Invitation to Search or Purchase? Optimal Multi-attribute Advertising

Yunfei (Jesse) Yao

The Chinese University of Hong Kong

Oct. 2025

Motivation

- Consumers often evaluate different attributes of a product before making a purchasing decision
- Learning requires time and effort
- Firms want to encourage consumers to purchase or continue seeking information rather than abandoning search
- May provide information through advertising

Motivation

- Advertising can raise consumers' interest
  - Tesla's ad highlighting its interior design featuring a large screen for a car buyer who values electronic entertainment systems
- Advertising may also create a negative impression
  - The same ad for a different buyer who prefers the mechanical feel of a car

Research Questions

- Whether the firm wants to advertise or not?
- What is the optimal advertising content - which attribute should the firm advertise?
- What is the role of advertising? Does the firm want to invite the consumer to purchase the product directly or to search for additional information?
- What should be the KPI for an advertising campaign?

Contribution

- Examining the choice of specific and feasible advertising content while allowing for the possibility of consumer search

Related Literature

- **Informational role of advertising**
  Nelson 1974, Anderson and Renault 2006, Lewis and Sappington 1994, Anderson and Renault 2006, 2009, Mayzlin and Shin 2011, Sun 2011, Despotakis and Yu 2023, Shin and Wang 2024, Yao 2024
- **Limited attention allocation**
  Arbatskaya 2007, Armstrong et al. 2009, Xu et al. 2010, Armstrong and Zhou 2011, Bordolo et al. 2013, Kőszegi and Szeidl 2013, Branco et al. 2016, Zhu and Dukes 2017, Jeziorski and Moorthy 2018, Ke and Lin 2020
- **Optimal information acquisition**
  Stigler 1961 and Weitzman 1979, Moscarini and Smith 2001, Branco et al. 2012, Liu and Dukes 2013, Ke et al. 2016, Ke and Villas-Boas 2019, Jerath and Ren 2023, Pease 2023

Results Preview

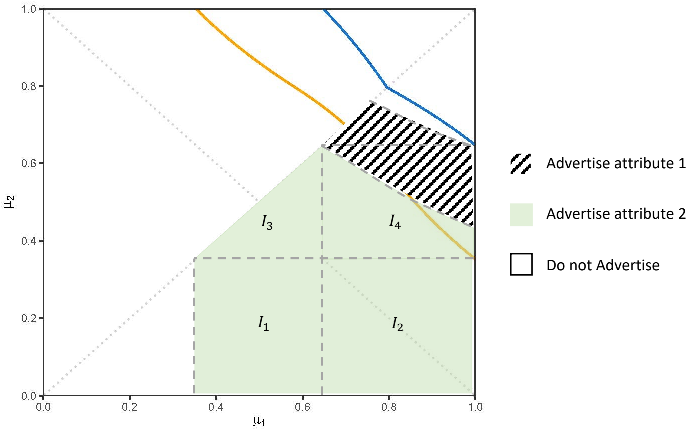


Figure: Optimal Advertising Strategy

Results Preview

Extreme prior beliefs about both attributes:

- The firm does not advertise
- No advertising: invitation to search (purchase) when the belief is high (very high);

Results Preview

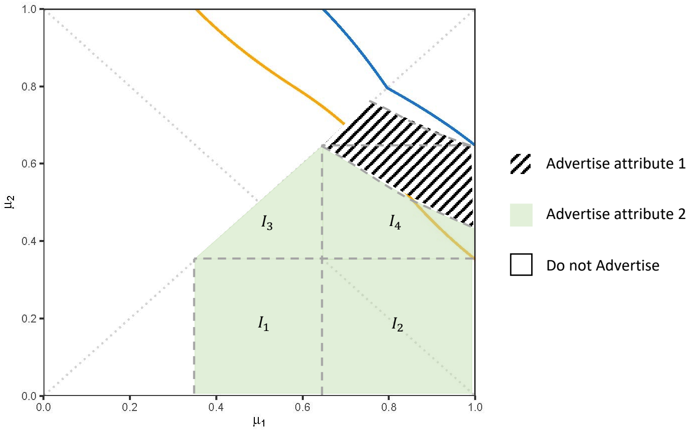


Figure: Optimal Advertising Strategy

Results Preview

Extreme prior beliefs about both attributes:

- The firm does not advertise

- No advertising: invitation to search (purchase) when the belief is high (very high);

Otherwise:

- Advertise the better attribute if the consumer is optimistic enough about the worse attribute;

  Advertise the worse attribute if the consumer is less optimistic

- Role of advertising: non-monotonic in the belief

  Reason: different binding incentives of the firm in different belief regions.

Model

- A firm and a consumer, both risk-neutral
- The consumer decides whether to purchase a product
- Two attributes with independent values (horizontal match)
  $U = U_1 + U_2,\ U_i \in \{0, 1\}$
  Consumer's prior belief that attribute i is good: $\mu_i(0)$
- Price $p \geq 3/2$: exogenous (so the consumer will not buy if one attribute is known to be bad)
- Time is continuous, $t \in [0, +\infty)$

Model

- The firm can reveal the value of one attribute by informative advertising (limited bandwidth)
- Firm advertises attribute $i \in \{1, 2\}$
  $$\Rightarrow U_i = \begin{cases} 1, \textit{with probability } \mu_i \\ 0, \textit{with probability } 1 - \mu_i \end{cases}$$
- Full disclosure: sharpest result (working on noisy disclosure extensions)

Model

- Consumers can learn more about the attributes
- If the firm advertises one attribute, they may search for information about the other attribute
- If the firm does not advertise, they may search for information about either attribute
- Flow search cost: c
  Noisy signal: $dS_i(t) = U_i dT_i(t) + \sigma dW_i(T_i(t))$
- Belief evolution:
  $d\mu_i(t) = \frac{1}{\sigma^2}\mu_i(t)[1 - \mu_i(t)]\{dS_i(t) - \mathbb{E}[U_i|\mathcal{F}_t]dT_i(t)\}$

Consumer's Search Strategy if the Firm Advertises One Attribute

- Firm advertises attribute $i \in \{1, 2\}$
  $$\Rightarrow U_i = \begin{cases} 1, \textit{with probability } \mu_i \\ 0, \textit{with probability } 1 - \mu_i \end{cases}$$
- Consumer may only search for information about the other attribute j: a one-dimensional search problem
- Equivalent to considering a single-attribute product whose value is U_j and whose price is $p' := p - U_i$.
- Consumer will quit if attribute i is bad
- Focus on the case in which attribute i is good

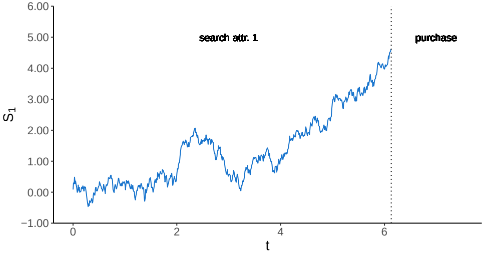
Consumer's Search Strategy if the Firm Advertises One Attribute

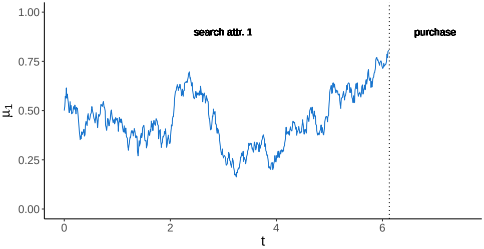
- Ke and Villas-Boas (2019) has characterized the optimal search strategy:

  The consumer

- searches for more information if $\mu_j \in (\underline{\mu}_j, \bar{\mu}_j)$,
- purchases the product if $\mu_j \geq \bar{\mu}_j$,
- and quits if $\mu_j \leq \underline{\mu}_j$.

Sample Path of Signals and Belief Evolution When Both Attributes Are Good and the Firm Advertises Attribute 2

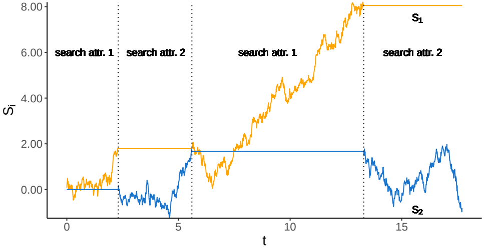


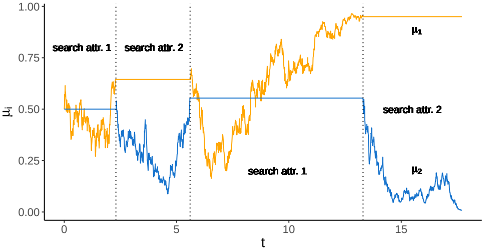


Consumer's Search Strategy if the Firm Does Not Advertise

- Consumers face a two-dimensional search problem
- They have uncertainty about both attributes
- Can search for information about either attribute

Sample Path of Signals and Belief Evolution When $U_1 = 1, U_2 = 0$, and the Firm Does Not Advertise





Consumer's Search Strategy if the Firm Does Not Advertise

- Derive the HJB equation
- Show the value function is a viscosity solution of the HJB equation (dynamic programming principle)
- Prove the viscosity solution of the HJB equation is unique
  $\Rightarrow$ the viscosity solution must be the value function
- Construct a learning rule and stopping time, and verify that the corresponding payoff is a viscosity solution of the HJB equation
  $\Rightarrow$ the learning rule and stopping time is optimal

Consumer's Search Strategy if the Firm Does Not Advertise

- When the consumer searches for information about attribute one:

$$V(\mu_1, \mu_2) = -cdt + \mathbb{E}[V(\mu_1 + d\mu_1, \mu_2)]$$

  Taylor's expansion and Ito's lemma $\Rightarrow$

$$\frac{\mu_1^2(1-\mu_1)^2}{2\sigma^2} V_{\mu_1\mu_1}(\mu_1, \mu_2) - c = 0$$

- When the consumer purchases the product:

$$V(\mu_1, \mu_2) = \mu_1 + \mu_2 - p$$

- When the consumer quits without purchasing:

$$V(\mu_1, \mu_2) = 0$$

Consumer's Search Strategy if the Firm Does Not Advertise

HJB equation:

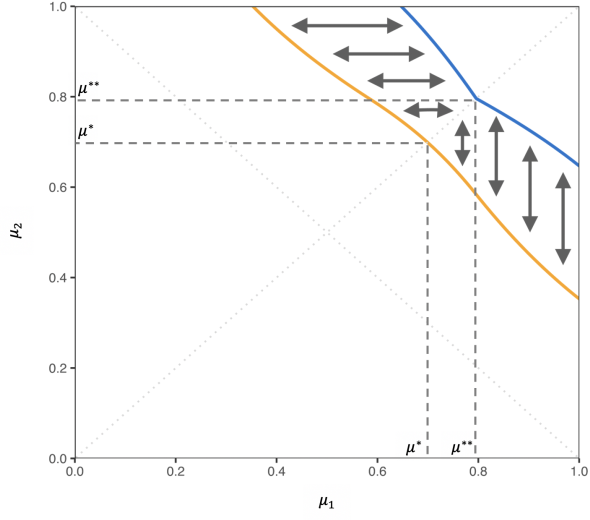
$$\max\left\{ \max_{i=1,2} \overbrace{\left[\frac{\mu_i^2(1-\mu_i)^2}{2\sigma^2} V_{\mu_i\mu_i}(\mu_1,\mu_2) - c \right]}^{\text{search for attribute i}}, \right.$$

$$\left. \max[\overbrace{\mu_1+\mu_2-p}^{\text{purchase}}, \overbrace{\ 0\ }^{\text{quit}}] - V(\mu_1,\mu_2) \right\} = 0$$

Proposition

There exists a unique viscosity solution to the HJB equation.

Consumer's Search Strategy if the Firm Does Not Advertise (Low Search Cost c)



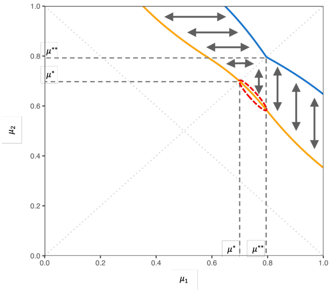
purchasing boundary
$(\mu_1, \bar{\mu}(\mu_1))$

quitting boundary
$(\mu_1, \underline{\mu}(\mu_1))$

, when $\mu_1 \geq \mu_2$

Characterization of the Quitting Boundary

$\mu_1 \in [\mu^*, \mu^{**}]$, $\mu_1 \geq \mu_2$



Searching for attribute two ($\mu_1 \geq \mu_2$):

$$\frac{\mu_2^2(1-\mu_2)^2}{2\sigma^2} V_{\mu_2\mu_2}(\mu_1, \mu_2) - c = 0$$

General solution:

$$V(\mu_1, \mu_2) = 2\sigma^2 c(1-2\mu_2)\ln\frac{1-\mu_2}{\mu_2} + B_1(\mu_1)\mu_2 + B_2(\mu_1)$$

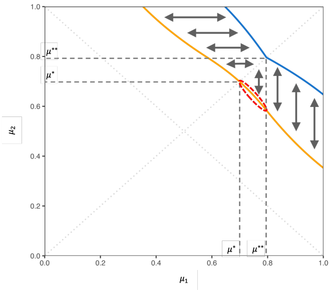
$V(\mu_1, \mu_2) = 0$ at the quitting boundary $(\mu_1, \underline{\mu}(\mu_1))$
Value matching and smooth pasting (wrt μ_2) implies

$$\frac{V(\mu_1, \mu_2)}{2\sigma^2 c} = (1-2\mu_2)\ln\frac{1-\mu_2}{\mu_2} + \phi(\underline{\mu}(\mu_1))\mu_2 - \psi(\underline{\mu}(\mu_1))$$

, $\phi(x) = 2\ln\frac{1-x}{x} + \frac{1}{x} - \frac{1}{1-x}, \psi(x) = \ln\frac{1-x}{x} + \frac{1-2x}{1-x}.$

Characterization of the Quitting Boundary

$\mu_1 \in [\mu^*, \mu^{**}]$, $\mu_1 \geq \mu_2$



Searching for attribute two ($\mu_1 \geq \mu_2$):

$$\frac{V(\mu_1, \mu_2)}{2\sigma^2 c} = (1 - 2\mu_2) \ln \frac{1 - \mu_2}{\mu_2} + \phi(\underline{\mu}(\mu_1))\mu_2 - \psi(\underline{\mu}(\mu_1))$$

Searching for attribute one ($\mu_1 < \mu_2$):

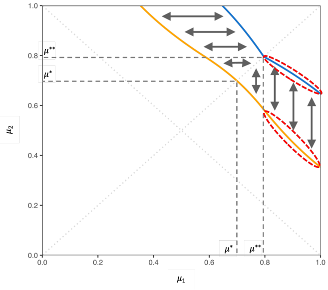
$$\frac{V(\mu_1, \mu_2)}{2\sigma^2 c} = (1 - 2\mu_1) \ln \frac{1 - \mu_1}{\mu_1} + \phi(\underline{\mu}(\mu_2))\mu_1 - \psi(\underline{\mu}(\mu_2))$$

Continuity of $V_{\mu_1}(\mu_1, \mu_2)$ at the boundary (main diagonal) implies:

$$\underline{\mu}'(\mu) = \frac{\phi(\underline{\mu}(\mu)) - \phi(\mu)}{\phi'(\underline{\mu}(\mu))[\mu - \underline{\mu}(\mu)]}, \quad \text{for } \mu \in [\mu^*, \mu^{**}] \quad (D_1)$$

Characterization of Purchasing and Quitting Boundaries

$\mu_1 \in [\mu^{**}, 1]$, $\mu_1 \geq \mu_2$



General solution:

$$V(\mu_1, \mu_2) = 2\sigma^2 c(1 - 2\mu_2) \ln \frac{1 - \mu_2}{\mu_2} + B_1(\mu_1)\mu_2 + B_2(\mu_1)$$

Value matching & smooth pasting at the quitting boundary:

$$\frac{V(\mu_1, \mu_2)}{2\sigma^2 c} = (1 - 2\mu_2) \ln \frac{1 - \mu_2}{\mu_2} + \phi(\underline{\mu}(\mu_1))\mu_2 - \psi(\underline{\mu}(\mu_1))$$

at the purchasing boundary:

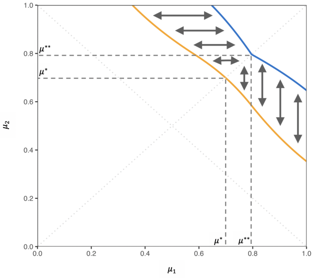
$$\frac{V(\mu_1, \mu_2)}{2\sigma^2 c} = (1 - 2\mu_2) \ln \frac{1 - \mu_2}{\mu_2} + \phi(\bar{\mu}(\mu_1))\mu_2 - \psi(\bar{\mu}(\mu_1)) + \frac{\mu_1 - \mu_2 - p}{2\sigma^2 c}$$

Equivalence in the search region implies:

$$\begin{cases} \phi(\underline{\mu}(\mu)) - \phi(\bar{\mu}(\mu)) = \frac{1}{2\sigma^2 c} \\ \psi(\underline{\mu}(\mu)) - \psi(\bar{\mu}(\mu)) = \frac{p-\mu}{2\sigma^2 c} \end{cases}, \text{ for } \mu \in [\mu^{**}, 1]$$

Characterization of Purchasing and Quitting Boundaries

Cutoff beliefs μ^* and μ^{**}



$$\begin{cases} \phi(\underline{\mu}(\mu)) - \phi(\bar{\mu}(\mu)) = \frac{1}{2\sigma^2 c} \\ \psi(\underline{\mu}(\mu)) - \psi(\bar{\mu}(\mu)) = \frac{p-\mu}{2\sigma^2 c} \end{cases}, \text{ for } \mu \in [\mu^{**}, 1]$$

(μ^{**}, μ^{**}) is on the purchasing boundary $(\mu_1, \bar{\mu}(\mu_1))$
$\Rightarrow \bar{\mu}(\mu^{**}) = \mu^{**}$

$$\Rightarrow \begin{cases} \phi(\underline{\mu}(\mu^{**})) - \phi(\mu^{**}) = \frac{1}{2\sigma^2 c} \\ \psi(\underline{\mu}(\mu^{**})) - \psi(\mu^{**}) = \frac{p-\mu^{**}}{2\sigma^2 c} \end{cases}$$

, which uniquely determines μ^{**}

The previous slides have shown:

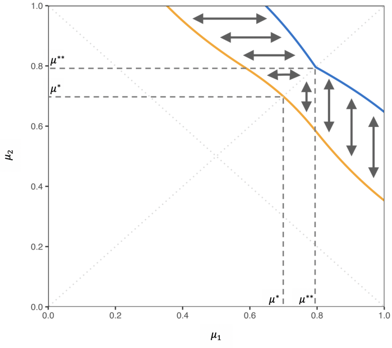
$$\underline{\mu}'(\mu) = \frac{\phi(\underline{\mu}(\mu)) - \phi(\mu)}{\phi'(\underline{\mu}(\mu))[\mu - \underline{\mu}(\mu)]}, \text{ for } \mu \in [\mu^*, \mu^{**}] \quad (D_1)$$

(μ^*, μ^*) is on the quitting boundary $\Rightarrow \mu^* = \underline{\mu}(\mu^*)$, which uniquely determines μ^*

Characterization of Purchasing and Quitting Boundaries

Proposition

Both $\underline{\mu}(\mu)$ and $\bar{\mu}(\mu)$ strictly decrease in μ, while the width of the search region, $\bar{\mu}(\mu) - \underline{\mu}(\mu)$, strictly increases in μ. In addition, if $\underline{\mu}(\mu) \geq 1/2$, then the slope of the quitting boundary is less than -1 and the slope of the purchasing boundary is greater than -1.



Key to the Verification

HJB equation:

$$\max\left\{\max_{i=1,2}\overbrace{\left[\frac{\mu_i^2(1-\mu_i)^2}{2\sigma^2}V_{\mu_i\mu_i}(\mu_1,\mu_2)-c\right]}^{\text{search for attribute i}},\right.$$

$$\left.\max[\overbrace{\mu_1+\mu_2-p}^{\text{purchase}},\ \overbrace{0}^{\text{quit}}]-V(\mu_1,\mu_2)\right\}=0$$

- In the search region, $\frac{\mu_1^2(1-\mu_1)^2}{2\sigma^2}V_{\mu_1\mu_1}(\mu_1,\mu_2)-c\leq 0$ if $\mu_1\geq\mu_2$
- In the search region, $\frac{\mu_2^2(1-\mu_2)^2}{2\sigma^2}V_{\mu_2\mu_2}(\mu_1,\mu_2)-c\leq 0$ if $\mu_1<\mu_2$
- New verification arguments for $\mu_1\in[\mu^{**},1]$

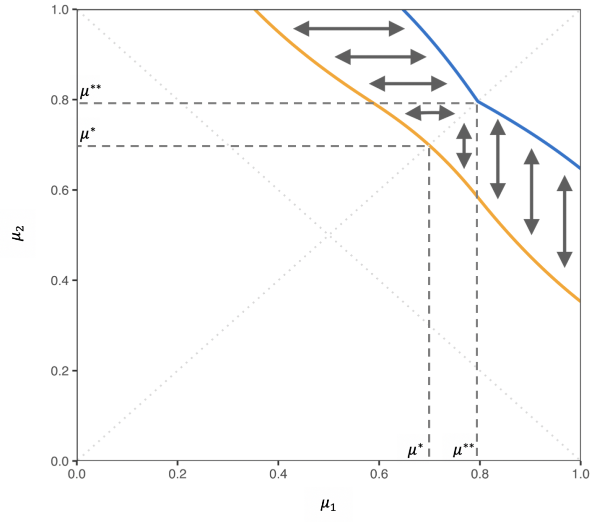
Firms' Advertising Strategy: Advertising One Attribute

Proposition

Suppose the firm advertises attribute one. The probability that the consumer purchases the product is:

$$
\begin{aligned}
& P_1(\mu_1, \mu_2) \\
:= \; & \mathbb{P}[\textit{purchasing} | \textit{firm advertises attribute one, prior belief } (\mu_1, \mu_2)] \\
= \; & \begin{cases} \mu_1, & \textit{if } \mu_2 \geq \bar{\mu}(1) \\ \mu_1 \cdot \frac{\mu_2 - \underline{\mu}(1)}{\bar{\mu}(1) - \underline{\mu}(1)}, & \textit{if } \mu_2 \in [\underline{\mu}(1), \bar{\mu}(1)] \\ 0, & \textit{if } \mu_2 \leq \underline{\mu}(1) \end{cases}
\end{aligned}
$$

Firms' Advertising Strategy: Advertising One Attribute



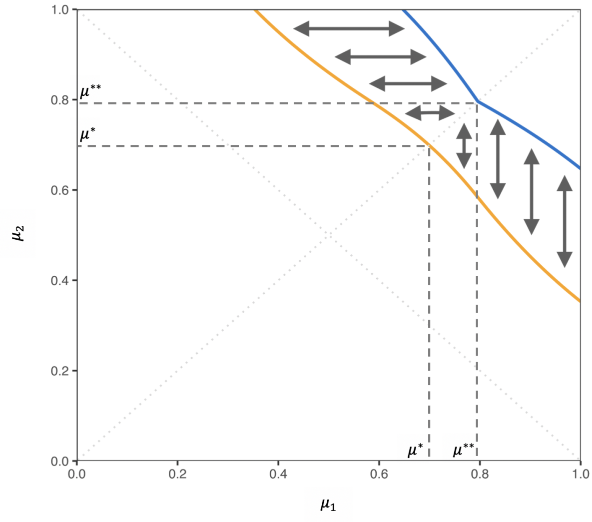
Firms' Advertising Strategy: Advertising One Attribute

Corollary

Suppose the firm advertises attribute two. The probability that the consumer purchases the product is:

$$
\begin{aligned}
& P_2(\mu_1, \mu_2) \\
:= \; & \mathbb{P}[\textit{purchasing} | \textit{firm advertises attribute two, prior belief } (\mu_1, \mu_2)] \\
= \; & \begin{cases} \mu_2, & \textit{if } \mu_1 \geq \bar{\mu}(1) \\ \mu_2 \cdot \frac{\mu_1 - \underline{\mu}(1)}{\bar{\mu}(1) - \underline{\mu}(1)}, & \textit{if } \mu_1 \in [\underline{\mu}(1), \bar{\mu}(1)] \\ 0, & \textit{if } \mu_1 \leq \underline{\mu}(1) \end{cases}
\end{aligned}
$$

Firms' Advertising Strategy: Not Advertising

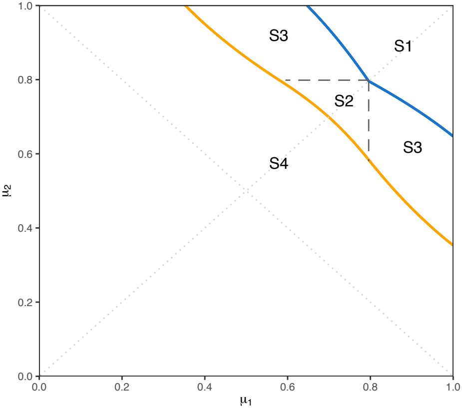


Firms' Advertising Strategy: Not Advertising

$$\mathbb{P}[\text{purchasing}|(\mu_1, \mu_2)]$$

$$= \begin{cases} 1, & \text{in } S_1 \\ \frac{\mu_2 - \underline{\mu}(\mu_1)}{\mu_1 - \underline{\mu}(\mu_1)} \; e^{-\int_{\mu_1}^{\mu^{**}} \frac{2}{x - \underline{\mu}(x)} dx}, & \text{in } S_2 \\ \frac{\mu_2 - \underline{\mu}(\mu_1)}{\bar{\mu}(\mu_1) - \underline{\mu}(\mu_1)}, & \text{in } S_3 \\ 0, & \text{in } S_4 \end{cases}$$

, when $\mu_1 \geq \mu_2$

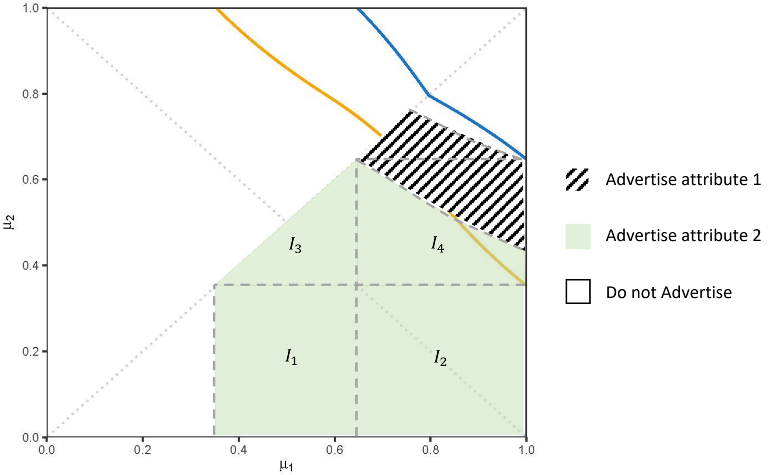


Optimal Advertising Strategy (when $\mu_1 \geq \mu_2$)

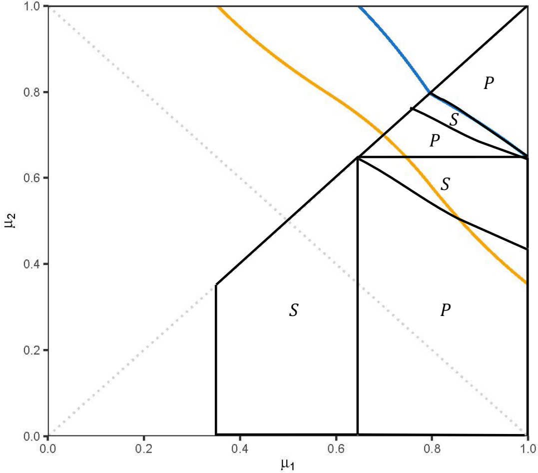
Two trade-offs:

- Whether (or not) to advertise
  - Pro: a higher likelihood of purchase if the advertised attribute is good
  - Con: no chance of sales if the advertised attribute is bad
- Whether to advertise the better attribute (or worse attribute)
  - Pro: a higher chance of keeping the consumer interested
  - Con: lower conditional evaluation and purchasing probability

Optimal Advertising Strategy



Consumer Behavior Induced by the Advertising Strategy



S Invitation to search

P Invitation to purchase

The Firm's Binding incentive

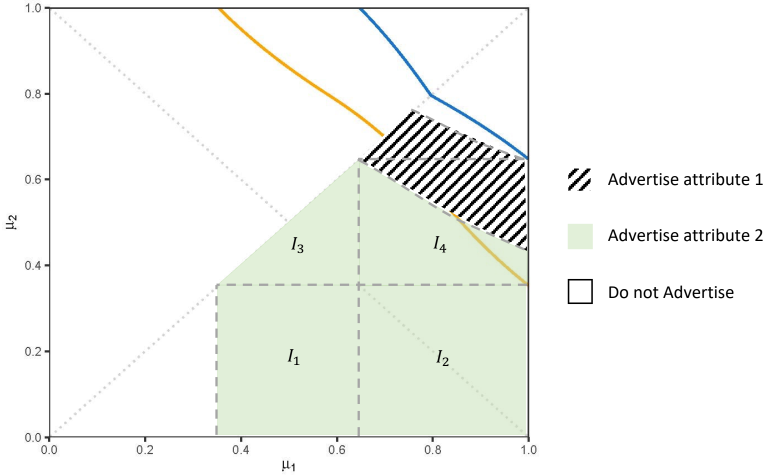
- Consumer is very pessimistic about an attribute:

  to convince the consumer to consider the product

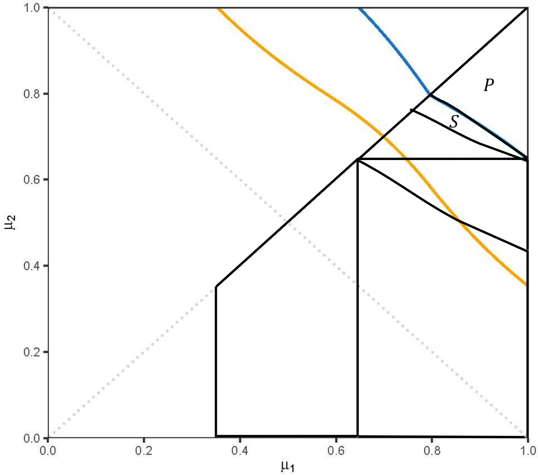
- Less extreme belief:

  to avoid or reduce the amunt of **wasted belief**

Optimal Advertising Strategy (Blank Region)



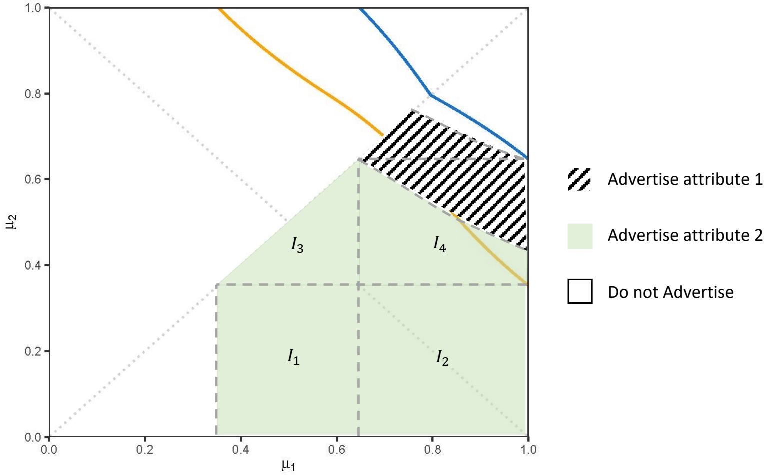
Optimal Advertising Strategy (Blank Region)



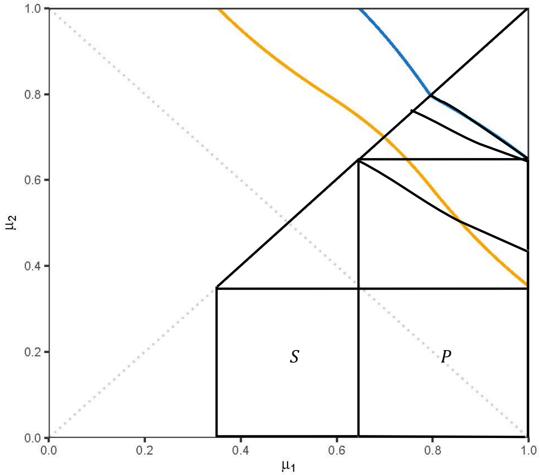
S Invitation to search

P Invitation to purchase

Optimal Advertising Strategy (Region I_1 & I_2)



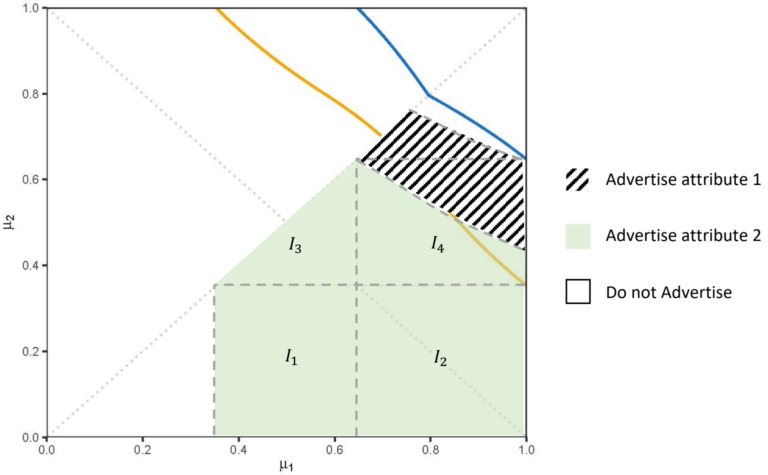
Optimal Advertising Strategy (Region I_1 & I_2)



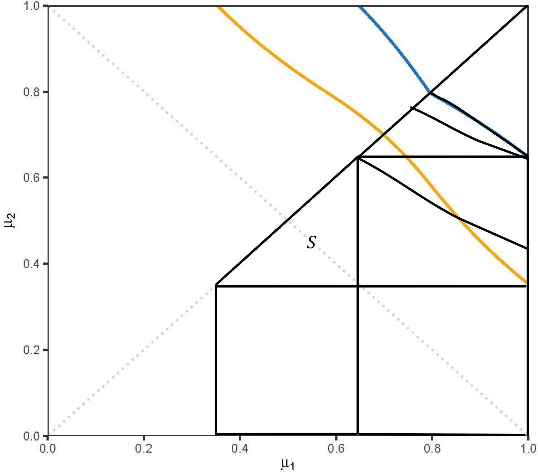
S Invitation to search

P Invitation to purchase

Optimal Advertising Strategy (Region I_3)



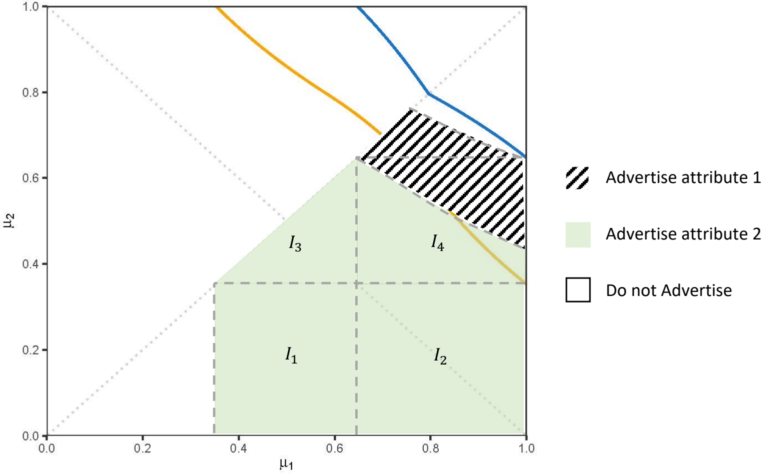
Optimal Advertising Strategy (Region I_3)



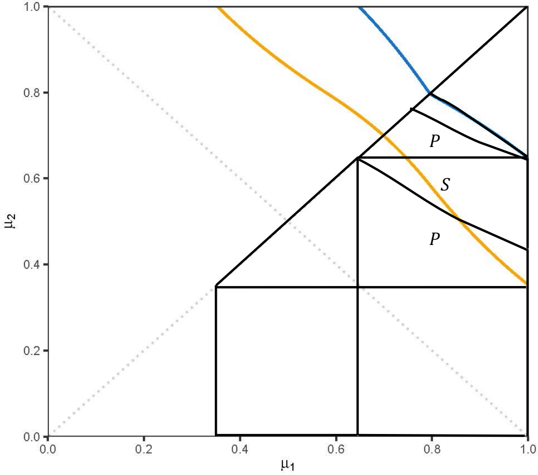
S Invitation to search

P Invitation to purchase

Optimal Advertising Strategy (Region I_4 and the Diagonal Striped Black Region)



Optimal Advertising Strategy (Region I_4 and the Diagonal Striped Black Region)



S Invitation to search

P Invitation to purchase

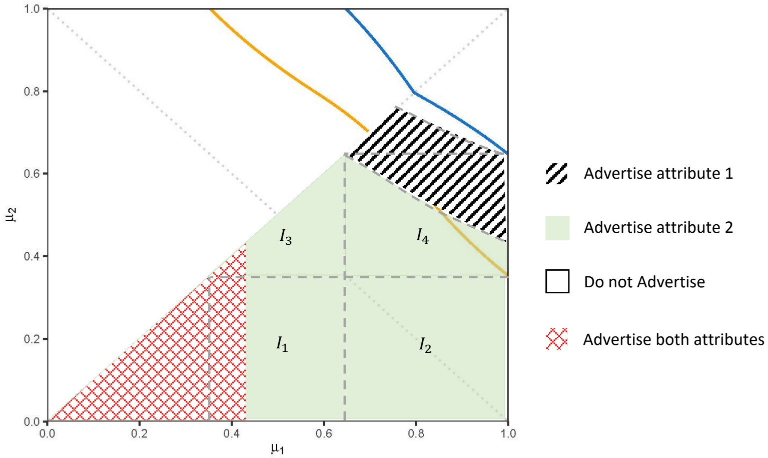
Extension: Allowing Advertising Both Attributes

- The same setup as before, except that:
- The firm can also advertise both attributes

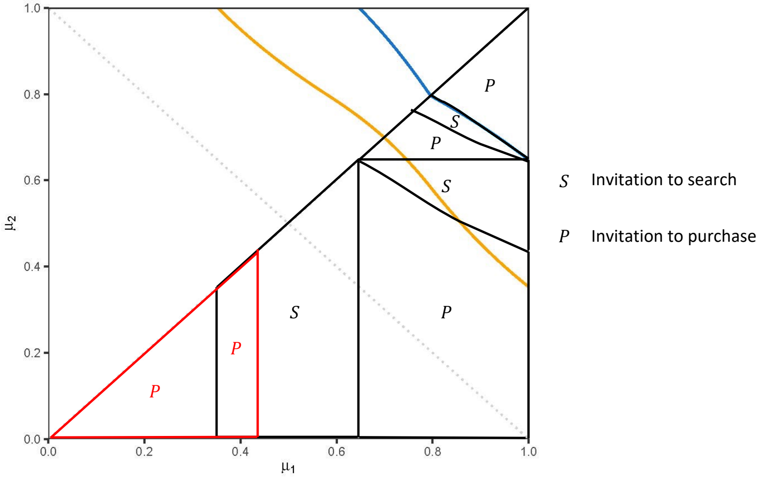
  Uncertainty about both attributes are resolved

  $\Rightarrow$ the consumer purchases iff both attributes are good

Extension: Allowing Advertising Both Attributes



Extension: Allowing Advertising Both Attributes



Thank you!