Search Fatigue, Choice Deferral, and Closure *

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Abstract

When gathering information to make decisions, individuals often have to delay making a decision because the process of gathering information is interrupted, and the individual is not yet ready to make a decision. The paper considers a model of choice deferral based on time-varying search costs, potentially based on search fatigue, in which individuals have to strategically decide whether to defer choice when information gathering is interrupted, taking into account the current available information, and when they will be able to resume gathering information. We find that individuals are more likely to defer choice when information gathering is interrupted less frequently, when individuals can resume gathering information sooner, and when they discount less the future. We also consider the case in which individuals incur costs of re-starting a process of information gathering, and cases in which the individual has greater or less information about the extent of search fatigue. The paper also considers optimal pricing, user interface design, and retargeting decisions, and shows how they should respond to the length of consumer browsing sessions, and gaps between browsing sessions. The paper illustrates the importance of modeling fatigue and interruptions in the search process.

1. Introduction

When gathering information to make decisions, an individual often has to delay making a decision because the process of gathering information is interrupted, and the individual is not yet ready to make a decision. This interruption can be caused by time-varying search costs, potentially based on search fatigue. When information gathering is interrupted, individuals have to strategically decide whether to make a choice based on current available information or defer choice until they have a chance to gather further information. Choice deferrals often occur in the health, food, financial, entertainment, and general consumption domains.

Consumers may search for information extensively before making a purchase, and there can be multiple interruptions to the search process as consumers engage in non-shopping-related activities such as answering phone calls or clicking on online ads and browsing information about other products. For example, when shopping for digital cameras, consumers, on average, search for information over 6 browsing sessions that span 15 days before making a purchase (Bronnenberg, Kim, and Mela 2016). The interruption could be based on search fatigue increasing the search costs, consumers being distracted during the search, or because the consumer has to perform another task, which temporarily increases the opportunity costs of searching for product information (Li, Capra, and Zhang 2020). Consumers then restart their search processes when they recover from search fatigue or when opportunity costs decrease. A research survey by Autobytel, a large internet automotive marketing services company, shows that more than 70% of US online searchers have experienced search engine fatigue, which drives them to distraction during car searches.[1] Consumers become impatient or frustrated during the search, and many of them leave their computers without finding the information. Related phenomena such as decision fatigue and daily deal fatigue have also raised significant attention from the popular press and companies.[2]

There is evidence of consumers' time-varying search costs: Using individual-level online click data, Koulayev (2014) and Ursu, Zhang, and Honka (2023) find that consumers' search costs increase as they search more. In addition, Ursu, Zhang, and Honka (2023) find that time-varying search costs have significant impact on the number of searches and the choice probability. They find, additionally, that a fatigue reduction (the time-varying part of the search cost) has a larger impact on market outcomes than a base search cost reduction (the time-invariant part of the search cost). The impact of time-varying search costs also depends on the consumer's likelihood of returning to search after an interruption, as indicated by the finding that larger and more popular websites (to which consumers are more likely to return after interruption) suffer less from high fatigue levels. The patterns of search fatigue and interruptions can also vary across channels and

[1] Source: https://www.marketingcharts.com/industries/automotive-industries-2009.

[2] Coverage of decision fatigue: https://www.optimizely.com/optimization-glossary/decision-fatigue/; and daily deal fatigue: https://www.foxbusiness.com/features/dont-fall-victim-to-daily-deal-fatigue.

product categories. For example, in e-commerce, consumers spend less time and view fewer pages per browsing session on mobile devices than on desktop computers, and each session is less likely to result in a purchase.[3] These statistics also vary across industries.[4] Thus, it is crucial for firms to understand how search fatigue and interruption affect consumer behavior.

When a decision-maker's search process is interrupted without having yet sufficient diagnostic information, the decision-maker has to decide whether to defer the choice. The decision-maker can use the available information to decide right away, or delay the decision until a future time when the decision-maker is again able to search for information. For example, the consumer starts gathering information about a product online and then receives a phone call from the boss about some new work task. The consumer can choose to either purchase the product right away given the available information, or delay choice until the consumer has again a chance to look for information after finishing the phone call and potentially the new tasks. Interruptions can also occur in offline shopping, when after shopping for a while the consumer may have to leave the store at some point, without having made a decision.

These interruptions to a decision-maker's search process are largely ignored in models of consumer search. One possible reason is that, while it is obvious that consumers also spend time on non-consumer-related activities, it is not obvious that explicitly modeling a consumer's time spent on non-consumer-related activities provides meaningful implications for the consumer's search and purchase behavior. In order to consider the possibility of search interruption and choice deferral, we formulate a model in which an individual gathers information gradually to decide whether to adopt an alternative. The individual can be either in a state of low search costs or a state of high search costs, and move across states at some hazard rate. In the consumer setting, these hazard rates could be relatively high for online shopping as consumers could frequently start and stop browsing sessions, but be lower for offline shopping, especially if it is a store that is of difficult access. To simplify the analysis, we consider that the individual has zero search costs in the low search costs state, and very high search costs in the high search costs state. Thus, the individual gathers information when she is in the low search costs state, and prefers not to gather information when she is in the high search costs state.

When gathering information (that is, in the low search costs state), the individual makes a choice if the individual obtains sufficiently diagnostic information. Suppose an individual has not gathered sufficient information to make a choice before the moment when the search costs increase, then at that moment, the individual has two options. The individual can either defer choice until she can resume gathering information at low search costs, or make an immediate choice using current information.

[3]Source: https://www.semrush.com/blog/mobile-vs-desktop-usage/
[4]Source: https://databox.com/google-analytics-4-industry-benchmarks

In order to obtain strategic effects at the time when the search process is interrupted, we consider that the individual either discounts the future or has a fixed cost of re-starting the search process. Either of these possibilities leads the individual, at the time when the search costs increase, to potentially decide not to defer choice, even though the individual has not made a choice up to that point, a phenomenon which is termed in the paper as choice closure.[5] The individual may decide to make a choice then (i.e., choice closure), because, even though the individual has not yet received sufficient positive information, the current evaluation is close enough such that it is better to make the choice now, than to wait for the search costs to come down again. If there is neither discounting nor fixed costs of re-starting the search, individuals would just defer choice automatically when the search costs increase. The strategic decision between choice deferral and choice closure is a novel result of the paper.

We find that individuals are more likely to defer choice when information gathering is interrupted less frequently, when individuals can resume gathering information sooner, and when they discount less the future. When individuals can resume gathering information sooner, or when individuals discount less the future, future information becomes in expectation more valuable when evaluated at the time when the deferral decision is made, and so individuals defer more choice. When information gathering is interrupted more frequently, individuals know that when they resume gathering information, they will be again interrupted quickly, therefore leading to a lower payoff from deferring choice. Seen the other way, individuals do more choice deferral when information gathering is interrupted less frequently. In terms of choice closure, we find that the extent of choice closure increases when information gathering is interrupted less frequently, when interruptions last longer, and when the individuals discount the future less.

If the interruption to information gathering is caused by search fatigue, then as the individual does more search, the individual may be aware that she is getting more tired of search over time. To capture this, we consider an extension with three states. The individual moves from the fully-rested search state to the fatigued search state, from the fatigued search state to the no-search state, and then from the no-search state back to the fully-rested search state. The individual expects information gathering to be interrupted sooner in the fatigued search state than in the fully-rested search state, reflecting her awareness of search fatigue. We show that the individual's choice deferral and choice closure behaviors are similar to those we find in the main model. We also find that the individual requires less positive information to make a choice in the fatigued search state than in the fully-rested search state, and the extent of this reduction is greater when the rate of fatigue is higher and when the individual discounts the future less.

We also consider the case in which individuals incur costs of re-starting a process of information

[5]For a related concept, see, for example, Webster and Kruglanski (1994), Choi et al. (2008).

gathering.[6] In that case, we do not need discounting for the choice deferral decision to be strategic. This case could be important empirically as the extent of time discounting between different opportunities to gather information may be relatively small (for example, days). The existence of costs of re-starting the information gathering phase can be seen as a possibility that leads to significant strategic effects at the time when the choice deferral decision is made. The model endogenously generates a distribution of consumer behavior for each search session, from buying before being fatigued, choice closure, and choice deferral, to quitting the search process. We also discuss how the model can be applied empirically.

We also derive a firm's optimal pricing strategy given the individuals' choice deferral behavior. If the initial expected value of adopting the alternative is low, the firm sets a price such that the consumer does not adopt it before gathering some information. In such a case, we find that the optimal price should be higher when the speed of information gathering is greater, when information gathering is interrupted less frequently, when the individuals can resume information gathering sooner, and when the individuals discount the future less. We also find that these comparatives statics are reversed if the initial expected value of adopting the alternative is sufficiently high. These results show how firms should use data on consumer browsing sessions to determine price, and provide managerial implications on how price should change following other interventions to reduce search fatigue or restart consumer search sooner, such as redesigning user interface, ad retargeting, email marketing, and push notifications.

Explicitly modeling search fatigue and interruptions also allows us to capture a new type of pricing strategy. When the initial expected value of adopting the alternative is in an intermediate range, the firm does not want consumers to buy without search, because the firm would have to charge too low a price for consumers to do so. The firm also does not want consumers to search for too long, because delaying the purchase is costly when the initial expected value of adopting the alternative is not too low. The firm optimally charges a price such that consumers would search initially, but would not defer choice when search costs increase as long as the expected value of adopting the alternative is close to the initial value. Thus, the firm takes advantage of the consumers' choice closure behavior to incentivize limited amount of search. Such pricing strategy does not exist in the benchmarks that do not explicitly model search fatigue and interruptions.

In addition to pricing, we also consider other managerial decisions that affect consumers' search environment. In particular, we study user interface design and retargeting. Firms can design the user interface to make the search process more or less likely to be interrupted. Existing research has made valuable contributions to understanding the impact of search frictions on firm profits. Ursu, Zhang, and Honka (2023) shed light on how an increased search fatigue can present challenge for firms, whereas Ngwe, Ferreira, and Teixeira (2019) identify potential benefits from higher search

[6] See Byrne and de Roos (2022) on evidence on the existence of start-up search costs.

frictions. By considering both choice deferral and choice closure, our approach aims to integrate these perspectives and offers a more comprehensive understanding of these varied findings. On the one hand, a higher rate of search fatigue keeps the consumer in the search mode for a shorter period, which is bad for the firm. On the other hand, it incentivizes the consumer to adopt the alternative more easily because of choice closure, which is good for the firm. We characterize when firms prefer a higher rate of search fatigue and when they prefer a lower level of search fatigue. A similar mechanism plays a critical role in firms' retargeting decisions. We show that, counter-intuitively, retargeting may backfire and hurt the firm even if it is costless, because it may reduce the positive effects of consumers' choice closure behaviors.

There is substantial work documenting the existence of choice deferrals by individuals, because of the inability to make a decision (see Anderson 2003, Chernev, Böckenholt, and Goodman 2015, Scheibehenne, Greifeneder, and Todd 2010, for reviews). This work has characterized the causes for choice deferral, and its consequences. For example, this work has investigated the role of dominance relations, option desirability, attribute commonality, and attribute alignability on choice deferral (e.g., Chernev and Hamilton 2009, Dhar 1997, Gourville and Soman 2005, Tversky and Shafir 1982), and that the option of choice deferral may affect individual choices and affect behavioral effects (e.g., Dhar and Simonson 2003). Bhatia and Mullet (2016) consider a sequential learning model with the possibility of choice deferral which provides an explanation for several of the behavioral effects obtained. A significant explanation for not choosing has been choice overload, the existence of too many options may deter choice, which can also been seen as deferral of choice. Examples of work providing explanations for this effect of choice overload include Kamenica (2008), Kuksov and Villas-Boas (2010), Villas-Boas (2009). In this paper, the existence of multiple alternatives is not going to play any role, and the decision of choice deferral comes from the difficulty of the decision being made, and from time-varying search costs (or, alternatively, time-varying information gained). In contrast, much less attention has been paid to choice closure, which speeds up consumers' decision-making when search costs increase. We show that this choice closure effect is important in guiding a firm's pricing and search intervention decisions.

In relation to the existing literature, a significant innovation of this paper is to formally consider future choice opportunities once choice is deferred. That is, while in the existing literature choice deferral is considered as no choice, here we formally consider the possibility of future choices when the individuals have again a chance to search for information. This formulation allows us to study choice deferral and choice closure as strategic decisions, and as applications, our study shows how optimal pricing, user interface design, and retargeting decisions in e-commerce should depend on the lengths of consumer browsing sessions and the gaps between browsing sessions.

The remainder of the paper is organized as follows. The next section introduces a base model of choice deferral with discounting. Section 3 presents the analysis and results of the consumer's search problem. Section 4 discusses optimal pricing. Section 5 examines marketing activities that affect the search environment. Section 6 considers two extensions to the main model, taking into account the effect of consumer awareness of search fatigue and the possibility of start-up search costs. Section 7 concludes. The online appendix collects the proofs of the results.

2. The Model

A decision-maker (DM) is gradually collecting information about whether to adopt an alternative. Suppose time is continuous. The DM can be either in a "search" mode or in a "no-search" mode. In the search mode, the DM has zero search costs, while in the no-search mode, the DM's search costs are sufficiently high such that the DM does not search for information.

Whether the DM is in the "search" or in the "no-search" mode is exogenous. If the DM is in the search mode, the DM moves to the no-search mode with a constant hazard rate of λ. If the DM is in the no-search mode, the DM moves to the search mode with a constant hazard rate of β. When the DM is in the search mode, the DM updates the expected value of adopting the alternative, and can choose to adopt the alternative at any time. In the no-search mode, the DM does not receive any information. At the instance when the DM moves from the search mode to the no-search mode, if the DM's beliefs about the alternative are not sufficiently high, the DM may choose to defer choice until the DM is again in the search mode.

This set-up captures the idea that the DM sometimes has the ability to search for information, and other times cannot search for information. This can also be interpreted as search fatigue, as the DM suddenly has high search costs after some periods of information gathering, stops getting information on the alternative, and decides to delay making a choice until the DM has again a chance to learn more information about the alternative (the DM gets sufficiently rested such that the DM returns to the search mode). Another interpretation is that, instead of higher search costs, search fatigue makes additional search uninformative, so the DM has to rest for some periods before gathering information again.

At each moment in time, the DM has some expected value of the payoff of the alternative, which we denote by x. The initial value x_0 is exogenous and summarizes all the information the DM has before searching. It can come from past experiences or word of mouth. With the increasingly rich data about individuals and marketing analytics tools, firms may potentially gain some information on x_0.[7] When the DM is in the search mode, x evolves as a Brownian motion with a constant variance σ^2. This can be interpreted as the DM learning over time about equally important and independent attributes, and there being an infinite number of attributes (e.g., Branco, Sun, and

[7] We take x_0 as exogenous in the analysis. If the firm has private information on the value of the product, the firm could potentially signal some average value of the product (i.e., x_0) through its market actions (e.g., price). Exploring this is beyond the scope of the paper.

Villas-Boas 2012).[8] When the DM is in the no-search mode, the expected value, x, stays fixed (as no information is gained). The payoff of not adopting the alternative is set at zero. The DM discounts the future at a continuous-time discount rate r and does not incur any on-going search costs when learning information. The discount rate can also be seen as the rate at which the alternative disappears. For example, a consumer considering purchasing a product may find the product out of stock, or a manager considering launching a product may find that the opportunity has passed. Table 1 presents the notation used throughout the paper.

Random Switching Between Search Modes

The main analysis considers random switching between search modes, as random switching captures the main effects at play and some of the potential uncertainty of when search interruptions occur, and facilitates the analysis. The case with no random switching between search states, which can be considered numerically, is presented in the online appendix.

There may be uncertainty about when the DM will be interrupted from searching due to fatigue or distractions. If we interpret the switching from the search mode to the no-search mode as search fatigue leading to the DM stopping to search, the DM could be endowed with a search fatigue limit when starting a search process, but would not know when that search fatigue limit is. With a constant hazard rate, the process is memoryless, and therefore, from the point of view of the DM, she gets search fatigue with the same likelihood, independent of how long the DM has been searching. This also fits with the interpretation in which the DM may be distracted by a phone call or online ads about other products. From the DM's point of view, it may be hard to know when she will be interrupted. So, the switching time can be seen as possibly random.

The existence of constant hazard rates of moving between the search and no-search mode allows the problem to be stationary so that the threshold of whether to adopt the alternative is constant over time. This helps to keep the model tractable. If the hazard rates of moving between the search mode and the no-search mode are not constant, then the thresholds of whether to adopt the alternative would also not be constant, leading to significant complications in the analysis (it could still be characterized numerically, but analytical results would be difficult to obtain). For example, if the DM understands that she is getting more fatigued over time from search, we would expect the hazard rate of moving from the search mode to the no-search mode to be increasing in the length of time that the DM has been in the search mode. This would lead the threshold to adopt the alternative to vary over time (in fact, to decrease with the length of time in the search mode). Similarly, we could expect the hazard rate of moving from the no-search mode to the search mode to be increasing in the length of time spent in the no-search mode, because a longer rest from search

[8] Alternatively, this case could be seen as the limit case when there is a finite but large number of attributes.

should lead to a greater likelihood of returning to search for information again. This possibility would not affect the results presented here, as the DM would prefer to continue waiting until the switch to the search mode, as that switch is expected to be sooner.

The two-search modes extension in Section 6 accommodates the case in which, rather than complete random switching, switching from the search mode to the no-search mode is more likely as the time in the search modes increases. We allow the DM to be aware of her increased fatigue over time by introducing an additional search mode. The DM switches first from the fully-rested search mode 1 to the fatigued search mode 2, and then to the no-search mode. Therefore, the switching from the search mode to the no-search mode is no longer stationary. As the DM searches for some time and has switched to the second search mode, she knows that she will be interrupted and not able to search sooner.

Other Assumptions and Extensions

If learning is done with signals about the overall value of the product, or if attributes have unequal importance with the DM checking first the most important attributes, or if there is non-zero correlation between the attributes, then we would have σ^2 to be decreasing over time, leading again to a threshold to adopt the alternative that is varying (decreasing) over time, which is a more complicated case to consider. The case presented here can be seen as the extreme case if the amount of information learned over time is constant, in contrast to the other extreme case in which all information about the alternative is learned in one shot. The real world would be somewhere between these two extreme cases.

The base model assumes an infinite horizon, so the DM can search indefinitely. In reality, the DM may face a deadline such that the decision becomes obsolete afterward. For example, a consumer shopping for a digital camera for an upcoming trip has to make a decision before the start of the trip. The existence of a deadline again makes the problem non-stationary, with adoption thresholds varying (decreasing) over time. We analyze this case numerically in the online appendix.

Note that discounting is crucial for the problem as presented. If there is no discounting, the DM would always defer choice when moving to the no-search mode, and choice deferral becomes non-strategic. One alternative to discounting is to have start-up search costs each time the search mode starts, and that case is considered in Section 6.

The base model assumes that that there are no ongoing search costs. If there are ongoing search costs when learning for information, then the DM would also have another threshold such that the DM permanently leaves the search process without adopting when the expected payoff of adopting the alternative drops below the threshold. We do not consider this case in the base model to simplify the analysis, as this case is not essential to obtain the strategic choice deferral effects. The ongoing search costs and the quitting threshold are considered in Section 6.

The assumption that the payoff of not adopting the alternative is zero is not without loss of generality. In fact, if the payoff of the outside option is positive, the DM has to consider the trade-off between losing the discounted payoff of the outside option and continuing to search for further information on the focal alternative. This would lead again to the existence of a lower threshold such that the DM leaves the search process by taking the outside option if the expected payoff of adopting the alternative drops below the threshold. We again do not consider this possibility to simplify the analysis, as this possibility is not essential to obtain the choice deferral effects.

3. Analysis

In order to consider the optimal decisions of the DM, we have to consider the expected present discounted value of the DM under the optimal decisions, depending on the state in which the DM is in. Let $V(x)$ be the expected discounted payoff for the DM if the DM is in the search mode, and $W(x)$ be the expected payoff for the DM if the DM is in the no-search mode, if the DM's current expected utility from adopting the alternative is x.

The optimal search behavior of the DM would be to adopt the alternative, when in the search mode, if the expected payoff of the alternative x reaches a threshold $\overline{x}$. When in the no-search mode, the DM would adopt the alternative if the expected payoff of the alternative is above some threshold $\widetilde{x}$.

Lemma 1. *The purchasing threshold in the search mode is larger than the purchasing threshold in the no-search mode, $\overline{x} > \widetilde{x}$.*

Note that at the instant at which the DM moves from the search mode to the no-search mode, if $x \in [\widetilde{x}, \overline{x})$, the DM chooses to adopt the alternative immediately because of the costly delay of getting any additional information. This is the case of choice closure. If $x < \widetilde{x}$ at the instance when the DM moves from the search mode to the no-search mode, the DM decides not to adopt the alternative then, and waits until the DM switches again to the search mode and gain further information then. This is the case in which the DM defers choice. Note that this means that there is a positive mass probability of the DM adopting the alternative at an instant when the DM moves from the search to the no-search mode.

The Bellman equation for $V(x)$ for $x < \widetilde{x}$ can be written as

$$V(x) = (1 - \lambda\, dt)e^{-r\, dt} EV(x + dx) + \lambda\, dt W(x). \tag{1}$$

(Note that we could have $e^{-r\,dt}EW(x+dx)$ instead of $W(x)$ in (1) and the subsequent analysis would not change, as the second order terms in $(dt)^2$ disappear as $dt \to 0$.) The Bellman equation for $V(x)$ for $x \in (\widetilde{x}, \overline{x})$ can be written as

$$V(x) = (1-\lambda\,dt)e^{-r\,dt}EV(x+dx) + \lambda\,dt x. \tag{2}$$

Applying Itô's Lemma to (2), we can obtain the following second-order differential equation in $V(x)$:

$$V(x) = \frac{\sigma^2}{2(r+\lambda)}V''(x) + \frac{\lambda}{r+\lambda}x \tag{3}$$

The Bellman equation for $W(x)$ can be written as

$$W(x) = \beta\,dtV(x) + (1-\beta\,dt)e^{-r\,dt}W(x), \tag{4}$$

from which one can obtain $W(x) = \frac{\beta}{r+\beta}V(x)$. Substituting $W(x)$ into (1), and using Itô's Lemma, we can obtain the second-order differential equation in $V(x)$ for $x < \widetilde{x}$ as

$$r\frac{r+\beta+\lambda}{r+\beta}V(x) = \frac{\sigma^2}{2}V''(x). \tag{5}$$

Solving the above second-order differential equations for $V(x)$, and using value matching and smooth pasting of $V(x)$ at $\widetilde{x}$ and $\overline{x}$, $V(\widetilde{x}^-) = V(\widetilde{x}^+), V'(\widetilde{x}^-) = V'(\widetilde{x}^+)$, $V(\overline{x}) = \overline{x}$, $V'(\overline{x}) = 1$, and $W(\widetilde{x}) = \widetilde{x}$, we obtain a system of five equations (presented in the online appendix) to obtain $\widetilde{x}$ and $\overline{x}$.

Defining, $\delta = \overline{x} - \widetilde{x}$, $\eta = \sqrt{\frac{2r}{\sigma^2}\frac{r+\beta+\lambda}{r+\beta}}$, and $\widetilde{\eta} = \sqrt{\frac{2(r+\lambda)}{\sigma^2}}$, we can obtain (see online appendix)

$$\begin{aligned}\beta(D-1)\left\{\eta(r+\lambda)\left[1+D-\frac{\delta\widetilde{\eta}}{D-1}(1+D^2)\right] + \widetilde{\eta}(r-\lambda)(D-1) - \delta\widetilde{\eta}^2 r(1+D)\right\} +\\ (r+\lambda)[r(D^2-1)(\eta-\widetilde{\eta}^2\delta) + r\widetilde{\eta}(1+D^2)(1-\eta\delta) + 2\widetilde{\eta}\lambda D] = 0\end{aligned} \tag{6}$$

which determines δ, where $D = e^{\widetilde{\eta}\delta}$. We can then also obtain $\widetilde{x}$ as a function of δ as

$$\widetilde{x} = \beta\frac{r + r\widetilde{\eta}\delta + \lambda D}{D[\widetilde{\eta}r(r+\beta+\lambda) + \eta(r+\beta)(r+\lambda)] - \widetilde{\eta}\beta r}. \tag{7}$$

Note that from (6) and (7) we can obtain that both δ/σ and $\widetilde{x}/\sigma$ are independent of σ. The reason is that the standard deviation of the DM's belief process in a unit of time is σ, and so all optimal thresholds are then proportional to σ.

In this model, $\widetilde{x}$ can be seen as a measure of the extent of choice deferral. When switching from the search mode to the no-search mode, the DM defers if and only if $x < \widetilde{x}$. On the other hand, $\delta = \overline{x} - \widetilde{x}$ can be seen as a measure of the extent of choice closure. When switching from the search mode to the no-search mode, the DM adopts the alternative immediately if $\widetilde{x} \leq x < \overline{x}$, even though the DM would not adopt the alternative if she is still in the search mode.

Figure 1 illustrates a sample path in which the individual makes the decision to take the alternative during the search mode after several choice deferrals. Figure 2 illustrates a sample path in which the individual makes the decision to take the alternative when switching from the search to the no-search mode (i.e., choice closure) after several choice deferrals.

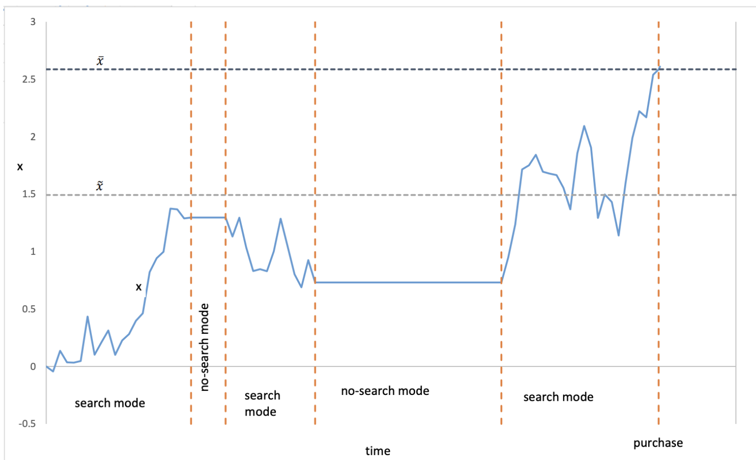


Figure 1: Example of sample path of individual expected payoff when making a decision during the search mode with $x_0 = 0, r = .05, \lambda = \beta = .5$, and $\sigma^2 = 1$. For these parameter values we have $\overline{x} \approx 2.59$ and $\widetilde{x} \approx 1.49$.

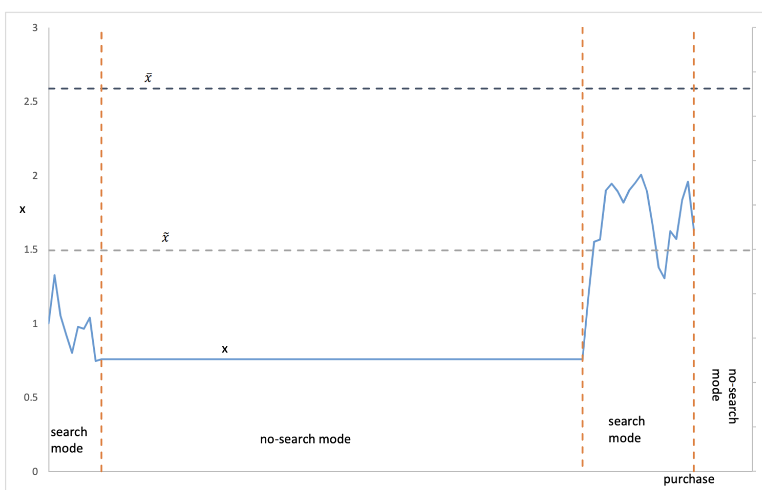


Figure 2: Example of sample path of individual expected payoff when making a decision when moving search to no-search mode (choice closure) with $x_0 = 1, r = .05, \lambda = \beta = .5$, and $\sigma^2 = 1$. For these parameter values we have $\overline{x} \approx 2.59$ and $\widetilde{x} \approx 1.49$.

We now first solve two benchmark models that do not directly have search fatigue and time-varying search costs. We then analyze the general case of the model. After that, we consider two limiting cases, $\beta \to 0$ and $\beta \to +\infty$, to obtain sharper comparative statics results.

Benchmarks

No-Fatigue Benchmark: In the first benchmark, we consider a variation of the model which assumes that consumers do not experience search fatigue ($\lambda \to 0$), and thus, the search cost is constant. In this case, the DM's behavior is governed by a single threshold, $\overline{x}_{nb} = \sqrt{\frac{\sigma^2}{2r}}$, where the subscript '*nb*' denotes the No-Fatigue Benchmark. The DM continues the search for $x < \overline{x}_{nb}$ and adopts the alternative when x reaches $\overline{x}_{nb}$.

The adoption threshold, $\overline{x}_{nb}$, does not depend on the rate of search interruption and recovery. Additionally, because $\widetilde{x}$ does not exist, there is no decision between choice closure and choice deferral in this benchmark.

Model-Free-Fatigue Benchmark: One might argue, however, that the No-Fatigue Benchmark is too naive as a benchmark to study search fatigue. Even if a researcher does not model search fatigue explicitly, the researcher can still be aware that the DM does not search for information 24 hours a day, and the DM's search fatigue still affects the observed behavior. For example, consider a busy car buyer who only searches for car information for half an hour on Sunday morning and half an hour on Saturday night. Starting from an initial search session on Sunday, one week of time elapses for each hour of information searched. If the consumer makes a decision after 4 hours of information search, then the researcher will observe that it takes the consumer 4 weeks from the initial search to make a decision.

To account for the difference in search time and total time, we can adjust the discount rate. In the long run, with hazard rates of λ and β, the DM's share of time in the search mode is $\frac{\beta}{\lambda+\beta}$. If the DM adopts the alternative after searching for T units of time, then we discount the final purchase by $e^{-\frac{\lambda+\beta}{\beta}rT}$. We can then write the DM's decision rule as $\overline{x}_{mb} = \sqrt{\frac{\sigma^2}{2r_{mb}}}$, where $r_{mb} = \frac{\lambda+\beta}{\beta}r$. The subscript '*mb*' denotes the Model-Free-Fatigue Benchmark, because this model captures search interruption and recovery parsimoniously without explicitly modelling them. The threshold $\overline{x}_{mb}$ can be viewed as the decision rule of the consumer who takes into account the time discount due to search interruptions in a reduced-form way. In contrast, $\overline{x}$ in the main model is the decision rule of the consumer who takes into account the impact of search interruptions strategically.

The benchmark adoption threshold, $\overline{x}_{mb}$, increases in both β and σ^2, and decreases in both λ and r. The rate of fatigue and recovery, λ and β, work through the adjusted discount rate r_{mb}. When the DM experiences more search interruptions (higher λ) or interruptions last longer (lower

β), the DM effectively has a higher discount rate and prefers to shorten the search process by lowering the adoption threshold $\overline{x}_{mb}$.

However, because $\widetilde{x}$ also does not exist in this benchmark, the model says nothing about the DM's choice closure behavior. The choice deferral behavior is also indistinguishable from the decision to search for more information while in the search state. As shown below, choice deferral and choice closure behaviors drive key results on pricing and search interventions. The lack of strategic decision between choice closure and choice deferral in both benchmarks highlights the importance of explicitly modeling search interruptions.

General Case

Consider now the general case. We first show the comparative statics of the purchasing thresholds in the search and no-search region with regard to different model parameters.

Proposition 1. *The purchase threshold in the search mode, $\overline{x}$, and the purchase threshold in the no-search mode, $\widetilde{x}$, are increasing in both β and σ^2, and decreasing in both λ and r. Moreover, $\overline{x}/\sigma, \widetilde{x}/\sigma$, and δ/σ, are independent of σ.*

As the likelihood of moving from the no-search mode to the search mode increases, the likelihood of being able to continue to search increases. Therefore, the DM prefers to search more and delay the adoption decision, which means that both purchase thresholds increase. As the information gained in the search mode, σ^2, is greater, the DM gains more from search, and chooses to search more, which results in both purchase thresholds to increase. When the discount rate increases, the present value of delaying purchase is reduced, and therefore the DM searches less, which means that both purchase thresholds fall. Similarly, when the likelihood of moving from the search mode to the no-search mode increases, the likelihood of being able to continue to search decreases, and therefore the DM prefers to make the adoption decision sooner, which means that both purchase thresholds fall.

For a fixed σ^2, the extent of choice deferral can be seen as increasing in $\widetilde{x}$ and, therefore, is increasing in β and decreasing in λ and r. The extent of choice deferral cannot be simply measured by the size of $\widetilde{x}$ when σ^2 changes. On the one hand, the region where the DM defers choice increases in $\widetilde{x}$. On the other hand, however, the DM's belief changes more quickly as σ^2 increases. So, a larger $\widetilde{x}$ does not necessarily imply a greater extent of choice deferral. Since the standard deviation of the DM's belief processes in a unit of time is σ, $\widetilde{x}$ normalized by $1/\sigma$, $\widetilde{x}/\sigma$, is more appropriate to measure the extent of choice deferral for different σ^2. As we noted previously, since $\widetilde{x}/\sigma$ is independent of σ, we have that the extent of choice deferral does not depend on σ^2. Similarly, the extent of choice closure does not depend on σ^2.

We illustrate the above results in Figures A.1-A.4 in the Appendix.

Intuitively, as the frequency of search interruption λ or the discount rate r increases, deferring search becomes less attractive and the DM is more likely to speed up the purchasing decision. Experimental evidence is consistent with our findings. Using lab experiments, Xia and Sudharshan (2002) find that "As interruption frequency increases, consumers with concrete goals will spend less time on the task."

Note that the above comparative statics on the purchase threshold in the search mode, $\overline{x}$, are in the same direction as those under the Model-Free-Fatigue Benchmark. The behaviors on choice closure and choice deferral, captured by $\overline{x} - \widetilde{x}$ and $\widetilde{x}$, are new. To better understand the extent of choice closure and the extent of choice deferral, we can examine two limiting cases where the rate of search recovery, β, is very small or very large.

Case of $\beta \to 0$:

In the case of $\beta \to 0$ we have that $\widetilde{x} \to 0$, such that when the search mode ends the DM adopts the alternative as long as $x \geq 0$. We can also then obtain that $\overline{x}$ in the limit solves

$$e^{\eta\overline{x}}(1 - \eta\overline{x}) + \frac{\lambda}{r} = 0. \tag{8}$$

From this we can obtain that $\overline{x} > 1/\eta$ and that at the limit $\overline{x}$ is decreasing in λ and r. Since $\widetilde{x} \to 0$, we also have that at the limit δ is decreasing in λ and r. We collect these results in the following proposition.

Proposition 2. *Suppose that β is sufficiently small. Then the difference between purchasing thresholds in the search and no-search region, $\delta = \overline{x} - \widetilde{x}$, is decreasing in both λ and r.*

The extent of choice closure can be seen as increasing in δ and, therefore, is decreasing in λ and r. As the discount rate, r, or the rate at which the DM moves from the search to the no-search mode, λ, increases, the DM has a stronger incentive to make a faster decision in the search mode while she always adopts anything positive in the no-search mode. So, the extent of choice closure decreases in λ and r. Notice that the extent of choice closure reflects a DM's incentive to speed up the purchasing decision in the no-search mode *relative to the search mode.* When λ or r increases, the DM wants to make a faster decision in both the search and no-search modes, with the effect stronger in the search mode. This implies that the extent of search closure will decrease instead of increase.

Case of $\beta \to \infty$:

In the case of $\beta \to \infty$ we have that $\delta \to 0$ and $\overline{x}, \widetilde{x} \to \sqrt{\frac{\sigma^2}{2r}}$. This shows that, as one may expect, when the DM is more likely to come back to the search mode, the DM is more demanding on the expected payoff of the alternative to decide to adopt it (in comparison to the case of $\beta \to 0$).

In this case of $\beta \to \infty$ it is also interesting to see the rate at which δ converges to zero, and the rate at which $\overline{x}$ and $\widetilde{x}$ converge to $\sqrt{\frac{\sigma^2}{2r}}$.

To see this note that as $\beta \to \infty$ we can obtain from (6) that

$$\beta(D-1)^2 \to 2(r+\lambda) \tag{9}$$

from which we can obtain that[9]

$$\delta\sqrt{\beta} \to \sigma, \tag{10}$$

which shows that δ is decreasing in the rate at which the DM returns to the search mode from the no-search mode, β. Therefore, the extent of choice closure can be seen as decreasing in β. As the DM becomes more likely to return to the search mode, the expected waiting time in the no-search mode and loss from discounting are lower. So, the DM has a weaker incentive to make a premature decision in the no-search mode.

We summarize these results in the following proposition.

Proposition 3. *Suppose that β is sufficiently large. Then the difference between purchasing thresholds in the search and no-search region, $\delta = \overline{x} - \widetilde{x}$, is decreasing in β.*

Intermediate β

We can consider the case of intermediate β numerically. The numerical analysis that we conducted indicates that the comparative statics derived in Proposition 2 and Proposition 3 also hold for intermediate values of β. This is illustrated by Figures A.1-A.4 in the Appendix.

Figure A.1 illustrates how the purchase thresholds $\overline{x}$ and $\widetilde{x}$ increase with the rate at which the individual switches from the no-search mode to the search mode, β, and that the difference $\overline{x} - \widetilde{x}$ decreases with β. Thus, the DM has a greater extent of choice deferral and a lesser extent of choice closure when the DM returns to the search mode sooner after interruptions to the search process.

Figure A.2 illustrates how the purchase thresholds $\overline{x}$ and $\widetilde{x}$ decrease with the discount rate r, for a case of β low ($\beta = .1$), and a case of β high ($\beta = 5$). The figure also illustrates that the difference $\overline{x} - \widetilde{x}$ decreases in r, as shown in Proposition 2.[10] As discussed in the limiting case of

[9] Please see the derivation in the online appendix.

[10] Note that for $\beta = 5$, $\overline{x} - \widetilde{x}$ still decreases in r even though the lines appear to be closer for r small in Figure A.2.

$\beta \to 0$, a higher discount rate has a greater effect on the purchase threshold in the search mode, $\overline{x}$, which leads to a decrease in the difference $\overline{x}-\widetilde{x}$, which means a lower extent of choice closure. Note that both the extent of choice deferral and the extent of choice closure decrease with r, because a less patient DM has a stronger incentive to make a decision before search interruptions arrive by lowering the purchase threshold $\overline{x}$. It emphasizes that choice deferral and choice closure are not two completely opposite concepts. We also observe that the effect of r on $\overline{x}-\widetilde{x}$ is smaller for a higher β, which corresponds to the finding that $\overline{x}-\widetilde{x}$ does not depend on r at the limit of $\beta \to \infty$.

Figure A.3 illustrates how the purchase thresholds $\overline{x}$ and $\widetilde{x}$ decrease with the rate at which the individual switches from the search mode to the no-search mode, λ, for a case of β low ($\beta = .1$), and a case of β high ($\beta = 5$). The figure also illustrates how the difference $\overline{x}-\widetilde{x}$ decreases in λ. Both the extent of choice deferral and the extent of choice closure decrease with λ. The rationale is similar to the one regarding the effect of the discount rate discussed above. When information gathering is interrupted more frequently, the DM has a stronger incentive to stop searching by lowering the purchase threshold in the search mode, $\overline{x}$. We also observe that the effect of λ on $\overline{x}-\widetilde{x}$ is smaller for a higher β, which corresponds to our finding that $\overline{x}-\widetilde{x}$ does not depend on λ at the limit of $\beta \to \infty$.

Figure A.4 illustrates how the purchase thresholds $\overline{x}$ and $\widetilde{x}$ increase with the amount of information learned in the search mode, σ^2, for a case of β low ($\beta = .1$), and a case of β high ($\beta = 5$). The figure illustrates how the difference $\overline{x}-\widetilde{x}$ also increases in σ^2. But as discussed previously, when σ^2 changes, the extent of choice deferral and the extent of choice closure are measured by $\widetilde{x}/\sigma$ and $(\overline{x}-\widetilde{x})/\sigma$, respectively, and both values do not change with σ^2. Thus, the extent of choice deferral and the extent of choice closure do not depend on σ^2.

4. Optimal Pricing

In this section, we derive the firm's optimal pricing strategy. The analysis for the model in Section 3 can be seen as describing the behavior of a DM facing a product with an exogenous price. Let P denote the price, let x denote the expected value of the payoff of the alternative as before, and let $y = x - P$ denote the expected payoff of the alternative minus the price. The DM then would adopt the alternative when y reaches $\overline{x}$ in the search mode, and would adopt the alternative when y reaches $\widetilde{x}$ in the no-search mode, where $\overline{x}$ and $\widetilde{x}$ are solutions to (6) and (7). Equivalently, the DM adopts when x reaches $\overline{x}+P$ in the search mode or when x reaches $\widetilde{x}+P$ in the no-search mode.

Let $V_f(x)$ be the expected discounted payoff for the firm if the DM is in the search mode, and $W_f(x)$ be the expected payoff for the firm if the DM is in the no-search mode. Since the consumer is in the search mode initially (she becomes fatigued only after gathering information for some time), the firm's objective is to choose a price that maximizes $V_f(x_0)$, $\max_P V_f(x_0)$.

The Bellman equation for $V_f(x)$ for $x < \widetilde{x} + P$ can be written as

$$V_f(x) = (1 - \lambda\, dt)e^{-r\,dt} EV(x + dx) + \lambda\, dt W_f(x). \tag{11}$$

The Bellman equation for $V_f(x)$ for $x \in (\widetilde{x} + P, \overline{x} + P)$ can be written as

$$V_f(x) = (1 - \lambda\, dt)e^{-r\,dt} EV_f(x + dx) + \lambda\, dt P. \tag{12}$$

Finally, the Bellman equation for $W_f(x)$ can be written as

$$W_f(x) = \beta\, dt V(x) + (1 - \beta\, dt)e^{-r\,dt} W_f(x), \tag{13}$$

from which one can obtain $W_f(x) = \frac{\beta}{r+\beta} V_f(x)$.

Given continuity of the value function at both $\overline{x} + P$ and $\widetilde{x} + P$, we have value matching of V_f at both these points:

$$\begin{aligned} V_f(\overline{x} + P) &= P \qquad (14) \\ V_f(\widetilde{x} + P^+) &= V_f(\widetilde{x} + P^-). \qquad (15) \end{aligned}$$

Furthermore, given infinite variation of x around $\widetilde{x} + P$, we also have smooth pasting at that point,

$$V_f'(\widetilde{x} + P^+) = V_f'(\widetilde{x} + P^-). \tag{16}$$

Applying Itô's Lemma to the Bellman equations, solving the corresponding differential equations, and using (14)-(16), we can solve for the firm's value function $V_f(x)$. The analysis is presented in the online appendix.

The optimal price, P^*, depends on the initial position, x_0. Suppose $P^* \le x_0 - \overline{x}$ (that is, $x_0 \ge \overline{x} + P^*$), then the DM adopts the alternative at x_0 in both the search mode and the no-search mode. In this case, because the DM purchases immediately at P^*, the firm's profit strictly increases in P^*. Thus, any price strictly below $x_0 - \overline{x}$ cannot be optimal. So we must have $P^* \ge x_0 - \overline{x}$. And there is a x_0^{**}, defined below, such that $P^* = x_0 - \overline{x}$ for $x_0 > x_0^{**}$.

Now consider the case where $P^* > x_0 - \widetilde{x}$ (that is, $x_0 < \widetilde{x} + P^*$). In this case, the DM does not adopt the alternative at x_0 in both the search mode and the no-search mode. And there is a x_0^*, defined below, when we will be in this case for $x_0 < x_0^*$.

We can obtain the value function of the firm in this region of x_0 as

$$V_f(x_0) = \frac{2r + \lambda(e^{\widetilde{\eta}\delta} + e^{-\widetilde{\eta}\delta})}{(\widetilde{\eta} + \eta)e^{\eta(\widetilde{x}+P)+\widetilde{\eta}\delta} + (\widetilde{\eta} - \eta)e^{\eta(\widetilde{x}+P)-\widetilde{\eta}\delta}} \frac{\widetilde{\eta}P}{r + \lambda} e^{\eta x_0} \tag{17}$$

Taking the derivative of $V_f(x_0)$ with respect to P, we have

$$\text{sign}\left\{\frac{\partial V_f(x_0)}{\partial P}\right\} = \text{sign}\{1 - \eta P\}.$$

We then have that for $x_0 < x_0^*$, the optimal price is $P^* = 1/\eta$. We can also then obtain that $x_0^* = \widetilde{x} + 1/\eta$.

Finally, consider the case where $P^* \in [x_0 - \overline{x}, x_0 - \widetilde{x}]$ (that is, $x_0 \in [\widetilde{x} + P^*, \overline{x} + P^*]$). In this case, the DM adopts at x_0 in the no-search mode but does not adopt at x_0 in the search mode. This is the case in which $x_0 \in [x_0^*, x_0^{**}]$. Let us denote $V_f(x, P)$ as the value function for $x \in (\widetilde{x} + P, \overline{x} + P)$, where we emphasize that that value function also depends on the price P.

In this case, the optimal interior price is obtained by differentiating $V_f(x, P)$ evaluated at $x = x_0$, with respect to price and making that derivative equal to zero. That equality determines the optimal price $P^*(x_0)$ implicitly by some function $h(P^*(x_0), x_0) = 0$, defined in the online appendix. We can then define x_0^{**} by making $x_0^{**} - \overline{x} = V_f(x_0^{**}, P^*(x_0^{**}))$ and $P^*(x_0^{**}) \in \arg\max_P V_f(x_0^{**}, P)$. As discussed in the online appendix we may, or may not, have continuity of the price function at x_0^{**}. If we have continuity of the price function at x_0^{**}, which it can be obtained to occur, for example, if λ/r and β are small enough, we also have that x_0^{**} satisfies $h(x_0^{**} - \overline{x}, x_0^{**}) = 0$. As also discussed in the online appendix, we have that the optimal price is declining in x_0 at any existing discontinuity, and will be declining in x_0 for some region of $x_0 \in [x_0^*, x_0^{**}]$ if the price function is continuous for β small. For β large we can obtain that the price function is continuous. Furthermore, the price function is monotonic for $\beta \to \infty$.[11]

We summarize the optimal pricing strategy and comparative statics in the following Proposition.

Proposition 4. *The optimal price depends on x_0 in the following way:*

1. ***(Inducing Deferral)*** *For x_0 sufficiently low ($x_0 < x_0^* = \widetilde{x} + 1/\eta$), the optimal price is $P^* = 1/\eta$, which does not depend on x_0. The DM does not adopt the alternative at x_0 in either the search mode or the no-search mode. In that case, the optimal price increases in σ^2 and β, and decreases in r and λ. If λ and β change simultaneously with a fixed ratio of λ/β, then the optimal price decreases in λ and β.*

2. ***(Inducing Closure)*** *For intermediate x_0 ($x_0 \in [x_0^*, x_0^{**}]$), the optimal price is in the range of $[1/\eta, x_0]$, and the DM does not adopt the alternative at x_0 in the search mode but adopts*

[11] Numerical analysis also suggests that the price function is monotonic in x_0 for β large.

the alternative at x_0 in the no-search mode. For β sufficiently small, the optimal price $P^*(x)$ decreases in x_0 at x_0^{**} and is thus non-monotonic. In addition, it is discontinuous at x_0^{**} if $\lambda/r > 2a^2 - 1$, where $a > 1$ satisfies $e^a(a-1) - 2a^2 + 1 = 0$.

3. ***(Inducing Purchase)*** *For x_0 sufficiently high ($x_0 > x_0^{**}$), the optimal price is $x_0 - \overline{x}$, and the DM adopts the alternative without searching. In that case, the optimal price decreases in σ^2 and β, and increases in r and λ.*

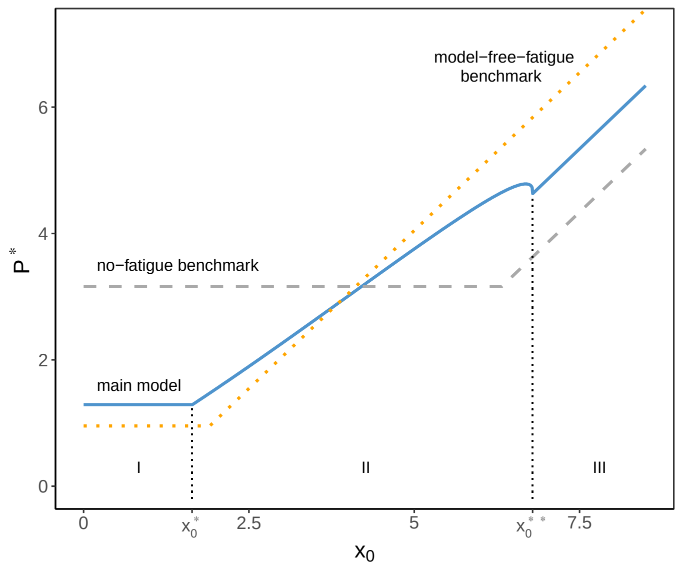


Figure 3: Example of the optimal price P^* as a function of x_0 for $r = .05, \lambda = .5, \beta = .05$ and $\sigma^2 = 1$. For these parameter values we have $x_0^* \approx 1.64$ and $x_0^{**} \approx 6.79$. Region I: $x_0 < x_0^*$ (the DM adopts in neither the search nor the no-search mode); Region II: $x_0 \in [x_0^*, x_0^{**}]$ (the DM adopts in the no-search mode only); Region III: $x_0 > x_0^{**}$ (the DM adopts in both the search and no-search mode).

Figure 3 illustrates how the optimal price varies with the initial position, x_0. Interestingly, under the optimal pricing, the firm uses price to induce different choice deferral behaviors at $x = x_0$. When x_0 is low (Proposition 4.1, Region I in the figure), the firm sets a price such that the DM defers choice if search is interrupted around $x = x_0$. For intermediate x_0 (Proposition 4.2, Region II in the figure), the firm sets a price to induce choice closure at $x = x_0$, i.e., the DM does not adopt the alternative in the search mode but adopts the alternative if search is interrupted around $x = x_0$. For higher x_0 (Proposition 4.3, Region III in the figure), the firm sets a price such that the DM adopts the alternative immediately even in the search mode.

Comparative Statics and Implications

The optimal price is constant if the DM's prior belief is sufficiently low, $x_0 < \widetilde{x} + 1/\eta$. The firm would need to charge too low a price (potentially even negative) to convince the DM to adopt the alternative without learning any information, which is not profitable. It is better to charge a higher price, hoping that the DM will receive enough positive signals and adopt the alternative at a high price. Therefore, the firm sets a constant price, $1/\eta$, such that the DM does not adopt the alternative at x_0 in either the search mode or the no-search mode. In this region, one can see that the optimal price increases in σ^2 and β, and decreases in r and λ. Intuitively, when λ increases or when β decreases, the DM is expected to spend a larger fraction of time in the no-search mode, exhibiting stronger search fatigue. When the DM faces more frequent and longer disruptions of information gathering, the firm should charge a lower price to prevent the DM from deferring choice. Online stores can often track consumers over different browsing sessions. The result suggests that firms should factor in the lengths of browsing sessions and gaps between browsing sessions in setting their prices.

Another implication of the findings is that the firm should change its price following its efforts to intervene with the DM's search/no-search pattern. For example, in online retail, firms may redesign interfaces to reduce consumer fatigue, so that consumers stay longer in a browsing session. Firms may also use instruments such as ad retargeting, push notifications, or email marketing to bring back previous visitors more quickly. The price should increase if these efforts are successful.

In this case, we also find that the optimal price decreases when the DM switches between the search mode and the no-search mode more frequently, even if the long-term fraction of time in each mode remains constant. That is, assuming $\lambda/\beta = \alpha$ for some fixed α, we find that P^* decreases in λ (or β) for low x_0. This is relevant when there is a change in the shopping environment such that consumers enter and exit search more or less frequently. For example, consumers shopping on mobile devices may have their browsing sessions disrupted and resumed more frequently than consumers shopping on computers. In that case, even if the overall time spent on shopping does not change for consumers on mobile devices, the firm should consider setting a lower price on mobile devices compared to the price on computers.

When the DM's prior belief about the alternative is higher, the optimal price depends on x_0 and the comparative statics may be reversed. This is because the firm can already obtain a high profit even if the DM does not receive additional information. The firm prefers to increase the adoption likelihood by setting a price such that the DM adopts the alternative at x_0 in the no-search mode and may even adopt it at x_0 in the search mode. In particular, if $x_0 > x_0^{**}$ the firm charges a price equal to $x_0 - \overline{x}$ to induce the DM to adopt the alternative without searching. In this case, the optimal price increases in x_0 linearly.

Comparison to Benchmarks

To better understand the new behaviors from modeling search fatigue, we derive the optimal pricing under the two benchmarks in the Appendix. In the No-Fatigue Benchmark, for $x_0 < 2\sqrt{\frac{\sigma^2}{2r}}$,the firm charges $p^*_{nb} = \sqrt{\frac{\sigma^2}{2r}}$ and the consumers search until x reaches $2\sqrt{\frac{\sigma^2}{2r}}$. For higher x_0, the firm charges $p^*_{nb} = x_0 - \sqrt{\frac{\sigma^2}{2r}}$ and the consumers buys immediately without search. The optimal pricing under the Model-Free-Fatigue Benchmark is similar. For $x_0 < 2\sqrt{\frac{\sigma^2}{2r}\frac{\beta}{\lambda+\beta}}$, the firm charges $p^*_{mb} = \sqrt{\frac{\sigma^2}{2r}\frac{\beta}{\lambda+\beta}}$ and the consumers search until x reaches $2\sqrt{\frac{\sigma^2}{2r}\frac{\beta}{\lambda+\beta}}$. For higher x_0, the firm charges $p^*_{mb} = x_0 - \sqrt{\frac{\sigma^2}{2r}\frac{\beta}{\lambda+\beta}}$ and the consumers buy immediately without search.

As Figure 3 illustrates, neither benchmark captures the pricing behavior in Region II. For intermediate values of x_0, the firm wants to incentivize some amount of search, but also does not want the consumers to delay purchase for too long. The firm thus can utilize the consumer's choice closure behavior to its benefit. The consumer is incentivized to search initially, but the pricing discourages search when her search cost increases unless she has acquired a significant amount of negative information before that point. This closure-inducing pricing strategy cannot be predicted without explicitly modeling the search interruptions.

Note also that the closure-inducing pricing strategy in Region II generates interesting non-monotonic behaviors in x_0, σ^2, and r while the optimal price in Region I and Region III is always monotonic in the model parameters. We explore the non-monotonicity below.

Non-monotonic Optimal Price

If $x_0 \in [x_0^*, x_0^{**}]$, the firm charges a price so that the DM would adopt the alternative at x_0 in the no-search mode but not in the search mode. In this case, the optimal price is in the interval $[x_0 - \overline{x}, x_0 - \widetilde{x}]$. One surprising finding is that the optimal price $P^*(x_0)$ may be non-monotonic in the initial belief x_0 for β small. The non-constant search cost drives this, as the optimal price always increases in the initial belief in both benchmark models.

The intuition is that there are two opposing effects of the prior belief x_0 on the price. On the one hand, consumers have a higher willingness to pay when the initial belief is higher, as reflected by the fact that both the upper bound $(x_0 - \widetilde{x})$ and the lower bound $(x_0 - \overline{x})$ of the optimal price increase in x_0. This effect drives the price higher. On the other hand, the firm can guarantee a payoff of $x_0 - \overline{x}$ by charging $P = x_0 - \overline{x}$, which induces an immediate purchase. For any time t passed by without conversion, the firm loses at least $(1 - e^{-rt})(x_0 - \overline{x})$ due to discounting. One can see that the firm's loss from non-adoption or delayed adoption increases in x_0. So, the firm has an incentive to induce the DM to adopt the alternative sooner. This effect drives the price lower. As a result, the optimal price can be non-monotonic in x_0.

For the optimal price to decrease in x_0, we need the second effect to be stronger than the first one. Since the firm's loss from non-adoption or delayed adoption $(1 - e^{-rt})(x_0 - \overline{x})$ is higher when x_0 is larger, the firm's incentive to induce the DM to adopt the alternative sooner by charging a lower price is stronger for larger x_0 (x_0 closer to x_0^{**} rather than x_0^{*}). Therefore, non-monotonicity of the optimal price happens near x_0^{**}. The existence of non-monotonic price also requires β to be small because it only appears in Region II where the price induces choice closure at x_0. Therefore, choice closure is essential to the non-monotonicity result. As we have discussed in section 3, the extent of choice closure decreases in β. So, non-monotonic price happens when β is small. The intuition for the opposing effects and non-monotonicity of the optimal price can also be seen more clearly in a two-period model, which we analyze in the online appendix.

Discussion on Evidence of Closure-Inducing Strategy

The comparison with the benchmarks shows that Region II is unique to the model with time-varying search costs. In Region II, the firm chooses a price that encourages the consumer to search but not to defer when the search cost rises.

While we cannot directly observe in practice whether a firm sets its price to induce closure, we can observe other actions that highlight the firm's desire to induce choice closure. One example consistent with such strategies can be found in online marketplaces such as travel service platforms. The firm may urge consumers to buy soon by charging an attractive price and stating that the offer is only valid for a limited time, even though the price often does not rise after the deadline.[12] Similarly, travel platforms may misleadingly mention a hotel's limited availability even when there is a large number of rooms left.[13] Both actions promote a sense of urgency to discourage consumers from deferring choices. This can be explained as a way to create hype or convey information (e.g., Subramanian and Rao 2016). Alternatively, this can be seen as the consumer still searching for some information before making a decision, but, when the search is interrupted by fatigue or distractions, the consumer would want to purchase the product rather than delay the decision.

Another example of a choice closure-inducing tactic is exit-intent popups. When a website detects that a consumer is about to leave, a popup is triggered to give the consumer a last-minute message to encourage action. These pop-ups often add urgency by highlighting inventory scarcity or the deadline for the current offer, or invoke observational learning by showing how many other consumers have bought and their testimonials.[14] Our model shows that whether a consumer defers

[12] Source: https://www.independent.co.uk/travel/news-and-advice/holiday-deals-limited-time-only-offers-which-investigation-fake-why-bookings-expedia-virgin-sandals-a8138311.html

[13] https://www.nbcnews.com/better/lifestyle/travel-website-you-re-using-says-there-s-only-1-ncna1073066

[14] For examples of exit-intent popups, see: https://www.nngroup.com/articles/exit-intent-good-ux/; https://optinmonster.com/40-exit-popup-hacks-that-will-grow-your-subscribers-and-revenue/#Urgency; and https://getsitecontrol.com/blog/exit-popups/.

or closes her choice upon fatigue depends crucially on the price. The firm can both encourage initial search and induce choice closure by pricing at an intermediate level, and doing so is optimal when the initial product value is in an intermediate range.

Other Comparative Statics

Figure 4 illustrates the comparative statics of P^* with regard to the speed of learning σ^2, the discount rate r, and the switching rates λ and β. Note that the DM behaves differently across different regions of the optimal prices.

Since $\widetilde{x}$ and $1/\eta$ increase in σ^2, the condition $x_0 < x^* = \widetilde{x} + 1/\eta$ (Region I) is more likely to be satisfied for larger σ^2. So, fixing x_0, the optimal price is $1/\eta$ and the DM adopts the alternative at x_0 in neither the search or no-search mode when σ^2 is large. Intuitively, the DM wants to search for information in a wider range of beliefs when the signal is more informative. As a result, the firm needs to charge a lower price to induce immediate purchase ($x_0 - \overline{x}$ decreases in σ^2). Since $1/\eta$ increases in σ^2, the optimal price increases in σ^2 in that region (Region I).

The intuition for the increasing price is the following. A higher price has two effects on the firm's profit (fixing other parameters). On one hand, the firm's current value of a purchase increases in price. This leads to a positive effect of price on the present value of the firm's expected profit. On the other hand, a higher price moves the DM's expected payoff of the alternative minus the price, $y = x - P$, further away from the purchasing threshold. This leads to slower purchases because the DM needs to obtain more positive signals to reach the purchasing threshold. So, a higher price has a negative effect on the present value of the firm's expected profit due to discounting. Notice that the extent of the positive effect depends only on the price. In contrast, a higher learning rate speeds up the purchasing decision and mitigates the loss from discounting. So, the extent of the negative effect falls in σ^2. As σ^2 increases, the positive effect of a higher price on the firm's profit remains the same while the negative effect becomes smaller. Therefore, the firm charges a higher price.

In contrast, the optimal price is $x_0 - \overline{x}$ and the DM adopts the alternative at x_0 without searching for small σ^2. Since $\overline{x}$ increases in σ^2, the optimal price decreases in σ^2 in that region (Region III). Intuitively, the firm wants to avoid search when the signal is not very informative and the consumer also wants to reduce search in that case. The less informative the search process (lower σ^2), the easier it is for the firm to convince the consumer to adopt the alternative without searching. So, the firm can charge a higher price as σ^2 decreases, for small σ^2. In the intermediate region (Region II), both forces are at play, and the optimal price can be non-monotonic in σ^2.

The firm's loss from delayed purchase is small when the discount rate r is small. So, it charges a price higher than $x_0 - \overline{x}, 1/\eta$, to increase the profit per purchase, which induces search. Since $1/\eta$

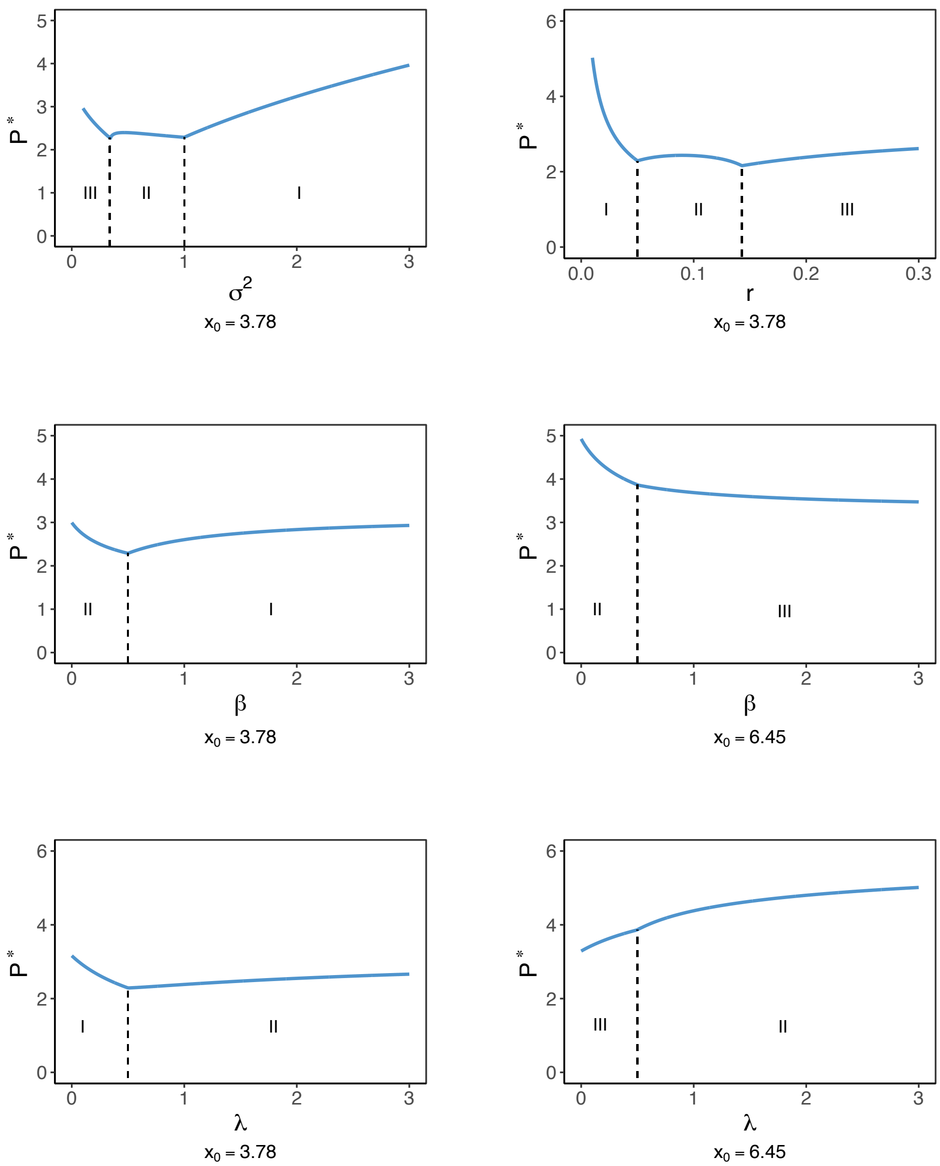


Figure 4: Example of the optimal price P^* as a function of σ^2, r, β, and λ, for $r = .05, \beta = .5$, $\lambda = .5$, and $\sigma^2 = 1$. Region I: $x_0 < x_0^*$ (the DM adopts in neither the search nor the no-search mode); Region II: $x_0 \in [x_0^*, x_0^{**}]$ (the DM adopts in the no-search mode only); Region III: $x_0 > x_0^{**}$ (the DM adopts in both the search and no-search mode).

decreases in r, the optimal price decreases in r in that region. The firm's loss from delayed purchase is high when the discount rate r is high. So, it prefers to charge $x_0 - \overline{x}$ to induce an immediate purchase in that case. Since $\overline{x}$ decreases in r, the optimal price increases in r in that region. The comparative statics with respect to the discount rate r may be non-monotonic for intermediate values of the parameters (Region II) because the optimal price is in the range of $[x_0 - \overline{x}, x_0 - \widetilde{x}]$ and there are two opposing effects in that region, as discussed above.

The optimal price can be non-monotonic in the recovery rate β when x_0 is low. When x_0 is low, the firm wants to encourage search, so it charges a price such that the DM does not adopt the alternative immediately in the search mode (Region I and Region II). When β is low, choice deferral is costly for both consumers and the firm, so the firm can charge a higher price and induce closure. In this region, a higher β makes deferral more attractive, so the firm has to charge a lower price to induce closure. The firm has a higher incentive to encourage search when the disruption of search is shorter (larger β). So, for β large, the firm charges a price such that the DM defers choice at x_0 (Region I). The shorter the disruption of search is, the easier it is for the firm to induce the DM to search. So, the optimal price increases in β in this region. As a result, the optimal price is non-monotonic in β as the strategy changes from inducing closure (Region II) to inducing deferral (Region I).

When x_0 is high, the firm's loss from delayed adoption is higher, and the firm prefers to induce choice closure at x_0 (Region II and Region III). It is harder to convince the DM to adopt the alternative in the search mode as the disruption of search becomes shorter (a higher β implies a higher $\overline{x}$). So, the firm has to lower the price further to discourage search in the search mode as β increases.

For similar reasons, the optimal price can be non-monotonic in the disruption rate λ when x_0 is low. The firm encourages the DM to search in both the search and no-search mode by charging $1/\eta$ for small λ when x_0 is low (Region I). The optimal price decreases in λ because it is harder to encourage search as it gets interrupted more frequently. For large λ, encouraging deferral is too costly, and the firm switches to a higher price to induce choice closure. More frequent interruptions make it easier to induce closure, so the firm can charge a higher price when λ increases in Region II. As a result, the optimal price is non-monotonic in λ as the strategy changes from inducing deferral (Region I) to inducing closure (Region II). In contrast, when x_0 is high, the firm charges $x_0 - \overline{x}$ for small λ (Region III), and the optimal price increases in λ.

Note that the model above assumes that the firm observes x_0 and commits to a fixed price going forward.[15] If the firm cannot commit to a fixed price, and is unable to observe the consumer's evolving state x_t, one has to consider the firm's belief about x_t. Consider any $t > 0$ when the

[15] This is similar to Branco, Sun, and Villas-Boas (2012) and Ning and Villas-Boas (2023), and see also discussion there.

consumer does not own the product, the firm's belief about x_t follows some continuous distribution, for which the consumer not searching for information can also provide information. The firm faces a skimming problem as in bargaining under incomplete information (e.g., Fudenberg, Levine, and Tirole, 1985). The firm may try to learn about the consumer's valuation for the product through successive price offers. The consumer's purchase threshold also depends on the consumer's expectation of all future price offers from the firm. After each price offer, if the consumer chooses not to buy the product, the firm's belief becomes truncated at the top. However, comparing to Fudenberg, Levine, and Tirole (1985), the current model has the additional features of time-varying search costs and evolving x_t, both of which significantly complicate the problem.

If the firm cannot commit to future prices and also knows about consumer beliefs, we are then in a situation similar to Ning (2021). The consumer may suffer from a hold-up problem, in which the firm may want to increase the price as x_t increases. As in Ning (2021), we would then potentially need to allow the firm to self-impose a price ceiling in the form of a list price, with the possibility of the firm offering dynamic discounts.

5. Search Interventions

In the standard search model, without search fatigue or interruption, firms can deter consumer search by providing consumers with a discount for immediate purchases (Armstrong and Zhou 2016). Relatedly, firms can increase consumers' purchase likelihood by taking advantage of choice closure and choice deferral. In addition to using price to affect the search behavior, they can also intervene in the consumer's search environment in the presence of search fatigue. By retargeting inactive consumers, firms can increase the rate of switching from the no-search mode to the search mode (β). By making it harder (easier) to search for relevant information on the website, firms can increase (decrease) the search friction, which leads to a higher (lower) rate of switching from the search mode to the no-search mode (λ).

5.1. Retargeting

Retargeting is a common practice where firms use email marketing, display ads, and other marketing tools to speed up consumers' purchase decisions. In the set-up presented here, retargeting can be viewed as increasing the consumer's switching rate from the no-search mode to the search mode (higher β).

Specifically, suppose the firm knows when the consumers are in the no search mode and can show consumers retargeting ads to raise their recovery rate from the non-search mode from β_0 to $\beta_r > \beta_0$ by incurring a retargeting cost of $k_r \geq 0$. The firm's objective is to maximize its expected payoff given the initial position of the consumer x_0 by choosing the optimal price and deciding whether to show retargeting ads, $\max_{P,\beta\in\{\beta_0,\beta_r\}} V_f(x_0) - k_r(\beta - \beta_0)$.[16] Because consumers will neither purchase nor gather new information in the no-search mode, one may think that firms always want to retarget consumers in the no-search mode to increase their likelihood of restarting the search, as long as the retargeting cost is sufficiently low. However, this only holds if consumers are oblivious to future retargeting. If a consumer is aware of the firm's retargeting strategy, then she expects to switch from the no-search mode to the search mode more frequently due to retargeting. As a result, she will want to search more before purchasing (higher $\overline{x}$ and $\widetilde{x}$) because she knows that she will stay in the search region for a longer proportion of the time due to retargeting. Because of discounting, a longer search time can be bad for the firm. It turns out that a higher rate of going back to the search mode can hurt the firm even if retargeting is costless. So, counter-intuitively, retargeting may backfire and hurt the firm even if it is free.

Proposition 5. *The firm does not retarget under any retargeting cost if the initial belief is high, $x_0 > x_0^{**}$. Suppose that λ is sufficiently large and the initial belief is low, $x_0 \leq 0$. Then, there exists a cutoff cost $\bar{k}$ such that the firm retargets if and only if the retargeting cost k_r is lower than $\bar{k}$.*

Several papers studying empirically the impact of retargeting use website (re)visit as the dependent variable, and they mainly find a positive result (e.g., Hoban and Bucklin 2015, Johnson, Lewis, and Nubbemeyer 2017, Sahni, Narayanan, and Kalyanam 2019). Our results show that the effect of retargeting on the time spent on search may be different from the effect of retargeting on profits. Spending too much time searching delays consumers' purchasing decisions and can hurt the firm. It suggests that empirical work on retargeting can benefit from examining multiple outcome variables.

5.2. User Interface Design

The firm can also design the user interface to make the search experience more or less likely to be distracted or interrupted. A higher likelihood of interruption corresponds to a higher switching rate from the search mode to the no-search mode (higher λ). The firm's objective is to choose the optimal $\lambda \in [\underline{\lambda}, \bar{\lambda}]$ that maximizes $V_f(x_0)$, $\max_{P,\lambda} V_f(x_0)$. One can view $[\underline{\lambda}, \bar{\lambda}]$ as the feasible space of λ. Due to constraints in user interface design and human limits, the firm can not completely avoid search interruptions. The firm also cannot distract consumers immediately, no matter how distracting the website is. Similar to the retargeting case, an increase in λ has both positive and negative effects on the firm's profit. On the one hand, more distractions will keep the consumer in the search mode for a shorter period of time, which is bad for the firm because the consumer

[16] It would be interesting to consider the case in which the firm is not fully aware if the consumer is in the no search mode. That case would lead however to a dynamic time-varying retargeting decision, which is beyond the scope of this paper. A detailed analysis in the absence of time-varying search costs can be found in Villas-Boas and Yao (2021).

neither purchases nor gathers new information in the no-search mode. On the other hand, the consumer will adopt the alternative more easily (lower $\overline{x}$ and $\widetilde{x}$) as she becomes more likely to be distracted. The consumer knows that she will more likely be interrupted and not able to search, and thus speeds up her decision. This can benefit the firm by pushing the consumer to purchase sooner.

It turns out that the effect of a higher likelihood of interruption can either hurt or benefit the firm.

Proposition 6. *The firm chooses $\lambda^* = \bar{\lambda}$ if the initial belief is high, $x_0 > x_0^{**}$. Suppose that β is sufficiently small. Then, the firm chooses $\lambda^* = \underline{\lambda}$ if the initial belief is low and the discount rate is small, $x_0 \leq 0$ and $r \leq (\sqrt{17} - 3)\lambda/4$.*

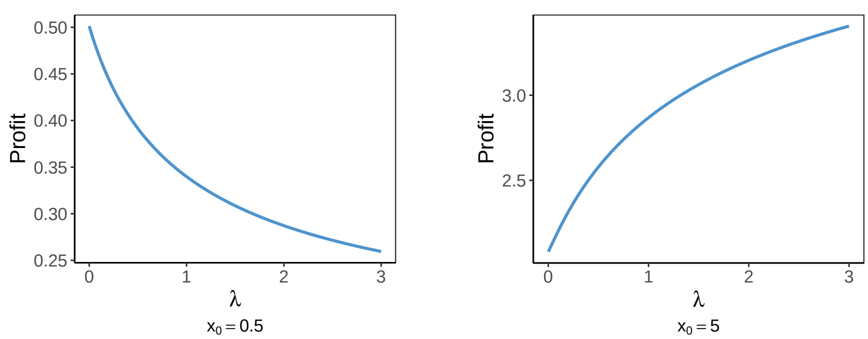


Figure 5: Example of the firm's profit as a function of λ, for $r = .05, \beta = .5$, and $\sigma^2 = 1$. On the right figure, x_0^{**} is always higher than x_0.

Figure 5 illustrates the firm's profit as a function of λ. As illustrated in the figure, the insights from Proposition 6 extend to a wider range of parameters. The firm's profit decreases in λ even if the initial belief is positive, as long as it is small. It increases in β even if the initial belief is lower than x_0^{**}, and the consumer does not adopt the alternative in the search mode, as long as it is large. Considering both choice deferral and choice closure gives us a more comprehensive understanding of the impact of search frictions on firm profits.

Propositions 5 and 6 are related to the effect of changing the deadline of an exploding offer to the consumer. An exploding offer with a given deadline can also be seen as a model with time-varying search costs where the cost goes to infinity after the deadline. Such time-varying costs change consumer's behavior. The main difference between our model and the exploding offer setting is that the consumer never goes back to search in the latter case, so there is no choice deferral in that case. Part of the results in Propositions 5 and 6 are driven by choice deferral, which delays the purchasing decision and hurts the firm's profits.

6. Extensions

We now consider extensions to the base model in which (i) the DM can become aware of increased fatigue, and (ii) there are start-up search costs.

6.1. Two Search Modes

We now consider a set-up in which the DM can become aware of her increased fatigue over time. We consider this possibility with the existence of two search modes, the fully-rested search mode 1 and the fatigued search mode 2. The DM moves from search mode 1 to search mode 2 at a hazard rate of λ_1, and then from search mode 2 to the no-search mode at a hazard rate of λ_2. For simplicity, we assume $\lambda_1 = \lambda_2 = \lambda$ in our analysis and discuss the case of $\lambda_1 \neq \lambda_2$ at the end of the section. Once in the no-search mode, the DM moves to search mode 1 at a hazard rate of β. This captures the idea that the DM is aware of search fatigue because the DM realizes that the no-search mode will arrive sooner when she is in search mode 2 than when she is in search mode 1.

In the construction of optimal decision-making, we are looking for three thresholds, $\overline{x}, \underline{x}$, and $\widetilde{x}$, with $\widetilde{x} < \underline{x} < \overline{x}$, such that in search mode 1 the DM adopts the alternative if $x \geq \overline{x}$, in search mode 2 the DM adopts the alternative if $x \geq \underline{x}$, and in the no-search mode the DM adopts the alternative if $x \geq \widetilde{x}$.

Let $V_1(x)$ be the expected payoff in search mode 1, $V_2(x)$ be the expected payoff in search mode 2, and $W(x)$ be the expected payoff in the no-search mode. Consider the Bellman equation in the no-search mode. We have

$$W(x) = \beta\, dt V_1(x) + (1 - \beta\, dt)e^{-r\, dt} W(x), \tag{18}$$

which leads to $W(x) = \frac{\beta}{r+\beta} V_1(x)$.

Consider now the Bellman equation in search mode 1. For $x \in (\underline{x}, \overline{x})$ we have

$$V_1(x) = (1 - \lambda\, dt)e^{-r\, dt} E V_1(x + dx) + \lambda\, dt x. \tag{19}$$

For $x < \underline{x}$ we have

$$V_1(x) = (1 - \lambda\, dt)e^{-r\, dt} E V_1(x + dx) + \lambda\, dt V_2(x). \tag{20}$$

Consider now the Bellman equation in search mode 2. For $x \in (\widetilde{x}, \underline{x})$ we have

$$V_2(x) = (1 - \lambda\, dt)e^{-r\, dt} E V_2(x + dx) + \lambda\, dt x. \tag{21}$$

Regarding the Bellman equation in search mode 2 for $x < \widetilde{x}$ we can obtain

$$V_2(x) = (1 - \lambda\, dt)e^{-r\, dt} EV_2(x + dx) + \lambda\, dt\, \frac{\beta}{r+\beta} V_1(x), \tag{22}$$

where we use that $W(x) = \frac{\beta}{r+\beta} V_1(x)$.

Applying Itô's Lemma to the Bellman equations, solving the corresponding differential equations, and using value matching and smooth pasting at the different thresholds, leads to a system of equations (presented and analyzed in the online appendix) to obtain $\overline{x}_1$, $\overline{x}_2$, and $\widetilde{x}$.

We illustrate the results for the general case in Figures A.5-A.8 in the Appendix. We observe that $\widetilde{x}$ decreases in λ, increases in β, and decreases in r. Thus, the extent of choice deferral is greater when the search process is interrupted less frequently, when the DM returns to search mode sooner after an interruption, and when the DM discounts the future less. We observe that $\overline{x} - \widetilde{x}$ decreases in λ, β, and r. Thus, the extent of choice closure is greater when the search process is interrupted less frequently, when search interruptions last longer, and when the DM discounts the future less. These observations match those from the base model.

Note that with two search modes, there are two types of choice closure. The first type of choice closure happens when the DM moves from search mode 1 to search mode 2. With $\underline{x} < \overline{x}$, the DM requires less positive information to adopt the alternative in the fatigued search mode 2 than in the fully-rested search mode 1, because the DM expects information gathering to be interrupted sooner. The extent of this choice closure is measured by $\overline{x} - \underline{x}$. The second type of choice closure happens when the DM moves from search mode 2 to the no-search mode. The extent of this choice closure is measured by $\underline{x} - \widetilde{x}$. From Figures A.5-A.8, we observe that the extent of choice closure in search mode 2, $\underline{x} - \widetilde{x}$, is greater than the extent of choice closure in search mode 1, $\overline{x} - \underline{x}$, for the parameter values considered, showing that greater fatigue leads to a greater extent of choice closure.

Choice Closure Behaviors for β and λ Small

To get sharper results on the DM's choice closure behaviors in different search modes, let us consider what happens when $\beta \to 0$, which makes $\widetilde{x} \to 0$. The online appendix presents the analysis for this case.

Proposition 7. *Consider the two search modes case, and assume that λ and β are sufficiently small. Then, $\overline{x} - \underline{x}$ increases in λ and $\underline{x} - \widetilde{x}$ decreases in λ. Both $\overline{x} - \underline{x}$ and $\underline{x} - \widetilde{x}$ decrease in the discount rate r while $\underline{x}/\sigma, \overline{x}/\sigma$, and $\frac{\overline{x}-\underline{x}}{\sigma}$ do not depend on the amount of information gained during search σ^2.*

The existence of two search modes leads to new insights in search mode 1. The extent of choice closure in search mode 1 behaves differently from the base model. A greater rate of search fatigue λ makes the DM more concerned about not being able to do further search. Thus, the DM has a stronger incentive to make a faster decision in both search modes, causing both $\overline{x}$ and $\underline{x}$ to decrease in λ. However, the effect is stronger in search mode 2 because a more fatigued DM expects search interruption to arrive sooner, causing $\overline{x} - \underline{x}$ to increase in λ. So, the extent of choice closure in search mode 1 increases in the rate of search fatigue, which is opposite to the comparative statics result in the base model.

The effects of λ, r, and σ^2 on the extent of choice closure in search mode 2 are similar to those in the base model. Intuitively, the consumer will switch from the search mode to the no-search mode when she becomes fatigued, just as in the base model.

Following the above discussion, we would then expect, in the case where the DM's rate of moving from search mode 1 to search mode 2, λ_1, is different from the DM's rate of moving from search mode 2 to the no-search mode, λ_2, the extent of choice closure in search mode 1 to increase in both λ_1 and λ_2, the extent of choice closure in search mode 2 to decrease in both λ_1 and λ_2, and the extent of choice deferral to decrease in both λ_1 and λ_2.

6.2. Start-Up Search Costs

The analysis above considered the strategic effects of choice deferral through discounting of future payoffs. We now consider the existence of start-up search costs in the beginning of the search mode and show that these start-up search costs yield strategic effects of choice deferral without discounting.

Consider the model of Section 2, but assume that the DM does not discount the future expected payoffs but has start-up search costs F when moving to the search mode from the no-search mode.[17] Furthermore, let us consider that the DM has ongoing search costs c per unit of time while in the search mode. The role of the search costs c is to give the DM an incentive to stop search and adopt the alternative in the search mode. Without the ongoing search costs and discounting, the DM would keep on learning information without making a decision until there would be a switch from the search to the no-search mode.

The optimal decision-making will involve the existence of four thresholds, $\overline{x}, \widetilde{x}, \widehat{x}$, and $\underline{\widehat{x}}$, with $\overline{x} > \widetilde{x} \geq 0 \geq \widehat{x} > \underline{\widehat{x}}$ such that the DM adopts the alternative in the search mode if $x \geq \overline{x}$, adopts the alternative when switching from the search mode to the no-search mode if $x \geq \widetilde{x}$, defers choice when switching from the search to the no-search mode if $x \in (\widehat{x}, \widetilde{x})$, stops search without adopting

[17] The start-up search costs F can also be seen as capturing in some way the possible effects of hyperbolic discounting (Laibson 1997).

the alternative when switching from the search to the no-search mode if $x \leq \widehat{x}$, and stops search in the search mode without adopting the alternative if $x < \underline{\widehat{x}}$.

Let $V(x)$ be the value function for the DM when in the search mode and $x \in (\widetilde{x}, \overline{x})$, $\widetilde{V}(x)$ be the value function for the DM when in the search mode and $x \in (\widehat{x}, \widetilde{x})$, and $\widehat{V}$ be the value function for the DM when in the search mode and $x \in (\underline{\widehat{x}}, \widehat{x})$. Furthermore, recall that $W(x)$ is the value function of the DM when in the no-search mode.

The Bellman equation of value function when the DM is in the no-search mode (which is relevant for $x \in (\widehat{x}, \widetilde{x})$) can be written as

$$W(x) = \beta\, dt[\widetilde{V}(x) - F] + (1 - \beta\, dt)W(x), \tag{23}$$

from which we can obtain $W(x) = \widetilde{V}(x) - F$.

When the DM is in the search mode and $x \in (\widehat{x}, \widetilde{x})$ we can then write the Bellman equation of the value function as

$$\widetilde{V}(x) = -c\, dt + (1 - \lambda\, dt)E\widetilde{V}(x + dx) + \lambda\, dt[\widetilde{V}(x) - F]. \tag{24}$$

The Bellman equation for $x \in (\widetilde{x}, \overline{x})$ can be written as

$$V(x) = -c\, dt + (1 - \lambda\, dt)EV(x + dx) + \lambda\, dt x. \tag{25}$$

The Bellman equation for $x \in (\underline{\widehat{x}}, \widehat{x})$ can be written as

$$\widehat{V}(x) = -c\, dt + (1 - \lambda\, dt)EV(x + dx). \tag{26}$$

Applying Itô's Lemma on the Bellman equations, solving the corresponding differential equations, and using value matching and smooth pasting at each threshold, leads to a system of equations, from which we can obtain, $\overline{x}$, $\widetilde{x}$, $\widehat{x}$, and $\underline{\widehat{x}}$.[18]

We can obtain $\overline{x} - \widetilde{x} = \widehat{x} - \underline{\widehat{x}}$, $\widetilde{x} = -\widehat{x}$, and

$$\begin{aligned} \widetilde{x} &= \max\left\{ \sqrt{\frac{\sigma^2}{2\lambda}} \frac{c}{2(\lambda F + c)} \frac{1 - H^2}{H} + \frac{\sigma^2}{4(\lambda F + c)}, 0 \right\} && (27) \\ \overline{x} &= \widetilde{x} + \frac{1}{\widehat{\eta}} \ln H, && (28) \end{aligned}$$

[18]The derivation of the solution is presented in the online appendix.

where

$$H = 1 + \frac{\lambda F}{c} + \sqrt{\left(\frac{\lambda F}{c} + 1\right)^2 - 1}. \tag{29}$$

and $\widehat{\eta} = \sqrt{2\lambda/\sigma^2}$.

Noting that $\widetilde{x} - \widehat{x}$ captures the extent of choice deferral and $\overline{x} - \widetilde{x}$ captures the extent of choice closure, we can obtain the following results.

Proposition 8. *Consider that there are start-up and ongoing search costs. Then the extent of choice deferral decreases in the current search costs c, in the start-up search costs F, and in the rate at which the DM switches from the search to the no-search mode, λ. Moreover, $\widetilde{x} = 0$ if the start-up search cost is high enough, $F \geq \sqrt{\frac{c^2}{\lambda^2} + \frac{\sigma^2}{8\lambda}}$. The extent of choice closure decreases in the ongoing search costs c and in the rate at which the DM switches from the search to the no-search mode, λ, and increases in the start-up search cost F.*

The ongoing search costs c play a similar role to the discount r in the base model with discounting. An increase in ongoing costs lowers the present value of future payoffs and encourages the DM to make a choice faster in the search mode. Thus, both the extent of choice deferral and the extent of choice closure decrease in c. An increase in start-up search costs F plays a similar role to a decrease in the rate of fatigue recovery β in the base model by decreasing the benefits of deferring choice. Thus, a higher F leads to a greater extent of choice closure and a lower extent of choice deferral. In particular, $\widetilde{x} = 0$ for F sufficiently high. Intuitively, if the start-up search costs are high enough, the DM does not restart search, and chooses to adopt the alternative if $x > 0$, when switching from the search to the no-search mode. Note that because there is no discounting in this case, the rate at which the DM switches from the no-search mode to the search mode, β, does not affect the extent of choice deferral or closure in this model.

The comparative statics on the rate at which the DM switches from the search to the no-search mode, λ, are in the same direction as in the base model. The DM expects more search interruptions at a higher λ which lowers the present value of future purchases and encourages the DM to make a choice faster in the search mode, causing both the extent of choice deferral and the extent of choice closure to decrease. This effect is consistent with the base model.

Given the above comparisons between the start-up costs model and the base model with discounting, if we consider adding start-up and ongoing search costs to the base model, we would expect the extent of choice deferral to decrease in the discount rate r, in the current search costs c, in the rate at which the DM switches from the search to the no-search mode λ, in the start-up search costs F, and increase in the rate at which the DM switches from the no-search mode to the search mode β. We would also expect the extent of choice closure to decrease in the discount rate r, in the current search costs c, in the rate at which the DM switches from the search to the no-search mode λ, in the rate at which the DM switches from the no-search mode to the search mode β, and increase in the start-up search costs F.

Figures A.9, A.10, and A.11 in the Appendix illustrate how the thresholds $\overline{x}, \widetilde{x}, \widehat{x}$, and $\underline{\widehat{x}}$ evolve as a function of the ongoing search costs, c, the hazard rate of switching from the search to the no-search mode, λ, and the start-up search costs, F. We observe that both the extent of choice deferral and the extent of choice closure decrease in the frequency of search interruptions λ, similar to the base model.

Empirical Applications

The model with start-up and continuing search costs can be more empirically relevant than the base model. First, the existence of search costs creates endogenous quitting behaviors that are ignored in the base model for tractability. Second, the strategic decision between deferral and closure in the base model relies on time discounting, but the extent of time discounting between different search opportunities may be relatively small. The existence of start-up search costs can create significant strategic effects without time discounting.

The model endogenously generates many consumer search and choice behaviors. For a given set of parameters, $(P, \lambda, \beta, c, F, \sigma^2, x_0)$, one can compute the distributions on the number of search sessions, the length of each session, and the portions of consumers who buy, defer, or quit after each search session through simulation. These moments allow one to estimate the model parameters using a dataset that contains browsing session-level information. The price P should be observable. The fatigue frequency λ can be inferred from the average length of search sessions from consumers who defer choice and resume search later. The recovery frequency β can be inferred from the average length of interruptions from consumers who defer choice and resume search later. The remaining parameters, (c, F, σ^2, x_0), can be estimated using the simulated method of moments (McFadden 1989, Lee and Ingram 1991, and Duffie and Singleton 1993).

The estimated model can be used to obtain prices close to the optimal. Managers may also be interested in the probability that a consumer will end her search session due to elevated search costs and the probability that the consumer will choose to defer instead of completing her choice.

7. Concluding Remarks

When searching for information to make a decision, an individual often faces interruptions to the information-gathering process due to time-varying search costs, potentially based on search fatigue. At the time when search is interrupted, decision-makers may decide to defer choices until they can gather information again because they do not have sufficient diagnostic information. Alternatively, when facing interruptions, decision-makers may strategically decide to make a choice immediately, even if they do not have sufficient diagnostic information, a behavior that the paper refers to as choice closure.

This paper investigates how the extent of choice deferral and the extent of choice closure respond to different environmental factors. We find that there is a greater extent of choice deferral when information gathering is interrupted less frequently, when individuals can resume gathering information sooner, and when individuals discount the future less. We also find that there is a greater extent of choice closure when information gathering is interrupted less frequently, when search interruptions last longer before individuals can resume gathering information, and when individuals discount the future less.

We investigate the effects of search fatigue by considering what happens when there are different stages in the search process with subsequently higher fatigue levels, showing that greater fatigue leads to a greater extent of choice closure. We also investigate the effects of start-up and ongoing search costs, in which case we can obtain strategic choice deferral and choice closure behaviors without time discounting. We find that the extent of choice deferral decreases in the ongoing search costs and in the start-up search costs, and the extent of choice closure decreases in the ongoing search costs but increases in the start-up search costs.

In terms of pricing, we find that the optimal price may be non-monotonic in consumers' initial beliefs about the product. For a low enough initial belief, we find that the optimal price increases when the speed of learning during information gathering is higher, when information gathering is interrupted less frequently, when consumers can resume gathering information sooner after interruptions, and when the firm and consumers discount the future less. These results suggest that firms should use data on consumer browsing sessions when determining price, and price should change following interventions to reduce search fatigue or restart consumer search sooner, such as redesigning user interface, ad retargeting, email marketing, and push notifications.

In addition to pricing, we also consider other managerial decisions that affect consumers' search environment. In particular, we study user interface design and retargeting. Firms can design the user interface to make the search process more or less likely to be interrupted. Existing research has had varied findings about the impact of search frictions on firm profits. Considering both choice deferral and choice closure gives us a more comprehensive understanding. On the one hand, a higher rate of search fatigue keeps the consumer in the search mode for a shorter period of time, which is bad for the firm. On the other hand, it incentivizes the consumer to adopt the alternative more easily because of choice closure, which is good for the firm. We characterize when firms prefer a higher rate of search fatigue and when they prefer a lower level of search fatigue. A similar mechanism plays a critical role in firms' retargeting decisions. We show that, counter-intuitively, retargeting may backfire and hurt the firm even if it is costless, because it reduces the positive effects of consumers' choice closure behaviors.

Table 1: Notation

Variable	Description
x	expected value of adopting the alternative
r	continuous discount rate
r_{mb}	adjusted discount rate in the Model-Free-Fatigue Benchmark
λ	hazard rate of the DM moving from the search mode to the no-search mode
β	hazard rate of the DM moving from the no-search mode to the search mode
$\overline{x}$	adoption threshold in the search mode
	adoption threshold in search mode 1 (two search modes model)
$\overline{x}_{nb}$	adoption threshold in the No-Fatigue Benchmark
$\overline{x}_{mb}$	adoption threshold in the Model-Free-Fatigue Benchmark
$\widetilde{x}$	adoption threshold in the no-search mode
$V(x)$	expected payoff for the DM in the search mode
	expected payoff for the DM in the search mode when $x > \widetilde{x}$ (search costs model)
$W(x)$	expected payoff for the DM in the no-search mode
δ	$\overline{x} - \widetilde{x}$
η	$\sqrt{\frac{2r}{\sigma^2}\frac{r+\beta+\lambda}{r+\beta}}$
$\widetilde{\eta}$	$\sqrt{\frac{2(r+\lambda)}{\sigma^2}}$
D	$e^{\widetilde{\eta}\delta}$
λ_1	hazard rate from search mode 1 to search mode 2 (two search modes model)
λ_2	hazard rate from search mode 2 to the no-search mode (two search modes model)
$\underline{x}$	adoption threshold in search mode 2 (two search modes model)
$V_1(x)$	expected payoff for the DM in search mode 1 (two search modes model)
$V_2(x)$	expected payoff for the DM in search mode 2 (two search modes model)
F	start-up search costs (search costs model)
c	ongoing search costs per unit of time (search costs model)
$\widehat{x}$	threshold to stop search in the no-search mode (search costs model)
$\underline{\widehat{x}}$	threshold to stop search in the search mode (search costs model)
$\widetilde{V}(x)$	expected payoff for the DM in the search mode for $x \in (\widehat{x}, \widetilde{x})$ (search costs model)
$\widehat{V}(x)$	expected payoff for the DM in the search mode for $x \in (\underline{\widehat{x}}, \widehat{x})$ (search costs model)
$\widehat{\eta}$	$\sqrt{\frac{2\lambda}{\sigma^2}}$
P	price
$P^*(x_0)$	the firm's optimal price if $x = x_0$ at time 0
y	$x - P$
$V_f(x)$	expected payoff for the firm if the DM is in the search mode
$W_f(x)$	expected payoff for the firm if the DM is in the no-search mode
$V_f(x, P)$	$V_f(x)$ for $x \in (\widetilde{x} + P, \overline{x} + P)$
x_0^*	$\widetilde{x} + 1/\eta$
$h(P, x)$	equation defined in (xxi)
x_0^{**}	the solution to the implicit equation $x_0^{**} - \overline{x} = V_f(x_0^{**}, P^*(x_0^{**}))$

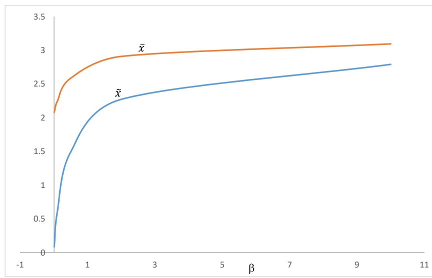
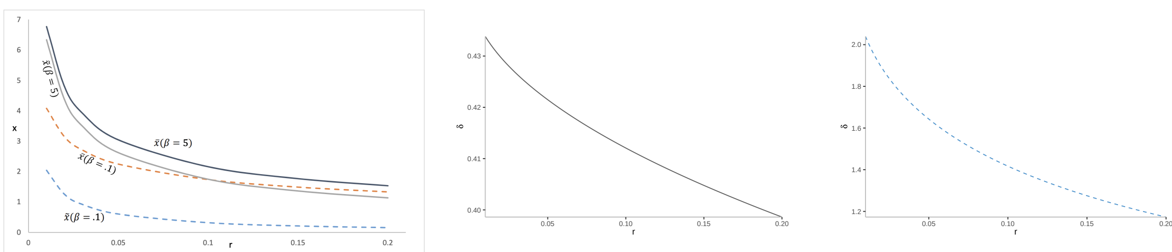


Figure A.1: Base model: Example of the purchase thresholds $\overline{x}$ and $\widetilde{x}$ as a function of β for $r = .05, \lambda = .5$, and $\sigma^2 = 1$.



(a) Purchase thresholds (b) δ when $\beta = 5$ (c) δ when $\beta = .1$

Figure A.2: Base model: Example of the purchase thresholds $\overline{x}$, $\widetilde{x}$, and their difference $delta = \overline{x} - \widetilde{x}$ as a function of r for $\lambda = .5$, $\sigma^2 = 1$, and $\beta = .1, 5$. (δ is the vertical distance between $\overline{x}$ and $\widetilde{x}$. Human eyes tend to view it as the straight line distance between $\overline{x}$ and $\widetilde{x}$, which leads to an optical illusion about the comparative statics of δ. So, we draw δ separately in two figures for clarity.)

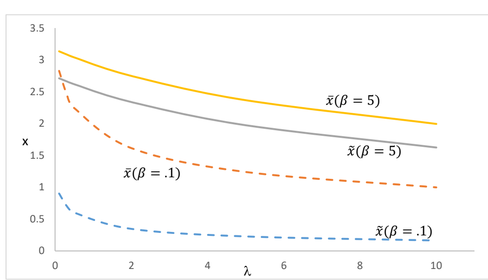


Figure A.3: Base model: Example of the purchase thresholds $\overline{x}$ and $\widetilde{x}$ as a function of λ for $r = .05$, $\sigma^2 = 1$, and $\beta = .1, 5$.

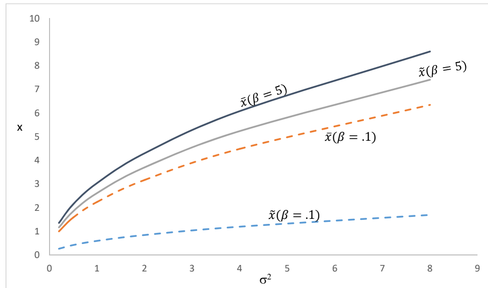


Figure A.4: Base model: Example of the purchase thresholds $\overline{x}$ and $\widetilde{x}$ as a function of σ^2 for $r = .05$, $\lambda = .5$, and $\beta = .1, 5$.

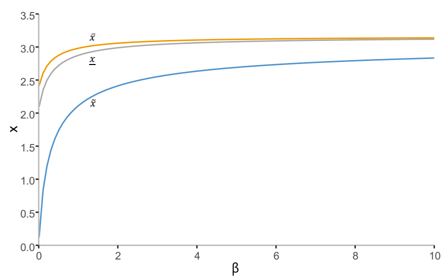


Figure A.5: Two search modes: Example of the purchase thresholds $\overline{x}$, $\underline{x}$, and $\widetilde{x}$ for the two search modes case as a function of β for $r = .05, \lambda = .5$, and $\sigma^2 = 1$.

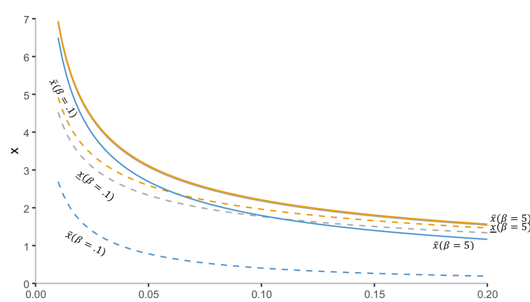


Figure A.6: Two search modes: Example of the purchase thresholds $\overline{x}$, $\underline{x}$, and $\widetilde{x}$ for the two search modes case as a function of r for $\lambda = .5$, $\sigma^2 = 1$, and $\beta = .1, 5$.

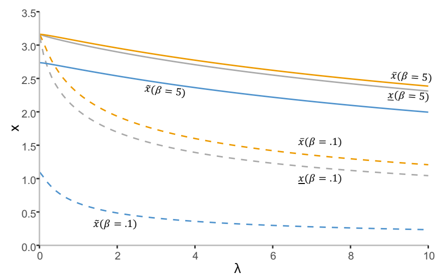


Figure A.7: Two search modes: Example of the purchase thresholds $\overline{x}$, $\underline{x}$, and $\widetilde{x}$ for the two search modes case as a function of λ for $r = .05$, $\sigma^2 = 1$, and $\beta = .1, 5$.

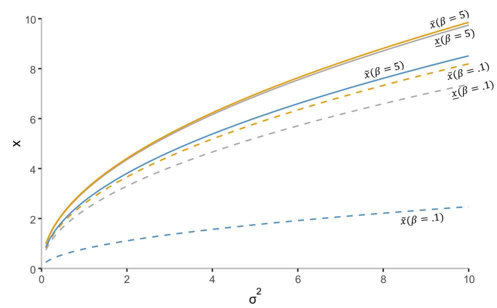


Figure A.8: Two search modes; Example of the purchase thresholds $\overline{x}$, $\underline{x}$, and $\widetilde{x}$ for the two search modes case as a function of σ^2 for $r = .05$, $\lambda = .5$, and $\beta = .1, 5$.

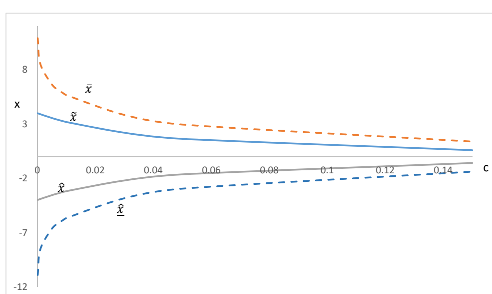


Figure A.9: Start-up search costs: Evolution of the stop/search thresholds for the start-up search costs case as a function of c for $\lambda = .5$, $\sigma^2 = 1$, and $F = .1$.

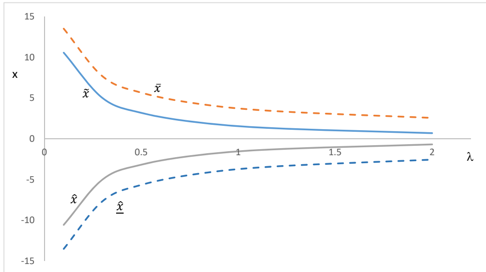


Figure A.10: Start-up search costs: Evolution of the stop/search thresholds for the start-up search costs case as a function of λ for $c = .01$, $\sigma^2 = 1$, and $F = .1$.

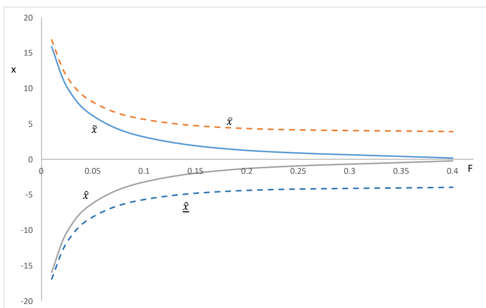


Figure A.11: Start-up search costs: Evolution of the stop/search thresholds for the start-up search costs case as a function of F for $\lambda = .5$, $\sigma^2 = 1$, and $c = .01$.

Declarations

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ONLINE APPENDIX

PROOF OF LEMMA 1:

First, note that the consumer's expected payoff of adopting the alternative in the search mode is non-positive if the purchasing threshold $\overline{x} \leq 0$. In contrast, because there is a positive probability of reaching any belief within a given period of time, the consumer's expected payoff of adopting the alternative in the search mode is strictly positive if the purchasing threshold $\overline{x} > 0$. Therefore, the optimal $\overline{x} > 0$.

Suppose the purchasing threshold in the no-search mode is higher than that in the search mode, $\widetilde{x} \geq \overline{x}$. Because the belief evolves continuously, one can see that the consumer never adopts the alternative in the no-search mode. We now show that the consumer can be strictly better off by using a lower purchasing threshold in the no-search region. Consider $\widetilde{x}' := \overline{x} - \epsilon$, where $\epsilon > 0$. Consider $x = \widetilde{x}'$ in the no-search mode. The consumer will adopt the alternative immediately and obtain a payoff of $\widetilde{x}'$ if the threshold in the no-search mode is $\tilde{x}'$.

Denote the consumer's expected payoff in the no-search mode by $W(x)$ and in the search mode by $V(x)$ if the threshold in the no-search mode is $\widetilde{x}$. We have $W(\widetilde{x}') = \mathbb{E}[e^{-rN_1}V(\widetilde{x}')]$, where N_1 is the time of the first arrival of a Poisson process with rate β. We have:

$$\begin{aligned} W(\widetilde{x}') =& \mathbb{E}[e^{-rN_1}V(\widetilde{x}')] \\ \leq& \mathbb{E}[e^{-rN_1}V(\overline{x})] \\ =& \mathbb{E}[e^{-rN_1}]\overline{x} \\ <& [\mathbb{P}(N_1 \leq 1) \cdot 1 + \mathbb{P}(N_1 > 1) \cdot e^{-r}]\overline{x} \\ =& [(1 - e^{-\beta}) \cdot 1 + e^{-\beta} \cdot e^{-r}]\overline{x} \\ =& \overline{x} - e^{-\beta}(1 - e^{-r})\overline{x} \end{aligned}$$

One can then see that $W(\widetilde{x}') < \widetilde{x}' = \overline{x} - \epsilon$ for ϵ small enough. Therefore, the threshold $\widetilde{x} \geq \overline{x}$ cannot be optimal.

DERIVATION OF SOLUTION TO BASE CASE WITH DISCOUNTING:

Given that $\lim_{x \to -\infty} V(x) = 0$, as the expected payoff of the DM has to approach zero if the expected payoff of the alternative approaches negative infinity, we have that the solution to (5) satisfies

$$V(x) = A_1 e^{\eta x} \tag{i}$$

where A_1 is a constant to be determined.

Similarly, applying Itô's Lemma to (2), we can obtain the solution to the second order differential equation in $V(x)$ for $x \in (\widetilde{x}, \overline{x})$ as

$$V(x) = A_2 e^{\widetilde{\eta} x} + A_3 e^{-\widetilde{\eta} x} + \frac{\lambda}{r+\lambda} x, \tag{ii}$$

where A_2 and A_3 are constants to be determined.

Using value matching and smooth pasting of $V(x)$ at $\widetilde{x}$ and $\overline{x}$, $V(\widetilde{x}^-) = V(\widetilde{x}^+), V'(\widetilde{x}^-) = V'(\widetilde{x}^+)$, $V(\overline{x}) = \overline{x}$, and $V'(\overline{x}) = 1$, and $W(\widetilde{x}) = \widetilde{x}$, we obtain the following system of five equations to obtain $\widetilde{x}, \overline{x}, A_1, A_2$, and A_3.

$$\begin{aligned}
A_2 e^{\widetilde{\eta}\overline{x}} + A_3 e^{-\widetilde{\eta}\overline{x}} + \frac{\lambda}{r+\lambda}\overline{x} &= \overline{x} && \text{(iii)} \\
\widetilde{\eta} A_2 e^{\widetilde{\eta}\overline{x}} - \widetilde{\eta} A_3 e^{-\widetilde{\eta}\overline{x}} + \frac{\lambda}{r+\lambda} &= 1 && \text{(iv)} \\
A_2 e^{\widetilde{\eta}\widetilde{x}} + A_3 e^{-\widetilde{\eta}\widetilde{x}} + \frac{\lambda}{r+\lambda}\widetilde{x} &= A_1 e^{\eta\widetilde{x}} && \text{(v)} \\
\widetilde{\eta} A_2 e^{\widetilde{\eta}\widetilde{x}} - \widetilde{\eta} A_3 e^{-\widetilde{\eta}\widetilde{x}} + \frac{\lambda}{r+\lambda} &= \eta A_1 e^{\eta\widetilde{x}} && \text{(vi)} \\
\frac{\beta}{r+\beta} A_1 e^{\eta\widetilde{x}} &= \widetilde{x}. && \text{(vii)}
\end{aligned}$$

Using (iii)-(vii), we can obtain a system of two equations to obtain $\widetilde{x}$ and $\overline{x}$ as

$$\begin{aligned}
e^{\widetilde{\eta}(\overline{x}-\widetilde{x})} &= \frac{\frac{r}{r+\lambda}\overline{x} + \frac{r}{\widetilde{\eta}(r+\lambda)}}{\widetilde{x}\left(\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right) + \frac{1}{\widetilde{\eta}}\left(\eta\widetilde{x}\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right)} && \text{(viii)} \\
e^{\widetilde{\eta}(\overline{x}-\widetilde{x})} &= \frac{\widetilde{x}\left(\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right) - \frac{1}{\widetilde{\eta}}\left(\eta\widetilde{x}\frac{r+\beta}{\beta} - \frac{\lambda}{r+\lambda}\right)}{\frac{r}{r+\lambda}\overline{x} - \frac{r}{\widetilde{\eta}(r+\lambda)}}. && \text{(ix)}
\end{aligned}$$

Using $\delta = \overline{x} - \widetilde{x}$ we can rewrite (viii) and (ix), as a system of equations for δ and $\widetilde{x}$ as

$$\begin{aligned}
\widetilde{x} &= \beta \frac{r + r\widetilde{\eta}\delta + \lambda D}{D[\widetilde{\eta} r(r+\beta+\lambda) + \eta(r+\beta)(r+\lambda)] - \widetilde{\eta}\beta r} && \text{(x)} \\
\widetilde{x} &= \beta \frac{\lambda + rD - \widetilde{\eta} r\delta D}{\widetilde{\eta} r\beta D + \eta(r+\beta)(r+\lambda) - \widetilde{\eta} r(r+\beta+\lambda)} && \text{(xi)}
\end{aligned}$$

where $D = e^{\widetilde{\eta}\delta} = e^{\widetilde{\eta}(\overline{x}-\widetilde{x})}$.[19] Using (x) and (xi) we can obtain (6) in the main text, from which we can obtain δ. We can then use (x) or (xi) to obtain $\widetilde{x}$. Lastly, we can obtain $\overline{x}$ from δ and $\widetilde{x}$ because $\overline{x} = \widetilde{x} + \delta$.

Derivation of equation (10) in the Base Case: Since $\delta \to 0$ as $\beta \to +\infty$ and $D = 1 + \widetilde{\eta}\delta + o(\delta)$,

[19] Since D depends on $\overline{x}$, (x) and (xi) can also be viewed as a system of equations for $\overline{x}$ and $\widetilde{x}$.

we have

$$
\begin{aligned}
&\beta(D-1)^2 \to 2(r+\lambda)\\
\Rightarrow\ &\beta[\widetilde{\eta}\delta + o(\delta)]^2 = 2(r+\lambda) + o(1)\\
\Rightarrow\ &\beta\widetilde{\eta}^2\delta^2 = 2(r+\lambda) + o(1)\\
\Rightarrow\ &\beta\delta^2 = \sigma^2 + o(1)\\
\Rightarrow\ &\sqrt{\beta}\delta \to \sigma,\ \text{as } \beta \to +\infty
\end{aligned}
$$

Proof of Proposition 1:

We provide proof for the comparative statics w.r.t. σ^2. The proofs for the comparative statics w.r.t. β, r, and λ are similar.

Given $\sigma_s^2 < \sigma_\ell^2$ and the corresponding cutoff beliefs $(\overline{x}_s, \widetilde{x}_s)$, $(\overline{x}_\ell, \widetilde{x}_\ell)$, we want to show that $\overline{x}_\ell \geq \overline{x}_s$ and $\widetilde{x}_\ell \geq \widetilde{x}_s$.

Suppose $\overline{x}_\ell < \overline{x}_s$. Then $V_s(\overline{x}_\ell) > \overline{x}_\ell$, because the DM keeps searching for information when the belief is $\overline{x}_\ell$ and $\sigma^2 = \sigma_s^2$. Also, $V_\ell(\overline{x}_\ell) = \overline{x}_\ell$, because the DM takes the alternative when the belief is $\overline{x}_\ell$ and $\sigma^2 = \sigma_\ell^2$. Therefore, $V_s(\overline{x}_\ell) > V_l(\bar{x}_l)$.

However, one can see that the DM can achieve a payoff of at least $V_s(\bar{x}_\ell)$ when $\sigma^2 = \sigma_\ell^2$ and the belief is $\overline{x}_\ell$ by using the optimal strategy when $\sigma^2 = \sigma_s^2$ (which may be sub-optimal when $\sigma^2 = \sigma_\ell^2$). Therefore, $V_s(\overline{x}_\ell) \leq V_\ell(\overline{x}_\ell)$, a contradiction. So, $\overline{x}_\ell \geq \overline{x}_s$.

Now suppose that $\widetilde{x}_\ell < \widetilde{x}_s$. Then $W_s(\widetilde{x}_\ell) > \widetilde{x}_\ell$, because the DM defers the choice when the belief is $\widetilde{x}_\ell$ and $\sigma^2 = \sigma_s^2$. Also, $W_\ell(\widetilde{x}_\ell) = \widetilde{x}_\ell$ because the DM takes the alternative in the no-search mode when the belief is $\widetilde{x}_\ell$ and $\sigma^2 = \sigma_\ell^2$. Therefore, $V_s(\widetilde{x}_\ell) = \frac{r+\beta}{\beta} W_s(\widetilde{x}_\ell) > \frac{r+\beta}{\beta} W_\ell(\widetilde{x}_\ell) = V_\ell(\widetilde{x}_\ell)$.

However, one can see that the DM can achieve a payoff of at least $V_s(\widetilde{x}_\ell)$ when $\sigma^2 = \sigma_\ell^2$ and the belief is $\widetilde{x}_\ell$ by using the optimal strategy when $\sigma^2 = \sigma_s^2$ (which may be sub-optimal when $\sigma^2 = \sigma_\ell^2$). Therefore, $V_s(\widetilde{x}_\ell) \leq V_\ell(\widetilde{x}_\ell)$, a contradiction. So, $\widetilde{x}_\ell \geq \widetilde{x}_s$.

Optimal Pricing for Benchmarks:

Let V_f^{nb} denote the firm's value function in the No-Fatigue Benchmark. The consumer buys when $x > \overline{x}_{nb} + P$. Note that if $x_0 \geq \overline{x}_{nb} + P$, the consumer buys immediately, thus the firm should charge $P = x_0 - \overline{x}_{nb}$. For $x < \overline{x}_{nb} + P$, we have

$$V_f^{nb}(x) = e^{-rdt} EV(x + dx) \tag{xii}$$

Applying Itô's Lemma and solving the resulting differential equation, we get

$$V_f^{nb}(x) = A_{nb}e^{\sqrt{\frac{2r}{\sigma}}x} + B_{nb}e^{-\sqrt{\frac{2r}{\sigma}}x} \tag{xiii}$$

where A_{nb} and B_{nb} are constants to be solved. Because $V_f^{nb}(x) \to 0$ as $x \to -\infty$, we must have $B_{nb} = 0$. The constant A_{nb} is solved by applying the boundary condition $V_f^{nb}(\overline{x}_{nb} + P) = P$. This produces

$$V_f^{nb}(x) = Pe^{\sqrt{\frac{2r}{\sigma}}(x-\overline{x}_{nb}-P)} \tag{xiv}$$

which is maximized at $P_{nb}^* = \overline{x}_{nb} = \sqrt{\frac{\sigma^2}{2r}}$. The optimal price is $P_{nb}^* = \sqrt{\frac{\sigma^2}{2r}}$ for $x_0 < 2\sqrt{\frac{\sigma^2}{2r}}$ and $P_{nb}^* = x_0 - \sqrt{\frac{\sigma^2}{2r}}$ for $x_0 \geq 2\sqrt{\frac{\sigma^2}{2r}}$.

A similar analysis of the Model-Free-Fatigue Benchmark shows that the optimal price is $P_{mb}^* = \sqrt{\frac{\sigma^2}{2r}\frac{\beta}{\lambda+\beta}}$ for $x_0 < 2\sqrt{\frac{\sigma^2}{2r}\frac{\beta}{\lambda+\beta}}$ and $P_{mb}^* = x_0 - \sqrt{\frac{\sigma^2}{2r}\frac{\beta}{\lambda+\beta}}$ for $x_0 \geq 2\sqrt{\frac{\sigma^2}{2r}\frac{\beta}{\lambda+\beta}}$.

Some Analysis of Optimal Pricing:

Substituting $W_f(x) = \frac{\beta}{r+\beta}V_f(x)$ into (11), and using Itô's Lemma, we can obtain the second order differential equation in $V_f(x)$ for $x < \widetilde{x} + P$ as

$$r\frac{r+\beta+\lambda}{r+\beta}V_f(x) = \frac{\sigma^2}{2}V_f''(x). \tag{xv}$$

Given that $\lim_{x\to-\infty} V_f(x) = 0$, as the expected payoff of the firm has to approach zero if the expected payoff of the alternative approaches negative infinity, we have that the solution to (xv) satisfies

$$V_f(x) = \widetilde{A}_1 e^{\eta x} \tag{xvi}$$

where $\widetilde{A}_1$ is a constant to be determined.[20]

Similarly, applying Itô's Lemma to (12), we can solve the resulting second order differential equation in $V_f(x)$ for $x \in (\widetilde{x} + P, \overline{x} + P)$ as

$$V_f(x) = \widetilde{A}_2 e^{\widetilde{\eta}x} + \widetilde{A}_3 e^{-\widetilde{\eta}x} + \frac{\lambda}{r+\lambda}P \tag{xvii}$$

where $\widetilde{A}_2$ and $\widetilde{A}_3$ are constants to be determined.

[20]Recall that $\eta = \sqrt{\frac{2r}{\sigma^2}\frac{r+\beta+\lambda}{r+\beta}}$ and $\widetilde{\eta} = \sqrt{\frac{2(r+\lambda)}{\sigma^2}}$.

Conditions (14)-(16) can be written as:

$$\begin{aligned}
P &= \widetilde{A}_2 e^{\widetilde{\eta}(\overline{x}+P)} + \widetilde{A}_3 e^{-\widetilde{\eta}(\overline{x}+P)} + \frac{\lambda}{r+\lambda}P && \text{(xviii)}\\
\widetilde{A}_1 e^{\eta(\widetilde{x}+P)} &= \widetilde{A}_2 e^{\widetilde{\eta}(\widetilde{x}+P)} + \widetilde{A}_3 e^{-\widetilde{\eta}(\widetilde{x}+P)} + \frac{\lambda}{r+\lambda}P && \text{(xix)}\\
\widetilde{A}_1 \eta e^{\eta(\widetilde{x}+P)} &= \widetilde{A}_2 \widetilde{\eta} e^{\widetilde{\eta}(\widetilde{x}+P)} - \widetilde{A}_3 \widetilde{\eta} e^{-\widetilde{\eta}(\widetilde{x}+P)}. && \text{(xx)}
\end{aligned}$$

Taking the derivative of (xvii) with respect to price and making it equal to zero, yields the optimal price for $x_0 \in [x_0^*, x_0^{**}]$. This yields an equation $h(P, x_0) = 0$ which is represented by

$$h(P, x_0) = \frac{\lambda}{r+\lambda}P^* + \widetilde{A}_2(1-\widetilde{\eta}P^*)e^{\widetilde{\eta}x_0} + \widetilde{A}_3(1+\widetilde{\eta}P^*)e^{-\widetilde{\eta}x_0} = 0, \qquad \text{(xxi)}$$

where $\widetilde{A}_2$ and $\widetilde{A}_3$ are both functions of price.

In order to obtain some more specific results we consider two particular cases.

The Case of $\beta \to 0$ for $x_0 \in [x_0^*, x_0^{**}]$:

When $\beta \to 0$, we have $\eta \to \widetilde{\eta}, \widetilde{x} \to 0$, and $e^{\eta\overline{x}}(1-\eta\overline{x}) + \frac{\lambda}{r} = 0$ (from which we recall that $\overline{x} > 1/\eta$).

From (xvii)-(xx) we can obtain that in the limit

$$\begin{aligned}
V_f(x_0) =& \frac{P}{r+\lambda}\left[\frac{r(\eta\overline{x}+1)}{2}e^{\eta(x_0-\overline{x}-P)} - \frac{\lambda}{2}e^{\eta(P-x_0)} + \lambda\right]\\
\text{sign}\left\{\frac{\partial V_f(x)}{\partial P}\right\} =& \text{sign}\left\{\lambda + (1-\eta P)\frac{r(\eta\overline{x}+1)}{2}e^{\eta(x_0-\overline{x}-P)} - (1+\eta P)\frac{\lambda}{2}e^{\eta(P-x_0)}\right\}.
\end{aligned}$$

Note that in this case we have $x_0^* \to 1/\eta$. So, for $x_0 > x_0^*$, we can obtain that $\frac{\partial V_f(x_0)}{\partial P} > 0$ for $P = 1/\eta$. Furthermore, we can obtain that $\frac{\partial V_f(x_0)}{\partial P} < 0$ for $P = x_0$. So, we have that for $x_0 \in [x_0^*, x_0^{**}]$ we have that $P^* \in [1/\eta, x_0]$. If the price function is continuous at x_0^{**}, then from the definition of x_0^{**} we can also obtain for this case of $\beta \to 0$ that $x_0^{**} \to \overline{x} + \frac{r+\lambda}{r\eta^2\overline{x}}$.

To check whether the price function is continuous at x_0^{**}, we can check whether for x_0^{**} obtained by $h(x_0^{**} - \overline{x}, x_0^{**}) = 0$ we have that $V_f(x_0^{**}, P)$ is concave in the price P when $P = x_0^{**} - \overline{x}$. This condition yields, using (8), $\lambda/r < 2a^2 - 1$ where $a > 1$ satisfies $e^a(a-1) - 2a^2 + 1 = 0$.[21] For $\lambda/r > 2a^2 - 1$ we then have then that the price function cannot be continuous at x_0^{**} and we then have x_0^{**} obtained by $x_0^{**} - \overline{x} = V_f(x_0^{**}, P^*(x_0^{**}))$ and $P^*(x_0^{**}) \in \arg\max_P V_f(x_0^{**}, P)$, and that the optimal price falls at the discontinuity, $\lim_{x_0 \nearrow x_0^{**}} P^*(x_0) > x_0^{**} - \overline{x}$.

[21]This yields $a \approx 1.94$ and $2a^2 - 1 \approx 6.51$.

For the case in which the price function is continuous at x_0^{**} we can also obtain that the price function is not monotonic in x_0 for $x_0 \in [x_0^*, x_0^{**}]$. Note that $\frac{\partial^2 V_f(x_0)}{\partial P \partial x_0}_{|x_0 = x_0^*} > 0$ so that the optimal price is increasing in x_0 for x_0 close to x_0^*. Note also that $\frac{\partial^2 V_f(x_0)}{\partial P \partial x_0}_{|x_0 = x_0^{**}}$ when the price function is continuous at x_0^{**} can be negative if $\eta \overline{x} < \sqrt{1 + \lambda / r}$. Using (8), we can then obtain that this condition always holds when the price function is continuous. In this case of $\beta \to 0$ and continuous price function, we then obtain that the optimal price is decreasing in x_0 for x_0 close to x_0^{**}.

Since $P^* < x_0 - \widetilde{x}$ for $x_0 > x_0^*$, the DM adopts the alternative at x_0 in the no-search mode. Since $P = x_0 - \overline{x}$ is optimal for $x_0 > x_0^{**}$, the DM adopts the alternative at x_0 in the search mode in that case. We can show that the firm's value function decreases in the price P when x_0 is high enough. When the DM's prior belief about the alternative is high enough, the firm can already obtain a high payoff by inducing the DM to adopt the alternative immediately without searching.

The Case of $\beta \to \infty$ for $x_0 \in [x_0^*, x_0^{**}]$:

When $\beta \to \infty$, we have $\overline{x}, \widetilde{x} \to \sqrt{\frac{\sigma^2}{2r}}$. Therefore, the interval $[x_0 - \overline{x}, x_0 - \widetilde{x}]$ disappears and the possible optimal prices are $P \geq x_0 - \widetilde{x}$. From the previous analysis, one can see that the optimal price is

$$P^* = \begin{cases} 1/\eta, \ if \ 1/\eta > x_0 - \widetilde{x} \\ x_0 - \widetilde{x}, \ otherwise. \end{cases}$$

To get that the price function is continuous for β large, we can obtain that $\frac{\partial^2 V_f(x,P)}{\partial P^2}_{|x = x_0^{**}, P = x_0^{**} - \overline{x}}$ is strictly negative for $\beta \to \infty$ and x_0^{**} satisfying $h(x_0^{**} - \overline{x}, x_0^{**}) = 0$. The result that the price function is monotonic for $\beta \to \infty$ is straightforward to obtain since we have $x_0^{**} - x_0^* \to 0$ for $\beta \to \infty$.

Optimal Pricing in a Two-period Model:

Consider a similar setup in discrete time. There are two periods, $t = 1, 2$. The DM can search for information about the alternative at most twice. Given the belief at the beginning of each period, x_t, the DM's belief will become $x_t + \Delta$ or $x_t - \Delta$ with equal probability if she is in the search mode and decides to search. The DM can adopt the alternative without searching, after searching once, or after searching twice. She may switch from the search to the no-search mode with probability λ at the end of the first period. The discount factor per period of both the firm and the DM is $\widehat{\delta}$. Let us first consider the optimal search strategy of the DM, where $y_t = x_t - P$.

Proposition 9. *Suppose* $4\widehat{\delta} + (1 - \lambda)\widehat{\delta}^2 > 4$.[22] *If* $y_0 \geq \frac{2(1-\lambda)\widehat{\delta}}{4 - 2\widehat{\delta} - 2\lambda\widehat{\delta} - (1-\lambda)\widehat{\delta}^2}\Delta$ *the DM adopts the*

[22] If this condition is not satisfied, the threshold of adopting the alternative without searching is different. But the intuition of the entire analysis is the same. We omit the presentation of that case for simplicity.

alternative without searching. If $y_0 \in [\Delta, \frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta)$ *the DM adopts the alternative after receiving a positive signal, receiving a negative signal and then a positive signal, or switching to the no-search mode at the end of period one. If* $y_0 \in [-\frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta, \Delta)$ *The DM adopts the alternative after receiving a positive signal or receiving a negative signal and then a positive signal. If* $y_0 \in [-\Delta, -\frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta)$ *the DM adopts the alternative after receiving two positive signals, or receiving a positive signal and then switching to the no-search mode. If* $y_0 \in [-2\Delta, -\Delta]$ *the DM adopts the alternative after receiving two positive signals.*

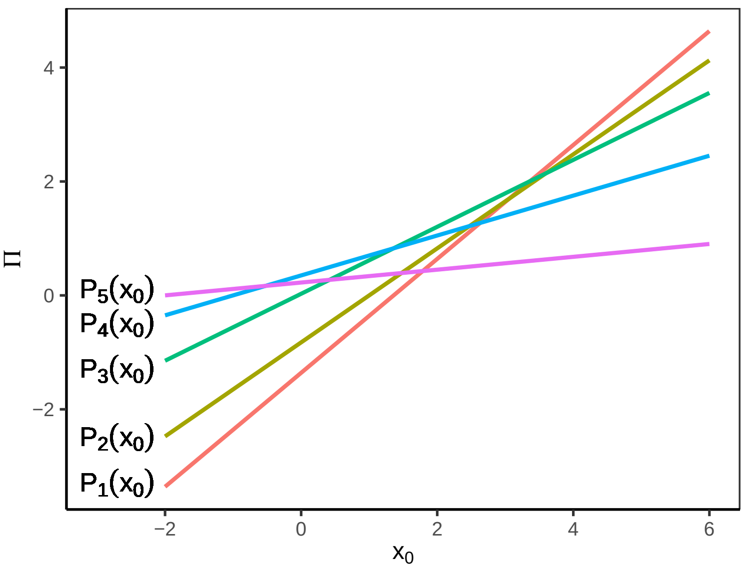

Figure A.1: Example of the firm's profit Π as a function of x_0 for $r = .95, \lambda = .5$, and $\Delta = 1$.

From this proposition, one can see that the DM's adoption likelihood is piecewise constant and non-decreasing in x_0. Therefore, the firm will only choose from prices such that $y_0 = x_0 - P \in \{\frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta, \Delta, -\frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta, -\Delta, -2\Delta\}$. Denote those price schemes by $P_1(x_0) = x_0 - \frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta, P_2(x_0) = x_0 - \Delta, P_3(x_0) = x_0 + \frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta, P_4(x_0) = x_0 + \Delta, P_5(x_0) = x_0 + 2\Delta$. Note that the price increases from $P_1(x_0)$ to $P_5(x_0)$ for a given x_0. The corresponding profits are: $\Pi_1(x_0) = x_0 - \frac{2(1-\lambda)\widehat{\delta}}{4-2\widehat{\delta}-2\lambda\widehat{\delta}-(1-\lambda)\widehat{\delta}^2}\Delta, \Pi_2(x_0) = (\frac{\widehat{\delta}}{2} + \frac{\lambda\widehat{\delta}}{2} + \frac{1-\lambda}{4}\widehat{\delta}^2)(x_0 - \Delta), \Pi_3(x_0) = (\frac{\widehat{\delta}}{2} + \frac{1-\lambda}{4}\widehat{\delta}^2)(x_0 + \frac{1-\widehat{\delta}}{2-\widehat{\delta}}\Delta), \Pi_4(x_0) = (\frac{\lambda\widehat{\delta}}{2} + \frac{1-\lambda}{4}\widehat{\delta}^2)(x_0 + \Delta), \Pi_5(x_0) = \frac{1-\lambda}{4}\widehat{\delta}^2(x_0 + 2\Delta)$. By plotting the firm's profits from all the candidate price schemes in Figure A.1, we can illustrate the optimal pricing strategy. The firm's expected payoff from charging each pricing scheme is linear in x_0. A lower pricing scheme leads to a higher adoption likelihood, and thus corresponds to a profit function with a higher slope and lower intercept. When the prior belief x_0 is low, the firm charges the highest candidate price $P_5(x_0)$, which increases in x_0 linearly. The intuition is that the DM only cares about $y_0 = x_0 - P$.

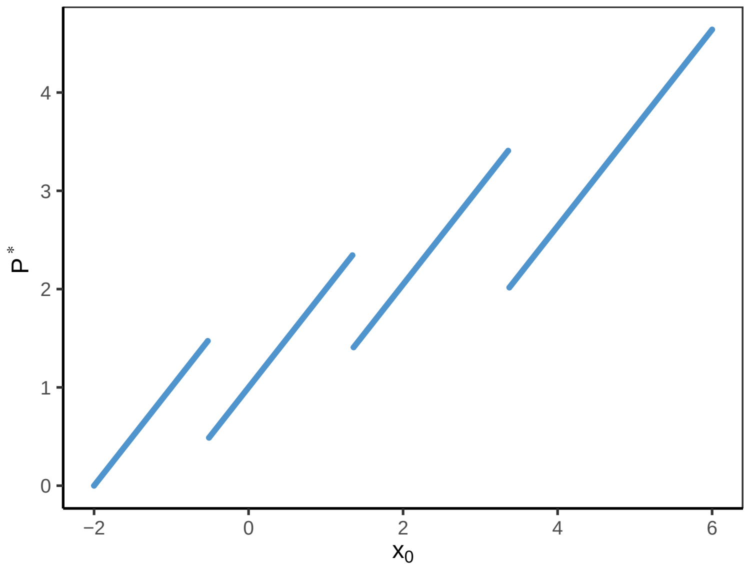

Figure A.2: Example of the optimal price P^* as a function of x_0 for $r = .95, \lambda = .5$, and $\Delta = 1$.

So, the firm can charge a higher price to induce the same adoption likelihood when the prior belief increases. As x_0 increases to a certain level, however, the firm switches from charging a price given by $P_5(x_0)$ to charging a lower price given by $P_4(x_0)$. Intuitively, as x_0 and the price increase, the firm's loss from non-adoption is larger. Therefore, the firm has a higher incentive to induce the DM to search less and adopt more while the cost of doing so, $P_5(x_0) - P_4(x_0)$, does not depend on x_0. When this incentive becomes strong enough, the optimal price has a discrete downward jump, as illustrated in Figure A.2. Then, the optimal price remains as given by $P_4(x_0)$ and increases in x_0 linearly until it switches from the pricing function $P_4(x_0)$ to $P_3(x_0)$ and decreases discontinuously. The optimal price then remains given $P_3(x_0)$ and increases in x_0 linearly until it switches from $P_3(x_0)$ to $P_1(x_0)$. For x_0 high enough, the optimal price is always given by $P_1(x_0)$, low enough such that the DM adopts the alternative without searching. In sum, each time the firm switches from one pricing scheme to another with a higher slope, the optimal price decreases discontinuously. In all other places, the optimal price increases in x_0 linearly.

The optimal price as a function of x_0 is smoother in continuous time. But non-monotonicity and discontinuity may still arise due to the effects we identify in discrete time.

Proof of Proposition 5:

When the initial belief is high, $x_0 > x_0^{**}$, Proposition 4 shows that the optimal price is $x_0 - \overline{x}$, and the DM adopts the alternative without searching. In this case, the firm's profit is $x_0 - \overline{x}$.

Proposition 1 shows that $\overline{x}$ increases in β. So, the firm's profit decreases in β. The firm does not retarget given any retargeting cost $k_r \geq 0$.

When the initial belief is low, $x_0 \leq 0$, Proposition 4 shows that the optimal price is $1/\eta$ and the DM does not adopt the alternative at x_0 in either the search mode or the no-search mode. The firm's profit is its value function evaluated at x_0. According to (17),

$$V_f(x_0) = \frac{2r + \lambda(e^{\widetilde{\eta}\delta} + e^{-\widetilde{\eta}\delta})}{(\widetilde{\eta} + \eta)e^{\eta(\widetilde{x}+P)+\widetilde{\eta}\delta} + (\widetilde{\eta} - \eta)e^{\eta(\widetilde{x}+P)-\widetilde{\eta}\delta}} \frac{\widetilde{\eta}P}{r + \lambda} e^{\eta x_0}.$$

One can see that $\eta = \sqrt{\frac{2r}{\sigma^2}\frac{r+\beta+\lambda}{r+\beta}}$ decreases in β. Because $x_0 \leq 0$, the last term of the value function $e^{\eta x_0}$ increases in β.

Now look at the terms before $e^{\eta x_0}$, $T_1 := \frac{2r+\lambda(e^{\widetilde{\eta}\delta}+e^{-\widetilde{\eta}\delta})}{(\widetilde{\eta}+\eta)e^{\eta(\widetilde{x}+P)+\widetilde{\eta}\delta}+(\widetilde{\eta}-\eta)e^{\eta(\widetilde{x}+P)-\widetilde{\eta}\delta}} \frac{\widetilde{\eta}P}{r+\lambda}$. One can see that $\lim_{\lambda\to+\infty} \eta/\widetilde{\eta} = \sqrt{r/(r+\beta)}$. One can also derive from (x) that $\lim_{\lambda\to+\infty} \widetilde{x}\sqrt{\lambda} = \beta\sigma/[\sqrt{2}(r + \sqrt{r(r+\beta)})]$. Therefore,

$$\begin{aligned}
\lim_{\lambda\to+\infty} \sqrt{\lambda}T_1 &= \lim_{\lambda\to+\infty} \sqrt{\lambda}\frac{2r + e^{\widetilde{\eta}\widetilde{x}}}{(\widetilde{\eta} + \eta)e^{1+\widetilde{\eta}\widetilde{x}}} \frac{\sqrt{(r+\beta)/r}}{r + \lambda} \\
&= \frac{\sqrt{(r+\beta)/r}}{(\sqrt{\frac{2}{\sigma^2}} + \sqrt{\frac{2r}{\sigma^2(r+\beta)}}) \cdot e} \\
&= \frac{\sigma}{\sqrt{2r}e} \frac{\sqrt{r+\beta}}{1 + \sqrt{\frac{r}{r+\beta}}},
\end{aligned}$$

which increases in β. Therefore, $V_f(x_0) = T_1 \cdot e^{\eta x_0}$ increases in β when λ is sufficiently large. Denote $\bar{k} = \frac{V_f(x_0|\beta_r) - V_f(x_0|\beta_0)}{\beta_r - \beta_0}$. One can see that $\bar{k} > 0$ and the firm retargets if and only if $k_r < \bar{k}$.

Proof of Proposition 6:

When the initial belief is high, $x_0 > x_0^{**}$, Proposition 4 shows that the optimal price is $x_0 - \overline{x}$, and the DM adopts the alternative without searching. In this case, the firm's profit is $x_0 - \overline{x}$. Proposition 1 shows that $\overline{x}$ decreases in λ. So, the firm's profit increases in λ. The firm chooses $\lambda^* = \bar{\lambda}$.

When the initial belief is low, $x_0 \leq 0$, Proposition 4 shows that the optimal price is $1/\eta$ and the DM does not adopt the alternative at x_0 in either the search mode or the no-search mode. The firm's profit is its value function evaluated at x_0. According to (17),

$$V_f(x_0) = \frac{2r + \lambda(e^{\widetilde{\eta}\delta} + e^{-\widetilde{\eta}\delta})}{(\widetilde{\eta} + \eta)e^{\eta(\widetilde{x}+P)+\widetilde{\eta}\delta} + (\widetilde{\eta} - \eta)e^{\eta(\widetilde{x}+P)-\widetilde{\eta}\delta}} \frac{\widetilde{\eta}P}{r + \lambda} e^{\eta x_0}$$

One can see that $\eta = \sqrt{\frac{2r}{\sigma^2}\frac{r+\beta+\lambda}{r+\beta}}$ increases in λ. Because $x_0 \leq 0$, the last term of the value function

$e^{\eta x_0}$ decreases in λ.

Now look at the terms before $e^{\eta x_0}$, $T_1 = \frac{2r+\lambda(e^{\tilde{\eta}\delta}+e^{-\tilde{\eta}\delta})}{(\tilde{\eta}+\eta)e^{\eta(\tilde{x}+P)+\tilde{\eta}\delta}+(\tilde{\eta}-\eta)e^{\eta(\tilde{x}+P)-\tilde{\eta}\delta}}\frac{\tilde{\eta}P}{r+\lambda}$. We have shown in the main model that as $\beta \to 0, \tilde{x} \to 0, \eta \to \tilde{\eta}$ and $e^{\eta\overline{x}}(1-\eta\overline{x})+\lambda/r = 0$. Therefore,

$$\begin{aligned}\lim_{\beta\to 0} T_1 &= \lim_{\beta\to 0} \frac{2r+\lambda(e^{\tilde{\eta}\overline{x}}+e^{-\tilde{\eta}\overline{x}})}{2\tilde{\eta}e^{1+\tilde{\eta}\overline{x}}(r+\lambda)} \\ &= \lim_{\beta\to 0} \frac{2r+\lambda[\frac{\lambda}{r(\tilde{\eta}\overline{x}-1)}+\frac{r(\tilde{\eta}\overline{x}-1)}{\lambda}]}{2\tilde{\eta}(r+\lambda)\frac{\lambda e}{r(\tilde{\eta}\overline{x}-1)}} \\ &= K\frac{(\lambda-r)\sigma^2+2r^2\overline{x}^2}{\lambda\sqrt{\lambda+r}},\end{aligned}$$

for some $K > 0$. Denote the last expression by $B(\lambda)$, and denote $K\frac{(\lambda-r)\sigma^2+2r^2k^2}{\lambda\sqrt{\lambda+r}}$ by $\widetilde{B}(\lambda)$, where $k > 0$ is an arbitrary fixed constant.

$$\widetilde{B}'(\lambda) = K\frac{(-\lambda^2+3\lambda r+2r^2)\sigma^2-2r^2(3\lambda+2r)k^2}{2\lambda^2(\lambda+r)^{\frac{3}{2}}}$$

One can see that $-\lambda^2+3\lambda r+2r^2 < 0$, and thus $\widetilde{B}'(\lambda) < 0$, when $r \le (\sqrt{17}-3)\lambda/4$. Therefore, $\widetilde{B}(\lambda)$ decreases in λ if $r \le (\sqrt{17}-3)\lambda/4$. Because $\overline{x}$ decreases in λ, when we replace the constant k in $\widetilde{B}(\lambda)$ by $\overline{x}$ in $B(\lambda)$, we also have $B(\lambda)$ decreases in λ if $r \le (\sqrt{17}-3)\lambda/4$. Therefore, $V_f(x_0) = T_1 \cdot e^{\eta x_0}$ decreases in λ when β is sufficiently small and r is small. The firm chooses $\lambda^* = \underline{\lambda}$.

Derivation of the Optimal Decision-Making in the Two Search Modes Case:

Applying Itô's Lemma to (19) and solve the differential equation, we can obtain

$$V_1(x) = B_3e^{\widetilde{\eta}x}+B_4e^{-\widetilde{\eta}x}+\frac{\lambda}{r+\lambda}x, \tag{xxii}$$

where B_3 and B_4 are constants to be determined.

Applying Itô's Lemma to (21) and solve the differential equation, we can obtain

$$V_2(x) = B_1e^{\widetilde{\eta}x}+B_2e^{-\widetilde{\eta}x}+\frac{\lambda}{r+\lambda}x, \tag{xxiii}$$

where B_1 and B_2 are constants to be determined. We can then use (xxiii) in (20) to obtain that for $x \in (\widetilde{x}, \underline{x})$, solving the corresponding differential equation,

$$V_1(x) = B_5e^{\widetilde{\eta}x}+B_6e^{-\widetilde{\eta}x}+\frac{\lambda^2}{(r+\lambda)^2}x+\frac{\lambda\widetilde{\eta}}{2(r+\lambda)}x[B_2e^{-\widetilde{\eta}x}-B_1e^{\widetilde{\eta}x}], \tag{xxiv}$$

where B_5 and B_6 are constant to be determined.

Putting together (20) and (22) for $x < \widetilde{x}$, we obtain a system of differential equations

$$(r+\lambda)V_2(x) = \frac{\sigma^2}{2}V_2''(x) + \lambda\frac{\beta}{r+\beta}V_1(x) \quad \text{(xxv)}$$

$$(r+\lambda)V_1(x) = \frac{\sigma^2}{2}V_1''(x) + \lambda V_2(x) \quad \text{(xxvi)}$$

which has the solution

$$V_2(x) = \widetilde{B}_1 e^{z_1 x} + \widetilde{B}_2 e^{z_2 x} \quad \text{(xxvii)}$$

$$V_1(x) = \sqrt{\frac{r+\beta}{\beta}}\left[\widetilde{B}_2 e^{z_2 x} - \widetilde{B}_1 e^{z_1 x}\right] \quad \text{(xxviii)}$$

where $z_1 = \sqrt{\widetilde{\eta}^2 + \frac{2\lambda}{\sigma^2}\sqrt{\frac{\beta}{r+\beta}}}$, and $z_2 = \sqrt{\widetilde{\eta}^2 - \frac{2\lambda}{\sigma^2}\sqrt{\frac{\beta}{r+\beta}}}$, and $\widetilde{B}_1$ and $\widetilde{B}_2$ are constants to be determined, where we use that $\lim_{x\to-\infty} V_1(x) = \lim_{x\to-\infty} V_2(x) = 0$.

Value matching and smooth pasting at the different thresholds, $V_1(\overline{x}) = \overline{x}, V_1'(\overline{x}) = 1, V_1(\underline{x}^+) = V_1(\underline{x}^-), V_1'(\underline{x}^+) = V_1'(\underline{x}^-), V_1(\widetilde{x}^+) = V_1(\widetilde{x}^-), V_1'(\widetilde{x}^+) = V_1'(\widetilde{x}^-), V_2(\widetilde{x}^+) = V_2(\widetilde{x}^-), V_2'(\widetilde{x}^+) = V_2'(\widetilde{x}^-), V_2(\underline{x}) = \underline{x}, V_2'(\underline{x}) = 1, \frac{\beta}{r+\beta}V_1(\widetilde{x}) = \widetilde{x}$, lead to the following system of 11 equations to obtain the 11 unknowns, $\overline{x}_1, \overline{x}_2, \widetilde{x}, B_1, B_2, B_3, B_4, B_5, B_6, \widetilde{B}_1$, and $\widetilde{B}_2$.

$$B_3\underline{X} + B_4/\underline{X} + \frac{\lambda}{r+\lambda}\underline{x} = B_5\underline{X} + B_6/\underline{X} + \frac{\lambda^2}{(r+\lambda)^2}\underline{x} - \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_1\underline{xX} + \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_2\underline{x}/\underline{X} \quad \text{(xxix)}$$

$$\widetilde{\eta}B_3\underline{X} - \widetilde{\eta}B_4/\underline{X} + \frac{\lambda}{r+\lambda} = \widetilde{\eta}B_5\underline{X} - \widetilde{\eta}B_6/\underline{X} + \frac{\lambda^2}{(r+\lambda)^2} - \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_1\underline{X} - \frac{\lambda\widetilde{\eta}^2}{2(r+\lambda)}B_1\underline{xX} + \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_2/\underline{X} - \frac{\lambda\widetilde{\eta}^2}{2(r+\lambda)}B_2\underline{x}/\underline{X} \quad \text{(xxx)}$$

$$B_5\widetilde{X} + B_6/\widetilde{X} + \frac{\lambda^2}{(r+\lambda)^2}\widetilde{x} - \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_1\widetilde{x}\widetilde{X} + \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_2\widetilde{x}/\widetilde{X} = \sqrt{\frac{r+\beta}{\beta}}\left[\widetilde{B}_2 e^{z_2\widetilde{x}} - \widetilde{B}_1 e^{z_1\widetilde{x}}\right] \quad \text{(xxxi)}$$

$$\widetilde{\eta}B_5\widetilde{X} - \widetilde{\eta}B_6/\widetilde{X} + \frac{\lambda^2}{(r+\lambda)^2} - \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_1\widetilde{X} - \frac{\lambda\widetilde{\eta}^2}{2(r+\lambda)}B_1\widetilde{x}\widetilde{X} + \frac{\lambda\widetilde{\eta}}{2(r+\lambda)}B_2/\widetilde{X} - \frac{\lambda\widetilde{\eta}^2}{2(r+\lambda)}B_2\widetilde{x}/\widetilde{X} = \sqrt{\frac{r+\beta}{\beta}}\left[z_2\widetilde{B}_2 e^{z_2\widetilde{x}} - z_1\widetilde{B}_1 e^{z_1\widetilde{x}}\right] \quad \text{(xxxii)}$$

$$
\begin{aligned}
B_1\widetilde{X} + B_2/\widetilde{X} + \frac{\lambda}{r+\lambda}\widetilde{x} &= \widetilde{B}_1 e^{z_1\widetilde{x}} + \widetilde{B}_2 e^{z_2\widetilde{x}} && \text{(xxxiii)}\\
\widetilde{\eta}B_1\widetilde{X} - \widetilde{\eta}B_2/\widetilde{X} + \frac{\lambda}{r+\lambda} &= z_1\widetilde{B}_1 e^{z_1\widetilde{x}} + z_2\widetilde{B}_2 e^{z_2\widetilde{x}} && \text{(xxxiv)}\\
B_3\overline{X} + B_4/\overline{X} + \frac{\lambda}{r+\lambda}\overline{x} &= \overline{x} && \text{(xxxv)}\\
\widetilde{\eta}B_3\overline{X} - \widetilde{\eta}B_4/\overline{X} + \frac{\lambda}{r+\lambda} &= 1 && \text{(xxxvi)}\\
B_1\underline{X} + B_2/\underline{X} + \frac{\lambda}{r+\lambda}\underline{x} &= \underline{x} && \text{(xxxvii)}\\
\widetilde{\eta}B_1\underline{X} - \widetilde{\eta}B_2/\underline{X} + \frac{\lambda}{r+\lambda} &= 1 && \text{(xxxviii)}\\
\sqrt{\frac{\beta}{r+\beta}}\left(-\widetilde{B}_1 e^{z_1\widetilde{x}} + \widetilde{B}_2 e^{z_2\widetilde{x}}\right) &= \widetilde{x}, && \text{(xxxix)}
\end{aligned}
$$

where $\overline{X} = e^{\widetilde{\eta}\overline{x}}$, $\underline{X} = e^{\widetilde{\eta}\underline{x}}$, and $\widetilde{X} = e^{\widetilde{\eta}\widetilde{x}}$.

Putting together (xxxv) and (xxxvi) one obtains

$$
\begin{aligned}
2B_3\overline{X} &= \frac{r}{r+\lambda}\left(\overline{x} + \frac{1}{\widetilde{\eta}}\right) && \text{(xl)}\\
2B_4/\overline{X} &= \frac{r}{r+\lambda}\left(\overline{x} - \frac{1}{\widetilde{\eta}}\right). && \text{(xli)}
\end{aligned}
$$

Putting together (xxxvii) and (xxxviii) one obtains

$$
\begin{aligned}
2B_1\underline{X} &= \frac{r}{r+\lambda}\left(\underline{x} + \frac{1}{\widetilde{\eta}}\right) && \text{(xlii)}\\
2B_2/\underline{X} &= \frac{r}{r+\lambda}\left(\underline{x} - \frac{1}{\widetilde{\eta}}\right). && \text{(xliii)}
\end{aligned}
$$

Putting together (xxix) and (xxx) one obtains

$$
\begin{aligned}
2B_3\underline{X} + \frac{\lambda}{r+\lambda}\left(\underline{x} + \frac{1}{\widetilde{\eta}}\right) = 2B_5\underline{X} &+ \frac{\lambda^2}{(r+\lambda)^2}\left(\underline{x} + \frac{1}{\widetilde{\eta}}\right) - \frac{\lambda}{2(r+\lambda)}B_1\underline{X}(1+2\widetilde{\eta}\underline{x})\\
&+ \frac{\lambda}{2(r+\lambda)}B_2/\underline{X} && \text{(xliv)}\\
2B_4/\underline{X} + \frac{\lambda}{r+\lambda}\left(\underline{x} - \frac{1}{\widetilde{\eta}}\right) = 2B_6/\underline{X} &+ \frac{\lambda^2}{(r+\lambda)^2}\left(\underline{x} - \frac{1}{\widetilde{\eta}}\right) - \frac{\lambda}{2(r+\lambda)}B_2/\underline{X}(1-2\widetilde{\eta}\underline{x})\\
&+ \frac{\lambda}{2(r+\lambda)}B_1\underline{X}. && \text{(xlv)}
\end{aligned}
$$

Putting together (xxxi) and (xxxii) one obtains

$$
\begin{aligned}
2B_5\widetilde{X} + \frac{\lambda^2}{(r+\lambda)^2}\left(\widetilde{x} + \frac{1}{\widetilde{\eta}}\right) \quad &- \quad \frac{\lambda}{2(r+\lambda)}B_1\widetilde{X}(1+2\widetilde{\eta}\widetilde{x}) + \frac{\lambda}{2(r+\lambda)}B_2/\widetilde{X} = \\
&\sqrt{\frac{r+\beta}{\beta}}\left(-\widetilde{\widetilde{B}}_1\left(1+\frac{z_1}{\widetilde{\eta}}\right) + \widetilde{\widetilde{B}}_2\left(1+\frac{z_2}{\widetilde{\eta}}\right)\right) \qquad \text{(xlvi)} \\
2B_6/\widetilde{X} + \frac{\lambda^2}{(r+\lambda)^2}\left(\widetilde{x} - \frac{1}{\widetilde{\eta}}\right) \quad &- \quad \frac{\lambda}{2(r+\lambda)}B_2/\widetilde{X}(1-2\widetilde{\eta}\widetilde{x}) + \frac{\lambda}{2(r+\lambda)}B_1\widetilde{X} = \\
&\sqrt{\frac{r+\beta}{\beta}}\left(-\widetilde{\widetilde{B}}_1\left(1-\frac{z_1}{\widetilde{\eta}}\right) + \widetilde{\widetilde{B}}_2\left(1-\frac{z_2}{\widetilde{\eta}}\right)\right) \qquad \text{(xlvii)}
\end{aligned}
$$

where $\widetilde{\widetilde{B}}_1 = \widetilde{B}_1 e^{z_1\widetilde{x}}$ and $\widetilde{\widetilde{B}}_2 = \widetilde{B}_2 e^{z_2\widetilde{x}}$. Putting together (xxxiii) and (xxxiv) one obtains

$$
\begin{aligned}
2B_1\widetilde{X} + \frac{\lambda}{r+\lambda}\left(\widetilde{x} + \frac{1}{\widetilde{\eta}}\right) \quad &= \quad \widetilde{\widetilde{B}}_1\left(1+\frac{z_1}{\widetilde{\eta}}\right) + \widetilde{\widetilde{B}}_2\left(1+\frac{z_2}{\widetilde{\eta}}\right) \qquad \text{(xlviii)} \\
2B_2/\widetilde{X} + \frac{\lambda}{r+\lambda}\left(\widetilde{x} - \frac{1}{\widetilde{\eta}}\right) \quad &= \quad \widetilde{\widetilde{B}}_1\left(1-\frac{z_1}{\widetilde{\eta}}\right) + \widetilde{\widetilde{B}}_2\left(1-\frac{z_2}{\widetilde{\eta}}\right). \qquad \text{(xlix)}
\end{aligned}
$$

Using (xxxix) we obtain $\widetilde{\widetilde{B}}_2 = \widetilde{\widetilde{B}}_1 + \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}$, which we can then substitute in (xlvi)-(xlix). Using the resulting equations (xlviii) and (xlix) we can obtain

$$
\begin{aligned}
\frac{2\widetilde{\eta}-z_1-z_2}{2\widetilde{\eta}+z_1+z_2}&\left[2B_1\widetilde{X} + \frac{\lambda}{r+\lambda}\left(\widetilde{x}+\frac{1}{\widetilde{\eta}}\right) - \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1+\frac{z_2}{\widetilde{\eta}}\right)\right] = \\
&2B_2/\widetilde{X} + \frac{\lambda}{r+\lambda}\left(\widetilde{x}-\frac{1}{\widetilde{\eta}}\right) - \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1-\frac{z_2}{\widetilde{\eta}}\right). \qquad \text{(l)}
\end{aligned}
$$

Using (xlii) and (xliii) in (l) one can then obtain

$$
\begin{aligned}
\frac{2\widetilde{\eta}-z_1-z_2}{2\widetilde{\eta}+z_1+z_2}&\left[\frac{\widetilde{X}}{\underline{X}}\frac{r}{r+\lambda}\left(\underline{x}+\frac{1}{\widetilde{\eta}}\right) + \frac{\lambda}{r+\lambda}\left(\widetilde{x}+\frac{1}{\widetilde{\eta}}\right) - \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1+\frac{z_2}{\widetilde{\eta}}\right)\right] = \\
&\frac{\underline{X}}{\widetilde{X}}\frac{r}{r+\lambda}\left(\underline{x}-\frac{1}{\widetilde{\eta}}\right) + \frac{\lambda}{r+\lambda}\left(\widetilde{x}-\frac{1}{\widetilde{\eta}}\right) - \sqrt{\frac{r+\beta}{\beta}}\widetilde{x}\left(1-\frac{z_2}{\widetilde{\eta}}\right), \qquad \text{(li)}
\end{aligned}
$$

which is an equation on only $\underline{x}$ and $\widetilde{x}$. Note that when $\beta \to \infty$ we have $\underline{x}, \widetilde{x} \to \sqrt{\frac{\sigma^2}{2r}}$ and (li) is satisfied.

Let $\delta_1 = \underline{x} - \widetilde{x}, \delta_2 = \overline{x} - \underline{x}, D_1 = e^{\widetilde{\eta}\delta_1}$, and $D_2 = e^{\widetilde{\eta}\delta_2}$. Using (xliv) and (xlvi) to take out B_5, and using B_1 from (xlii), B_2 from (xliii), B_3 from (xl), and $\widetilde{C}_1$ from (xlviii), we can obtain

$$
\frac{1}{D_2}\left(\overline{x}+\frac{1}{\widetilde{\eta}}\right) = \frac{\lambda+r}{r}G_1(\underline{x},\widetilde{x}), \qquad \text{(lii)}
$$

where

$$
\begin{aligned}
G_1(\underline{x},\widetilde{x}) &= D_1\Big[-\frac{\lambda^2}{(r+\lambda)^2}\Big(\widetilde{x}+\frac{1}{\widetilde{\eta}}\Big)+\frac{r+\beta}{\beta}\widetilde{x}\Big(1+\frac{z_2}{\widetilde{\eta}}\Big)+\sqrt{\frac{r+\beta}{\beta}}\frac{z_2-z_1}{2\widetilde{\eta}+z_1+z_2}\Big[\frac{1}{D_1}\frac{r}{r+\lambda}\Big(\underline{x}+\frac{1}{\widetilde{\eta}}\Big)+ \\
&\quad \frac{\lambda}{r+\lambda}\Big(\widetilde{x}+\frac{1}{\widetilde{\eta}}\Big)-\widetilde{x}\sqrt{\frac{r+\beta}{\beta}}\Big(1+\frac{z_2}{\widetilde{\eta}}\Big)\Big]\Big]-\frac{\lambda r}{4(r+\lambda)^2}\Big(\underline{x}(3+2\widetilde{\eta}\delta_1+D_1^2)+ \\
&\quad \frac{1}{\widetilde{\eta}}(5+2\widetilde{\eta}\delta_1-D_1^2)\Big). \qquad \text{(liii)}
\end{aligned}
$$

Similarly, using (xlv) and (xlvii) to take out B_6, and using B_1 from (xlii), B_2 from (xliii), B_4 from (xli), and $\widetilde{C}_1$ from (xlviii), we can obtain

$$
D_2\Big(\overline{x}-\frac{1}{\widetilde{\eta}}\Big)=\frac{\lambda+r}{r}G_2(\underline{x},\widetilde{x}), \qquad \text{(liv)}
$$

where

$$
\begin{aligned}
G_2(\underline{x},\widetilde{x}) &= \frac{1}{D_1}\Big[-\frac{\lambda^2}{(r+\lambda)^2}\Big(\widetilde{x}-\frac{1}{\widetilde{\eta}}\Big)+\frac{r+\beta}{\beta}\widetilde{x}\Big(1-\frac{z_2}{\widetilde{\eta}}\Big)+\sqrt{\frac{r+\beta}{\beta}}\frac{z_1-z_2}{2\widetilde{\eta}+z_1+z_2}\Big[\frac{1}{D_1}\frac{r}{r+\lambda}\Big(\underline{x}+ \\
&\quad \frac{1}{\widetilde{\eta}}\Big)+\frac{\lambda}{r+\lambda}\Big(\widetilde{x}+\frac{1}{\widetilde{\eta}}\Big)-\widetilde{x}\sqrt{\frac{r+\beta}{\beta}}\Big(1+\frac{z_2}{\widetilde{\eta}}\Big)\Big]\Big]-\frac{\lambda r}{4(r+\lambda)^2}\Big(\underline{x}(3-2\widetilde{\eta}\delta_1+\frac{1}{D_1^2})-\frac{1}{\widetilde{\eta}}(5- \\
&\quad 2\widetilde{\eta}\delta_1-\frac{1}{D_1^2})\Big). \qquad \text{(lv)}
\end{aligned}
$$

Note then that (li), (lii), and (liv) is a system of equations for $\overline{x},\underline{x}$, and $\widetilde{x}$. Note also that putting (lii) and (liv) together one obtains

$$
\overline{x}^2=\frac{(\lambda+r)^2}{r^2}G_1G_2+\frac{1}{\widetilde{\eta}^2}, \qquad \text{(lvi)}
$$

which determines $\overline{x}$ as a function of $\underline{x}$ and $\widetilde{x}$. Plugging it in (lii), we can then use (li) and (lii) to solve for $\underline{x}$ and $\widetilde{x}$.

Derivation of Optimal Decision-Making for $\beta = 0$ in the Two Search Modes Case:

In the case of $\beta\to 0$ and $\widetilde{x}\to 0$, we obtain $z_1,z_2\to\widetilde{\eta}$, and for $x<\widetilde{x}=0$ we obtain

$$
\begin{aligned}
V_2(x) &= \widehat{B}_1 e^{\widetilde{\eta}x} && \text{(lvii)}\\
V_1(x) &= \widehat{B}_2 e^{\widetilde{\eta}x}-\frac{\lambda\widehat{B}_1}{\sigma^2\widetilde{\eta}}e^{\widetilde{\eta}x}, && \text{(lviii)}
\end{aligned}
$$

where $\widehat{B}_1$ and $\widehat{B}_2$ are constants to be determined.

We then have that the condition $\frac{\beta}{r+\beta}V_1(\widetilde{x})=\widetilde{x}$, (xxxix), is no longer required, and that conditions (xxxi)-(xxxiv), are replaced by the conditions

$$\begin{aligned}
B_5 + B_6 &= \widehat{B}_2 && \text{(lix)}\\
\widetilde{\eta} B_5 - \widetilde{\eta} B_6 + \frac{\lambda^2}{(r+\lambda)^2} &- \frac{\lambda\widetilde{\eta}}{2(r+\lambda)} B_1 + \frac{\lambda\widetilde{\eta}}{2(r+\lambda)} B_2 &&\\
&= \widetilde{\eta}\widehat{B}_2 - \frac{\lambda\widehat{B}_1}{\sigma^2\widetilde{\eta}} && \text{(lx)}\\
B_1 + B_2 &= \widehat{B}_1 && \text{(lxi)}\\
\widetilde{\eta} B_1 - \widetilde{\eta} B_2 + \frac{\lambda}{r+\lambda} &= \widetilde{\eta}\widehat{B}_1, && \text{(lxii)}
\end{aligned}$$

respectively.

Using (lxi) and (lxii) we can obtain $B_2 = \frac{\lambda}{2\widetilde{\eta}(r+\lambda)}$. Using this in (xliii), we can obtain the condition for the optimal $\underline{x}$ as

$$e^{\eta\underline{x}}(1-\eta) + \frac{\lambda}{r} = 0, \tag{lxiii}$$

as $\eta = \widetilde{\eta}$ for $\beta = 0$, which is intuitively the same condition as (8). Using (xlii) and (xxxiii) we can then also obtain $\widehat{B}_1 = \frac{r}{2(r+\lambda)}\left(\underline{x} + \frac{1}{\widetilde{\eta}}\right)\frac{1}{\underline{X}} + \frac{\lambda}{2\widetilde{\eta}(r+\lambda)}$.

Note also that in this case (xlvii) is replaced by

$$2B_6 - \frac{\lambda^2}{\widetilde{\eta}(r+\lambda^2)} - \frac{\lambda}{2(r+\lambda)}B_2 + \frac{\lambda}{2(r+\lambda)}B_1 = \frac{\lambda\widehat{B}_1}{2(r+\lambda)}. \tag{lxiv}$$

Using (lxiv) and (xlv) to substitute away B_6, we can then use B_1, B_2, and B_4 obtained above to yield

$$\frac{\lambda}{4\widetilde{\eta}} + \frac{\lambda}{\widetilde{\eta}} + \frac{r}{4}\underline{X}\left(\underline{x} + \frac{1}{\widetilde{\eta}}\right) = \frac{r+\lambda}{\lambda}\overline{X}\left(\overline{x} - \frac{1}{\widetilde{\eta}}\right) + \frac{\lambda(1-\lambda)(r+\lambda)}{r\widetilde{\eta}} - \frac{\lambda}{2}\underline{x}, \tag{lxv}$$

which determines $\overline{x}$ as a function of $\underline{x}$.

Proof of Proposition 7:

We can obtain the condition for the optimal $\underline{x}$ as

$$e^{\eta\underline{x}}(1-\eta\underline{x}) + \frac{\lambda}{r} = 0, \tag{lxvi}$$

as $\eta = \widetilde{\eta}$ for $\beta = 0$, which is intuitively the same condition as (8). Furthermore, we can obtain

$$\frac{\lambda}{4\widetilde{\eta}} + \frac{\lambda}{\widetilde{\eta}} + \frac{r}{4}\underline{X}\left(\underline{x} + \frac{1}{\widetilde{\eta}}\right) = \frac{r+\lambda}{\lambda}\overline{X}\left(\overline{x} - \frac{1}{\widetilde{\eta}}\right) + \frac{\lambda(1-\lambda)(r+\lambda)}{r\widetilde{\eta}} - \frac{\lambda}{2}\underline{x}, \tag{lxvii}$$

where $\overline{X} = e^{\widetilde{\eta}\overline{x}}$ and $\underline{X} = e^{\widetilde{\eta}\underline{x}}$, which determines $\overline{x}$ as a function of $\underline{x}$. For $\lambda \to 0$, we can then obtain

$\underline{x}, \overline{x} \to \sqrt{\frac{\sigma^2}{2r}}$, and

$$\frac{\underline{x} - 1/\eta}{\lambda} \to \frac{1}{r\eta e} \tag{lxviii}$$

$$\frac{\overline{x} - 1/\eta}{\lambda} \to \frac{1}{2r\eta}. \tag{lxix}$$

For $\beta = 0$ and λ sufficiently small, the extent of choice closure in search mode 1 is approximated by

$$\overline{x} - \underline{x} \approx \frac{\lambda}{r\eta}\left(\frac{1}{2} - \frac{1}{e}\right) = \sqrt{\frac{\lambda}{r+\lambda}\frac{\sigma^2}{2r}}\frac{1}{\sqrt{r}}\left(\frac{1}{2} - \frac{1}{e}\right) \tag{lxx}$$

The extent of choice closure in search mode 2 is approximated by

$$\underline{x} - \widetilde{x} \approx \frac{1}{\eta}\left(\frac{\lambda}{re} + 1\right) = \frac{\sigma^2}{2r}\left(\sqrt{\frac{\lambda}{r+\lambda}}\frac{1}{\sqrt{r}}\frac{1}{e} + \sqrt{\frac{r+\lambda}{r}}\right) \tag{lxxi}$$

The comparative statics follow immediately.

Derivation of Solution for Start-Up Search Costs Case:

Using Itô's Lemma on equation (24) and solving the corresponding differential equation, we can obtain

$$\widetilde{V}(x) = \frac{\lambda F + c}{\sigma^2}x^2 + a_1 x + a_0, \tag{lxxii}$$

where a_1 and a_0 are constants to be determined.

Using Itô's Lemma on equation (25) and solving the corresponding differential equation, one obtains

$$V(x) = C_1 e^{\widehat{\eta}x} + C_2 e^{-\widehat{\eta}x} + x - c/\lambda, \tag{lxxiii}$$

Using Itô's Lemma on equation (26) and solving the corresponding differential equation, one obtains

$$\widehat{V}(x) = C_3 e^{\widehat{\eta}x} + C_4 e^{-\widehat{\eta}x} - c/\lambda. \tag{lxxiv}$$

If $\widetilde{x} > 0$, then value matching and smooth pasting at $\overline{x}, \widetilde{x}$, $\widehat{x}$, and $\underline{\widehat{x}}$ leads to $V(\overline{x}) = \overline{x}, V'(\overline{x}) = 1, V(\widetilde{x}) = \widetilde{V}(\widetilde{x}), V'(\widetilde{x}) = \widetilde{V}'(\widetilde{x}), V(\widetilde{x}) - F = \widetilde{x}, \widetilde{V}(\widehat{x}) - F = 0, \widetilde{V}(\widehat{x}) = \widehat{V}(\widehat{x}), \widetilde{V}'(\widehat{x}) = \widehat{V}'(\widehat{x}), \widehat{V}(\underline{\widehat{x}}) = 0,$

and $\widehat{V}'(\underline{\widehat{x}}) = 0$, which are the conditions

$$
\begin{aligned}
C_1 e^{\widehat{\eta}\overline{x}} + C_2 e^{-\widehat{\eta}\overline{x}} &= c/\lambda && \text{(lxxv)}\\
C_1 e^{\widehat{\eta}\overline{x}} - C_2 e^{-\widehat{\eta}\overline{x}} &= 0 && \text{(lxxvi)}\\
C_1 e^{\widehat{\eta}\widetilde{x}} + C_2 e^{-\widehat{\eta}\widetilde{x}} &= F + c/\lambda && \text{(lxxvii)}\\
\widehat{\eta}[C_1 e^{\widehat{\eta}\widetilde{x}} - C_2 e^{-\widehat{\eta}\widetilde{x}}] + 1 &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widetilde{x} && \text{(lxxviii)}\\
C_3 e^{\widehat{\eta}\widehat{x}} + C_4 e^{-\widehat{\eta}\widehat{x}} &= F + c/\lambda && \text{(lxxix)}\\
\widehat{\eta}[C_3 e^{\widehat{\eta}\widehat{x}} - C_4 e^{-\widehat{\eta}\widehat{x}}] &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widehat{x} && \text{(lxxx)}\\
C_3 e^{\widehat{\eta}\underline{\widehat{x}}} + C_4 e^{-\widehat{\eta}\underline{\widehat{x}}} &= c/\lambda && \text{(lxxxi)}\\
C_3 e^{\widehat{\eta}\underline{\widehat{x}}} - C_4 e^{-\widehat{\eta}\underline{\widehat{x}}} &= 0 && \text{(lxxxii)}\\
a_0 + a_1\widetilde{x} + \frac{\lambda F + c}{\sigma^2}\widetilde{x}^2 &= \widetilde{x} + F && \text{(lxxxiii)}\\
a_0 + a_1\widehat{x} + \frac{\lambda F + c}{\sigma^2}\widehat{x}^2 &= F. && \text{(lxxxiv)}
\end{aligned}
$$

From (lxxv) and (lxxvi) we can obtain $C_1 = \frac{c}{2\lambda}e^{-\widehat{\eta}\overline{x}}$ and $C_2 = \frac{c}{2\lambda}e^{\widehat{\eta}\overline{x}}$. Similarly, (lxxxi) and (lxxxii) we can obtain $C_3 = \frac{c}{2\lambda}e^{-\widehat{\eta}\underline{\widehat{x}}}$ and $C_4 = \frac{c}{2\lambda}e^{\widehat{\eta}\underline{\widehat{x}}}$. Using this in the other equations we can then obtain

$$
\begin{aligned}
\frac{c}{2\lambda}e^{\widehat{\eta}(\widetilde{x}-\overline{x})} + \frac{c}{2\lambda}e^{-\widehat{\eta}(\widetilde{x}-\overline{x})} &= F + \frac{c}{\lambda} && \text{(lxxxv)}\\
\widehat{\eta}\left[\frac{c}{2\lambda}e^{\widehat{\eta}(\widetilde{x}-\overline{x})} - \frac{c}{2\lambda}e^{-\widehat{\eta}(\widetilde{x}-\overline{x})}\right] + 1 &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widetilde{x} && \text{(lxxxvi)}\\
\frac{c}{2\lambda}e^{\widehat{\eta}(\widehat{x}-\underline{\widehat{x}})} + \frac{c}{2\lambda}e^{-\widehat{\eta}(\widehat{x}-\underline{\widehat{x}})} &= F + \frac{c}{\lambda} && \text{(lxxxvii)}\\
\widehat{\eta}\left[\frac{c}{2\lambda}e^{\widehat{\eta}(\widehat{x}-\underline{\widehat{x}})} - \frac{c}{2\lambda}e^{-\widehat{\eta}(\widehat{x}-\underline{\widehat{x}})}\right] &= a_1 + 2\frac{\lambda F + c}{\sigma^2}\widehat{x} && \text{(lxxxviii)}\\
\frac{\lambda F + c}{\sigma^2}(\widetilde{x}^2 - \widehat{x}^2) + a_1(\widetilde{x} - \widehat{x}) &= \widetilde{x}. && \text{(lxxxix)}
\end{aligned}
$$

From (lxxxv) and (lxxxvii) we can obtain $\overline{x} - \widetilde{x} = \widehat{x} - \underline{\widehat{x}}$. Using (lxxxv) we can also obtain $e^{\widehat{\eta}(\widetilde{x}-\overline{x})} = 1/H$, where

$$
H = 1 + \frac{\lambda F}{c} + \sqrt{\left(\frac{\lambda F}{c} + 1\right)^2 - 1}. \tag{xc}
$$

Using $\overline{x} - \widetilde{x} = \widehat{x} - \underline{\widehat{x}}$ in (lxxxvi) and (lxxxviii) we can obtain

$$
a_1 = \frac{1}{2} - \frac{\lambda F + c}{\sigma^2}(\widetilde{x} + \widehat{x}). \tag{xci}
$$

Substituting in (lxxxix) one obtains $\widetilde{x} = -\widehat{x}$ and $a_1 = 1/2$. Using this in (lxxxvi) one obtains

$$\widetilde{x} = \sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{2(\lambda F + c)}\frac{1-H^2}{H} + \frac{\sigma^2}{4(\lambda F + c)}}. \tag{xcii}$$

If $\widetilde{x} = 0$, the DM may strictly prefer stopping search without adopting the alternative to deferring choice. So, the value matching condition $V(\widetilde{x}) - F = \widetilde{x}$ needs to be replaced by $V(\widetilde{x}) - F \geq \widetilde{x}$. In that case, $\widetilde{x}$ will be 0 rather than $\sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{2(\lambda F+c)}\frac{1-H^2}{H} + \frac{\sigma^2}{4(\lambda F+c)}}$. Therefore, in general,

$$\widetilde{x} = \max\left\{\sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{2(\lambda F + c)}\frac{1-H^2}{H} + \frac{\sigma^2}{4(\lambda F + c)}}, 0\right\} \tag{xciii}$$

Proof of Proposition 8:

The derivations for the comparative statics with regard to F and the comparative statics of the extent of choice closure with regard to c are straightforward.

According to (27) and $\widetilde{x} = -\widehat{x}$, if $\widetilde{x} > 0$, then

$$\operatorname{sign}\left\{\frac{\partial(\widetilde{x} - \widehat{x})}{\partial c}\right\} = \operatorname{sign}\left\{\frac{8Fc^2}{2c + \lambda F} - \sigma^2\right\} \tag{xciv}$$

First consider $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 < 0$. Since $\widetilde{x} > 0$ is equivalent to $\sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{2(\lambda F+c)}\frac{1-H^2}{H} + \frac{\sigma^2}{4(\lambda F+c)}} > 0 \Leftrightarrow 8F(2c + \lambda F) - \sigma^2 < 0 \Leftrightarrow c < \frac{\sigma^2}{16F} - \frac{\lambda F}{2}$, we have $\frac{\partial(\widetilde{x}-\widehat{x})}{\partial c} < 0$ if $c < \frac{\sigma^2}{16F} - \frac{\lambda F}{2}$ and $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 < 0$, according to (xciv). $\widetilde{x} = 0$ and thus $\frac{\partial(\widetilde{x}-\widehat{x})}{\partial c} = 0$ if $c \geq \frac{\sigma^2}{16F} - \frac{\lambda F}{2}$ and $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 < 0$.

Now consider $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 \geq 0$. We have shown in the previous case that $\widetilde{x} = 0$ is equivalent to $8F(2c+\lambda F) - \sigma^2 \geq 0$. Since $8F(2c+\lambda F) - \sigma^2 > \frac{8Fc^2}{2c+\lambda F} - \sigma^2 \geq 0$, $\widetilde{x}$ is always 0 when $\frac{8Fc^2}{2c+\lambda F} - \sigma^2 > 0$.

In sum, the extent of choice closure always (weakly) decreases in c.

Now let's look at the comparative statics with regard to λ. One can see that H is increasing in λ. Therefore, $\frac{1-H^2}{H} = \frac{1}{H} - H$ is decreasing in λ and $\widetilde{x} - \widehat{x} = 2\widetilde{x} = \sqrt{\frac{\sigma^2}{2\lambda}\frac{c}{(\lambda F+c)}\frac{1-H^2}{H} + \frac{\sigma^2}{2(\lambda F+c)}}$ is decreasing in λ. So, the extent of choice deferral decreases in λ.

$$\begin{aligned}(28) \Rightarrow & \overline{x} - \widetilde{x} = \frac{1}{\widehat{\eta}}\ln H = \sqrt{\frac{\sigma^2}{2\lambda}}\ln\left[1 + \frac{\lambda F}{c} + \sqrt{\left(\frac{\lambda F}{c} + 1\right)^2 - 1}\right] \\ \Rightarrow & \operatorname{sign}\left\{\frac{\partial(\overline{x} - \widetilde{x})}{\partial \lambda}\right\} = \operatorname{sign}\left\{-\ln H + \frac{2\lambda F}{c\sqrt{\left(\frac{\lambda F}{c} + 1\right)^2 - 1}}\right\} = \operatorname{sign}[G(\lambda)],\end{aligned} \tag{xcv}$$

where $G(\lambda) := -\ln H + \frac{2\lambda F}{c\sqrt{(\frac{\lambda F}{c}+1)^2-1}}$. One can see that $G(0) = 0$ and $G'(\lambda)$ is proportional to

$-(\frac{\lambda F}{c})^2 < 0$. Therefore, $G(\lambda) < 0, \forall \lambda > 0$. We then have that (xcv) implies that $\frac{\partial(\overline{x}-\widetilde{x})}{\partial \lambda} < 0$. So, the extent of choice closure decreases in λ.

Non-Random Switching Between Search and No-Search Modes:

In this section we study the DM's optimal search strategy under non-random switching between search states numerically. The length of time that the DM spends in each mode is deterministic. Specifically, the DM moves from the search mode to the no-search mode after $1/\lambda$ units of time and moves back from the no-search mode to the search mode after $1/\beta$ units of time.

We approximate continuous time using a discrete-time grid with increments of $dt = 0.001$. We simulate the stochastic movement of x using a discrete-time approximation to Brownian motion (Dixit 1993).

Because the DM knows whether she is able to search or not in the next "period", there is only one relevant adoption threshold at each t. Figure A.3 illustrates the decision thresholds under non-random switching. The DM's optimal search strategy is no longer stationary. The adoption threshold in the search mode, $\overline{x}$, decreases in time during a search period, so the DM is more likely to adopt the alternative the longer she searches within a session. The extent of choice deferral, $\widetilde{x}$, increases in time during the no-search period. When the DM recovers from the no-search mode to the search model, the decision threshold resets. One can interpret the difference between $\overline{x}$ at the beginning of the search mode and the $\widetilde{x}$ at the beginning of the no-search mode as the extent of choice closure.

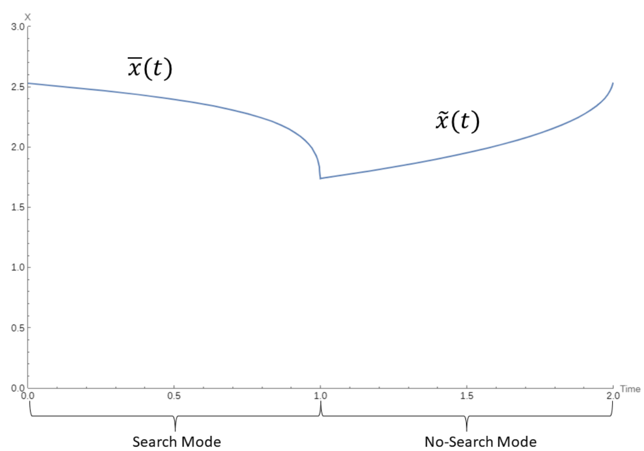

Figure A.3: Example of the adoption threshold as a function of time for $r = .05$, $\lambda = 1$, $\beta = 1$, and $\sigma = 1$.

For a fixed t, we also observe that $\overline{x}$ and $\widetilde{x}$ move in the same directions as they do under the base model when β, λ, r, and σ^2 change.

Search Strategy Under Deadline:

In this section we study the case with a decision deadline numerically. If the DM has not adopted the alternative after time T, then the decision becomes obsolete, and the DM receives a utility of 0.

We approximate continuous time using a discrete-time grid with increments of $dt = 0.001$. We simulate the stochastic movement of x using a discrete-time approximation to Brownian motion.

Figure A.4 illustrates the decision thresholds. The DM's optimal search strategy is no longer stationary. Both the extent of deferral, $\widetilde{x}$, and the extent of closure, $\overline{x} - \widetilde{x}$, decrease in time and approach 0 as $t \to T$. The DM is more likely to adopt the alternative over time.

For a fixed t, we observe that $\overline{x}$, $\widetilde{x}$, and $\overline{x} - \widetilde{x}$ move in the same directions as they do under the base model when β, λ, r, and σ^2 change.

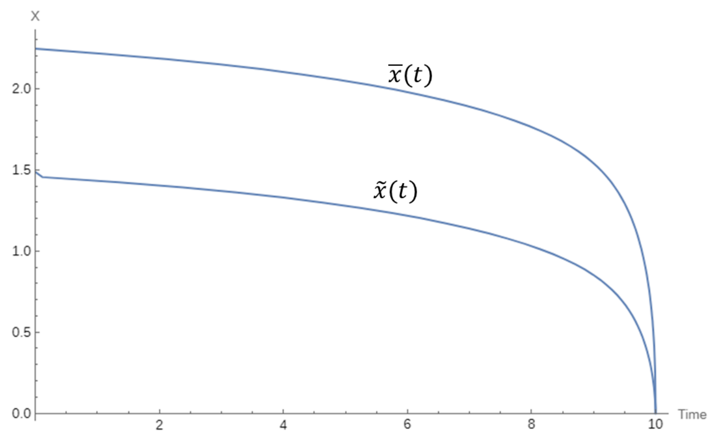

Figure A.4: Example of the adoption thresholds as a function of time for $r = .05$, $\lambda = 1$, $\beta = 1$, $\sigma = 1$, and $T = 10$.