Precision-Recall Tradeoff in Competitive Targeting

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Abstract

Firms face a fundamental precision-recall trade-off in choosing their targeting strategies. They can choose to target a smaller set of consumers with a high probability of conversion (precision) but miss out on many consumers who might still be interested in their product. Conversely, firms can target a larger set of consumers (recall), but this results in a greater probability that their targeting is wasted on uninterested consumers. This trade-off is particularly relevant in contemporary targeting contexts when firms use machine learning algorithms to predict the probability with which individual consumers will be interested. We study this precision-recall trade-off under competition between firms who strategically choose their algorithmic targeting policies. We show that competing firms favor a targeting policy that has higher precision but lower recall as compared to a monopoly. Firms target fewer consumers when their algorithms are more correlated. They also strategically decrease the precision of their targeting policies in order to reduce competition. If firms endogenously choose their algorithmic correlation, then there is an equilibrium incentive to decrease the correlation.

Keywords Precision-Recall Trade-off, Targeted Advertising, Machine Learning, Algorithms

JEL Codes D43, L13, M37

I Introduction

Targeting is fundamental to marketing strategy. One of the general challenges in targeted marketing is reducing wastage: Firms must focus their marketing budget on consumers who are most likely to be interested in their products, which requires them to make predictions of consumer types based on observable characteristics and behavior. At the core the prediction of consumer types for targeting can be interpreted as a classification problem - i.e., trying to predict whether a consumer is interested in the product and the likelihood that she will make a purchase if targeted by the firms' marketing activities. The traditional targeting examined in the literature typically involves targeting based on simple binary prediction signals of the consumers' interest in the product. However, in the contemporary digital economy firms have rich information on consumer characteristics, opinions, behaviors, and social interactions. As part of their data analytics strategy, firms can increasingly use AI and machine learning algorithms to produce individual-level predictions for targeting consumers.

In general, any algorithm, ranging from a simple Logit regression to a more complex neural network, takes data on observed consumer characteristics and behaviors as input and produces a probability or likelihood of conversion that firms can use for targeting consumers as the output. In other words, in most contexts involving algorithmic targeting, firms do not target based on binary signals as is typically modeled in the literature. Instead, they face a *distribution of probabilities* for which they need to decide whom to target. For every individual consumer, the algorithm produces a probability with which the consumer will be interested in purchasing. For example, Shi et al. (2022) describe how Alibaba decides which customers to target with promotional messages based on the predicted likelihood of visits and purchases (i.e., algorithmic scores), which are generated using machine learning algorithms using high-dimensional consumer data. Our framework captures this aspect of the classification problem and shows that it is important for characterizing the precision-recall trade-off in the competitive algorithmic targeting strategies of firms.

In reality, the classification of consumers by algorithms is always imperfect because of data limitations, privacy concerns, or model selection constraints. Given this, the firm faces a fundamental trade-off associated with classification problems relevant to all machine learning models, namely the "precision-recall" trade-off. In the context of targeting, precision is defined as the share of interested consumers among all consumers targeted by the firm - i.e., it measures to what extent consumers targeted are indeed interested in the product. On the other hand, recall is defined as the share of interested consumers targeted out of all interested consumers.

Firms can either focus on a smaller set of consumers with high predicted probabilities of conversion (precision) so that most targeted consumers indeed turn out to be those who are interested and purchase the product, or they can target a larger set of consumers so that most of the interested consumers are targeted (recall), but not both. For the intermediate consumers who have moderate likelihoods, there will be some consumers who are interested but whom the firm will miss out on targeting if it prioritizes precision; whereas there will be others who are uninterested and on whom the firm will waste its advertising budget if it prioritizes recall. In practice, there is evidence that firms are well aware of this trade-off. For example, LinkedIn suggests advertisers to "balance precision with volume" in their targeting policies[1].

In this paper, we examine the precision-recall trade-off in competitive targeting. How should a firm choose its targeting policy, and what are the equilibrium targeting policies of competing firms? In practice the algorithmic predictions of firms might be correlated either because firms may use public data or similar data analytics tools. What is the impact of the correlation in the firms' algorithms on their precision and recall choices? The model considers competition between two firms for a set of consumers. Consumers have a binary type: they are either interested in

[1] https://www.linkedin.com/business/marketing/blog/content-marketing/what-is-b2b-marketing-definition-strategy-and-trends

the product and will make a purchase if targeted, or are not interested and will not make a purchase even if they are targeted. Firms do not know the type of consumers and they rely on algorithms to make predictions. The algorithm takes as input the available consumer and market information and generates as output a distribution of probabilities that consumers are interested in purchasing. Firms then simultaneously decide who to target based on the predictions. Each firm makes a profit for every interested consumer that they end up targeting, and they also incur a cost for every consumer targeted. If a consumer is targeted by both firms, the profit they generate is lower than the monopoly profit when the consumer is targeted by only one firm.

Competing firms favor a targeting policy that has greater precision but lower recall relative to a monopoly. In other words, firms strategically respond in equilibrium by concentrating their targeting efforts on consumers who have a higher likelihood of conversion, resulting in higher precision of their targeting. For the competing firms, given possible correlation in their predictions, the equilibrium expected profit of a firm depends on the other firm's targeting policy. We first show that there exists a unique mixed-strategy symmetric equilibrium. In this equilibrium, when firms' algorithmic predictions have a high enough correlation, they will never target any consumer for sure. Consumers with higher predicted probabilities of being interested will be targeted with a higher probability. When the correlation is not that high, both firms target consumers who they predict to have a sufficiently high likelihood of being interested for sure, while mixing for consumers who have moderately high conversion probabilities.

The correlation in the firms' algorithms has a nuanced effect on the precision-recall trade-off in targeting. The equilibrium targeting intensity - i.e., the overall number of consumers targeted - decreases in the correlation of firms' predictions because a greater overlap in targeting reduces the expected profits. Thus, both firms strategically target fewer consumers as their predictions become more correlated, and this helps to soften competition.

How does correlation change the precision and recall choices? We find that as firms' predictions converge, recall decreases as firms target fewer consumers. Surprisingly, precision also decreases. In other words, firms on average target consumers who are less likely to be interested. The total number of consumers being targeted generally decreases with competition even if firms' predictions are uncorrelated, implying that competition may reduce consumer welfare by creating less information value for consumers. As firms' predictions become more correlated, the information value further decreases.

We extend the model to allow for endogenous correlation of predictions. We find firms have an incentive to invest in lowering the correlation of their predictions. In another extension, we also consider asymmetric pure strategy equilibria, to show that our main result on the precision-recall trade-off under competition continues to hold.

I.A Related Research

We contribute to the classic research stream on the competitive effects of targeted advertising (Chen et al., 2001; Chen and Iyer, 2002; Iyer et al., 2005; Bergemann and Bonatti, 2011; Zhang and Katona, 2012; Johnson, 2013; Chen et al., 2017; Lauga et al., 2018; Shin and Yu, 2021; Ke et al., 2022; Choi et al., 2023; Ning et al., 2023). Contrary to the standard assumption in the literature that firms receive a binary signal of the consumer's type, our model directly represents the output of a machine learning algorithm as a distribution of the probability of conversion across consumers. By modeling the micro foundation of the signal structure from machine learning algorithms we are able to study the precision-recall trade-off and describe how algorithmic predictions affect targeting policies. There are also some recent papers that examine precision and recall in the context of marketing strategy choices: Berman et al. (2023) examine a recommendation algorithm's choice of precision and recall in designing the optimal information structure to persuade consumers who are uncertain about product fit. Jerath and Ren (2021) consider the consumer's attention allocation to favorable and unfavorable information which has precision recall type effects. Overall the focus of these studies is the design of the information structure to influence consumers, whereas our paper examines the choice of precision and recall in the design of firms' competitive targeting strategies.

We also contribute to the growing literature on strategic interactions of algorithms and machine learning models(Liang, 2019; Miklós-Thal and Tucker, 2019; Salant and Cherry, 2020; Calvano et al., 2020; O'Connor and Wilson, 2021; Montiel Olea et al., 2022). Closely related to our work, Iyer and Ke (2023) consider the model selection problem and the bias-variance trade-off in the context of targeting. They find competition favors bias because it can help firms manage overlaps in targeting, thereby softening competition. We focus on a different trade-off that is also a general feature of machine learning models. The precision-recall trade-off is not about model selection, but rather about firms' decision rule given the prediction that the model generates. In other words, Iyer and Ke (2023) studies how to pick an algorithmic model, while we focus on the micro-foundations of how to deploy algorithmic targeting - i.e., who and how to target given the model's predictions. This is a feature that would apply, in general, to the deployment of any machine learning model design that the firm may end up choosing.

II Model

II.A Basic Setup

There is a market with up to two firms and a unit mass of consumers. Consumers are of two types: interested or uninterested. Interested consumers, if targeted, will make a purchase. Uninterested consumers, as well as interested consumers who are not targeted by the firm, will not make a purchase.[2] The prior probability that a consumer is interested is μ_0. Firms are uncertain about the type of any individual consumer. They rely on predictions from an information system to identify interested consumers. In the context of contemporary big data environments, such a predictive information system can be a machine learning algorithm that provides predictions to identify interested consumers at the individual level. An algorithm, e.g., Logit regression or a more complicated neural network, uses data on consumer and market characteristics, and yields a prediction for each individual consumer, which is the likelihood with which the consumer is interested in the product. Accordingly, we assume that the prediction for consumer i is the probability p_i of that consumer i being interested. The distribution of the predictions $f(p)$ is defined on the support $[0, 1]$. This distribution characterizes the informativeness of the algorithm, which is determined by the limits of the data and the sophistication of the algorithm. The distribution is Bayesian consistent, i.e., the expected probability of a consumer being interested is $\int pf(p)\mathrm{d}p = \mu_0$, the prior probability. At one extreme, a perfect algorithm has a bimodal distribution with two mass points at $p = 0$ and $p = 1$. In contrast, a fully uninformative algorithm has a unimodal distribution with one mass point at $p = \mu_0$.

Firms produce competing products and they reach consumers through targeted advertising. If an interested consumer is targeted by only one firm, they will generate value v for the firm. If both firms target the same consumer, they will generate an expected value $w < v$ for each firm. The unit cost of targeting is c, which is also assumed to be less than v, as otherwise no firm has an incentive to target any consumer. The firm decides which consumers to target based on the predictions of its algorithm. Specifically, consider a targeting policy of $q(p) \in [0, 1]$, in which $q(p)$ is the share of probability-p consumers being targeted. Therefore, we allow firms to use mixed strategies, which can be interpreted in the standard manner as different

[2] A standard interpretation of this assumption is advertising as information - that advertising creates awareness of the existence of the product.

advertising intensities (for example, across time or media channels).

This is a general framework that accommodates the standard models in the targeting literature. For example, the Chen et al. (2001) model assumes that firms can receive noisy binary signals about consumer types. Mapping this to our framework, the signal structure represented by the targetability measure in Chen et al. (2001) is a degenerate bimodal posterior distribution of consumer types (loyal and switcher). As firms' targetability increases, the two mass points of the bimodal distribution shift away from the prior. The perfect targetability case corresponds to the bimodal distribution at 0 and 1.[3]

II.B Predictions of Competing Firms

When both firms use algorithms to predict consumer types, their predictions may be correlated. In practice, the algorithmic predictions of firms might be correlated because firms may use the same public data in addition to their private company data or because they use similar data analytics models and tools. Our analysis focuses on the case where the two algorithms have identically distributed predictions but may be positively correlated. This allows us to derive general results on how the degree of correlations in predictions affects targeting policies.

Specifically, we assume that for any given consumer, if the prediction from firm 1 is p_1, then with probability ρ, the prediction from firm 2 is also p_1, and with probability $1-\rho$, the prediction is drawn from the distribution with probability density function $p_1 \cdot pf(p)/\mu_0 + (1-p_1)\cdot(1-p)f(p)/(1-\mu_0)$. In other words, for any given consumer, the prediction of firm 2 agrees with firm 1 with probability ρ. Otherwise, firm 2 will assign this consumer a probability according to a weighted average of two

[3]Note that what is labeled as the precision in Chen et al. (2001) is different from our definition as in the classification and machine learning literature. In Chen et al. (2001) precision is not about a firm's targeting policy, but rather about the informativeness of the predictions (which is analogous to the function $f(p)$ in our framework) irrespective of the targeting policy.

distributions. One can interpret $pf(p)/\mu_0$ as the distribution of predictions conditional on the consumer being interested, and $(1-p)f(p)/(1-\mu_0)$ as the distribution of predictions conditional on the consumer being uninterested. When the consumer is indeed interested, the prediction of firm 2 for that consumer follows distribution $pf(p)/\mu_0$. When the consumer is indeed uninterested, the prediction of firm 2 for that consumer follows distribution $(1-p)f(p)/(1-\mu_0)$. The distribution of the prediction shifts to the right tail of $f(p)$ when the consumer is interested, and shifts to the left tail of $f(p)$ when the consumer is uninterested.

Alternatively, one can consider a simpler correlation structure. For example, suppose firm 1's prediction for a given consumer is p_1. Then, with probability ρ, firm 2's prediction is also p_1, and with probability $1-\rho$, firm two's prediction is uniformly drawn from $f(p)$. Under this alternative setup, a firm's prediction is completely uncorrelated with the other firm's with probability $1-\rho$. This might seem to be a simpler correlation structure. However, this alternative correlation can have some less than desirable and practically unappealing properties. To see this, suppose that the firms' algorithms are close to perfect in their prediction abilities (i.e., informativeness). In such a case, it is possible that one firm has a prediction probability of 1 while the other firm has 0 with a high likelihood (close to $(1-\rho)/2$ when $\mu_0 = 1/2$). In contrast, under the correlation structure that we adopt in our main model described above, a firm's prediction is still correlated with the other firm's prediction (even if they are not an exact match) with probability $1-\rho$. This implies the logical and intuitively appealing outcome that the two firms' predictions will almost always be identical regardless of ρ when their algorithms are almost perfect. However, the specific choice of the correlation structure does not affect our results. In the online appendix, we show that all the main results in the paper hold under the alternative simpler correlation structure.

III Analysis

III.A Monopoly Benchmark

The monopoly firm's targeting strategy will be to choose a threshold policy $\underline{p}_m \in (0,1)$, such that it targets all consumers with $p \geq \underline{p}_m$: $q(p) = 1_{p \geq \underline{p}_m}$. At the threshold $\underline{p}_m$, the marginal revenue of targeting $\underline{p}_m v$ equals the targeting cost c, thus $\underline{p}_m = c/v$. This threshold policy is also consistent with industry practices. For example, Shi et al. (2022) note that Alibaba targets consumers whose aggregated algorithmic scores, which combine both benefits and costs of targeting, are above a threshold.

In monopoly, the precision of the optimal targeting policy is:

$$\frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\int_{\underline{p}_m}^{1} f(p)dp} = \frac{\int_{\underline{p}_m}^{1} pf(p)dp}{a_m} \tag{1}$$

The recall of the optimal targeting policy is:

$$\frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\int_{0}^{1} pf(p)dp} = \frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\mu_0} \tag{2}$$

Both the precision and recall depend on the firm's targeting policy $\underline{p}_m$ and the informativeness of the algorithm, as characterized by the distribution of the predictions $f(\cdot)$. Notice that higher precision implies that the algorithm targets more interested consumers rather than uninterested ones, while higher recall implies that the algorithm targets more interested consumers out of the total interested pool. Given the algorithm, the precision increases while the recall decreases in the targeting threshold $\underline{p}_m$. By targeting only a small subset of consumers who are highly likely to be interested, the firm generates a low false negative rate and achieves high precision. However, the firm sacrifices recall because it does not target a larger set of consumers who are likely to be interested. So, in this set, the firm generates a high false negative rate, resulting in lower recall. At one extreme, as $\underline{p}_m \to 1$, Precision $\to 1$, and Recall $\to 0$. In contrast, the opposite applies when the targeting threshold $\underline{p}_m$ is low. As $\underline{p}_m \to 0$, Precision $\to \mu_0$, and Recall $\to 1$.

To what extent the firm's optimal targeting policy prioritizes precision vs. recall depends on the profits and cost of targeting. If the targeting cost is high relative to its benefit (high c/v), the monopoly only wants to target high-probability consumers to save targeting costs (precision over recall). In contrast, if the targeting cost is low relative to its benefit (low c/v), the monopoly does not want to miss out on consumers who may be moderately likely to be interested (recall over precision).

Mapping From an Algorithm and Targeting Policy to Precision and Recall

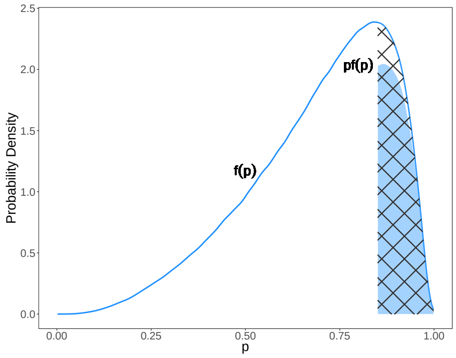
The equations (1) and (2) give the formulas of precision and recall. Figure 1 illustrates the precision and recall for a given algorithm and a given targeting policy. Consider a simple prediction algorithm - the logistic regression with one explanatory variable x (consumer characteristic), which is distributed normally with zero mean and unit standard deviation among all consumers. Formally, the logit model is $\mathbb{P}(x) = 1/[1 + e^{-(\beta_0+\beta_1 x)}] + \epsilon$, where ϵ is the unobserved shocks and follows a logistic distribution. The firm will first estimate the model coefficients, β_0 and β_1, using historical data. Suppose that the estimates are $\widehat{\beta}_0 = 1$ and $\widehat{\beta}_1 = 1$. Then, it can predict the probability of interest for each potential consumer based on the individual characteristics x according to $\widehat{\mathbb{P}}(x) = 1/[1 + e^{-(\widehat{\beta}_0+\widehat{\beta}_1 x)}]$. Consumers with different characteristics will be predicted to have different likelihood of interest. By aggregating the individual-level predictions, the firm can obtain a distribution of probabilities for its potential consumers, $f(p)$.

In Figure 1a and 1b the monopoly targets all consumers whose probability of interest is at least 85 percent ($\underline{p}_m = 0.85$). In both figures, there are $f(p)$ mass of probability p consumers, and the firm targets all of them if $p \geq 0.85$. Among those consumers, $pf(p)$ mass of probability p consumers are interested and targeted by the firm. The solid region measures the total number of interested consumers targeted by the firm. The cross-hatched region in Figure 1a measures the total mass of consumers targeted by the firm. The precision equals the area of the solid region divided by the area of the cross-hatched region. As we can see from the figure, the precision is high in this case because the firm targets only high-probability consumers. The majority of those consumers turn out to be interested. The cross-hatched region in Figure 1b measures the total mass of interested consumers. The recall equals the area of the solid region divided by the area of the cross-hatched region. The recall is low in this case because the firm misses out on many potential consumers by targeting very selectively.

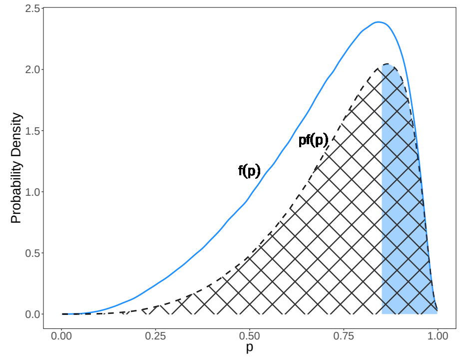
In contrast, the monopoly targets all consumers whose probability of interest is at least 25 percent in Figure 1c and 1d ($\underline{p}_m = 0.25$). The precision and the recall still equal the area of the solid region divided by the area of the cross-hatched region in these two figures, respectively. By targeting a larger set of consumers, the firm induces a lower precision and a higher recall. There are not many missing opportunities because most interested consumers are targeted. However, the firm also wastes more of its marketing budget on uninterested consumers due to extensive mis-targeting.

Linking Precision-Recall and Hypothesis Testing

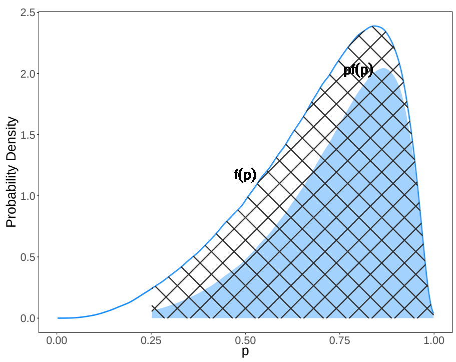
Precision-recall as a classification concept in machine learning is closely connected to hypothesis testing. We use the monopoly case to illustrate it. Let the null hypothesis be that a given consumer is not interested. The firm decides whether to reject the null hypothesis based on the available information. If the firm rejects the null hypothesis, it will target the consumer because the alternative hypothesis is that the consumer is interested. False negatives in this context mean that the null hypothesis is wrong (the consumer is in truth interested) but the firm fails to reject the null hypothesis (therefore does not target the consumer). Under perfect recall, the firm targets all interested consumers. So, there are no false negatives. In general, recall equals the power of the test. False positives in this context mean that the null hypothesis is correct (the consumer is not interested) but the firm rejects the null hypothesis (and



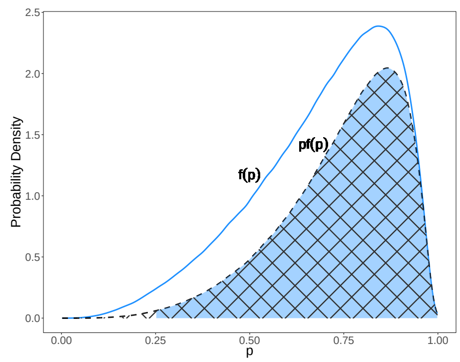
(a) Precision = 0.904 when the firm targets all consumers whose $p \geq 0.85$.



(b) Recall = 0.300 when the firm targets all consumers whose $p \geq 0.85$.



(c) Precision = 0.706 when the firm targets all consumers whose $p \geq 0.25$.



(d) Recall = 0.995 when the firm targets all consumers whose $p \geq 0.25$.

Figure 1: Precision and Recall under different targeting policies.

therefore targets the consumer). Under perfect precision, the firm does not target any uninterested consumers. So, there are no false positives. Unlike recall which is equivalent to the complement of the type II error rate, precision is not identical to any single statistical property in hypothesis testing, though it is related to the type I error rate. The reason is that precision also depends on the prior probability that a consumer is interested.

III.B Competitive Algorithmic Targeting

In this section, we consider the case of the competing firms choosing their targeting policies $q_i(p)$ simultaneously. In the main analysis that follows, we derive the implications of the symmetric equilibrium. Later on in section IV.B we analyze the robustness of the main insights to the case of possible asymmetric equilibria.

We first note that there does not exist a symmetric pure strategy equilibrium for any positive ρ.[4] We thus consider the *symmetric mixed-strategy equilibrium*. Suppose each firm targets probability p consumers with probability $q(p)$. Let $\underline{p} = \inf\{p : q(p) > 0\}$, i.e., the lowest probability consumer that firms target for strictly positive probability. Denote the recall of the targeting policy given $q(p)$ by R and the aggregate targeting intensity given $q(p)$ by $a = \int_{\underline{p}}^{1} f(p)q(p)dp$.

The expected payoff of a firm from targeting a probability p consumer is:

$$[\rho q(p) + (1-\rho)R]pw + [1 - \rho q(p) - (1-\rho)R]pv - c$$

$[\rho q(p) + (1-\rho)R]p$ is the probability that a probability p-consumer is interested and targeted by both firms, in which case the focal firm gets a payoff of w. There are two such cases: i) when the predictions coincide and the other firm also targets this consumer, which happens with probability $p\rho q(p)$, or ii) when the two predictions differ but the algorithm of the other firm assigns a sufficiently high probability (i.e., above the threshold $\underline{p}$) and targets this consumer regardless, which happens with probability $p(1-\rho)\int_0^1 q(p)pf(p)/\mu_0 dp = p(1-\rho)\int_{\underline{p}}^1 q(p)pf(p)/\mu_0 dp = p(1-\rho)R$. Similarly, the consumer is interested and targeted by only one firm with probability $[1 - \rho q(p) - (1-\rho)R]p$, which will result in a payoff of v.

[4] Suppose there is such an equilibrium. Consider $\underline{p}$ as the lowest probability for which firms target. Clearly, $\underline{p} \geq \underline{p}_m$, the monopoly targeting threshold. We now have $\underline{p}\{[\rho + (1-\rho)R]w + [1 - \rho - (1-\rho)R]v\} - c \geq 0$. By targeting probability $p - \epsilon$ consumer, the firm obtains an expected payoff of $(\underline{p} - \epsilon)\{[(1-\rho)R]w + [1-(1-\rho)R]v\} - c = \underline{p}\{[\rho + (1-\rho)R]w + [1-\rho-(1-\rho)R]v\} - c + \rho(v - w)\underline{p} - \epsilon\{[(1-\rho)R]w + [1-(1-\rho)R]v\} \geq 0 + \rho(v-w)\underline{p}_m - \epsilon > 0$ for ϵ small enough. Therefore, firms have an incentive to deviate.

Assumption 1 $0 \leq w < c < v$. *A firm prefers to target an interested consumer by itself, but does not prefer to target an interested consumer who is also targeted by the competitor.*

In equilibrium, there are two types of targeting strategies. Either the firm never targets any consumer for sure, or it targets a set of consumers with the highest prediction probabilities for sure, and the consumers with lower prediction probabilities partially.

Lemma 1 *In any symmetric equilibrium, either* $q(p) \in (0,1), \forall p \in (\underline{p}, 1]$ *or there exists* $\overline{p} \in (\underline{p}, 1)$ *such that* $q(p) = 1, \forall p > \overline{p}$, *and* $q(p) \in (0,1), \forall p \in (\underline{p}, \overline{p})$.

For notational ease, we let $\overline{p} = 1$ if the firm never targets any consumer for sure. Under competition, the precision of the optimal targeting policy is:

$$\frac{\int_{\underline{p}}^{1} pq(p)f(p)dp}{\int_{\underline{p}}^{1} q(p)f(p)dp} = \frac{\int_{\underline{p}}^{1} pq(p)f(p)dp}{a} \tag{3}$$

The recall of the optimal targeting policy is:

$$\frac{\int_{\underline{p}}^{1} pq(p)f(p)dp}{\int_{0}^{1} pf(p)dp} = \frac{\int_{\underline{p}}^{1} pq(p)f(p)dp}{\mu_0} \tag{4}$$

The firm is indifferent between targeting or not for $p \in (\underline{p}, \overline{p})$. The firm's expected profit for those consumers is 0, its payoff from not targeting.

$$\begin{aligned} & [\rho q(p) + (1-\rho)R]pw + [1 - \rho q(p) - (1-\rho)R]pv - c = 0 \\ \Rightarrow\ & q(p) = \frac{v - (1-\rho)(v-w)R - c/p}{\rho(v-w)}, \ \forall p \in (\underline{p}, \overline{p}) \end{aligned} \tag{5}$$

$$\begin{aligned} & q(\underline{p}) = 0 \\ \Rightarrow\ & \underline{p} = \frac{c}{v - (1-\rho)(v-w)R} \end{aligned} \tag{6}$$

The firm strictly prefers targeting to not targeting for $p > \overline{p}$. So, it targets those high-probability consumers for sure and obtains a positive profit. If $\overline{p} = 1$, then no consumers will be targeted by the firm for sure. To emphasize that $\underline{p}$, a, and R depend on ρ, we use the notation $\underline{p}(\rho)$, $a(\rho)$, and $R(\rho)$ when necessary to avoid confusion. Assumption 1 implies that $\underline{p}(1) = c/v \in (0, 1)$. Lemma 2 establishes that such an equilibrium exists.

Lemma 2 *There exists a unique mixed-strategy symmetric equilibrium. The firm targets some consumers with a positive probability in equilibrium. Furthermore, there exists $\widehat{\rho} \in [0, \frac{v-c}{v-w}]$ such that firms never target any consumer for sure if and only if $\rho \geq \widehat{\rho}$. Otherwise, the firm targets consumers with probabilities $p \geq \overline{p}$ for sure and mix for consumers with probabilities between $\underline{p}$ and $\overline{p}$.*

Figure 2 illustrates the targeting probability of the optimal targeting policy for different correlations ρ. When the correlation is high, a firm never targets any consumer for sure because the rival firm has a high likelihood of having the same prediction for the consumer. In contrast, when the correlation is low, a firm targets high-probability consumers for sure because the other firm has a high chance of having a different prediction and not targeting that consumer. However, for lower-probability consumers, the potential gain for a firm from targeting decreases. At the same time, the probability of overlapping in targeting stays the same if firms were to target that consumer for sure. Thus, when the predicted probability for a consumer is below a threshold, the firms in equilibrium mix between targeting and not targeting that consumer in order to soften the competition.

On the extensive margin (i.e., the lowest probability consumer being targeted), firms become less selective as their predictions become more correlated. The intuition is as follows. Consider marginal consumers whose probability of being targeted is very low. When the predictions are highly correlated, both firms' predictions are largely aligned on the identity of the marginal consumers. Each firm will be more confident that those marginal consumers will be targeted by the other firm with a low probability. Thus, a firm becomes more inclined to target some of those consumers.

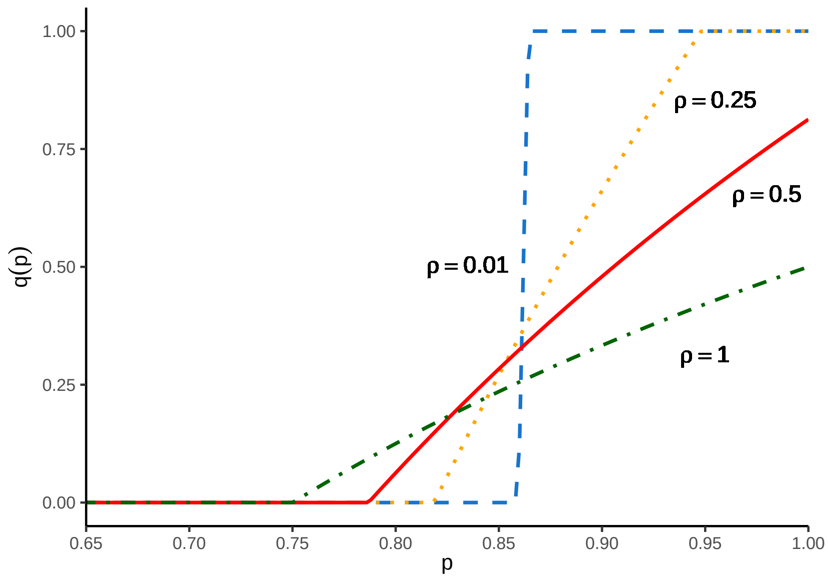


Figure 2: Targeting probability of the Optimal Targeting Policy for $c = .75, v = 1, w = .5, f \sim U[0, 1]$.

In contrast, a consumer with a predicted probability of one will be more likely to be highly valued by the other firm as well when the predictions become more correlated. Therefore, a firm becomes less inclined to target such a consumer. Therefore, the targeting probability $q(p)$ becomes flatter as the correlation increases.

We now proceed to establish one of the main results of this paper by comparing the equilibrium targeting policies of the monopoly benchmark with the duopoly case in the following proposition. It shows that compared with monopoly, firms lean towards precision under competition.

Proposition 1 *The duopoly's optimal targeting policy has a higher precision and a lower recall than the monopoly's.*

This proposition connects the precision-recall trade-off that is a fundamental aspect of the deployment of machine-learning algorithms to the equilibrium incentives of competitive firms. Relative to a monopoly, competition favors precision over recall in algorithmic targeting. The intuition is that competition reduces the expected profit from targeting due to overlaps in the set of targeted consumers of the rival firms. Firms, therefore, strategically respond in equilibrium by concentrating their targeting efforts on consumers who are predicted by their algorithms to have a high likelihood of conversion, thereby leading to an increase in the precision of their targeting.

We now discuss the details of equilibrium and the implications of the important comparative static predictions. To begin with, consider the effect of the correlation in the firms' predictions on the equilibrium profits: When the correlation is sufficiently high ($\rho > \hat{\rho}$) then as shown in Lemma 2 the equilibrium targeting policy involves firms not targeting any consumers for sure. This implies that the equilibrium profits are zero. Conversely, when the correlation is lower, a set of high-probability consumers will be targeted for sure, and this implies positive equilibrium profits. Further, in this case, lower values of ρ are associated with increasing profits. Thus, firms will have the incentive to choose uncorrelated algorithms ($\rho = 0$) if it is costless. Note that in reality, the correlation between the algorithms can be due to the extent to which the firms use the same public data or because they use the targeting tools offered by a common data analytics intermediary. This result, therefore, has an organizational strategy implication that under competition, firms have greater incentives to develop their internal private data systems and their data analytics capabilities. In section IV.A, we will consider the implications of endogenizing ρ.

Next, as illustrated in Figure 3, as ρ increases and the predictions of the firm converge, firms end up targeting fewer consumers leading to a decrease in the recall. As the predictions converge, the targeting of the firms is more likely to coincide, reducing the returns from targeting. Interestingly, and somewhat counter to expectation, we prove that under competition, as long as the level of correlation is high enough ($\rho > \hat{\rho}$), the precision also decreases as ρ increases. As the correlation increases, the high-probability consumers become less attractive because both firms increasingly identify these consumers as the more valuable high-probability consumers and compete by setting higher targeting intensity. Conversely, the more moderate-probability consumers become relatively more attractive because they are now less likely to be targeted. With higher correlation, when a firm predicts a moderate probability consumer, it can expect that the consumer is less likely to be predicted as a high probability consumer by the rival, implying that the rival will have the incentive to target moderate-probability consumers less aggressively. Overall, this leads to $q(p)$ becoming flatter, decreasing the precision.

As the Figure illustrates, firms continuously adjust the precision and recall of their targeting policies as the correlation in their predictions changes. This feature is driven by the fact that each firm has access to individual-level predictions and, therefore, has a large targeting policy space to choose from. This contrasts with the traditional targeting contexts wherein firms would only have a separate signal at the level of a consumer segment and cannot further distinguish individual consumers within the segment. In such a case the precision and recall targeting policy of a firm will be less flexible and would stay constant for an interval of the prediction correlation.

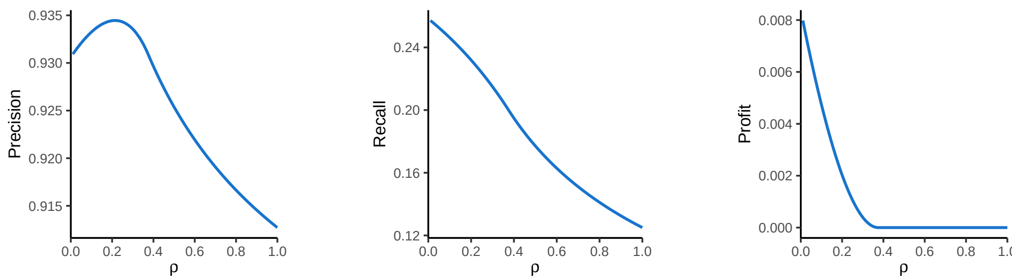


Figure 3: Comparative Statics with regard to ρ for $c = .75, v = 1, w = .5, f \sim U[0, 1]$.

III.B.1 Information Value to Consumers

Targeting provides information value to consumers if the firm's targeting policy correctly targets interested consumers. Targeted consumers who find the product fits their needs will buy and obtain a positive surplus. The total information value of a targeting policy is proportional to $\int_{\underline{p}}^{1} pq(p)f(p)dp$. We find that a higher correlation decreases the information value because fewer interested consumers end up being targeted. Since $\int_{\underline{p}}^{1} pq(p)f(p)dp$ is the recall multiplied by the prior probability, μ_0, and the prior probability does not depend on the targeting policy, the information value of a targeting policy is proportional to its recall. The comparative statics of the information value of the optimal targeting policy is the same as the comparative statics of the recall, which decreases in ρ. Thus an increase in the correlation decreases the information value of consumers.

IV Extensions

IV.A Endogenous Prediction Correlations

We now examine the implications when firms are able to endogenously choose the correlations in their predictions. When firms rely on public data or common data analytics providers, their targeting predictions are likely to be more correlated. Conversely, a firm may also invest in internal analytics organization and in acquiring private consumer and market data, which may lead to a decrease in the prediction correlations between the firms. For example, for a fixed size of the training sample and thus the probability distribution f, a firm may use more private data as opposed to publicly available data in order to differentiate their prediction models. In this sense we can interpret the endogenous choice of algorithmic correlation as resulting from the organizational strategy for data analytics by firms in competitive markets.

Consider the following model that endogenizes the prediction correlation ρ: In the first period, both firms simultaneously make costly investments in lowering ρ. This may represent each firm's decision to invest in generating private data and data analytics within the organization. Nevertheless, such investments will be a public good since any firm's investment will help determine the prediction correlation between the firms. Specifically, the correlation ρ is decreasing in the investments of both firms: $\rho = \rho(I_1, I_2)$, in which I_1, I_2 are the first-period investment costs of firms 1 and 2. Denote the initial correlation as $\rho_0 := \rho(0, 0)$. In the second period, both firms simultaneously choose the targeting policy exactly as in the main model. We make the following assumptions on the investment technology.

Assumption 2 *$\rho(I_1, I_2)$ is smooth and strictly decreasing in $I_j, j = 1, 2$. $\rho(I_1, I_2) > 0,\ \forall I_1, I_2 < v$. Define $\rho_1(I_1) = \rho_0 - \rho(I_1, 0)$, $\rho_2(I_2) = \rho_0 - \rho(0, I_2)$, and $K_j(\Delta\rho) = \rho_j^{-1}(\Delta\rho)$. $K_j'(0) = 0, j = 1, 2$.*

Note that $\rho_j(I_j)$ measures how much correlation a firm can unilaterally reduce by investing I_j when the other firm incurs zero costs. It is important to recognize the public production of the prediction correlation – i.e., any given firm's investment reduces the correlation in the predictions for both firms. Denote $K_j(\Delta\rho)$ as the inverse which measures how much cost a firm needs to incur to reduce the correlation by $\Delta\rho$ when the other firm does not invest and incurs zero costs. The assumptions $K_j'(0) = 0$ and $\rho(I_1, I_2) > 0,\ \forall\, I_1, I_2 < v$ mean that it is easy to reduce the correlation by a little bit, but very costly to reduce the correlation all the way to zero. In reality, firms may easily reduce ρ a bit by investing in a little more private data. However, it can be very difficult for firms to achieve zero correlation in their predictions in a competitive market,

Proposition 2 *Firms invest and earn a positive expected profit if $\rho_0 < \widehat{\rho}$. They will not invest and earn zero profits if $\rho_0 > \widehat{\rho}$.*

One may think that firms have a stronger incentive to invest in reducing the correlation if the initial correlation is high because otherwise they will get zero profits without the investment. It turns out to be the opposite. Firms will invest if the initial correlation is already low enough, such that they will earn a positive profit without investment. The low-cost assumption for small investment does not directly imply that firms will invest because the benefit of a small investment may also be very low. We prove that the benefit of investing a little bit is roughly linear in the investment cost, which outweighs the investment cost. In contrast, as Figure 3 shows, the profit is zero for an interval of $\rho \in [\widehat{\rho}, 1]$. So, a lower ρ will increase the profit (not taking into account the investment costs) only if firms invest a lot such that $\rho < \widehat{\rho}$. In that case, the investment cost may be so high that firms prefer not to invest.

The proposition provides a structure to consider the industry scenarios which are likely to encourage investments by firms in private company data analytics. For example, when there is already a well-entrenched public data system which is used by the firms, the incentive to invest within the organization's data analytics is attenuated.

Indeed, given that profits with a high correlation are zero, this would precisely be the situation in which such investments by a firm would have been beneficial. Therefore, there is a suggestion in the model that there can be under-investment in private data because the investment by a firm to soften competition through reduced correlation is a public good.

IV.B Robustness: Pure Strategy Asymmetric Equilibria

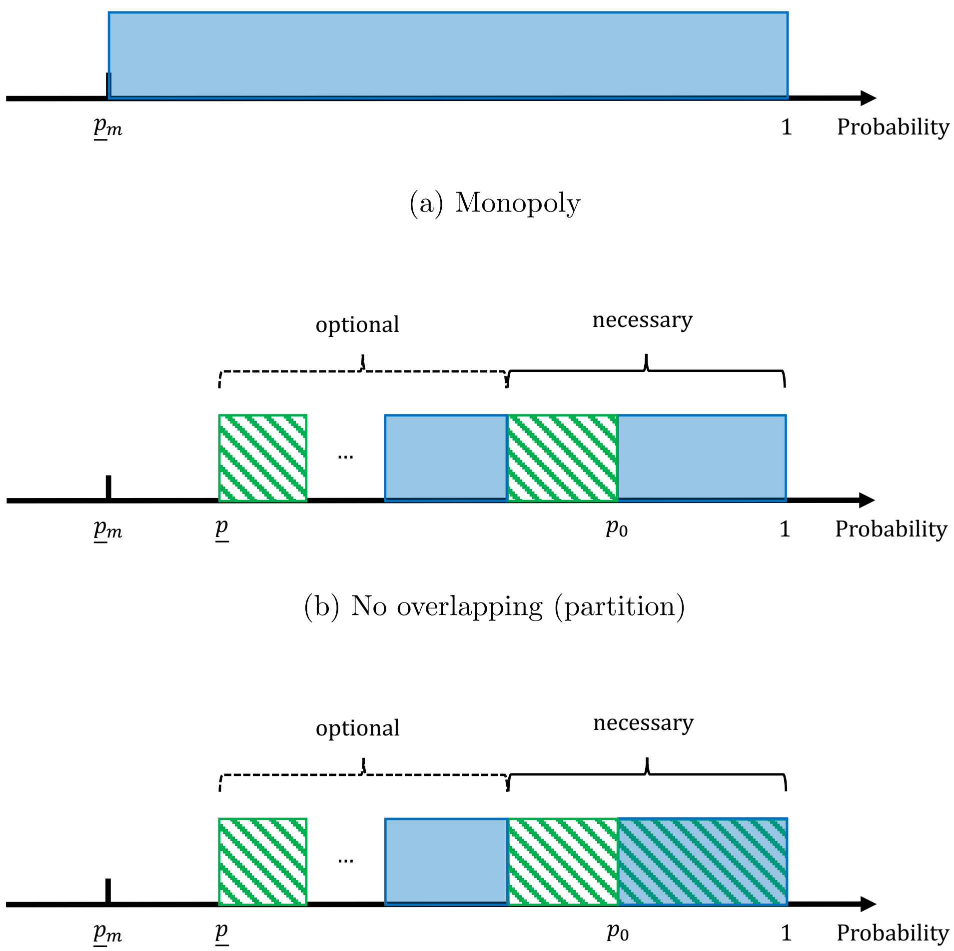
In our main analysis, we focus on the symmetric mixed-strategy equilibrium - this is because both firms are symmetric, and thus it has a natural interpretation. We also note that there are also a multiplicity of asymmetric pure strategy equilibria. So our objective in this section is to show the robustness and the generality of our main result on the precision-recall choice under competition to the asymmetric equilibria and highlight some common features.

Define $\underline{p} := \inf\{p \in [0,1] : q_1(p) > 0 \textit{ or } q_2(p) > 0\}$, where $q_i(p)$ is firm i's targeting probability of a type p consumer. Then, one can see that at least one firm targets at any $p > \underline{p}$. In addition, pure strategy symmetric equilibrium does not exist. If both firms target consumers whose probability is larger than or equal to $\underline{p}$, then each firm wants to deviate by targeting $\underline{p} - \epsilon$. Lastly, $\underline{p} \geq \underline{p}_m$. This is because a monopoly does not target below $\underline{p}_m$, and the presence of competition makes targeting at any probability less attractive.

Proposition 3 *There exists an equilibrium in which one firm targets and acts like a monopoly if* $\rho \geq \frac{R_m w + (1-R_m)v - c}{(1-R_m)(v-w)}$. *In any other pure strategy asymmetric equilibria, there exists* $p_0 \in (\underline{p}, 1)$ *such that exactly one firm targets consumers whose* $p \in (\underline{p}, p_0)$. *Either one or both firms target every consumer whose* $p > p_0$.

The recall of either firm is lower than the monopoly case. The precision of at least one firm is higher than the monopoly case.

Figure 4 illustrates the equilibrium strategy. When the prediction correlation is



(a) Monopoly

(b) No overlapping (partition)

(c) Overlapping at high-probability consumers

Figure 4: Pure Strategy Asymmetric Equilibria

high (Figure 4a), it is possible that a firm acts like a monopoly and drives the opponent out of the market. This is akin to a foreclosure equilibrium where the opponent cannot operate with positive profits. Even if the opponent predicts a consumer to have probability one, it anticipates that the rival that acts like a monopoly is highly likely to target that consumer as well. The expected equilibrium payoff of targeting is negative, and thus, it does not enter the market. Other than this special case, both firms target some consumers. For consumers moderately interested, firms partition their targeting region to soften competition. For really valuable consumers (high p), either one (Figure 4b) or both firms (Figure 4c) target them. Whether one or both firms target those consumers depends on the correlation of their prediction and the overall targeting probability of each firm. There is at least one partitioned region where only one firm targets consumers, and there can be more than one partitioned region.

The main message that firms prioritize precision over recall under competition still holds in all asymmetric equilibria. Except for the really valuable consumers, firms do not simultaneously target consumers predicted to have the same probability, which avoids overlaps in targeting the same consumer due to having the same predictions.[5] Thus, firms reduce their recall and focus on precision instead.

V Discussion and Concluding Remarks

It is hard to overstate the importance of targeting in marketing strategy. While the advances in data analytics allow firms to have unprecedented and at-scale targeting abilities, algorithmic targeting predictions may nevertheless not be perfect. Thus, firms invariably have to choose between precision - targeting a few consumers with high probabilities of conversion, or recall - targeting a large set of consumers, many

[5]Notice that there are still overlaps in targeting even for moderately interested consumers because firms may have different predictions for the same consumer.

of whom may not convert. This trade-off is also a fundamental feature of all machine learning classification algorithms.

This paper examines the precision-recall trade-off as part of the strategic deployment of algorithmic targeting by firms. It introduces a micro-founded model to study competitive targeting policies in the presence of the precision-recall trade-off. We find that compared to a monopoly, the competition between firms lowers recall but increases precision. However, as competing firms have more correlated predictions, they tend to not only target fewer consumers as the return to targeting decreases (lower recall) but also target less probable consumers (lower precision) in order to avoid head-to-head competition. This may decrease consumer welfare through fewer interested consumers ending up being targeted, as well as through more uninterested consumers receiving targeting ads. Firms have an incentive to invest in decreasing the correlations in their predictions, but such investments, being a public good, also suffer from the free-rider problem.

The results have some important managerial and policy implications. Our main result on the precision-recall trade-off provides prescriptions on how firms should deploy their data analytics operations under competition. Furthermore, the equilibrium targeting policies under competition imply that firms will need to account for how much the predictions differ across competing firms in designing their targeting policies. Policy-wise, the results suggest that privacy regulations may have an unintended side effect of preventing competing firms from using private data to differentiate their predictions. This could have negative consequences both for the industry and consumers because firms may strategically change their targeting policies using less precision and recall.

There are several interesting issues that may be further explored in future research. The analysis of the paper was conducted for a given prediction distribution $f(p)$. We now use the Beta distribution to illustrate the implications of different prediction distributions $f(p)$, with the parameters of $v = 1, w = 0.5, c = 0.75$. As the parameters of the Beta distribution α and β increase (while keeping the prior constant), the distribution becomes less informative since the mass shifts towards the center instead of 0 and 1. As a result, both precision and recall decrease in the monopoly case (Table 1 Panel A). In the case of the duopoly, we numerically solve the equilibrium targeting policies and again find similar patterns for any given correlation ρ (Table 1 Panels B-D): as the prediction becomes more informative, both precision and recall increase (and vice-versa).

We discuss the information value of targeting in section III.B.1. In addition to this positive effect, targeting may also have a negative effect on consumer welfare. Consumers may obtain a negative surplus (hassle costs) if they see information about irrelevant products. Alternatively there could be privacy costs that arise from being mis-targeted. Future studies that explicitly model the mis-targeting cost to consumers will help us understand the aggregate impact of targeting on consumer welfare.

While we focus on targeted advertising, the framework can be potentially applied to other marketing decisions such as product design, promotion, and pricing. For example, it would be interesting to study whether competition, as measured by firms' predictions, favors more distinct product designs. Another potential generalization is to extend the model beyond the binary type assumption. Finally, our precision-recall targeting framework can be used to study algorithmic discrimination in targeting in markets where the protected characteristics (e.g., race or gender) of consumers are salient and can be discriminated against.

Table 1: Precision and Recall in the Beta Distribution $f(p)$ Example

Panel A: Monopoly

alpha	beta	precision	recall
0.3	0.45	0.918025	0.604371
0.4	0.60	0.902381	0.527402
0.5	0.75	0.889673	0.464304

Panel B: Duopoly, $\rho = 0.3$

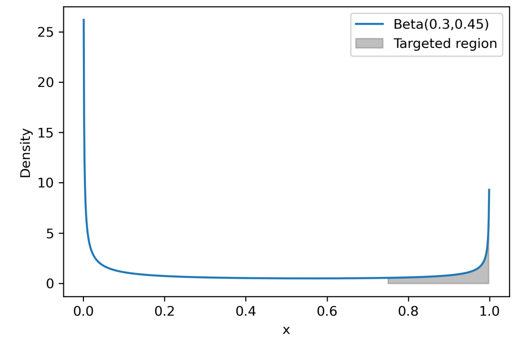
alpha	beta	rho	precision	recall
0.3	0.45	0.3	0.965718	0.318260
0.4	0.60	0.3	0.956560	0.279639
0.5	0.75	0.3	0.946341	0.243850

Panel B: Duopoly, $\rho = 0.5$

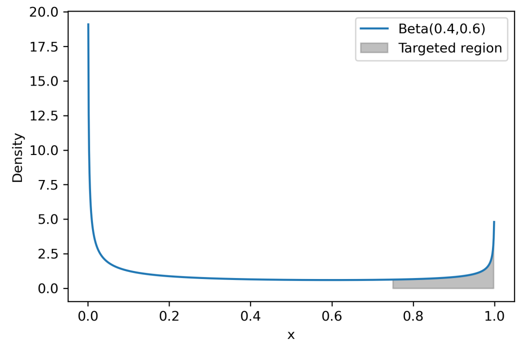
alpha	beta	rho	precision	recall
0.3	0.45	0.5	0.957185	0.278751
0.4	0.60	0.5	0.947413	0.237471
0.5	0.75	0.5	0.937732	0.203032

Panel B: Duopoly, $\rho = 0.7$

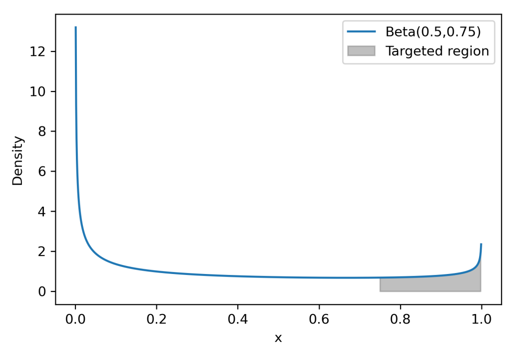
alpha	beta	rho	precision	recall
0.3	0.45	0.7	0.950975	0.251179
0.4	0.60	0.7	0.941057	0.208947
0.5	0.75	0.7	0.931453	0.175061



(a) Beta Distribution ($\alpha = 0.3, \beta = 0.45$)



(b) Beta Distribution ($\alpha = 0.4, \beta = 0.6$)



(c) Beta Distribution ($\alpha = 0.5, \beta = 0.75$)

Figure 5: Illustration of Different Prediction Distribution $f(p)$

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Appendix

Proof of Lemma 1. The lemma is equivalent to the following two statements:

Claim 1: If firms target a type p_1 consumer with a positive probability ($q(p_1) > 0$), then they target any higher-type consumer with a positive probability ($q(p) > 0,\ \forall p > p_1$).

Claim 2: If firms target a type p_2 consumer for sure ($q(p_2) = 1$), then they target any higher-type consumer for sure ($q(p) = 1,\ \forall p > p_2$).

Proof of Claim 1: If firms target a type p_1 consumer with a positive probability, then the expected payoff from targeting such a consumer is no less than the expected payoff from not targeting: $[\rho q(p_1) + (1-\rho)R]p_1 w + [1 - \rho q(p_1) - (1-\rho)R]p_1 v - c \geq 0$. Suppose they do not target a type $p > p_1$ consumer at all, $q(p) = 0$. By deviating and targeting a type p consumer for sure, the deviating firm can obtain an expected payoff of $[(1-\rho)R]pw + [1-(1-\rho)R]pv - c > [\rho q(p_1) + (1-\rho)R]p_1 w + [1 - \rho q(p_1) - (1-\rho)R]p_1 v - c \geq 0$. So, firms will deviate. A contradiction.

Proof of Claim 2: If firms target a type p_2 consumer for sure, then the expected payoff from targeting such a consumer is no less than the expected payoff from not targeting: $[\rho q(p_2) + (1-\rho)R]p_2 w + [1 - \rho q(p_2) - (1-\rho)R]p_2 v - c \geq 0$. Suppose they do not target a type $p > p_2$ consumer for sure, $q(p) < 1$. By deviating and targeting a type p consumer for sure, the deviating firm can obtain an expected payoff of $[(1-\rho)R]pw + [1-(1-\rho)R]pv - c > [\rho q(p_2) + (1-\rho)R]p_2 w + [1 - \rho q(p_2) - (1-\rho)R]p_2 v - c \geq 0$. So, firms will deviate. A contradiction. ■

Proof of Lemma 2.

Existence:

The theorem in Becker and Damianov (2006) states that any symmetric game with a finite number of players, whose strategy space is compact and Hausdorff and payoff function is continuous, has a symmetric Nash equilibrium in mixed strategies. It is natural to define the (pure) strategy space of the firm as the standard topology on

[0,1], which is compact and Hausdorff. Also, the firm's payoff function is continuous. Hence, there exists a symmetric Nash equilibrium in mixed strategies. One can see that no firm targeting cannot be a Nash Equilibrium, and the equilibrium strategy must be a cutoff strategy (target probability p consumers if and only if p is higher than a cutoff value). So, there exists a mixed-strategy symmetric equilibrium.

Uniqueness:

1. Firms never target any consumer for sure ($\overline{p} = 1$)

$$
\begin{aligned}
R &= \frac{\int_{\underline{p}}^{1} pf(p)q(p)dp}{\mu_0} \\
&= \frac{\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} \frac{pf(p)}{\rho(v-w)}[-c/p + v - (1-\rho)(v-w)R]dp}{\mu_0}
\end{aligned}
$$

For any fixed ρ, define:

$$
\begin{aligned}
G(R) &:= \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} \frac{pf(p)}{\rho(v-w)}[-c/p + v - (1-\rho)(v-w)R]dp - \mu_0 R \\
\text{Then, } G(0) &= \int_{\frac{c}{v}}^{1} \frac{pf(p)}{\rho(v-w)}[v - c/p]dp > 0 \\
G'(R) &= \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} -(1-\rho)\frac{pf(p)}{\rho}dp - \mu_0 < 0
\end{aligned}
$$

So, there is a unique R^* such that $G(R^*) = 0$ (we have previously shown the existence part).

2. Firms target high-probability consumers for sure ($\overline{p} < 1$)

$$
\begin{aligned}
& [\rho + (1-\rho)R]\overline{p}w + [1-\rho-(1-\rho)R]\overline{p}v - c = 0 \\
\Rightarrow\ & \overline{p} = \frac{c}{[\rho + (1-\rho)R]w + [1-\rho-(1-\rho)R]v} \qquad (7)
\end{aligned}
$$

$$R = [\int_{\overline{p}}^{1} pf(p)dp + \int_{\underline{p}}^{\overline{p}} pf(p)q(p)dp]/\mu_0$$
$$= [\int_{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}}^{1} pf(p)dp + \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} pf(p)q(p)dp]/\mu_0$$

For any fixing ρ, define:

$$H(R) := \int_{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}}^{1} pf(p)dp + \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} pf(p)q(p)dp - \mu_0 R$$

Then, $H(0) > 0$.

$$H'(R) = -\frac{d\overline{p}}{dR}\overline{p}f(\overline{p}) + \frac{d\overline{p}}{dR}\overline{p}f(\overline{p}) \cdot 1 +$$
$$\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} -(1-\rho)(v-w)\frac{f(p)}{\rho(v-w)}dp - \frac{d\underline{p}}{dR}\underline{p}f(\underline{p}) \cdot 0 - \mu_0$$
$$= -\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} \frac{(1-\rho)f(p)}{\rho}dp - \mu_0 < 0$$

So, there is a unique R^* such that $H(R^*) = 0$.

To show that there exists $\widehat{\rho} \in [0, \frac{v-c}{v-w}]$ such that firms never target any consumer for sure if and only if $\rho \geq \widehat{\rho}$, we just need to show the following claim:

If firms never target any consumer for sure for $\rho = \rho_s$, then they also never target any consumer for sure for any $\rho_l > \rho_s$.

Suppose not. When $\rho = \rho_l$, Lemma 1 implies that there exists $\overline{p}_l$ and $\underline{p}_l$ such that firms target consumers with probabilities $p \geq \overline{p}_l$ for sure and mix for consumers with probabilities between $\underline{p}_l$ and $\overline{p}_l$. Thus, $q_l(p) = 1 > q_s(p),\ \forall p \geq \overline{p}_l$. The proof of the comparative statics results will show that $q_l'(p) < q_s'(p),\ \forall p \in (\underline{p}_s, \overline{p}_l)$. So, $q_l(p) > q_s(p),\ \forall p \in [\underline{p}_s, 1] \Rightarrow R_l > R_s$. A contradiction to the comparative statics that the recall decreases in ρ, which will be shown in the proof of the comparative statics results. ■

Proof of Proposition 1. The result of the recall is implied by the fact that each duopolistic firm only targets a subset of the consumers that the monopoly targets. We now show that each duopolistic firm has a higher targeting precision than the monopoly.

The precision of the monopoly's optimal targeting policy is:

$$\frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\int_{\underline{p}_m}^{1} f(p)dp} = \int_{\underline{p}_m}^{1} p\frac{f(p)}{\int_{\underline{p}_m}^{1} f(p)dp}dp$$

Observe that $\int_{\underline{p}_m}^{1} \frac{f(p)}{\int_{\underline{p}_m}^{1} f(p)dp}dp = 1$. Hence, $f_m(p) := \frac{f(p)}{\int_{\underline{p}_m}^{1} f(p)dp}$ is a p.d.f. of a random variable $X_m \in [\underline{p}_m, 1]$. The monopoly's precision is the expectation of X_m.

The precision of the duopoly's optimal targeting policy is:

$$\frac{\int_{\underline{p}_d}^{1} pq(p)f(p)dp}{\int_{\underline{p}_d}^{1} q(p)f(p)dp} = \frac{\int_{\underline{p}_m}^{1} pq(p)f(p)dp}{\int_{\underline{p}_m}^{1} q(p)f(p)dp} = \int_{\underline{p}_m}^{1} p\frac{q(p)f(p)}{\int_{\underline{p}_m}^{1} q(p)f(p)dp}dp$$

, where the first equality comes from the fact that $\underline{p}_d \geq \underline{p}_m$ and $q(p) = 0,\ \forall p \in [\underline{p}_m, \underline{p}_d)$.

Observe that $\int_{\underline{p}_m}^{1} \frac{q(p)f(p)}{\int_{\underline{p}_m}^{1} q(p)f(p)dp}dp = 1$. Hence, $f_d(p) := \frac{q(p)f(p)}{\int_{\underline{p}_m}^{1} q(p)f(p)dp}$ is a p.d.f. of a random variable $X_d \in [\underline{p}_m, 1]$. The duopoly's precision is the expectation of X_d.

We proceed by showing that X_d first-order stochastic dominates X_m. As a result, the expectation of X_d is larger than the expectation of X_m, which concludes the proof.

Lemma 3 *Given two continuously distributed random variables Z_1 and $Z_2 \in [\underline{z}, \overline{z}] \subset \mathbb{R}$, whose p.d.f. (c.d.f.) are f_1 and f_2 (F_1 and F_2), respectively. Z_1 first-order stochastic dominates Z_2 if $h(z) := f_1(z)/f_2(z)$ increases in z. The stochastic dominance is strict if $h(z)$ is not a constant.*

Proof of Lemma 3. We have $1 = \int_{\underline{z}}^{\overline{z}} f_1(z)dz = \int_{\underline{z}}^{\overline{z}} h(z)f_2(z)dz$.

If $h(z)$ is a constant, $h(z) = h_0$, then $1 = \int_{\underline{z}}^{\overline{z}} h(z)f_2(z)dz = h_0 \int_{\underline{z}}^{\overline{z}} f_2(z)dz = h_0 \Rightarrow f_1(z) = f_2(z),\ \forall z$. So, Z_1 first-order stochastic dominates Z_2.

If $h(z)$ is not a constant, then $h(\underline{z}) < h(\overline{z})$. Since $h(z)$ increases in z, we have $h(\underline{z}) = h(\underline{z}) \int_{\underline{z}}^{\overline{z}} f_2(z)dz < \int_{\underline{z}}^{\overline{z}} h(z)f_2(z)dz = 1 < h(\overline{z}) \int_{\underline{z}}^{\overline{z}} f_2(z)dz = h(\overline{z})$. Therefore, there exists $\hat{z}_1 \leq \hat{z}_2 \in (\underline{z}, \overline{z})$ such that $h(z) < 1$ if $z < \hat{z}_1$ and $h(z) > 1$ if $z > \hat{z}_2$.

For any $z \in (\underline{z}, \overline{z})$, we want to show that $F_1(z) \leq F_2(z)$.

Define $H(z) := F_2(z) - F_1(z) = \int_{\underline{z}}^{z} f_2(z)dz - \int_{\underline{z}}^{z} f_1(z)dz = \int_{\underline{z}}^{z} f_2(z)[1-h(z)]dz$.

One can see that $H(\underline{z}) = H(\overline{z}) = 0$. $H'(z) = f_2(z)[1-h(z)] \begin{cases} > 0, \ if \ z \in (\underline{z}, \hat{z}_1) \\ = 0, \ if \ z \in (\hat{z}_1, \hat{z}_2) \\ < 0, \ if \ z \in (\hat{z}_2, \overline{z}) \end{cases}$.

So, $H(z) = F_2(z) - F_1(z) > 0, \forall z \in (\underline{z}, \overline{z})$. For any $z \in [\underline{z}, \overline{z}]$, one can see that $F_2(z) \geq F_1(z)$, with the inequality strict when $z \notin \{\underline{z}, \overline{z}\} \Rightarrow Z_1$ strictly first-order stochastic dominates Z_2. ■

Note that $q(p)$ weakly increases in p according to our characterization of the optimal targeting policy. Therefore, $\frac{q(p)f(p)}{\int_{\underline{p}}^{1} q(p)f(p)dp} / \frac{f(p)}{\int_{\underline{p}}^{1} f(p)dp} = q(p)\frac{\int_{\underline{p}}^{1} f(p)dp}{\int_{\underline{p}}^{1} q(p)f(p)dp}$ weakly increases in p. Lemma 3 then implies that X_d first-order stochastic dominates X_m. As a result, the expectation of X_d is larger than the expectation of X_m, which concludes the proof. ■

Proof of the comparative statics results.

1. Firms never target any consumer for sure ($\overline{p} = 1$)

   Comparative statics of R w.r.t. ρ:

   In equilibrium, $G(R) = 0$. By the implicit function theorem, $\frac{\partial R}{\partial \rho} = -\frac{\frac{\partial G}{\partial \rho}}{\frac{\partial G}{\partial R}}$. We have shown that $\frac{\partial G}{\partial R}$ is negative.

$$\begin{aligned}
\frac{\partial G}{\partial \rho} &= \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} \frac{pf(p)}{v-w}[-c/p + v - (1-\rho)(v-w)R](-1/\rho^2) + \frac{pf(p)R}{\rho}dp \\
&= \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} \frac{pf(p)}{\rho^2(v-w)}[c/p - v + (1-\rho)(v-w)R + \rho R(v-w)]dp \\
&\propto \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} pf(p)Rdp - \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} pf(p)q(p)dp \\
&= \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} pf(p)Rdp - R\mu_0 \\
&= R[\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} pf(p)dp - \int_{0}^{1} pf(p)dp] < 0.
\end{aligned}$$

Therefore, $\frac{\partial R}{\partial \rho} < 0$.

Comparative statics of $\underline{p}$ w.r.t. ρ:

We have shown that the recall R decreases in ρ. Therefore, $v-(1-\rho)(v-w)R$ increases. Equation (6) then implies that $\underline{p}$ decreases in ρ.

Comparative statics of the precision wrt ρ :

Suppose $\rho_l > \rho_s > \frac{v-c}{v-w}$. The precision of the optimal targeting policy when $\rho = \rho_s$ is:

$$\frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp} = \int_{\underline{p}_s}^1 p\frac{q_s(p)f(p)}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp}dp$$

Observe that $\int_{\underline{p}_s}^1 \frac{q_s(p)f(p)}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp}dp = 1$. Hence, $f_s(p) := \frac{q_s(p)f(p)}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp}$ is a p.d.f. of a random variable $X_s \in [\underline{p}_s, 1]$. The precision is the expectation of X_s.

The precision of the optimal targeting policy when $\rho = \rho_l$ is:

$$\begin{aligned}\frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{\int_{\underline{p}_l}^1 q_l(p)f(p)dp} &= \frac{\int_{\underline{p}_l}^{\underline{p}_s} pq_l(p)f(p)dp + \int_{\underline{p}_s}^1 pq_l(p)f(p)dp}{\int_{\underline{p}_l}^{\underline{p}_s} q_l(p)f(p)dp + \int_{\underline{p}_s}^1 q_l(p)f(p)dp} \\ &< \frac{\int_{\underline{p}_s}^1 pq_l(p)f(p)dp}{\int_{\underline{p}_s}^1 q_l(p)f(p)dp} \\ &= \int_{\underline{p}_s}^1 p\frac{q_l(p)f(p)}{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}dp\end{aligned}$$

, where the inequality comes from the fact that $\frac{\int_{\underline{p}_l}^{\underline{p}_s} pq_l(p)f(p)dp}{\int_{\underline{p}_l}^{\underline{p}_s} q_l(p)f(p)dp} < \underline{p}_s < \frac{\int_{\underline{p}_s}^1 pq_l(p)f(p)dp}{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}$. Observe that $\int_{\underline{p}_s}^1 \frac{q_l(p)f(p)}{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}dp = 1$. Hence, $f_l(p) := \frac{q_l(p)f(p)}{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}$ is a p.d.f. of a random variable $X_l \in [\underline{p}_s, 1]$. The upper bound of the precision is the expectation of X_l.

We proceed by showing that X_s first-order stochastic dominates X_l. As a result, the expectation of X_s is larger than the expectation of X_l, which concludes the proof.

In order to apply lemma 3 to obtain the final result, we need to show that $f_s(p)/f_l(p) = \frac{\frac{q_s(p)f(p)}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp}}{\frac{q_l(p)f(p)}{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}} = \frac{q_s(p)}{q_l(p)}\frac{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp}$ increases in p. Since $\frac{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp}$ is a constant, we only need to show that $\frac{q_s(p)}{q_l(p)}$ strictly increases in p.

According to (5), $q_s'(p) = \frac{c}{p^2\rho_s(v-w)} > \frac{c}{p^2\rho_l(v-w)} = q_l'(p), \forall p \in (\underline{p}_s, 1)$. One can see that $\int_{\underline{p}_s}^1 q_s(p)pf(p)dp = R_s\mu_0 > R_l\mu_0 = \int_{\underline{p}_l}^1 q_l(p)pf(p)dp > \int_{\underline{p}_s}^1 q_l(p)pf(p)dp$. Since $q_l(\underline{p}_s) > q_l(\underline{p}_l) = 0 = q_s(\underline{p}_s)$, there must exist $\tilde{p} \in (\underline{p}_s, 1)$ such that

$$\begin{cases} q_s(p) > q_l(p), \ if \ p \in (\tilde{p}, 1] \\ \tilde{q} := q_s(\tilde{p}) = q_l(\tilde{p}) \\ q_s(p) < q_l(p), \ if \ p \in [\underline{p}_s, \tilde{p}) \end{cases}.$$

$$\begin{aligned} \Xi(p) :=& \frac{q_s(p)}{q_l(p)} \overset{(5)}{=} \frac{\tilde{q} + \frac{c}{\rho_s(v-w)}(\frac{1}{\tilde{p}} - \frac{1}{p})}{\tilde{q} + \frac{c}{\rho_l(v-w)}(\frac{1}{\tilde{p}} - \frac{1}{p})} \\ \Rightarrow Sgn(\Xi'(p)) =& Sgn(\frac{1}{\rho_s} - \frac{1}{\rho_l}) > 0 \end{aligned}$$

Therefore, $\frac{q_s(p)}{q_l(p)}$ strictly increases in p. Lemma 3 then implies that X_s first-order stochastic dominates X_l. As a result, the expectation of X_s is larger than the expectation of X_l, which concludes the proof.

2. Firms target high-probability consumers for sure ($\overline{p} < 1$)

   In this case, the precision of the optimal targeting policy is:

$$\frac{\int_{\underline{p}}^{\overline{p}} pq(p)f(p)dp + \int_{\overline{p}}^1 pf(p)dp}{\int_{\underline{p}}^{\overline{p}} q(p)f(p)dp + \int_{\overline{p}}^1 f(p)dp} = \frac{\int_{\underline{p}}^{\overline{p}} pq(p)f(p)dp + \int_{\overline{p}}^1 pf(p)dp}{a}$$

   The recall of the optimal targeting policy is:

$$\frac{\int_{\underline{p}}^{\overline{p}} pq(p)f(p)dp + \int_{\overline{p}}^1 pf(p)dp}{\int_0^1 pf(p)dp} = \frac{\int_{\underline{p}}^{\overline{p}} pq(p)f(p)dp + \int_{\overline{p}}^1 pf(p)dp}{\mu_0}$$

   Comparative statics of R w.r.t. ρ:

In equilibrium, $H(R)=0$. By the implicit function theorem, $\frac{\partial R}{\partial \rho}=-\frac{\frac{\partial H}{\partial \rho}}{\frac{\partial H}{\partial R}}$. We have shown that $\frac{\partial H}{\partial R}$ is negative.

$$\begin{aligned}
\frac{\partial H}{\partial \rho}=&-\frac{d\overline{p}}{d\rho}\overline{p}f(\overline{p})+\frac{d\overline{p}}{d\rho}\overline{p}f(\overline{p})\cdot 1-\frac{d\underline{p}}{d\rho}\underline{p}f(\underline{p})\cdot 0+\\
&\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}}\frac{pf(p)}{v-w}[-c/p+v-(1-\rho)(v-w)R](-1/\rho^2)+\frac{pf(p)R}{\rho}dp\\
=&\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}}\frac{pf(p)}{\rho^2(v-w)}[c/p-v+(1-\rho)(v-w)R+\rho R(v-w)]dp\\
\propto&\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}}pf(p)Rdp-\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}}pf(p)q(p)dp\\
=&\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}}pf(p)Rdp-R\mu_0\\
=&R[\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}}pf(p)dp-\int_0^1 pf(p)dp]<0.
\end{aligned}$$

Therefore, $\frac{\partial R}{\partial \rho}<0$.

Comparative statics of $\underline{p}$ w.r.t. ρ:

We have shown that the overall targeting probability a decreases when ρ increases. Therefore, $v-(1-\rho)(v-w)R$ increases. Equation (6) then implies that $\underline{p}$ decreases in ρ.

Comparative statics of $\overline{p}$ w.r.t. ρ:

Consider any given $\rho_l>\rho_s$ such that the corresponding $\overline{p}_l$ and $\overline{p}_s$ are lower than 1. Suppose $\overline{p}_l\le\overline{p}_s$. The recall corresponding to ρ_l is $R_l=[\int_{\overline{p}_l}^1 pf(p)dp+\int_{\underline{p}_l}^{\overline{p}_l}q_l(p)pf(p)dp]/\mu_0$. The recall corresponding to ρ_s is $R_s=[\int_{\overline{p}_s}^1 pf(p)dp+\int_{\underline{p}_s}^{\overline{p}_s}q_s(p)pf(p)dp]/\mu_0$. We first show that $q_l(p)>q_s(p),\forall p\in(\underline{p}_s,\overline{p}_l)$.

Observe that $q_l(\overline{p}_l)=1>q_s(\overline{p}_l)$. For any $p\in(\underline{p}_s,\overline{p}_l)$, we have

$$q_l(p)=q_l(\overline{p}_l)+\frac{c}{\rho_l(v-w)}(1/\overline{p}_l-1/p)>q_s(\overline{p}_l)+\frac{c}{\rho_s(v-w)}(1/\overline{p}_l-1/p)=q_s(\overline{p}_l).$$

Hence,

$$
\begin{aligned}
R_l\mu_0 &= \int_{\overline{p}_s}^{1} pf(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} pf(p)dp + \int_{\underline{p}_l}^{\overline{p}_l} q_l(p)pf(p)dp \\
&> \int_{\overline{p}_s}^{1} pf(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} q_s(p)pf(p)dp + \int_{\underline{p}_s}^{\overline{p}_l} q_l(p)pf(p)dp \\
&> \int_{\overline{p}_s}^{1} pf(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} q_s(p)pf(p)dp + \int_{\underline{p}_s}^{\overline{p}_l} q_s(p)pf(p)dp \\
&= R_s\mu_0.
\end{aligned}
$$

But we have shown that R decreases in ρ. A contradiction. Therefore, $\overline{p}_l > \overline{p}_s$.

Comparative statics of the profit w.r.t. ρ:

Denote the firm's total profit by Π and the per-unit profit for probability p consumer by $\pi(p)$. Then, $\Pi = \int_{\overline{p}}^{1} \pi(p)f(p)dp$, where $\pi(p) = p\{[\rho + (1-\rho)R]w + [1-\rho-(1-\rho)R]v\} - c$. One can see that $\pi(p)$ is strictly increasing and linear in p for $p \in [\overline{p}, 1]$. Consider any given $\rho_l > \rho_s$ such that the corresponding $\overline{p}_l$ and $\overline{p}_s$ are lower than 1. We have shown that $\overline{p}_s < \overline{p}_l$. Therefore, $\pi_s(\overline{p}_l) > \pi_s(\overline{p}_s) = 0 = \pi_l(\overline{p}_l) \Rightarrow [\rho_s + (1-\rho_s)a_s]w + [1-\rho_s-(1-\rho_s)a_s]v > [\rho_l + (1-\rho_l)a_l]w + [1-\rho_l-(1-\rho_l)a_l]v \Rightarrow \pi_s(p) > \pi_l(p),\ \forall p \in [\overline{p}_l, 1] \Rightarrow \Pi_s > \Pi_l$.[6]

■

Declarations

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[6]From the expression of $\pi(p)$ and $\pi(\overline{p}) = 0$, one can derive that $\pi(p) = \frac{p-\overline{p}}{\overline{p}}c,\ \forall p \geq \overline{p}$.

ONLINE APPENDIX

Extensions

Proof of Proposition 2. We first consider the case in which $\rho_0 > \widehat{\rho}$. We have shown that the profit is zero for any $\rho \geq \widehat{\rho}$. Therefore, if firms invest in equilibrium, it must be that $\rho(I_1, I_2) < \widehat{\rho}$. The investment cost may be higher than the benefit. For example, suppose $\rho(v, v) \geq \widehat{\rho}$. Then no firm will invest.

We then show that no firm investing is not an equilibrium if $\rho_0 < \widehat{\rho}$. Suppose no firm investing is an equilibrium. We want to show that either firm has an incentive to deviate. Consider without loss of generality firm 1.

We first compute how much firm 1's payoff in the second period increases as ρ decreases.

Equation (8) implies that:

$$Q(\rho, \overline{p}) := \overline{p}\{[\rho + (1-\rho)R(\rho)]w + [1 - \rho - (1-\rho)R(\rho)]v\} - c = 0$$

By implicit function theorem,

$$\frac{\partial \overline{p}}{\partial \rho} = -\frac{\partial Q/\partial \rho}{\partial Q/\partial \overline{p}} = \frac{\overline{p}[1 - R + (1-\rho)\frac{\partial R}{\partial \rho}](v - w)}{[\rho + (1-\rho)R]w + [1 - \rho - (1-\rho)R]v}$$

We have shown in the proof of the comparative statics results that $\overline{p}$ strictly increases in ρ for $\rho < \widehat{\rho}$. Therefore, $\overline{p}[1 - R + (1-\rho)\frac{\partial R}{\partial \rho}](v - w) > 0$. Denote $\frac{\partial \overline{p}}{\partial \rho}|_{\rho=\rho_0}$ by D_0 ($D_0 > 0$). Consider an investment by firm 1 such that the correlation decreases from ρ_0 to $\rho_1 := \rho_0 - \epsilon$. Denote the $\overline{p}$ corresponding to ρ_0 by $\overline{p}_0$ and the $\overline{p}$ corresponding to ρ_1 by $\overline{p}_1$. We use similar notations for π and Π.[7]

By Taylor expansion, $\overline{p}_1 = \overline{p}_0 - D_0\epsilon + o(\epsilon)$. Firm 1's increase in profit (not taking into account the investment cost) is:

$$\Pi_1 - \Pi_0 = \int_{\overline{p}_1}^{1} \pi_1(p) f(p) dp - \int_{\overline{p}_0}^{1} \pi_0(p) f(p) dp$$

[7] Both have been defined in the proof of the comparative statics results.

$$
\begin{aligned}
&> \int_{\overline{p}_0}^{1} \pi_1(p) f(p) dp - \int_{\overline{p}_0}^{1} \pi_0(p) f(p) dp \\
&= \int_{\overline{p}_0}^{1} [\pi_1(p) - \pi_0(p)] f(p) dp \\
&= \int_{\overline{p}_0}^{1} [\frac{p - \overline{p}_1}{\overline{p}_1} c - \frac{p - \overline{p}_0}{\overline{p}_0} c] f(p) dp \\
&= \int_{\overline{p}_0}^{1} p \frac{\overline{p}_0 - \overline{p}_1}{\overline{p}_0 \overline{p}_1} c f(p) dp \\
&> \int_{\overline{p}_0}^{1} \overline{p}_0 \frac{D_0 \epsilon + o(\epsilon)}{\overline{p}_0 \overline{p}_1} c f(p) dp \\
&= \frac{D_0 \epsilon + o(\epsilon)}{\overline{p}_1} c \int_{\overline{p}_0}^{1} f(p) dp \\
&= \frac{D_0 \epsilon + o(\epsilon)}{\overline{p}_1} c [1 - F(\overline{p}_0)] \\
&= \frac{D_0 c [1 - F(\overline{p}_0)]}{\overline{p}_1} \epsilon + o(\epsilon)
\end{aligned}
$$

Thus, the profit increase is linear in the decrease of ρ, ϵ, if we ignore the higher-order term $o(\epsilon)$.

We then compute firm 1's investment cost in the first period. One can see that $K_j(0) = 0$.

$$
\begin{aligned}
K_1(\epsilon) &= K_1(0) + K_1'(0)\epsilon + o(\epsilon) \\
&= 0 + 0 \cdot \epsilon + o(\epsilon) \\
&= o(\epsilon)
\end{aligned}
$$

Therefore, $\Pi_1 - \Pi_0 > K_1(\epsilon)$ for ϵ small enough. It shows that firm 1 will deviate if no firm invests. ■

Proof of Proposition 3. In order for a firm acting as a monopoly to be an equilibrium, the other firm must not want to target probability 1 consumer, which is equivalent to:

$$\rho w + (1-\rho)[R_m w + (1-R_m)v] - c \leq 0$$
$$\Leftrightarrow \;\; \rho \geq \frac{R_m w + (1-R_m)v - c}{(1-R_m)(v-w)}$$

The statement that in any other pure strategy asymmetric equilibria, there exists $p_0 \in (\underline{p}, 1)$ such that exactly one firm targets consumers whose $p \in (\underline{p}, p_0)$, and either one or both firms target every consumer whose $p > p_0$ is equivalent to the following two claims.

Claim 1: At least one firm targets consumers whose $p \in (\underline{p}, 1]$.

Claim 2: If both firms target a probability p' consumer, then they also target any consumer whose $p > p'$.

Proof of Claim 1: For any $p \in (\underline{p}, 1]$, there exists $\tilde{p} \in [\underline{p}, p)$ such that $q_1(\tilde{p}) > 0$ or $q_2(\tilde{p}) > 0$. Assume without loss of generality that $q_1(\tilde{p}) > 0$. Firm 1's expected payoff from targeting such a consumer is no less than the expected payoff from not targeting: $[\rho + (1-\rho)R_2]\tilde{p}w + [1-\rho-(1-\rho)R_2]\tilde{p}v - c \geq 0$. Suppose neither firm targets a type $p > \tilde{p}$ consumer. By deviating and targeting that consumer, firm 1 can obtain an expected payoff of $[(1-\rho)R_2]pw + [1-(1-\rho)R_2]pv - c > [\rho + (1-\rho)R_2]\tilde{p}w + [1-\rho-(1-\rho)R_2]\tilde{p}v - c \geq 0$. So, it will deviate. A contradiction.

Proof of Claim 2: If both firms target a probability p' consumer, then firm i's expected payoff from targeting such a consumer is no less than the expected payoff from not targeting: $[\rho + (1-\rho)R_j]p'w + [1-\rho-(1-\rho)R_j]p'v - c \geq 0$, where $j \neq i$. Claim 1 says that at least one firm targets any consumer whose $p > p'$. Suppose one of the firms does not target probability p consumer. Assume without loss of generality that firm 1 does not target that consumer. By deviating and targeting that consumer, it can obtain an expected payoff of $[(1-\rho)R_2]pw + [1-(1-\rho)R_2]pv - c > [\rho + (1-\rho)R_2]p'w + [1-\rho-(1-\rho)R_2]p'v - c \geq 0$. So, firm 1 will deviate. A contradiction.

Lastly, we prove that the recall of either firm is lower than the monopoly case, and the precision of at least one firm is higher than the monopoly case.

The monopoly equilibrium is straightforward. Now let's look at other equilibria where both firms target some consumers (the case of Figure 4b and 4c). $\underline{p} \geq \underline{p}_m$ implies that the recall of either firm is lower than the monopoly case.

The precision of firm i's targeting policy is:

$$\frac{\int_{\underline{p}}^{1} p q_i(p) f(p) dp}{\int_{\underline{p}}^{1} q_i(p) f(p) dp} = \frac{\int_{\underline{p}}^{1} p q_i(p) f(p) dp}{a_i}$$

We first consider the case where there is no overlap in firms' targeting regions (the case of Figure 4b). In that case, $q_1(p) + q_2(p) = 1, \forall p \in (\underline{p}, 1]$. Since both firms target some consumers, one can see that $\underline{p} > \underline{p}_m$. We have:

$$\begin{aligned}
&\frac{\int_{\underline{p}}^{1} p q_1(p) f(p) dp + \int_{\underline{p}}^{1} p q_2(p) f(p) dp}{\int_{\underline{p}}^{1} q_1(p) f(p) dp + \int_{\underline{p}}^{1} q_2(p) f(p) dp} \\
=&\frac{\int_{\underline{p}}^{1} p f(p) dp}{\int_{\underline{p}}^{1} f(p) dp} \\
=&\int_{\underline{p}}^{1} p \frac{f(p)}{\int_{\underline{p}}^{1} f(p) dp} dp \\
=&\int_{\underline{p}}^{1} p \frac{f(p)}{\int_{\underline{p}}^{1} f(p) dp} dp + \int_{\underline{p}_m}^{\underline{p}} p \cdot 0 \ dp
\end{aligned}$$

The above formula is the expectation of a random variable $X \in [\underline{p}_m, 1]$ with a p.d.f. of 0 for $p \in [\underline{p}_m, \underline{p}]$ and a p.d.f. of $\frac{f(p)}{\int_{\underline{p}}^{1} f(p) dp}$ for $p \in (\underline{p}, 1]$.

The precision of the monopoly's targeting policy is:

$$\frac{\int_{\underline{p}_m}^{1} p f(p) dp}{\int_{\underline{p}_m}^{1} f(p) dp} = \int_{\underline{p}_m}^{1} p \frac{f(p)}{\int_{\underline{p}_m}^{1} f(p) dp} dp$$

The above formula is the expectation of a random variable $Y \in [\underline{p}_m, 1]$ with a p.d.f. of $\frac{f(p)}{\int_{\underline{p}_m}^{1} f(p) dp}$ for $p \in [\underline{p}_m, 1]$. Lemma 3 implies that X strictly first-order stochastic dominates Y. Therefore, $\frac{\int_{\underline{p}}^{1} p q_1(p) f(p) dp + \int_{\underline{p}}^{1} p q_2(p) f(p) dp}{\int_{\underline{p}}^{1} q_1(p) f(p) dp + \int_{\underline{p}}^{1} q_2(p) f(p) dp} > \frac{\int_{\underline{p}_m}^{1} p f(p) dp}{\int_{\underline{p}_m}^{1} f(p) dp}$.

Now suppose $\frac{\int_{\underline{p}}^1 pq_1(p)f(p)dp}{\int_{\underline{p}}^1 q_1(p)f(p)dp} \leq \frac{\int_{\underline{p}_m}^1 pf(p)dp}{\int_{\underline{p}_m}^1 f(p)dp}$ and $\frac{\int_{\underline{p}}^1 pq_2(p)f(p)dp}{\int_{\underline{p}}^1 q_2(p)f(p)dp} \leq \frac{\int_{\underline{p}_m}^1 pf(p)dp}{\int_{\underline{p}_m}^1 f(p)dp}$. We then have:

$$
\begin{aligned}
&\frac{\int_{\underline{p}}^1 pq_1(p)f(p)dp + \int_{\underline{p}}^1 pq_2(p)f(p)dp}{\int_{\underline{p}}^1 q_1(p)f(p)dp + \int_{\underline{p}}^1 q_2(p)f(p)dp} \\
=&\frac{\int_{\underline{p}}^1 pq_1(p)f(p)dp}{\int_{\underline{p}}^1 q_1(p)f(p)dp + \int_{\underline{p}}^1 q_2(p)f(p)dp} + \frac{\int_{\underline{p}}^1 pq_2(p)f(p)dp}{\int_{\underline{p}}^1 q_1(p)f(p)dp + \int_{\underline{p}}^1 q_2(p)f(p)dp} \\
=&\frac{\int_{\underline{p}}^1 q_1(p)f(p)dp}{\int_{\underline{p}}^1 q_1(p)f(p)dp + \int_{\underline{p}}^1 q_2(p)f(p)dp} \frac{\int_{\underline{p}}^1 pq_1(p)f(p)dp}{\int_{\underline{p}}^1 q_1(p)f(p)dp} + \\
&\frac{\int_{\underline{p}}^1 q_2(p)f(p)dp}{\int_{\underline{p}}^1 q_1(p)f(p)dp + \int_{\underline{p}}^1 q_2(p)f(p)dp} \frac{\int_{\underline{p}}^1 pq_2(p)f(p)dp}{\int_{\underline{p}}^1 q_2(p)f(p)dp} \\
\leq&\frac{\int_{\underline{p}}^1 q_1(p)f(p)dp}{\int_{\underline{p}}^1 q_1(p)f(p)dp + \int_{\underline{p}}^1 q_2(p)f(p)dp} \frac{\int_{\underline{p}_m}^1 pf(p)dp}{\int_{\underline{p}_m}^1 f(p)dp} + \\
&\frac{\int_{\underline{p}}^1 q_2(p)f(p)dp}{\int_{\underline{p}}^1 q_1(p)f(p)dp + \int_{\underline{p}}^1 q_2(p)f(p)dp} \frac{\int_{\underline{p}_m}^1 pf(p)dp}{\int_{\underline{p}_m}^1 f(p)dp} \\
=&\frac{\int_{\underline{p}_m}^1 pf(p)dp}{\int_{\underline{p}_m}^1 f(p)dp}
\end{aligned}
$$

A contradiction. So, the precision of at least one firm is higher than the monopoly case.

We then consider the case where there is overlap in firms' targeting regions (the case of Figure 4c). In that case, $q_1(p)+q_2(p) = 1, \forall p \in (\underline{p}, p_0)$ and $q_1(p)+q_2(p) = 2, \forall p > p_0$.

The following lemma relates the overlapping case with the non-overlapping case so that the previous argument applies.

Lemma 4 *Suppose $a, b, A, B > 0$ and $A/a < B/b$, then $\frac{A+2B}{a+2b} > \frac{A+B}{a+b}$.*

Proof of Lemma 4.

$$\begin{aligned}\frac{A+2B}{a+2b} &= \frac{A+B}{a+2b} + \frac{B}{a+2b}\\ &= \frac{a+b}{a+2b}\frac{A+B}{a+b} + \frac{b}{a+2b}\frac{B}{b}\\ &< \frac{a+b}{a+2b}\frac{B}{b} + \frac{b}{a+2b}\frac{B}{b}\\ &= \frac{B}{b}\end{aligned}$$

, where the last inequality holds because $\frac{B}{b} = \frac{a\frac{B}{b}+B}{a+b} > \frac{a\frac{A}{a}+B}{a+b} = \frac{A+B}{a+b}$. ■

Let $A = \int_{\underline{p}}^{p_0} pf(p)dp, B = \int_{p_0}^{1} pf(p)dp, a = \int_{\underline{p}}^{p_0} f(p)dp, b = \int_{p_0}^{1} f(p)dp$. Notice that $A/a < p_0 < B/b$. So, Lemma 4 implies that

$$\frac{A+2B}{a+2b} > \frac{A+B}{a+b}$$

Notice that $\frac{\int_{\underline{p}}^{1} pq_1(p)f(p)dp+\int_{\underline{p}}^{1} pq_2(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp+\int_{\underline{p}}^{1} q_2(p)f(p)dp} = \frac{A+2B}{a+2b}$ in the overlapping case. We have shown in the non-overlapping case that the RHS is larger than the precision of the monopoly. So, in the overlapping case, $\frac{\int_{\underline{p}}^{1} pq_1(p)f(p)dp+\int_{\underline{p}}^{1} pq_2(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp+\int_{\underline{p}}^{1} q_2(p)f(p)dp}$ is also larger than the precision of the monopoly. By the same proof-by-contradiction argument as before, one can see that the precision of at least one firm is higher than the monopoly case. ■

Alternative Correlation Structure

We now consider an alternative correlation structure. Suppose firm one's prediction for a given consumer is p_1. Then, with probability ρ, firm two's prediction is also p_1, and with probability $1-\rho$, firm two's prediction is uniformly drawn from $f(p)$. Under this alternative correlation structure, the expected payoff of a firm from targeting a probability p consumer is $[\rho q(p) + (1-\rho)a]pw + [1 - \rho q(p) - (1-\rho)a]pv - c$ rather than $[\rho q(p) + (1-\rho)R]pw + [1 - \rho q(p) - (1-\rho)R]pv - c$.

All the main results in the paper hold under this alternative correlation structure. We present proof of those results below.

Proof of Lemma 1. The proof is the same as the proof in the appendix, except that one needs to replace R with a in the formulas. ■

Proof of Lemma 2.

Existence:

The proof is the same as the proof of Lemma 2 in the appendix.

Uniqueness:

1. Firms never target any consumer for sure ($\overline{p} = 1$)

$$\begin{aligned}
a &= \int_{\underline{p}}^{1} f(p)q(p)dp \\
&= \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p)\frac{-c/p + (1-\rho)aw + [1-(1-\rho)a]v}{\rho(v-w)}dp \\
&= \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} \frac{f(p)}{\rho(v-w)}[-c/p + v - (1-\rho)(v-w)a]dp
\end{aligned}$$

For any fixed ρ, define:

$$G(a) := \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} \frac{f(p)}{\rho(v-w)}[-c/p + v - (1-\rho)(v-w)a]dp - a$$

Then, $G(0) = \int_{\frac{c}{v}}^{1} \frac{f(p)}{\rho(v-w)}[v - c/p]dp > 0$

$$\begin{aligned}
G'(a) &= \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} -(1-\rho)(v-w)\frac{f(p)}{\rho(v-w)}dp - \\
&\quad c\Big(-\frac{-(1-\rho)(v-w)}{[v-(1-\rho)(v-w)a]^2}\Big)\Big[v-(1-\rho)(v-w)a - \frac{c}{\frac{c}{v-(1-\rho)(v-w)a}}\Big]\frac{f\big(\frac{c}{v-(1-\rho)(v-w)a}\big)}{\rho(v-w)} \\
&= -\frac{1-\rho}{\rho}\Big[1 - F\Big(\frac{c}{v-(1-\rho)(v-w)a}\Big)\Big] - 1 < 0
\end{aligned}$$

Uniqueness then follows.

2. Firms target high-probability consumers for sure ($\overline{p} < 1$)

$$\begin{aligned}
&[\rho + (1-\rho)a(\rho)]\overline{p}w + [1-\rho-(1-\rho)a(\rho)]\overline{p}v - c = 0 \\
\Rightarrow\ &\overline{p} = \frac{c}{[\rho + (1-\rho)a(\rho)]w + [1-\rho-(1-\rho)a(\rho)]v} \qquad (8)
\end{aligned}$$

$$
\begin{aligned}
a &= \int_{\overline{p}}^{1} f(p)dp + \int_{\underline{p}}^{\overline{p}} f(p)q(p)dp \\
&= 1 - F(\overline{p}) + \int_{\underline{p}}^{\overline{p}} f(p)q(p)dp
\end{aligned}
$$

For any fixing ρ, define:

$$
H(a) := 1 - F(\overline{p}) + \int_{\underline{p}}^{\overline{p}} f(p)q(p)dp - a
$$

Then, $H(0) = 1 - F(\overline{p}) + \int_{c/v}^{\overline{p}} f(p)q(p)dp > 0$

$$
\begin{aligned}
H'(a) =& c\frac{-(1-\rho)(v-w)}{[v-(1-\rho)(v-w)a]^2} q\left(\frac{c}{v-(1-\rho)(v-w)a}\right) f\left(\frac{c}{v-(1-\rho)(v-w)a}\right) + \\
& \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{\overline{p}} -(1-\rho)(v-w)\frac{f(p)}{\rho(v-w)} dp - 1 \\
=& -(1-\rho)(v-w)\left[\frac{cq\left(\frac{c}{v-(1-\rho)(v-w)a}\right) f\left(\frac{c}{v-(1-\rho)(v-w)a}\right)}{[v-(1-\rho)(v-w)a]^2} + \frac{F(\overline{p}) - F\left(\frac{c}{v-(1-\rho)(v-w)a}\right)}{\rho(v-w)}\right] - 1 \\
& < 0
\end{aligned}
$$

Uniqueness then follows.

To show that there exists $\widehat{\rho} \in [0, \frac{v-c}{v-w}]$ such that firms never target any consumer for sure if and only if $\rho \geq \widehat{\rho}$, we just need to show the following claim:

If firms never target any consumer for sure for $\rho = \rho_s$, then they also never target any consumer for sure for any $\rho_l > \rho_s$.

Suppose not. When $\rho = \rho_l$, Lemma 1 implies that there exists $\overline{p}_l$ and $\underline{p}_l$ such that firms target consumers with probabilities $p \geq \overline{p}_l$ for sure and mix for consumers with probabilities between $\underline{p}_l$ and $\overline{p}_l$. Thus, $q_l(p) = 1 > q_s(p),\ \forall p \geq \overline{p}_l$. The proof of the comparative statics results will show that $q_l'(p) < q_s'(p),\ \forall p \in (\underline{p}_s, \overline{p}_l)$. So, $q_l(p) > q_s(p),\ \forall p \in [\underline{p}_s, 1] \Rightarrow a_l > a_s$. A contradiction to the comparative statics that the overall targeting probability decreases in ρ, which will be shown in the proof of the comparative statics results. ■

Proof of Proposition 1. The same as the proof of Proposition 1 in the appendix. ■

Proof of the comparative statics results.

1. Firms never target any consumer for sure ($\overline{p} = 1$)

   Comparative statics of a w.r.t. ρ:

   In equilibrium, $G(a) = 0$. By the implicit function theorem, $\frac{\partial a}{\partial \rho} = -\frac{\frac{\partial G}{\partial \rho}}{\frac{\partial G}{\partial a}}$. We have shown that $\frac{\partial G}{\partial a}$ is negative.

$$
\begin{aligned}
\frac{\partial G}{\partial \rho} =& \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} \frac{f(p)}{v-w}[-c/p + v - (1-\rho)(v-w)a](-1/\rho^2) + \frac{f(p)a}{\rho} dp \\
=& \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} \frac{f(p)}{\rho^2(v-w)}[c/p - v + (1-\rho)(v-w)a + \rho a(v-w)]dp \\
\propto& \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p)[-q(p) + a]dp \\
=& a\int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p)dp - \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p)q(p)dp \\
=& a\int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p)dp - a \\
=& -F(\frac{c}{v-(1-\rho)(v-w)a})a < 0.
\end{aligned}
$$

   Therefore, $\frac{\partial a}{\partial \rho} < 0$.

   Comparative statics of $\underline{p}$ w.r.t. ρ:

   We have shown that the overall targeting probability a decreases when ρ increases. Therefore, $v - (1-\rho)(v-w)a$ increases. Equation (6) then implies that $\underline{p}$ decreases in ρ.

   Comparative statics of the precision wrt ρ :

   The same as the proof in the appendix except that we use $\int_{\underline{p}_s}^1 q_s(p)f(p)dp = a_s > a_l = \int_{\underline{p}_l}^1 q_l(p)f(p)dp > \int_{\underline{p}_s}^1 q_l(p)f(p)dp$ and $q_l(\underline{p}_s) > q_l(\underline{p}_l) = 0 = q_s(\underline{p}_s)$ to argue that there must exist $\tilde{p} \in (\underline{p}_s, 1)$ such that $\begin{cases} q_s(p) > q_l(p), \;\; if \;\; p \in (\tilde{p}, 1] \\ \tilde{q} := q_s(\tilde{p}) = q_l(\tilde{p}) \\ q_s(p) < q_l(p), \;\; if \;\; p \in [\underline{p}_s, \tilde{p}) \end{cases}$.

Proof of the Comparative statics of the recall wrt ρ :

Suppose $\rho > \frac{v-c}{v-w}$. We have shown that $\frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{a_l} < \frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{a_s}$. Since $a_s > a_l$, we have $\frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{a_l} = \frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{a_s}\frac{a_s}{a_l} < \frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{a_s} \Rightarrow \frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{a_s} < \frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{a_s} \Rightarrow \frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{\mu_0} < \frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{\mu_0}$.

2. Firms target high-probability consumers for sure ($\overline{p} < 1$)

Comparative statics of a w.r.t. ρ:

In equilibrium, $H(a) = 0$. By the implicit function theorem, $\frac{\partial a}{\partial \rho} = -\frac{\frac{\partial H}{\partial \rho}}{\frac{\partial H}{\partial a}}$. We have shown that $\frac{\partial H}{\partial a}$ is negative.

$$\begin{aligned}\frac{\partial H}{\partial \rho} =& - f(\overline{p})\frac{\partial \overline{p}}{\partial \rho} + \frac{\partial \overline{p}}{\partial \rho}q(\overline{p})f(\overline{p}) - \\ & \frac{c}{[v-(1-\rho)(v-w)a]^2}(v-w)aq(\frac{c}{v-(1-\rho)(v-w)a})f(\frac{c}{v-(1-\rho)(v-w)a}) \\ =& - \frac{c}{[v-(1-\rho)(v-w)a]^2}(v-w)aq(\frac{c}{v-(1-\rho)(v-w)a})f(\frac{c}{v-(1-\rho)(v-w)a}) \\ <&0\end{aligned}$$

Therefore, $\frac{\partial a}{\partial \rho} < 0$.

Comparative statics of $\underline{p}$ w.r.t. ρ:

We have shown that the overall targeting probability a decreases when ρ increases. Therefore, $v - (1-\rho)(v-w)a$ increases. Equation (6) then implies that $\underline{p}$ decreases in ρ.

Comparative statics of $\overline{p}$ w.r.t. ρ:

Consider any given $\rho_l > \rho_s$ such that the corresponding $\overline{p}_l$ and $\overline{p}_s$ are lower than 1. We prove by contradiction. suppose $\overline{p}_l \leq \overline{p}_s$. The overall targeting probability corresponding to ρ_l is $a_l = \int_{\overline{p}_l}^{1} f(p)dp + \int_{\underline{p}_l}^{\overline{p}_l} q_l(p)f(p)dp$. The overall targeting probability corresponding to ρ_s is $a_s = \int_{\overline{p}_s}^{1} f(p)dp + \int_{\underline{p}_s}^{\overline{p}_s} q_s(p)f(p)dp$. We first show that $q_l(p) > q_s(p), \forall p \in (\underline{p}_s, \overline{p}_l)$.

Observe that $q_l(\overline{p}_l) = 1 > q_s(\overline{p}_l)$. For any $p \in (\underline{p}_s, \overline{p}_l)$, we have

$$\begin{aligned} q_l(p) =& q_l(\overline{p}_l) + \frac{c}{\rho_l(v-w)}(1/\overline{p}_l - 1/p) \\ >& q_s(\overline{p}_l) + \frac{c}{\rho_s(v-w)}(1/\overline{p}_l - 1/p) = q_s(\overline{p}_l) \end{aligned}$$

Hence,

$$\begin{aligned} a_l &= \int_{\overline{p}_s}^{1} f(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} f(p)dp + \int_{\underline{p}_l}^{\overline{p}_l} q_l(p)f(p)dp \\ &> \int_{\overline{p}_s}^{1} f(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} q_s(p)f(p)dp + \int_{\underline{p}_s}^{\overline{p}_l} q_l(p)f(p)dp \\ &> \int_{\overline{p}_s}^{1} f(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} q_s(p)f(p)dp + \int_{\underline{p}_s}^{\overline{p}_l} q_s(p)f(p)dp = a_s \end{aligned}$$

But we have shown that a decreases in ρ. A contradiction. Therefore, $\overline{p}_l > \overline{p}_s$.

Comparative statics of the profit w.r.t. ρ:

The same as the proof in the appendix, except that one needs to replace R with a in the formulas.

■

Proof of Proposition 2. The same as the proof at the beginning of the online appendix, except that one needs to replace R with a in the formulas. ■

Proof of Proposition 3. The threshold for a monopoly equilibrium to exist is now $\rho \geq \frac{a_m w + (1-a_m)v - c}{(1-a_m)(v-w)}$ rather than $\rho \geq \frac{R_m w + (1-R_m)v - c}{(1-R_m)(v-w)}$.

The proof is the same as the proof at the beginning of the online appendix, except that one needs to replace R with a in the formulas. ■