Algorithmic Targeting and the Precision-Recall Trade-off

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Abstract

We examine the implications of competitive algorithmic targeting when outcomes of targeting algorithms are the individual consumer-level predicted probabilities of conversion. In these situations, firms implicitly face the well-known precision-recall trade-off while choosing their targeting strategies. They can choose to target a smaller set of consumers with a high probability of conversion (precision) but miss out on many consumers who might still be interested in their product. Conversely, firms can target a larger set of consumers (recall), but this results in a greater probability that their targeting is wasted on uninterested consumers. We analyze this precision-recall trade-off under competition between firms that strategically choose their algorithmic targeting policies. We show that competing firms favor a targeting policy that has higher precision but lower recall compared to a monopoly. Firms target fewer consumers when their algorithms are more correlated. They also have the incentive to strategically decrease the precision of their targeting policies in order to reduce competition. If firms endogenously choose their algorithmic correlation, then there is an equilibrium incentive to decrease the correlation.

Keywords Precision-Recall Trade-off, Targeted Advertising, Machine Learning, Algorithms

JEL Codes D43, L13, M37

I Introduction

Targeting is fundamental to marketing strategy. One of the general challenges in targeted marketing is reducing wastage: Firms must focus their marketing budget on consumers who are most likely to be interested in their products, which requires them to make predictions of consumer types based on observable characteristics and behavior. At the core, the prediction of consumer types for targeting can be interpreted as a classification problem - i.e., trying to predict whether a consumer is interested in the product and the likelihood that she will make a purchase if targeted by the firms' marketing activities. The traditional targeting examined in the literature typically involves targeting based on simple binary prediction signals of the consumers' interest in the product. However, in the contemporary digital economy, firms have rich information on consumer characteristics, opinions, behaviors, and social interactions. As part of their data analytics strategy, firms can increasingly use AI and machine learning algorithms to produce individual-level predictions to target consumers.

In general, any algorithm, ranging from a simple logit regression to a more complex neural network, takes data on observed consumer characteristics and behaviors as input and produces a probability or likelihood of conversion that firms can use for targeting consumers as the output. In other words, in most contexts involving algorithmic targeting, firms do not target based on binary signals as is typically modeled in the literature. Instead, they face a *distribution of probabilities* for which they need to decide who to target. For every individual consumer, the algorithm produces a probability with which the consumer will be interested in purchasing. For example, Shi et al. (2022) describe how Alibaba decides which customers to target with promotional messages based on the predicted likelihood of visits and purchases (i.e., algorithmic scores), which are generated using machine learning algorithms using high-dimensional consumer data. Our framework captures this aspect of the targeting problem and its linkage to the precision-recall trade-off in classification problems while analyzing the competitive algorithmic targeting strategies of firms.

In reality, the classification of consumers by algorithms is always imperfect due to data limitations, privacy concerns, or model selection constraints. This is reflected in the probabilistic targeting output of targeting algorithms. Given this, the firm faces a fundamental trade-off associated with classification problems relevant to all machine learning models, namely the "precision-recall" trade-off. In the context of targeting, precision is defined as the share of interested consumers among all consumers targeted by the firm - i.e., it measures to what extent consumers targeted are indeed interested in the product. On the other hand, recall is defined as the share of interested consumers targeted out of all interested consumers.

Firms can either focus on a smaller set of consumers with high predicted probabilities of conversion (precision) so that most targeted consumers indeed turn out to be those who are interested and purchase the product, or they can target a larger set of consumers so that most of the interested consumers are targeted (recall), but not both. For the intermediate consumers who have moderate likelihoods, there will be some consumers who are interested but whom the firm will miss out on targeting if it prioritizes precision; whereas there will be others who are uninterested and on whom the firm will waste its advertising budget if it prioritizes recall. In practice, there is evidence that firms are well aware of this trade-off. For example, LinkedIn suggests advertisers to "balance precision with volume" in their targeting policies[1]. As another example, Admetrics, a B2B marketing technology company, advises advertisers to prioritize precision over volume in their targeting decisions, as aggressive bidding by Temu and Shein intensifies competition in online advertising.[2]

We examine the implications of competitive algorithmic targeting. Given that the typical outcome of targeting algorithms used by firms are the individual-level pre-

[1] The source is https://www.linkedin.com/business/marketing/blog/content-marketing/what-is-b2b-marketing-definition-strategy-and-trends.

[2] The source is https://www.admetrics.io/en/post/ecommerce-strategies-to-keep-up-with-temu-and-shein-in-europe.

dicted probabilities for consumers, we examine the implications for the well-known precision-recall trade-off in statistical learning in the context of competitive targeting. How should a firm choose its targeting policy, and what are the equilibrium targeting policies of competing firms? In practice the algorithmic predictions of firms might be correlated either because firms may use public data or similar data analytics tools. What is the impact of the correlation in the firms' algorithms on their precision and recall choices? The model considers competition between two firms for a set of consumers. Consumers have a binary type: they are either interested in the product and will make a purchase if targeted, or are not interested and will not make a purchase even if they are targeted. Firms do not know the type of consumers and they rely on algorithms to make predictions. The algorithm takes as input the available consumer and market information and generates as output a distribution of probabilities that consumers are interested in purchasing. Firms then simultaneously decide who to target based on the predictions. Each firm makes a profit for every interested consumer that they end up targeting, and they also incur a cost for every consumer targeted. If a consumer is targeted by both firms, the profit they generate is lower than the monopoly profit when the consumer is targeted by only one firm.

Competing firms favor a targeting policy that has greater precision but lower recall relative to a monopoly. In other words, firms respond strategically in equilibrium by concentrating their targeting efforts on consumers who have a higher likelihood of conversion, resulting in higher precision of their targeting. For the competing firms, given possible correlation in their predictions, the equilibrium expected profit of a firm depends on the other firm's targeting policy. We first show that there exists a unique mixed-strategy symmetric equilibrium. In this equilibrium, when firms' algorithmic predictions have a high enough correlation, they will never target any consumer for sure. Consumers with higher predicted probabilities of being interested will be targeted with a higher probability. When the correlation is not that high, both firms target consumers who they predict to have a sufficiently high likelihood of being interested for sure, while mixing for consumers who have moderately high conversion probabilities.

The correlation in the firms' algorithms has a nuanced effect on the precision-recall trade-off in targeting. The equilibrium targeting intensity - i.e., the overall number of consumers targeted - decreases in the correlation of firms' predictions because a greater overlap in targeting reduces the expected profits. Thus, both firms strategically target fewer consumers as their predictions become more correlated, and this helps to soften competition.

How does correlation change the precision and recall choices? We find that as firms' predictions converge, recall decreases as firms target fewer consumers. Surprisingly, precision also decreases. In other words, firms on average target consumers who are less likely to be interested. The total number of consumers being targeted generally decreases with competition even if firms' predictions are uncorrelated, implying that competition may reduce consumer welfare by creating less information value for consumers. As firms' predictions become more correlated, the information value further decreases.

We extend the model to allow for endogenous correlation of predictions. We find firms have an incentive to invest in lowering the correlation of their predictions. In two other extensions, we also consider endogenous pricing and asymmetric pure strategy equilibria, to show that our main result on the precision-recall trade-off under competition continues to hold.

I.A Related Research

We contribute to the classic research stream on the competitive effects of targeted advertising (Chen et al., 2001; Chen and Iyer, 2002; Iyer et al., 2005; Bergemann and Bonatti, 2011; Zhang and Katona, 2012; Johnson, 2013; Chen et al., 2017; Lauga et al., 2018; Shin and Yu, 2021; Ke et al., 2022; Choi et al., 2023; Ning et al., 2023). Contrary to the standard assumption in the literature that firms receive a binary signal of the consumer's type, our model directly represents the output of a machine learning algorithm as a distribution of the probability of conversion across consumers. Modeling the signal structure from machine learning algorithms as a distribution of predicted probabilities allows us to naturally link it to the precision-recall trade-off and describe how algorithmic predictions affect targeting policies. There are also some recent papers that examine precision and recall in the context of marketing strategy choices: Berman et al. (2023) examine a recommendation algorithm's choice of precision and recall in designing the optimal information structure to persuade consumers who are uncertain about product fit. Jerath and Ren (2021) consider the consumer's attention allocation to favorable and unfavorable information which has precision recall type effects. The focus of these studies is the design of the information structure to influence consumers, whereas the present paper examines precision and recall in the design choices of firms' competitive targeting strategies.

We also contribute to the growing literature on strategic interactions of algorithms and machine learning models(Liang, 2019; Miklós-Thal and Tucker, 2019; Salant and Cherry, 2020; Calvano et al., 2020; O'Connor and Wilson, 2021; Montiel Olea et al., 2022; Yang et al., 2024). Closely related to our work, Iyer and Ke (2024) consider the model selection problem and the bias-variance trade-off in the context of targeting. They find competition favors bias because it can help firms manage overlaps in targeting, thereby softening competition. We focus on a different trade-off that is also a general feature of machine learning models. The precision-recall trade-off is not about model selection, but rather about firms' decision rule given the prediction that the model generates. In other words, Iyer and Ke (2024) studies how to pick an algorithmic model, while we focus on how to deploy algorithmic targeting - i.e., who and how to target given the model's predictions. This is a feature that would apply, in general, to the deployment of any machine learning model design that the firm may end up choosing.

II Model

II.A Basic Setup

There is a market with up to two firms and a unit mass of consumers. Consumers are of two types: interested or uninterested. Interested consumers, if targeted, will make a purchase. Uninterested consumers, as well as interested consumers who are not targeted by the firm, will not make a purchase.[3] The prior probability that a consumer is interested is μ_0.

Firms are uncertain about the type of any individual consumer. They rely on predictions from an information system to identify interested consumers. In the context of contemporary big data environments, such a predictive information system can be a machine learning algorithm that provides predictions to identify interested consumers at the individual level. An algorithm, e.g., Logit regression or a more complicated neural network, uses data on consumer and market characteristics, and yields a prediction for each individual consumer, which is the likelihood with which the consumer is interested in the product. Accordingly, we assume that the prediction for consumer i is the probability p_i of that consumer i being interested. The distribution of the predictions $f(p)$ is defined on the support $[0, 1]$. This distribution characterizes the informativeness of the algorithm, which is determined by the limits of the data and the sophistication of the algorithm. The distribution is Bayesian consistent, i.e., the expected probability of a consumer being interested is $\int p f(p) \mathrm{d}p = \mu_0$, the prior probability. At one extreme, a perfect algorithm has a bimodal distribution with two mass points at $p = 0$ and $p = 1$. In contrast, a fully uninformative algorithm has a unimodal distribution with one mass point at $p = \mu_0$.

Firms produce competing products and they reach consumers through targeted advertising. If an interested consumer is targeted by only one firm, they will generate

[3] A standard interpretation of this assumption is advertising as information - that advertising creates awareness of the existence and the characteristics of the product.

value v for the firm. If both firms target the same consumer, they will generate an expected value $w < v$ for each firm. The unit cost of targeting is c, which is also assumed to be less than v, as otherwise no firm has an incentive to target any consumer. The firm decides which consumers to target based on the predictions of its algorithm. Specifically, consider a targeting policy of $q(p) \in [0, 1]$, in which $q(p)$ is the share of probability-p consumers being targeted. Therefore, we allow firms to use mixed strategies, which can be interpreted in the standard manner as different advertising intensities (for example, across time or media channels).

This is a general framework that accommodates the standard models in the targeting literature. For example, the Chen et al. (2001) model assumes that firms can receive noisy binary signals about consumer types. Mapping this to our framework, the signal structure represented by the targetability measure in Chen et al. (2001) is a degenerate bimodal posterior distribution of consumer types (loyal and switcher). As firms' targetability increases, the two mass points of the bimodal distribution shift away from the prior. The perfect targetability case corresponds to the bimodal distribution at 0 and 1.[4]

II.B Predictions of Competing Firms

When both firms use algorithms to predict consumer types, their predictions may be correlated. In practice, the algorithmic predictions of firms might be correlated because firms may use the same public data in addition to their private company data or because they use similar data analytics models and tools. Our analysis focuses on the case where the two algorithms have identical (unconditional) distributions of the predictions, $f(p)$, but may be positively correlated. This allows us to derive general

[4]Note that what is labeled as the precision in Chen et al. (2001) is different from our definition as in the classification and machine learning literature. In Chen et al. (2001) precision is not about a firm's targeting policy, but rather about the informativeness of the predictions (which is analogous to the function $f(p)$ in our framework) irrespective of the targeting policy.

results on how the degree of correlations in predictions affects targeting policies.

Specifically, Consider a consumer, for whom firm 1 makes a prediction of p_1. We divide into two cases firm 2's prediction for this consumer. With probability p_1, the consumer is indeed interested. In this case, we assume that with probability ρ, the prediction from firm 2 coincides with the prediction from firm 1, p_1. A larger ρ implies that firms' predictions are more likely to be aligned. This happens when they use the same public/third-party data for this particular consumer.

With probability $1-\rho$, firm 2's prediction differs from firm 1's because the firms use private or first-party data. The prediction of firm 2 is drawn from the distribution with probability density function $pf(p)/\mu_0$. The conditional distribution $pf(p)/\mu_0$ shifts to the right tail of the unconditional distribution $f(p)$ to capture the idea that a firm's prediction should be closer to 1 in distribution when the consumer is indeed interested.

With probability $1-p_1$, the consumer is indeed uninterested. In this case, we also assume that with probability ρ, the prediction of firm 2 coincides with the prediction from firm 1, p_1, due to the use of the same public data. With probability $1-\rho$, firm 2's prediction differs from firm 1's and is drawn from the distribution with probability density function $(1-p)f(p)/(1-\mu_0)$. The conditional distribution $(1-p)f(p)/(1-\mu_0)$ shifts to the left tail of the unconditional distribution $f(p)$ to capture the idea that a firm's prediction should be closer to 0 in distribution when the consumer is indeed uninterested.

Putting these pieces together, for a consumer whose predicted probability by firm 1 is p_1, the distribution of the prediction for that consumer by firm 2 is $f(p|p_1) := p_1[\rho\delta(p_1)+(1-\rho)pf(p)/\mu_0]+(1-p_1)[\rho\delta(p_1)+(1-\rho)(1-p)f(p)/(1-\mu_0)] = \rho\delta(p_1)+(1-\rho)[p_1\cdot pf(p)/\mu_0+(1-p_1)\cdot(1-p)f(p)/(1-\mu_0)]$, where δ is the Dirac delta function.[5] In other words, for any given consumer, the prediction of firm 2 agrees with firm 1 with

[5]Note that we will recover the unconditional distribution $f(p)$ if we integrate the conditional distribution function $f(p|p_1)$ over p_1.

probability ρ. Otherwise, firm 2 will assign this consumer a probability according to a weighted average of two conditional distributions. The specific functional form of the two conditional distributions, which shift to the right and the left tail of the unconditional distribution $f(p)$, means that a firm's prediction is still correlated with the other firm's prediction even if they are not an exact match (with probability $1-\rho$). This implies the logical and intuitively appealing outcome that the two firms' predictions will almost always be identical regardless of ρ when their algorithms are almost perfect.[6]

We emphasize that ρ is the probability that the predictions of competing firms match exactly, which is not the typical Pearson correlation coefficient.

III Analysis

III.A Monopoly Benchmark

The monopoly firm's targeting strategy will be to choose a threshold policy $\underline{p}_m \in (0,1)$, such that it targets all consumers with $p \geq \underline{p}_m$: $q(p) = 1_{p \geq \underline{p}_m}$. At the threshold

[6]Alternatively, one can consider a simpler correlation structure. Suppose that firm 1's prediction for a given consumer is p_1. Then, with probability ρ, firm 2's prediction is also p_1, and with probability $1-\rho$, firm two's prediction is uniformly drawn from $f(p)$. Under this alternative setup, a firm's prediction is completely uncorrelated with the other firm's with probability $1-\rho$. This might seem to be a simpler correlation structure but it can have some less-than-desirable and practically unappealing properties. To see this, suppose that the firms' algorithms are close to perfect in their prediction abilities (i.e., informativeness). In such a case, it is possible that one firm has a prediction probability of 1 while the other firm has 0 with a high likelihood (close to $(1-\rho)/2$ when $\mu_0 = 1/2$). In contrast, under the correlation structure that we adopt in our main model described above, the two firms' predictions will almost always be identical regardless of ρ when their algorithms are almost perfect. Nevertheless, the specific choice of the correlation structure does not affect our results. In the online appendix, we show that all the main results in the paper hold under the alternative simpler correlation structure.

$\underline{p}_m$, the marginal revenue of targeting $\underline{p}_m v$ equals the targeting cost c, thus $\underline{p}_m = c/v$. This threshold policy is also consistent with industry practices. For example, Shi et al. (2022) note that Alibaba targets consumers whose aggregated algorithmic scores, which combine both benefits and costs of targeting, are above a threshold.

In monopoly, the precision of the optimal targeting policy is:

$$\frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\int_{\underline{p}_m}^{1} f(p)dp} = \frac{\int_{\underline{p}_m}^{1} pf(p)dp}{a_m} \tag{1}$$

The recall of the optimal targeting policy is:

$$\frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\int_{0}^{1} pf(p)dp} = \frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\mu_0} \tag{2}$$

Both the precision and recall depend on the firm's targeting policy $\underline{p}_m$ and the informativeness of the algorithm, as characterized by the distribution of the predictions $f(\cdot)$. Notice that higher precision implies that the algorithm targets more interested consumers rather than uninterested ones, while higher recall implies that the algorithm targets more interested consumers out of the total interested pool. Given the algorithm, the precision increases while the recall decreases in the targeting threshold $\underline{p}_m$. By targeting only a small subset of consumers who are highly likely to be interested, the firm generates a low false negative rate and achieves high precision. However, the firm sacrifices recall because it does not target a larger set of consumers who are likely to be interested. So, in this set, the firm generates a high false negative rate, resulting in lower recall. At one extreme, as $\underline{p}_m \to 1$, Precision $\to 1$, and Recall $\to 0$. In contrast, the opposite applies when the targeting threshold $\underline{p}_m$ is low. As $\underline{p}_m \to 0$, Precision $\to \mu_0$, and Recall $\to 1$.

To what extent the firm's optimal targeting policy prioritizes precision vs. recall depends on the profits and cost of targeting. If the targeting cost is high relative to its benefit (high c/v), the monopoly only wants to target high-probability consumers to save targeting costs (precision over recall). In contrast, if the targeting cost is low relative to its benefit (low c/v), the monopoly does not want to miss out on consumers who may be moderately likely to be interested (recall over precision).

In the appendix, we discuss and visually illustrate the mapping from an algorithm and targeting policy to precision and recall, as well as the link between precision-recall and hypothesis testing.

III.B Competitive Algorithmic Targeting

In this section, we consider the case of the competing firms choosing their targeting policies $q_i(p)$ simultaneously. In the main analysis that follows, we derive the implications of the symmetric equilibrium. Later on in section IV.C we analyze the robustness of the main insights to the case of possible asymmetric equilibria.

We first note that there does not exist a symmetric pure strategy equilibrium for any positive ρ.[7] We thus consider the *symmetric mixed-strategy equilibrium*. Suppose each firm targets probability p consumers with probability $q(p)$. Let $\underline{p} = \inf\{p : q(p) > 0\}$, i.e., the lowest probability consumer that firms target for strictly positive probability. Denote the recall of the targeting policy given $q(p)$ by R and the aggregate targeting intensity given $q(p)$ by $a = \int_{\underline{p}}^{1} f(p)q(p)dp$.7

The expected payoff of a firm from targeting a probability p consumer is:

$$[\rho q(p) + (1-\rho)R]pw + [1-\rho q(p) - (1-\rho)R]pv - c$$

$[\rho q(p) + (1-\rho)R]p$ is the probability that a probability p-consumer is interested and targeted by both firms, in which case the focal firm gets a payoff of w. There are

[7]One can see that no firm targeting cannot be a Nash Equilibrium. Now suppose there is a symmetric pure strategy equilibrium where firms target some consumers. Consider $\underline{p} \leq 1$ as the lowest probability for which firms target. Clearly, $\underline{p} \geq \underline{p}_m$, the monopoly targeting threshold. We now have $\underline{p}\{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v\}-c \geq 0$. By deviating and targeting probability $\underline{p}-\epsilon$ consumer, the firm obtains an expected payoff of $(\underline{p}-\epsilon)\{[(1-\rho)R]w+[1-(1-\rho)R]v\}-c = \underline{p}\{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v\}-c+\rho(v-w)\underline{p}-\epsilon\{[(1-\rho)R]w+[1-(1-\rho)R]v\} \geq 0+\rho(v-w)\underline{p}_m-\epsilon > 0$ for ϵ small enough. Therefore, firms have an incentive to deviate.

two such cases: i) when the predictions coincide and the other firm also targets this consumer, which happens with probability $p\rho q(p)$, or ii) when the two predictions differ but the algorithm of the other firm assigns a sufficiently high probability (i.e., above the threshold $\underline{p}$) and targets this consumer regardless, which happens with probability $p(1-\rho)\int_0^1 q(p)pf(p)/\mu_0 dp = p(1-\rho)\int_{\underline{p}}^1 q(p)pf(p)/\mu_0 dp = p(1-\rho)R$. Similarly, the consumer is interested and targeted by only one firm with probability $[1-\rho q(p)-(1-\rho)R]p$, which will result in a payoff of v.

Assumption 1 $0 \leq w < c < v$. *A firm prefers to target an interested consumer by itself, but does not prefer to target an interested consumer who is also targeted by the competitor.*

In equilibrium, there are two types of targeting strategies. Either the firm never targets any consumer for sure, or it targets a set of consumers with the highest prediction probabilities for sure, and the consumers with lower prediction probabilities partially.

Lemma 1 *In any symmetric equilibrium, either* $q(p) \in (0,1), \forall p \in (\underline{p}, 1]$ *or there exists* $\overline{p} \in (\underline{p}, 1)$ *such that* $q(p) = 1, \forall p > \overline{p}$, *and* $q(p) \in (0,1), \forall p \in (\underline{p}, \overline{p})$.

For notational ease, we let $\overline{p} = 1$ if the firm never targets any consumer for sure. Under competition, the precision of the optimal targeting policy is:

$$\frac{\int_{\underline{p}}^1 pq(p)f(p)dp}{\int_{\underline{p}}^1 q(p)f(p)dp} = \frac{\int_{\underline{p}}^1 pq(p)f(p)dp}{a} \tag{3}$$

The recall of the optimal targeting policy is:

$$\frac{\int_{\underline{p}}^1 pq(p)f(p)dp}{\int_0^1 pf(p)dp} = \frac{\int_{\underline{p}}^1 pq(p)f(p)dp}{\mu_0} \tag{4}$$

Figure 1 illustrates the possible symmetric equilibrium ($\overline{p} = 1$ in the left figure). The firm is indifferent between targeting or not and adopts a mixed strategy for

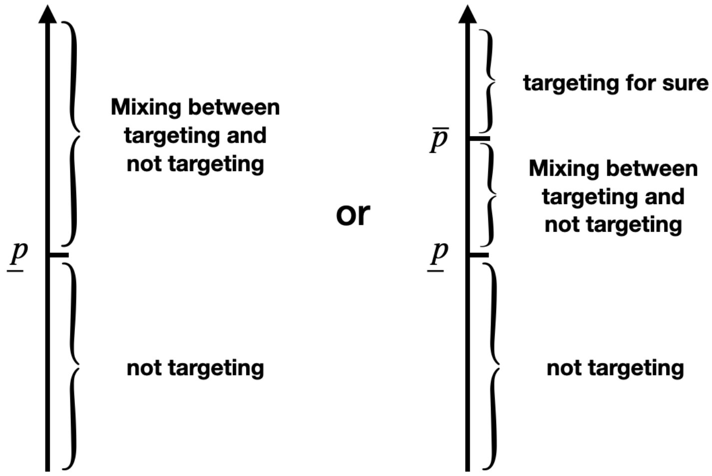


Figure 1: Possibile symmetric equilibrium

$p \in (\underline{p}, \overline{p})$. The firm's expected profit for those consumers is 0, its payoff from not targeting.

$$
\begin{aligned}
& [\rho q(p) + (1-\rho)R]pw + [1 - \rho q(p) - (1-\rho)R]pv - c = 0 \\
\Rightarrow\ & q(p) = \frac{v - (1-\rho)(v-w)R - c/p}{\rho(v-w)},\ \forall p \in (\underline{p}, \overline{p}) \qquad (5) \\
& q(\underline{p}) = 0 \\
\Rightarrow\ & \underline{p} = \frac{c}{v - (1-\rho)(v-w)R} \qquad (6)
\end{aligned}
$$

According to equation 5, the targeting probability $q(p)$ is non-linear in p. The reason is that the benefit of targeting is linear in p, whereas the cost of targeting is a constant c. So, the targeting probability contains a c/p term, which is non-linear.

The firm strictly prefers targeting to not targeting for $p > \overline{p}$. So, it targets those high-probability consumers for sure and obtains a positive profit. If $\overline{p} = 1$, then no consumers will be targeted by the firm for sure. To emphasize that $\underline{p}$, a, and R depend on ρ, we use the notation $\underline{p}(\rho)$, $a(\rho)$, and $R(\rho)$ when necessary to avoid confusion. Assumption 1 implies that $\underline{p}(1) = c/v \in (0, 1)$. Lemma 2 establishes that such an equilibrium exists and is unique.

Lemma 2 *There exists a unique mixed-strategy symmetric equilibrium. The firm targets some consumers with a positive probability in equilibrium. Furthermore, there exists* $\widehat{\rho} \in [0, (v-c)/(v-w)]$ *such that firms never target any consumer for sure if and only if* $\rho \geq \widehat{\rho}$*. Otherwise, the firm targets consumers with probabilities* $p \geq \overline{p}$ *for sure and mix for consumers with probabilities between* $\underline{p}$ *and* $\overline{p}$*.*

When the probability of interest is high and the correlation is low, each consumer is very valuable and gives the firm a strictly positive expected profit. So, each firm targets such consumers for sure. For a moderate probability consumer, targeting the consumer for sure results in a significant probability of overlap in targeting, thereby a net loss. So, each firm targets the consumer with some probability and is indifferent between targeting or not. As the consumer's probability of interest p increases, a firm's expected payoff from either solely targeting the consumer or jointly targeting the consumer increases. To maintain the indifference, it must be the case that the likelihood of both firms targeting the same consumer increases. So, each firm's targeting probability $q(p)$ increases in p. This indifference condition pins down the unique symmetric mixed-strategy equilibrium.

Figure 2 illustrates the targeting probability of the optimal targeting policy for different ρ. When ρ is high, a firm never targets any consumer for sure because the rival firm has a high likelihood of having the same prediction for the consumer. In contrast, when ρ is low, a firm targets high-probability consumers for sure because the other firm has a high chance of having a different prediction and not targeting that consumer. However, for lower-probability consumers, the potential gain for a firm from targeting decreases. At the same time, the probability of overlapping in targeting stays the same if firms were to target that consumer for sure. Thus, when the predicted probability for a consumer is below a threshold, the firms in equilibrium mix between targeting and not targeting that consumer in order to soften the competition.

On the extensive margin (i.e., the lowest probability consumer being targeted), firms become less selective as their predictions become more correlated. The intuition

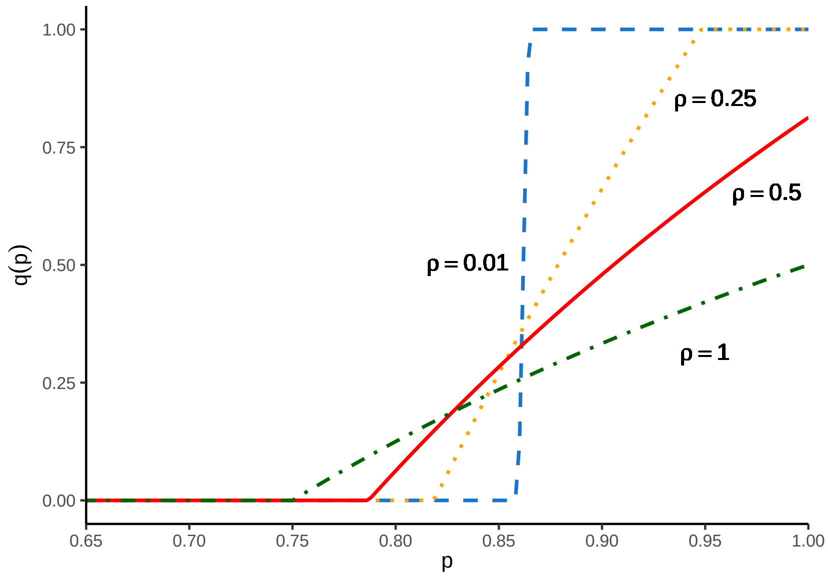


Figure 2: Targeting probability of the Optimal Targeting Policy for $c = .75, v = 1, w = .5, f \sim U[0,1]$.

is as follows. Consider marginal consumers whose probability of being targeted is very low. When the predictions are highly correlated, both firms' predictions are largely aligned on the identity of the marginal consumers. Each firm will be more confident that those marginal consumers will be targeted by the other firm with a low probability. Thus, a firm becomes more inclined to target some of those consumers. In contrast, a consumer with a predicted probability of one will be more likely to be highly valued by the other firm as well when the predictions become more correlated. Therefore, a firm becomes less inclined to target such a consumer. Therefore, the targeting probability $q(p)$ becomes flatter as the correlation increases.

We now proceed to establish one of the main results of this paper by comparing the equilibrium targeting policies of the monopoly benchmark with the duopoly case in the following proposition.

Proposition 1 *The duopoly's optimal targeting policy has a higher precision and a lower recall than the monopoly's.*

This proposition connects the precision-recall trade-off that is a fundamental aspect of the deployment of machine-learning algorithms to the equilibrium incentives of competitive firms. Relative to a monopoly, competition favors precision over recall in algorithmic targeting. The intuition is that competition reduces the expected profit from targeting due to overlaps in the set of targeted consumers of the rival firms. Firms, therefore, strategically respond in equilibrium by concentrating their targeting efforts on consumers who are predicted by their algorithms to have a high likelihood of conversion, thereby leading to an increase in the precision of their targeting.

Comparative Statics With Regard to the Correlation

We now discuss the details of equilibrium and the implications of the important comparative static predictions (the proofs are left in the Appendix). To begin with, consider the effect of the correlation in the firms' predictions on the equilibrium profits: When the correlation is sufficiently high ($\rho > \hat{\rho}$) then as shown in Lemma 2 the equilibrium targeting policy involves firms not targeting any consumers for sure. This implies that the equilibrium profits are zero. Conversely, when the correlation is lower, a set of high-probability consumers will be targeted for sure, and this implies positive equilibrium profits. Further, in this case, lower values of ρ are associated with increasing profits. Thus, firms will have the incentive to choose uncorrelated algorithms ($\rho = 0$) if it is costless. Note that in reality, the correlation between the algorithms can be due to the extent to which the firms use the same public data or because they use the targeting tools offered by a common data analytics intermediary. This result, therefore, has an organizational strategy implication that under competition, firms have greater incentives to develop their internal private data systems and their data analytics capabilities. In section IV.A, we will consider the implications of endogenizing ρ.

Next, as illustrated in Figure 3, we show that as ρ increases and the predictions of the firm converge, firms end up targeting fewer consumers leading to a decrease in the recall. As the predictions converge, the targeting of the firms is more likely to coincide, reducing the returns from targeting. Interestingly, and somewhat counter to expectation, we prove that under competition, as long as the level of correlation is high enough ($\rho > \hat{\rho}$), the precision also decreases as ρ increases. As the correlation increases, the high-probability consumers become less attractive because both firms increasingly identify these consumers as the more valuable high-probability consumers and compete by setting higher targeting intensity. Conversely, the more moderate-probability consumers become relatively more attractive because they are now less likely to be targeted. With higher correlation, when a firm predicts a moderate probability consumer, it can expect that the consumer is less likely to be predicted as a high probability consumer by the rival, implying that the rival will have the incentive to target moderate-probability consumers less aggressively. Overall, this leads to $q(p)$ becoming flatter, decreasing the precision.

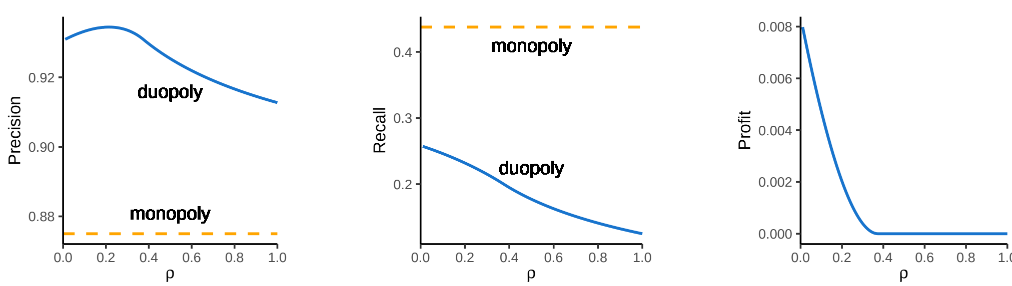


Figure 3: Comparative Statics with regard to ρ for $c = .75, v = 1, w = .5, f \sim U[0, 1]$. The profit under monopoly is 0.3125.

Comparison to Traditional Targeting with Segment-level Predictions

The difference between this paper and the traditional targeting contexts, such as Chen et al. (2001), is that firms face a distribution of probabilities instead of only having a separate signal at the level of a consumer segment, and cannot further distinguish individual consumers within the segment. With the advancement of algorithmic targeting technologies, the ability to generate predictions at the individual level is becoming increasingly prevalent. So, it is important to understand the implications of individual level probabilistic targeting. With individual-level predictions, as Figure 3 illustrates, firms continuously adjust the precision and recall of their targeting policies as the correlation in their predictions changes. They can respond to strategic incentives by adjusting the targeting strategies given an arbitrarily small change to the competitive environment. With individual-level predictions firms have a large targeting policy space to choose from. This contrasts with traditional targeting contexts in which firms would only have a separate signal at the level of a discrete consumer segment and cannot further distinguish individual consumers within the segment. In such a case, the precision and recall targeting policy of a firm will be less flexible and would stay constant for an interval of the prediction correlation. Firms may not be able to respond smoothly to strategic incentives when the competitive environment changes.

The more flexible targeting ability enabled by the individual-level predictions implies that firms do not need to distort their precision-recall trade-off because of constraints in the targeting policy space. In the traditional targeting context with segment-level predictions, we can get the somewhat unnatural outcome that firms may target more consumers and increase their recall even as the correlation in their predictions increases, which is driven by their constrained targeting ability. In contrast, with individual-level predictions and the larger targeting policy space, such an artifact disappears, and the recall is always decreasing in the correlation.

Other Comparative Statics

We now consider comparative statics with regard to other parameters. The recall decreases in c and increases in w. It can either increase or decrease in v. Intuitively, the firm targets more consumers when the targeting cost c is lower. With a higher w, the firm's expected payoff increases even if a consumer is targeted by both firms. Therefore, for a consumer whose probability is above the targeting threshold, each firm targets the consumer with a higher likelihood, which leads to a higher recall. When v is higher, however, the firm does not necessarily want to increase its recall. Though targeting an interested consumer by itself becomes more valuable, firms may increase the competition and target more consumers together if they increase the recall, which may hurt their profits. Numerical simulations also suggest that the precision decreases in v because it encourages firms to target consumers with lower probabilities. The opposite happens with c. Regarding w, we find the precision decreases in w but it may be invariant to w when correlation is high such that firms never target any consumer for sure. In this case, although a firm gets more from targeting consumers with a given probability, such an effect is offset by the increase in the recall of the competing firm.

IV Extensions

IV.A Endogenous Prediction Correlations

We now examine the implications when firms are able to endogenously choose the correlations in their predictions. When firms rely on public data or common data analytics providers, their targeting predictions are likely to be more correlated. Conversely, a firm may also invest in internal analytics organization and in acquiring private consumer and market data, which may lead to a decrease in the prediction correlations between the firms. For example, for a fixed size of the training sample and thus the probability distribution f, a firm may use more private data as opposed to publicly available data in order to differentiate their prediction models. In this sense, we can interpret the endogenous choice of algorithmic correlation as resulting from the organizational strategy for data analytics by firms in competitive markets.

Consider the following model that endogenizes ρ. Each consumer i is characterized by a vector of characteristics x_i. They are also binary in type $y_i \in \{0, 1\}$, indicating whether they are uninterested or interested in the firm's product. Firms are uncertain about y_i. They observe a signal of consumer characteristics x_i such that $x_{ij} = x_i + \epsilon_{ij}$, in which $\epsilon_{ij} \sim H()$ is the noise that are identically distributed across firms j. Upon observing x_{ij}, firm j's prediction is $P_j(y_i = 1|x_{ij}) = g(x_{ij})$. Over the distribution of x_{ij}, firm j's distribution of predictions is given by $f_j(p) = f(p)$.

For every consumer i, there exist three signals. First, there is a public signal $\hat{x}_{i0} = x_i + \hat{\epsilon}_{i0}$, $\hat{\epsilon}_{i0} \sim H()$. This corresponds to public data that are commonly available to both firms. Second, there are private signals $\hat{x}_{ij} = x_i + \hat{\epsilon}_{ij}$, in which $\hat{\epsilon}_{ij} \sim H()$. $\hat{\epsilon}_{ij}$ will be independent across $j \in \{0, 1, 2\}$. For a consumer i, if a firm chooses public data, they will get a signal of $x_{ij} = \hat{x}_{i0} = x_i + \hat{\epsilon}_{i0}$. If firm i chooses private data, they will get a signal of $x_{ij} = \hat{x}_{ij} = x_i + \hat{\epsilon}_{ij}$. Thus, the correlation between firms' predictions depends on to what extent firms rely on public data versus private data: ρ is the probability of both firms getting x_{i0} draws as both use public signal for this focal consumer. Thus, their predictions will coincide. Otherwise, the probability that their predictions are equal to each other has a measure of zero.

Both firms have access to the public signal at no cost. Suppose that initially both firms are also endowed with private data of $1 - \sqrt{\rho_0}$ units of consumers. A special case of $\rho_0 = 1$ applies when only public data are initially available. Firms can decide to invest $I \geq 0$ to gain access to private data for $d(I)$ extra units of consumers. We assume $d(I)$ is increasing and concave in I, with $d(0) = 0$ and $d(I) < \sqrt{\rho_0}, \forall I$. We then have $\rho(I_1, I_2) = (\sqrt{\rho_0} - d(I_1))(\sqrt{\rho_0} - d(I_2))$ when the investments in private data are uncorrelated among consumers. Thus, depending on the investments $\rho(I_1, I_2)$

becomes the probability that both firms draw the common signal x_{i0}.

An assumption of the above framework is that for a given consumer i, a firm j can choose to use only public data (observing $\hat{x}_{i0}$) or only private data (observing $\hat{x}_{ij}$), but it cannot merge these two data sources to potentially improve accuracy. There are some important reasons. For example, combining data sources amounts to creating new personal data profiles, which require fresh consent or a re-evaluation of the legal basis under regulations. It may also risk re-identifying individuals who may be subject to additional data protection rules. Furthermore, some data sources, such as social media platforms, explicitly forbid data scraping or combining user profile data for commercial targeting. Even when purchasing "public" demographic data from a broker, the contractual license can restrict how the firm can merge it with the private data, especially if it becomes personally identifiable.

In the first period, both firms simultaneously make costly investments in lowering ρ. In the second period, both firms simultaneously choose the targeting policy exactly as in the main model. We note that this setup satisfies the following conditions on the investment technology.

Assumption 2 *$\rho(I_1, I_2)$ is smooth and strictly decreasing in $I_j, j = 1, 2$. $\rho(I_1, I_2) > 0,\ \forall I_1, I_2 < v$. Define $\rho_1(I_1) = \rho_0 - \rho(I_1, 0)$, $\rho_2(I_2) = \rho_0 - \rho(0, I_2)$, and $K_j(\Delta\rho) = \rho_j^{-1}(\Delta\rho)$. $K_j'(0) = 0, j = 1, 2$.*

Note that $\rho_j(I_j)$ measures how much correlation a firm can unilaterally reduce by investing I_j when the other firm incurs zero costs. It is important to recognize the public production of the prediction correlation – i.e., any given firm's investment reduces the correlation in the predictions for both firms. Denote $K_j(\Delta\rho)$ as the inverse which measures how much cost a firm needs to incur to reduce the correlation by $\Delta\rho$ when the other firm does not invest and incurs zero costs. The assumptions $K_j'(0) = 0$ and $\rho(I_1, I_2) > 0,\ \forall\ I_1, I_2 < v$ mean that it is easy to reduce the correlation by a little bit, but very costly to reduce the correlation all the way to zero. In reality, firms may easily reduce ρ a bit by investing in a little more private data. However, it can be very difficult for firms to achieve zero correlation in their predictions in a competitive market.

Proposition 2 *Firms invest and earn a positive expected profit if* $\rho_0 < \widehat{\rho}$. *They may not invest and earn zero profits if* $\rho_0 > \widehat{\rho}$.

One may think that firms have a stronger incentive to invest in reducing the correlation if the initial correlation is high because otherwise they will get zero profits without the investment. It turns out to be the opposite. Firms will invest if the initial correlation is already low enough, such that they will earn a positive profit without investment. The low-cost assumption for small investment does not directly imply that firms will invest because the benefit of a small investment may also be very low. We prove that the benefit of investing a little bit is roughly linear in the investment cost, which outweighs the investment cost. In contrast, as Figure 3 shows, the profit is zero for an interval of $\rho \in [\widehat{\rho}, 1]$. So, a lower ρ will increase the profit (not taking into account the investment costs) only if firms invest a lot such that $\rho < \widehat{\rho}$. In that case, the investment cost may be so high that firms prefer not to invest.

The proposition provides a structure to consider the industry scenarios which are likely to encourage investments by firms in private company data analytics. For example, when there is already a well-entrenched public data system which is used by the firms, the incentive to invest in internal data analytics within the organization is attenuated. Indeed, given that profits with a high correlation are zero, this would precisely be the situation in which such investments by a firm would have been beneficial. Therefore, there is a suggestion in the model that there can be under-investment in private data because the investment by a firm to soften competition through reduced correlation is a public good. That said, investment in private data can also lead to higher accuracy. If so, one might expect a higher level of investment and thus more reduction in correlation. Still, one would expect that the free-riding problem in competitive targeting would lead to underinvestment. In our analysis, we assume the same accuracy to separate the effect of correlation. This allows us to study when firms may decide to invest in private data even without any improvement in accuracy.

IV.B Pricing

The main model focuses on the targeting strategy by fixing an exogenous pair of expected targeting values, v and w. This setup can be interpreted as both firms charging a symmetric price, P, which is exogenously given. If an interested consumer is targeted by only one firm, then the expected value of targeting is $v = P$. If both firms target the same consumer, the consumer randomly chooses a product from one of the firms, leading to an expected value of targeting of $w = P/2$.

In order to gain understanding of the role of endogenous prices we conduct a series of numerical simulations of a model with two feasible price levels, $P_H > P_L$. Each firm simultaneously selects both its price and the targeting rule. Focusing on pure strategies, we illustrate our findings in Table 1, where we denote the equilibrium pricing strategy of the firms by (P_1, P_2). As shown in the table, both firms charge a high price when ρ is low. As ρ increases and price competition intensifies, firms transition to an asymmetric price equilibrium. These qualitative properties hold across a broad range of parameters. For example, fixing $P_H = 1$ and $c = .75$, we obtain the same insights for $P_L \in [0.76, 0.86]$.[8]

The intuition behind this finding is that by deviating to a low price P_L, a firm gains higher profits from the overlapping consumers, but earns lower profits from consumers it exclusively targets. The latter effect dominates when firms are more likely to focus on different sets of consumers - i.e., when the correlation in their predictions

[8]When P_L is sufficiently close to P_H, a (P_H, P_H) equilibrium does not exist because a firm can attract all overlapping consumers by deviating to P_L, sacrificing only a small portion of profit per consumer.

ρ	equilibrium prices
$\rho < 0.2217$	(P_H, P_H)
$\rho > 0.2217$	(P_H, P_L) or (P_L, P_H)

Table 1: Equilibrium with pricing for $P_H = 1, P_L = .8$, and $c = .75$.

is low. Consequently, each firm has little incentive to lower its price, thereby sustaining the high-price equilibrium. In this case, each firm's targeting rule coincides with the main model, where $v = P_H$ and $w = P_H/2$. As the correlation increases and firms increasingly target the same consumers, the incentive to soften competition strengthens, prompting one firm to cut its price. When this occurs, the low-price firm attracts all interested overlapping consumers, implying that its targeting decision is not affected by the high-price firm. Its targeting strategy then resembles that of a monopoly in the main model, where $v = P_L$, targeting all consumers with a predicted probability exceeding c/P_L. In response, the high-price firm targets some of the low-probability consumers ($p < c/P_L$). It may also target high-probability consumers (p close to 1) when the correlation is low.

Regarding equilibrium targeting strategies, we find that the recall of each firm is lower than the monopoly case, where the monopoly charges P_H. The precision of at least one firm is higher than the monopoly case. So, the main message - that firms prioritize precision over recall in competitive settings - remains valid.

IV.C Robustness: Pure Strategy Asymmetric Equilibria

In our main analysis, we focus on the symmetric mixed-strategy equilibrium - this is because both firms are symmetric, and thus it has a natural interpretation. We also note that there are also a multiplicity of asymmetric pure strategy equilibria. So our objective in this section is to show the robustness and the generality of our main result on the precision-recall choice under competition to the asymmetric equilibria and highlight some common features.

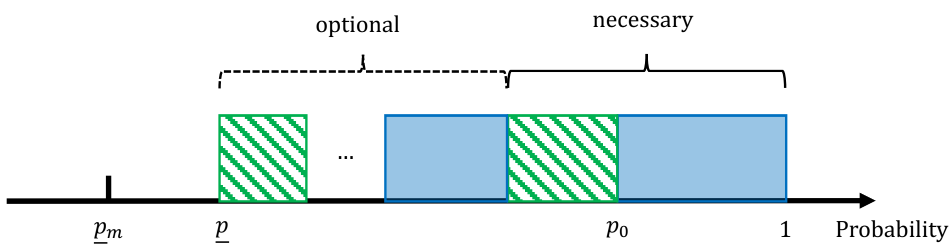
Define $\underline{p} := \inf\{p \in [0,1] : q_1(p) > 0 \textit{ or } q_2(p) > 0\}$, where $q_i(p)$ is firm i's targeting probability of a type p consumer. Then, one can see that at least one firm targets at any $p > \underline{p}$. In addition, pure strategy symmetric equilibrium does not exist. If both firms target consumers whose probability is larger than or equal to $\underline{p}$, then each firm wants to deviate by targeting $\underline{p} - \epsilon$. Lastly, $\underline{p} \geq \underline{p}_m$. This is because a monopoly does not target below $\underline{p}_m$, and the presence of competition makes targeting at any probability less attractive.

Proposition 3 *There exists an equilibrium in which one firm targets and acts like a monopoly if* $\rho \geq [R_m w + (1 - R_m)v - c]/[(1 - R_m)(v - w)]$. *In any other pure strategy asymmetric equilibria, there exists* $p_0 \in (\underline{p}, 1)$ *such that exactly one firm targets consumers whose* $p \in (\underline{p}, p_0)$. *Either one or both firms target every consumer whose* $p > p_0$.

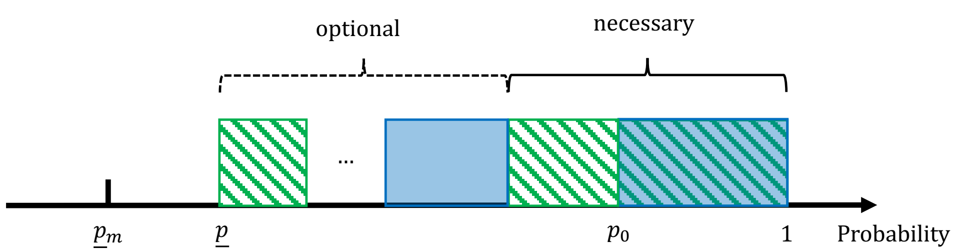
The recall of either firm is lower than the monopoly case. The precision of at least one firm is higher than the monopoly case.

Figure 4 illustrates the equilibrium strategy. When the prediction correlation is high (Figure 4a), it is possible that a firm acts like a monopoly and drives the opponent out of the market. This is akin to a foreclosure equilibrium where the opponent cannot operate with positive profits. Even if the opponent predicts a consumer to have probability one, it anticipates that the rival that acts like a monopoly is highly likely to target that consumer as well. The expected equilibrium payoff of targeting is negative, and thus, it does not enter the market. Other than this special case, both firms target some consumers. For consumers moderately interested, firms partition their targeting region to soften competition. For really valuable consumers (high p), either one (Figure 4b) or both firms (Figure 4c) target them. Whether one or both firms target those consumers depends on the correlation of their prediction and the overall targeting probability of each firm. There is at least one partitioned region

(a) Monopoly



(b) No overlapping (partition)



(c) Overlapping at high-probability consumers

Figure 4: Pure Strategy Asymmetric Equilibria

where only one firm targets consumers, and there can be more than one partitioned region.

The main message that firms prioritize precision over recall under competition still holds in all asymmetric equilibria. Except for the really valuable consumers, firms do not simultaneously target consumers predicted to have the same probability, which avoids overlaps in targeting the same consumer due to having the same predictions.[9] Thus, firms reduce their recall and focus on precision instead.

V Discussion and Concluding Remarks

It is hard to overstate the importance of targeting in marketing strategy. While the advances in data analytics allow firms to have unprecedented and at-scale targeting abilities, algorithmic targeting predictions may nevertheless not be perfect. In reality, the typical output of a targeting algorithm is a distribution of predicted probabilities of conversion across consumers. Given this probabilistic targeting output, firms invariably have to choose between precision - targeting a few consumers with high probabilities of conversion, or recall - targeting a large set of consumers, many of whom may not convert. This trade-off is also a fundamental feature of all machine learning classification algorithms.

Given that the outcome of targeting algorithms used by firms are the individual-level predicted probabilities for consumers, this ppaer analyzes the precision-recall trade-off as part of the strategic deployment of algorithmic targeting by firms. It introduces a micro-founded model to study competitive targeting policies in the presence of the precision-recall trade-off. We find that compared to a monopoly, the competition between firms lowers recall but increases precision. However, as competing firms have more correlated predictions, they tend to not only target fewer consumers

[9]Notice that there are still overlaps in targeting even for moderately interested consumers because firms may have different predictions for the same consumer.

as the return to targeting decreases (lower recall) but also target less probable consumers (lower precision) in order to avoid head-to-head competition. Firms have an incentive to invest in decreasing the correlations in their predictions, but such investments, being a public good, also suffer from the free-rider problem.

The results have some important policy and managerial implications. Our main result on the precision-recall trade-off provides prescriptions on how firms should deploy their data analytics operations under competition. Furthermore, the equilibrium targeting policies under competition imply that firms will need to account for how much the predictions differ across competing firms in designing their targeting policies. Policy-wise, the results suggest that privacy regulations may have an unintended side effect of preventing competing firms from using private data to differentiate their predictions. This could have negative consequences for both the industry and consumers because firms can strategically change their targeting policies using less precision and recall.

There are several interesting issues that may be further explored in future research. The analysis of the paper was conducted for a given prediction distribution $f(p)$. We now use the Beta distribution to illustrate the implications of different prediction distributions $f(p)$, with the parameters of $v = 1, w = 0.5, c = 0.75$. As the parameters of the Beta distribution α and β increase (while keeping the prior constant), the distribution becomes less informative since the mass shifts towards the center instead of 0 and 1. As a result, both precision and recall decrease in the monopoly case (Table 2 Panel A). In the case of the duopoly, we numerically solve the equilibrium targeting policies and again find similar patterns for any given ρ (Table 2 Panels B-D): as the prediction becomes more informative, both precision and recall increase (and vice-versa).

As discussed in the model setup, targeting creates awareness of the existence of the product. Without being targeted, a consumer will not be aware and will never buy the product. In contrast, targeted consumers will buy the product and obtain a positive surplus if they find the product fits their needs. So, targeting creates value for consumers if the firm correctly targets the interested consumers. For monopoly, the total information value a targeting policy creates for consumers is proportional to $\int_{\underline{p}}^{1} pq(p)f(p)dp$, or its recall. With two firms, the information value becomes more complicated as it involves not only the recall of both firms, but to what degree do their targeting policies overlap. We conjecture that higher correlation may lower information value since it leads to lower recall for both firms. Moreover, higher correlation implies their targeting are more likely to coincide. To the extend that the benefit of being targeted by two firms is weakly lower than twice of that of being targeted by one firm, such coincidence will also lower the value of targeting to consumers.

Although we focus on targeted advertising, the framework can potentially be applied to other marketing decisions such as product design, promotion, and pricing. For example, it would be interesting to study whether competition, as measured by firms' predictions, favors more distinct product designs. Another potential generalization is to extend the model beyond the binary type assumption. Finally, our precision-recall targeting framework can be used to study algorithmic discrimination in targeting in markets where the protected characteristics (e.g., race or gender) of consumers are salient and can be discriminated against.

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Table 2: Precision and Recall in the Beta Distribution $f(p)$ Example

Panel A: Monopoly

alpha	beta	precision	recall
0.3	0.45	0.918025	0.604371
0.4	0.60	0.902381	0.527402
0.5	0.75	0.889673	0.464304

Panel B: Duopoly, $\rho = 0.3$

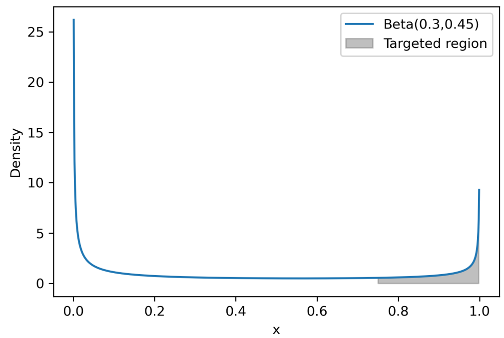
alpha	beta	rho	precision	recall
0.3	0.45	0.3	0.965718	0.318260
0.4	0.60	0.3	0.956560	0.279639
0.5	0.75	0.3	0.946341	0.243850

Panel B: Duopoly, $\rho = 0.5$

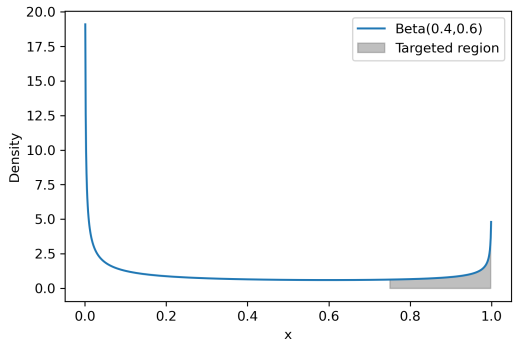
alpha	beta	rho	precision	recall
0.3	0.45	0.5	0.957185	0.278751
0.4	0.60	0.5	0.947413	0.237471
0.5	0.75	0.5	0.937732	0.203032

Panel B: Duopoly, $\rho = 0.7$

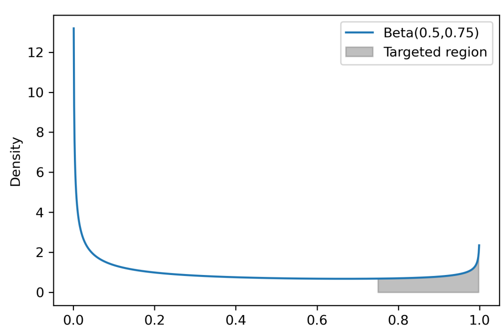
alpha	beta	rho	precision	recall
0.3	0.45	0.7	0.950975	0.251179
0.4	0.60	0.7	0.941057	0.208947
0.5	0.75	0.7	0.931453	0.175061



(a) Beta Distribution ($\alpha = 0.3, \beta = 0.45$)



(b) Beta Distribution ($\alpha = 0.4, \beta = 0.6$)



(c) Beta Distribution ($\alpha = 0.5, \beta = 0.75$)

Figure 5: Illustration of Different Prediction Distribution $f(p)$

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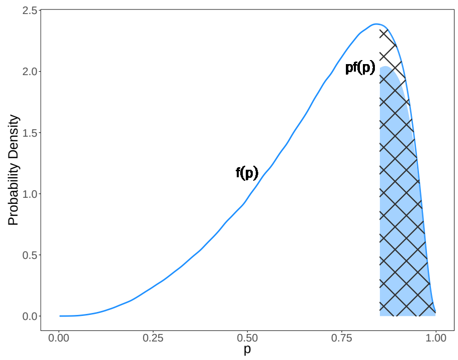
Appendix

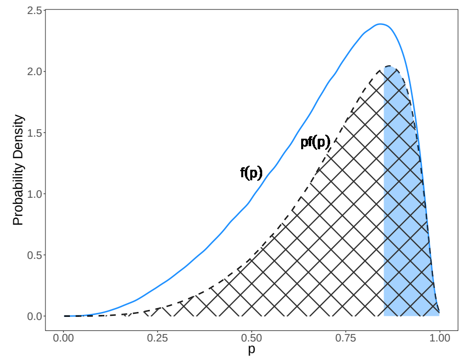
Mapping From an Algorithm and Targeting Policy to Precision and Recall

The equations (1) and (2) give the formulas of precision and recall. Figure 6 illustrates the precision and recall for a given algorithm and a given targeting policy. Consider a simple prediction algorithm - the logistic regression with one explanatory variable x (consumer characteristic), which is distributed normally with zero mean and unit standard deviation among all consumers. Formally, the logit model is $\mathbb{P}(x) = 1/[1 + e^{-(\beta_0+\beta_1 x)}] + \epsilon$, where ϵ is the unobserved shocks and follows a logistic distribution. The firm will first estimate the model coefficients, β_0 and β_1, using historical data. Suppose that the estimates are $\widehat{\beta}_0 = 1$ and $\widehat{\beta}_1 = 1$. Then, it can predict the probability of interest for each potential consumer based on the individual characteristics x according to $\widehat{\mathbb{P}}(x) = 1/[1+e^{-(\widehat{\beta}_0+\widehat{\beta}_1 x)}]$. Consumers with different characteristics will be predicted to have different likelihood of interest. By aggregating the individual-level predictions, the firm can obtain a distribution of probabilities for its potential consumers, $f(p)$.

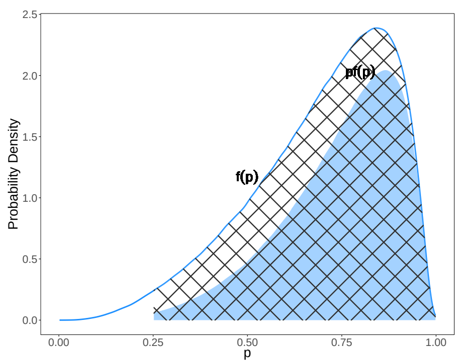
In Figure 6a and 6b the monopoly targets all consumers whose probability of interest is at least 85 percent ($\underline{p}_m = 0.85$). In both figures, there are $f(p)$ mass of probability p consumers, and the firm targets all of them if $p \geq 0.85$. Among those consumers, $pf(p)$ mass of probability p consumers are interested and targeted by the firm. The solid region measures the total number of interested consumers targeted by the firm. The cross-hatched region in Figure 6a measures the total mass of consumers targeted by the firm. The precision equals the area of the solid region divided by the area of the cross-hatched region. As we can see from the figure, the precision is high in this case because the firm targets only high-probability consumers. The majority of those consumers turn out to be interested. The cross-hatched region in Figure 6b measures the total mass of interested consumers. The recall equals the area of the solid region divided by the area of the cross-hatched region. The recall is low in



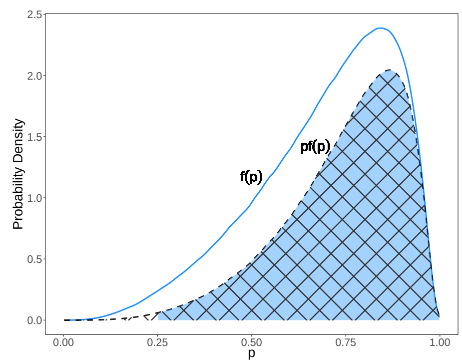
(a) Precision = 0.904 when the firm targets all consumers whose $p \geq 0.85$.



(b) Recall = 0.300 when the firm targets all consumers whose $p \geq 0.85$.



(c) Precision = 0.706 when the firm targets all consumers whose $p \geq 0.25$.



(d) Recall = 0.995 when the firm targets all consumers whose $p \geq 0.25$.

Figure 6: Precision and Recall under different targeting policies.

this case because the firm misses out on many potential consumers by targeting very selectively.

In contrast, the monopoly targets all consumers whose probability of interest is at least 25 percent in Figure 6c and 6d ($\underline{p}_m = 0.25$). The precision and the recall still equal the area of the solid region divided by the area of the cross-hatched region in these two figures, respectively. By targeting a larger set of consumers, the firm induces a lower precision and a higher recall. There are not many missing opportunities because most interested consumers are targeted. However, the firm also wastes more of its marketing budget on uninterested consumers due to extensive mis-targeting.

Linking Precision-Recall and Hypothesis Testing

Precision-recall as a classification concept in machine learning is closely connected to hypothesis testing. We use the monopoly case to illustrate it. Let the null hypothesis be that a given consumer is not interested. The firm decides whether to reject the null hypothesis based on the available information. If the firm rejects the null hypothesis, it will target the consumer because the alternative hypothesis is that the consumer is interested. False negatives in this context mean that the null hypothesis is wrong (the consumer is in truth interested) but the firm fails to reject the null hypothesis (therefore does not target the consumer). Under perfect recall, the firm targets all interested consumers. So, there are no false negatives. In general, recall equals the power of the test. False positives in this context mean that the null hypothesis is correct (the consumer is not interested) but the firm rejects the null hypothesis (and therefore targets the consumer). Under perfect precision, the firm does not target any uninterested consumers. So, there are no false positives. Unlike recall which is equivalent to the complement of the type II error rate, precision is not identical to any single statistical property in hypothesis testing, though it is related to the type I error rate. The reason is that precision also depends on the prior probability that a consumer is interested.

Proof of Lemma 1. The lemma is equivalent to the following two statements:

Claim 1: If firms target a type p_1 consumer with a positive probability ($q(p_1) > 0$), then they target any higher-type consumer with a positive probability ($q(p) > 0,\ \forall p > p_1$).

Claim 2: If firms target a type p_2 consumer for sure ($q(p_2) = 1$), then they target any higher-type consumer for sure ($q(p) = 1,\ \forall p > p_2$).

Proof of Claim 1: If firms target a type p_1 consumer with a positive probability, then the expected payoff from targeting such a consumer is no less than the expected payoff from not targeting: $[\rho q(p_1)+(1-\rho)R]p_1w+[1-\rho q(p_1)-(1-\rho)R]p_1v-c \geq 0$. Suppose they do not target a type $p > p_1$ consumer at all, $q(p) = 0$. By deviating and targeting a type p consumer for sure, the deviating firm can obtain an expected payoff of $[(1-\rho)R]pw+[1-(1-\rho)R]pv-c > [\rho q(p_1)+(1-\rho)R]p_1w+[1-\rho q(p_1)-(1-\rho)R]p_1v-c \geq 0$. So, firms will deviate. A contradiction.

Proof of Claim 2: If firms target a type p_2 consumer for sure, then the expected payoff from targeting such a consumer is no less than the expected payoff from not targeting: $[\rho q(p_2)+(1-\rho)R]p_2w+[1-\rho q(p_2)-(1-\rho)R]p_2v-c \geq 0$. Suppose they do not target a type $p > p_2$ consumer for sure, $q(p) < 1$. By deviating and targeting a type p consumer for sure, the deviating firm can obtain an expected payoff of $[(1-\rho)R]pw+[1-(1-\rho)R]pv-c > [\rho q(p_2)+(1-\rho)R]p_2w+[1-\rho q(p_2)-(1-\rho)R]p_2v-c \geq 0$. So, firms will deviate. A contradiction. ■

Proof of Lemma 2.

Existence:

The theorem in Becker and Damianov (2006) states that any symmetric game with a finite number of players, whose strategy space is compact and Hausdorff and payoff function is continuous, has a symmetric Nash equilibrium (including both pure and mixed strategies). It is natural to define the (pure) strategy space of the firm as the standard topology on [0,1], which is compact and Hausdorff. Also, the firm's payoff function is continuous. Hence, there exists a symmetric Nash equilibrium in pure or mixed strategies. Footnote 7 has shown that there does not exist a symmetric Nash equilibrium in pure strategy. So, there must exist a mixed-strategy symmetric equilibrium.

Uniqueness:

1. Firms never target any consumer for sure ($\overline{p} = 1$)

$$
\begin{aligned}
R &= \frac{\int_{\underline{p}}^{1} pf(p)q(p)dp}{\mu_0} \\
&= \frac{\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} \frac{pf(p)}{\rho(v-w)}[-c/p+v-(1-\rho)(v-w)R]dp}{\mu_0}
\end{aligned}
$$

For any fixed ρ, define:

$$
\begin{aligned}
G(R) &:= \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} \frac{pf(p)}{\rho(v-w)}[-c/p+v-(1-\rho)(v-w)R]dp - \mu_0 R \\
\text{Then, } G(0) &= \int_{\frac{c}{v}}^{1} \frac{pf(p)}{\rho(v-w)}[v-c/p]dp > 0 \\
G'(R) &= \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} -(1-\rho)\frac{pf(p)}{\rho}dp - \mu_0 < 0
\end{aligned}
$$

So, there is a unique R^* such that $G(R^*) = 0$ (we have previously shown the existence part).

2. Firms target high-probability consumers for sure ($\overline{p} < 1$)

$$
\begin{aligned}
&[\rho + (1-\rho)R]\overline{p}w + [1-\rho-(1-\rho)R]\overline{p}v - c = 0 \\
\Rightarrow\ &\overline{p} = \frac{c}{[\rho+(1-\rho)R]w + [1-\rho-(1-\rho)R]v} \qquad (7)
\end{aligned}
$$

$$
\begin{aligned}
R &= [\int_{\overline{p}}^{1} pf(p)dp + \int_{\underline{p}}^{\overline{p}} pf(p)q(p)dp]/\mu_0 \\
&= [\int_{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}}^{1} pf(p)dp + \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} pf(p)q(p)dp]/\mu_0
\end{aligned}
$$

For any fixing ρ, define:

$$H(R) := \int_{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}}^{1} pf(p)dp + \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} pf(p)q(p)dp - \mu_0 R$$

Then, $H(0) > 0$.

$$\begin{aligned} H'(R) =& -\frac{d\overline{p}}{dR}\overline{p}f(\overline{p}) + \frac{d\overline{p}}{dR}\overline{p}f(\overline{p}) \cdot 1 + \\ & \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} -(1-\rho)(v-w)\frac{f(p)}{\rho(v-w)}dp - \frac{d\underline{p}}{dR}\underline{p}f(\underline{p}) \cdot 0 - \mu_0 \\ =& -\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} \frac{(1-\rho)f(p)}{\rho}dp - \mu_0 < 0 \end{aligned}$$

So, there is a unique R^* such that $H(R^*) = 0$.

To show that there exists $\widehat{\rho} \in [0, \frac{v-c}{v-w}]$ such that firms never target any consumer for sure if and only if $\rho \geq \widehat{\rho}$, we just need to show the following claim:

If firms never target any consumer for sure for $\rho = \rho_s$, then they also never target any consumer for sure for any $\rho_l > \rho_s$.

Suppose not. When $\rho = \rho_l$, Lemma 1 implies that there exists $\overline{p}_l$ and $\underline{p}_l$ such that firms target consumers with probabilities $p \geq \overline{p}_l$ for sure and mix for consumers with probabilities between $\underline{p}_l$ and $\overline{p}_l$. Thus, $q_l(p) = 1 > q_s(p)$, $\forall p \geq \overline{p}_l$. The proof of the comparative statics results will show that $q_l'(p) < q_s'(p)$, $\forall p \in (\underline{p}_s, \overline{p}_l)$. So, $q_l(p) > q_s(p)$, $\forall p \in [\underline{p}_s, 1] \Rightarrow R_l > R_s$. A contradiction to the comparative statics that the recall decreases in ρ, which will be shown in the proof of the comparative statics results. ■

Proof of Proposition 1. The result of the recall is implied by the fact that each duopolistic firm only targets a subset of the consumers that the monopoly targets. We now show that each duopolistic firm has a higher targeting precision than the monopoly.

The precision of the monopoly's optimal targeting policy is:

$$\frac{\int_{\underline{p}_m}^1 pf(p)dp}{\int_{\underline{p}_m}^1 f(p)dp} = \int_{\underline{p}_m}^1 p\frac{f(p)}{\int_{\underline{p}_m}^1 f(p)dp}dp$$

Observe that $\int_{\underline{p}_m}^1 \frac{f(p)}{\int_{\underline{p}_m}^1 f(p)dp}dp = 1$. Hence, $f_m(p) := \frac{f(p)}{\int_{\underline{p}_m}^1 f(p)dp}$ is a p.d.f. of a random variable $X_m \in [\underline{p}_m, 1]$. The monopoly's precision is the expectation of X_m.

The precision of the duopoly's optimal targeting policy is:

$$\frac{\int_{\underline{p}_d}^1 pq(p)f(p)dp}{\int_{\underline{p}_d}^1 q(p)f(p)dp} = \frac{\int_{\underline{p}_m}^1 pq(p)f(p)dp}{\int_{\underline{p}_m}^1 q(p)f(p)dp} = \int_{\underline{p}_m}^1 p\frac{q(p)f(p)}{\int_{\underline{p}_m}^1 q(p)f(p)dp}dp$$

, where the first equality comes from the fact that $\underline{p}_d \geq \underline{p}_m$ and $q(p) = 0,\ \forall p \in [\underline{p}_m, \underline{p}_d)$.

Observe that $\int_{\underline{p}_m}^1 \frac{q(p)f(p)}{\int_{\underline{p}_m}^1 q(p)f(p)dp}dp = 1$. Hence, $f_d(p) := \frac{q(p)f(p)}{\int_{\underline{p}_m}^1 q(p)f(p)dp}$ is a p.d.f. of a random variable $X_d \in [\underline{p}_m, 1]$. The duopoly's precision is the expectation of X_d.

We proceed by showing that X_d first-order stochastic dominates X_m. As a result, the expectation of X_d is larger than the expectation of X_m, which concludes the proof.

Lemma 3 *Given two continuously distributed random variables Z_1 and $Z_2 \in [\underline{z}, \overline{z}] \subset \mathbb{R}$, whose p.d.f. (c.d.f.) are f_1 and f_2 (F_1 and F_2), respectively. Z_1 first-order stochastic dominates Z_2 if $h(z) := f_1(z)/f_2(z)$ increases in z. The stochastic dominance is strict if $h(z)$ is not a constant.*

Proof of Lemma 3. We have $1 = \int_{\underline{z}}^{\overline{z}} f_1(z)dz = \int_{\underline{z}}^{\overline{z}} h(z)f_2(z)dz$.

If $h(z)$ is a constant, $h(z) = h_0$, then $1 = \int_{\underline{z}}^{\overline{z}} h(z)f_2(z)dz = h_0 \int_{\underline{z}}^{\overline{z}} f_2(z)dz = h_0 \Rightarrow f_1(z) = f_2(z),\ \forall z$. So, Z_1 first-order stochastic dominates Z_2.

If $h(z)$ is not a constant, then $h(\underline{z}) < h(\overline{z})$. Since $h(z)$ increases in z, we have $h(\underline{z}) = h(\underline{z}) \int_{\underline{z}}^{\overline{z}} f_2(z)dz < \int_{\underline{z}}^{\overline{z}} h(z)f_2(z)dz = 1 < h(\overline{z}) \int_{\underline{z}}^{\overline{z}} f_2(z)dz = h(\overline{z})$. Therefore, there exists $\hat{z}_1 \leq \hat{z}_2 \in (\underline{z}, \overline{z})$ such that $h(z) < 1$ if $z < \hat{z}_1$ and $h(z) > 1$ if $z > \hat{z}_2$.

For any $z \in (\underline{z}, \overline{z})$, we want to show that $F_1(z) \leq F_2(z)$.

Define $H(z) := F_2(z) - F_1(z) = \int_{\underline{z}}^{z} f_2(z)dz - \int_{\underline{z}}^{z} f_1(z)dz = \int_{\underline{z}}^{z} f_2(z)[1 - h(z)]dz$.

One can see that $H(\underline{z}) = H(\overline{z}) = 0$. $H'(z) = f_2(z)[1-h(z)] \begin{cases} > 0, & if\ z \in (\underline{z}, \hat{z}_1) \\ = 0, & if\ z \in (\hat{z}_1, \hat{z}_2) \\ < 0, & if\ z \in (\hat{z}_2, \overline{z}) \end{cases}$.

So, $H(z) = F_2(z) - F_1(z) > 0, \forall z \in (\underline{z}, \overline{z})$. For any $z \in [\underline{z}, \overline{z}]$, one can see that

$F_2(z) \geq F_1(z)$, with the inequality strict when $z \notin \{\underline{z}, \overline{z}\} \Rightarrow Z_1$ strictly first-order stochastic dominates Z_2. ■

Note that $q(p)$ weakly increases in p according to our characterization of the optimal targeting policy. Therefore, $\frac{q(p)f(p)}{\int_{\underline{p}}^1 q(p)f(p)dp} / \frac{f(p)}{\int_{\underline{p}}^1 f(p)dp} = q(p)\frac{\int_{\underline{p}}^1 f(p)dp}{\int_{\underline{p}}^1 q(p)f(p)dp}$ weakly increases in p. Lemma 3 then implies that X_d first-order stochastic dominates X_m. As a result, the expectation of X_d is larger than the expectation of X_m, which concludes the proof. ■

Proof of the comparative statics results.

1. Firms never target any consumer for sure ($\overline{p} = 1$)

   Comparative statics of R w.r.t. ρ:

   In equilibrium, $G(R) = 0$. By the implicit function theorem, $\frac{\partial R}{\partial \rho} = -\frac{\frac{\partial G}{\partial \rho}}{\frac{\partial G}{\partial R}}$. We have shown that $\frac{\partial G}{\partial R}$ is negative.

$$
\begin{aligned}
\frac{\partial G}{\partial \rho} =& \int_{\frac{c}{v-(1-\rho)(v-w)R}}^1 \frac{pf(p)}{v-w}[-c/p + v - (1-\rho)(v-w)R](-1/\rho^2) + \frac{pf(p)R}{\rho}dp \\
=& \int_{\frac{c}{v-(1-\rho)(v-w)R}}^1 \frac{pf(p)}{\rho^2(v-w)}[c/p - v + (1-\rho)(v-w)R + \rho R(v-w)]dp \\
\propto& \int_{\frac{c}{v-(1-\rho)(v-w)R}}^1 pf(p)Rdp - \int_{\frac{c}{v-(1-\rho)(v-w)R}}^1 pf(p)q(p)dp \\
=& \int_{\frac{c}{v-(1-\rho)(v-w)R}}^1 pf(p)Rdp - R\mu_0 \\
=& R[\int_{\frac{c}{v-(1-\rho)(v-w)R}}^1 pf(p)dp - \int_0^1 pf(p)dp] < 0.
\end{aligned}
$$

   Therefore, $\frac{\partial R}{\partial \rho} < 0$.

   Comparative statics of $\underline{p}$ w.r.t. ρ:

   We have shown that the recall R decreases in ρ. Therefore, $v - (1-\rho)(v-w)R$ increases. Equation (6) then implies that $\underline{p}$ decreases in ρ.

   Comparative statics of the precision wrt ρ :

Suppose $\rho_l > \rho_s > \frac{v-c}{v-w}$. The precision of the optimal targeting policy when $\rho = \rho_s$ is:

$$\frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp} = \int_{\underline{p}_s}^1 p\frac{q_s(p)f(p)}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp}dp$$

Observe that $\int_{\underline{p}_s}^1 \frac{q_s(p)f(p)}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp}dp = 1$. Hence, $f_s(p) := \frac{q_s(p)f(p)}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp}$ is a p.d.f. of a random variable $X_s \in [\underline{p}_s, 1]$. The precision is the expectation of X_s.

The precision of the optimal targeting policy when $\rho = \rho_l$ is:

$$\begin{aligned}\frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{\int_{\underline{p}_l}^1 q_l(p)f(p)dp} &= \frac{\int_{\underline{p}_l}^{\underline{p}_s} pq_l(p)f(p)dp + \int_{\underline{p}_s}^1 pq_l(p)f(p)dp}{\int_{\underline{p}_l}^{\underline{p}_s} q_l(p)f(p)dp + \int_{\underline{p}_s}^1 q_l(p)f(p)dp}\\ &< \frac{\int_{\underline{p}_s}^1 pq_l(p)f(p)dp}{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}\\ &= \int_{\underline{p}_s}^1 p\frac{q_l(p)f(p)}{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}dp\end{aligned}$$

, where the inequality comes from the fact that $\frac{\int_{\underline{p}_l}^{\underline{p}_s} pq_l(p)f(p)dp}{\int_{\underline{p}_l}^{\underline{p}_s} q_l(p)f(p)dp} < \underline{p}_s < \frac{\int_{\underline{p}_s}^1 pq_l(p)f(p)dp}{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}$. Observe that $\int_{\underline{p}_s}^1 \frac{q_l(p)f(p)}{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}dp = 1$. Hence, $f_l(p) := \frac{q_l(p)f(p)}{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}$ is a p.d.f. of a random variable $X_l \in [\underline{p}_s, 1]$. The upper bound of the precision is the expectation of X_l.

We proceed by showing that X_s first-order stochastic dominates X_l. As a result, the expectation of X_s is larger than the expectation of X_l, which concludes the proof.

In order to apply lemma 3 to obtain the final result, we need to show that $f_s(p)/f_l(p) = \frac{\frac{q_s(p)f(p)}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp}}{\frac{q_l(p)f(p)}{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}} = \frac{q_s(p)}{q_l(p)}\frac{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp}$ increases in p. Since $\frac{\int_{\underline{p}_s}^1 q_l(p)f(p)dp}{\int_{\underline{p}_s}^1 q_s(p)f(p)dp}$ is a constant, we only need to show that $\frac{q_s(p)}{q_l(p)}$ strictly increases in p.

According to (5), $q_s'(p) = \frac{c}{p^2\rho_s(v-w)} > \frac{c}{p^2\rho_l(v-w)} = q_l'(p), \forall p \in (\underline{p}_s, 1)$. One can see that $\int_{\underline{p}_s}^1 q_s(p)pf(p)dp = R_s\mu_0 > R_l\mu_0 = \int_{\underline{p}_l}^1 q_l(p)pf(p)dp > \int_{\underline{p}_s}^1 q_l(p)pf(p)dp$.

Since $q_l(\underline{p}_s) > q_l(\underline{p}_l) = 0 = q_s(\underline{p}_s)$, there must exist $\tilde{p} \in (\underline{p}_s, 1)$ such that
$$\begin{cases} q_s(p) > q_l(p), \; if \; p \in (\tilde{p}, 1] \\ \tilde{q} := q_s(\tilde{p}) = q_l(\tilde{p}) \\ q_s(p) < q_l(p), \; if \; p \in [\underline{p}_s, \tilde{p}) \end{cases}.$$

$$\begin{aligned} \Xi(p) :=& \frac{q_s(p)}{q_l(p)} \overset{(5)}{=} \frac{\tilde{q} + \frac{c}{\rho_s(v-w)}(\frac{1}{\tilde{p}} - \frac{1}{p})}{\tilde{q} + \frac{c}{\rho_l(v-w)}(\frac{1}{\tilde{p}} - \frac{1}{p})} \\ \Rightarrow Sgn(\Xi'(p)) =& Sgn(\frac{1}{\rho_s} - \frac{1}{\rho_l}) > 0 \end{aligned}$$

Therefore, $\frac{q_s(p)}{q_l(p)}$ strictly increases in p. Lemma 3 then implies that X_s first-order stochastic dominates X_l. As a result, the expectation of X_s is larger than the expectation of X_l, which concludes the proof.

Comparative statics of R w.r.t. c:

In equilibrium, $G(R) = 0$. By the implicit function theorem, $\frac{\partial R}{\partial c} = -\frac{\frac{\partial G}{\partial c}}{\frac{\partial G}{\partial R}}$. We have shown that $\frac{\partial G}{\partial R}$ is negative.

$$\frac{\partial G}{\partial c} = \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} \frac{pf(p)}{\rho(v-w)} \cdot (-\frac{1}{p})dp < 0$$

Therefore, $\frac{\partial R}{\partial c} < 0$.

Comparative statics of R w.r.t. w:

In equilibrium, $G(R) = 0$. By the implicit function theorem, $\frac{\partial R}{\partial w} = -\frac{\frac{\partial G}{\partial w}}{\frac{\partial G}{\partial R}}$. We have shown that $\frac{\partial G}{\partial R}$ is negative.

$$\frac{\partial G}{\partial w} = \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{1} \frac{pf(p)}{\rho} \frac{-c/p + v - (1-\rho)(v-w)R}{(v-w)^2} + \frac{pf(p)}{\rho(v-w)}(1-\rho)Rdp > 0$$

Therefore, $\frac{\partial R}{\partial w} > 0$.

2. Firms target high-probability consumers for sure ($\overline{p} < 1$)

In this case, the precision of the optimal targeting policy is:

$$\frac{\int_{\underline{p}}^{\overline{p}} pq(p)f(p)dp + \int_{\overline{p}}^{1} pf(p)dp}{\int_{\underline{p}}^{\overline{p}} q(p)f(p)dp + \int_{\overline{p}}^{1} f(p)dp} = \frac{\int_{\underline{p}}^{\overline{p}} pq(p)f(p)dp + \int_{\overline{p}}^{1} pf(p)dp}{a}$$

The recall of the optimal targeting policy is:

$$\frac{\int_{\underline{p}}^{\overline{p}} pq(p)f(p)dp + \int_{\overline{p}}^{1} pf(p)dp}{\int_{0}^{1} pf(p)dp} = \frac{\int_{\underline{p}}^{\overline{p}} pq(p)f(p)dp + \int_{\overline{p}}^{1} pf(p)dp}{\mu_0}$$

Comparative statics of R w.r.t. ρ:

In equilibrium, $H(R) = 0$. By the implicit function theorem, $\frac{\partial R}{\partial \rho} = -\frac{\frac{\partial H}{\partial \rho}}{\frac{\partial H}{\partial R}}$. We have shown that $\frac{\partial H}{\partial R}$ is negative.

$$\begin{aligned}
\frac{\partial H}{\partial \rho} =& -\frac{d\overline{p}}{d\rho}\overline{p}f(\overline{p}) + \frac{d\overline{p}}{d\rho}\overline{p}f(\overline{p}) \cdot 1 - \frac{d\underline{p}}{d\rho}\underline{p}f(\underline{p}) \cdot 0 + \\
& \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} \frac{pf(p)}{v-w}[-c/p + v - (1-\rho)(v-w)R](-1/\rho^2) + \frac{pf(p)R}{\rho}dp \\
=& \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} \frac{pf(p)}{\rho^2(v-w)}[c/p - v + (1-\rho)(v-w)R + \rho R(v-w)]dp \\
\propto& \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} pf(p)Rdp - \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} pf(p)q(p)dp \\
=& \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} pf(p)Rdp - R\mu_0 \\
=& R[\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} pf(p)dp - \int_0^1 pf(p)dp] < 0.
\end{aligned}$$

Therefore, $\frac{\partial R}{\partial \rho} < 0$.

Comparative statics of $\underline{p}$ w.r.t. ρ:

We have shown that the overall targeting probability a decreases when ρ increases. Therefore, $v - (1-\rho)(v-w)R$ increases. Equation (6) then implies that $\underline{p}$ decreases in ρ.

Comparative statics of $\overline{p}$ w.r.t. ρ:

Consider any given $\rho_l > \rho_s$ such that the corresponding $\overline{p}_l$ and $\overline{p}_s$ are lower than 1. Suppose $\overline{p}_l \leq \overline{p}_s$. The recall corresponding to ρ_l is $R_l = [\int_{\overline{p}_l}^1 pf(p)dp + \int_{\underline{p}_l}^{\overline{p}_l} q_l(p)pf(p)dp]/\mu_0$. The recall corresponding to ρ_s is $R_s = [\int_{\overline{p}_s}^1 pf(p)dp + \int_{\underline{p}_s}^{\overline{p}_s} q_s(p)pf(p)dp]/\mu_0$. We first show that $q_l(p) > q_s(p), \forall p \in (\underline{p}_s, \overline{p}_l)$.

Observe that $q_l(\overline{p}_l) = 1 > q_s(\overline{p}_l)$. For any $p \in (\underline{p}_s, \overline{p}_l)$, we have

$$q_l(p) = q_l(\overline{p}_l) + \frac{c}{\rho_l(v-w)}(1/\overline{p}_l - 1/p) > q_s(\overline{p}_l) + \frac{c}{\rho_s(v-w)}(1/\overline{p}_l - 1/p) = q_s(\overline{p}_l).$$

Hence,

$$\begin{aligned}
R_l\mu_0 =& \int_{\overline{p}_s}^1 pf(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} pf(p)dp + \int_{\underline{p}_l}^{\overline{p}_l} q_l(p)pf(p)dp \\
>& \int_{\overline{p}_s}^1 pf(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} q_s(p)pf(p)dp + \int_{\underline{p}_s}^{\overline{p}_l} q_l(p)pf(p)dp \\
>& \int_{\overline{p}_s}^1 pf(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} q_s(p)pf(p)dp + \int_{\underline{p}_s}^{\overline{p}_l} q_s(p)pf(p)dp \\
=& R_s\mu_0.
\end{aligned}$$

But we have shown that R decreases in ρ. A contradiction. Therefore, $\overline{p}_l > \overline{p}_s$.

Comparative statics of the profit w.r.t. ρ:

Denote the firm's total profit by Π and the per-unit profit for probability p consumer by $\pi(p)$. Then, $\Pi = \int_{\overline{p}}^1 \pi(p)f(p)dp$, where $\pi(p) = p\{[\rho + (1-\rho)R]w + [1-\rho-(1-\rho)R]v\} - c$. One can see that $\pi(p)$ is strictly increasing and linear in p for $p \in [\overline{p}, 1]$. Consider any given $\rho_l > \rho_s$ such that the corresponding $\overline{p}_l$ and $\overline{p}_s$ are lower than 1. We have shown that $\overline{p}_s < \overline{p}_l$. Therefore, $\pi_s(\overline{p}_l) > \pi_s(\overline{p}_s) = 0 = \pi_l(\overline{p}_l) \Rightarrow [\rho_s + (1-\rho_s)a_s]w + [1-\rho_s-(1-\rho_s)a_s]v > [\rho_l + (1-\rho_l)a_l]w + [1-\rho_l-(1-\rho_l)a_l]v \Rightarrow \pi_s(p) > \pi_l(p),\ \forall p \in [\overline{p}_l, 1] \Rightarrow \Pi_s > \Pi_l$.[10]

[10]From the expression of $\pi(p)$ and $\pi(\overline{p}) = 0$, one can derive that $\pi(p) = \frac{p-\overline{p}}{\overline{p}}c,\ \forall p \geq \overline{p}$.

Comparative statics of R w.r.t. c:

In equilibrium, $H(R) = 0$. By the implicit function theorem, $\frac{\partial R}{\partial c} = -\frac{\frac{\partial H}{\partial c}}{\frac{\partial H}{\partial R}}$. We have shown that $\frac{\partial H}{\partial R}$ is negative.

$$\frac{\partial H}{\partial c} = -\frac{d\overline{p}}{dc}\overline{p}f(\overline{p}) + \frac{d\overline{p}}{dc}\overline{p}f(\overline{p}) \cdot 1 - \frac{d\underline{p}}{dc}\underline{p}f(\underline{p}) \cdot 0 + \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} \frac{pf(p)}{\rho(v-w)} \cdot (-\frac{1}{p})dp$$
$$= \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} \frac{pf(p)}{\rho(v-w)} \cdot (-\frac{1}{p})dp < 0.$$

Therefore, $\frac{\partial R}{\partial c} < 0$.

Comparative statics of R w.r.t. w:

In equilibrium, $H(R) = 0$. By the implicit function theorem, $\frac{\partial R}{\partial w} = -\frac{\frac{\partial H}{\partial w}}{\frac{\partial H}{\partial R}}$. We have shown that $\frac{\partial H}{\partial R}$ is negative.

$$\frac{\partial H}{\partial w} = -\frac{d\overline{p}}{dw}\overline{p}f(\overline{p}) + \frac{d\overline{p}}{dw}\overline{p}f(\overline{p}) \cdot 1 - \frac{d\underline{p}}{dw}\underline{p}f(\underline{p}) \cdot 0 +$$
$$\int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} \frac{pf(p)}{\rho}\frac{-c/p+v-(1-\rho)(v-w)R}{(v-w)^2} + \frac{pf(p)}{\rho(v-w)}(1-\rho)Rdp$$
$$= \int_{\frac{c}{v-(1-\rho)(v-w)R}}^{\frac{c}{[\rho+(1-\rho)R]w+[1-\rho-(1-\rho)R]v}} \frac{pf(p)}{\rho}\frac{-c/p+v-(1-\rho)(v-w)R}{(v-w)^2} + \frac{pf(p)}{\rho(v-w)}(1-\rho)Rdp > 0.$$

Therefore, $\frac{\partial R}{\partial w} > 0$.

■

ONLINE APPENDIX

Extensions

Proof of Proposition 2. We first consider the case in which $\rho_0 > \widehat{\rho}$. We have shown that the profit is zero for any $\rho \geq \widehat{\rho}$. Therefore, if firms invest in equilibrium, it must be that $\rho(I_1, I_2) < \widehat{\rho}$. The investment cost may be higher than the benefit. For example, suppose $\rho(v, v) \geq \widehat{\rho}$. Then no firm will invest.

We then show that no firm investing is not an equilibrium if $\rho_0 < \widehat{\rho}$. Suppose no firm investing is an equilibrium. We want to show that either firm has an incentive to deviate. Consider without loss of generality firm 1.

We first compute how much firm 1's payoff in the second period increases as ρ decreases.

Equation (8) implies that:

$$Q(\rho, \overline{p}) := \overline{p}\{[\rho + (1-\rho)R(\rho)]w + [1 - \rho - (1-\rho)R(\rho)]v\} - c = 0$$

By implicit function theorem,

$$\frac{\partial \overline{p}}{\partial \rho} = -\frac{\partial Q/\partial \rho}{\partial Q/\partial \overline{p}} = \frac{\overline{p}[1 - R + (1-\rho)\frac{\partial R}{\partial \rho}](v - w)}{[\rho + (1-\rho)R]w + [1 - \rho - (1-\rho)R]v}$$

We have shown in the proof of the comparative statics results that $\overline{p}$ strictly increases in ρ for $\rho < \widehat{\rho}$. Therefore, $\overline{p}[1 - R + (1-\rho)\frac{\partial R}{\partial \rho}](v - w) > 0$. Denote $\frac{\partial \overline{p}}{\partial \rho}|_{\rho=\rho_0}$ by D_0 ($D_0 > 0$). Consider an investment by firm 1 such that the correlation decreases from ρ_0 to $\rho_1 := \rho_0 - \epsilon$. Denote the $\overline{p}$ corresponding to ρ_0 by $\overline{p}_0$ and the $\overline{p}$ corresponding to ρ_1 by $\overline{p}_1$. We use similar notations for π and Π.[11]

By Taylor expansion, $\overline{p}_1 = \overline{p}_0 - D_0\epsilon + o(\epsilon)$. Firm 1's increase in profit (not taking into account the investment cost) is:

$$\Pi_1 - \Pi_0 = \int_{\overline{p}_1}^{1} \pi_1(p) f(p) dp - \int_{\overline{p}_0}^{1} \pi_0(p) f(p) dp$$

[11] Both have been defined in the proof of the comparative statics results.

$$
\begin{aligned}
&> \int_{\overline{p}_0}^{1} \pi_1(p) f(p) dp - \int_{\overline{p}_0}^{1} \pi_0(p) f(p) dp \\
&= \int_{\overline{p}_0}^{1} [\pi_1(p) - \pi_0(p)] f(p) dp \\
&= \int_{\overline{p}_0}^{1} [\frac{p - \overline{p}_1}{\overline{p}_1} c - \frac{p - \overline{p}_0}{\overline{p}_0} c] f(p) dp \\
&= \int_{\overline{p}_0}^{1} p \frac{\overline{p}_0 - \overline{p}_1}{\overline{p}_0 \overline{p}_1} c f(p) dp \\
&> \int_{\overline{p}_0}^{1} \overline{p}_0 \frac{D_0 \epsilon + o(\epsilon)}{\overline{p}_0 \overline{p}_1} c f(p) dp \\
&= \frac{D_0 \epsilon + o(\epsilon)}{\overline{p}_1} c \int_{\overline{p}_0}^{1} f(p) dp \\
&= \frac{D_0 \epsilon + o(\epsilon)}{\overline{p}_1} c [1 - F(\overline{p}_0)] \\
&= \frac{D_0 c [1 - F(\overline{p}_0)]}{\overline{p}_1} \epsilon + o(\epsilon)
\end{aligned}
$$

Thus, the profit increase is linear in the decrease of ρ, ϵ, if we ignore the higher-order term $o(\epsilon)$.

We then compute firm 1's investment cost in the first period. One can see that $K_j(0) = 0$.

$$
\begin{aligned}
K_1(\epsilon) &= K_1(0) + K_1'(0)\epsilon + o(\epsilon) \\
&= 0 + 0 \cdot \epsilon + o(\epsilon) \\
&= o(\epsilon)
\end{aligned}
$$

Therefore, $\Pi_1 - \Pi_0 > K_1(\epsilon)$ for ϵ small enough. It shows that firm 1 will deviate if no firm invests. ■

Proof of Proposition 3. In order for a firm acting as a monopoly to be an equilibrium, the other firm must not want to target probability 1 consumer, which is equivalent to:

$$\rho w + (1-\rho)[R_m w + (1-R_m)v] - c \leq 0$$
$$\Leftrightarrow \;\; \rho \geq \frac{R_m w + (1-R_m)v - c}{(1-R_m)(v-w)}$$

The statement that in any other pure strategy asymmetric equilibria, there exists $p_0 \in (\underline{p}, 1)$ such that exactly one firm targets consumers whose $p \in (\underline{p}, p_0)$, and either one or both firms target every consumer whose $p > p_0$ is equivalent to the following two claims.

Claim 1: At least one firm targets consumers whose $p \in (\underline{p}, 1]$.

Claim 2: If both firms target a probability p' consumer, then they also target any consumer whose $p > p'$.

Proof of Claim 1: For any $p \in (\underline{p}, 1]$, there exists $\tilde{p} \in [\underline{p}, p)$ such that $q_1(\tilde{p}) > 0$ or $q_2(\tilde{p}) > 0$. Assume without loss of generality that $q_1(\tilde{p}) > 0$. Firm 1's expected payoff from targeting such a consumer is no less than the expected payoff from not targeting: $[\rho + (1-\rho)R_2]\tilde{p}w + [1-\rho-(1-\rho)R_2]\tilde{p}v - c \geq 0$. Suppose neither firm targets a type $p > \tilde{p}$ consumer. By deviating and targeting that consumer, firm 1 can obtain an expected payoff of $[(1-\rho)R_2]pw + [1-(1-\rho)R_2]pv - c > [\rho+(1-\rho)R_2]\tilde{p}w + [1-\rho-(1-\rho)R_2]\tilde{p}v - c \geq 0$. So, it will deviate. A contradiction.

Proof of Claim 2: If both firms target a probability p' consumer, then firm i's expected payoff from targeting such a consumer is no less than the expected payoff from not targeting: $[\rho + (1-\rho)R_j]p'w + [1-\rho-(1-\rho)R_j]p'v - c \geq 0$, where $j \neq i$. Claim 1 says that at least one firm targets any consumer whose $p > p'$. Suppose one of the firms does not target probability p consumer. Assume without loss of generality that firm 1 does not target that consumer. By deviating and targeting that consumer, it can obtain an expected payoff of $[(1-\rho)R_2]pw + [1-(1-\rho)R_2]pv - c > [\rho + (1-\rho)R_2]p'w + [1-\rho-(1-\rho)R_2]p'v - c \geq 0$. So, firm 1 will deviate. A contradiction.

Lastly, we prove that the recall of either firm is lower than the monopoly case, and the precision of at least one firm is higher than the monopoly case.

The monopoly equilibrium is straightforward. Now let's look at other equilibria where both firms target some consumers (the case of Figure 4b and 4c). $\underline{p} \geq \underline{p}_m$ implies that the recall of either firm is lower than the monopoly case.

The precision of firm i's targeting policy is:

$$\frac{\int_{\underline{p}}^{1} pq_i(p)f(p)dp}{\int_{\underline{p}}^{1} q_i(p)f(p)dp} = \frac{\int_{\underline{p}}^{1} pq_i(p)f(p)dp}{a_i}$$

We first consider the case where there is no overlap in firms' targeting regions (the case of Figure 4b). In that case, $q_1(p) + q_2(p) = 1, \forall p \in (\underline{p}, 1]$. Since both firms target some consumers, one can see that $\underline{p} > \underline{p}_m$. We have:

$$\begin{aligned}
&\frac{\int_{\underline{p}}^{1} pq_1(p)f(p)dp + \int_{\underline{p}}^{1} pq_2(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp + \int_{\underline{p}}^{1} q_2(p)f(p)dp} \\
=&\frac{\int_{\underline{p}}^{1} pf(p)dp}{\int_{\underline{p}}^{1} f(p)dp} \\
=&\int_{\underline{p}}^{1} p\frac{f(p)}{\int_{\underline{p}}^{1} f(p)dp}dp \\
=&\int_{\underline{p}}^{1} p\frac{f(p)}{\int_{\underline{p}}^{1} f(p)dp}dp + \int_{\underline{p}_m}^{\underline{p}} p \cdot 0 \; dp
\end{aligned}$$

The above formula is the expectation of a random variable $X \in [\underline{p}_m, 1]$ with a p.d.f. of 0 for $p \in [\underline{p}_m, \underline{p}]$ and a p.d.f. of $\frac{f(p)}{\int_{\underline{p}}^{1} f(p)dp}$ for $p \in (\underline{p}, 1]$.

The precision of the monopoly's targeting policy is:

$$\frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\int_{\underline{p}_m}^{1} f(p)dp} = \int_{\underline{p}_m}^{1} p\frac{f(p)}{\int_{\underline{p}_m}^{1} f(p)dp}dp$$

The above formula is the expectation of a random variable $Y \in [\underline{p}_m, 1]$ with a p.d.f. of $\frac{f(p)}{\int_{\underline{p}_m}^{1} f(p)dp}$ for $p \in [\underline{p}_m, 1]$. Lemma 3 implies that X strictly first-order stochastic dominates Y. Therefore, $\frac{\int_{\underline{p}}^{1} pq_1(p)f(p)dp+\int_{\underline{p}}^{1} pq_2(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp+\int_{\underline{p}}^{1} q_2(p)f(p)dp} > \frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\int_{\underline{p}_m}^{1} f(p)dp}$.

Now suppose $\frac{\int_{\underline{p}}^{1} pq_1(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp} \leq \frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\int_{\underline{p}_m}^{1} f(p)dp}$ and $\frac{\int_{\underline{p}}^{1} pq_2(p)f(p)dp}{\int_{\underline{p}}^{1} q_2(p)f(p)dp} \leq \frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\int_{\underline{p}_m}^{1} f(p)dp}$. We then have:

$$
\begin{aligned}
&\frac{\int_{\underline{p}}^{1} pq_1(p)f(p)dp + \int_{\underline{p}}^{1} pq_2(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp + \int_{\underline{p}}^{1} q_2(p)f(p)dp} \\
=&\frac{\int_{\underline{p}}^{1} pq_1(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp + \int_{\underline{p}}^{1} q_2(p)f(p)dp} + \frac{\int_{\underline{p}}^{1} pq_2(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp + \int_{\underline{p}}^{1} q_2(p)f(p)dp} \\
=&\frac{\int_{\underline{p}}^{1} q_1(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp + \int_{\underline{p}}^{1} q_2(p)f(p)dp} \frac{\int_{\underline{p}}^{1} pq_1(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp} + \\
&\frac{\int_{\underline{p}}^{1} q_2(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp + \int_{\underline{p}}^{1} q_2(p)f(p)dp} \frac{\int_{\underline{p}}^{1} pq_2(p)f(p)dp}{\int_{\underline{p}}^{1} q_2(p)f(p)dp} \\
\leq&\frac{\int_{\underline{p}}^{1} q_1(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp + \int_{\underline{p}}^{1} q_2(p)f(p)dp} \frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\int_{\underline{p}_m}^{1} f(p)dp} + \\
&\frac{\int_{\underline{p}}^{1} q_2(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp + \int_{\underline{p}}^{1} q_2(p)f(p)dp} \frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\int_{\underline{p}_m}^{1} f(p)dp} \\
=&\frac{\int_{\underline{p}_m}^{1} pf(p)dp}{\int_{\underline{p}_m}^{1} f(p)dp}
\end{aligned}
$$

A contradiction. So, the precision of at least one firm is higher than the monopoly case.

We then consider the case where there is overlap in firms' targeting regions (the case of Figure 4c). In that case, $q_1(p)+q_2(p) = 1, \forall p \in (\underline{p}, p_0)$ and $q_1(p)+q_2(p) = 2, \forall p > p_0$.

The following lemma relates the overlapping case with the non-overlapping case so that the previous argument applies.

Lemma 4 *Suppose $a, b, A, B > 0$ and $A/a < B/b$, then $\frac{A+2B}{a+2b} > \frac{A+B}{a+b}$.*

Proof of Lemma 4.

$$\begin{aligned}\frac{A+2B}{a+2b} =& \frac{A+B}{a+2b} + \frac{B}{a+2b} \\ =& \frac{a+b}{a+2b}\frac{A+B}{a+b} + \frac{b}{a+2b}\frac{B}{b} \\ <& \frac{a+b}{a+2b}\frac{B}{b} + \frac{b}{a+2b}\frac{B}{b} \\ =& \frac{B}{b}\end{aligned}$$

, where the last inequality holds because $\frac{B}{b} = \frac{a\frac{B}{b}+B}{a+b} > \frac{a\frac{A}{a}+B}{a+b} = \frac{A+B}{a+b}$. ■

Let $A = \int_{\underline{p}}^{p_0} pf(p)dp, B = \int_{p_0}^{1} pf(p)dp, a = \int_{\underline{p}}^{p_0} f(p)dp, b = \int_{p_0}^{1} f(p)dp$. Notice that $A/a < p_0 < B/b$. So, Lemma 4 implies that

$$\frac{A+2B}{a+2b} > \frac{A+B}{a+b}$$

Notice that $\frac{\int_{\underline{p}}^{1} pq_1(p)f(p)dp + \int_{\underline{p}}^{1} pq_2(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp + \int_{\underline{p}}^{1} q_2(p)f(p)dp} = \frac{A+2B}{a+2b}$ in the overlapping case. We have shown in the non-overlapping case that the RHS is larger than the precision of the monopoly. So, in the overlapping case, $\frac{\int_{\underline{p}}^{1} pq_1(p)f(p)dp + \int_{\underline{p}}^{1} pq_2(p)f(p)dp}{\int_{\underline{p}}^{1} q_1(p)f(p)dp + \int_{\underline{p}}^{1} q_2(p)f(p)dp}$ is also larger than the precision of the monopoly. By the same proof-by-contradiction argument as before, one can see that the precision of at least one firm is higher than the monopoly case. ■

Alternative Correlation Structure

We now consider an alternative correlation structure. Suppose firm one's prediction for a given consumer is p_1. Then, with probability ρ, firm two's prediction is also p_1, and with probability $1-\rho$, firm two's prediction is uniformly drawn from $f(p)$. Under this alternative correlation structure, the expected payoff of a firm from targeting a probability p consumer is $[\rho q(p) + (1-\rho)a]pw + [1 - \rho q(p) - (1-\rho)a]pv - c$ rather than $[\rho q(p) + (1-\rho)R]pw + [1 - \rho q(p) - (1-\rho)R]pv - c$.

All the main results in the paper hold under this alternative correlation structure. We present proof of those results below.

Proof of Lemma 1. The proof is the same as the proof in the appendix, except that one needs to replace R with a in the formulas. ■

Proof of Lemma 2.

Existence:

The proof is the same as the proof of Lemma 2 in the appendix.

Uniqueness:

1. Firms never target any consumer for sure ($\overline{p} = 1$)

$$\begin{aligned}
a &= \int_{\underline{p}}^{1} f(p)q(p)dp \\
&= \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p)\frac{-c/p + (1-\rho)aw + [1-(1-\rho)a]v}{\rho(v-w)}dp \\
&= \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} \frac{f(p)}{\rho(v-w)}[-c/p + v - (1-\rho)(v-w)a]dp
\end{aligned}$$

For any fixed ρ, define:

$$G(a) := \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} \frac{f(p)}{\rho(v-w)}[-c/p + v - (1-\rho)(v-w)a]dp - a$$

Then, $G(0) = \int_{\frac{c}{v}}^{1} \frac{f(p)}{\rho(v-w)}[v - c/p]dp > 0$

$$\begin{aligned}
G'(a) &= \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} -(1-\rho)(v-w)\frac{f(p)}{\rho(v-w)}dp - \\
&\quad c\left(-\frac{-(1-\rho)(v-w)}{[v-(1-\rho)(v-w)a]^2}\right)\left[v-(1-\rho)(v-w)a - \frac{c}{\frac{c}{v-(1-\rho)(v-w)a}}\right]\frac{f(\frac{c}{v-(1-\rho)(v-w)a})}{\rho(v-w)} \\
&= -\frac{1-\rho}{\rho}\left[1 - F\left(\frac{c}{v-(1-\rho)(v-w)a}\right)\right] - 1 < 0
\end{aligned}$$

Uniqueness then follows.

2. Firms target high-probability consumers for sure ($\overline{p} < 1$)

$$[\rho + (1-\rho)a(\rho)]\overline{p}w + [1-\rho-(1-\rho)a(\rho)]\overline{p}v - c = 0$$

$$\Rightarrow \ \overline{p} = \frac{c}{[\rho + (1-\rho)a(\rho)]w + [1-\rho-(1-\rho)a(\rho)]v} \tag{8}$$

$$
\begin{aligned}
a &= \int_{\overline{p}}^{1} f(p)dp + \int_{\underline{p}}^{\overline{p}} f(p)q(p)dp \\
&= 1 - F(\overline{p}) + \int_{\underline{p}}^{\overline{p}} f(p)q(p)dp
\end{aligned}
$$

For any fixing ρ, define:

$$
H(a) := 1 - F(\overline{p}) + \int_{\underline{p}}^{\overline{p}} f(p)q(p)dp - a
$$

Then, $H(0) = 1 - F(\overline{p}) + \int_{c/v}^{\overline{p}} f(p)q(p)dp > 0$

$$
\begin{aligned}
H'(a) =& c\frac{-(1-\rho)(v-w)}{[v-(1-\rho)(v-w)a]^2} q(\frac{c}{v-(1-\rho)(v-w)a}) f(\frac{c}{v-(1-\rho)(v-w)a}) + \\
& \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{\overline{p}} -(1-\rho)(v-w)\frac{f(p)}{\rho(v-w)} dp - 1 \\
=& -(1-\rho)(v-w)[\frac{cq(\frac{c}{v-(1-\rho)(v-w)a}) f(\frac{c}{v-(1-\rho)(v-w)a})}{[v-(1-\rho)(v-w)a]^2} + \frac{F(\overline{p}) - F(\frac{c}{v-(1-\rho)(v-w)a})}{\rho(v-w)}] - 1 \\
& < 0
\end{aligned}
$$

Uniqueness then follows.

To show that there exists $\widehat{\rho} \in [0, \frac{v-c}{v-w}]$ such that firms never target any consumer for sure if and only if $\rho \geq \widehat{\rho}$, we just need to show the following claim:

If firms never target any consumer for sure for $\rho = \rho_s$, then they also never target any consumer for sure for any $\rho_l > \rho_s$.

Suppose not. When $\rho = \rho_l$, Lemma 1 implies that there exists $\overline{p}_l$ and $\underline{p}_l$ such that firms target consumers with probabilities $p \geq \overline{p}_l$ for sure and mix for consumers with probabilities between $\underline{p}_l$ and $\overline{p}_l$. Thus, $q_l(p) = 1 > q_s(p),\ \forall p \geq \overline{p}_l$. The proof of the comparative statics results will show that $q_l'(p) < q_s'(p),\ \forall p \in (\underline{p}_s, \overline{p}_l)$. So, $q_l(p) > q_s(p),\ \forall p \in [\underline{p}_s, 1] \Rightarrow a_l > a_s$. A contradiction to the comparative statics that the overall targeting probability decreases in ρ, which will be shown in the proof of the comparative statics results. ■

Proof of Proposition 1. The same as the proof of Proposition 1 in the appendix. ■

Proof of the comparative statics results.

1. Firms never target any consumer for sure ($\overline{p} = 1$)

   Comparative statics of a w.r.t. ρ:

   In equilibrium, $G(a) = 0$. By the implicit function theorem, $\frac{\partial a}{\partial \rho} = -\frac{\frac{\partial G}{\partial \rho}}{\frac{\partial G}{\partial a}}$. We have shown that $\frac{\partial G}{\partial a}$ is negative.

$$\begin{aligned}
\frac{\partial G}{\partial \rho} =& \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} \frac{f(p)}{v-w}[-c/p + v - (1-\rho)(v-w)a](-1/\rho^2) + \frac{f(p)a}{\rho} dp \\
=& \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} \frac{f(p)}{\rho^2(v-w)}[c/p - v + (1-\rho)(v-w)a + \rho a(v-w)] dp \\
\propto& \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p)[-q(p) + a] dp \\
=& a \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p) dp - \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p)q(p) dp \\
=& a \int_{\frac{c}{v-(1-\rho)(v-w)a}}^{1} f(p) dp - a \\
=& -F\left(\frac{c}{v-(1-\rho)(v-w)a}\right) a < 0.
\end{aligned}$$

   Therefore, $\frac{\partial a}{\partial \rho} < 0$.

   Comparative statics of $\underline{p}$ w.r.t. ρ:

   We have shown that the overall targeting probability a decreases when ρ increases. Therefore, $v - (1-\rho)(v-w)a$ increases. Equation (6) then implies that $\underline{p}$ decreases in ρ.

   Comparative statics of the precision wrt ρ :

   The same as the proof in the appendix except that we use $\int_{\underline{p}_s}^{1} q_s(p)f(p)dp = a_s > a_l = \int_{\underline{p}_l}^{1} q_l(p)f(p)dp > \int_{\underline{p}_s}^{1} q_l(p)f(p)dp$ and $q_l(\underline{p}_s) > q_l(\underline{p}_l) = 0 = q_s(\underline{p}_s)$ to argue that there must exist $\tilde{p} \in (\underline{p}_s, 1)$ such that $\begin{cases} q_s(p) > q_l(p),\ \ if\ \ p \in (\tilde{p}, 1] \\ \tilde{q} := q_s(\tilde{p}) = q_l(\tilde{p}) \\ q_s(p) < q_l(p),\ \ if\ \ p \in [\underline{p}_s, \tilde{p}) \end{cases}$.

Proof of the Comparative statics of the recall wrt ρ :

Suppose $\rho > \frac{v-c}{v-w}$. We have shown that $\frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{a_l} < \frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{a_s}$. Since $a_s > a_l$, we have $\frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{a_l} = \frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{a_s}\frac{a_s}{a_l} < \frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{a_s} \Rightarrow \frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{a_s} < \frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{a_s} \Rightarrow \frac{\int_{\underline{p}_l}^1 pq_l(p)f(p)dp}{\mu_0} < \frac{\int_{\underline{p}_s}^1 pq_s(p)f(p)dp}{\mu_0}$.

2. Firms target high-probability consumers for sure ($\overline{p} < 1$)

   Comparative statics of a w.r.t. ρ:

   In equilibrium, $H(a) = 0$. By the implicit function theorem, $\frac{\partial a}{\partial \rho} = -\frac{\frac{\partial H}{\partial \rho}}{\frac{\partial H}{\partial a}}$. We have shown that $\frac{\partial H}{\partial a}$ is negative.

$$\begin{aligned}\frac{\partial H}{\partial \rho} =& - f(\overline{p})\frac{\partial \overline{p}}{\partial \rho} + \frac{\partial \overline{p}}{\partial \rho}q(\overline{p})f(\overline{p}) - \\ & \frac{c}{[v-(1-\rho)(v-w)a]^2}(v-w)aq(\frac{c}{v-(1-\rho)(v-w)a})f(\frac{c}{v-(1-\rho)(v-w)a}) \\ =& - \frac{c}{[v-(1-\rho)(v-w)a]^2}(v-w)aq(\frac{c}{v-(1-\rho)(v-w)a})f(\frac{c}{v-(1-\rho)(v-w)a}) \\ &<0\end{aligned}$$

   Therefore, $\frac{\partial a}{\partial \rho} < 0$.

   Comparative statics of $\underline{p}$ w.r.t. ρ:

   We have shown that the overall targeting probability a decreases when ρ increases. Therefore, $v - (1-\rho)(v-w)a$ increases. Equation (6) then implies that $\underline{p}$ decreases in ρ.

   Comparative statics of $\overline{p}$ w.r.t. ρ:

Consider any given $\rho_l > \rho_s$ such that the corresponding $\overline{p}_l$ and $\overline{p}_s$ are lower than 1. We prove by contradiction. suppose $\overline{p}_l \leq \overline{p}_s$. The overall targeting probability corresponding to ρ_l is $a_l = \int_{\overline{p}_l}^{1} f(p)dp + \int_{\underline{p}_l}^{\overline{p}_l} q_l(p)f(p)dp$. The overall targeting probability corresponding to ρ_s is $a_s = \int_{\overline{p}_s}^{1} f(p)dp + \int_{\underline{p}_s}^{\overline{p}_s} q_s(p)f(p)dp$. We first show that $q_l(p) > q_s(p), \forall p \in (\underline{p}_s, \overline{p}_l)$.

Observe that $q_l(\overline{p}_l) = 1 > q_s(\overline{p}_l)$. For any $p \in (\underline{p}_s, \overline{p}_l)$, we have

$$\begin{aligned} q_l(p) =& q_l(\overline{p}_l) + \frac{c}{\rho_l(v-w)}(1/\overline{p}_l - 1/p) \\ >& q_s(\overline{p}_l) + \frac{c}{\rho_s(v-w)}(1/\overline{p}_l - 1/p) = q_s(\overline{p}_l) \end{aligned}$$

Hence,

$$\begin{aligned} a_l &= \int_{\overline{p}_s}^{1} f(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} f(p)dp + \int_{\underline{p}_l}^{\overline{p}_l} q_l(p)f(p)dp \\ &> \int_{\overline{p}_s}^{1} f(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} q_s(p)f(p)dp + \int_{\underline{p}_s}^{\overline{p}_l} q_l(p)f(p)dp \\ &> \int_{\overline{p}_s}^{1} f(p)dp + \int_{\overline{p}_l}^{\overline{p}_s} q_s(p)f(p)dp + \int_{\underline{p}_s}^{\overline{p}_l} q_s(p)f(p)dp = a_s \end{aligned}$$

But we have shown that a decreases in ρ. A contradiction. Therefore, $\overline{p}_l > \overline{p}_s$.

Comparative statics of the profit w.r.t. ρ:

The same as the proof in the appendix, except that one needs to replace R with a in the formulas.

■

Proof of Proposition 2. The same as the proof at the beginning of the online appendix, except that one needs to replace R with a in the formulas. ■

Proof of Proposition 3. The threshold for a monopoly equilibrium to exist is now $\rho \geq \frac{a_m w + (1-a_m)v - c}{(1-a_m)(v-w)}$ rather than $\rho \geq \frac{R_m w + (1-R_m)v - c}{(1-R_m)(v-w)}$.

The proof is the same as the proof at the beginning of the online appendix, except that one needs to replace R with a in the formulas. ■