_2(x)$ be the expected payoff in search mode 2, and $W(x)$ be the expected payoff in the no-search mode.

Consider the Bellman equation in the no-search mode. We have

$$W(x) = \beta\, dt V_1(x) + (1 - \beta\, dt)e^{-r\, dt}W(x), \tag{14}$$

which leads to $W(x) = \frac{\beta}{r+\beta}V_1(x)$.

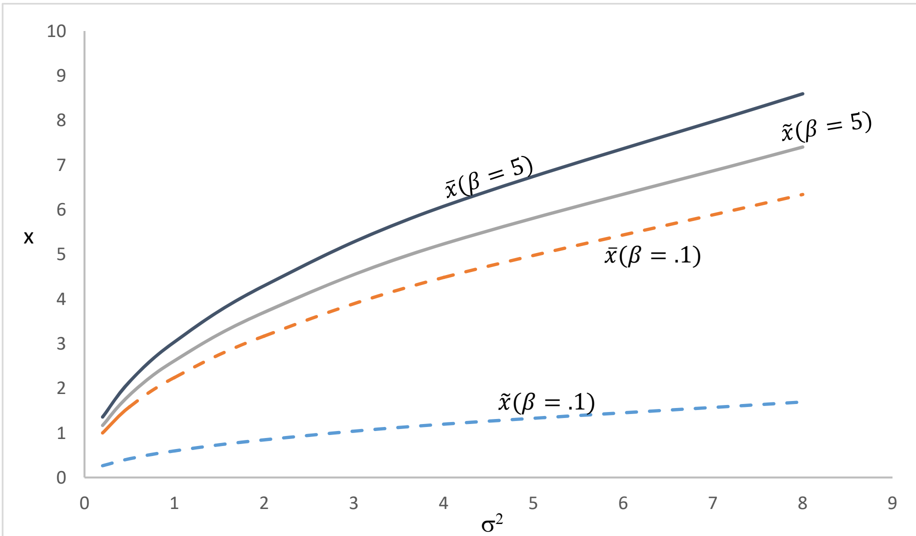

Figure 6: Example of the purchase thresholds $\overline{x}$ and $\widetilde{x}$ as a function of σ^2 for $r = .05$, $\lambda = .5$, and $\beta = .1, 5$.

Consider now the Bellman equation in search mode 1. For $x \in (\underline{x}, \overline{x})$ we have

$$V_1(x) = (1 - \lambda\, dt)e^{-r\, dt} E V_1(x + dx) + \lambda\, dt x \tag{15}$$

which reduces to

$$V_1(x) = B_3 e^{\widetilde{\mu} x} + B_4 e^{-\widetilde{\mu} x} + \frac{\lambda}{r + \lambda} x, \tag{16}$$

where B_3 and B_4 are constants to be determined. For $x < \underline{x}$ we have

$$V_1(x) = (1 - \lambda\, dt)e^{-r\, dt} E V_1(x + dx) + \lambda\, dt V_2(x). \tag{17}$$

Consider now the Bellman equation in search mode 2. For $x \in (\widetilde{x}, \underline{x})$ we have

$$V_2(x) = (1 - \lambda\, dt)e^{-r\, dt} E V_2(x + dx) + \lambda\, dt x \tag{18}$$

which reduces to

$$V_2(x) = B_1 e^{\widetilde{\mu} x} + B_2 e^{-\widetilde{\mu} x} + \frac{\lambda}{r+\lambda} x, \tag{19}$$

where B_1 and B_2 are constants to be determined. We can then use (19) in (17) to obtain that for $x \in (\widetilde{x}, \underline{x})$, solving the corresponding differential equation,

$$V_1(x) = B_5 e^{\widetilde{\mu} x} + B_6 e^{-\widetilde{\mu} x} + \frac{\lambda^2}{(r+\lambda)^2} x + \frac{\lambda \widetilde{\mu}}{2(r+\lambda)} x [B_2 e^{-\widetilde{\mu} x} - B_1 e^{\widetilde{\mu} x}], \tag{20}$$

where B_5 and B_6 are constant to be determined.

Regarding the Bellman equation in search mode 2 for $x < \widetilde{x}$ we can obtain

$$V_2(x) = (1 - \lambda\, dt) e^{-r\, dt} E V_2(x + dx) + \lambda\, dt\, \frac{\beta}{r+\beta} V_1(x), \tag{21}$$

where we use that $W(x) = \frac{\beta}{r+\beta} V_1(x)$.

Putting together (17) and (21) for $x < \widetilde{x}$, we obtain a system of differential equations

$$(r+\lambda) V_2(x) = \frac{\sigma^2}{2} V_2''(x) + \lambda \frac{\beta}{r+\beta} V_1(x) \tag{22}$$

$$(r+\lambda) V_1(x) = \frac{\sigma^2}{2} V_1''(x) + \lambda V_2(x) \tag{23}$$

which has the solution

$$V_2(x) = C_1 e^{z_1 x} + C_2 e^{z_2 x} \tag{24}$$

$$V_1(x) = \sqrt{\frac{r+\beta}{\beta}} \, [C_2 e^{z_2 x} - C_1 e^{z_1 x}] \tag{25}$$

where $z_1 = \sqrt{\widetilde{\mu}^2 + \frac{2\lambda}{\sigma^2} \sqrt{\frac{\beta}{r+\beta}}}$, and $z_2 = \sqrt{\widetilde{\mu}^2 - \frac{2\lambda}{\sigma^2} \sqrt{\frac{\beta}{r+\beta}}}$, and C_1 and C_2 are constants to be determined, where we use that $\lim_{x \to -\infty} V_1(x) = \lim_{x \to -\infty} V_2(x) = 0$.

Value matching and smooth pasting at the different thresholds, $V_1(\overline{x}) = \overline{x}, V_1'(\overline{x}) = 1, V_1(\underline{x}^+) = V_1(\underline{x}^-), V_1'(\underline{x}^+) = V_1'(\underline{x}^-), V_1(\widetilde{x}^+) = V_1(\widetilde{x}^-), V_1'(\widetilde{x}^+) = V_1'(\widetilde{x}^-), V_2(\widetilde{x}^+) = V_2(\widetilde{x}^-), V_2'(\widetilde{x}^+) = V_2'(\widetilde{x}^-), V_2(\underline{x}) = \underline{x}, V_2'(\underline{x}) = 1, \frac{\beta}{r+\beta} V_1(\widetilde{x}) = \widetilde{x}$, leads to a system of 11 equations (presented and analyzed in the Appendix) to obtain the 11 unknowns, $\overline{x}_1, \overline{x}_2, \widetilde{x}, B_1, B_2, B_3, B_4, B_5, B_6, C_1$, and C_2.

We illustrate the results for the general case in Figures 7-10.

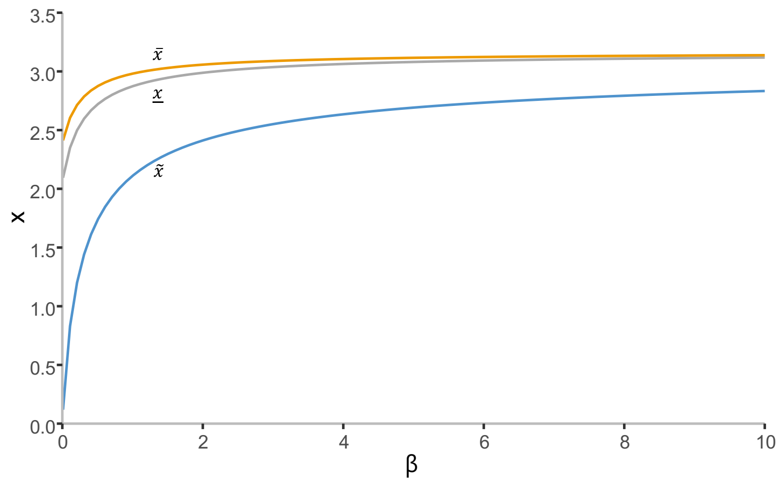

Figure 7: Example of the purchase thresholds $\overline{x}$, $\underline{x}$, and $\widetilde{x}$ for the two search modes case as a function of β for $r = .05, \lambda = .5$, and $\sigma^2 = 1$.

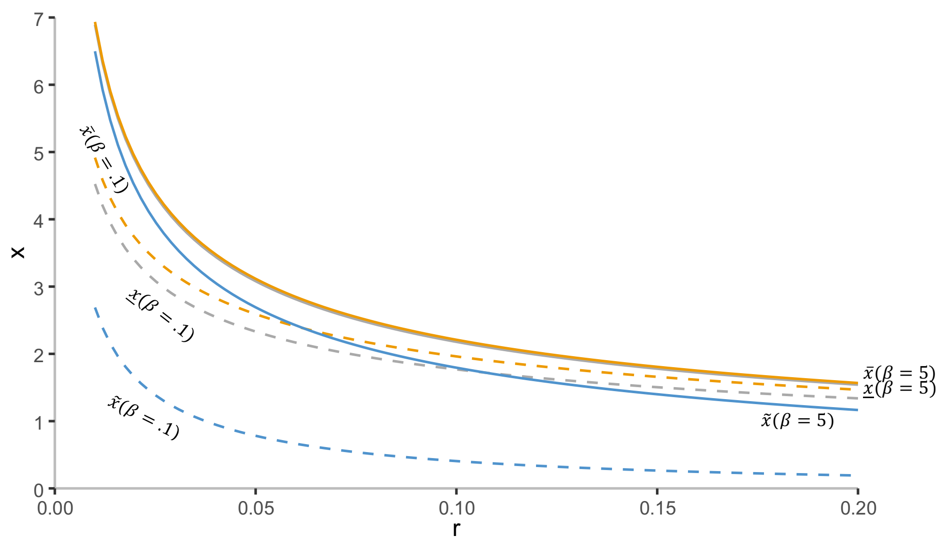

Figure 8: Example of the purchase thresholds $\overline{x}$, $\underline{x}$, and $\widetilde{x}$ for the two search modes case as a function of r for $\lambda = .5$, $\sigma^2 = 1$, and $\beta = .1, 5$.

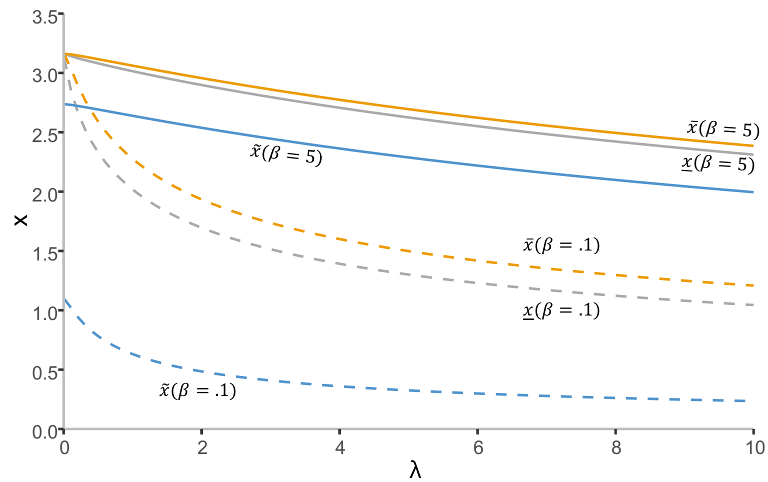

Figure 9: Example of the purchase thresholds $\overline{x}$, $\underline{x}$, and $\widetilde{x}$ for the two search modes case as a function of λ for $r = .05$, $\sigma^2 = 1$, and $\beta = .1, 5$.

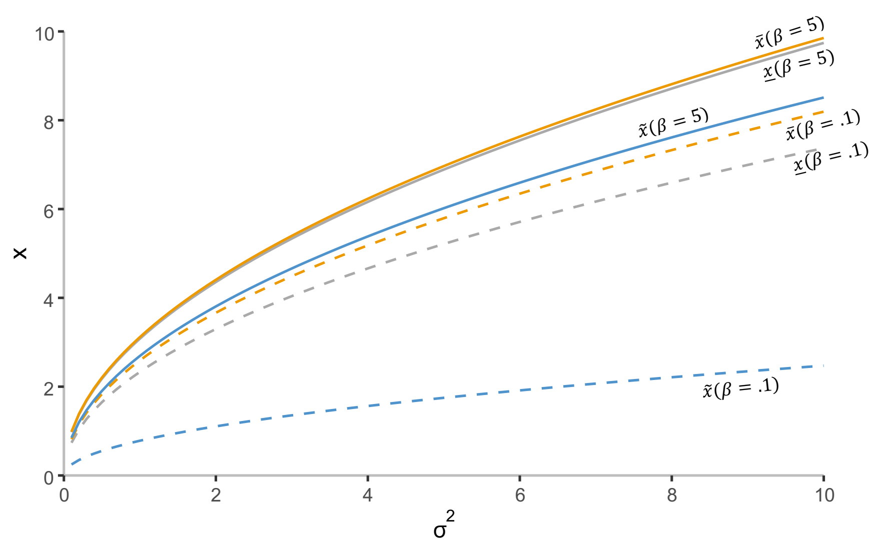

Figure 10: Example of the purchase thresholds $\overline{x}$, $\underline{x}$, and $\widetilde{x}$ for the two search modes case as a function of σ^2 for $r = .05$, $\lambda = .5$, and $\beta = .1, 5$.

The Case of $\beta \to 0$.

To get sharper results let us consider what happens when $\beta \to 0$. In this case, we have $\widetilde{x} \to 0$, and to get the values at the limit of $\underline{x}$ and $\overline{x}$ we can focus on the case of $\beta = 0$.

In this case, we obtain $z_1, z_2 \to \widetilde{\mu}$, and for $x < \widetilde{x} = 0$ we obtain

$$V_2(x) = C_1 e^{\widetilde{\mu}x} \tag{26}$$

$$V_1(x) = C_2 e^{\widetilde{\mu}x} - \frac{\lambda C_1}{\sigma^2 \widetilde{\mu}} e^{\widetilde{\mu}x}, \tag{27}$$

where C_1 and C_2 are constants to be determined.

We then have that the condition $\frac{\beta}{r+\beta} V_1(\widetilde{x}) = \widetilde{x}$, is no longer required. The Appendix presents the analysis in this case.

We can obtain the condition for the optimal $\underline{x}$ as

$$e^{\mu \underline{x}}(1-\mu) + \frac{\lambda}{r} = 0, \tag{28}$$

as $\mu = \widetilde{\mu}$ for $\beta = 0$, which is intuitively the same condition as (9). Furthermore, we can obtain

$$\frac{\lambda}{4\widetilde{\mu}} + \frac{\lambda}{\widetilde{\mu}} + \frac{r}{4}\underline{X}\left(\underline{x} + \frac{1}{\widetilde{\mu}}\right) = \frac{r+\lambda}{\lambda}\overline{X}\left(\overline{x} - \frac{1}{\widetilde{\mu}}\right) + \frac{\lambda(1-\lambda)(r+\lambda)}{r\widetilde{\mu}} - \frac{\lambda}{2}\underline{x}, \tag{29}$$

which determines $\overline{x}$ as a function of $\underline{x}$. For $\lambda \to 0$, we can then obtain $\underline{x}, \overline{x} \to \sqrt{\frac{\sigma^2}{2r}}$, and

$$\frac{\underline{x} - 1/\mu}{\lambda} \to \frac{1}{r\mu e} \tag{30}$$

$$\frac{\overline{x} - 1/\mu}{\lambda} \to \frac{1}{2r\mu}. \tag{31}$$

This then yields that when λ and β are relatively small, both $\underline{x}.\overline{x}$, and $\overline{x} - \underline{x}$ are increasing in the amount of information gained during search σ^2, and decreasing in the discount rate r. We can also obtain that the ratio $\frac{\overline{x}-1/\mu}{\underline{x}-1/\mu}$ converges to $e/2$ and that both $\overline{x}$ and $\underline{x}$ increase linearly in λ at $\lambda \to 0$. We state these results in the following proposition.

Proposition 3. *Consider the two search modes case, and assume that λ and β are relatively small. Then, both $\underline{x}.\overline{x}$, and $\overline{x} - \underline{x}$ are increasing in the amount of information gained during search σ^2, and decreasing in the discount rate r.*

5. Start-Up Search Costs

The analysis above considered the strategic effects of choice deferral through discounting of future payoffs. We now consider the existence of start-up search costs in the beginning of the search mode and show these search-up search costs yield strategic effects of choice deferral without discounting.

Consider the model of Section 2, but assume that the DM does not discount the future expected payoffs but has start-up search costs F when moving to the search mode from the no-search mode and decides to continue learning information. Furthermore, let us consider that the DM has on-going search costs c per unit of time while in the search mode. The role of these search costs c is for there to be optimal for the DM to stop search and adopt the alternative while in the search mode. Without these search costs per unit of time and no discounting, the DM would keep on learning information without making a decision until there would be a switch from the search to the no-search mode.

The optimal decision-making will involve the existence of four thresholds, $\overline{x}, \widetilde{x}, \widehat{x}$, and $\underline{\widehat{x}}$, with $\overline{x} > \widetilde{x} > 0 > \widehat{x} > \underline{\widehat{x}}$ such that the DM adopts immediately the alternative if $x \geq \overline{x}$, decides to adopt the alternative when switching from the search mode to the no-search mode if $x \geq \widetilde{x}$, defers choice when switching from the search to the no-search mode if $x \in (\widehat{x}, \widetilde{x})$, stops search without adopting the alternative when swtiching from the search to the no-search mode if $x \leq \widehat{x}$, and stops search without adopting the alternative if $x < \underline{\widehat{x}}$.

Let $V(x)$ be the value function for the DM when in the search mode and $x \in (\widetilde{x}, \overline{x})$, $\widetilde{V}(x)$ be the value function for the DM when in the search mode and $x \in (\widehat{x}, \widetilde{x})$, and $\widehat{V}$ be the value function for the DM when in the search mode and $x \in (\underline{\widehat{x}}, \widehat{x})$. Furthermore, remember that $W(x)$ is the value function of the DM when in the no-search mode.

The Bellman equation of value function when in the no-search mode (which is relevant for $x \in (\widehat{x}, \widetilde{x})$) can be written as

$$W(x) = \beta\, dt[\widetilde{V}(x) - F] + (1 - \beta\, dt)W(x), \tag{32}$$

from which we can obtain $W(x) = \widetilde{V}(x) - F$.

When the DM is in the search mode and $x \in (\widehat{x}, \widetilde{x})$ we can then write the Bellman equation of the value function as

$$\widetilde{V}(x) = -c\, dt + (1 - \lambda\, dt)E\widetilde{V}(x + dx) + \lambda\, dt[\widetilde{V}(x) - F]. \tag{33}$$

Using Itô's Lemma, and solving the corresponding differential equation, we can obtain

$$\widetilde{V}(x) = \frac{\lambda F + c}{\sigma^2} x^2 + a_1 x + a_0, \tag{34}$$

where a_1 and a_0 are constants to be determined.

The Bellman equation for $x \in (\widetilde{x}, \overline{x})$ can be written as

$$V(x) = -c\,dt + (1 - \lambda\,dt)EV(x + dx) + \lambda\,dtx. \tag{35}$$

Using Itô's Lemma and solving the corresponding differential equation, one obtains

$$V(x) = C_1 e^{\widehat{\mu}x} + C_2 e^{-\widehat{\mu}x} + x - c/\lambda, \tag{36}$$

where $\widehat{\mu} = \sqrt{2\lambda/\sigma^2}$.

The Bellman equation for $x \in (\underline{\widehat{x}}, \widehat{x})$ can be written as

$$\widehat{V}(x) = -c\,dt + (1 - \lambda\,dt)EV(x + dx). \tag{37}$$

Using Itô's Lemma and solving the corresponding differential equation, one obtains

$$\widehat{V}(x) = C_3 e^{\widehat{\mu}x} + C_4 e^{-\widehat{\mu}x} - c/\lambda. \tag{38}$$

Value matching and smooth pasting at $\overline{x}, \widetilde{x}$, $\widehat{x}$, and $\underline{\widehat{x}}$, leads to $V(\overline{x}) = \overline{x}, V'(\overline{x}) = 1, V(\widetilde{x}) = \widetilde{V}(\widetilde{x}), V'(\widetilde{x}) = \widetilde{V}'(\widetilde{x}), V(\widetilde{x}) - F = \widetilde{x}, \widetilde{V}(\widehat{x}) - F = 0, \widetilde{V}(\widehat{x}) = \widehat{V}(\widehat{x}), \widetilde{V}'(\widehat{x}) = \widehat{V}'(\widehat{x}), \widehat{V}(\underline{\widehat{x}}) = 0$, and $\widehat{V}'(\underline{\widehat{x}}) = 0$. This is a system of 10 equations, from which we can obtain, $\overline{x}, \widetilde{x}, \widehat{x}, \underline{\widehat{x}}, a_0, a_1, C_1, C_2, C_3$, and C_4. (The derivation of the solution is presented in the Appendix.)

We can obtain $\overline{x} - \widetilde{x} = \widehat{x} - \underline{\widehat{x}}$, $\widetilde{x} = -\widehat{x}$, and

$$\begin{aligned} \widetilde{x} &= \sqrt{\frac{\sigma^2}{2\lambda}} \frac{c}{2(\lambda F + c)} \frac{1 - H^2}{H} + \frac{\sigma^2}{4(\lambda F + c)} && (39) \\ \overline{x} &= \widetilde{x} + \frac{1}{\widehat{\mu}} \ln H, && (40) \end{aligned}$$

where

$$H = 1 + \frac{\lambda F}{c} + \sqrt{\left(\frac{\lambda F}{c} + 1\right)^2 - 1}. \tag{41}$$

Noting that $(\widetilde{x} - \widehat{x})$ can be seen as related to the extent of choice deferral, we can obtain that choice deferral is decreasing in the current search costs c, in the start-up search costs F, and in the rate at which the DM switches from the search to the no-search mode. The increase in the search costs, either current or start-up, decreases the benefits of choice deferral, and therefore the DM decreases choice deferral. We state these results in the following proposition.

Proposition 4. *Consider that there are start-up search costs. Then the extent of choice deferral decreases in the current search costs c, on the start-up search costs F, and on the rate at which the DM switches from the search to the no-search mode, λ.*

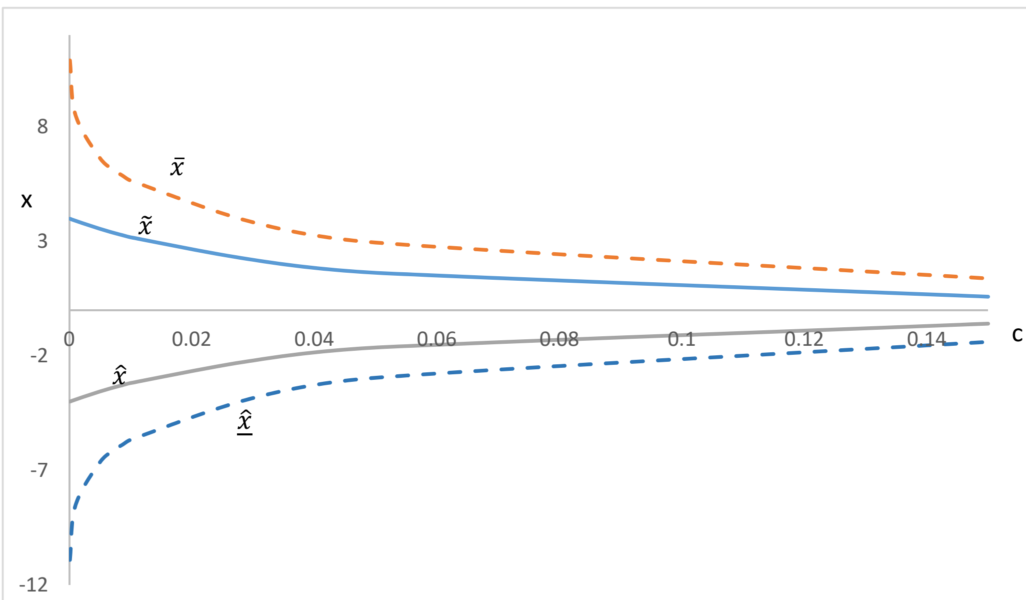

Figure 11: Evolution of the stop-search thresholds for the start-up search costs case as a function of c for $\lambda = .5$, $\sigma^2 = 1$, and $F = .1$.

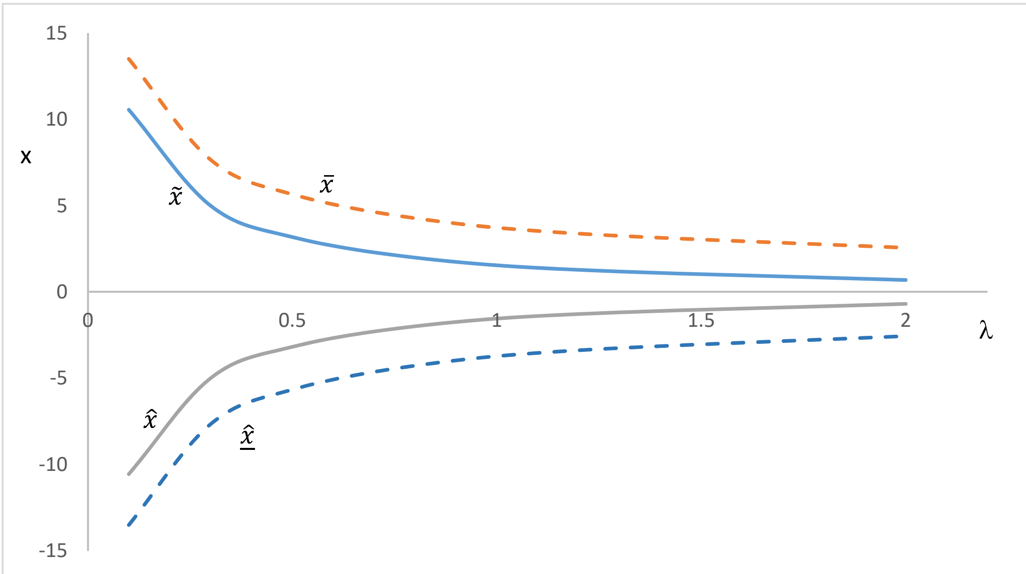

Figure 12: Evolution of the stop-search thresholds for the start-up search costs case as a function of λ for $c = .01$, $\sigma^2 = 1$, and $F = .1$.

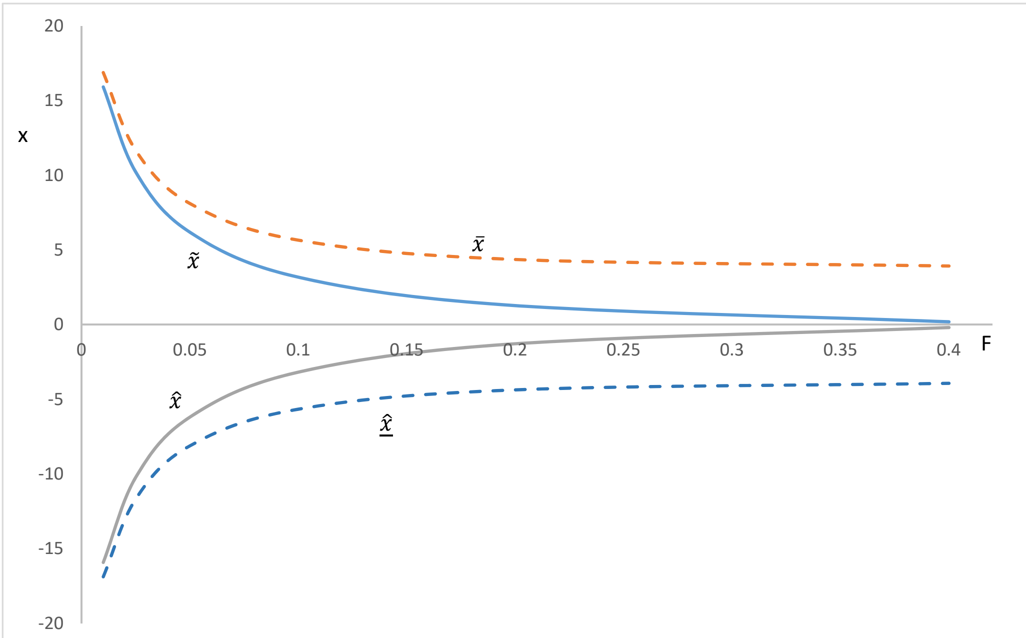

Figure 13: Evolution of the stop-search thresholds for the start-up search costs case as a function of F for $\lambda = .5$, $\sigma^2 = 1$, and $c = .01$.

Figures 11, 12, and 13 illustrate how the thresholds $\overline{x}, \widetilde{x}, \widehat{x}$, and $\underline{\widehat{x}}$ evolve as a function of the current search search costs, c, the hazard rate of switching from the search to the no-search mode, λ, and the start-up search costs, F. In particular note that, intuitively, if the start-up search costs are high enough the DM does not restart search, and chooses to adopt the alternative if $x > 0$, when switching from the search to the no-search mode.

6. Optimal Pricing

In this section, we derive the firm's optimal pricing strategy for the baseline model with discounting and one search mode. The analysis for the baseline model in Section 3 can be seen as describing the behavior of a DM facing a product with a price of 0. Let P denote the price, let x denote the expected value of the payoff of the alternative as before, and let $y = x - P$ denote the expected payoff of the alternative minus the price. The DM then would adopt the alternative when y reaches $\overline{x}$ in the search mode, and would adopt the alternative when y reaches $\widetilde{x}$ in the no-search mode, where $\overline{x}$ and $\widetilde{x}$ are solutions to 7 and 8. Equivalently, the DM adopts when x reaches $\overline{x}+P$ in the search mode or when x reaches $\widetilde{x}+P$ in the no-search mode.

Let $V_f(x)$ be the expected discounted payoff for the firm if the DM is in the search mode, and $W_f(x)$ be the expected payoff for the firm if the DM is in the no-search mode.

The Bellman equation for $V_f(x)$ for $x < \widetilde{x}+P$ can be written as

$$V_f(x) = (1-\lambda\, dt)e^{-r\,dt}EV(x+dx) + \lambda\, dtW_f(x). \tag{42}$$

(Note that we could have $e^{-r\,dt}EW_f(x+dx)$ instead of $W_f(x)$ in (42) and the subsequent analysis would not change, as the second order terms in dt disappear.) The Bellman equation for $V_f(x)$ for $x \in (\widetilde{x}+P, \overline{x}+P)$ can be written as

$$V_f(x) = (1-\lambda\, dt)e^{-r\,dt}EV_f(x+dx) + \lambda\, dtP. \tag{43}$$

Finally, the Bellman equation for $W_f(x)$ can be obtained to be

$$W_f(x) = \beta\, dtV(x) + (1-\beta\, dt)e^{-r\,dt}W_f(x), \tag{44}$$

from which one can obtain $W_f(x) = \frac{\beta}{r+\beta}V_f(x)$. Substituting into (42), and using Itô's Lemma, we can obtain the second order differential equation in $V_f(x)$ for $x < \widetilde{x}+P$ as

$$r\frac{r+\beta+\lambda}{r+\beta}V_f(x) = \frac{\sigma^2}{2}V_f''(x). \tag{45}$$

Given that $\lim_{x\to-\infty} V_f(x) = 0$, as the expected payoff of the firm has to approach zero if the expected payoff of the alternative approaches negative infinity, we have that the solution to (45) satisfies

$$V_f(x) = C_1 e^{\mu x} \tag{46}$$

where C_1 is a constant to be determined.

Similarly, applying Itô's Lemma to (43), we can obtain the second order differential equation in $V_f(x)$ for $x \in (\widetilde{x}+P, \overline{x}+P)$ as

$$V_f(x) = C_2 e^{\widetilde{\mu}x} + C_3 e^{-\widetilde{\mu}x} + \frac{\lambda}{r+\lambda}P \tag{47}$$

where C_2 and C_3 are constants to be determined.

The optimal price, P^*, depends on the initial position, x_0. Suppose $P^* \leq x_0 - \overline{x}$, or $x_0 \geq \overline{x} + P^*$, then the DM adopts the alternative at x_0 in both search mode and no-search mode. In this case, because the DM purchase immediately at P^*, the firm's profit strictly increases in P^*. Thus any price strictly below $x_0 - \overline{x}$ cannot be optimal. We must have $P^* \geq x_0 - \overline{x}$.

Now consider the case where $P^* > x_0 - \widetilde{x}$, or $x_0 < \widetilde{x} + P^*$. In this case, the DM does not adopt the alternative at x_0 in both search mode and no-search mode. Using value matching of $W_f(x)$ at $\widetilde{x} + P^*$, $W(\widetilde{x} + P^*) = P^*$, we obtain:

$$\frac{\beta}{r+\beta} C_1 e^{\mu(\widetilde{x}+P)} = P \tag{48}$$

which gives $C_1 = \frac{r+\beta}{\beta} P e^{-\mu(\widetilde{x}+P)}$. Plugging C_1 into equation 46, we get:

$$V_f(x_0) = \frac{r+\beta}{\beta} \cdot P^* \cdot e^{\mu x_0 - \widetilde{x} - P^*} \tag{49}$$

Taking derivative of $V_f(x_0)$ with respect to P^*, we find that the optimal price is

$$P^* = \frac{1}{\mu} = \sqrt{\frac{\sigma^2}{2r} \cdot \frac{r+\beta}{r+\beta+\lambda}} \tag{50}$$

For $P^* = \frac{1}{\mu}$ to be optimal, the initial position has to be sufficiently low such that the DM does not adopt the alternative at x_0 in both search mode and no-search mode, i.e., $x_0 < \widetilde{x} + \frac{1}{\mu}$.

Finally, consider the case where $P^* \in (x_0 - \overline{x}, x_0 - \widetilde{x}]$, or $x_0 \in [\widetilde{x} + P^*, \overline{x} + P^*)$. In this case, the DM adopts at x_0 in no-search mode but does not adopt at x_0 in search mode. Using value matching of $V_f(x)$ at $\widetilde{x} + P^*$, $V_f(\{\widetilde{x} + P^*\}^-) = V(\{\widetilde{x} + P^*\}^+)$, value matching of $W_f(x)$ at $\widetilde{x} + P^*$, $W(\widetilde{x} + P^*) = P^*$, and value matching of $V_f(x)$ at $\overline{x} + P^*$, $V_f(\overline{x} + P^*) = P^*$, we can obtain (see Appendix):

$$V_f(x_0) = \frac{\lambda}{r+\lambda} P^* + C_4 P^* e^{-\widetilde{\mu}(x_0 - \widetilde{x} - P^*)} + C_5 P^* e^{\widetilde{\mu}(\widetilde{x} + P^* - x_0)} \tag{51}$$

where

$$C_4 = \frac{r}{\beta} \frac{1}{1 - e^{2\widetilde{\mu\delta}}} + \frac{r}{r+\lambda} \frac{1}{1 + e^{\widetilde{\mu\delta}}}$$

and

$$C_5 = \frac{r}{\beta}\frac{1}{1-e^{-2\widetilde{\mu}\delta}} + \frac{r}{r+\lambda}\frac{1}{1+e^{-\widetilde{\mu}\delta}}$$

and $\delta = \overline{x} - \widetilde{x}$.

Note that from (51) we get that if the price is $P = x_0 - \widetilde{x}$, then firm's profit is $\frac{r+\beta}{\beta}(x_0 - \widetilde{x})$. If the price is $P = x_0 - \overline{x}$, the DM adopts immediately at time 0 and the profit is $x_0 - \overline{x}$. By comparing the two profits, we see that the profit under $P = x_0 - \widetilde{x}$ is strictly higher than the the profit under $P = x_0 - \overline{x}$. Thus for $x_0 \geq \widetilde{x} + \frac{1}{\mu}$, the optimal price must be such that the DM adopts at x_0 in no-search mode but does not adopt at x_0 in search mode, i.e.,

$$P^* = \underset{P \in (x_0 - \overline{x}, x_0 - \widetilde{x}]}{\operatorname{argmax}} \frac{\lambda}{r+\lambda}P + C_4 P e^{\widetilde{\mu}(x_0 - \widetilde{x} - P)} + C_5 P e^{\widetilde{\mu}(\widetilde{x} + P - x_0)} \tag{52}$$

We summarize the optimal pricing strategy in the following Proposition.

Proposition 5. *If x_0 is sufficiently low, i.e., $x_0 < \widetilde{x} + \frac{1}{\mu}$, then the optimal price is $P^* = \sqrt{\frac{\sigma^2}{2r} \cdot \frac{r+\beta}{r+\beta+\lambda}}$, which does not depend on x_0. The DM does not adopt the alternative at x_0 in the search mode or the no-search mode. For higher values of x_0, the optimal price is in the range of $(x_0 - \overline{x}, x_0 - \widetilde{x}]$. The DM does not adopt the alternative at x_0 in the search mode but adopts the alternative at x_0 in the no-search mode. Under optimal pricing, the DM never adopts the alternative at x_0 in the search mode.*

Figure 14 illustrates how the optimal price varies with initial position, x_0. When x_0 is low, the DM does not adopt initially in both the search mode or the no-search mode, and the optimal price does not depend on x_0. For higher values of x_0, the optimal price increases in x_0.

We can compute how the optimal price varies with σ^2, r, λ, and β for the case where x_0 is sufficiently low such that the DM does no adopt before acquiring some positive information about the alternative.

Proposition 6. *If $x_0 < \widetilde{x} + \frac{1}{\mu}$, then the optimal price increases in σ^2 and β, and decreases in r and λ. If λ and β change simultaneously with a fixed ratio of $\frac{\lambda}{\beta}$, then the optimal price decreases in λ and β.*

Proposition 6 shows that the optimal price depends on the speed of the DM's information acquisition, as well as the frequencies at which the DM enters and exits search. Intuitively, when λ increases or when β decreases, the DM is expected to spend a larger fraction of time

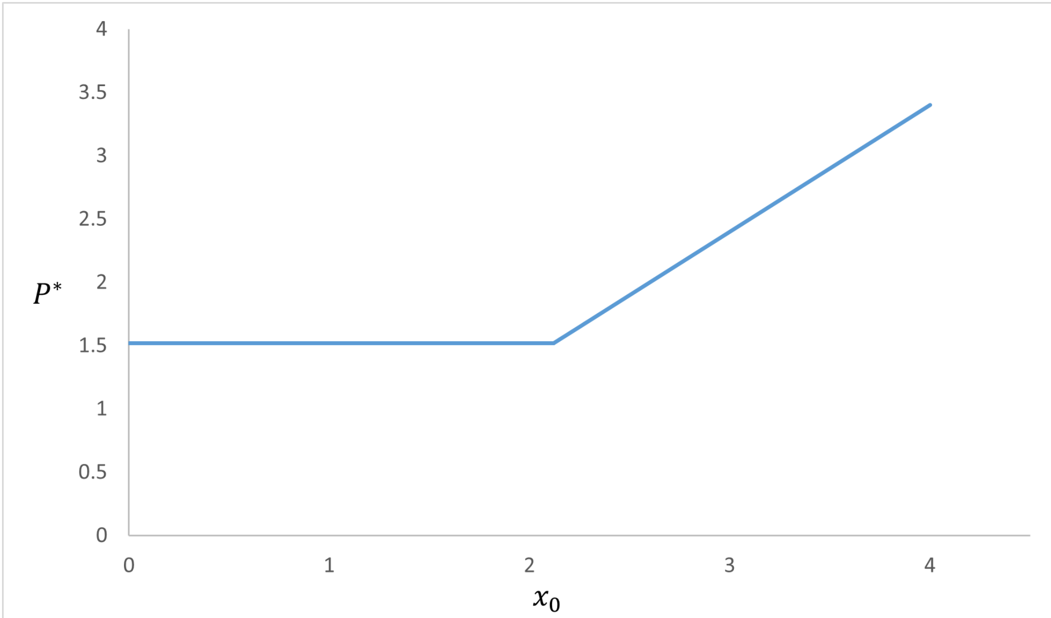

Figure 14: Example of the optimal price P^* as a function of x_0 for $r = .05, \lambda = .5$, $\beta = .1$, and $\sigma^2 = 1$.

in no-search mode, exhibiting stronger search fatigue. When the DM faces more frequent and longer disruptions of information gathering, the firm should charge a lower price, to prevent the DM from deferring choice. Online stores can often track consumers over different browsing sessions. Our result suggests that firms should factor in the lengths of browsing sessions and gaps between browsing sessions in setting their prices.

Another implication of Proposition 6 is that the firm should change its price following efforts by the firm to intervene with the DM's search/no-search pattern. For example, in online retail, firms may re-design interfaces to reduce consumer fatigue, so that consumers stay longer in a browsing session. Firms may also use instruments such as ad re-targeting, push notifications, or email marketing to bring back previous visitors more quickly. Proposition 6 suggests that price should increase if these efforts are successful.

Proposition 6 also shows that the optimal price decreases when the DM switches between the search mode and the no-search mode more frequently, even if the long-term fraction of time in each mode remains constant. This is relevant when there is a change in the shopping environment such that consumers enters and exits search more or less frequently. For example, consumers shopping on mobile devices may have their browsing sessions disrupted and resumed more frequently than consumers shopping on computers [reference?]. In that case, even if the overall time spent on shopping does not change for consumers on mobile devices, the firm should consider setting a lower price on mobile devices compared to the price on computers.

However, note that, for higher x_0, the comparative statics maybe be reversed. For $x_0 \geq \widetilde{x} + \frac{1}{\mu}$, the optimal price P^* must be in the range of $(x_0 - \overline{x}, x_0 - \widetilde{x}]$. In Section 3, we observe that $\overline{x}$ and $\widetilde{x}$ increase in β and σ^2 and decrease in r and λ. If so, then both the upper bound and the lower bound of the optimal price would decrease in β and σ^2 and increase in r and λ, opposite from the comparative statics on P^* when x_0 is low.

Figures 15 to 18 show the effects of model parameters on P^* in the general case. Note the existence of non-monotonicity for each parameter under a fixed x_0. As β and σ^2 increase, or as r and λ decrease, $\widetilde{x} + \frac{1}{\mu}$ can decrease from above x_0 to below x_0.

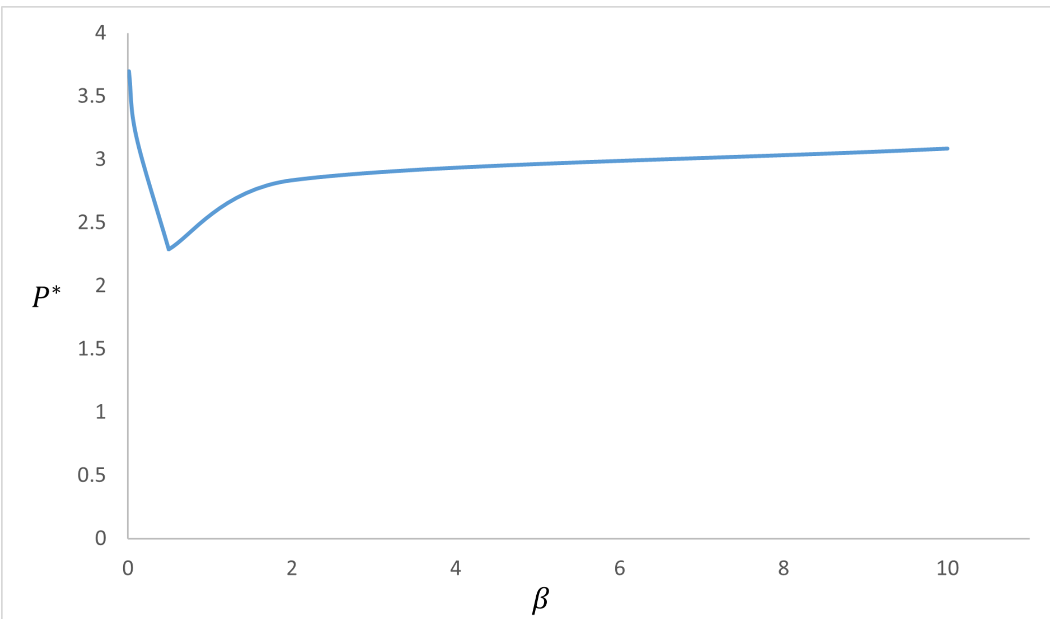

Figure 15: Example of the optimal price P^* as a function of β for $r = .05, \lambda = .5$, $\sigma^2 = 1$, and $x_0 = 3.78$

7. Concluding Remarks

This paper investigates the extent to which a DM may decide to defer choice when the search conditions change, and the DM does not have significant diagnostic information. We investigate the possibility of search fatigue, and what happens when there are start-up search costs.

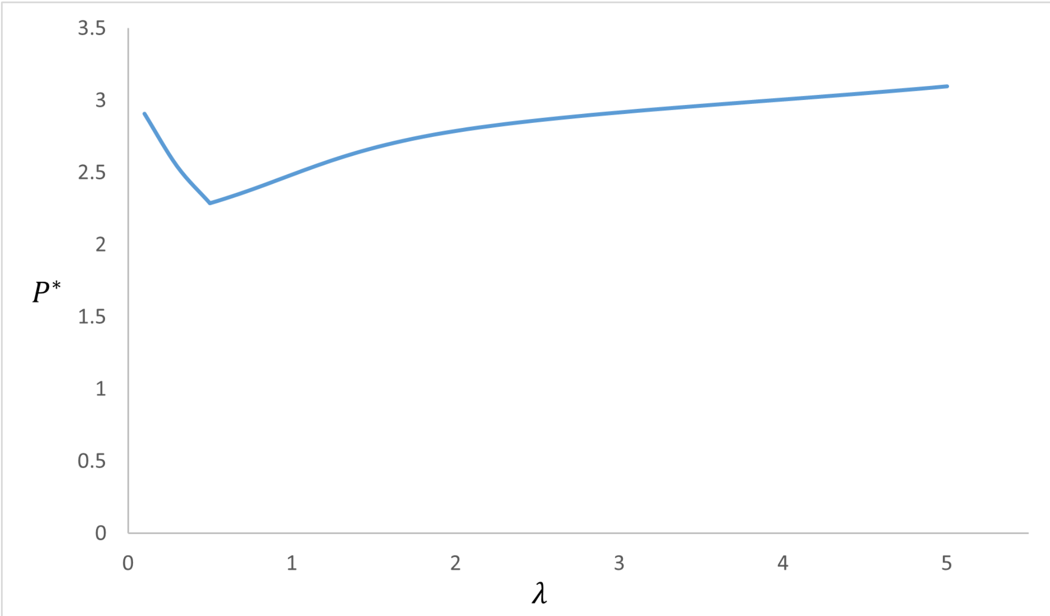

Figure 16: Example of the optimal price P^* as a function of λ for $r = .05, \beta = .5$, $\sigma^2 = 1$, and $x_0 = 3.78$

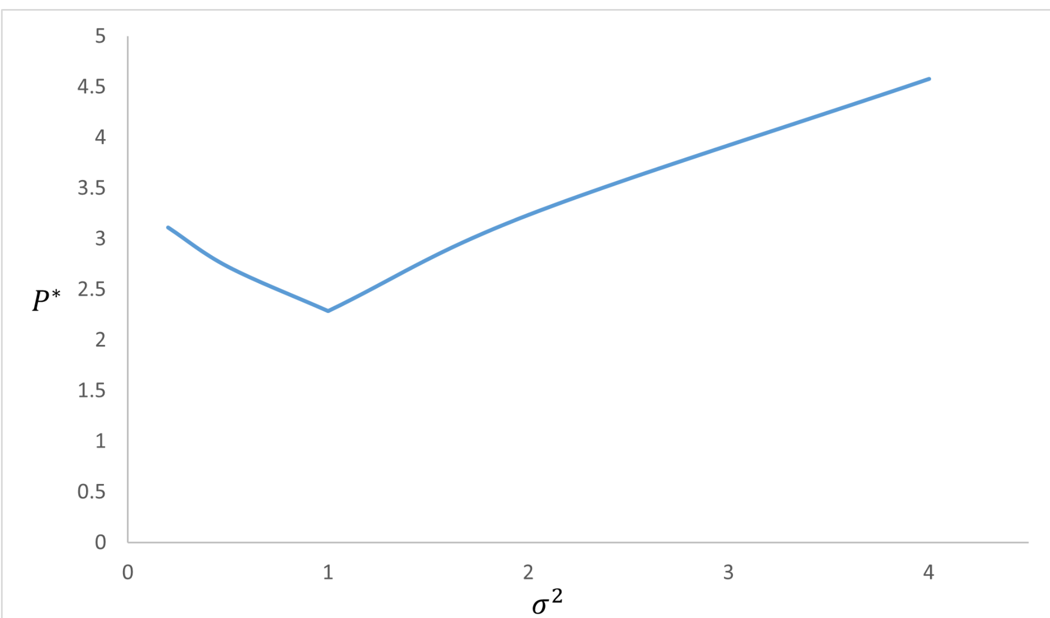

Figure 17: Example of the optimal price P^* as a function of σ^2 for $r = .05, \lambda = .5$, $\beta = .5$, and $x_0 = 3.78$

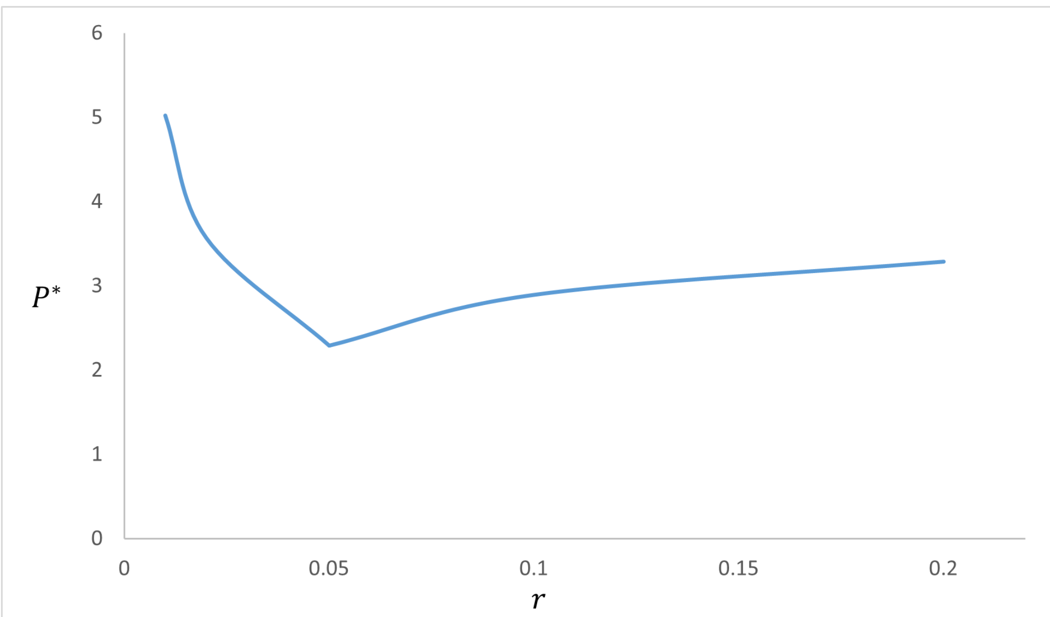

Figure 18: Example of the optimal price P^* as a function of r for $\lambda = .5$, $\beta = .5$, $\sigma^2 = 1$, and $x_0 = 3.78$

APPENDIX

Derivation of Solution to Base Case with Discounting:

Using value matching and smooth pasting of $V(x)$ at $\widetilde{x}$ and $\overline{x}$, $V(\widetilde{x}^-) = V(\widetilde{x}^+), V'(\widetilde{x}^-) = V'(\widetilde{x}^+)$, $V(\overline{x}) = \overline{x}$, and $V'(\overline{x}) = 1$, and $W(\widetilde{x}) = \widetilde{x}$, we obtain the following system of five equations to obtain $\widetilde{x}, \overline{x}, A_1, A_2$, and A_3.

$$\begin{aligned}
A_2 e^{\widetilde{\mu}\overline{x}} + A_3 e^{-\widetilde{\mu}\overline{x}} + \frac{\lambda}{r+\lambda}\overline{x} &= \overline{x} && \text{(i)}\\
\widetilde{\mu}A_2 e^{\widetilde{\mu}\overline{x}} - \widetilde{\mu}A_3 e^{-\widetilde{\mu}\overline{x}} + \frac{\lambda}{r+\lambda} &= 1 && \text{(ii)}\\
A_2 e^{\widetilde{\mu}\widetilde{x}} + A_3 e^{-\widetilde{\mu}\widetilde{x}} + \frac{\lambda}{r+\lambda}\widetilde{x} &= A_1 e^{\mu\widetilde{x}} && \text{(iii)}\\
\widetilde{\mu}A_2 e^{\widetilde{\mu}\widetilde{x}} - \widetilde{\mu}A_3 e^{-\widetilde{\mu}\widetilde{x}} + \frac{\lambda}{r+\lambda} &= \mu A_1 e^{\mu\widetilde{x}} && \text{(iv)}\\
\frac{\beta}{r+\beta}A_1 e^{\mu\widetilde{x}} &= \widetilde{x}. && \text{(v)}
\end{aligned}$$

Using (i)-(v), we can obtain a system of two equations to obtain $\widetilde{x}$ and $\overline{x}$ as

$$\begin{aligned}
e^{\widetilde{\mu}(\overline{x}-\widetilde{x})} &= \frac{\frac{r}{r+\lambda}\overline{x} + \frac{r}{\widetilde{\mu}(r+\lambda)}}{\widetilde{x}\left(\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right) + \frac{1}{\widetilde{\mu}}\left(\mu\widetilde{x}\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right)} && \text{(vi)}\\
e^{\widetilde{\mu}(\overline{x}-\widetilde{x})} &= \frac{\widetilde{x}\left(\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right) - \frac{1}{\widetilde{\mu}}\left(\mu\widetilde{x}\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right)}{\frac{r}{r+\lambda}\overline{x} - \frac{r}{\widetilde{\mu}(r+\lambda)}}. && \text{(vii)}
\end{aligned}$$

Using $\delta = \overline{x} - \widetilde{x}$ we can rewrite (vi) and (vii), as a system of equations for δ and $\widetilde{x}$ as

$$\begin{aligned}
\widetilde{x} &= \beta\frac{r + r\widetilde{\mu}\delta + \lambda D}{D[\widetilde{\mu}r(r+\beta+\lambda) + \mu(r+\beta)(r+\lambda)] - \widetilde{\mu}\beta r} && \text{(viii)}\\
\widetilde{x} &= \beta\frac{\lambda + rD - \widetilde{\mu}r\delta D}{\widetilde{\mu}r\beta D + \mu(r+\beta)(r+\lambda) - \widetilde{\mu}r(r+\beta+\lambda)} && \text{(ix)}
\end{aligned}$$

where $D = e^{\widetilde{\mu}\delta}$. Using (viii) and (ix) we can obtain (7) in the main text, from which we can obtain δ. We can then use (viii) or (ix) to obtain $\widetilde{x}$.

Derivation of the Optimal Decision-Making in the Two Search Modes Case:

As noted above, value matching and smooth pasting at the different thresholds leads to the following system of 11 equations to obtain the 11 unknowns, $\overline{x}_1, \overline{x}_2, \widetilde{x}, B_1, B_2, B_3, B_4, B_5, B_6, C_1$, and C_2.

$$\begin{aligned}
B_3\underline{X} + B_4/\underline{X} + \frac{\lambda}{r+\lambda}\underline{x} &= B_5\underline{X} + B_6/\underline{X} + \frac{\lambda^2}{(r+\lambda)^2}\underline{x} - \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_1\underline{xX} \\
&\quad + \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_2\underline{x}/\underline{X} \qquad \text{(x)} \\
\widetilde{\mu}B_3\underline{X} - \widetilde{\mu}B_4/\underline{X} + \frac{\lambda}{r+\lambda} &= \widetilde{\mu}B_5\underline{X} - \widetilde{\mu}B_6/\underline{X} + \frac{\lambda^2}{(r+\lambda)^2} - \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_1\underline{X} - \frac{\lambda\widetilde{\mu}^2}{2(r+\lambda)}B_1\underline{xX} \\
&\quad + \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_2/\underline{X} - \frac{\lambda\widetilde{\mu}^2}{2(r+\lambda)}B_2\underline{x}/\underline{X} \qquad \text{(xi)} \\
B_5\widetilde{X} + B_6/\widetilde{X} + \frac{\lambda^2}{(r+\lambda)^2}\widetilde{x} &- \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_1\widetilde{x}\widetilde{X} + \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_2\widetilde{x}/\widetilde{X} = \\
&\quad \sqrt{\frac{r+\beta}{\beta}}\left[C_2e^{z_2\widetilde{x}} - C_1e^{z_1\widetilde{x}}\right] \qquad \text{(xii)} \\
\widetilde{\mu}B_5\widetilde{X} - \widetilde{\mu}B_6/\widetilde{X} + \frac{\lambda^2}{(r+\lambda)^2} &- \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_1\widetilde{X} - \frac{\lambda\widetilde{\mu}^2}{2(r+\lambda)}B_1\widetilde{x}\widetilde{X} + \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_2/\widetilde{X} \\
&\quad - \frac{\lambda\widetilde{\mu}^2}{2(r+\lambda)}B_2\widetilde{x}/\widetilde{X} = \sqrt{\frac{r+\beta}{\beta}}\left[z_2C_2e^{z_2\widetilde{x}} - z_1C_1e^{z_1\widetilde{x}}\right] \qquad \text{(xiii)} \\
B_1\widetilde{X} + B_2/\widetilde{X} + \frac{\lambda}{r+\lambda}\widetilde{x} &= C_1e^{z_1\widetilde{x}} + C_2e^{z_2\widetilde{x}} \qquad \text{(xiv)} \\
\widetilde{\mu}B_1\widetilde{X} - \widetilde{\mu}B_2/\widetilde{X} + \frac{\lambda}{r+\lambda} &= z_1C_1e^{z_1\widetilde{x}} + z_2C_2e^{z_2\widetilde{x}} \qquad \text{(xv)} \\
B_3\overline{X} + B_4/\overline{X} + \frac{\lambda}{r+\lambda}\overline{x} &= \overline{x} \qquad \text{(xvi)} \\
\widetilde{\mu}B_3\overline{X} - \widetilde{\mu}B_4/\overline{X} + \frac{\lambda}{r+\lambda} &= 1 \qquad \text{(xvii)} \\
B_1\underline{X} + B_2/\underline{X} + \frac{\lambda}{r+\lambda}\underline{x} &= \underline{x} \qquad \text{(xviii)} \\
\widetilde{\mu}B_1\underline{X} - \widetilde{\mu}B_2/\underline{X} + \frac{\lambda}{r+\lambda} &= 1 \qquad \text{(xix)} \\
\sqrt{\frac{\beta}{r+\beta}}\left(-C_1e^{z_1\widetilde{x}} + C_2e^{z_2\widetilde{x}}\right) &= \widetilde{x}, \qquad \text{(xx)}
\end{aligned}$$

where $\overline{X} = e^{\widetilde{\mu}\overline{x}}$, $\underline{X} = e^{\widetilde{\mu}\underline{x}}$, and $\widetilde{X} = e^{\widetilde{\mu}\widetilde{x}}$.

Putting together (xvi) and (xvii) one obtains

$$\begin{aligned}
2B_3\overline{X} &= \frac{r}{r+\lambda}\left(\overline{x} + \frac{1}{\widetilde{\mu}}\right) \qquad \text{(xxi)} \\
2B_4/\overline{X} &= \frac{r}{r+\lambda}\left(\overline{x} - \frac{1}{\widetilde{\mu}}\right). \qquad \text{(xxii)}
\end{aligned}$$

Putting together (xviii) and (xix) one obtains

$$
\begin{aligned}
2B_1\underline{X} &= \frac{r}{r+\lambda}\left(\underline{x}+\frac{1}{\widetilde{\mu}}\right) && \text{(xxiii)}\\
2B_2/\underline{X} &= \frac{r}{r+\lambda}\left(\underline{x}-\frac{1}{\widetilde{\mu}}\right). && \text{(xxiv)}
\end{aligned}
$$

Putting together (x) and (xi) one obtains

$$
\begin{aligned}
2B_3\underline{X}+\frac{\lambda}{r+\lambda}\left(\underline{x}+\frac{1}{\widetilde{\mu}}\right)=2B_5\underline{X} &+ \frac{\lambda^2}{(r+\lambda)^2}\left(\underline{x}+\frac{1}{\widetilde{\mu}}\right)-\frac{\lambda}{2(r+\lambda)}B_1\underline{X}(1+2\widetilde{\mu}\underline{x})\\
&+\frac{\lambda}{2(r+\lambda)}B_2/\underline{X} && \text{(xxv)}\\
2B_4/\underline{X}+\frac{\lambda}{r+\lambda}\left(\underline{x}-\frac{1}{\widetilde{\mu}}\right)=2B_6/\underline{X} &+ \frac{\lambda^2}{(r+\lambda)^2}\left(\underline{x}-\frac{1}{\widetilde{\mu}}\right)-\frac{\lambda}{2(r+\lambda)}B_2/\underline{X}(1-2\widetilde{\mu}\underline{x})\\
&+\frac{\lambda}{2(r+\lambda)}B_1\underline{X}. && \text{(xxvi)}
\end{aligned}
$$

Putting together (xii) and (xiii) one obtains

$$
\begin{aligned}
2B_5\widetilde{X}+\frac{\lambda^2}{(r+\lambda)^2}\left(\widetilde{x}+\frac{1}{\widetilde{\mu}}\right) &- \frac{\lambda}{2(r+\lambda)}B_1\widetilde{X}(1+2\widetilde{\mu}\widetilde{x})+\frac{\lambda}{2(r+\lambda)}B_2/\widetilde{X}=\\
&\sqrt{\frac{r+\beta}{\beta}}\left(-\widetilde{C}_1\left(1+\frac{z_1}{\widetilde{\mu}}\right)+\widetilde{C}_2\left(1+\frac{z_2}{\widetilde{\mu}}\right)\right) && \text{(xxvii)}\\
2B_6/\widetilde{X}+\frac{\lambda^2}{(r+\lambda)^2}\left(\widetilde{x}-\frac{1}{\widetilde{\mu}}\right) &- \frac{\lambda}{2(r+\lambda)}B_2/\widetilde{X}(1-2\widetilde{\mu}\widetilde{x})+\frac{\lambda}{2(r+\lambda)}B_1\widetilde{X}=\\
&\sqrt{\frac{r+\beta}{\beta}}\left(-\widetilde{C}_1\left(1-\frac{z_1}{\widetilde{\mu}}\right)+\widetilde{C}_2\left(1-\frac{z_2}{\widetilde{\mu}}\right)\right) && \text{(xxviii)}
\end{aligned}
$$

where $\widetilde{C}_1=C_1e^{z_1\widetilde{x}}$ and $\widetilde{C}_2=C_2e^{z_2\widetilde{x}}$. Putting together (xiv) and (xv) one obtains

$$
\begin{aligned}
2B_1\widetilde{X}+\frac{\lambda}{r+\lambda}\left(\widetilde{x}+\frac{1}{\widetilde{\mu}}\right) &= \widetilde{C}_1\left(1+\frac{z_1}{\widetilde{\mu}}\right)+\widetilde{C}_2\left(1+\frac{z_2}{\widetilde{\mu}}\right) && \text{(xxix)}\\
2B_2/\widetilde{X}+\frac{\lambda}{r+\lambda}\left(\widetilde{x}-\frac{1}{\widetilde{\mu}}\right) &= \widetilde{C}_1\left(1-\frac{z_1}{\widetilde{\mu}}\right)+\widetilde{C}_2\left(1-\frac{z_2}{\widetilde{\mu}}\right). && \text{(xxx)}
\end{aligned}
$$

Using (xx) we obtain $\widetilde{C}_2=\widetilde{C}_1+\sqrt{\frac{r+\beta}{\beta}}\widetilde{x}$, which we can then substitute in (xxvii)-(xxx).

Using the resulting equations (xxix) and (xxx) we can obtain

$$\frac{2\widetilde{\mu}-z_1-z_2}{2\widetilde{\mu}+z_1+z_2}\left[2B_1\widetilde{X}+\frac{\lambda}{r+\lambda}\left(\widetilde{x}+\frac{1}{\widetilde{\mu}}\right)-\sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1+\frac{z_2}{\widetilde{\mu}}\right)\right]=$$
$$2B_2/\widetilde{X}+\frac{\lambda}{r+\lambda}\left(\widetilde{x}-\frac{1}{\widetilde{\mu}}\right)-\sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1-\frac{z_2}{\widetilde{\mu}}\right). \qquad \text{(xxxi)}$$

Using (xxiii) and (xxiv) in (xxxi) one can then obtain

$$\frac{2\widetilde{\mu}-z_1-z_2}{2\widetilde{\mu}+z_1+z_2}\left[\frac{\widetilde{X}}{\underline{X}}\frac{r}{r+\lambda}\left(\underline{x}+\frac{1}{\widetilde{\mu}}\right)+\frac{\lambda}{r+\lambda}\left(\widetilde{x}+\frac{1}{\widetilde{\mu}}\right)-\sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1+\frac{z_2}{\widetilde{\mu}}\right)\right]=$$
$$\frac{\underline{X}}{\widetilde{X}}\frac{r}{r+\lambda}\left(\underline{x}-\frac{1}{\widetilde{\mu}}\right)+\frac{\lambda}{r+\lambda}\left(\widetilde{x}-\frac{1}{\widetilde{\mu}}\right)-\sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1-\frac{z_2}{\widetilde{\mu}}\right), \qquad \text{(xxxii)}$$

which is an equation on only $\underline{x}$ and $\widetilde{x}$. Note that when $\beta\to\infty$ we have $\underline{x},\widetilde{x}\to\sqrt{\frac{\sigma^2}{2r}}$ and (xxxii) is satisfied.

Let $\delta_1=\underline{x}-\widetilde{x}, \delta_2=\overline{x}-\underline{x}, D_1=e^{\widetilde{\mu}\delta_1}$, and $D_2=e^{\widetilde{\mu}\delta_2}$. Using (xxv) and (xxvii) to take out B_5, and using B_1 from (xxiii), B_2 from (xxiv), B_3 from (xxi), and $\widetilde{C}_1$ from (xxix), we can obtain

$$\frac{1}{D_2}\left(\overline{x}+\frac{1}{\widetilde{\mu}}\right)=\frac{\lambda+r}{r}G_1(\underline{x},\widetilde{x}), \qquad \text{(xxxiii)}$$

where

$$\begin{aligned}G_1(\underline{x},\widetilde{x}) &= D_1\Bigg[-\frac{\lambda^2}{(r+\lambda)^2}\left(\widetilde{x}+\frac{1}{\widetilde{\mu}}\right)+\frac{r+\beta}{\beta}\widetilde{x}\left(1+\frac{z_2}{\widetilde{\mu}}\right)+\sqrt{\frac{r+\beta}{\beta}}\frac{z_2-z_1}{2\widetilde{\mu}+z_1+z_2}\Bigg[\frac{1}{D_1}\frac{r}{r+\lambda}\left(\underline{x}+\frac{1}{\widetilde{\mu}}\right)+\\ &\quad \frac{\lambda}{r+\lambda}\left(\widetilde{x}+\frac{1}{\widetilde{\mu}}\right)-\widetilde{x}\sqrt{\frac{r+\beta}{\beta}}\left(1+\frac{z_2}{\widetilde{\mu}}\right)\Bigg]\Bigg]-\frac{\lambda r}{4(r+\lambda)^2}\Bigg(\underline{x}(3+2\widetilde{\mu}\delta_1+D_1^2)+\\ &\quad \frac{1}{\widetilde{\mu}}(5+2\widetilde{\mu}\delta_1-D_1^2)\Bigg). \qquad \text{(xxxiv)}\end{aligned}$$

Similarly, using (xxvi) and (xxviii) to take out B_6, and using B_1 from (xxiii), B_2 from (xxiv), B_4 from (xxii), and $\widetilde{C}_1$ from (xxix), we can obtain

$$D_2\left(\overline{x}-\frac{1}{\widetilde{\mu}}\right)=\frac{\lambda+r}{r}G_2(\underline{x},\widetilde{x}), \qquad \text{(xxxv)}$$

where

$$G_2(\underline{x},\widetilde{x}) = \frac{1}{D_1}\Bigg[-\frac{\lambda^2}{(r+\lambda)^2}\left(\widetilde{x}-\frac{1}{\widetilde{\mu}}\right)+\frac{r+\beta}{\beta}\widetilde{x}\left(1-\frac{z_2}{\widetilde{\mu}}\right)+\sqrt{\frac{r+\beta}{\beta}}\frac{z_1-z_2}{2\widetilde{\mu}+z_1+z_2}\Bigg[\frac{1}{D_1}\frac{r}{r+\lambda}\Bigg(\underline{x}+\frac{1}{\widetilde{\mu}}\Bigg)+\frac{\lambda}{r+\lambda}\left(\widetilde{x}+\frac{1}{\widetilde{\mu}}\right)-\widetilde{x}\sqrt{\frac{r+\beta}{\beta}}\left(1+\frac{z_2}{\widetilde{\mu}}\right)\Bigg]\Bigg]-\frac{\lambda r}{4(r+\lambda)^2}\Bigg(\underline{x}(3-2\widetilde{\mu}\delta_1+\frac{1}{D_1^2})-\frac{1}{\widetilde{\mu}}(5-2\widetilde{\mu}\delta_1-\frac{1}{D_1^2})\Bigg). \tag{xxxvi}$$

Note then that (xxxii), (xxxiii), and (xxxv) is a system of equations for $\overline{x},\underline{x}$, and $\widetilde{x}$. Note also that putting (xxxiii) and (xxxv) together one obtains

$$\overline{x}^2 = \frac{(\lambda+r)^2}{r^2}G_1G_2+\frac{1}{\widetilde{\mu}^2}, \tag{xxxvii}$$

which determines $\overline{x}$ as a function of $\underline{x}$ and $\widetilde{x}$. Plugging it in (xxxiii), we can then use (xxxii) and (xxxiii) to solve for $\underline{x}$ and $\widetilde{x}$.

Derivation of Optimal Decision-Making for $\beta = 0$ in the Two Search Modes Case:

We have that the condition $\frac{\beta}{r+\beta}V_1(\widetilde{x}) = \widetilde{x}$, (xx), is no longer required, and that conditions (xii)-(xv), are replaced by the conditions

$$\begin{aligned}
B_5 + B_6 &= C_2 && \text{(xxxviii)}\\
\widetilde{\mu}B_5 - \widetilde{\mu}B_6 + \frac{\lambda^2}{(r+\lambda)^2} &- \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_1 + \frac{\lambda\widetilde{\mu}}{2(r+\lambda)}B_2 &&\\
&= \widetilde{\mu}C_2 - \frac{\lambda C_1}{\sigma^2\widetilde{\mu}} && \text{(xxxix)}\\
B_1 + B_2 &= C_1 && \text{(xl)}\\
\widetilde{\mu}B_1 - \widetilde{\mu}B_2 + \frac{\lambda}{r+\lambda} &= \widetilde{\mu}C_1, && \text{(xli)}
\end{aligned}$$

respectively.

Using (lv) and (lvi) we can obtain $B_2 = \frac{\lambda}{2\widetilde{\mu}(r+\lambda}$. Using this in (xxiv), we can obtain the condition for the optimal $\underline{x}$ as

$$e^{\mu\underline{x}}(1-\mu)+\frac{\lambda}{r} = 0, \tag{xlii}$$

as $\mu = \widetilde{\mu}$ for $\beta = 0$, which is intuitively the same condition as (9). Using (xxiii) and (xiv) we can then also obtain $C_1 = \frac{r}{2(r+\lambda)}\left(\underline{x} + \frac{1}{\widetilde{\mu}}\right)\frac{1}{\underline{X}} + \frac{\lambda}{2\widetilde{\mu}(r+\lambda)}$.

Note also that in this case (xxviii) is replaced by

$$2B_6 - \frac{\lambda^2}{\widetilde{\mu}(r+\lambda^2)} - \frac{\lambda}{2(r+\lambda}B_2 + \frac{\lambda}{2(r+\lambda)}B_1 = \frac{\lambda C_1}{2(r+\lambda)}. \tag{xliii}$$

Using (xliii) and (xxvi) to substitute away B_6, we can then use B_1, B_2, and B_4 obtained above to yield

$$\frac{\lambda}{4\widetilde{\mu}} + \frac{\lambda}{\widetilde{\mu}} + \frac{r}{4}\underline{X}\left(\underline{x} + \frac{1}{\widetilde{\mu}}\right) = \frac{r+\lambda}{\lambda}\overline{X}\left(\overline{x} - \frac{1}{\widetilde{\mu}}\right) + \frac{\lambda(1-\lambda)(r+\lambda)}{r\widetilde{\mu}} - \frac{\lambda}{2}\underline{x}, \tag{xliv}$$

which determines $\overline{x}$ as a function of $\underline{x}$.

Derivation of Solution for Start-Up Search Costs Case:

Value matching and smooth pasting at $\overline{x}, \widetilde{x}$, $\widehat{x}$, and $\underline{\widehat{x}}$, leads to $V(\overline{x}) = \overline{x}, V'(\overline{x}) = 1, V(\widetilde{x}) = \widetilde{V}(\widetilde{x}), V'(\widetilde{x}) = \widetilde{V}'(\widetilde{x}), V(\widetilde{x}) - F = \widetilde{x}, \widetilde{V}(\widehat{x}) - F = 0, \widetilde{V}(\widehat{x}) = \widehat{V}(\widehat{x}), \widetilde{V}'(\widehat{x}) = \widehat{V}'(\widehat{x}), \widehat{V}(\underline{\widehat{x}}) = 0$, and $\widehat{V}'(\underline{\widehat{x}}) = 0$, which are the conditions

$$\begin{aligned}
C_1 e^{\widehat{\mu\overline{x}}} + C_2 e^{-\widehat{\mu\overline{x}}} &= c/\lambda && \text{(xlv)}\\
C_1 e^{\widehat{\mu\overline{x}}} - C_2 e^{-\widehat{\mu\overline{x}}} &= 0 && \text{(xlvi)}\\
C_1 e^{\widehat{\mu\widetilde{x}}} + C_2 e^{-\widehat{\mu\widetilde{x}}} &= F + c/\lambda && \text{(xlvii)}\\
\widehat{\mu}[C_1 e^{\widehat{\mu\widetilde{x}}} - C_2 e^{-\widehat{\mu\widetilde{x}}}] + 1 &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widetilde{x} && \text{(xlviii)}\\
C_3 e^{\widehat{\mu\widehat{x}}} + C_4 e^{-\widehat{\mu\widehat{x}}} &= F + c/\lambda && \text{(xlix)}\\
\widehat{\mu}[C_3 e^{\widehat{\mu\widehat{x}}} - C_4 e^{-\widehat{\mu\widehat{x}}}] &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widehat{x} && \text{(l)}\\
C_3 e^{\widehat{\mu\underline{\widehat{x}}}} + C_4 e^{-\widehat{\mu\underline{\widehat{x}}}} &= c/\lambda && \text{(li)}\\
C_3 e^{\widehat{\mu\underline{\widehat{x}}}} - C_4 e^{-\widehat{\mu\underline{\widehat{x}}}} &= 0 && \text{(lii)}\\
a_0 + a_1\widetilde{x} + \frac{\lambda F + c}{\sigma^2}\widetilde{x}^2 &= \widetilde{x} + F && \text{(liii)}\\
a_0 + a_1\widehat{x} + \frac{\lambda F + c}{\sigma^2}\widehat{x}^2 &= F. && \text{(liv)}
\end{aligned}$$

From (xlv) and (xlvi) we can obtain $C_1 = \frac{c}{2\lambda}e^{-\widehat{\mu\overline{x}}}$ and $C_2 = \frac{c}{2\lambda}e^{\widehat{\mu\overline{x}}}$. Similarly, (li) and (lii) we can obtain $C_3 = \frac{c}{2\lambda}e^{-\widehat{\mu\underline{\widehat{x}}}}$ and $C_4 = \frac{c}{2\lambda}e^{\widehat{\mu\underline{\widehat{x}}}}$. Using this in the other equations we can then obtain

$$
\begin{aligned}
\frac{c}{2\lambda} e^{\widehat{\mu}(\widetilde{x}-\overline{x})} + \frac{c}{2\lambda} e^{-\widehat{\mu}(\widetilde{x}-\overline{x})} &= F + \frac{c}{\lambda} && \text{(lv)} \\
\widehat{\mu}\left[\frac{c}{2\lambda} e^{\widehat{\mu}(\widetilde{x}-\overline{x})} - \frac{c}{2\lambda} e^{-\widehat{\mu}(\widetilde{x}-\overline{x})}\right] + 1 &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widetilde{x} && \text{(lvi)} \\
\frac{c}{2\lambda} e^{\widehat{\mu}(\widehat{x}-\underline{\widehat{x}})} + \frac{c}{2\lambda} e^{-\widehat{\mu}(\widehat{x}-\underline{\widehat{x}})} &= F + \frac{c}{\lambda} && \text{(lvii)} \\
\widehat{\mu}\left[\frac{c}{2\lambda} e^{\widehat{\mu}(\widehat{x}-\underline{\widehat{x}})} - \frac{c}{2\lambda} e^{-\widehat{\mu}(\widehat{x}-\underline{\widehat{x}})}\right] &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widehat{x} && \text{(lviii)} \\
\frac{\lambda F + c}{\sigma^2}(\widetilde{x}^2 - \widehat{x}^2) + a_1(\widetilde{x} - \widehat{x}) &= \widetilde{x}. && \text{(lix)}
\end{aligned}
$$

From (lv) and (lvii) we can obtain $\overline{x} - \widetilde{x} = \widehat{x} - \underline{\widehat{x}}$. Using (lv) we can also obtain $e^{\widehat{\mu}(\widetilde{x}-\overline{x})} = 1/H$, where

$$H = 1 + \frac{\lambda F}{c} + \sqrt{\left(\frac{\lambda F}{c} + 1\right)^2 - 1}. \tag{lx}$$

Using $\overline{x} - \widetilde{x} = \widehat{x} - \underline{\widehat{x}}$ in (lvi) and (lviii) we can obtain

$$a_1 = \frac{1}{2} - \frac{\lambda F + c}{\sigma^2}(\widetilde{x} + \widehat{x}). \tag{lxi}$$

Substituting in (lix) one obtains $\widetilde{x} = -\widehat{x}$ and $a_1 = 1/2$. Using this in (lvi) one obtains

$$\widetilde{x} = \sqrt{\frac{\sigma^2}{2\lambda} \frac{c}{2(\lambda F + c)} \frac{1 - H^2}{H}} + \frac{\sigma^2}{4(\lambda F + c)}. \tag{lxii}$$

Derivation of Optimal Pricing for the Base Case:

Using value matching of $V_f(x)$ at $\widetilde{x} + P^*$, $V_f(\{\widetilde{x} + P^*\}^-) = V(\{\widetilde{x} + P^*\}^+)$, value matching of $V_f(x)$ at $\overline{x} + P^*$, $V_f(\overline{x} + P^*) = P^*$, and value matching of $W_f(x)$ at $\widetilde{x} + P^*$, $W(\widetilde{x} + P^*) = P^*$, we have the following system of equations:

$$
\begin{aligned}
C_1 e^{\mu(\widetilde{x}+P)} &= \frac{\lambda}{r + \lambda} P + C_2 e^{\widetilde{\mu}(\widetilde{x}+P)} + C_3 e^{-\widetilde{\mu}(\widetilde{x}+P)} && \text{(lxiii)} \\
\frac{\lambda}{r + \lambda} P + C_2 e^{\widetilde{\mu}(\overline{x}+P)} + C_3 e^{-\widetilde{\mu}(\overline{x}+P)} &= P && \text{(lxiv)} \\
\frac{\beta}{r + \beta} C_1 e^{\mu(\widetilde{x}+P)} &= P && \text{(lxv)}
\end{aligned}
$$

Using (lxv) in (lxiii), we get:

$$C_2 e^{\widetilde{\mu}(\widetilde{x}+P)} + C_3 e^{-\widetilde{\mu}(\widetilde{x}+P)} = \frac{r}{r+\lambda}\frac{r+\lambda+\beta}{\beta}P \tag{lxvi}$$

Using $\delta = \overline{x} - \widetilde{x}$, we can rewrite (lxiv) as

$$C_2 e^{\widetilde{\mu}(\widetilde{x}+P)} + C_3 e^{-\widetilde{\mu}(\widetilde{x}+P)} e^{-2\widetilde{\mu}\delta} = \frac{r}{r+\lambda} P e^{-\widetilde{\mu}\delta} \tag{lxvii}$$

or

$$C_2 e^{\widetilde{\mu}(\widetilde{x}+P)} e^{2\widetilde{\mu}\delta} + C_3 e^{-\widetilde{\mu}(\widetilde{x}+P)} = \frac{r}{r+\lambda} P e^{\widetilde{\mu}\delta} \tag{lxviii}$$

Subtract (lxvii) from (lxvi), we get

$$C_3 = \left[\frac{r}{\beta}\frac{1}{1-e^{-2\widetilde{\mu\delta}}} + \frac{r}{r+\lambda}\frac{1}{1+e^{-\widetilde{\mu\delta}}}\right] e^{\widetilde{\mu}(\widetilde{x}+P)} \tag{lxix}$$

Subtract (lxviii) from (lxvi), we get

$$C_2 = \left[\frac{r}{\beta}\frac{1}{1-e^{2\widetilde{\mu\delta}}} + \frac{r}{r+\lambda}\frac{1}{1+e^{\widetilde{\mu\delta}}}\right] P e^{-\widetilde{\mu}(\widetilde{x}+P)} \tag{lxx}$$

Finally, plugging C_2 and C_3 into (47) produces (51).

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