Reputation for Privacy

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Abstract

As consumers become increasingly concerned about their privacy, firms can benefit from committing not to sell consumer data. However, the holdup problem prevents firms from doing so in a static setting. This paper studies whether the reputation consideration of the firm can serve as a commitment device in a long-run game when consumers have imperfect monitoring technology. We find that a patient enough monopoly can commit because its reputation will suffer from a persistent punishment if consumers detect the data sale. In contrast, reputation may fail to serve as a commitment device when there are multiple firms. The penalty for selling data is temporary when consumers do not know exactly which firm sold the data. In addition, selling data imposes a negative externality on other firms, but each firm does not take the externality into account in equilibrium. We characterize conditions under which duopolistic firms lose the ability to commit even if they are arbitrarily patient. Furthermore, the firm is penalized less for selling the data but not rewarded more for not doing so when the number of firms increases. Reputation cannot serve as a commitment device for privacy under any conditions when there are many firms.

1 Introduction

An information market has emerged in the digital era. The business of collecting and selling consumer data is estimated to be worth around $200 billion.[1] Firms use detailed information about individuals to offer a personalized product, price discriminate, show targeted ads, etc. Aware of the costs of revealing information, consumers are becoming increasingly concerned about their privacy. People started to raise concerns about their privacy in the 1990s. About .01% of the US population opted out of the database of Lotus MarketPlace.[2] But most people at that time were either not aware of privacy issues or did not care much about them. By contrast, a 2021 survey of the general US population by KPMG found that 86% of consumers viewed data privacy as a growing concern.[3]

One reason people worry about a firm collecting their data is that they do not know how the firm will use it. According to the same survey, 40% of consumers do not trust firms to use their data ethically. Taylor (2004) shows that a firm can be better off by not protecting consumer privacy if consumers are naive and unaware that the firm sells their data. However, selling data can backfire if consumers are sophisticated and expect the firm to sell their data. Since then, a large body of literature has documented that the ability to commit to consumer privacy benefits the firm. As a result, companies pay increasing attention to privacy. Apple, for instance, has invested heavily in operating systems to protect consumer privacy and spent a good deal on advertising its progress in privacy protection. In this particular setting, the firm desires to commit to protecting consumer privacy by not selling consumer data. However, the non-verifiable nature of digital data makes it hard for a firm to commit. Despite the emerging regulations about consumer privacy, such as GDPR and CCPA, there are concerns about the credibility of such policies. Even if firms do not sell data in the presence of such regulation, consumers cannot easily verify it. The firm will not

[1] The source is https://www.latimes.com/business/story/2019-11-05/column-data-brokers.
[2] The source is https://www.forbes.com/sites/forbestechcouncil/2020/12/14/the-rising-concern-around-consumer-data-and-privacy/?sh=73c76330487e.
[3] The source is https://kpmg.com/us/en/articles/2023/bridging-the-trust-chasm.html.

benefit from protecting consumer privacy if it fails to obtain consumers' trust in how firms handle their data. This paper looks at one possible solution - building trust by reputation. Protecting consumer privacy is more credible and convincing from the consumer's perspective if it is in the interest of the firm and is an endogenous equilibrium outcome.

The main contribution of the paper is to characterize conditions such that reputation considerations can or cannot serve as a commitment device for privacy. When a monopoly is patient enough, reputation enables it to commit never to sell data. It achieves the Stackelberg payoff (optimal payoff if the firm can commit to any action) in all but a finite number of periods. However, when there are multiple firms, reputation may fail to enable any firms to commit. We find conditions under which the incentive to deviate is so strong that duopolistic firms lose the ability to commit even if they are arbitrarily close to perfectly patient. Furthermore, reputation consideration of the firm cannot serve as a commitment device for privacy under any conditions when there are many firms. Such reputation failure hurts all the firms.

We consider long-lived firms interacting with short-lived consumers repeatedly in two markets. In the product market, the consumer decides how much information to reveal. Each firm infers consumer preferences based on the revealed information and offers a personalized product and price. The consumer then makes the purchase decision. By revealing more information, they get a better recommendation.[4] However, the firm will charge a higher price when it collects more information from the consumer, knowing that they have a higher expected valuation for the product. So, the consumer faces a tradeoff between better product fit and lower price. In the information market, the firm can sell consumer data to third parties (e.g., data intermediaries). Consumers may suffer disutility from the sale of their data because of both intrinsic and instrumental reasons (Lin 2022). They are more vulnerable to data sales if they reveal more information to the firm. Therefore, the consumer's decision as to how much information to reveal in the product market depends on their belief about

[4] We refer to the consumer as the inclusive singular "they" throughout the paper.

the firm's behavior in the information market. If the consumer thinks the firm will sell their data, they will reveal no information, to minimize privacy loss. If they trust the firm not to sell their data, they will reveal some information, to get a better product recommendation. The Stackelberg action (optimal action if the firm can commit to any action) of the firm is not to sell data. But whether to sell data or not is decided after the consumer reveals the information. Hence, the *holdup problem* prevents the firm from not selling data in a static setting.

This paper studies whether the reputation consideration of the firm can serve as a commitment device in a long-run game when consumers have imperfect monitoring technology. Reputation can be a commitment device for a patient enough monopoly but may fail to be one when there are multiple firms, even when firms are arbitrarily close to perfectly patient. The intuition is that a monopoly's reputation depends solely on its own actions. The monopoly will never restore its reputation by deviating from selling the data and being caught. The persistent reputation cost strongly incentivizes the monopoly to commit to privacy. In contrast, when there are multiple firms, consumers do not know exactly which firm sold the data, even if they observe data sales. One firm's reputation depends on other firms' actions. Selling data by one firm imposes a negative externality on other firms. Firms do not take this externality into account in equilibrium. The benefit of not selling data is lower because other firms' behavior may still hurt the firm's reputation. Anticipating this externality, consumers penalize each firm less when observing data sales. The penalty for selling data is temporary and the reputation can be restored. In addition, the likelihood that the deviation changes the consumer's future information-revelation decision decreases in the number of firms. Therefore, the cost of selling data is lower, and a firm's reputation may be hurt even if it does not sell data. In this case, the firm has more incentive to deviate.

We find conditions under which the incentive to deviate is so strong that duopolistic firms lose the ability to commit even if they are arbitrarily close to perfectly patient. Furthermore, the firm is penalized less for selling the data but not rewarded more for not doing so when the number of firms increases. As a result, reputation cannot serve as a commitment device for privacy under any conditions when there are many firms.

We consider several regulatory interventions and extensions. Liability fines with regulator monitoring can help restore firms' commitment power if the monitoring technology is good enough. In such cases, the regulation always increases consumer surplus, and improves social welfare if the regulator's monitoring cost is low. On the contrary, reputation failure still occurs even if the regulator limits the amount of data a firm can sell because it does not transform the reputation cost of selling the data from a temporary punishment into a persistent one.

The consumer never knows the exact identity of the data leakage in the main model. In reality, sometimes the consumer may observe information about not only evidence of data leakage but also the identity of data leakage due to media or regulatory exposure. Knowledge about the identity of data leakage does not affect the monopoly case where there is no uncertainty about which firm sold the data. In the duopoly case, we assume in an extension that the consumer will be able further to identify the identity of data leakage with some probability, conditional on detecting data sales. If such ability is perfect, then reputation consideration can serve as a commitment device, and the regulator does not need additional regulations to sustain the equilibrium of privacy protection. In contrast, if the likelihood of knowing the exact identity of data leakage is low, we still have the reputation failure results under the same set of conditions as in the main model. In such cases, additional regulations are necessary to sustain the equilibrium of privacy protection.

We also consider false detection in another extension. In the main model, we consider the possibility of false negatives by allowing the consumer not to detect all the data sales. However, there could also be false positives - sometimes consumers may think that their data were sold, but in reality, they were not sold. Compared to the main model, a rational firm has a weaker incentive to commit to consumer data protection in the presence of false detection. Therefore, the reputation failure result in the multi-firm case still holds. In the monopoly case, the equilibrium impact of false detection depends on the sophistication of consumers. If the consumer is naive and thinks that at least one firm sold their data after observing a signal of data sales, a patient enough rational firm can still commit to protecting privacy if the false detection rate is low. If the consumer is sophisticated and updates their belief with the idea that the signal may be false-positive, the possibility of false detection moves a monopoly's incentive closer to the multiple-firm case in the main model. A rational firm loses its commitment power because of the higher incentive to deviate.

This paper contributes to the literature on the economics of privacy (see Acquisti, Taylor, and Wagman 2016 for a survey). Goldfarb and Tucker (2012) and Lin (2022) document the existence of substantial consumer privacy concerns. Absent an information market and the possibility of selling data, Chen and Iyer (2002) study competing firms' incentives to collect data. They find that firms may voluntarily collect less information about consumers to mitigate price competition. Closely related to our paper, Jullien, Lefouili, and Riordan (2020) study a website's incentive to sell consumer information in a two-period model. Unlike our setup, the website in their paper does not try to change consumers' beliefs about its type. Instead, the website wants to affect consumer behavior based on their vulnerability and bad experiences due to data sales. In a static framework, Ichihashi (2020) shows that sellers prefer to commit to the price of the good so that buyers will reveal more information. We investigate when such commitment is feasible without an external commitment device. Recent papers have paid much attention to the economic impact of regulations such as GDPR, CCPA, and AdChoices, which seek to protect consumers' privacy and give them more control over their data (Goldfarb and Tucker 2011, Conitzer, Taylor, and Wagman 2012, Campbell, Goldfarb, and Tucker 2015, Athey, Catalini, and Tucker 2017, Gardete and Bart 2018, Anderson, Baik, and Larson 2019, Goldberg, Johnson, and Shriver 2019, Montes, Sand-Zantman, and Valletti 2019, Choi, Jerath, and Sarvary 2020, 2023, Johnson, Shriver, and Du 2020, Rafieian and Yoganarasimhan 2021, Choi and Jerath 2022, Hu, Momot, and Wang 2022, Ke and Sudhir 2022, Lei, Miao, and Momot 2022, Momot and Salikhov 2022, Bonatti, Huang, and Villas-

Boas 2023, Fainmesser, Galeotti, Momot 2023, Johnson, Shriver, and Goldberg 2023, Bondi, Omid, and Yao 2024, Miklós-Thal et al. 2024, Ning, Shin, and Yu 2024).

There are two reasons why reputation is essential despite various regulations. First, the main focus of those regulations is to give consumers more control over their data usage rather than to provide the firm with commitment power. Second, the opacity and non-verifiability of data transactions raise concerns about the credibility of such policies. Even if a firm does not sell data in the presence of such regulation, consumers still may not believe it. The firm will not benefit from protecting consumer privacy if it fails to obtain consumers' trust in how firms handle their data. Protecting consumer privacy is more credible and convincing from the consumer's perspective if it is in the interest of the firm and is an endogenous equilibrium outcome.

This paper is also related to the reputation literature. The idea of modeling reputation by incomplete information originates from Kreps et al. (1982), Kreps and Wilson (1982), and Milgrom and Roberts (1982). Fudenberg and Levine (1989) show that a patient long-run player will commit to the Stackelberg action in the presence of a behavioral type and perfect monitoring. Reputation serves as a commitment device and selects away bad equilibria for the long-lived player. In contrast, a strand of literature on bad reputation, including Ely and Välimäki (2003) and Morris (2001), shows that reputation concerns may hurt the firm under imperfect monitoring. Different from our papers where a firm can build reputation by always taking the Stackelberg action (optimal action with commitment power) in the absence of other firms, reputation failure in the bad reputation literature relies on a player's inability to build reputation by always taking the Stackelberg action. Substantively, the paper most closely related to ours is Phelan (2006), which studies a problem where the government builds a reputation for trust. Despite perfect monitoring, the reputation shock is non-permanent because the government's type can change over time.

Tirole (1996) initiates the study of collective reputation, where individual reputation and incentive depend on both one's own and other players' past behavior. Unlike our paper which looks at the entire equilibrium, Tirole (1996) focuses on the steady state and enforces bad behavior in the initial period. Neeman, Öry, and Yu (2019) compares collective reputation and individual reputation. They characterize conditions such that socially optimal actions are more likely to be sustained and conditions such that socially optimal actions are less likely to be sustained under collective reputation rather than individual reputation. Collective reputation is also studied in the umbrella branding literature (Wernerfelt 1988, Cabral 2000, Kuksov 2007, Miklós-Thal 2012, Moorthy 2012, Kuksov, Shachar, and Wang 2013, Zhang 2015, Klein et al. 2019, Yu 2021, Ke, Shin, and Yu 2023). Despite the long development of the reputation literature, researchers have not paid much attention to the reputation for privacy. Our paper contributes to the reputation literature by investigating the economic and managerial implications of reputation for privacy, and by connecting bad reputation and collective reputation. The underlying mechanism that drives reputation failure is a combination of bad reputation and collective reputation, and is qualitatively different from either the bad reputation literature or the collective reputation literature.

Lastly, by building its reputation for privacy, a firm wishes to separate itself from other firms who do not care about consumer privacy. So, our paper is also related to the literature on signaling (Gal-Or, Geylani, and Dukes 2008, Iyer and Kuksov 2010, Kuksov and Wang 2013, Miklós-Thal and Zhang 2013, Jiang, Ni, and Srinivasan 2014, Dai and Singh 2020, Cao, Chen, and Ke 2023, Yang, Shi, and Lin 2023, Ke et al. 2024).

The remainder of the paper is organized as follows. Section 2 presents the main model. Section 3 solves the equilibrium of the static game. Section 4 characterizes the equilibrium of the repeated game when there is one firm. Section 5 solves the equilibrium of the repeated game when there are multiple firms. Section 6 examines whether some regulatory interventions can restore firms'commitment power. Section 7 considers two extensions to the main model. Section 8 concludes.

2 Model

Time is infinite, $t = 0, 1, 2, ...$, and the discount factor is $\delta < 1$. There are N long-lived firms and a short-lived consumer in each period. The consumer interacts with all the firms. The firm's payoff is $(1-\delta)\sum_{t=0}^{+\infty} \delta^t u_t$, where u_t is the stage payoff at time t. There is a product market and an information market.

2.1 Product Market

Consumers have different horizontal preferences and are located uniformly on a circle with a circumference of 1. When a consumer visits a firm in the product market, they choose how much information to reveal. If a consumer located at $x \sim [0, 1)$ reveals $\eta \in [0, 1]$ proportion of information, the firm gets a noisy signal $l^c \sim U[x-(1-\eta)/2 \mod 1, x+(1-\eta)/2 \mod 1]$ about the consumer's location, as illustrated by Figure 1. This implies that the firm will offer a product located at $l^f = l^c$ given the signal. The firm offers a personalized product and sets the price p based on the signal. The consumer then makes the purchase decision. Denote the distance between the product's location, y, and the consumer's location, x, by $d = |x - y|$. We have $d \sim U[0, (1 - \eta)/2]$. The baseline valuation of the product is v, and the disutility from the mismatch of the recommended product and the consumer's horizontal taste is td. Therefore, the consumer gets $v - td - p$ if they buy and 0 if they do not buy. Different firms offer different products, so the consumer may buy from multiple firms as long as the product offered by each firm gives the consumer a positive expected payoff. We assume that there is enough horizontal differentiation, $t > v$.

The setting of the firm offering one personalized product and price to each consumer based on the information the consumer reveals is analogous to the setting in Ichihashi (2020). The implicit assumption that each firm offers only one product and a consumer cannot purchase products not recommended by the firm can be justified by the consumer's limited attention. We acknowledge that the modeling choice does not fully capture the real-world details.

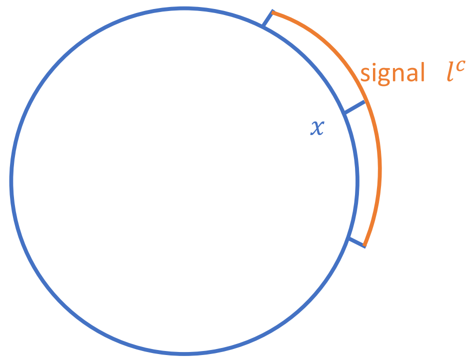

Figure 1: Consumer's location x and the signal l^c

However, the product market is not the focus of this paper. This simple setup can concisely reflect the consumer's trade-off between better recommendation and higher price due to the hold-up problem by revealing more information, which is the main economic force in the product market and will be discussed in section 3.1.

2.2 Information Market

A firm can sell consumer data to third parties (e.g., data intermediaries) in the information market. It has the data directly revealed by the consumer, as well as the consumer's behavioral data (whether they purchase given the product and price offered). It can infer consumers' willingness to pay from the data they reveal directly (Bergemann, Bonatti, and Gan 2022), or from their behavioral data (Shen and Villas-Boas 2018, Taylor 2004, Villas-Boas 1999, 2004). A firm gets $D(\eta) > 0$ by selling data. We assume that $D(\eta)$ increases in η to reflect that more accurate information is more valuable. We do not impose other assumptions on the specific form of $D(\eta)$.

Consumers may suffer disutility from the sale of their data because of both the intrinsic preference for privacy and the expected economic loss (Lin 2022). They will be more vulnerable to data sales if they reveal more information. So, we assume that the expected privacy cost of the consumer is ηu_b, where u_b is a constant.[5] Consumers can imperfectly

[5] The main property we need is that the consumer's expected privacy loss is larger when the consumer reveals more information. We choose this particular form for simplicity.

monitor firms' behavior in the information market. If a firm sold their data, the consumer detects data sales with probability q.[6] The consumer receives a signal $s = y$ if they caught any of the sales and $s = n$ if they did not detect any sales. The assumption that consumers cannot distinguish which firm sold the data gives the sharpest illustration of the main idea. In section 7.1, we relax the assumption by allowing consumers to sometimes know the exact identity of data leakage.

2.3 Reputation

We follow the standard way of modeling reputation by incompete information, originated from a series of papers by Kreps et al. (1982), Kreps and Roberts (1982), and Milgrom and Roberts (1982). There are two types of firms. A rational type (type R) maximizes the expected sum of discounted utilities, whereas a behavioral type (type B) always sells consumer data. One can view the behavioral type as a myopic firm who only cares about the present. An alternative and better interpretation is not to think about the behavioral type literally. Rather, it is a standard modeling device in the reputation literature that introduces incomplete information in order to model reputation. Our focus is on the equilibrium behavior of the rational firm. The firm's reputation is consumers' belief about the probability that the firm is type B. The common prior that each firm is type B is $\mu_0 \in (0, 1)$. Consumers update their belief about the firm's type by Bayes' rule. Denote the belief about firm i's type at time t by $\mu_{i,t}$. Reputation for privacy in this paper refers to the reputation for protecting consumer privacy in the information market. The consumer's current belief is a measurement of the firm's reputation.

2.4 Timing

At the beginning of each period, a short-lived consumer arrives. They choose the amount of information to reveal to each firm based on the firm's reputation (the consumer's belief

[6] This simple informaton structure is analogous to the setting in Tirole (1996).

about the type of each firm). Then, the firm receives a noisy signal about the location (preference) of the consumer. The firm offers the consumer a personalized product and price based on the signal. The consumer makes the purchase decision. Then, the firm decides whether to sell the consumer's data to third parties in the information market. The consumer receives an imperfect signal about firms' data-selling behavior and updates her belief about each firm's type. This belief captures the firm's reputation and is observed by the newly arrived consumer in the next period. Firms' reputations can be communicated to future consumers through various channels such as word of mouth, media coverage, third-party grading, etc. The entire history of the game, in contrast, is hard to be communicated to future consumers because it involves too much information. So, the period-t consumer observes the current reputation of the firm $\vec{\mu}_t = (\mu_{1,t}, \mu_{2,t}, ..., \mu_{N,t})$ rather than the entire history. Figure 2 illustrates the timing of the game.

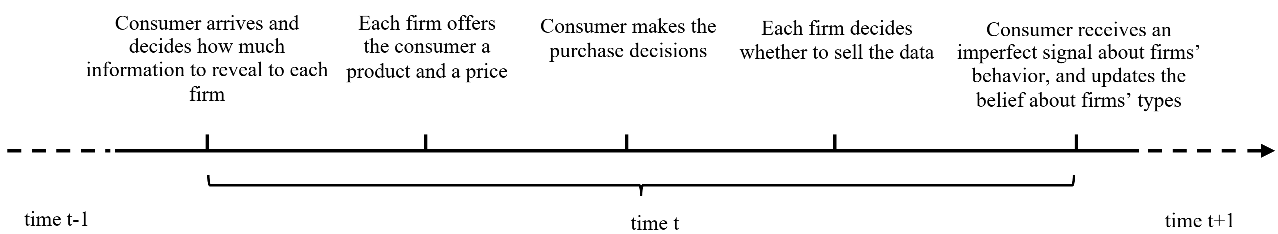

Figure 2: Timing of the Game

2.5 Solution Concept

We focus on whether a Markov Perfect Equilibrium (MPE) exists where rational firms could commit to never selling the data.[7] MPE requires that firms' and consumers' strategies depend only on the current state. It is widely used in the reputation literature (e.g., Mailath and Samuelson 2001, Bar-Isaac 2003, Phelan 2006, Board and Meyer-ter-Vehn 2013, Pei

[7] Technically, the solution concept we use is the Markov Perfect Bayesian Equilibrium, because there is incomplete information about the firm's type. However, the reputation literature usually uses the notion of MPE for this kind of equilibrium.

2016, Marinovic and Szydlowski 2023). The belief $\vec{\mu}_t$ is the natural state variable. In the symmetric equilibrium where every firm always has the same reputation, we simplifies the notation by using $\mu_t = \mu_{i,t}$ to denote the reputation of each firm. Importantly, a player's strategy in each period is a function of the current belief (and the observations in the current period) rather than the whole history. In other words, each player's strategy depends only on the current state, not on the history of how the state was reached. Heuristically, it is easier for a firm's reputation to be carried over time through an aggregate summary (the consumer's current belief), but it is hard for the present consumer to know the entire history in the past. Formally, the MPE of this game is defined as follows.

Definition 1. *Denote the state space as $K = \{\vec{\mu} \in [0,1]^N\}$, the consumer's strategy as σ^c, and the firm's strategy as σ^f. The MPE of this game is a profile of strategies σ^{c^*}, σ^{f^*} such that:*

1. *Each player's strategy is a Markov strategy, which depends only on the current state (and the observable action in the current period), not on the history of how the state was reached.*

2. *For every state, the strategy of each player maximizes the player's expected continuation payoff conditional on other players following the equilibrium strategies.*

3. *At the end of each period, the state is updated by Bayes' rule, based on the previous state, the players' equilibrium strategies, and the signal s about the firm's imperfectly monitored data-selling action.*[8]

[8] Section 4.1 and 5.1 present the specific belief updating (state trainsition) formulas, which depend on signal s. Usually in a MPE, the state transition probability is a function of the previous state and players' actions. We can express the state transition in the same way because a firm's different actions (sell data or not) induce different distribution of the signal s, thereby different distributions of the updated state.

3 Equilibrium in a Static Game With One Firm

We first analyze the equilibrium outcome of a static game with one period and a single firm to prepare us for the analyses of the entire game.

3.1 Firm's Pricing Decision

Given the product recommendation and price, the consumer purchases if and only if the expected payoff is positive, $v - td - p \geq 0$. Thus, the firm's pricing problem and the optimal price are:

$$\max_{p} \; p \cdot \mathbb{P}[v - td - p \geq 0] = p \cdot \min\left\{\frac{2(v-p)}{(1-\eta)t}, 1\right\}$$
$$\Rightarrow \; p^* = \begin{cases} \frac{v}{2}, \; if \; \eta \leq 1 - \frac{v}{t} \\ v - \frac{(1-\eta)t}{2}, \; if \; \eta > 1 - \frac{v}{t} \end{cases}$$

When the consumer reveals a lot of information, $\eta > 1 - \frac{v}{t}$, the firm accurately knows their preference. The recommended product is always close to the consumer's actual location, and the firm can extract a high surplus, even from the consumer located farthest away from the recommended product. Therefore, the firm sets a price such that the consumer always purchases. When the consumer reveals less information, the firm gets a noisier signal about their preference. The profit from each purchase will be too low if the firm wants the consumer to always buy the product. Therefore, only consumers with a high enough valuation for the recommended product purchase it at the optimal price. If the recommended product is located too far away from the consumer, the consumer will not buy it.

3.2 Consumer's Information Revelation Decision

From the consumer's perspective, revealing more information has two opposite consequences in the product market. On one hand, the firm can offer a better-matched product, which benefits the consumer. On the other hand, the firm will charge a higher price, knowing that the consumer has a higher expected valuation, which hurts the consumer.[9] One can see that the consumer's expected payoff in the product market is $v^2/[4(1-\eta)t]$ if $\eta \leq 1-v/t$ and is $(1-\eta)t/4$ if $\eta > 1-v/t$. In an extreme case, if the firm knows the consumer's preference perfectly, it will extract all the consumer surplus. Therefore, the consumer never reveals everything. In the other extreme case, if the firm knows nothing about the consumer's preference, it can only recommend a random product. The poor match also hurts the consumer. So, it is optimal for the consumer to reveal partial information if they only consider the product market.

The consumer also needs to consider the effect of information revelation in the information market. Disclosing more information to the firm always hurts the consumer there, as they are more vulnerable when the firm sells their data. Conditional on the firm selling data, the consumer's expected privacy cost is ηu_b. The probability that a firm sells data is endogenously determined in equilibrium. Therefore, given a probability of selling data μ_s, the consumer's ex-ante expected privacy loss is $-\mu_s \eta u_b$.[10]

Putting everything together, the consumer's ex-ante expected payoff is:

$$U_0(\eta) = \begin{cases} \frac{v^2}{4(1-\eta)t} - \mu_s \eta u_b, \ if \ \eta \leq 1 - \frac{v}{t} \\ \frac{(1-\eta)t}{4} - \mu_s \eta u_b, \ if \ \eta > 1 - \frac{v}{t} \end{cases} \tag{U}$$

Considering both the product and information markets, it is never optimal for the consumer to reveal too much information, as Figure 3 illustrates. The firm can charge a high price because the product recommendation is pretty accurate, with lots of information about the consumer. In addition, the privacy loss from data sales in the information market is high. Consumers may, however, prefer revealing a moderate amount of information to revealing nothing. By revealing some information, consumers benefit from a better recommendation

[9] For more discussion about this kind of holdup problem, see Villas-Boas (2009) and Wernerfelt (1994).
[10] The probability that a firm sells data, μ_s, is endogenously determined in equilibrium.

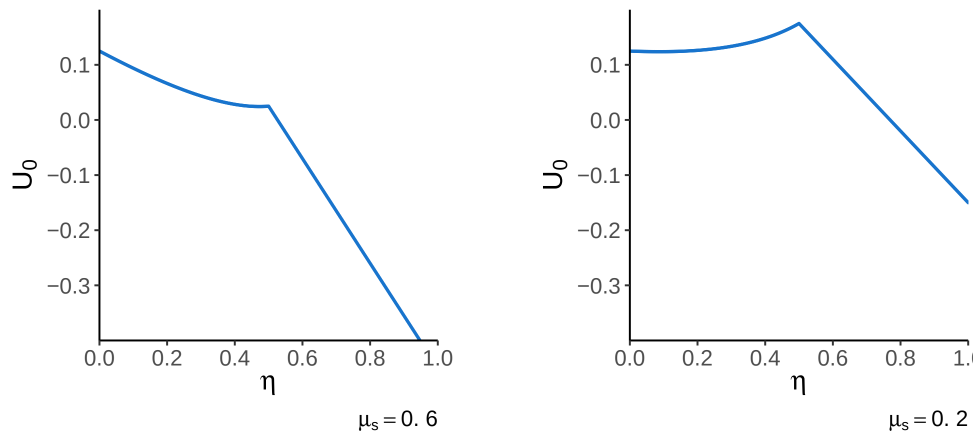

Figure 3: Ex-ante consumer payoff as a function of η for $v = 1,\ t = 2,\ u_b = .75$, and $\mu_s = .6$ (left) or .2 (right).

in the product market but suffer a privacy cost if the firm sells it in the information market. If the firm's likelihood of selling the data is high, the high expected privacy loss in the information market outweighs the gain from the better match in the product market. In that case, the consumer reveals no information. On the contrary, the consumer partially reveals their preference for a better recommendation if the firm's likelihood of selling data is low. The following result formalizes our intuition. For the problem to be interesting, we assume that the threshold $\widehat{\mu} \in (0, 1)$. We also assume that they choose $\eta = 1 - v/t$ when they are indifferent between $\eta = 1 - v/t$ and 0, which does not affect any analyses.

Proposition 1. *The consumer does not reveal any information if $\mu_s > \widehat{\mu}$, and reveals $\eta^* = 1 - \frac{v}{t}$ amount of information if $\mu_s \leq \widehat{\mu}$, where $\widehat{\mu} = \frac{v}{4u_b}$.*

Corollary 1. *The firm's profit in the product market is $v/2$ if $\mu_s \leq \widehat{\mu}$ and is $v^2/2t$ if $\mu_s > \widehat{\mu}$.*

If the firm could commit not to sell consumer data, the consumer would choose $\eta = 1-v/t$, which gives the firm a stage payoff of $v/2$. If, instead, the firm always sells consumer data, the consumer will not reveal any information (i.e., they will choose $\eta = 0$). In that case, the firm obtains a stage payoff of $v^2/2t + D(0)$. Denote by Δu the difference between the firm's expected stage payoff with and without commitment.

$$\Delta u := v/2 - v^2/2t - D(0) \tag{Δ}$$

The firm will always sell consumer data if $\Delta u < 0$ and the entire problem becomes trivial. So, we focus on the interesting case in which $\Delta u > 0$ throughout the remaining analyses.

3.3 Some Benchmarks

We now analyze the property of the stage game of a single firm by comparing the equilibrium outcome with some benchmarks. If the firm can commit to any action in the information market (e.g., by moving first), the firm takes the *Stackelberg action* and obtains the *Stackelberg payoff*.

Definition 2. *Suppose player 1 chooses action* $a_1 \in A$ *and player 2 chooses action* $a_2 \in A_2$*. Player* $i \in \{1,2\}$*'s stage-game payoff is* $u_i(a_1, a_2)$*.* $BR_2(a_1) \subset A_2$ *is player 2's best response correspondence to* a_1*. Then, player 1's Stackelberg action is* $\arg\max_{a_1 \in A_1}[\min_{a_2 \in BR_2(a_1)} u_1(a_1, a_2)]$*, and player 1's Stackelberg payoff is* $\max_{a_1 \in A_1}[\min_{a_2 \in BR_2(a_1)} u_1(a_1, a_2)]$*.*

One can see that the Stackelberg action for the firm is not to sell the data. The consumer will reveal $\eta = 1 - v/t$ proportion of information, and the firm gets the Stackelberg payoff of $v/2$.

If the consumer acts first and minimizes the firm's payoff, the firm gets the *minmax payoff*, which is the payoff the firm can guarantee regardless of the consumer's action.

Definition 3. *Suppose player 1 chooses action* $a_1 \in A_1$ *and player 2 chooses action* $a_2 \in A_2$*. Player* $i \in \{1,2\}$*'s stage-game payoff is* $u_i(a_1, a_2)$*. Then, player 1's minmax payoff is* $\min_{\alpha_2 \in \Delta(A_2)}[\max_{a_1 \in A_1} u_1(a_1, \alpha_2)]$*, where* $\Delta(A_2)$ *denotes a set of probability distributions (mixed strategies) over pure strategies* $a_2 \in A_2$ *and* $u_1(a_1, \alpha_2) := \sum_{a_2 \in A_2} \alpha_2(a_2) u_1(a_1, a_2)$*.*[11]

[11] The notation $\alpha_2(a_2)$ denotes the probability that a player using mixed strategy $\alpha_2 \in \Delta(A_2)$ plays the specific strategy $a_2 \in A_2$.

To minimize the firm's payoff, one can see that the consumer will reveal no information. In response, the firm sells data and gets the minmax payoff of $v^2/2t + D(0)$.

In a static game, the firm always sells data because it decides whether to sell data after the consumer reveals information. Anticipating this, the consumer reveals nothing. The firm receives the minmax payoff. Our objective in this paper is to study whether reputation considerations in the dynamic game enable the firm to commit to the Stackelberg action and receive the Stackelberg payoff.

4 Monopoly

4.1 Belief Updating

We first derive the consumer's belief updating processes about a monopoly's type given the observed signal s, using the Bayes' rule. Because the behavioral type always sells the data, detection of data sales (signal y) will increase the consumer's belief that the firm is the bad type, whereas a lack of evidence of data sales (signal n) will reduce the belief. Notice that even if $s = n$, it is possible that the firm is the bad type but the consumer fails to detect data sales. So, the consumer's belief decreases but is still positive.

Proposition 2. *For a given belief μ_t, the consumer's updated belief μ_{t+1} is $(1-q)\mu_t/\{(1-q)\mu_t + [1-\sigma_{sell}^{f^*}(\mu_t)q](1-\mu_t)\}$ if they observe signal $s = n$ and is $\mu_t/[\mu_t + \sigma_{sell}^{f^*}(\mu_t)(1-\mu_t)]$ if they observe signal $s = y$, where $\sigma_{sell}^{f^*}(\mu_t)$ is the rational firm's equilibrium probability of selling data under state μ_t.*

The proposition directly implies the following sharper belief-updating process if the rational firm's equilibrium strategy is to never sell the data.

Corollary 2. *Suppose the rational type never sells the data in equilibrium. For a given belief μ_t, after the consumer receives signal n for k consecutive periods, the belief becomes $\mu_{t+k} = (1-q)^k\mu_t/[(1-q)^k\mu_t + 1 - \mu_t]$, which approaches 0 as $k \to +\infty$.*

In this case, if the consumer observes $s = y$, then they know for sure that the firm sold the data in the previous period. So, the firm must be a bad type because the rational type does not sell the data. If the monopoly continues not to sell consumer data, the consumer's belief will keep decreasing. After enough time, the consumer is almost certain that the firm is not the bad type.

4.2 Equilibrium

Suppose consumers expect the rational firm never to sell the data in equilibrium. In that case, a signal n will destroy the firm's reputation by making the consumer believe that the firm is the bad type in all current and future periods. Then, the firm is stuck with the minmax payoff. By deviating, the firm risks being detected by the consumer with a positive probability. The persistent punishment strongly incentivizes the firm not to sell the data for short-term benefit. As a result, regardless of the monitoring technology or the price of data in the information market, reputation can always serve as a commitment device as long as the monopoly is patient enough.

Proposition 3. *There exists a $\widehat{\delta} < 1$ such that, for any $\delta > \widehat{\delta}$, there exists a MPE where the rational firm never sells consumer data and the consumer always reveals $\eta = 1 - v/t$ proportion of information after a finite period.*

By protecting consumer privacy, the rational firm keeps reducing the consumer's belief that it is a bad type. When the belief is below a threshold, the consumer is willing to reveal some information, which benefits the firm. A patient firm does not want to deviate, because the consumer may observe the deviation and believe that the firm is the bad type. If that happens, future consumers will never reveal any information. So, the firm permanently suffers from less revenue in the product market. This severe punishment provides a strong incentive for the firm to trade the short-term benefit of selling data in the information market for the long-term benefit of earning a higher profit in the product market.

5 Multiple Firms

Consumers update their beliefs about firms collectively rather than individually in the presence of multiple firms. The collective belief-updating process generates two opposing effects on a firm's incentive to sacrifice short-term benefits for long-term gains. On one hand, there is a smaller incentive for the firm to build its reputation if the belief is very high or very low because the short-term return is low in such cases. The presence of other firms makes the consumer's belief updating about one firm slower. The less extreme belief implies that the firm has a stronger incentive. On the other hand, the presence of multiple firms and collective reputation leads to noisier signals and worse monitoring. It incentivizes a firm to free-ride on other firms, which implies that the firm has a weaker incentive to build its reputation.

The two opposing forces make it non-obvious whether the reputation consideration of the firm can serve as a commitment device when there are multiple firms. For instance, in a different setting with one market, Neeman, Öry, and Yu (2019) show that, compared to individual reputation, collective reputation can be either more likely or less likely to sustain the socially optimal equilibrium.

5.1 Belief Updating

When there are at least two firms, the belief updating is qualitatively different from the monopoly case. Consider the consumer's belief about firm 1's type after observing a signal s, assuming that a rational firm never sells the data. If $s = y$, the consumer knows that at least one firm sold the data in the previous period but is unsure whether firm 1 sold it. So, the belief that firm 1 is a bad type increases but is still lower than 1, unlike the monopoly case. The reputation shock is temporary, and firm 1 can rebuild its reputation. Conditional on other firms' behavior, the likelihood that $s = n$ if firm 1 is a rational type and did not sell the data is higher than the case where firm 1 is a bad type, but consumers cannot directly observe the data sales. Consequently, the belief that firm 1 is a bad type decreases but is still positive. Formally, the belief updating is as follows.

Proposition 4. *Suppose the rational type never sells the data in equilibrium. For a given firm i, the posterior belief μ_{t+1} is $(1-q)\mu_t/(1-q\mu_t)$ if $s=n$ and is $[1-(1-q)(1-q\mu_t)^{N-1}]\mu_t/[1-(1-q\mu_t)^N]$ if $s=y$. It does not depend on the number of firms N if $s=n$ and decreases in N if $s=y$.*[12]

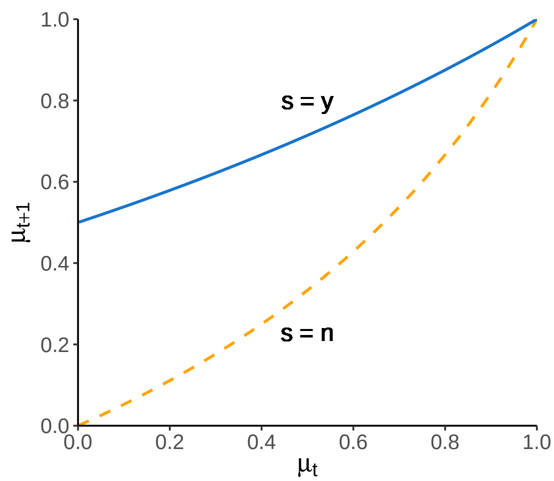

Figure 4: Belief updating as a function of μ_t for $q=.5$ and $N=2$.

Even if firm 1 sold the data and the consumer observes a signal y, they know that at least one firm sold the data but does not know which firm. Therefore, they penalize firm 1 less than they do in the monopoly case. The reputation cost is temporary, and the consumer's belief will decrease if they receive signal n in the future. When there are more firms, the signal's noise is larger, and the consumer has less idea about which firm sold the data. Therefore, there will be smaller increases in belief in response to signal y. If the consumer observes a signal n, the belief reduction does not depend on the number of firms. So, the firm is *penalized less* for selling the data but *not rewarded more* for not doing so when the number of firms increases. In addition, the realization of the signal depends little on a single

[12] We ignore the subscript i in the belief $\mu_{i,t}$ for simplicity.

firm's action when there are many firms. So, the likelihood that firm 1's action changes the consumer's future information-revelation decision decreases in the number of firms. Figure 4 illustrates the belief updating when there are two firms.

5.2 Two Firms

In this section, we study whether it is possible to achieve commitment by reputation when there are two firms.

Proposition 5. *Suppose there are two firms. There does not exist any MPE in which any rational firm could commit to never sell data, even when $\delta \to 1$, if the following conditions hold:*

$$\Delta u < \overline{\Delta u} := \frac{vq(t-v)}{2t}, \tag{1}$$

$$q(1-q) < \frac{2D(0)}{v}, \tag{2}$$

$$v/u_b < 2, \tag{3}$$

where Δu, the difference between the firm's expected stage payoff with and without commitment, has been defined in equation (Δ).

This proposition identifies sufficient conditions under which firms cannot commit even if they are arbitrarily close to perfectly patient. The role of each condition is the following.

- Condition (1)

  The whole idea of building reputation is to sacrifice short-term gains for long-term benefits. This condition means the commitment payoff is not much higher than the one without commitment. The left-hand side of the condition, Δu, is a measure of long-term benefits. Proposition 3 has shown that reputation consideration provides a monopoly with a strong enough incentive to commit not to sell consumer data when there is any long-term benefit,

$\Delta u > 0$. In contrast, this condition shows that, when there are two firms, reputation consideration may not serve as a commitment device even if there is a positive long-term benefit, as long as it is not too large, $\Delta u \in (0, \overline{\Delta u})$.

- Condition (2)

  This condition means that the noise of the monitoring technology is either low or high. Because of the imperfect monitoring technology, consumers may get a false-positive signal y if a firm did not sell data or a false-negative signal n if a firm sold data. If q is high, the consumer will likely get a signal y even if a firm did not sell data, because the other firm sold data. If q is low, the consumer will likely get a signal n even if a firm sold data, because of poor monitoring. In either case, the likelihood that selling data changes the consumer's future information-revelation decision is low, which creates a stronger incentive for the firm to deviate.

- Condition (3)

  The consumer enjoys a positive utility from consuming good products in the product market. The valuation of the product, v, is related to the benefit of revealing some information to the firm. The consumer incurs a disutility if the firm sells their data. The cost of revealing a certain amount of information increases in u_B. This condition means that the consumer's benefit from revealing some information to the firm is not too high relative to its cost. A lower benefit-to-cost ratio leads to a lower cutoff belief of revealing information, $\widehat{\mu}$. In particular, a lower $\widehat{\mu}$ implies that a good reputation is less persistent. Even if the firm does not sell data and reduces the belief below the threshold, signal y in future periods will more easily raise the belief above $\widehat{\mu}$, inducing the consumer to stop revealing any information. The fast depreciation of reputation makes building reputation less attractive.

When the above conditions hold, reputation considerations provide no commitment power to rational firms, even if they are arbitrarily close to perfectly patient. The finding is in stark contrast to the monopoly case. It shows that consideration for bad reputation itself does not drive the reputation failure result in the privacy setting. The joint effect of bad reputation and collective reputation leads to reputation failure.

The mechanism of the reputation failure is the following. When there are two firms, consumers do not know exactly which firm sold the data, even if they observe data sales. One firm's reputation depends on other firms' actions. Selling data by one firm imposes a negative externality on other firms. Firms do not take this into account in equilibrium. The benefit of not selling data is lower because other firms' behavior may still hurt the firm's reputation. Anticipating this externality, consumers penalize each firm less when observing data sales. When the noise of the monitoring technology is either low or high, a firm's deviation is less likely to change the consumer's future information-revelation decision. Therefore, the cost of selling data is lower. In addition, a firm's reputation may be hurt even if it does not sell data. Consequently, the firm has more incentive to deviate.

5.3 Many Firms

In the previous section, we have characterized a set of conditions under which reputation consideration of the firm fails to serve as a commitment device for privacy even if duopolistic firms are arbitrarily close to perfectly patient. In this section, we show that it is even harder for each firm to commit not to sell consumer data when there are more firms. In particular, the next result shows that reputation consideration of the firm cannot serve as a commitment device for privacy *under any conditions* when there are many firms.

Proposition 6. *For any $\delta \in (0, 1)$, $\exists N_\delta$ s.t. $\forall N \geq N_\delta$, firms always sell data, and consumers reveal nothing in the unique MPE. Consumers' belief about each firm's type is always μ_0.*

No matter how patient firms are, they cannot build a reputation for privacy. The intuition for the failure of reputation as a commitment device is the following. On one hand, the consumer has a noisy signal about which firm sold the data. Even if the firm deviates and the consumer observes it, the penalty for that particular firm is less than that for the monopoly firm. Moreover, the penalty decreases in the number of firms. On the other hand, even if none of rational firms deviates, other firms may be the bad type and sell the data. So, as the number of firms increases, it becomes less likely that an individual firm's sale of data changes the consumer's future information-revelation decision. Moreover, the reputation of the firm when the consumer observes a signal n does not improve at a faster rate when the number of firms increases, as section 5.1 has shown.

Both forces give the firm more incentive to deviate and sell the data. So, it becomes harder to commit when the number of firms increases. Eventually, the firm loses all the commitment power and sells data in every period. Anticipating that the firm will always sell data, consumers do not reveal anything. The belief about each firm's type remains the same over time, and there is no reputation building.

To summarize, the monopoly can get Stackelberg payoffs in all but a finite number of periods under substantial punishment for selling data. In contrast, each firm can only get the minmax payoff under weaker punishment when there are multiple firms, even if there is no competition.

5.4 Relationship With Previous Work

Prisoners' Dilemma

One may think the result that a monopoly can commit but more firms cannot commit even though they would be collectively better off with commitment is similar to prisoners' dilemma. However, there are key differences between our results and mechanisms and those in the prisoners' dilemma.

The prisoner's dilemma arises in a static game. The key economic force for each player to deviate from cooperation is the lack of future rewards or punishments. Cooperation can happen when we consider a repeated version of the game because non-cooperation has a long-term impact on the player. Our findings are all based on repeated interactions. Even if there are an infinite number of periods and firms are arbitrarily close to perfectly patient, no firm can commit to never selling consumer data. This result is qualitatively different from the prisoners' dilemma. Our mechanism comes from the joint effect of bad reputation and collective reputation, which has yet to be studied to our knowledge.

Bad Reputation

In the bad reputation literature, initiated by Morris (2001) and Ely and Välimäki (2003), a rational player's incentive to avoid a bad reputation eliminates all the welfare because the player may need to distort the action from the Pareto-optimal one to the action that hurts everyone's short-term payoffs due to reputation considerations. The reputation failure results appear even if there is only one long-run player. By contrast, in this paper, a patient enough firm can build reputation and achieve the Stackelberg payoff without other long-lived firms and collective reputation.

The key difference is that the firm's reputation may be hurt even if it takes the optimal action for the consumer in the bad reputation literature, whereas a monopoly always improves its reputation by doing the right thing for the consumer in our paper. Fundamentally, the difference comes from the different market structures. Usually, in the literature on bad reputation, there is a single market, and the incentive of the bad type is partially misaligned with the consumer. The partial misalignment implies that the bad type's action may sometimes be optimal for the consumer. Consequently, the rational/good type may need to take an action that hurts the consumer and itself to distinguish itself from the bad type. In contrast, the unique feature of privacy is that there are two markets. The firm and the consumer's incentives are partially misaligned in the product market: the firm always benefits from more information, whereas the consumer wants to reveal partial information. In the information market, however, the bad type's incentive is completely misaligned with the consumer: the firm wants to sell the data while the consumer does not. As a result, the rational type can always separate itself from the bad type by taking the optimal action for the consumer in the information market. Therefore, multiple firms and collective reputation are critical to the reputation failure results.

5.5 Practical Implications

Even though commitment may be desirable for the firm, it may not be possible without strict external regulations. In addition, the opacity and non-verifiability of data transactions raise concerns about the credibility of such policies. Even if a firm does not sell data in the presence of such regulation, consumers still may not believe it. The firm will not benefit from protecting consumer privacy if it fails to obtain consumers' trust in how firms handle their data. Protecting consumer privacy is more credible and convincing from the consumer's perspective if it is in the interest of the firm and is an endogenous equilibrium outcome.

A monopoly can always build a reputation for caring about consumer privacy by not selling data. After a finite period, consumers will reward such behavior by sharing more information. The monopoly can enjoy a higher profit by recommending better-fit products and charging a premium. However, when there are multiple firms in the market, it may not be in the firm's best interest to protect consumer privacy. Even if a firm never sells consumer data, it may not be able to build a reputation for privacy. So, it loses the revenue from selling consumer information, while does not have any (or enough) gain. Because firms benefit from committing never to sell consumer data, they need to think about other ways of achieving the commitment.

Our model suggests that the key to commitment power is the tradeoff between the short-term benefit in the information market and the long-term benefit in the product market. From the firm's perspective, a potential solution is to improve the recommendation algorithm so that it has a higher marginal benefit from consumer information. The firm will have a stronger incentive to maintain a good reputation in order to profit from the product market. It can also consider compensating consumers if the signal is y. In that case, the firm will face an additional penalty for selling consumer data. Therefore, the "free lunch" in the information market is more costly for the firm. This idea is related to restoring truthful communication between an advisor and a decision-maker through monetary incentives, studied by Durbin and Iyer (2009). In reality, firms actively lobby in favor of or against government regulations. Another implication of our results is that, in markets where firms earn most of their revenue from the actual products rather than advertising,[13] oligopoly firms should support privacy regulations more than monopoly ones because the equilibrium consideration of reputation may not be enough to serve as a commitment device when there are multiple firms.

From the regulator's perspective, it may invest in additional monitoring efforts and charge liability fines if it catches data sales by a specific firm, increasing the firm's cost of selling data. It could also limit the amount of data a firm can sell, reducing the benefit of selling data. In the next section, we analyze these regulatory interventions in detail.

6 Regulatory Interventions

The main model shows the delicate nature of a firm's ability to commit to consumer data protection when there is more than one firm. This section studies several regulatory interventions and their impact on the equilibrium outcome. In particular, we want to examine whether each regulatory intervention can help restore firms' commitment power, thereby enhancing privacy protection. We also study the welfare implications of the regulatory intervention. Because reputation consideration on its own can serve as a commitment device for a patient enough monopoly, we will focus on the duopoly case in this section.

[13] In our model, we do not consider advertising revenue, which depends crucially on the amount of information about individual consumers a firm can collect.

6.1 Liability Fines with Regulator Monitoring

A common form of regulation is liability fines. In the main model, the consumer can never know if a particular firm sold their data when there are multiple firms. For this regulatory intervention to be feasible, the regulator needs additional information to sometimes know for sure that a specific firm sold the data.[14] So, we assume that the regulator incurs a cost c_r per period of time to monitor the data sales and that every time a firm sold the data, the regulator detects it with probability q_r and charges the firm a constant fine $f > 0$.

Proposition 7. *There exists a $\widehat{\delta} < 1$ such that, for any $\delta > \widehat{\delta}$, there exists a MPE where any rational firm never sells consumer data if the monitoring technology is good enough, $q_r > (1-q)/(2-q)$. In such cases, this regulation can improve consumer welfare. There exists a threshold $\widehat{c}_r$ such that this regulation can improve social welfare if the regulator's monitoring cost is lower than the threshold, $c_r < \widehat{c}_r$.*[15]

The proposition shows that liability fines can help restore firms' commitment power as long as the regulator's monitoring technology is good enough. We make several comments here.

The sufficient condition we identified in the above proposition does not depend on the amount of the fine. So, the recovery of commitment power relies on the information conveyed by the liability fine (a firm is a bad type). To be clear, a higher fine will further increase the incentive of a rational firm not to sell the data, and could potentially restore the commitment power for a wider range of monitoring technology q_r. But if q_r is higher than $(1-q)/(2-q)$, then the mere informational value of the fine suffices to restore the commitment power without the help of the monetary incentive. Also, the threshold $(1-q)/(2-q)$ decreases in q. So, the consumer's detection ability and the regulator's monitoring ability are substitutes for each other.

[14] Naturally, the regulator has access to better monitoring tools and resources than consumers.

[15] Because the behavioral type is a modeling device for reputation, we only include the payoff of the consumers and rational firms in the calculation of social welfare.

Because this regulation can restore the commitment power and sustain the equilibrium of privacy protection, rational firms will not be fined in equilibrium. Therefore, the amount of the fine does not affect social welfare. The cost of this regulation from the social welfare perspective is the monitoring cost the regulator needs to incur. So, the impact of the regulation on social welfare depends on the magnitude of the monitoring cost. The regulation improves social welfare if the monitoring cost is low because it helps sustain the privacy protection equilibrium at a low cost. It reduces social welfare if the monitoring cost is high because the monitoring itself generates neither consumer surplus nor firm surplus.

6.2 Limiting the Amount of Data A Firm Can Sell

Another potential regulatory intervention is to limit the amount of data a firm can sell. Some information about the consumers may be more sensitive than others. The leakage of such sensitive information can equip the consumer with a much higher risk. The regulator may strictly restrict the transaction of this kind of information. In this extension, we consider a regulatory intervention that limits the amount of data a firm can possibly sell from all the information revealed to the firm, η, to be less than a threshold, $\bar{\eta}$.[16]

Despite the reduction of a firm's potential benefit from selling data, the reputation failure results still hold. The mechanism of Proposition 5 still works for the following reason. A rational firm is trading off the benefit and cost of selling consumer data when deciding whether to deviate from the privacy-protection equilibrium. The data-limitation regulation affects neither the (reputation) cost of selling data nor the benefit of selling data when the belief about a firm is higher than $\widehat{\mu}$ because the consumer reveals no information to the firm in that case.[17] For a rational firm, there is a positive probability that the belief about it will exceed $\widehat{\mu}$ at some point. So, reputation failure, as identified in the main model, still occurs.

[16] An alternative setup is to limit the amount of data a firm can sell from η to $\psi\eta$, where $\psi \in (0, 1)$ is a constant. The result and intuition remain the same under this alternative setup.

[17] The regulation reduces the benefit of selling data when the belief about a firm is lower than $\widehat{\mu}$ because the consumer reveals a positive amount of information while the firm can only sell part of the information.

Fundamentally, this regulation does not transform the reputation cost of selling the data from a temporary punishment into a persistent one.

7 Extensions

7.1 Ability to Identify the Identity of Data Leakage

A key mechanism in the main model is that reputation as a vehicle for commitment requires repeated interactions with some degree of monitoring. Establishing such a reputation for privacy requires traceability, which is lost when there are multiple firms. In reality, sometimes the consumer may observe information about not only evidence of data leakage but also the identity of data leakage. For example, in Facebook's Cambridge Analytica data scandal, a British consulting firm collected personal data of many Facebook users without their consent and used it for political advertising.[18] This data leakage received much attention and many people are aware of it due to the extensive media coverage. In this case, the identity of data leakage, Facebook, is known to the users.

The ability to identify the identity of data leakage does not affect the monopoly case, where the consumer always knows the firm's identity after observing data sales. So, we focus on the duopoly case in this extension. To capture the phenomenon that a specific firm may be exposed by the media or regulator, we assume that conditional on detecting data sales (with probability q for each firm that sold data), the consumer will be able to further identify the exact identity of data leakage with probability ϕ (for each firm). Note that the media or regulator may not find out the identity of all the firms who sold data simultaneously, so we allow for the possibility that only one firm is identified though both firms sold the data.

Proposition 8. *Suppose there are two firms. There does not exist any MPE in which any rational firm could commit to never sell data, even when $\delta \to 1$, if conditions* (1), (2), *and*

[18] The coverage of this example is available at https://www.businessinsider.com/cambridge-analytica-whistleblower-christopher-wylie-facebook-data-2019-10.

(3) *in Proposition 5 hold and if the probability of identifying the identity of data leakage, ϕ, is low. In contrast, if $\phi = 1$, then there exists a $\widehat{\delta} < 1$ such that, for any $\delta > \widehat{\delta}$, there exists a MPE where a rational firm never sells consumer data.*

If the consumer can always identify the identity of data leakage upon catching data sales ($\phi = 1$), then the reputation failure results in the main model (Proposition 5) will break down. In such cases, a rational firm faces a high likelihood of persistent reputation costs, which strongly incentivizes it to commit to privacy protection. Therefore, the reputation consideration itself can serve as a commitment device, and the regulator does not need additional regulations to sustain the equilibrium of privacy protection.

In contrast, if the likelihood that the media or regulator exposes the identity of data leakage is low (small ϕ), we still have the reputation failure results under the same set of conditions (plus small ϕ, which is the new parameter in this extension) as in Proposition 5 of the main model. Though a rational firm can never restore its reputation if it deviates from protecting consumer privacy and gets exposed by the media or regulator, the punishment is not strong enough to prevent data sales because of the low likelihood of identifying the specific firm. Most of the time, even if the consumer caught the data sales, they do not know the identity of the data leakage, which corresponds to the duopoly case in the main model where the penalty for selling data is temporary and the reputation can be restored. For example, even though the media may catch a firm like Facebook in the Cambridge Analytica data scandal, such media exposure happens infrequently and does not lead to enough commitment power. In fact, Meta is still under regulatory scrutiny for data sharing despite the above scandal.[19] In such cases, additional regulations are needed to sustain the equilibrium of privacy protection.

[19] The source is https://www.cnn.com/2024/02/29/tech/meta-data-processing-europe-gdpr/index.html. We thank an anonymous reviewer for suggesting this extension and providing related examples.

7.2 False Detection of Data Sales

The consumer has imperfect detection ability and receives a noisy signal s about whether the firm sold the data. In the main model, we consider the possibility of false negatives by allowing the signal to be n with some probability even if a firm sold the data. This reflects that a consumer may be unable to detect all the data sales. However, there could also be false positives - sometimes consumers may think that their data were sold, but in reality, they were not sold.[20] For example, a consumer receiving direct mail may think that one of the food delivery companies they have interacted with sold their address. Nevertheless, there may also be no data sales - it could be that an advertiser is doing direct mail advertising in the entire neighborhood. In this extension, we examine the impact of allowing for false detection on the equilibrium outcome. A key difference between the main model and the previous literature is that the firm's reputation may be hurt even if it takes the optimal action for the consumer in the bad reputation literature, whereas a monopoly always improves its reputation by doing the right thing for the consumer in our paper. By considering the possibility of false detection, we also move the paper closer to the bad reputation literature.

In the presence of false detection, the benefit of not selling the data is lower because the firm's reputation may still be hurt due to the potential false-positive signal. Similarly, the cost of selling the data is lower. So, compared to the main model without the possibility of false detection, a rational firm has a smaller incentive to commit to consumer data protection. Therefore, false detection does not qualitatively change the reputation failure result in the multi-firm case, and we focus on the monopoly case in this extension.

The only difference between this extension and the main model is that the consumer may falsely detect data sales with probability $q' \in (0, q)$ if a firm does not sell the data, corresponding to a false-positive error. The equilibrium depends crucially on the sophistication of consumers. We consider two cases.

[20] Yang et al. (2024) study the implication of false-positive alarms under disinformation detection and show that the possibility of false positives can lead to qualitatively new equilibrium outcomes. We thank an anonymous reviewer for suggesting the possibility of false detection in this setting.

1. Naive consumer: Not aware of the possibility of false-positive signals, consumers update their belief in the same way as in the main model (they think that at least one firm sold their data after observing signal y).

2. Sophisticated consumer: Consumers are fully aware of the possibility of false-positive signals and the likelihood of false detection, and update their belief accordingly (they know that signal y may be false positive).

Proposition 9. *Consider a monopoly.*

1. *(Naive consumer) There exists* $\widehat{\delta} < 1$ *and* $\widehat{q}' > 0$ *such that there exists a MPE in which a rational firm never sells the data without being falsely detected if* $\delta \geq \widehat{\delta}$ *and* $q' \leq \widehat{q}'$*.*

2. *(Sophisticated consumer) There does not exist any MPE in which a rational firm could commit to never selling data without being falsely detected, even when* $\delta \to 1$*.*

If consumers are naive, the detection of data sales, $s = y$, regardless of true positive or false positive, will permanently destroy the firm's reputation. Therefore, there is still a high cost of being caught for selling the data. The false detection rate q' can be viewed as an additional source of noise in monitoring. When the level of this noise is low, a patient enough rational firm can still commit due to the mechanism we identified in the monopoly case of the main model.

If consumers are sophisticated, however, a signal $s = y$ never permanently destroys the firm's reputation because consumers know that the signal may be false-positive when they update their belief about the firm. Anticipating the possibility of false detection, consumers penalize the firm less when observing $s = y$. The penalty for selling data is temporary and can be restored. It moves the firm's incentive closer to the multiple-firm case in the main model. A rational firm loses its commitment power because of the higher incentive to deviate.

8 Conclusion

This paper studies whether reputation consideration can serve as a commitment device for privacy. We show that it depends on the market structure. For a patient enough monopoly, reputation enables it to commit to the Stackelberg action of not selling consumers' data. This is because when consumers observe data sales, they know that the monopoly sold it. So, the firm will never restore its reputation if it is caught selling data. The high and permanent reputation cost strongly incentivizes the monopoly to commit to privacy.

In contrast, reputation may fail to serve as a commitment device when there are multiple firms. The penalty for selling data is temporary when consumers do not know which firm sold the data. In addition, selling data imposes a negative externality on other firms, but each firm does not take the externality into account in equilibrium. We find conditions under which the incentive to deviate is so strong that duopolistic firms lose the ability to commit even if they are arbitrarily close to perfectly patient. Furthermore, the firm is penalized less for selling the data but not rewarded more for not doing so when the number of firms increases. Reputation cannot serve as a commitment device for privacy under any conditions when there are many firms. Reputation failure in the presence of many firms hurts all the firms.

We consider several regulatory interventions and extensions. Reputation failure in the presence of multiple firms persists when the regulator limits the amount of data a firm can sell or when the consumer observes with a low probability the exact identity of data leakage due to media or regulatory exposure. Rational firms restore their commitment power when there are liability fines with good enough regulator monitoring or when the consumer always observes the exact identity of data leakage conditional on detecting data sales. Compared to the main model, a rational firm has a smaller incentive to commit in the presence of false detection. Therefore, the reputation failure result in the multi-firm case still holds when we allow for false-positive signals. In the monopoly case, a patient enough rational firm can still commit to consumer data protection if the false detection rate is low and the consumer is naive, whereas it loses the commitment power if the consumer is sophisticated.

There are a couple of limitations to the current work. Consumers can reveal an arbitrary amount of information in the product market. However, a firm sometimes restricts the communication space. So, the consumer can only choose from a menu of the amount of information to disclose. It will be interesting to study the optimal design of the menu and how much advantage a firm could gain by offering such a contract. Also, the consumer's privacy loss from data sales is exogenous in this paper. Endonenizing the privacy cost in a game theoretic model can provide further insights. We leave these topics for future research.

Appendix

Proof of Proposition 1. The consumer's expected ex-ante payoff by choosing to reveal η proportion of information is:

$$U_0(\eta) = \begin{cases} -\mu_s \eta u_b + \frac{v^2}{4(1-\eta)t}, \ if \ \eta \leq 1 - \frac{v}{t} \\ -\mu_s \eta u_b + \frac{(1-\eta)t}{4}, \ if \ \eta > 1 - \frac{v}{t} \end{cases}$$

$U_0(\eta)$ decreases in η for $\eta > 1-\frac{v}{t}$, so the consumer will not reveal more than $1-\frac{v}{t}$ proportion of information. Consider $\eta \in [0, 1-\frac{v}{t}]$. $dU_0(\eta)/d\eta = -\mu_s u_b + \frac{v^2}{4t(1-\eta)^2}$, which increases in η. So, the optimal η is either 0 or $1-\frac{v}{t}$. $U_0(1-\frac{v}{t}) \geq U_0(0) \Leftrightarrow \mu_s \leq \widehat{\mu}$, where $\widehat{\mu} = \frac{v}{4u_b}$. □

Proof of Proposition 2. By Bayes' rule,

$$\begin{aligned} P(typeB|s=n) =& \frac{P(s=n|typeB)P(typeB)}{P(s=n|typeB)P(typeB) + P(s=n|typeR)P(typeR)} \\ =& \frac{(1-q)\mu_t}{(1-q)\mu_t + [1-\sigma^{f^*}_{sell}(\mu_t)q](1-\mu_t)}, \\ P(typeB|s=y) =& \frac{P(s=y|typeB)P(typeB)}{P(s=y|typeB)P(typeB) + P(s=y|typeR)P(typeR)} \\ =& \frac{\mu_t}{\mu_t + \sigma^{f^*}_{sell}(\mu_t)(1-\mu_t)}. \end{aligned}$$

□

Proof of Corollary 2. Because the rational type does not sell the data, a signal $s = y$ implies the firm is the behavioral type. So, $\mu_{t+1} = 1$. Now consider $s = n$. By Baye's rule,

$$\begin{aligned} \mathbb{P}[type\ B|s=n] &= \frac{\mathbb{P}[s=n|type\ B]\mathbb{P}[type\ B]}{\mathbb{P}[s=n|type\ B]\mathbb{P}[type\ B] + \mathbb{P}[s=n|type\ R]\mathbb{P}[type\ R]} \\ &= \frac{(1-q)\mu_t}{(1-q)\mu_t + 1\cdot(1-\mu_t)} = \frac{1-q}{1-q\mu_t}\mu_t. \end{aligned}$$

By induction, we have $\mu_{t+1} = \frac{(1-q)\mu_t}{(1-q)\mu_t+1-\mu_t}$ after receiving signal n once. Suppose $\mu_{t+k} =$

$\frac{(1-q)^k}{(1-q)^k\mu_t+1-\mu_t}\mu_t$ after receiving signal n for k consecutive periods. After receiving signal n for $k+1$ consecutive periods, we have $\mu_{t+k+1} = \frac{(1-q)\mu_{t+k}}{(1-q)\mu_{t+k}+1-\mu_{t+k}} = \frac{(1-q)^{k+1}}{(1-q)^{k+1}\mu_t+1-\mu_t}\mu_t$. So, it shows that $\mu_{t+k} = \frac{(1-q)^k}{(1-q)^k\mu_t+1-\mu_t}\mu_t$ after receiving signal n for k consecutive periods. One can see that $\frac{(1-q)^k}{(1-q)^k\mu_t+1-\mu_t}\mu_t$ approaches 0 as $k \to +\infty$. □

Proof of Proposition 3. Let $\widehat{k} = \left\lceil \frac{ln\frac{v(1-\mu_0)}{(4u_b-v)\mu_0}}{ln(1-q)} \right\rceil$. We first show that the consumer reveals $\eta = 1 - v/t$ proportion of information after $\widehat{k}$ periods, if the firm never sells the data in equilibrium. By Corollary 2, the belief after not selling data for k consecutive periods is $\mu_k = \frac{(1-q)^k}{(1-q)^k\mu_0+1-\mu_0}\mu_0$. By Proposition 1, consumer reveals $\eta = 1 - v/t$ proportion of information if and only if $\mu_k \le \widehat{\mu} \Leftrightarrow k \ge \frac{ln\frac{v(1-\mu_0)}{(4u_b-v)\mu_0}}{ln(1-q)}$.

We now show that the rational firm has no incentive to deviate to selling data at any time. The game is continuous at infinity because of discounting. So, we can use the single-deviation property. Suppose the firm deviates once at period t when the belief is μ_t. There are two cases.

1. $\mu \le \widehat{\mu}$. The value function of the equilibrium strategy (never sell data) is:

   $V(\mu_t) = (1-\delta)\frac{v}{2}\frac{1}{1-\delta} = \frac{v}{2}$

   The value function of deviating once at period t is (assuming the firm sells data when the belief is 1, which maximizes the payoff):

   $\tilde{V}(\mu_t) = (1-\delta)[\frac{v}{2} + D(1-\frac{v}{t}) + \delta(q\frac{\frac{v^2}{2t}+D(0)}{1-\delta} + (1-q)\frac{\frac{v}{2}}{1-\delta})]$

   The rational firm will not deviate if $V(\mu_t) > \tilde{V}(\mu_t) \Leftrightarrow \frac{\delta}{1-\delta} > \frac{D(1-\frac{v}{t})}{q(v/2-v^2/2t-D(0))}$. One can see that $\exists \delta_1 \in (0,1)$ s.t. the inequality holds for any $\delta \ge \delta_1$.

2. $\mu > \widehat{\mu}$. The value function of the equilibrium strategy (never sell data) is:

   $V(\mu_t) = (1-\delta)[\sum_{k=0}^{\widehat{k}-1}\delta^k\frac{v^2}{2t} + \sum_{k=\widehat{k}}^{+\infty}\delta^k\frac{v}{2}]$

   The value function of deviating once at period t is (assuming the firm sells data when the belief is 1, which maximizes the payoff):

$$\tilde{V}(\mu_t) = (1-\delta)[\tfrac{v^2}{2t} + D(0) + \delta\left(q\frac{\frac{v^2}{2t}+D(0)}{1-\delta} + (1-q)[\textstyle\sum_{k=0}^{\widehat{k}-2}\delta^k\frac{v^2}{2t} + \sum_{k=\widehat{k}-1}^{+\infty}\delta^k\frac{v}{2}]\right)]$$

The rational firm will not deviate if $V(\mu_t) > \tilde{V}(\mu_t) \Leftrightarrow \frac{\delta^{\widehat{k}}}{(1-\delta)[1-(1-q)\delta]} > \frac{D(0)}{q(v/2-v^2/2t)}$. One can see that $\exists\delta_2 \in (0,1)$ s.t. the inequality holds for any $\delta \geq \delta_2$.

Let $\widehat{\delta} = \max\{\delta_1, \delta_2\}$. One can see that $\widehat{\delta} < 1$ and for any $\delta > \widehat{\delta}$, the firm never sells consmer data, $\eta = 0$ in the first $\widehat{k}$ periods, and $\eta = 1 - v/t$ after $\widehat{k}$ periods is a MPE. □

Proof of Proposition 4. By Baye's rule, for a given firm,

$$\mathbb{P}[type\ B|s=n] = \frac{\mathbb{P}[s=n|type\ B]\mathbb{P}[type\ B]}{\mathbb{P}[s=n|type\ B]\mathbb{P}[type\ B] + \mathbb{P}[s=n|type\ R]\mathbb{P}[type\ R]}$$
$$= \frac{(1-q)[1\cdot(1-\mu_t) + (1-q)\mu_t]^{N-1}\mu_t}{(1-q)[1\cdot(1-\mu_t)+(1-q)\mu_t]^{N-1}\mu_t + 1\cdot[1\cdot(1-\mu_t)+(1-q)\mu_t]^{N-1}(1-\mu_t)}$$
$$= \frac{1-q}{1-q\mu_t}\mu_t, \text{ which does not depend on } N.$$
$$\mathbb{P}[type\ B|s=y] = \frac{\mathbb{P}[s=y|type\ B]\mathbb{P}[type\ B]}{\mathbb{P}[s=y|type\ B]\mathbb{P}[type\ B] + \mathbb{P}[s=y|type\ R]\mathbb{P}[type\ R]}$$
$$= \frac{\left[1-(1-q)[1\cdot(1-\mu_t)+(1-q)\mu_t]^{N-1}\right]\mu_t}{\left[1-(1-q)[1\cdot(1-\mu_t)+(1-q)\mu_t]^{N-1}\right]\mu_t + \left[1-1\cdot[1\cdot(1-\mu_t)+(1-q)\mu_t]^{N-1}\right](1-\mu_t)}$$
$$= \frac{1-(1-q)(1-q\mu_t)^{N-1}}{1-(1-q\mu_t)^N}\mu_t, \text{ which decreases in } N \text{ by checking the derivative.}$$

□

Proof of Proposition 5. Consider firm 1 WLOG. The updated belief after one signal is:

$$\begin{cases} \mu^y = \mathbb{P}(\text{firm 1 is bad type}|s=y,\ \text{initial belief is } \mu) = \frac{1-(1-q)(1-q\mu)}{1-(1-q\mu)^2}\mu \\ \mu^n = \mathbb{P}(\text{firm 1 is bad type}|s=n,\ \text{initial belief is } \mu) = \frac{1-q}{1-q\mu}\mu \end{cases}$$

Both μ^y and μ^n increase in μ. $\mu^y \geq 1/2,\ \forall\mu$. Suppose there exists an equilibrium in which rational firms never sells the data. Then consumers have identical beliefs about both firms. Denote the corresponding value function by $V(\cdot)$. Consider a belief $\mu > \widehat{\mu}$. $V(\mu) = (1-\delta)\frac{v^2}{2t} + \delta\left[q\mu V(\mu^y) + (1-q\mu)V(\mu^n)\right]$. The value function of deviating once in the current period is $V_{dev}(\mu) = (1-\delta)\left(\frac{v^2}{2t} + D(0)\right) + \delta\left[q[1+(1-q)\mu]V(\mu^y) + (1-q)(1-q\mu)V(\mu^n)\right]$.

$$V_{dev}(\mu) - V(\mu) = (1-\delta)D(0) - \delta\left[V(\mu^n) - V(\mu^y)\right]q(1-q\mu). \tag{4}$$

Condition (3) ($v/u_b < 2$) $\Rightarrow \widehat{\mu} < 1/2$. $\mu^y \geq 1/2$, $\forall \mu$ implies that consumer will reveal no information after one signal y, which gives the rational firm a stage equilibrium payoff of $\frac{v^2}{2t}$. If the signal is n and $\mu^n \leq \widehat{\mu}$, rational firm gets a stage payoff of $v/2$; If the signal is n and $\mu^n > \widehat{\mu}$, rational firm gets a stage payoff of $\frac{v^2}{2t}$. So, we gets an upper bound of $V(\mu^n)$ by assuming that the belief is always no greater than $\widehat{\mu}$:

$$V(\mu^n) \leq (1-\delta)\left[\frac{v}{2} + \sum_{k=1}^{+\infty}\delta^k[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}]\right] = (1-\delta)\frac{v}{2} + \delta[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}]$$

Always selling consumer data gives a lower bound on the value function:

$$V(\mu^y) \geq (1-\delta)\sum_{k=0}^{+\infty}\delta^k[\frac{v^2}{2t} + D(0)] = \frac{v^2}{2t} + D(0)$$

Hence, $V(\mu^n) - V(\mu^y) \leq (1-\delta)\frac{v}{2} + \delta[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}] - [\frac{v^2}{2t} + D(0)]$. Plug it back to (4),

$$\begin{aligned} &V_{dev}(\mu) - V(\mu) \\ \geq &(1-\delta)[D(0) - \delta q(1-q\mu)\frac{v}{2}] - \delta q(1-q\mu)\left[\delta[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}] - [\frac{v^2}{2t} + D(0)]\right] \end{aligned} \tag{5}$$

With a strictly positive probability, the signal will be y for k consecutive periods, $\forall k$. Denote the belief after k consecutive signal y by μ^{y^k}. One can see that $\mu^y \in (\mu, 1), \forall \mu \in (0,1)$. So, μ^{y^k} strictly increases in k and is bounded by 1. Thus, $\{\mu^{y^k}\}_{k=1}^{+\infty}$ has a limit. Denote the limit by $\mu^{y^{+\infty}}$. We have $(\mu^{y^{+\infty}})^y = \mu^{y^{+\infty}} \Rightarrow \mu^{y^{+\infty}} = 1$. So, μ^{y^k} could be arbitrarily close to 1 with a strictly positive probability. If condition (1) holds ($\Delta u < \overline{\Delta u}$), then $(1-q)(v/2 - v^2/2t) < D(0)$. For large enough δ and μ, we have $\delta[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}] - [\frac{v^2}{2t} + D(0)] < 0$. If condition (2) holds ($q(1-q)v/2 < D(0)$), for large enough δ and μ, we have $D(0) - \delta q(1-q\mu)\frac{v}{2} > 0$. Together, we get that $(1-\delta)[D(0) - \delta q(1-q\mu)\frac{v}{2}] - \delta q(1-q\mu)\left[\delta[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}] - [\frac{v^2}{2t} + D(0)]\right] > 0, \overset{(5)}{\Rightarrow} V_{dev}(\mu) - V(\mu) > 0$. Therefore, rational firm will sell the data when the belief is μ and the discount factor is high enough. A contradiction. □

Proof of Proposition 6. Fix $\delta \in (0,1)$. Suppose $\forall N_\delta,\ \exists N \geq N_\delta$ s.t. there exists a MPE in which a rational firm (label it by firm 1 WLOG) does not sell the data at $t = 0$. Denote the value function of firm 1 by $V_1(\cdot)$. The prior belief is $\vec{\mu}_0 = (\mu_0, \mu_0, ..., \mu_0)$. Denote the posterior belief upon observing signal y (n) by $\vec{\mu}^y$ ($\vec{\mu}^n$) when the initial belief is $\vec{\mu}$ and the equilibrium strategy is σ.

Suppose $\mu_0 > \widehat{\mu}$. $V_1(\vec{\mu_0}) = (1-\delta)\frac{v^2}{2t} + \delta\left[\mathbb{P}(s=y|\sigma^{f^*})V_1(\vec{\mu}_0^y) + \mathbb{P}(s=n|\sigma^{f^*})V_1(\vec{\mu}_0^n)\right]$, where $\mathbb{P}(s=n|\sigma^{f^*}) \leq (1-q\mu_0)^{N-1}$ and $\mathbb{P}(s=y|\sigma^{f^*}) = 1 - \mathbb{P}(s=n|\sigma^{f^*}) \geq 1-(1-q\mu_0)^{N-1}$. The upper bound of the probability of signal n, $\mathbb{P}(s=n|\sigma^{f^*})$, is obtained when no rational firm sells data under σ^{f^*} given belief $\vec{\mu}_0$.

The value function of firm 1 if it deviates once in the first period (denote the strategy by $\sigma^{f'}$) is $V_{1,dev}(\vec{\mu}_0) = (1-\delta)\left(\frac{v^2}{2t} + D(0)\right) + \delta\left[\mathbb{P}(s=y|\sigma^{f'})V_1(\vec{\mu}_0^y) + \mathbb{P}(s=n|\sigma^{f'})V_1(\vec{\mu}_0^n)\right]$, where $\mathbb{P}(s=n|\sigma^{f'}) = (1-q)\mathbb{P}(s=n|\sigma^{f^*})$ and $\mathbb{P}(s=y|\sigma^{f'}) = 1 - \mathbb{P}(s=n|\sigma^{f'})$. Therefore, we have $V_{1,dev}(\mu_0) - V_1(\mu_0) = (1-\delta)D(0) - \delta\left[V_1(\mu_0^n) - V_1(\mu_0^y)\right]q\mathbb{P}(s=n|\sigma^{f^*})$. Because $V_1(\cdot) \in [\frac{v^2}{2t}, \frac{v}{2} + D(1-\frac{v}{t})]$, $V_1(\mu_0^n) - V_1(\mu_0^y) \leq \frac{v}{2} + D(1-\frac{v}{t}) - \frac{v^2}{2t}$, which is a constant. $\mathbb{P}(s=n|\sigma^{f^*}) \leq (1-q\mu_0)^{N-1} \to 0$ $(N \to +\infty)$. Hence, given δ, $\exists N_\delta$ s.t. $\forall N \geq N_\delta,\ V_{1,dev}(\mu_0) - V_1(\mu_0) > 0$. But we assume that firm 1 does not sell the data at $t = 0$. A contradiction.

In sum, firms always sell data and consumers reveal nothing at $t = 0$ in equilibrium. Anticipating that, the consumer's posterior belief about each firm's type is μ_0. This repeats in every period. Therefore, for any $\delta \in (0,1), \exists N_\delta\ s.t.\ \forall N \geq N_\delta$, firms always sell data and consumers reveal nothing in the unique MPE. Consumer's belief about the firm's type is always μ_0.

Suppose $\mu_0 \leq \widehat{\mu}$. The first period payoff will be $\frac{v}{2}$ in equilibrium and $\frac{v}{2} + D(1-\frac{v}{t})$ if the firm deviates. All the remaining proof is the same as above. □

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Online Appendix for Reputation for Privacy

Proof of Proposition 7. We assume WLOG that firm 1 is a rational firm. Denote the regulator's signal about whether firm i sold the data by $s_i^r \in \{+, -\}$, where $s_i^r = +$ means the regulator detects firm i's data sales (the regulator will charge the firm a constant fine f) and $s_i^r = -$ means the regulator does not detect data sales by firm i. The consumer can perfectly infer the regulator's signal according to whether the regulator fines a firm. So, we assume WLOG that the consumer also observes s_i^r to avoid introducing another notation about whether the regulator fines a firm. We consider two cases, distinguished by whether a firm has been fined by the regulator, and show that a rational firm has no incentive to deviate to selling data.

1. A firm has been fined.

   Things are trivial if firm 1 has been fined. Its belief will stay at 1. The consumer will reveal nothing to firm 1 and firm 1 will always sell the consumer data. But, as we will show, firm 1 will not deviate to selling the data in equilibrium, and therefore will not be fined in equilibrium. Now consider the case where firm 2 has been fined and firm 1 has not been fined, which implies that the consumer's belief about firm 2 stays at 1. Denote the current current belief about firm 1 by μ_1. Denote the consumer's belief in the next period after observing signal s and s_1^r by μ_1^{s,s_1^r}.[21] By Bayes' rule,

$$
\begin{aligned}
\mu_1^{y,-} =& \frac{P(s=y, s_1^r=-|firm\ 1\ B)P(firm\ 1\ B)}{P(s=y, s_1^r=-|firm\ 1\ B)P(firm\ 1\ B) + P(s=y, s_1^r=-|firm\ 1\ R)P(firm\ 1\ R)} \\
=& \frac{q(2-q)(1-q_r)\mu_1}{q(2-q)(1-q_r)\mu_1 + q(1-\mu_1)},
\end{aligned}
$$

   where $P(s=y, s_1^r=-|firm\ 1\ B)$

$$
=q^2(1-q_r) + q(1-q)(1-q_r) + (1-q)q(1-q_r) = q(2-q)(1-q_r),
$$

$$
P(s=y, s_1^r=-|firm\ 1\ R) = q.
$$

[21] The signal s_2^r does not affect the consumer's belief because the consumer knows that firm 2 is bad type, $\mu_2 = 1$.

One can check that $q_r > (1-q)/(2-q) \Leftrightarrow \mu_1^{y,-} < \mu_1$. One can also see that $\mu_1^{n,-} < \mu_1^{y,-}$. By induction, the consumer's belief after observing signal $s = y$ and $s_1^r = -$ for k consecutive periods is

$$\mu_1^{y,-^k} = \frac{q^k(2-q)^k(1-q_r)^k\mu_1}{q^k(2-q)^k(1-q_r)^k\mu_1 + q^k(1-\mu_1)}, \tag{6}$$

which decreases in k.

We now show that firm 1 will not deviate from the equilibrium of privacy protection under this circumstances. The game is continuous at infinity because of discounting. So, we can use the single-deviation property.

We first consider the case where $\mu_1 \leq \widehat{\mu}$. By not deviating, firm 1 will never be fined by the regulator. Because $\mu_1^{n,-} < \mu_1^{y,-} < \mu_1 \leq \widehat{\mu}$, the belief about firm 1 will stay below $\widehat{\mu}$ regardless of the consumer's signal s. Firm 1's value function is $V_1(\mu_1, 1) = (1-\delta)[(v/2)/(1-\delta)] = v/2$.

By deviating only once in the current period, firm 1 will be fined by the regulator with probability q_r. Its belief will stay at 1 and the consumer will never reveal anything to it. With the complementary probability, the regulator does not fine firm 1. As shown above, the belief about firm 1 will stay below $\widehat{\mu}$ regardless of the consumer's signal s. In this case, firm 1's value function is $V_{1,dev}(\mu_1, 1) = q_r(1-\delta)\left\{D(1-v/t) + \frac{v}{2} + \delta[v^2/2t + D(0)]/(1-\delta)\right\} + (1-q_r)(1-\delta)[D(1-v/t) + (v/2)/(1-\delta)]$. Therefore,

$$\begin{aligned}
&V_1(\mu_1, 1) - V_{1,dev}(\mu_1, 1) \\
&= q_r v/2 - \delta q_r[v^2/2t + D(0)] - (1-\delta)\{q_r[D(1-v/t) + (v/2)] + (1-q_r)D(1-v/t)\} \\
&> q_r\Delta u - (1-\delta)\{q_r[D(1-v/t) + (v/2)] + (1-q_r)D(1-v/t)\} \\
&> 0\ , \forall \delta > 1 - \frac{q_r\Delta u}{q_r[D(1-v/t) + (v/2)] + (1-q_r)D(1-v/t)}.
\end{aligned}$$

So, a patient enough rational firm will not deviate in this case.

We then consider the case where $\mu_1 > \widehat{\mu}$. The belief about firm 1 after k period is no greater than $\mu_1^{y,-k}$ if firm 1 does not deviate. In addition,

$$\mu_1^{y,-k} < \widehat{\mu} \Leftrightarrow k > \frac{ln[\widehat{\mu}(1-\mu_1)/\mu_1(1-\widehat{\mu})]}{ln[(2-q)(1-q_r)]}. \tag{7}$$

Let $\widehat{k} := \lceil \frac{ln[\widehat{\mu}(1-\mu_1)/\mu_1(1-\widehat{\mu})]}{ln[(2-q)(1-q_r)]} \rceil$. One can see that the belief about firm 1 after $\widehat{k}$ period will be lower than $\widehat{\mu}$ if firm 1 does not deviate. Therefore, similar to the argument in the $\mu_1 \leq \widehat{\mu}$ case, we have $V_1(\mu_1, 1) \geq (1-\delta)[(v^2/2t)(1+\delta+\delta^2+...+\delta^{\widehat{k}-1}) + \delta^{\widehat{k}}(v/2)/(1-\delta)]$ and $V_{1,dev}(\mu_1, 1) \leq q_r(1-\delta)\left\{D(0) + \frac{v^2}{2t} + \delta[v^2/2t + D(0)]/(1-\delta)\right\} + (1-q_r)(1-\delta)[D(0) + (v^2/2t) + \delta(v/2)/(1-\delta)]$.

$$\begin{aligned}
&V_1(\mu_1, 1) - V_{1,dev}(\mu_1, 1) \\
&> \delta^{\widehat{k}} v/2 - q_r\delta[v^2/2t + D(0)] - (1-q_r)\delta \cdot v/2 - (1-\delta)[D(0) + (v^2/2t)] \\
&> q_r \Delta u - (1-\delta^{\widehat{k}})[v/2 + D(0) + v^2/2t] \\
&> 0 \ , \forall \delta > [1 - \frac{q_r \Delta u}{v/2 + D(0) + v^2/2t}]^{1/k}.
\end{aligned}$$

So, a patient enough rational firm will not deviate in this case.

2. No firm has been fined.

The consumer's belief about each firm will be identical in this case. Denote the current belief about each firm by μ. The game is continuous at infinity because of discounting. So, we can use the single-deviation property.

We first consider the case where $\mu < \widehat{\mu}$. By deviating only once in the current period, firm 1's value function is $V_{1,dev}(\mu) \leq q_r(1-\delta)\left\{D(1-v/t) + \frac{v}{2} + \delta[v^2/2t + D(0)]/(1-\delta)\right\} + (1-q_r)(1-\delta)[D(1-v/t) + (v/2)/(1-\delta)]$. If firm 1 does not deviate, its value function is $V_1(\mu) = \mu V_1(\mu|\text{firm 2 is bad}) + (1-\mu)V_1(\mu|\text{firm 2 is good})$.

If firm 2 is good, then no firm sells consumer data without deviation. In such cases, the signal is always $s = n, s_i^r = -$. So, the belief stays below $\widehat{\mu}$ and $V_1(\mu|\text{firm 2 is good}) = (1-\delta)(v/2)/(1-\delta) = v/2$. If firm 2 is bad, for any integer k_1, firm 2 will be detected and fined by the regulator with probability $1-(1-q_r)^{k_1}$ within the first k_1 period. The belief about firm 1 after k_1 period is the highest if the consumer receives $s = y$ in each period and $s_2^r = -$ in the first $k_1 - 1$ period. Denote this upper bound of the belief about firm 1 after k_1 period by $\bar{\mu}_1(k_1, \mu)$, which is strictly lower than 1. Equation (7) implies that the belief about firm 1 will be lower than $\widehat{\mu}$ after another $k_2(k_1, \mu) := \lceil \frac{ln[\widehat{\mu}(1-\bar{\mu}_1(k_1,\mu))/\bar{\mu}_1(k_1,\mu)(1-\widehat{\mu})]}{ln[(2-q)(1-q_r)]} \rceil$ periods if firm 2 has been fined and $\mu_1 \leq \bar{\mu}_1(k_1, \mu)$ after k_1 periods. Note that $k_2(k_1, \mu)$ does not depend on the discount factor δ.

Firm 1 can always gurantee a flow payoff of $(1-\delta)v^2/2t$, and can keep getting a flow payoff of $(1-\delta)v/2$ after $k_1 + k_2(k_1, \mu)$ periods if firm 2 is bad and has been fined within the first k_1 periods. Therefore, firm 1's value function by not deviating is:

$$
\begin{aligned}
&V_1(\mu) \\
=&\mu V_1(\mu|\text{firm 2 is bad}) + (1-\mu)V_1(\mu|\text{firm 2 is good}) \\
\geq&\mu(1-\delta)\left\{[1-(1-q_r)^{k_1}][\frac{v^2}{2t}(1+\delta+...+\delta^{k_1+k_2(k_1,\mu)-1}) + \delta^{k_1+k_2(k_1,\mu)}\frac{v/2}{1-\delta}] + (1-q_r)^{k_1}\frac{v^2/2t}{1-\delta}\right\} + \\
&(1-\mu)(1-\delta)\frac{v/2}{1-\delta}, \\
&V_1(\mu) - V_{1,dev}(\mu) \\
>&\left\{\mu[1-(1-q_r)^{k_1}]\delta^{k_1+k_2(k_1,\mu)} + q_r - \mu\right\}\frac{v}{2} - \delta q_r[v^2/2t + D(0)] - (1-\delta)[D(1-v/t) + q_r v/2] \\
>&\left\{\mu[1-(1-q_r)^{k_1}]\delta^{k_1+k_2(k_1,\mu)} + q_r - \mu\right\}\frac{v}{2} - q_r[v^2/2t + D(0)] - (1-\delta)[D(1-v/t) + q_r v/2] \\
=&q_r\Delta u - \frac{\mu v}{2}\{1-[1-(1-q_r)^{k_1}]\delta^{k_1+k_2(k_1,\mu)}\} - (1-\delta)[D(1-v/t) + q_r v/2], \qquad (*)
\end{aligned}
$$

which holds for any $k_1 \in \mathbb{N}_+$. One can see that $q_r\Delta u - \frac{\mu v}{2}\{1-[1-(1-q_r)^{k_1}] \to q_r\Delta u$ as $k_1 \to +\infty$. So, there exists a $\bar{k}_1 \in \mathbb{N}_+$ such that $q_r\Delta u - \frac{\mu v}{2}\{1-[1-(1-q_r)^{\bar{k}_1}] > q_r\Delta u/2$. Plug in $k_1 = \bar{k}_1$ to $(*)$, we have $V_1(\mu) - V_{1,dev}(\mu) > q_r\Delta u - \frac{\mu v}{2}\{1-[1-(1-q_r)^{\bar{k}_1}]\delta^{\bar{k}_1+k_2(\bar{k}_1,\mu)}\} - (1-\delta)[D(1-v/t)+q_r v/2]$, which approaches $q_r\Delta u - \frac{\mu v}{2}\{1-[1-(1-q_r)^{\bar{k}_1}]\} > q_r\Delta u/2$ as $\delta \to 1$. Because $q_r\Delta u - \frac{\mu v}{2}\{1-[1-(1-q_r)^{\bar{k}_1}]\delta^{\bar{k}_1+k_2(\bar{k}_1,\mu)}\} - (1-$

$\delta)[D(1-v/t)+q_r v/2]$ is continuous, there exists a $\bar{\delta} \in (0,1)$ such that $V_1(\mu)-V_{1,dev}(\mu) > q_r\Delta u - \frac{\mu v}{2}\{1-[1-(1-q_r)^{\bar{k}_1}]\delta^{\bar{k}_1+k_2(\bar{k}_1,\mu)}\} - (1-\delta)[D(1-v/t)+q_r v/2] > q_r\Delta u/4 > 0$ for any $\delta \geq \bar{\delta}$.

So, a patient enough rational firm will not deviate in this case.

The idea for the proof of the case where $\mu_1 > \widehat{\mu}$ is very similar.

One can see that the regulation can improve consumer welfare when it sustains the equilibrium of privacy protection. In terms of social welfare, let the sum of consumer welfare and firm welfare be V_w when there is such a regulation and V_{wo} when there is no regulation. The regulation improves social welfare if $V_w - (1-\delta)\sum_{t=0}^{+\infty}\delta^t c_r > V_{wo} \Leftrightarrow c_r < V_w - V_{wo}$. □

Proof of Proposition 8. Suppose there exists an equilibrium in which a rational firm never sells the data. The belief about a firm will become 1 if it sold data and the consumer identifies its identity. Before consumers identify any identity of data leakage, the consumers have identical beliefs about both firms. Denote the consumer's signal by n if they did not detect any data sales, by y if they caught any of the sales but did not identify any identity of data leakage, by $y,1$ if they caught any of the sales and identify firm 1 selling data, by $y,2$ if they caught any of the sales and identify firm 2 selling data, and by $y,12$ if they caught any of the sales and identify both firms selling data. We assume WLOG that firm 1 is a rational firm and consider two cases.

1. imperfect identification $\phi < 1$

   Consider the circumstance where consumers have not identified any identity of data leakage and the belief about each firm is $\mu > \widehat{\mu}$. Denote by μ^n the belief about both firms after signal n, by μ^y the belief about both firms after signal y, by $\mu^{y,1}$ the belief about firm 2 after signal $y,1$, by $\mu^{y,2}$ the belief about firm 1 after signal $y,2$.[22] Then, the value function of firm 1 is $V_1(\mu,\mu) = (1-\delta)\frac{v^2}{2t} + \delta[(1-\mu q)V_1(\mu^n,\mu^n) + \mu q\phi V_1(\mu^{y,2},1) + \mu q(1-\phi)V_1(\mu^y,\mu^y)]$. The value function of deviating once in the current period is

[22] The belief about firm 1/firm 2/both firms is 1 after signal $y,1/y,2/y,12$.

$V_{1,dev}(\mu) = (1-\delta)[\frac{v^2}{2t} + D(0)] + \delta[(1-q)(1-\mu q)V_1(\mu^n, \mu^n) + \mu q\phi(1-q\phi)V_1(\mu^{y,2}, 1) + q(1-\phi)[1+\mu-\mu q(1+\phi)]V_1(\mu^y, \mu^y) + q\phi(1-\mu q\phi)V_1(1, \mu^{y,1}) + \mu q^2\phi^2 V_1(1,1)]$. Hence, $V_{1,dev}(\mu,\mu) - V_1(\mu,\mu) = (1-\delta)D(0) + \delta\{q(1-\phi)[1-\mu q(1+\phi)]V_1(\mu^y,\mu^y) + q\phi(1-\mu q\phi)V_1(1,\mu^{y,1}) + \mu q^2\phi^2 V_1(1,1) - q(1-\mu q)V_1(\mu^n,\mu^n) - \mu q^2\phi^2 V_1(\mu^{y,2},1)\}$.

By Bayes' rule, the consumer's belief about both firms after observing signal y is:

$$
\begin{aligned}
\mu^y =& \frac{P(signal\ y|firm\ 1\ B)P(firm\ 1\ B)}{P(signal\ y|firm\ 1\ B)P(firm\ 1\ B) + P(signal\ y|firm\ 1\ R)P(firm\ 1\ R)} \\
=& \frac{q(1-\phi)[1+\mu-\mu q(1+\phi)]\cdot\mu}{q(1-\phi)[1+\mu-\mu q(1+\phi)]\cdot\mu + \mu q(1-\phi)\cdot(1-\mu)} \\
=& \frac{1+\mu-\mu q(1+\phi)}{2-\mu q(1+\phi)} \geq 1/2.
\end{aligned}
$$

Condition (3) ($v/u_b < 2$) $\Rightarrow \widehat{\mu} < 1/2$. $\mu^y \geq 1/2$, $\forall\mu$ implies that consumer will reveal no information after one signal y, which gives firm 1 a stage equilibrium payoff of $\frac{v^2}{2t}$. In other cases, firm 1 gets a stage payoff of at most $v/2$. So,

$$
\begin{aligned}
V_1(\mu^n,\mu^n) &\leq (1-\delta)\left[\frac{v}{2} + \sum_{k=1}^{+\infty}\delta^k[\mu q(1-\phi)\frac{v^2}{2t} + [1-\mu q(1-\phi)]\frac{v}{2}]\right] \\
&= (1-\delta)\frac{v}{2} + \delta\left[\mu q(1-\phi)\frac{v^2}{2t} + [1-\mu q(1-\phi)]\frac{v}{2}\right].
\end{aligned}
$$

Because firm 1 gets a stage payoff of at most $v/2$ in equilibrium, $V_1(\mu^{y,2}, 1) \leq v/2$.

Always selling consumer data gives a lower bound on the value function:

$$V_1(\mu^y,\mu^y), V_1(1,\mu^{y,1}), V_1(1,1) \geq (1-\delta)\sum_{k=0}^{+\infty}\delta^k[\frac{v^2}{2t} + D(0)] = \frac{v^2}{2t} + D(0).$$

Therefore,

$$
\begin{aligned}
&V_{1,dev}(\mu,\mu) - V_1(\mu,\mu) \\
\geq&(1-\delta)D(0) + \delta\left\{q(1-\phi)[1-\mu q(1+\phi)] + q\phi(1-\mu q\phi) + \mu q^2\phi^2\right\}[\frac{v^2}{2t} + D(0)] -
\end{aligned}
$$

$$\delta q(1-\mu q)\left\{(1-\delta)\frac{v}{2}+\delta\left[\mu q(1-\phi)\frac{v^2}{2t}+[1-\mu q(1-\phi)]\frac{v}{2}\right]\right\}-\delta\mu q^2\phi^2\frac{v}{2}$$
$$=(1-\delta)[D(0)-\delta q(1-\mu q)\frac{v}{2}]+$$
$$\delta\{\left[q(1-\phi)[1-\mu q(1+\phi)]+q\phi(1-\mu q\phi)+\mu q^2\phi^2\right][\frac{v^2}{2t}+D(0)]-$$
$$q(1-\mu q)\left\{(1-\delta)\frac{v}{2}+\delta\left[\mu q(1-\phi)\frac{v^2}{2t}+[1-\mu q(1-\phi)]\frac{v}{2}\right]\right\}-\mu q^2\phi^2\frac{v}{2}\} \tag{8}$$

With a strictly positive probability, the signal will be y for k consecutive periods, $\forall k$. Denote the belief after k consecutive signal y by μ^{y^k}. One can see that $\mu^y \in (\mu, 1), \forall \mu \in (0,1)$. So, μ^{y^k} strictly increases in k and is bounded by 1. Thus, $\{\mu^{y^k}\}_{k=1}^{+\infty}$ has a limit. Denote the limit by $\mu^{y^{+\infty}}$. We have $(\mu^{y^{+\infty}})^y = \mu^{y^{+\infty}} \Rightarrow \mu^{y^{+\infty}} = 1$. So, μ^{y^k} could be arbitrarily close to 1 with a strictly positive probability. If condition (2) holds ($q(1-q)v/2 < D(0)$), for large enough δ and μ, we have $D(0)-\delta q(1-q\mu)\frac{v}{2} > 0$. So, the term following $(1-\delta)$ in equation (8) is positive. If condition (1) holds ($\Delta u < \overline{\Delta u}$), then $(1-q)(v/2 - v^2/2t) < D(0)$. For large enough δ and μ and small enough ϕ, we have $\left[q(1-\phi)[1-\mu q(1+\phi)]+q\phi(1-\mu q\phi)+\mu q^2\phi^2\right][\frac{v^2}{2t}+D(0)]-q(1-\mu q)\left\{(1-\delta)\frac{v}{2}+\delta\left[\mu q(1-\phi)\frac{v^2}{2t}+[1-\mu q(1-\phi)]\frac{v}{2}\right]\right\}-\mu q^2\phi^2\frac{v}{2} > 0$. So, the term following δ in equation (8) is positive. Together, we get that $V_{1,dev}(\mu,\mu)-V_1(\mu,\mu) > 0$. Therefore, firm 1 will sell the data. A contradiction.

2. perfect identification $\phi = 1$

The game is continuous at infinity because of discounting. So, we can use the single-deviation property. There are four cases.

(a) No identity has been identified, $(\mu_1,\mu_2) = (\mu,\mu)$ and $\mu \leq \widehat{\mu}$.

Firm 1's value function of not deviating is $V_1(\mu,\mu) = (1-\delta)v/2+\delta[(1-\mu q)V_1(\mu^n,\mu^n)+\mu q V_1(\mu^{y,2},1)]$. The value function of deviating once in the current period is $V_{1,dev}(\mu) = (1-\delta)[v/2+D(1-v/t)]+\delta[(1-q)(1-\mu q)V_1(\mu^n,\mu^n)+(1-q)\mu q V_1(\mu^{y,2},1)+q(1-\mu q)V_1(1,\mu^{y,1})+\mu q^2 V_1(1,1)]$. By Bayes' rule, $\mu^n = \mu^{y,2} = (1-q)\mu/(1-q\mu) <$

$\mu \le \widehat{\mu}$. Therefore, the consumer's belief about firm 1 is always lower than $\widehat{\mu}$ if firm 1 does not deivate. So, $V_1(\mu^n, \mu^n) = V_1(\mu^{y,2}, 1) = v/2$. One can also see that $V_1(1, \mu^{y,1}) = V_1(1, 1) = v^2/2t + D(0)$.

Hence, $V_1(\mu, \mu) - V_{1,dev}(\mu, \mu) = -(1-\delta)D(1-v/t) + \delta q \Delta u$, which is positive for δ large enough. Therefore, firm 1 does not deviate.

(b) Firm 2 has been identified as a bad type, $(\mu_1, \mu_2) = (\mu_1, 1)$ and $\mu_1 \le \widehat{\mu}$.

Firm 1's value function of not deviating is $V_1(\mu, \mu) = v/2$. The value function of deviating once in the current period is $V_{1,dev}(\mu) = (1-\delta)[v/2 + D(1-v/t)] + \delta[q(v^2/2t + D(0)) + (1-q)v/2]$. Hence, $V_1(\mu, \mu) - V_{1,dev}(\mu, \mu) = -(1-\delta)D(1 - v/t) + \delta q \Delta u$, which is positive for δ large enough. Therefore, firm 1 does not deviate.

(c) Firm 2 has been identified as a bad type, $(\mu_1, \mu_2) = (\mu_1, 1)$ and $\mu_1 > \widehat{\mu}$.

If firm 1 does not deviate, then the only possible signals are n and $y, 2$. By Bayes' rule, $\mu_1^n = \mu_1^{y,2} = (1-q)\mu_1/(1-q\mu_1)$. Denote the belief after k periods by $\mu_{1,k}$. One can see that $\mu_{1,k}$ strictly decreases in k and is bounded by 0. Thus, $\{\mu_{1,k}\}_{k=1}^{+\infty}$ has a limit. Denote the limit by $\mu_{1,+\infty}$. We have $(\mu_{1,+\infty})^n = \mu_{1,+\infty} \Rightarrow \mu_{1,+\infty} = 0$. Therefore, there exists $\widehat{k} \in \mathbb{N}_+$ such that the belief about firm 1 will be lower than $\widehat{\mu}$ after k periods, for any $k \ge \widehat{k}$.

Firm 1's value function of not deviating is

$$\begin{aligned} V_1(\mu_1, 1) \ge & (1-\delta)\sum_{j=0}^{\widehat{k}} \delta^j \frac{v^2}{2t} + (1-\delta)\delta^{\widehat{k}+1} \sum_{j=0}^{+\infty} \delta^j \frac{v}{2} \\ = & (1-\delta^{\widehat{k}+1})\frac{v^2}{2t} + \delta^{\widehat{k}+1}\frac{v}{2} \end{aligned} \tag{9}$$

The value function of deviating once in the current period is

$$V_{1,dev}(\mu_1, 1) \le (1-\delta)[\frac{v^2}{2t} + D(0)] + \delta\{q[\frac{v^2}{2t} + D(0)] + (1-q)\frac{v}{2}\}$$

Hence,

$$
\begin{aligned}
&V_1(\mu_1, 1) - V_{1,dev}(\mu_1, 1) \\
&\geq -(1-\delta)D(0) + (1-\delta^{\widehat{k}+1})\frac{v^2}{2t} + \delta^{\widehat{k}+1}\frac{v}{2} - \delta q[\frac{v^2}{2t} + D(0)] - \delta(1-q)\frac{v}{2} \\
&= -(1-\delta)D(0) + (1-\delta^{\widehat{k}+1})\frac{v^2}{2t} - \delta(1-\delta^{\widehat{k}})\frac{v}{2} + \delta q\Delta u \\
&\geq -(1-\delta)D(0) - \delta(1-\delta^{\widehat{k}})\frac{v}{2} + \delta q\Delta u,
\end{aligned}
$$

which is positive for δ large enough. Therefore, firm 1 does not deviate.

(d) No identity has been identified, $(\mu_1, \mu_2) = (\mu, \mu)$ and $\mu > \widehat{\mu}$.

Firm 1's value function of not deviating is $V_1(\mu, \mu) = (1-\delta)v^2/2t + \delta[(1-\mu q)V_1(\mu^n, \mu^n) + \mu q V_1(\mu^{y,2}, 1)]$. The value function of deviating once in the current period is $V_{1,dev}(\mu, \mu) = (1-\delta)[v^2/2t + D(0)] + \delta[(1-q)(1-\mu q)V_1(\mu^n, \mu^n) + (1-q)\mu q V_1(\mu^{y,2}, 1) + q(1-\mu q)V_1(1, \mu^{y,1}) + \mu q^2 V_1(1,1)] = (1-\delta)[v^2/2t + D(0)] + \delta\{(1-q)(1-\mu q)V_1(\mu^n, \mu^n) + (1-q)\mu q V_1(\mu^{y,2}, 1) + q[v^2/2t + D(0)]\}$.

So, $V_1(\mu, \mu) - V_{1,dev}(\mu, \mu) = -(1-\delta)D(0) + \delta q(1-\mu q)V_1(\mu^n, \mu^n) + \delta\mu q^2 V_1(\mu^{y,2}, 1) - \delta q[v^2/2t + D(0)]$. Because $\mu^{y,2} = (1-q)\mu/(1-q\mu) < \mu$, the same argument as in equation (9) implies that $V_1(\mu, 1) \geq (1-\delta^{\widehat{k}+1})v^2/2t + \delta^{\widehat{k}+1}v/2$. Also, $V_1(\mu^n, \mu^n) \geq v^2/2t + D(0)$. Therefore,

$$
\begin{aligned}
V_1(\mu, \mu) - V_{1,dev}(\mu, \mu) &\geq -(1-\delta)D(0) + \delta\mu q^2[(1-\delta^{\widehat{k}+1})\frac{v^2}{2t} + \delta^{\widehat{k}+1}\frac{v}{2} - \frac{v^2}{2t} - D(0)] \\
&= -(1-\delta)D(0) + \delta\mu q^2[\delta^{\widehat{k}+1}\Delta u - (1-\delta^{\widehat{k}+1})D(0)],
\end{aligned}
$$

which is positive for δ large enough. Therefore, firm 1 does not deviate.

□

Proof of Proposition 9. Suppose there exists an equilibrium in which a rational firm never sells the data without being falsely detected. Denote the current belief about the firm by μ.

Consider the incentive of a rational firm.

1. Naive consumer

   In this case, consumers are not aware of the possibility of false-positive signals, and update their belief in the same way as in the main model. According to the proof of Corolloary 2, $\mu^n = P(typeB|s = n) = (1-q)\mu/(1-q\mu), \mu^y = P(typeB|s = y) = 1$. The game is continuous at infinity because of discounting. So, we can use the single-deviation property. There are two cases.

   (a) $\mu \leq \widehat{\mu}$. The value function of the equilibrium strategy is: $V(\mu_t) = (1-\delta)\frac{v}{2} + \delta[(1-q')V(\mu^n) + q'V(\mu^y)]$. The value function of deviating once in the current period is (assuming the firm sells data when the belief is 1, which maximizes the payoff): $V_{dev}(\mu) = (1-\delta)[\frac{v}{2} + D(1-v/t)] + \delta[(1-q)V(\mu^n) + qV(\mu^y)]$.

$$V_{dev}(\mu) - V(\mu) = (1-\delta)D(1-v/t) - (q-q')\delta[V(\mu^n) - V(\mu^y)] \qquad (10)$$

$$V(\mu^y) = v^2/2t + D(0)$$

$$V(\mu^n) = (1-\delta)v/2 + \delta[q'(v^2/2t + D(0)) + (1-q')V(\mu^{nn})]$$

$$= (1-\delta)v/2 + \delta[q'(v^2/2t + D(0)) + (1-q')V(\mu^n)] \qquad (11)$$

$$\Rightarrow V(\mu^n) = \frac{(1-\delta)v/2 + \delta q'[v^2/2t + D(0)]}{1-\delta(1-q')} \qquad (12)$$

$$\Rightarrow V(\mu^n) - V(\mu^y) = \frac{1-\delta}{1-\delta(1-q')}\Delta u$$

$$\overset{(10)}{\Rightarrow} V_{dev}(\mu) - V(\mu) = (1-\delta)\left[D(1-v/t) - \frac{\delta(q-q')\Delta u}{1-\delta(1-q')}\right]$$

$$\Rightarrow V_{dev}(\mu) - V(\mu) < 0 \Leftrightarrow \delta > \delta_1 := \frac{D(1-v/t)}{(q-q')\Delta u + (1-q')D(1-v/t)},$$

$$\text{which is less than 1 if } \; q' < q_1' := \frac{q\Delta u}{D(1-v/t) + \Delta u}.$$

   Equation (11) holds because both μ^n and μ^{nn} are lower than $\widehat{\mu}$, and a single signal $s = y$ moves the belief to 1 permanently, and thereby $V(\mu^{nn}) = V(\mu^n)$.

   Therefore, a rational firm will not deviate if the discount factor is high enough,

$\delta > \delta_1$, and the false detection rate is low enough, $q' < q'_1$.

(b) $\mu > \widehat{\mu}$. Similar to the previous case, we have

$$V_{dev}(\mu) - V(\mu) = (1-\delta)D(0) - (q-q')\delta[V(\mu^n) - V(\mu^y)]. \tag{13}$$

According to Corolloary 2, $\mu^{n^k} = (1-q)^k\mu/[(1-q)^k\mu + 1 - \mu]$, which implies that $\mu^{n^k} < \widehat{\mu} \Leftrightarrow k > ln[\widehat{\mu}(1-\mu)/\mu(1-\widehat{\mu})]/ln(1-q)$. Let $\widehat{k}(\mu) = \lceil ln[\widehat{\mu}(1-\mu)/\mu(1-\widehat{\mu})]/ln(1-q)\rceil$. One can see that the belief after $\widehat{k}(\mu)$ period will be lower than $\widehat{\mu}$ if the rational firm does not deviate and there is no false detection. Because the belief either keeps decreasing or jumps to 1 and stays there forever, we have $\widehat{k}(\mu) \leq \widehat{k}(\mu_0)$. Let $\widehat{k}_0 = \widehat{k}(\mu_0)$.

$$\begin{aligned}
V(\mu^n) \geq &(1-\delta)\frac{v^2}{2t}(1+\delta+...+\delta^{\widehat{k}_0}) + [1-(1-q')^{\widehat{k}_0+1}]\delta^{\widehat{k}_0+1}[\frac{v^2}{2t}+D(0)] + \\
&(1-q')^{\widehat{k}_0+1}\delta^{\widehat{k}_0+1}\left\{(1-\delta)\frac{v}{2} + \delta[(1-q')V(\mu^{n^{\widehat{k}_0+2}}) + q'\left(\frac{v^2}{2t}+D(0)\right)]\right\} \\
V(\mu^y) =&v^2/2t + D(0) \\
\Rightarrow \quad &[V(\mu^n) - V(\mu^y)]/(1-\delta) \\
\geq &(1-q')^{\widehat{k}_0+1}\delta^{\widehat{k}_0+1}\Delta u + \frac{(1-q')^{\widehat{k}_0+2}\delta^{\widehat{k}_0+2}\Delta u}{1-\delta(1-q')} - (1+\delta+...+\delta^{\widehat{k}_0})D(0) \\
\geq &(1-q')^{\widehat{k}_0+1}\delta^{\widehat{k}_0+1}\Delta u + \frac{(1-q')^{\widehat{k}_0+2}\delta^{\widehat{k}_0+2}\Delta u}{1-\delta(1-q')} - (\widehat{k}_0+1)D(0) \\
\overset{(13)}{\Rightarrow} \quad &[V_{dev}(\mu) - V(\mu)]/(1-\delta) \\
\leq &D(0) - (q-q')\delta\left[(1-q')^{\widehat{k}_0+1}\delta^{\widehat{k}_0+1}\Delta u + \frac{(1-q')^{\widehat{k}_0+2}\delta^{\widehat{k}_0+2}\Delta u}{1-\delta(1-q')} - (\widehat{k}_0+1)D(0)\right].
\end{aligned}$$

Denote the above upper bound of $[V_{dev}(\mu) - V(\mu)]/(1-\delta)$ by $J(q',\delta)$. One can see that $J(q',\delta)$ is continous, increases in q', decreases in δ, and approaches $I(q') := D(0) - (q-q')[(1-q')^{\widehat{k}_0+1}\Delta u + [(1-q')^{\widehat{k}_0+2}\Delta u]/q' - (\widehat{k}_0+1)D(0)]$ as $\delta \to 1$. Because $I(q') \to -\infty$ as $q' \to 0^+$, there exists a $q'_2 > 0$ such that $I(q') < -2$, $\forall q' \leq q'_2$.

Consequently, there exists $\delta_2 < 1$ such that $J(q'_2, \delta) < -1, \ \forall \delta \geq \delta_2$. Because $J(q', \delta)$ increases in q', $J(q', \delta) \leq J(q'_2, \delta), \ \forall q' \leq q'_2$. Therefore, $J(q', \delta) < -1 < 0$ if $\delta \geq \delta_2$ and $q' \leq q'_2$.[23]

Therefore, a rational firm will not deviate if the discount factor is high enough and the false detection rate is low enough.

In sum, let $\widehat{\delta} = \max\{\delta_1, \delta_2\}, \ \widehat{q'} = \max\{q'_1, q'_2\}$. Then, there exists an equilibrium in which a rational firm never sells the data without being falsely detected if $\delta \geq \widehat{\delta}$ and $q' \leq \widehat{q'}$.

2. Sophistcated consumer

In this case, consumers are fully aware of the possibility of false-positive signals and the likelihood of false detection, and update their belief accordingly. By Bayes' rule,

$$\begin{aligned}
\mu^n = P(typeB|s=n) =& \frac{P(s=n|typeB)P(typeB)}{P(s=n|typeB)P(typeB) + P(s=n|typeR)P(typeR)} \\
=& \frac{(1-q)\mu}{(1-q)\mu + (1-q')(1-\mu)}, \\
\mu^y = P(typeB|s=y) =& \frac{P(s=y|typeB)P(typeB)}{P(s=y|typeB)P(typeB) + P(s=y|typeR)P(typeR)} \\
=& \frac{q\mu}{q\mu + q'(1-\mu)}.
\end{aligned}$$

By induction, the belief after observing k consecutive signal y is $\mu^{y^k} = q^k\mu/[q^k\mu + q'^k(1-\mu)]$, which increases in k. Hence,

$$\mu^{y^k} < \widehat{\mu} \Leftrightarrow k < \frac{ln[\mu(1-\widehat{\mu})/\widehat{\mu}(1-\mu)]}{ln(q'/q)}.$$

Let $\widehat{k}(\mu) := \lfloor \frac{ln[\mu(1-\widehat{\mu})/\widehat{\mu}(1-\mu)]}{ln(q'/q)} \rfloor$. One can see that the belief within the first $\widehat{k}(\mu)$ periods is always lower than $\widehat{\mu}$ along any possibile history.

Consider any belief $\mu < \widehat{\mu}$. The value function of a rational firm is $V(\mu) = (1-\delta)v/2+$

[23] Note that the values of q'_2 and δ_2 do not depend on each other.

$\delta[q'V(\mu^y) + (1-q')V(\mu^n)]$. The value function of deviating once in the current period is $V_{dev}(\mu) = (1-\delta)[v/2 + D(1-v/t)] + \delta[qV(\mu^y) + (1-q)V(\mu^n)]$. Therefore,

$$\begin{aligned}
V_{dev}(\mu) - V(\mu) &= (1-\delta)D(1-v/t) - (q-q')\delta[V(\mu^n) - V(\mu^y)] \qquad (14)\\
V(\mu^y) &\geq (1-\delta)\frac{v}{2}[1+\delta+...+\delta^{\widehat{k}(\mu)}] = (1-\delta^{\widehat{k}(\mu)+1})\frac{v}{2}\\
V(\mu^n) &\leq v/2\\
\Rightarrow V(\mu^n) - V(\mu^y) &\leq \delta^{\widehat{k}(\mu)+1}\frac{v}{2}\\
\overset{(14)}{\Rightarrow} V_{dev}(\mu) - V(\mu) &\geq (1-\delta)D(1-v/t) - (q-q')\delta^{\widehat{k}(\mu)+2}\frac{v}{2},\\
\Rightarrow V_{dev}(\mu) - V(\mu) > 0 \Leftrightarrow \widehat{k}(\mu) &> ln\frac{2(1-\delta)D(1-v/t)}{(q-q')v}/ln\delta - 2 \qquad (15)
\end{aligned}$$

Because $\widehat{k}(\mu)$ increases in μ and $\lim_{\mu\to 0^+}\widehat{k}(\mu) = +\infty$, for any $\delta \in (0,1)$, there exists a $\mu \in (0,\widehat{\mu})$ such that condition (15) holds. Hence, the firm has an incentive to deviate and there does not exist an equilibrium in which a rational firm never sells the data.

□