

Privacy and Polarization: An Inference-Based Framework

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Abstract

Advances in behavioral targeting allow media firms to monetize based on advertising. However, behavioral targeting requires consumer tracking, which has heightened privacy concerns among consumers and regulators. We examine how stricter privacy regulations that ban consumer tracking affect media firms' content strategies and ideological positioning. We consider a model where media firms choose their ideological positioning and advertising, while ideologically heterogeneous consumers select their preferred content based on both their ideology and idiosyncratic taste shocks. We compare two salient informational environments: (1) behavioral targeting, where perfect inference about consumers is allowed, and (2) contextual targeting, where consumer tracking is banned due to privacy regulations, and publishers can only infer consumer types based on their media choice. We show that privacy regulations that ban behavioral targeting incentivize publishers to shift toward more extreme and polarizing positioning in order to draw better inferences about consumer types, even though the shift to more extreme ideological positions can hurt both demand and consumer welfare from content consumption. Compared to the monopoly case, competition increases firms' inference motives and leads to more polarized content over a wider range of parameters due to an inferential complementarity effect arising from consumer self-selection. Our research uncovers a previously unexplored relationship between privacy and polarization, shedding light on the potential unintended consequences of privacy regulations in media markets.

JEL Codes: M37, L82, L13, D83.

Keywords: advertising, targeting, privacy, polarization.

1 Introduction

Digital publishers increasingly use advertising as a monetization strategy. At the core of ad-based monetization is behavioral ad targeting that creates a sustainable revenue stream for publishers and keeps online content mostly free. However, behavioral ad targeting requires the collection and use of consumer-level data, which leads to privacy concerns among consumers. According to a recent survey by Pew Research, over 80% of US adults are concerned about how companies use the data they collect from them, making the need for privacy regulation an issue with bipartisan support (McClain et al., 2023). In response to consumers’ privacy concerns in the US and globally, regulatory bodies and even some private firms have started taking actions to restrict consumer tracking online and protect consumer privacy. A few prominent examples include the California Consumer Privacy Act (CCPA) and the European Union’s General Data Protection Regulation (GDPR) in the public sector, and Apple’s App Tracking Transparency (ATT) and Google’s Privacy Sandbox in the private sector.

Although the main intent of privacy regulations is to safeguard consumer privacy, a consistent finding from past empirical research is that privacy regulation hurts digital publishers (Goldfarb and Tucker, 2011; Alcobendas et al., 2021; Johnson et al., 2023). Specifically, prior research suggests that the revenue loss due to privacy regulations is more pronounced for general interest (vs. specialized) publishers, who have greater uncertainty about their consumer types without consumer tracking (Goldfarb and Tucker, 2011). For example, in the absence of consumer tracking, a mainstream news website such as the New York Times has a harder time inferring consumer types and interests in order to show them relevant ads, compared to a niche, ideologically extreme website such as Infowars, which has more precise information about their consumers. Thus, the negative impact of privacy regulation on news publishers and media firms largely depends on their ideological positioning and content strategies.

The content-dependent impact of privacy regulation on media firms gives rise to an important question: does privacy regulation affect media firms’ ideological positioning and content strategies? To counter the loss imposed by privacy regulations, a media firm can shift from a mainstream positioning and content strategy (e.g., general news coverage) to a more specific and niche posi-

tioning and content strategy (e.g., polarizing opinion programs), thereby drawing more accurate inferences about consumers who choose their content and showing them more relevant ads. Such a shift in news content has important implications in the media landscape, especially given the rise in political polarization and media bias. Nevertheless, the extant research on privacy has focused on measuring the impact of regulations on publishers’ market *outcomes* and largely ignored their equilibrium response in terms of positioning and content strategies.

In this paper, we bridge this gap and endogenize media firms’ positioning in markets with and without privacy regulations that ban consumer tracking. We explicitly model the choice of mainstream and niche positioning using a simple Hotelling model of firm positioning and consumer demand. Specifically, we seek to answer the following questions:

1. How does privacy regulation affect the positioning and content strategies employed by media firms who monetize based on advertising in monopoly and duopoly settings?
2. What are the implications in terms of polarization? When do media firms have a greater incentive to shift to a more polarizing positioning?
3. What are the implications in terms of demand, consumers’ consumption utility, and publisher profits?

To answer these questions, we develop a model in which profit-maximizing media firms choose their ideological positioning and advertising, while ideologically heterogeneous readers consume the media content based on both its match with their ideological preferences and their own idiosyncratic taste shocks. In our model, media firms monetize by selling ad impressions to advertisers. As such, their profits are the product of two separate components: (1) an extensive margin that captures the *quantity of impressions*, and (2) an intensive margin that captures the *quality of impressions*.

To study the impact of privacy regulations that ban consumer tracking (e.g., GDPR), we compare two different information environments: *behavioral targeting* and *contextual targeting*. The difference between the two comes from the possibility of tracking. Under *behavioral targeting*, media firms possess perfect knowledge of each consumer’s type because they are allowed to track

consumers. The *contextual targeting* scenario mimics the situation in the presence of privacy regulations that ban consumer tracking, where media firms are only allowed to use contextual information to target their ads to consumers. The comparison between contextual and behavioral targeting allows us to examine the downstream impact of privacy regulations on media firms' equilibrium choices and overall market outcomes.

Under behavioral targeting, since media firms can perfectly match ads to consumers on a one-to-one basis, they can achieve maximum quality of impressions regardless of consumers' media choice. As a result, the media firm's profit-maximization problem is greatly simplified. Because perfect ads-to-readers matching is possible, the profit-maximizing positioning is the one that maximizes the quantity of impressions.

Conversely, under contextual targeting, media firms no longer have access to perfect information about consumer types, so they need to rely solely on a single piece of information to infer consumer types: consumer's self-selection into media. In this case, media firms have the incentive to deviate from the demand-maximizing positioning (the equilibrium under behavioral targeting) and shift to positioning and content strategies that increase the quality of impressions. In other words, publishers may benefit from creating content with lower demand but sharper signals about consumers, thereby achieving higher profits by balancing the quantity and quality of impressions.

We first consider a monopolist media firm and examine its equilibrium strategies under both behavioral and contextual targeting regimes. We show that deviating from the demand-maximizing positioning and content strategy can be an equilibrium under contextual targeting when consumers are sufficiently ideologically differentiated and when their sensitivity to imperfect ad targeting (mismatches between ad and consumer type) is sufficiently high. In particular, we find that the deviation is more toward extreme and polarizing ideological positioning because publishers can sharpen the signal about consumer types by moving toward the extreme ends of consumers' ideological preferences. Notably, this incentive persists even when such a shift toward extreme content reduces both consumer welfare from content consumption and total demand.

We then study the duopoly case to examine the equilibrium outcomes under the two targeting regimes in a competitive environment. Studying the duopoly case is important because competition

naturally incentivizes media firms to differentiate and move towards more polarizing positions. As such, it is not clear whether the inferential motives bolstered by privacy regulation are strong enough to shift the equilibrium to further polarization under contextual targeting compared to behavioral targeting, as in the monopoly case. Our equilibrium analysis in the duopoly case reveals important insights. First, we find that the media firms’ ideological positioning in the equilibrium under contextual targeting is at least as polarizing as that under behavioral targeting, which establishes the positive link between privacy regulation and polarization in a competitive environment.

Second, compared to the monopoly setting, we document that privacy regulation leads to more polarization even when consumers are less ideologically differentiated and when their sensitivity to imperfect ad targeting is lower. A key underlying mechanism we identify is an *inferential complementarity* effect between the two firms, which indicates that the presence of one firm aids the other firm’s inference, making the inferential motives stronger for the media firms. Therefore, inferential complementarity, combined with regular product differentiation incentives under competition, strengthen the shift to more polarization due to privacy regulation under duopoly.

In summary, our paper makes several contributions to the literature. We study the link between privacy and polarization and highlight the possibility of an unintended consequence of privacy regulations in increasing content polarization. Specifically, we show that privacy regulation can lead to greater content polarization, even when it reduces total demand and consumer welfare from content consumption. Our finding is important as it goes against extensive media speculations and policy memos that cite personalization as a key contributor to the increased polarization over the past few decades (Pariser, 2011). In our model, we find the opposite: it is the *inability to personalize* that leads firms to move to more extreme and polarizing ideological positioning to sharpen their inferences about consumers. A key innovation of our framework is in explicitly modeling privacy as an inference problem. In particular, our inference-based framework suggests that by limiting what firms *know* about consumers, privacy regulations shift firms’ focus to what they *want to know* about consumers, which creates an incentive to use content polarization as a device to draw more accurate inference. From a game theoretical standpoint, this modeling framework allows us to examine players’ equilibrium responses to the privacy shifts. For empirical studies, this provides

a framework to quantify the magnitude of information gain/loss in various data environments. Finally, we extend the literature on media bias by introducing inference motives as an important determinant of actions chosen by strategic players, which is largely ignored in the prior literature on media markets: notably, we show privacy leads firms to polarize *beyond* the underlying ideological differences among consumers; thus, the average consumer is exposed to *more ideologically extreme* content than he would experience under perfect targeting.

2 Literature Review

Our work relates to the literature on media markets. Early theoretical work in this domain considers the competition between broadcasters in the presence of the advertising market and shares insights into equilibrium outcomes in this market in terms of content provision and advertising strategies (Dukes and Gal-Or, 2003; Gal-Or and Dukes, 2003; Anderson and Coate, 2005; Godes et al., 2009). A separate stream of work in this literature has examined content strategies as they relate to media bias and polarization (Mullainathan and Shleifer, 2005; Gentzkow and Shapiro, 2006; Xiang and Sarvary, 2007). With the growth of digital news consumption, a series of recent studies have focused on the specific aspects of the digital context and examined pricing and content strategies by media markets (Johar et al., 2012; Sun and Zhu, 2013; Ambrus et al., 2016; Athey et al., 2018; Berman et al., 2019; Lin, 2020; Amaldoss et al., 2021; Jain and Qian, 2021; Amaldoss et al., 2023; Amaldoss and Du, 2023). Our paper adds to this stream of work by studying a key aspect of the digital context: behavioral targeting and the possibility of privacy regulations. In particular, we study the impact of privacy regulations on media markets and examine the equilibrium outcomes in terms of ideological positioning and content strategies and their implications for media bias and polarization.

Our work relates to the literature on behavioral targeting, personalization, and, more broadly, online advertising. With the advancements in targeting technologies in advertising markets, a series of papers have studied the impact of targeting accuracy or advertising strategy on equilibrium market outcomes (Chen et al., 2001; Iyer et al., 2005; Levin and Milgrom, 2010; Bergemann and Bonatti, 2011; Mayzlin and Shin, 2011; Kuksov et al., 2013; Amaldoss et al., 2016; Sayedi, 2018;

Berman and Katona, 2020; Rafeian and Yoganarasimhan, 2021; Shin and Yu, 2021; Lauga et al., 2022; Berman et al., 2023; Chaimanowong et al., 2023; Ning et al., 2023; Shin and Shin, 2023; Johnson et al., 2024). In the news media context, many have speculated that greater personalization results in more polarization, citing the positive correlation between the rise of political polarization in the US and the surge in personalized content delivery through online platforms (Pariser, 2011). Despite the widespread lay belief that causally connects personalization and polarization, empirical findings in this domain do not present a consistent viewpoint. Surprisingly, the demographic groups in the US that are least likely to use the Internet experienced the greatest increase in polarization (Boxell et al., 2017). Moreover, studies analyzing users’ browsing histories reveal that, despite social media and the Internet being associated with greater ideological divergence among users, they also increase exposure to opposing views (Flaxman et al., 2016). Investigations focused on specific platforms and their personalized features offer conflicting results concerning the link between personalization and polarization. Notably, studies investigating Facebook’s news feed algorithm, Google’s search personalization, and YouTube have found limited evidence suggesting that personalization contributes to content bias (Bakshy et al., 2015; Ribeiro et al., 2020; Hosseinmardi et al., 2021). In our paper, we build on the theoretical literature on ad targeting and develop a model to study the impact of personalization on the supply of polarizing content. Our work extends this literature by providing an inference-based theoretical account that presents a more nuanced view of the link between personalization and polarization.

Our paper also relates to the literature on privacy. A vast body of theoretical work has examined different issues related to consumer identification, privacy, and information markets (Villas-Boas, 1999, 2004; Taylor, 2004; Acquisti and Varian, 2005; Bergemann and Bonatti, 2015; Bergemann et al., 2018; Choi et al., 2020; Rhodes and Zhou, 2021; Choi and Jerath, 2022; Yang, 2022; Bonatti et al., 2023; Choi et al., 2023; Ke and Sudhir, 2023; Yao, 2023; Strack and Yang, 2023; Miklós-Thal et al., 2023). Empirical papers in this domain have studied the impact of privacy regulations on market outcomes in different settings (Goldfarb and Tucker, 2011; Johnson, 2022; Johnson et al., 2023). In particular, Goldfarb and Tucker (2011) study a change in tracking and targeting regulations and document lower response rates to ads and, therefore, lower ad revenues for publishers.

Notably, they demonstrate a heterogeneous effect of privacy regulation on digital publishers, with general interest publishers such as the New York Times experiencing higher revenue loss than specialized publishers such as Car and Driver Magazine. We extend this literature by endogenizing media firms’ ideological positioning and content strategies, to allow them to respond optimally to the change in privacy policies. We present a generic theoretical framework that characterizes privacy as an inference problem, in which stricter privacy policies have a negative impact on the accuracy of inference about consumers. Importantly, we identify the possibility of increased polarization as an unintended consequence of privacy regulations in digital markets.

3 Model

We theoretically characterize a market where media firms (e.g., news publishers) create content for consumers and monetize by placing ads. To reflect consumers’ ideological preferences, we assume three discrete consumer types $\{0, 1/2, 1\}$, where 0 and 1 refer to the opposing ideological ends (e.g., left vs. right) and $1/2$ refers to the centrist position. Consistent with consumer preferences, $\{0, 1/2, 1\}$ defines the action set for the media firm’s ideological positioning or slant, which allows us to characterize the match between the consumer type and the media firm’s content. The media firm’s ideological positioning informs its general content strategies. As such, we refer to this decision with terms such as ideological positioning, content strategy, and slant interchangeably throughout the paper. While politics is a natural application for our model, our framework encompasses broader settings with ideological preferences about issues (e.g., animal rights vs. hunting websites). We unify our concept by referring to the positioning in the middle ($= 1/2$) as *mainstream* and the positioning at the two extremes ($\in \{0, 1\}$) as *niche*.¹

We denote the consumer type by θ . We assume that the consumer type has a symmetric distribution that depends on a single parameter λ , with $1 - 2\lambda$ proportion of users having a centrist or mainstream position ($\theta = 1/2$), and λ proportion of consumers being at each extreme or niche end of consumer types ($\theta \in \{0, 1\}$). As such, we can view λ as a measure of consumer polarization

¹Our use of “niche” and “mainstream” is in line with Johnson and Myatt (2006), and needs not have a market share interpretation. In other words, niche strategy is always more polarizing than mainstream strategy, but might have lower demand.

and ideological partisanship, which is the key *polarization parameter* in our study. Since we want to study media firms' incentive to polarize in equilibrium with and without privacy regulations, we focus on cases where mainstream consumers are in minority: $\lambda \geq 1/3$. It is clear that cases with low levels of consumer polarization will lead to minimal content polarization regardless of the targeting regime. In contrast, our focal cases where $\lambda \in [1/3, 1/2]$ represent settings where content polarization can be viewed as a major possibility.²

We denote the media firm's ideological positioning choice by x , which comes from $\{0, 1/2, 1\}$. If the consumer chooses the media firm, an impression will be generated and the media firm can place an ad $a \in [0, 1]$ in that impression catering to different consumer types. As such, although the ideological positioning x is the same for all consumers, ad a can be targeted at the impression level depending on the information available about the consumer.

We consider two types of information environment, *behavioral targeting* and *contextual targeting*. The information environment is characterized by the observability of each consumer's type, which depends on whether behavioral tracking and targeting are allowed. When *behavioral targeting* is allowed, we assume that the media firm knows the consumer type for each consumer on its platform, as it has rich prior information about the consumer through tracking. On the other hand, when behavioral targeting is banned due to privacy regulations, the media firm needs to engage in *contextual targeting* and infer consumer type based on the context, e.g., the content consumed by the consumer.

Figure 1 presents the timeline of our game in four stages. We present the details of each stage as follows:

1. In the first stage, consumer type θ is realized. Without privacy regulation (under behavioral targeting), it is known by the media firm(s). With privacy regulation (under contextual targeting), it is the consumer's private information.
2. In the second stage, each media firm chooses an ideological positioning strategy x to maximize its expected profits. The media firm's positioning choice x informs its content strategy. The

²It is worth noting that our qualitative insight holds for $\lambda < 1/3$. The only reason we focus on $\lambda \in [1/3, 1/2]$ is that the insights from these cases have clearer implications for polarization.

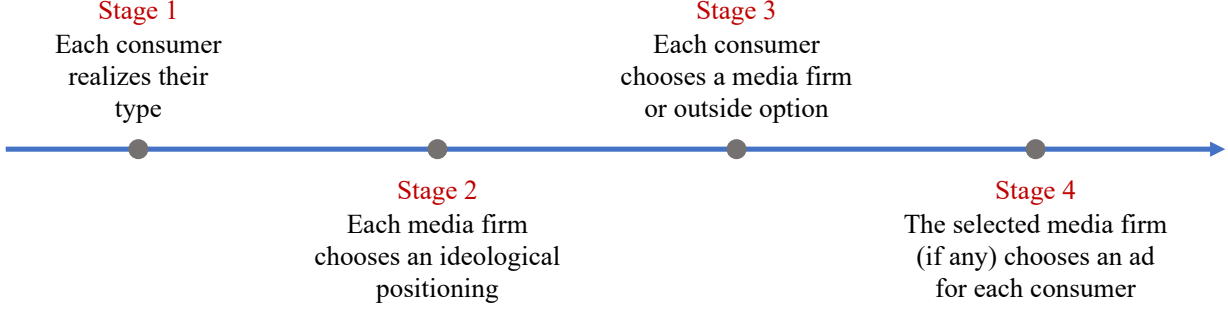


Figure 1: Timeline of the game

media firm can choose among one mainstream position at $x = 1/2$ and two niche positions at $x = 0$ and $x = 1$. In the political context, the niche positions can be interpreted as left- and right-leaning ideological positions. The media firm's positioning choice is broadcast to all consumers through their content. Consistent with the literature on media slant, the ideological positioning or slanting strategy is a single choice, so the firm cannot be right- and left-leaning at the same time (Mullainathan and Shleifer, 2005; Gentzkow and Shapiro, 2010).

3. In the third stage, each consumer consumes at most one content unit.³ We define a type- θ consumer's utility from consuming the content by the media firm with ideological positioning x as follows:

$$U(x; \theta) = 1/2 - |x - \theta| + \epsilon, \quad (1)$$

where the first term is the base utility from media consumption that is set to $1/2$, the second term is the consumer's distaste for ideological mismatch as measured by the distance of the media firm's ideological positioning from the consumer type, and the third term is an idiosyncratic error term with mean zero that comes from the distribution $f(\cdot)$ such that $f(\epsilon) > 0$, $\forall \epsilon \in [-1/2, 1/2]$. We can generalize the base utility to any v (see §5.4), but fixing it at $v = 1/2$ conveys the main insights while keeping the model simple. We normalize the utility from consumer's outside option to 0.

The utility framework in Equation (1) implies that a consumer located at one extreme ($\theta = 0$)

³The assumption that only one content unit is consumed is without loss of generality. As long as individual consumption does not depend on θ , all of our findings remain unchanged.

or 1) will never consume the content by a media firm located at the other extreme ($x = 1$ or 0), but may consume the content by a mainstream media firm ($x = 1/2$) if her idiosyncratic term is sufficiently high.⁴ Mainstream consumers ($\theta = 1/2$) may consume the mainstream or either niche media firms, depending on the relative values of the idiosyncratic terms.

4. In the final stage, the media firm chooses ad a if the consumer generates an impression by choosing the media firm's content. The expected value generated from showing ad a to a consumer of type θ is given by the match value function \mathcal{M} as follows:

$$\mathcal{M}(a; \theta) = 1 - \gamma(a - \theta)^2, \quad (2)$$

where the match value convexly decreases in the mismatch between ad and consumer $|a - \theta|$, and the parameter γ captures the *targeting value* and is an instrumental parameter for our analysis.⁵

We assume that the match value function $\mathcal{M}(\cdot; \cdot)$ is a continuous and smooth function in both a and θ . As is clear from Equation (2), the maximum value of the match value function is one. We do not restrict ourselves to the case of $\gamma \leq 1$; therefore, a large ad-consumer mismatch ($(a - \theta)^2 > 1/\gamma$) can result in negative ad profits. These could be interpreted, for instance, as the ad being repulsive to consumers (e.g., a hunting rifle ad for a vegetarian consumer), alienating them in the future.

We now define the media firm's profit maximization problem. Let $D(x)$ denote the total consumer demand for a media firm at position x . This is the *quantity* of impressions the media firm is able to generate. For each impression, the firm can place an ad and obtain ad revenue that depends on the *quality* of the impression characterized by the match value function. We assume that there is a perfectly competitive ad marketplace, so the revenue-per-impression is the same as the expected

⁴In the general base utility extension in §5.4, the consumer located at one extreme may consume content from the opposite extreme and the main insights are robust.

⁵We remark that this formulation essentially reduces consumer types to one dimensional or, in other words, assumes that ideological and advertising preferences are perfectly correlated. While this is obviously a simplification (motivated by analytical convenience), we believe it is well justified by recent empirical research documenting the high correlation between these two dimensions (Schoenmueller et al. (2023)).

match value of the ad for a given impression.⁶ Let \mathcal{I} denote the information available for each impression. We define the firm’s optimal profits π given information \mathcal{I} as follows:

$$\pi(x, a; \mathcal{I}) = \max_{x, a} D(x) \cdot \mathbf{E}[\mathcal{M}(a; \theta) \mid \mathcal{I}], \quad (3)$$

where the firm jointly maximizes content strategy x and ad a given the information \mathcal{I} available. The firm’s contextual information about an impression is that the consumer has chosen the media firm’s content x . In any event, the firm has contextual information about an impression, so we have $x \in \mathcal{I}$. Under behavioral targeting, we assume that the firm also has information about the exact consumer type θ . Since we are interested in cases under *behavioral* and *contextual* targeting, we characterize the information available under each case as follows:

- The information under *behavioral* targeting is defined as \mathcal{I}_b and includes the contextual information x as well as behavioral information θ , i.e., $\mathcal{I}_b = \{x, \theta\}$.
- The information under *contextual* targeting is defined as \mathcal{I}_c and includes the contextual information x , i.e., $\mathcal{I}_c = \{x\}$.

We specifically model the information under behavioral and contextual targeting as extreme cases to obtain cleaner results and better understand the incentives under these two information environments. However, our framework is flexible, and one could easily consider more middle-ground cases where there is more contextual information or less behavioral information.

4 Consumer Privacy and Firm’s Inference Problem

A natural way to think about consumer privacy in our setting is to examine how uncertain the firm is about the consumer’s type given the information \mathcal{I} . We present our definition of consumer privacy as follows:

⁶The prior literature suggests the possibility of thin markets under behavioral targeting (Levin and Milgrom, 2010; Rafeian and Yoganarasimhan, 2021). However, the assumption of a perfectly competitive ad market is reasonable in our context because we have three broad consumer types. As such, if the market is thick enough for each, our assumption holds. Further, even in the presence of a thin market, the media firm can still extract a higher revenue from a better ad match by setting a reservation price, which is fundamentally the assumption we make.

Definition 1 (Consumer Privacy). *The consumer's privacy given the information \mathcal{I} is defined as $\text{Privacy}(\mathcal{I})$ and is equal to the conditional variance of consumer type given information \mathcal{I} , that is, $\text{Privacy}(\mathcal{I}) = \text{Var}(\theta|\mathcal{I})$.*

We can easily verify that the consumer's privacy is equal to zero under behavioral targeting, as \mathcal{I}_b fully reveals the consumer's type. On the other hand, the consumer's privacy under contextual targeting depends on the consumer's content/media choice. Some choices can reveal more about the consumer type, thereby reducing the posterior variance of type and consumer privacy. This is an important consideration for the media firm when choosing the ideological positioning and content strategy. We characterize the relationship between consumer privacy and content choice in the following definition:

Definition 2 (Privacy Reducing Choice). *Consumer's content choice x is privacy-reducing over content choice x' if $\text{Privacy}(\{x\}) < \text{Privacy}(\{x'\})$.*

This definition reflects the insight that a consumer who chooses Newsmax or Democracy Now reveals more about their type than a consumer who chooses Wall Street Journal or CNN. To see how the privacy measure enters the media firm's problem, we revisit the firm's profit maximization problem as presented in Equation (3). Since the ad choice is the final decision in the timeline, we start with the firm's optimal ad choice. For ad choice a , the firm needs to infer the consumer type given information \mathcal{I} . The following lemma characterizes the media firm's optimal ad choice:

Lemma 1. *The firm's optimal ad choice a^* is the posterior mean of consumer type given information \mathcal{I} , i.e., $a^* = \mathbf{E}[\theta | \mathcal{I}]$.*

This lemma follows from the property of the variance and the quadratic specification of \mathcal{M} .⁷ It indicates that the optimal ad a^* is the same as consumer type θ under behavioral targeting: $a^*(\theta) = \theta$, while it is equal to the mean of θ given the consumer's content choice x under contextual targeting: $a^*(x) = \mathbf{E}(\theta|x)$. Given the ad choice characterized in Lemma 1, we can establish the following result about the media firm's optimal ideological positioning as follows:

⁷In other words, with a different functional form for $\mathcal{M}(a, \theta)$, the firm might find it optimal to deviate from $a^* = \mathbf{E}[\theta | \mathcal{I}]$ in the contextual targeting case.

Corollary 1. *The media firm chooses the ideological positioning and content strategy x that maximizes $D(x) \cdot [1 - \gamma \text{Privacy}(\mathcal{I})]$.*

The corollary implies that the firm's profits are negatively correlated with consumer privacy. Under behavioral targeting, the firm fully observes consumer type (i.e., $\text{Privacy}(\mathcal{I}_b) = 0$), so the firm chooses the ideological positioning that maximizes its total demand:

$$\pi^b(x) = \max_x D(x), \quad (4)$$

where $\pi^b(x)$ is shorthand for the profits under behavioral targeting, and the firm is only interested in maximizing the *quantity* of impressions. Under contextual targeting, the firm needs to infer consumer type given the consumer's self-selection into the media firm's content, so the profit maximization problem simplifies to:

$$\pi^c(x) = \max_x D(x)[1 - \gamma \text{Privacy}(\{x\})], \quad (5)$$

where $\pi^c(x)$ is shorthand for the profits under contextual targeting, and $\text{Privacy}(x)$ is the privacy of consumer type under contextual targeting where the only information available about the consumer is their content choice x . As shown in Equation (5), the media firm has to trade off the *quantity* of impressions with the *quality* of impressions. As a consequence, the demand-maximizing ideological positioning is not necessarily the optimal one; the firm may trade off more popular positioning with a more accurate inference about the consumer. In our equilibrium analysis, we examine this inference incentive as it relates to polarization.

5 Equilibrium

5.1 Equilibrium Concept

Due to the setup of a multi-stage game with incomplete information, we consider Perfect Bayesian Equilibrium. Specifically, when the firm chooses the advertising location in the last period, it uses Bayes' rule to update its belief about consumer type based on the consumer's equilibrium content

choice.

5.2 Monopoly

5.2.1 Demand

When there is only one firm, the consumer compares the utility of consuming the monopoly's content with the utility of choosing the outside option. Thus, she will consume the content if and only if the utility from content consumption is positive. The following lemma calculates consumer demand given different content choices.

Lemma 2. *The total demand for a monopolist media firm depends on their ideological positioning strategy as follows:*

- (a) *The total demand for a monopoly's mainstream positioning, $x = 1/2$, is $1 - 2F(0)\lambda$. Among them, $1 - 2\lambda$ are mainstream consumers, $[1 - F(0)]\lambda$ are type 0 consumers, and $[1 - F(0)]\lambda$ are type 1 consumers.*
- (b) *The total demand for a monopoly's niche positioning, $x = 0$ or 1 , is $1 - F(0) + [2F(0) - 1]\lambda$. Among them, λ are type $\theta = x$ consumers and $[1 - F(0)](1 - 2\lambda)$ are mainstream consumers.*

This lemma highlights an important, general, and natural property. On average, a random consumer is farther from a niche position than a mainstream position. Hence, if the media firm chooses the mainstream ideological positioning, $x = 1/2$, both types of niche consumers, $\theta \in \{0, 1\}$, may consume it because they are not located too far away from the content. In contrast, if the firm chooses the niche ideological positioning, say $x = 0$, mainstream consumers may consume it, while type $\theta = 1$ consumers will not, independent of their idiosyncratic taste shock, because they are too ideologically distant from the content. This endogenous selection leads to an asymmetry in the firm's inference ability. By choosing niche ideological positioning $x = 0$, the firm knows for sure that it does not attract any type $\theta = 1$ consumers, and can better infer the consumer's type by Bayesian updating.

5.2.2 Media Firm's Ideological Positioning

The monopolist's positioning strategy is straightforward under behavioral targeting. It chooses the ideological positioning that maximizes total demand because there is no inference problem, and profit simply equals total demand, as indicated in Equation (4). The following proposition characterizes the monopolist equilibrium under behavioral targeting:

Proposition 1. *Under behavioral targeting, the monopolist media firm chooses the mainstream ideological positioning if $[4F(0)-1]\lambda < F(0)$ and niche ideological positioning if $[4F(0)-1]\lambda > F(0)$.*

This proposition characterizes the equilibrium strategy under the general case of behavioral targeting. It is easy to verify that the optimality condition for the mainstream positioning can easily be satisfied. For example, if we make the common assumption that the distribution of the idiosyncratic term is symmetric, we find that the mainstream positioning is always an equilibrium strategy. This is because $F(0) = 1/2$ under a symmetric distribution, which implies that for all possible values of λ , mainstream positioning is the equilibrium strategy.

As discussed earlier, the firm's perfect inference under behavioral targeting implies that the firm does not strategically consider inference. In contrast, under contextual targeting, it relies on Bayesian updating to infer consumer type and considers inference strategically. As such, it may have an incentive to deviate from the demand-maximizing strategy (i.e., equilibrium under behavioral targeting) and choose an ideological positioning that allows more accurate ad targeting. Hence, the question is under which strategy the firm knows more about consumer type in an impression. The following lemma helps characterize the decision-making trade-off for the media firm:

Lemma 3. *The consumer's choice of niche content is privacy-reducing over the choice of mainstream content, i.e., $\text{Privacy}(\{1/2\}) > \text{Privacy}(\{x\})$, such that $x \in \{0, 1\}$.*

This lemma indicates that a mainstream positioning leads to a lower intensive margin (ad revenue) on each impression, because the firm will have greater uncertainty about the consumer type given their content choice. Therefore, there is a natural incentive for media firms to move to more niche ideological positions. If the niche positioning is demand-maximizing and thus the equilibrium under behavioral targeting (when $[4F(0) - 1]\lambda > F(0)$), Lemma 3 implies that it will

be the equilibrium under contextual targeting, because niche strategy does better on both elements in the expected profit equation in Equation (5): *quantity* and *quality* of impressions.

When the demand-maximizing strategy is mainstream ideological positioning, the trade-off between quantity and quality of impressions becomes more interesting. Naturally, we expect the equilibrium to depend on the value of targeting that is captured by the parameter γ . The following proposition characterizes this relationship:

Proposition 2. *Under contextual targeting, the monopolist media firm's equilibrium strategy depends on the values of λ and γ as follows:*

- (a) *If $[4F(0) - 1]\lambda > F(0)$, the media firm chooses niche ideological positioning in equilibrium.*
- (b) *If $[4F(0) - 1]\lambda < F(0)$, there exists $\gamma^m > 0$ for any value of λ such that the media firm chooses niche ideological positioning if $\gamma > \gamma^m$ and mainstream ideological positioning if $\gamma < \gamma^m$. The threshold γ^m is decreasing in λ .*

Figure 2 combines the equilibrium under behavioral and contextual targeting and presents three regions. We find that equilibrium strategies are only different in region B (shaded) where $[4F(0) - 1]\lambda < F(0)$ and $\gamma > \gamma^m$. In this case, the monopolist media firm chooses a niche ideological positioning in equilibrium under contextual targeting, but a mainstream positioning under behavioral targeting. Although the media firm generates fewer impressions by choosing a niche ideological positioning, it faces lower uncertainty about consumer type in these impressions. This lower uncertainty translates into a higher revenue-per-impression, which makes up for the lower demand when the value of targeting is sufficiently high, i.e., $\gamma > \gamma^m$. Together, the ideological positioning that the media firm chooses under contextual targeting is at least as polarizing as that under behavioral targeting.

From a privacy standpoint, our analysis shows that a privacy regulation that bans consumer tracking can lead to increased polarization and partisanship in the supply side. Intuitively, this is because consumer tracking better separates the media firm's ideological positioning choice from its advertising choice. Moreover, our results shed light on the well-studied link between consumer polarization (demand) and content polarization (supply), where the prior literature finds a natural

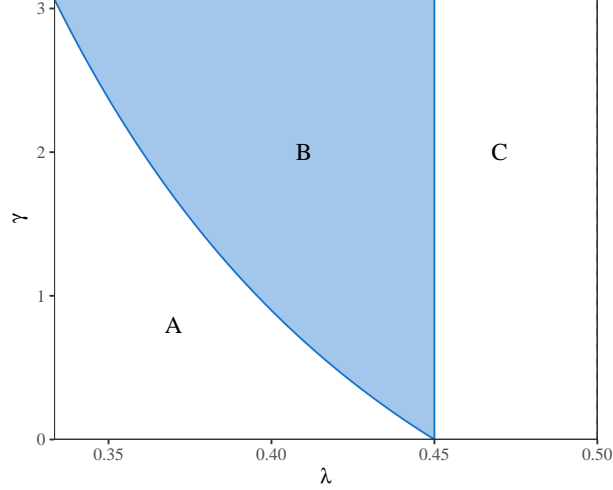


Figure 2: Equilibrium regions in monopoly under behavioral and contextual targeting

Note: This figure shows equilibrium regions when $F(0) = 9/16$. Region A shows the cases where mainstream positioning is the equilibrium under both targeting regimes. Region B (shaded) illustrates cases where privacy regulation leads to higher polarization because the firm chooses niche positioning under contextual targeting, but a mainstream positioning under behavioral targeting. Region C shows cases where the firm chooses niche positioning under both targeting regimes.

incentive for media firms to slant their content (Gentzkow and Shapiro, 2010). We revisit this link with and without privacy regulations and find that when consumers are more polarized (higher λ), privacy regulation can change the media firm's equilibrium strategy from moderate (mainstream) to partisan (niche) for a wider range of γ , because the threshold γ^m is decreasing in λ . Thus, we find that privacy regulation strengthens the link between consumer polarization and content polarization.

5.2.3 Consumer Utility from Content Consumption

We have shown that the monopolist media firm may trade off the total demand for a more accurate inference when behavioral tracking is banned. In this section, we ask the following question: how does the switch from mainstream to niche content strategy affect consumer's utility from content consumption? Intuitively, niche consumers receive a higher utility from consuming niche content, while mainstream consumers receive a higher utility from consuming mainstream content. Hence, niche content strategy leads to lower consumer utility from content consumption if most consumers

are mainstream (low λ), while it leads to higher consumer utility from content consumption if most consumers are niche (high λ). We formalize this insight in the following proposition:

Proposition 3. *Niche content strategy leads to lower consumer utility from content consumption if and only if*

$$\lambda < \hat{\lambda} := \frac{1-2[1-F(0)]\mathbf{E}[\epsilon|\epsilon>0]}{3-8[1-F(0)]\mathbf{E}[\epsilon|\epsilon>0]}.$$

We can now combine this proposition with Propositions 1 and 2 and examine when the equilibrium strategy under contextual targeting leads to higher or lower consumer utility from content consumption, compared to the equilibrium under behavioral targeting. This is an important comparison from a privacy policy standpoint because it helps us understand a key welfare consequence of privacy regulations. To illustrate this aspect, we need to focus on where the equilibrium strategies are different with and without privacy regulation ($[4F(0) - 1]\lambda < F(0)$ and $\gamma > \gamma^m$) and examine under what condition privacy regulation has unintended consequences in terms of consumer welfare from content consumption. The following corollary highlights this insight:

Corollary 2. *Consumer utility from content consumption is lower under contextual targeting than under behavioral targeting if $\lambda < \hat{\lambda}$, $[4F(0) - 1]\lambda < F(0)$, and $\gamma > \gamma^m$.*

The cutoff threshold $\hat{\lambda}$ determines the region where privacy regulation has an unintended consequence in terms of consumer welfare. This cutoff depends critically on the distribution of the idiosyncratic term. In particular, $\hat{\lambda}$ is increasing in $\mathbf{E}(\epsilon|\epsilon > 0)$.⁸ Therefore, understanding $\mathbf{E}(\epsilon|\epsilon > 0)$ is essential to the interpretation of our results.

In general, a distribution of ϵ that is concentrated around 0 results in low values of $\mathbf{E}(\epsilon|\epsilon > 0)$, while the opposite is true for a bimodal distribution that concentrates mass near the extremes $-1/2$ and $1/2$, for which $\mathbf{E}(\epsilon|\epsilon > 0) \rightarrow 1/2$. As such, $\mathbf{E}(\epsilon|\epsilon > 0)$ has a close relationship with the variance of the idiosyncratic term ϵ , which captures consumers' media preferences that are unexplained by ideology. In that sense, the higher the variance of the idiosyncratic term, the more open the consumer is to consuming content farther from their own ideology. This is an important construct because many behavioral nudges to encourage people to read from other sides attempt to reduce the relative importance of ideology in consumer utility.

⁸The first derivative is $2[1 - F(0)]/(8[1 - F(0)]\mathbf{E}(\epsilon|\epsilon > 0) - 3)^2$, which is always non-negative.

Interestingly, our results suggest that as consumers' openness to ideologically different content increases (higher variance of ϵ), the region where privacy regulation reduces consumer welfare from content consumption grows (higher $\hat{\lambda}$). Focusing on the two extreme cases is illuminating. First, if there is no variance in the idiosyncratic term ($\epsilon = 0$), the cutoff $\hat{\lambda}$ will be equal to $1/3$, implying that the privacy regulation always increases consumer welfare whenever it changes the equilibrium under behavioral targeting. In contrast, in the limit case of a bimodal ϵ where $\mathbf{E}(\epsilon|\epsilon > 0) \rightarrow 1/2$, we have $\hat{\lambda} = 1/2$, which indicates that the equilibrium shift caused by privacy regulation reduces consumer welfare from content consumption. We illustrate this point in Figure 3, where we use three different error distributions by varying the shape parameters of the Beta distribution. As shown in all figures, as the relative importance of ideology in consumer preferences decreases through changing the variance of the error term, the region that reduces consumer utility from content consumption expands.

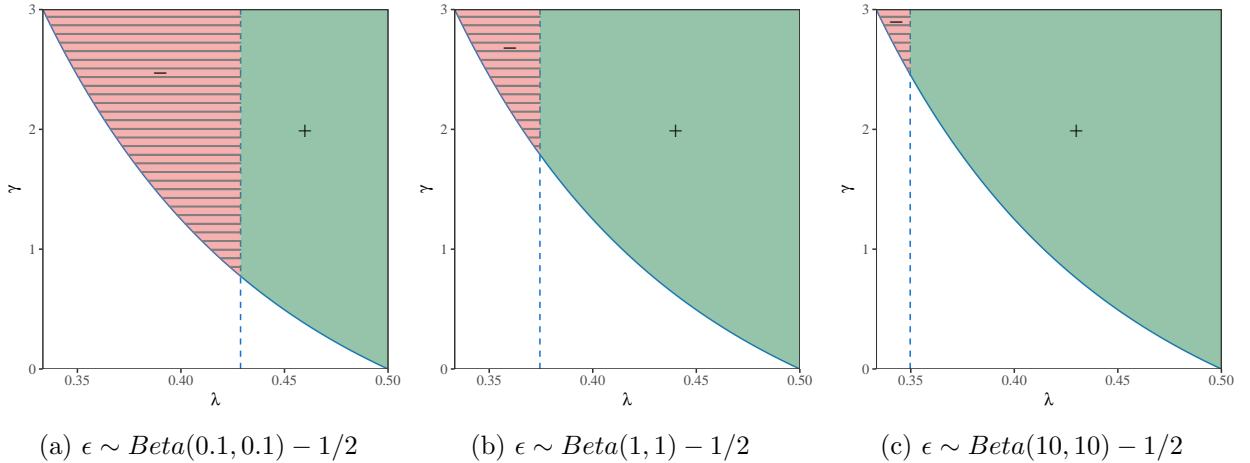


Figure 3: Impact of privacy regulation on consumer utility from content consumption.

Note: The green shaded region with the plus sign shows the region where consumer utility from content consumption is higher under contextual targeting than behavioral targeting. The red striped region with the minus sign shows the region where the equilibrium under contextual targeting leads to lower consumer utility from content consumption. The blank region shows the area where the consumer utility is the same under both behavioral and contextual targeting.

In sum, our analysis shows several important welfare implications. First, we show that for any non-deterministic idiosyncratic term, there always exists a parameter range such that privacy regulation leads to a lower consumer utility from content consumption. This implies that banning behavioral tracking may drive the monopolist media firm to switch from mainstream to niche

ideological positioning, even when doing so reduces both total demand and consumer welfare from utility consumption. Second, we show that the negative impact on consumer welfare from utility consumption is more pronounced when ideology plays a smaller role in determining consumers' media preferences, a case that is often the target of depolarization efforts.

5.3 Duopoly

We now introduce competition in a duopoly setting. The impact of competition in our setting is both *a priori* unclear and conceptually interesting: are the two firms' inference motives complements or substitutes? Further, given that competition generally leads to an increase in differentiation (even absent any inference motive), is it still true that contextual advertising leads to *more* content polarization compared to behavioral advertising? Moreover, provided that increase in differentiation occurs following a ban on individual data tracking, what are its implications for consumer demand and utility from media consumption? To answer these questions, we adapt our monopoly model to a duopoly setting.

5.3.1 Demand

In contrast to the monopoly case, consumers compare the utility of consuming a firm's content not only with the outside option but also with the utility of consuming the other firm's content. Suppose that the first media firm chooses x_1 ideological positioning, while the second one chooses x_2 ideological positioning. Let (x_1, x_2) denote firms' positioning strategies. Without loss of generality, we assume that $x_1 \leq x_2$. By symmetry, $(0,0)$ is equivalent to $(1,1)$, and $(0,1/2)$ is equivalent to $(1/2,1)$. Hence, we only need to consider $(0,0)$, $(0,1/2)$, $(0,1)$, and $(1/2,1/2)$. One can see that $(0,1)$ dominates $(0,0)$, so $(0,0)$ will never be an equilibrium. Hence, we need to only consider three possible equilibria: $(0,1)$, $(1/2,1/2)$ and $(0,1/2)$. Lemma 4 fully characterizes consumer demand under each possible equilibrium strategies for the two firms as follows:

Lemma 4. *The media firms' demand under three possible equilibria $(0,1)$, $(1/2,1/2)$ and $(0,1/2)$ are characterized as follows:*

- (a) *Under $(0,1)$ strategy profile, Firm 1's total demand is $\lambda + \frac{1-F(0)^2}{2}(1-2\lambda)$. Among them, λ are*

type 0 consumers and $\frac{1-F(0)^2}{2}(1-2\lambda)$ are mainstream consumers. Firm 2's total demand is $\lambda + \frac{1-F(0)^2}{2}(1-2\lambda)$. Among them, λ are type 1 consumers and $\frac{1-F(0)^2}{2}(1-2\lambda)$ are mainstream consumers.

(b) Under $(1/2, 1/2)$ strategy profile, each firm's total demand is $\frac{1-2\lambda}{2} + [1 - F(0)^2]\lambda$. Among them, $\frac{1-2\lambda}{2}$ are mainstream consumers, $\frac{1-F(0)^2}{2}\lambda$ are type 0 consumers, and $\frac{1-F(0)^2}{2}\lambda$ are type 1 consumers.

(c) Under $(0, 1/2)$ strategy profile, Firm 1's total demand is $\lambda[1 - \int_{-1/2}^0 F(\epsilon_0)f(\epsilon + 1/2)d\epsilon] + (1 - 2\lambda) \int_{-1/2}^0 F(\epsilon)f(\epsilon + 1/2)d\epsilon$. Among them, $\lambda[1 - \int_{-1/2}^0 F(\epsilon)f(\epsilon + 1/2)d\epsilon]$ are type 0 consumers and $(1 - 2\lambda) \int_{-1/2}^0 F(\epsilon)f(\epsilon + 1/2)d\epsilon$ are mainstream consumers. Firm 2's total demand is $\lambda \int_{-1/2}^0 F(\epsilon)f(\epsilon + 1/2)d\epsilon + (1 - 2\lambda)[1 - \int_{-1/2}^0 F(\epsilon)f(\epsilon + 1/2)d\epsilon] + [1 - F(0)]\lambda$. Among them, $\lambda \int_{-1/2}^0 F(\epsilon)f(\epsilon + 1/2)d\epsilon$ are type 0 consumers, $(1 - 2\lambda)[1 - \int_{-1/2}^0 F(\epsilon)f(\epsilon + 1/2)d\epsilon]$ are type 1/2 consumers, and $[1 - F(0)]\lambda$ are type 1 consumers.

It is important to note that, even without inference motivation (i.e., in the case of behavioral advertising), competing firms have an incentive to choose a more polarized content strategy to increase differentiation and soften the competition, which increases the total demand. Therefore, it is, in some sense, harder for them to choose a *more* polarized content strategy (compared to the baseline level of differentiation) when the information environment shifts from behavioral targeting to contextual targeting. Nevertheless, the next section shows that the inference motivation becomes stronger with competition.

5.3.2 Media Firms' Ideological Positioning

For tractability, we make an additional assumption that $\epsilon \sim U[-1/2, 1/2]$, which simplifies the demand function characterized in Lemma 4. The following propositions summarize the equilibria under behavioral and contextual advertising.

Proposition 4. *Suppose $\epsilon \sim U[-1/2, 1/2]$. Under behavioral advertising, the equilibrium is $(1/2, 1/2)$ if $\lambda < \lambda_1$, $(1/2, 1/2)$ or $(0, 1)$ if $\lambda_1 < \lambda < \lambda_2$, and $(0, 1)$ if $\lambda > \lambda_2$, where $1/3 < \lambda_1 < \lambda_2 = 3/7$.*

Proposition 5. Suppose $\epsilon \sim U[-1/2, 1/2]$. The equilibria are (weakly) more polarized under contextual advertising than under behavioral advertising.⁹ The following table summarizes cases where the media firms choose more polarized content strategies under contextual advertising than under behavioral advertising, where $1/3 < \lambda_0 < \lambda_1$, $\gamma^d > 0$ for $\lambda < \lambda_2$, and $\gamma^{d'} > 0$ for $\lambda < \lambda_1$.

Condition	Equilibrium Under	
	Behavioral Advertising	Contextual Advertising
$1/3 < \lambda < \lambda_0$ & $\gamma^d < \gamma < \gamma^{d'}$	$(1/2, 1/2)$	$(0, 1/2)$
$\lambda_0 < \lambda < \lambda_1$ & $\gamma^{d'} < \gamma < \gamma^d$	$(1/2, 1/2)$	$(1/2, 1/2)$ or $(0, 1)$
$1/3 < \lambda < \lambda_1$ & $\gamma > \max\{\gamma^d, \gamma^{d'}\}$	$(1/2, 1/2)$	$(0, 1)$
$\lambda_1 < \lambda < \lambda_2$ & $\gamma > \gamma^d$	$(1/2, 1/2)$ or $(0, 1)$	$(0, 1)$

Figure 4 illustrates media firms' content strategies in duopoly under behavioral (left figure) and contextual (right figure) advertising. The content strategies are the same under both behavioral and contextual advertising in the blank region, while the extent of media polarization is larger under contextual advertising than under behavioral advertising in the solid and striped-shaded regions.

When $\lambda > \lambda_2$, there are enough niche consumers (at least 6/7 of the total) that $(0, 1)$ is the equilibrium in duopoly even under behavioral advertising. As such, there is no room for the duopoly to be more polarized if the advertising environment switches from behavioral advertising to contextual advertising. Now consider the interesting case in which $\lambda < \lambda_2$, so that at least 1/7 of consumers are mainstream. In both monopoly and duopoly, contextual advertising leads to more content polarization when $\gamma > \gamma^m$ (the striped shaded region in Figure 4b). In addition, contextual advertising leads to more polarization when $\min\{\gamma^d, \gamma^{d'}\} < \gamma < \gamma^m$ and $\lambda < \lambda_1$ or when $\gamma^d < \gamma < \gamma^m$ and $\lambda_1 < \lambda < \lambda_2$ (the solid shaded region in Figure 4b) under duopoly but not under monopoly. In particular, when $1/3 < \lambda < \lambda_1$ and $\gamma > \max\{\gamma^d, \gamma^{d'}\}$, equilibria across the two regimes display opposite properties: equilibrium under behavioral advertising features both firms choosing the mainstream ideological positioning, while equilibrium under contextual advertising features the two firms picking opposite niche ideological positions. Thus, privacy regulation leads firms to switch from “underproviding” to “overproviding” (compared to the underlying consumer

⁹The extent of media polarization increases from $(1/2, 1/2)$ to $(0, 1/2)$ to $(0, 1)$.

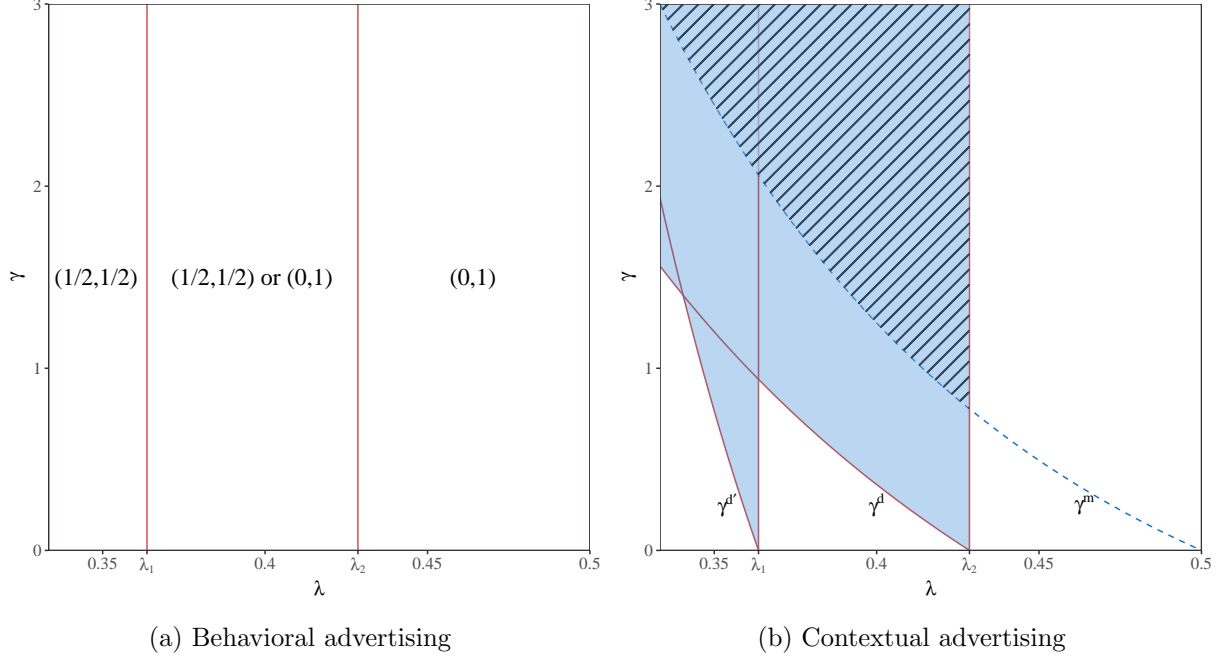


Figure 4: Media firms' equilibrium content strategies in duopoly when $\epsilon \sim U[-1/2, 1/2]$.

Note: The blank region shows areas where the equilibrium is the same under behavioral and contextual targeting in duopoly. The shaded area shows the total area where the equilibrium is more polarized under contextual targeting than behavioral targeting. The striped region shows the part of the shaded area where the equilibrium under contextual targeting is more polarizing than the equilibrium under behavioral targeting in monopoly too.

ideology distribution) highly partisan positions.

Therefore, counter to intuition, conditional on differentiation motives due to competition, duopolistic firms have a *stronger* incentive than a monopoly to polarize due to inference motives. In other words, compared to a monopolist, duopolistic firms choose a more polarized strategy under contextual rather than behavioral advertising in a wider range of parameters, as long as the competition is not strong enough that they already have chosen the most polarized strategy under behavioral targeting.

The underlying mechanism is the following. On the one hand, competing media firms directly cannibalize each other's demand if they choose the mainstream content strategy. In contrast, they can cover different niche consumers by choosing the opposite ideological positions and thereby soften the competition. As a result, the benefit of higher total demand in the market by choosing mainstream content strategy is lower in the duopoly case than in the monopoly case. On the other hand, the benefit of a more accurate inference is not affected by competition - a monopolistic firm

and a duopolistic firm extract the same amount of surplus in the advertising market for a given level of consumer privacy. In addition, by choosing niche ideological positioning, a duopolistic firm can obtain a more accurate inference than a monopoly due to an *inferential complementarity* effect. The presence of one firm aids the other firm's inference because competition improves the selection of consumers: on average, consumers of each firm are more ideologically aligned with that firm, compared to the monopoly case. For instance, in the $(0, 1)$ case, both firms lose relatively more demand from consumers of opposite ideologies, thus decreasing the variance in consumer types for each firm and increasing profits per impression in the contextual targeting regime.¹⁰ Therefore, firms lean more toward quality in the quantity-quality trade-off under a duopoly than under a monopoly.

The fact that an increase in privacy results in an increase in equilibrium content polarization is interesting in light of the well-discussed link between content personalization (that is, lack of privacy) and polarization. It is important to stress that, according to this view, personalized content does not *polarize* consumers; rather, it simply *matches* their prior ideological positions. In this case, one could say that content positioning is the *consequence*, rather than the *cause*, of polarization.

The drivers of Proposition 5 are quite different: the desire to make precise inferences to increase advertising profits pushes two competing firms away from the standard Hotelling equilibrium, $(1/2, 1/2)$, and toward $(0, 1)$ that is more ideologically polarized than the underlying distribution of the consumers' ideological preferences, for any value of $\lambda < 1/2$. Therefore, more privacy leads to more content polarization on the supply side. Content polarization is driven by firms' desire to monetize ads, not just by consumer ideology; in fact, content distribution often becomes more polarized than consumers.

As we have shown in the monopoly case, a more polarized content strategy may lead to lower consumer utility from content consumption. However, with two firms, polarization becomes desirable for consumers. The next proposition shows that consumer utility from content consumption

¹⁰This complementarity motive arises as long as at least one firm chooses niche content. When both firms choose mainstream strategies, $(1/2, 1/2)$, each firm's demand is simply cut in half, and their inferential ability is the same as a monopoly, as no additional consumer ideological self-selection arises.

always increases in the extent of polarization under duopoly.

Proposition 6. *For any $\lambda \in (1/3, 1/2)$ and $\epsilon \sim U[-1/2, 1/2]$, a more polarizing equilibrium in duopoly leads to a higher consumer utility from consumption: $(0, 1)$ leads to higher consumer utility from content consumption than $(0, 1/2)$, and $(0, 1/2)$ leads to higher consumer utility from content consumption than $(1/2, 1/2)$.*

This proposition helps us examine the implications of a privacy regulation that enforces a move from behavioral to contextual advertising. As shown in Proposition 5, the equilibrium under contextual advertising is at least as polarizing as the equilibrium under behavioral advertising for the entire parameter space. Thus, we can write the following corollary:

Corollary 3. *For any $\lambda \in (1/3, 1/2)$ and $\epsilon \sim U[-1/2, 1/2]$, privacy regulation leads to higher consumer utility from content consumption in a duopoly.*

This corollary shows that a privacy regulation in the duopoly context does not have unintended consequences in terms of the content consumption aspect of consumer welfare. This is different from our findings in the monopoly setting that shows the shift to greater content polarization under privacy regulation can reduce consumers' utility from content consumption. Intuitively, our findings in the duopoly setting indicate that consumers enjoy higher content polarization, which is expected given that we consider cases where consumers are polarized to begin with ($\lambda > 1/3$). In summary, we find that in a competitive environment, privacy regulation helps consumer receive higher utility from content consumption, although at the expense of an increase in content polarization.

5.4 Extension: General Base Utility

Throughout our exposition so far, the consumer's base utility from media consumption was fixed at $1/2$. This allows us to convey all the main insights about the inference role of content strategy and the quantity-quality trade-off while keeping the model simple. Nevertheless, it is interesting to study whether and how the incentives to opt for mainstream or niche positioning differ for firms of different quality. In this extension, we generalize the base utility to v , and show the robustness of

our findings. In addition, we demonstrate how the size of the base utility strengthens/weakens the firm's inference incentive.

Proposition 7. *Suppose $\epsilon \sim U[-1/2, 1/2]$. There exists $\underline{v} \leq 7/25, \bar{v} \geq 5/6, \gamma^m > 0, \gamma^{d''}, \lambda_1 < \lambda_2$ and $\bar{\lambda}(v) \in (1/3, 1/2]$ such that the following holds if $\underline{v} < v < \bar{v}$:*

- (a) *(Monopoly) The firm chooses mainstream content under behavioral advertising and niche content under contextual advertising if $1/3 < \lambda < \bar{\lambda}(v)$ and $\gamma > \gamma^m$. The threshold γ^m decreases in λ and increases in v . In addition, there exists $\hat{\lambda}(v) > 1/3$ such that niche content strategy leads to lower consumer welfare if $\lambda < \hat{\lambda}(v)$.*
- (b) *(Duopoly) The equilibrium under behavioral advertising is $(1/2, 1/2)$ if $\lambda < \lambda_1$, $(1/2, 1/2)$ or $(0, 1)$ if $\lambda_1 < \lambda < \lambda_2$, and $(0, 1)$ if $\lambda > \lambda_2$. The equilibrium under contextual advertising is more polarized than that under behavioral advertising if $\gamma > \gamma^{d''}$ and $1/3 < \lambda < \lambda_2$. Moreover, $\gamma^{d''} < \gamma^m$ for all $\lambda \in (1/3, \lambda_2)$.*

As we can see, for an interval range of the base utility that includes $1/2$, we recover all the main insights from Propositions 1 to 5 in the main model. It is still the case that the media firm's ideological positioning has an inference role for the firm under contextual advertising, that a media firm will choose more polarized content in equilibrium to increase profit per consumer despite the lower total demand and potentially lower consumer welfare, and that competition strengthens firms' inference incentive.

In the main model, a consumer located at one extreme will never consume the content located at the other extreme. In contrast, a consumer located at one extreme may consume the opposite content when the base utility v is larger than $1/2$. In particular, a type 0 consumer will obtain a positive utility from consuming $x = 1$ content with probability $\mathbf{P}(v - |x - \theta| + \epsilon > 0) = 1/3$ if $v = 5/6$. Therefore, our extension to the general base utility establishes the robustness of our results to settings where a consumer located at one extreme may consume content from the opposite extreme.

The general base utility setup provides additional insight into how the base utility affects the firm's inference incentive. The firm has a smaller incentive to improve its inference by choosing more

polarized ideological positioning as the base utility increases because γ^m increases in v . The base utility can be viewed as the vertical quality of the media firm, whereas the match between consumer type and media/ad location is horizontal. When the vertical quality is high, the media firm attracts consumers with ideological preferences far from the firm’s position. As a result, the horizontal match quality becomes less important and it becomes harder for the firm to distinguish different consumer types based on their self-selection. The firm is more incentivized to favor quantity in the quantity-quality trade-off by choosing mainstream content with a larger total demand.

In contrast, when the base utility is low, consumers are more selective in their consumption decisions. Mainstream consumers will not consume niche content that has a low vertical quality and poor horizontal match unless the random shock is very high (a large idiosyncratic term). So, by choosing the more polarized niche content strategy, the firm can screen consumer types very accurately and thus obtain a high profit per consumer by offering well-matched ads to most consumers. In this case, the firm has a stronger incentive to favor quality in the quantity-quality trade-off by choosing a niche content strategy.

In summary, our extension to the general base utility goes beyond an extension and shares important insights into the link between privacy and content polarization. In particular, we find that the impact of privacy regulation on content polarization is stronger when the base utility is lower. On the other hand, the link becomes weaker when the base utility is higher. Thus, policymakers can focus their attention on policies that help increase the vertical content quality.

6 Conclusion

The increased use of behavioral targeting by websites has heightened concerns by consumer privacy advocates and regulators. In this paper, we study the consequences of privacy regulations in the media landscape and ask the following question: how does a privacy regulation affect the equilibrium strategies of media firms in terms of ideological positioning and content polarization? We theoretically examine this question by building a model of product positioning and consumer demand. In our model, media firms choose their ideological positioning and create content, and consumers select their preferred content based on their ideology and idiosyncratic shocks. De-

pending on the eyeballs media firms generate, these firms extract advertising revenue based on the expected efficacy of their ads.

To examine the impact of privacy regulations, we focus on two primary information environments: behavioral and contextual ad targeting. Under *behavioral targeting*, media firms can perfectly track consumers and target their ads, reflecting the case without privacy regulation. In contrast, under *contextual targeting*, publishers cannot track readers and are therefore limited to tailoring ads to content, relying on the information contained in consumers' self-selection into the media firm's content. We show that banning behavioral targeting incentivizes publishers to shift towards niche (or polarizing) content strategies to improve their ability to draw inferences about consumers. In monopoly, this holds even when the increase in content polarization decreases total demand and consumer utility from content consumption. In duopoly, we document a stronger shift toward content polarization due to privacy regulation, though this shift does not reduce consumer utility from content consumption.

The findings of this study shed light on the complex connection between privacy and polarization. In recent years, many in the press have underscored the pivotal role played by increasingly precise personalization and behavioral targeting in the surge of political polarization within the United States Pariser (2011). This perspective posits that hyper-personalization due to the lack of privacy regulation led to the creation of echo chambers and increased polarization. However, our research presents an alternative reversed link between privacy and polarization. In our model, the inability to personalize content compels firms to use their content strategy as a targeting strategy that relies on consumers' self-selection into content. This strategic shift creates a unique incentive for producing more partisan content in both monopoly and duopoly scenarios.

Our paper contributes to the literature in several ways. First, we identify a counter-intuitive and theoretically robust link between privacy and polarization. We show that a privacy regulation leads to greater content polarization in equilibrium when media firms monetize based on advertising, even when content polarization reduces total demand and consumer utility from content consumption. Therefore, our paper identifies the firm's inference motive as another driver of content polarization beyond consumer demand and highlights the unintended consequences of privacy regulations in

media markets. Second, we bring a novel inference-based framework to study the equilibrium effects of privacy regulation. In particular, our framework suggests that by limiting what firms *know* about consumers, privacy regulations shift firms' focus to what they *want to know* about consumers, thereby creating an incentive to use content strategies as means for more accurate inference. Lastly, our paper offers important implications for managers and policymakers. In particular, we present the equilibrium effect of privacy regulations in media markets as a function of interpretable model parameters. We show that privacy regulations can have an unintended consequence of increased content polarization. Further, we propose that investments that increase content quality can mitigate the negative effects of privacy regulations on content polarization.

Nevertheless, our paper has certain limitations that serve as great avenues for future research. First, we focused on discrete consumer types for tractability and interpretability reasons. Future research can extend our framework to a continuous setting. Second, in order to highlight the media firm's problem, we did not explicitly model the advertising marketplace. One could endogenize the advertising marketplace and obtain new insights for the advertising side of the market. Finally, our paper abstracted away from the dynamics between content polarization and opinion polarization. Future research can incorporate this dynamic and examine the long-run equilibrium.

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Appendix

Proof of Lemma 1. Let p_j denote the posterior probability of $\theta = j$ conditional on the information \mathcal{I} , that is, $p_j = P(\theta = j \mid \mathcal{I})$. Therefore, we can write:

$$\mathbf{E}[\theta \mid \mathcal{I}] = p_0 \times 0 + p_{1/2} \times \frac{1}{2} + p_1 \times 1 = \frac{p_{1/2} + 2p_1}{2}$$

Now, we write the optimal ad choice as follows:

$$\begin{aligned} \max_a \mathbf{E}[\mathcal{M}(a; \theta) \mid \mathcal{I}] &= \max_a [1 - \gamma \mathbf{E}[(a - \theta)^2 \mid \mathcal{I}]] \\ &= \max_a \left[1 - \gamma \left(p_0 a^2 + p_{1/2} \left(a - \frac{1}{2} \right)^2 + p_1 (a - 1)^2 \right) \right] \end{aligned}$$

We can now write the first-order condition as follows:

$$\begin{aligned} \frac{\partial \mathbf{E}[\mathcal{M}(a; \theta) \mid \mathcal{I}]}{\partial a} &= -\gamma \left(2p_0 a + 2p_{1/2} \left(a - \frac{1}{2} \right) + 2p_1 (a - 1) \right) \\ &= -\gamma (a(2p_0 + 2p_{1/2} + 2p_1) - p_{1/2} - 2p_1) \\ &= -\gamma (2a - p_{1/2} - 2p_1) \\ &= 0 \\ \Rightarrow a^* &= \frac{p_{1/2} + 2p_1}{2} = \mathbf{E}[\theta \mid \mathcal{I}] \end{aligned}$$

■

Proof of Corollary 1. The media firm's profit maximization problem is the following:

$$\begin{aligned} \pi(x, a; \mathcal{I}) &= \max_{x, a} D(x) \cdot \mathbf{E}[\mathcal{M}(a; \theta) \mid \mathcal{I}] \\ &= \max_x D(x) \cdot \mathbf{E}[\mathcal{M}(a^*; \theta) \mid \mathcal{I}] \\ &= \max_x D(x) \cdot \mathbf{E}[1 - \gamma(a^* - \theta)^2 \mid \mathcal{I}] \\ &= \max_x D(x) \cdot (1 - \gamma \mathbf{E}[(a^* - \theta)^2 \mid \mathcal{I}]) \end{aligned}$$

$$\begin{aligned}
&= \max_x D(x) \cdot (1 - \gamma \mathbf{E}[(\mathbf{E}[\theta | \mathcal{I}] - \theta)^2 | \mathcal{I}]) \\
&= \max_x D(x) \cdot [1 - \gamma \mathbf{Var}(\theta | \mathcal{I})] \\
&= \max_x D(x) \cdot [1 - \gamma \text{Privacy}(\mathcal{I})]
\end{aligned}$$

■

Proof of Lemma 2. One can see that consumers whose type matches exactly the media firm's location will consume for sure. So, we just need to determine whether other consumers prefer consuming the content to the outside option.

1. Mainstream content strategy $x = 1/2$

The demand from mainstream consumers is $1 - 2\lambda$.

Consider a type 0 consumer. Her utility from consuming the content is $U(1/2; 0) = \epsilon$. She will consume the content if and only if her consumption utility is greater than that of the outside option, i.e., $\epsilon > 0$. Therefore, the consumption probability is $P(\epsilon > 0) = 1 - F(0)$. Hence, the demand from type 0 consumers is $\lambda[1 - F(0)]$. By symmetry, the demand from type 1 consumers is also $\lambda[1 - F(0)]$.

In sum, the total demand is $1 - 2\lambda + 2\lambda[1 - F(0)] = 1 - 2F(0)\lambda$.

2. Niche content strategy $x = 0$

The demand from type 0 consumers is λ and the demand from type 1 consumers is 0.

Consider a type 1/2 consumer. Her utility from consuming the content is $U(0, 1/2) = \epsilon$. She will consume the content if and only if $\epsilon > 0$, her utility from the outside option. By the same argument in the previous case, one can see that the demand from type 1/2 consumers is $(1 - 2\lambda)[1 - F(0)]$.

In sum, the total demand is $\lambda + (1 - 2\lambda)[1 - F(0)] = 1 - F(0) + [2F(0) - 1]\lambda$.

3. Niche content strategy $x = 1$

It is symmetric to the previous $x = 0$ case.

■

Proof of Proposition 1. Under behavioral advertising, the match between the ad and the consumer type is always 1. Therefore, the media firm chooses the content strategy that leads to the highest demand. According to Lemma 2, a mainstream positioning generates a higher demand than niche content does if and only if $1 - 2F(0)\lambda > 1 - F(0) + [2F(0) - 1]\lambda \Leftrightarrow [4F(0) - 1]\lambda > F(0)$. ■

Proof of Lemma 3. Due to the symmetry between the two types of niche content, we only need to compare $\text{Privacy}(\{0\})$ and $\text{Privacy}(\{1/2\})$.

1. Mainstream content $x = 1/2$

According to Lemma 2, the demand from type 0 and from type 1 consumers are identical.

So, $\mathbf{E}[\theta|x = 1/2] = 1/2$.

$$\begin{aligned} \text{Privacy}(\{1/2\}) &= \mathbf{Var}[\theta|x = 1/2] \\ &= \mathbf{E}[(\theta - 1/2)^2|x = 1/2] \\ &= 2 \cdot \frac{\lambda[1 - F(0)]}{1 - 2\lambda F(0)} \cdot (1/2)^2 \\ &= \frac{1}{2} \cdot \frac{\lambda[1 - F(0)]}{1 - 2\lambda F(0)} \end{aligned}$$

2. Niche content $x = 0$

$$\begin{aligned} \text{Privacy}(\{0\}) &= \mathbf{Var}[\theta|x = 0] \\ &= \mathbf{E}[\theta^2|x = 0] - \mathbf{E}[\theta|x = 0]^2 \\ &= \frac{[1 - F(0)](1 - 2\lambda)}{\lambda + [1 - F(0)](1 - 2\lambda)} \cdot \left(\frac{1}{2}\right)^2 - \left\{ \frac{[1 - F(0)](1 - 2\lambda)}{1 - F(0) + [2F(0) - 1]\lambda} \cdot \frac{1}{2} \right\}^2 \\ &= \frac{1}{4} \cdot \frac{[1 - F(0)](1 - 2\lambda)}{\lambda + [1 - F(0)](1 - 2\lambda)} - \frac{1}{4} \cdot \frac{[1 - F(0)]^2(1 - 2\lambda)^2}{\{\lambda + [1 - F(0)](1 - 2\lambda)\}^2} \\ &= \frac{1}{4} \cdot \frac{[1 - F(0)](1 - 2\lambda)\lambda}{\{\lambda + [1 - F(0)](1 - 2\lambda)\}^2} \end{aligned}$$

$$\begin{aligned}
& \text{Privacy}(\{0\}) < \text{Privacy}(\{1/2\}) \\
& \Leftrightarrow \frac{1}{4} \cdot \frac{[1 - F(0)](1 - 2\lambda)\lambda}{\{\lambda + [1 - F(0)](1 - 2\lambda)\}^2} < \frac{1}{2} \cdot \frac{\lambda[1 - F(0)]}{1 - 2\lambda F(0)} \\
& \Leftrightarrow (1 - 2\lambda)[1 - 2F(0)\lambda] < 2\{\lambda + [1 - F(0)](1 - 2\lambda)\}^2 \\
& \Leftrightarrow \lambda + \lambda(1 - 2\lambda)F(0) + 2\lambda(1 - 2\lambda)[1 - F(0)] + \lambda^2 + [1 - F(0)]^2(1 - 2\lambda)^2 > \frac{1}{2}
\end{aligned}$$

The LHS is greater than $\lambda + \lambda(1 - 2\lambda)F(0) + \lambda(1 - 2\lambda)[1 - F(0)] + \lambda^2 = \lambda(2 - \lambda)$. Since $\lambda \in (1/3, 1/2)$ and $\lambda(2 - \lambda)$ increases in λ for $\lambda < 1$, we have the LHS $> \lambda(2 - \lambda) > 1/3(2 - 1/3) = 5/9 > 1/2$. Therefore, $\text{Privacy}(\{0\}) < \text{Privacy}(\{1/2\})$ always holds. By definition, niche content is privacy-reducing over mainstream content. \blacksquare

Proof of Proposition 2. Consider contextual advertising. By symmetry, we only need to compare the firm's expected profits from type-0 niche content strategy and mainstream content strategy.

$$\begin{aligned}
\pi^c(0) > \pi^c(1/2) & \Leftrightarrow D(0) \cdot [1 - \gamma \mathbf{Var}(\theta|x=0)] > D(1/2) \cdot [1 - \gamma \mathbf{Var}(\theta|x=1/2)] \\
& \Leftrightarrow \gamma > \frac{\frac{D(1/2)}{D(0)} - 1}{\mathbf{Var}(\theta|x=1/2) \frac{D(1/2)}{D(0)} - \mathbf{Var}(\theta|x=0)}
\end{aligned}$$

Denote $\frac{\frac{D(1/2)}{D(0)} - 1}{\mathbf{Var}(\theta|x=1/2) \frac{D(1/2)}{D(0)} - \mathbf{Var}(\theta|x=0)}$ by γ^m . Since $D(1/2) > D(0)$ and $\mathbf{Var}[\theta|x=0] < \mathbf{Var}[\theta|x=1/2]$, we have $\gamma^m > 0$. So, the media firm chooses niche content strategy if $\gamma > \gamma^m$ and mainstream content strategy if $\gamma < \gamma^m$ under contextual advertising.

Lastly, we show that $\gamma^m = \frac{\frac{D(1/2)}{D(0)} - 1}{\mathbf{Var}(\theta|x=1/2) \frac{D(1/2)}{D(0)} - \mathbf{Var}(\theta|x=0)} = \frac{4\{F(0) + [1 - 4F(0)]\lambda\}\{1 - F(0) + [2F(0) - 1]\lambda\}}{[1 - F(0)]\lambda[1 - 2F(0) + 4F(0)\lambda]}$ decreases in λ .

(1) $F(0) < 1/4$

In this case, $4\{1 - F(0) + [2F(0) - 1]\lambda\}$ in the numerator decreases in λ while $[1 - F(0)][1 - 2F(0) + 4F(0)\lambda]$ in the denominator increases in λ . Therefore, we only need to show that the remaining part of γ^m , $\frac{F(0) + [1 - 4F(0)]\lambda}{\lambda} = \frac{F(0)}{\lambda} + 1 - 4F(0)$, decreases in λ , which is straightforward.

(2) $1/4 \leq F(0) \leq 1/2$

In this case, both $F(0) + [1 - 4F(0)]\lambda$ and $1 - F(0) + [2F(0) - 1]\lambda$ decreases in λ . So, the numerator of γ^m decreases in λ . One can see that the denominator of γ^m increases in λ . Therefore, γ^m decreases in λ .

(3) $F(0) > 1/2$

In this case, $4\{F(0) + [1 - 4F(0)]\lambda\}$ in the numerator decreases in λ while $[1 - F(0)][1 - 2F(0) + 4F(0)\lambda]$ in the denominator increases in λ . Therefore, we only need to show that the remaining part of γ^m , $\frac{1-F(0)+[2F(0)-1]\lambda}{\lambda} = \frac{1-F(0)}{\lambda} + 2F(0) - 1$, decreases in λ , which is straightforward. ■

Proof of Proposition 3. We first compute the consumer welfare for each content strategy.

1. Mainstream content strategy $x = 1/2$

The consumer welfare is $(1 - 2\lambda)[1/2 + \mathbf{E}[\epsilon]] + 2[1 - F(0)]\lambda\mathbf{E}[\epsilon|\epsilon > 0] = 1/2 - \lambda + 2[1 - F(0)]\lambda\mathbf{E}[\epsilon|\epsilon > 0]$.

2. Niche content strategy $x = 0$

The consumer welfare is $\lambda[1/2 + \mathbf{E}[\epsilon]] + [1 - F(0)](1 - 2\lambda)\mathbf{E}[\epsilon|\epsilon > 0] = \lambda/2 + [1 - F(0)](1 - 2\lambda)\mathbf{E}[\epsilon|\epsilon > 0]$.

3. Niche content strategy $x = 1$

It is symmetric to the previous $x = 0$ case.

Niche content strategy leads to lower consumer welfare if and only if

$$\begin{aligned} \lambda/2 + [1 - F(0)](1 - 2\lambda)\mathbf{E}[\epsilon|\epsilon > 0] &< 1/2 - \lambda + 2[1 - F(0)]\lambda\mathbf{E}[\epsilon|\epsilon > 0] \\ \Leftrightarrow \lambda < \hat{\lambda} &= \frac{1 - 2[1 - F(0)]\mathbf{E}[\epsilon|\epsilon > 0]}{3 - 8[1 - F(0)]\mathbf{E}[\epsilon|\epsilon > 0]} \end{aligned}$$

In addition, $\hat{\lambda} > 1/3 \Leftrightarrow 2[1 - F(0)]\mathbf{E}[\epsilon|\epsilon > 0] > 0$, which holds for any distribution of ϵ . ■

Proof of Lemma 4. There are three candidate equilibria: $(0, 1/2)$, $(1/2, 1/2)$, $(0, 1)$.

1. (0,1)

The demands for type 0 and type 1 content are symmetric. So, we only need to examine type 0 content. Each consumer will choose type 0 content if and only if her utility from type 0 content is positive (higher than the outside option) and higher than her utility from type 1 content.

One can see that all type 0 consumers will choose type 0 content. Now consider type 1/2 consumers. A mainstream consumer will consume one of the contents if and only if her utility from at least one content is positive. By symmetry, her overall probabilities of choosing type 0 and type 1 content are identical.

$$\begin{aligned}
 & P(\text{a mainstream consumer chooses type 0 content}) \\
 &= \frac{1}{2} \cdot P(\max\{\epsilon_0, \epsilon_1\} > 0) \\
 &\stackrel{\text{independence of } \epsilon_j}{=} \frac{1}{2} \cdot [1 - P(\epsilon_0 \leq 0)P(\epsilon_1 \leq 0)] \\
 &= \frac{1}{2} \cdot [1 - F(0)^2]
 \end{aligned}$$

In sum, the demand of type 0 content from type 0 consumers is λ and from type 1/2 consumers is $\frac{1-F(0)^2}{2}(1-2\lambda)$. The total demand of type 0 content is $\lambda + \frac{1-F(0)^2}{2}(1-2\lambda)$.

2. (1/2, 1/2)

One can see that all mainstream consumers will consume the content by one of the media firms. The demands from type 0 and type 1 consumers are symmetric. Consider type 0 consumers. She will consume the content by one of the firms if and only if her utility from at least one content is positive. By symmetry, her overall probabilities of choosing either

mainstream content are identical.

$$\begin{aligned}
& P(\text{a type 0 consumer chooses firm } i) \\
&= \frac{1}{2} \cdot P(\max\{\epsilon_{1/2}, \epsilon_{1/2'}\} > 0) \\
&\stackrel{\text{independence of } \epsilon_j}{=} \frac{1}{2} \cdot [1 - P(\epsilon_{1/2} \leq 0)P(\epsilon_{1/2'} \leq 0)] \\
&= \frac{1 - F(0)^2}{2}
\end{aligned}$$

In sum, the demand of either firm from mainstream consumers is $\frac{1-2\lambda}{2}$, from type 0 consumers is $\frac{1-F(0)^2}{2}\lambda$, and from type 1 consumers is $\frac{1-F(0)^2}{2}\lambda$. The total demand of either firm is $\frac{1-2\lambda}{2} + [1 - F(0)^2]\lambda$.

3. (0, 1/2)

Consider first the demand for type 0 content. Since a type 0 consumer's utility from consuming type 0 content is always positive, we only need to compare her utility from type 0 and type 1/2 contents.

$$\begin{aligned}
& P(\text{a type 0 consumer chooses type 0 content}) \\
&= P(1/2 + \epsilon_0 > 0 + \epsilon_{1/2}) \\
&= P(\epsilon_{1/2} < \epsilon_0 + 1/2) \\
&= 1 - F(0) + \int_{-1/2}^0 \int_{-1/2}^{\epsilon_0+1/2} f(\epsilon_{1/2})d\epsilon_{1/2}f(\epsilon_0)d\epsilon_0 \\
&= 1 - F(0) + \int_{-1/2}^0 F(\epsilon_0 + 1/2)f(\epsilon_0)d\epsilon_0 \\
&\stackrel{\text{integral by parts}}{=} 1 - \int_{-1/2}^0 F(\epsilon_0)f(\epsilon_0 + 1/2)d\epsilon_0
\end{aligned}$$

A type 1/2 consumer will choose type 0 content if and only if her utility of consuming it is positive and higher than her utility of consuming mainstream content. Since her utility of consuming mainstream content is always positive, we only need the condition that her utility

of consuming type 0 content is higher than her utility of consuming mainstream content.

$$\begin{aligned}
& P(\text{a type } 1/2 \text{ consumer chooses type 0 content}) \\
&= P(0 + \epsilon_0 > 1/2 + \epsilon_{1/2}) \\
&= P(\epsilon_0 > \epsilon_{1/2} + 1/2) \\
&\stackrel{\text{symmetry}}{=} P(\epsilon_{1/2} > \epsilon_0 + 1/2) \\
&= 1 - P(\epsilon_{1/2} < \epsilon_0 + 1/2) \\
&\stackrel{\text{previous case}}{=} 1 - [1 - \int_{-1/2}^0 F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0] \\
&= \int_{-1/2}^0 F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0
\end{aligned}$$

Therefore, the demand of type 0 content from type 0 consumers is $\lambda[1 - \int_{-1/2}^0 F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0]$ and from type 1/2 consumers is $(1 - 2\lambda) \int_{-1/2}^0 F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0$. The total demand of type 0 content is $\lambda[1 - \int_{-1/2}^0 F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0] + (1 - 2\lambda) \int_{-1/2}^0 F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0$.

Consider now the demand for type 1/2 content.

$$\begin{aligned}
& P(\text{a type 0 consumer chooses type } 1/2 \text{ content}) \\
&= 1 - P(\text{a type 0 consumer chooses type 0 content}) \\
&= \int_{-1/2}^0 F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0
\end{aligned}$$

$$\begin{aligned}
& P(\text{a type } 1/2 \text{ consumer chooses type } 1/2 \text{ content}) \\
&= 1 - P(\text{a type } 1/2 \text{ consumer chooses type 0 content}) \\
&= 1 - \int_{-1/2}^0 F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0
\end{aligned}$$

$$\begin{aligned}
& P(\text{a type 1 consumer chooses type 1/2 content}) \\
&= P(U(1/2, 1) > 0) \\
&= P(\epsilon_{1/2} > 0) \\
&= 1 - F(0)
\end{aligned}$$

Therefore, the demand of type 1/2 content from type 0 consumers is $\lambda \int_{-1/2}^0 F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0$, from type 1/2 consumers is $(1 - 2\lambda)[1 - \int_{-1/2}^0 F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0]$, and from type 1 consumers is $[1 - F(0)]\lambda$. The total demand of type 1/2 content is $\lambda \int_{-1/2}^0 F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0 + (1 - 2\lambda)[1 - \int_{-1/2}^0 F(\epsilon_0) f(\epsilon_0 + 1/2) d\epsilon_0] + [1 - F(0)]\lambda$.

■

Proof of Proposition 4. We first simplify the demand in Lemma 4 when $\epsilon \sim U[-1/2, 1/2]$.

Corollary 4. *Suppose $\epsilon \sim U[-1/2, 1/2]$. The demand under each possible equilibrium is characterized as follows:*

1. *Under (0,1) strategy profile, Firm 1's total demand is $3/8 + \lambda/4$. Among them, λ are type 0 consumers and $3/8 - 3\lambda/4$ are mainstream consumers. Firm 2's total demand is $3/8 + \lambda/4$. Among them, λ are type 1 consumers and $3/8 - 3\lambda/4$ are mainstream consumers.*
2. *Under (1/2,1/2) strategy profile, each firm's total demand is $1/2 - \lambda/4$. Among them, $1/2 - \lambda$ are mainstream consumers, $3\lambda/8$ are type 0 consumers, and $3\lambda/8$ are type 1 consumers.*
3. *Under (0,1/2) strategy profile, Firm 1's total demand is $1/8 + 5\lambda/8$. Among them, $7\lambda/8$ are type 0 consumers and $(1 - 2\lambda)/8$ are mainstream consumers. Firm 2's total demand is $7/8 - 9\lambda/8$. Among them, $\lambda/8$ are type 0 consumers, $\frac{7}{8}(1 - 2\lambda)$ are type 1/2 consumers, and $\lambda/2$ are type 1 consumers.*

Now consider the equilibrium strategy under behavioral advertising. Under behavioral advertising, the advertising revenue per consumer is 1. So, each firm's profit equals its total demand.

Suppose the equilibrium is $(0, 1/2)$. Firm 1 will deviate from $x = 0$ to $x = 1/2$ if and only if:

$$\begin{aligned} 1/2 - \lambda/4 &> 1/8 + 5\lambda/8 \\ \Leftrightarrow \lambda &< \lambda_2 = 3/7 \end{aligned} \tag{6}$$

Firm 2 will deviate from $x = 1/2$ to $x = 1$ if and only if:

$$\begin{aligned} 3/8 + \lambda/4 &> 7/8 - 9\lambda/8 \\ \Leftrightarrow \lambda &> \lambda_1 = 4/11 \end{aligned} \tag{7}$$

From the above conditions and noting that $\lambda_1 < \lambda_2$, one can see that $(0, 1/2)$ will never be an equilibrium.

Now suppose the equilibrium is $(1/2, 1/2)$. Equation 6 implies that the firm will deviate to 0 or 1 if $\lambda > \lambda_2$. Similarly, if the equilibrium is $(0, 1)$, Equation 7 implies that the firm will deviate to $1/2$ if $\lambda < \lambda_1$.

In sum, the equilibrium is $(1/2, 1/2)$ if $\lambda < \lambda_1$, $(1/2, 1/2)$ or $(0, 1)$ if $\lambda_1 < \lambda < \lambda_2$, and $(0, 1)$ if $\lambda > \lambda_2$. ■

Proof of Proposition 5. We first characterize the equilibrium ad choices and profits for each firm under contextual advertising.

1. $(1/2, 1/2)$

By symmetry, $a = \mathbf{E}(\theta) = 1/2$. $\mathbf{Var}(\theta) = \mathbf{E}[\theta - \mathbf{E}(\theta)]^2 = \frac{3\lambda}{4(2-\lambda)}$.

$$\begin{aligned} \pi(1/2, 1/2) &= D(1/2, 1/2) \cdot [1 - \gamma \mathbf{Var}(\theta)] \\ &= \frac{2 - \lambda}{4} - \frac{3\gamma\lambda}{16} \end{aligned}$$

2. $(0, 1)$

By symmetry, we only need to consider firm 1. Firm 2's ad choice will be $1 - a_1$, and firm 2's profit will be identical to firm 1's.

$$a_1 = \mathbf{E}(\theta) = \frac{3/8 - 3\lambda/4}{\lambda/4 + 3/8} \cdot \frac{1}{2} = \frac{3(1-2\lambda)}{2(3+2\lambda)}. \quad \mathbf{Var}(\theta) = \mathbf{E}[\theta - \mathbf{E}(\theta)]^2 = \frac{6\lambda(1-2\lambda)}{(3+2\lambda)^2}.$$

$$\begin{aligned} \pi(0, 1) &= D(0, 1) \cdot [1 - \gamma \mathbf{Var}(\theta)] \\ &= \frac{2\lambda + 3}{8} - \frac{3\gamma\lambda(1-2\lambda)}{4(2\lambda + 3)} \end{aligned}$$

3. (0, 1/2)

$$\text{Consider firm 1 first. } a_1 = \mathbf{E}_1(\theta) = \frac{1-2\lambda}{2(1+5\lambda)}. \quad \mathbf{Var}_1(\theta) = \mathbf{E}_1[\theta - \mathbf{E}_1(\theta)]^2 = \frac{7\lambda(1-2\lambda)}{4(1+5\lambda)^2}.$$

$$\begin{aligned} \pi_1(0, 1/2) &= D_1(0, 1/2) \cdot [1 - \gamma \mathbf{Var}_1(\theta)] \\ &= \frac{1 + 5\lambda}{8} - \frac{7\gamma\lambda(1-2\lambda)}{32(1+5\lambda)} \end{aligned}$$

$$\text{Consider firm 2 then. } a_2 = \mathbf{E}_2(\theta) = \frac{7-6\lambda}{2(7-9\lambda)}. \quad \mathbf{Var}_2(\theta) = \mathbf{E}_2[\theta - \mathbf{E}_2(\theta)]^2 = \frac{35\lambda - 54\lambda^2}{4(7-9\lambda)^2}.$$

$$\begin{aligned} \pi_2(0, 1/2) &= D_2(0, 1/2) \cdot [1 - \gamma \mathbf{Var}_2(\theta)] \\ &= \frac{7 - 9\lambda}{8} - \frac{\gamma(35\lambda - 54\lambda^2)}{32(7 - 9\lambda)} \end{aligned}$$

Suppose the equilibrium is (0, 1/2). Firm 1 will deviate from $x = 0$ to $x = 1/2$ if and only if:

$$\begin{aligned} \pi(1/2, 1/2) &> \pi_1(0, 1/2) \\ \Leftrightarrow \gamma < \gamma^d &= (\lambda_2 - \lambda) \frac{28(1 + 5\lambda)}{\lambda(44\lambda - 1)} \end{aligned} \tag{8}$$

An immediate implication is that this deviation never happens if $\lambda \geq \lambda_2$.

Firm 2 will deviate from $x = 1/2$ to $x = 1$ if and only if:¹¹

$$\begin{aligned} \pi(0, 1) &> \pi_2(0, 1/2) \\ \Leftrightarrow \gamma > \gamma^{d'} &= (\lambda_1 - \lambda) \frac{44(2\lambda + 3)(7 - 9\lambda)}{\lambda(-540\lambda^2 + 460\lambda - 63)} \end{aligned} \tag{9}$$

¹¹Firm 2 has a stronger incentive to deviate to $x = 1$ rather than $x = 0$ to soften competition. So, we only need to consider its deviation to $x = 1$.

An immediate implication is that deviation always happens if $\lambda \geq \lambda_1$.¹² Therefore, $(0,1/2)$ may only be an equilibrium if $\lambda < \lambda_1$. Now suppose $\lambda < \lambda_1$. In this case, we have shown that firm 1 will deviate if $\gamma < \gamma^d$ and firm 2 will deviate if $\gamma > \gamma^{d'}$. Some calculation yields that there exists a unique $\lambda_0 \in (1/3, \lambda_1)$ such that $\gamma^d < (>) \gamma^{d'}$ if $\lambda < (>) \lambda_0$. Therefore, at least one deviation happens and $(0,1/2)$ will not be an equilibrium if $\lambda > \lambda_0$. There is no deviation and $(0,1/2)$ is an equilibrium if $\lambda < \lambda_0$ and $\gamma^d < \gamma < \gamma^{d'}$.

Now suppose the equilibrium is $(1/2,1/2)$. Equation 8 implies that the firm will deviate to 0 or 1 if $\gamma > \gamma^d$, which may hold if $\lambda < \lambda_2$ and always holds if $\lambda > \lambda_2$. Similarly, if the equilibrium is $(0,1)$, Equation 9 implies that the firm will deviate to $1/2$ if $\gamma < \gamma^{d'}$, which may hold if $\lambda < \lambda_1$ and never holds if $\lambda > \lambda_1$.

In sum, $(0,1)$ is an equilibrium if $\lambda > \lambda_1$ or if $\lambda < \lambda_1$ & $\gamma > \gamma^{d'}$. $(1/2,1/2)$ is an equilibrium if $\lambda < \lambda_2$ & $\gamma < \gamma^d$.

We summarize the equilibria under behavioral and contextual advertising in the following table.

Condition	Equilibrium Under	
	Behavioral Advertising	Contextual Advertising
$1/3 < \lambda < \lambda_1$ & $\gamma < \min\{\gamma^d, \gamma^{d'}\}$	$(1/2,1/2)$	$(1/2,1/2)$
$1/3 < \lambda < \lambda_0$ & $\gamma^d < \gamma < \gamma^{d'}$	$(1/2,1/2)$	$(0,1/2)$
$\lambda_0 < \lambda < \lambda_1$ & $\gamma^{d'} < \gamma < \gamma^d$	$(1/2,1/2)$	$(1/2,1/2)$ or $(0,1)$
$1/3 < \lambda < \lambda_1$ & $\gamma > \max\{\gamma^d, \gamma^{d'}\}$	$(1/2,1/2)$	$(0,1)$
$\lambda_0 < \lambda_1 < \lambda < \lambda_2$ & $\gamma < \gamma^d$	$(1/2,1/2)$ or $(0,1)$	$(1/2,1/2)$ or $(0,1)$
$\lambda_1 < \lambda < \lambda_2$ & $\gamma > \gamma^d$	$(1/2,1/2)$ or $(0,1)$	$(0,1)$
$\lambda > \lambda_2$	$(0,1)$	$(0,1)$

¹² $\frac{44(2\lambda+3)(7-9\lambda)}{\lambda(-540\lambda^2+460\lambda-63)} > 0$ for $\lambda \in (1/3, 1/2)$.

Online Appendix for Privacy and Polarization: An Inference-Based Framework

May 2024

Proof of Proposition 6. We first compute the consumer utility from content consumption for each possible equilibrium strategy profile.

1. (0,1)

The consumer utility from consuming each firm's content is:

$$\begin{aligned} & \lambda(1/2 + \mathbf{E}[\epsilon_0]) + (3/8 - 3\lambda/4)\mathbf{E}[\epsilon_0 | \epsilon_0 > 0 \text{ and } \epsilon_0 > \epsilon_1]) \\ &= \lambda/2 + (3/8 - 3\lambda/4) \cdot 5/18 \\ &= \frac{5 - \lambda}{18} \end{aligned}$$

Hence, the total consumer utility from content consumption is $2(5 - \lambda)/18 = (5 - \lambda)/9$.

2. (1/2,1/2)

The consumer utility from consuming each firm's content is:

$$\begin{aligned} & \frac{1 - 2\lambda}{2}(1/2 + \mathbf{E}[\epsilon_{1/2}]) + 2\left(\frac{1 - F(0)^2}{2}\right)\lambda\mathbf{E}[\epsilon_{1/2} | \epsilon_{1/2} > 0 \text{ and } \epsilon_{1/2} > \epsilon_{1/2'}]) \\ &= \frac{1 - 2\lambda}{4} + \frac{3\lambda}{4} \frac{5}{18} \\ &= \frac{6 - 7\lambda}{24} \end{aligned}$$

Hence, the total consumer utility from content consumption is $2(6 - 7\lambda)/24 = (6 - 7\lambda)/12$.

3. (0,1/2)

The consumer utility from consuming firm 1's content is:

$$\begin{aligned}
& \frac{7\lambda}{8}(1/2 + \mathbf{E}[\epsilon_0 | \epsilon_0 + 1/2 > \epsilon_{1/2}]) + \frac{1-2\lambda}{8}\mathbf{E}[\epsilon_0 | \epsilon_0 > 0 \text{ and } \epsilon_0 > 1/2 + \epsilon_{1/2}] \\
&= \frac{7\lambda}{8} \cdot \left(\frac{1}{2} + \frac{1}{21}\right) + \frac{1-2\lambda}{8} \cdot \frac{1}{3} \\
&= \frac{2+19\lambda}{48}
\end{aligned}$$

The consumer utility from consuming firm 2's content is:

$$\begin{aligned}
& \frac{\lambda}{8}\mathbf{E}[\epsilon_{1/2} | \epsilon_{1/2} > 0 \text{ and } \epsilon_{1/2} > 1/2 + \epsilon_0] + \frac{7(1-2\lambda)}{8}(1/2 + \mathbf{E}[\epsilon_{1/2} | \epsilon_{1/2} > 1/2 + \epsilon_0]) + \\
& \frac{\lambda}{2}\mathbf{E}[\epsilon_{1/2} | \epsilon_{1/2} > 0] \\
&= \frac{\lambda}{8} \cdot \frac{1}{3} + \frac{7(1-2\lambda)}{8}(1/2 + 1/3) + \frac{\lambda}{2} \cdot \frac{1}{4} \\
&= \frac{35-62\lambda}{48}
\end{aligned}$$

Hence, the total consumer utility from content consumption is $\frac{2+19\lambda}{48} + \frac{35-62\lambda}{48} = \frac{37-43\lambda}{48}$.

The total consumer utility from content consumption under (0,1) strategy profile is higher than that under (0,1/2) strategy profile if and only if $2 \cdot \frac{5-\lambda}{18} > \frac{37-43\lambda}{48} \Leftrightarrow \lambda > 31/113$, which always holds. The total consumer utility from content consumption under (0,1/2) strategy profile is higher than that under (1/2,1/2) strategy profile if and only if $\frac{37-43\lambda}{48} > 2 \cdot \frac{6-7\lambda}{24} \Leftrightarrow \lambda < 13/15$, which always holds. ■

Proof of Proposition 7. We consider two cases:

1. $v < 1/2$:

In this case, the consumer may not consume the content even if the content location perfectly match the consumer type. In addition, the consumer never consumes the content if the content location is on the opposite end of the consumer's location.

(a) Monopoly

We first summarize the demand for a monopoly by choosing a particular content.

Suppose the monopoly chooses niche content strategy $x = 0$.¹ One can show that the demand from type 0 consumer is $\lambda(1/2 + v)$, from type 1/2 consumer is $(1 - 2\lambda)v$, and from type 1 consumer is 0. Hence, the total demand is $\lambda(1/2 + v) + (1 - 2\lambda)v$.

Suppose the monopoly chooses mainstream content strategy $x = 1/2$. One can show that the demand from type 0 consumer is λv , from type 1/2 consumer is $(1 - 2\lambda)(1/2 + v)$, and from type 1 consumer is λv . Hence, the total demand is $(1 - 2\lambda)(1/2 + v) + 2\lambda v$.

$$D(0) < D(1/2) \Leftrightarrow \lambda < \frac{1}{3 - 2v}$$

Consequently, the monopoly chooses mainstream positioning under behavioral advertising if and only if $\lambda < 1/(3 - 2v)$.

Now let us study the equilibrium under contextual advertising. We restrict the attention to the case where $1/3 < \lambda < 1/(3 - 2v)$ because niche positioning gives the firm higher demand and better inference if $\lambda \geq 1/(3 - 2v)$ whereas mainstream positioning dominates niche positioning if $\lambda \leq 1/3$.

Some calculations yield that $\mathbf{Var}(\theta|x = 0) = \frac{1}{4} \frac{v(1-2\lambda)(\lambda v + \lambda/2)}{(\lambda/2 - \lambda v + v)^2}$ and $\mathbf{Var}(\theta|x = 1/2) = \frac{1}{2} \frac{\lambda v}{1/2 + v - \lambda}$. Niche positioning gives the firm better inference if and only if $\mathbf{Var}(\theta|x = 0) < \mathbf{Var}(\theta|x = 1/2)$, which always hold for $\lambda \in (1/3, 1/(3 - 2v))$.

The monopoly prefers niche position to mainstream position if:

$$\begin{aligned} \pi_{con}(x = 0) &> \pi_{con}(x = 1/2) \\ \Leftrightarrow D(0)[1 - \gamma \mathbf{Var}(\theta|x = 0)] &> D(1/2)[1 - \gamma \mathbf{Var}(\theta|x = 1/2)] \\ \Leftrightarrow \gamma &> \frac{16[1 + \lambda(2v - 3)]}{\lambda v[2\lambda^3(1 - 2v)^2(1 + 2v) + 4\lambda v(-1 + 2v + 8v^2) + \lambda^2(-1 + 10v + 4v^2 - 40v^3) - 4(-4 + v^2 + 2v^3)]} \end{aligned}$$

Denote the threshold as γ^m . One can show that γ^m decreases in λ and increases in v .²

Now consider consumer utility from content consumption. Denote the consumer utility

¹The case where $x = 1$ is symmetric to this case. We only need to consider one of these cases.

²Observe that γ^m can be negative for small v . In that case, we let $\gamma^m = 0$ for convenience.

from content consumption given content strategy x as $CW(x)$. We have:

$$\begin{aligned}
CW(1/2) &= (1 - 2\lambda)\mathbf{E}[v + \epsilon | v + \epsilon > 0]P(v + \epsilon > 0) + \\
&\quad 2\lambda\mathbf{E}[v - 1/2 + \epsilon | v - 1/2 + \epsilon > 0]P(v - 1/2 + \epsilon > 0) \\
&= \frac{(1 - 2\lambda)(v + 1/2)^2}{2} + \lambda v^2 \\
CW(0) &= \lambda\mathbf{E}[v + \epsilon | v + \epsilon > 0]P(v + \epsilon > 0) + \\
&\quad (1 - 2\lambda)\mathbf{E}[v - 1/2 + \epsilon | v - 1/2 + \epsilon > 0]P(v - 1/2 + \epsilon > 0) \\
&= \frac{\lambda(v + 1/2)^2}{2} + \frac{(1 - 2\lambda)v^2}{2}
\end{aligned}$$

Niche content strategy leads to lower consumer welfare if: $CW(0) < CW(1/2) \Leftrightarrow \lambda < \hat{\lambda} := \frac{v+1/4}{-v^2+3v+1/4}$. One can see that $\hat{\lambda} > 1/3$.

(b) Duopoly

We first compute the demand given different strategy profiles.

i. (0,1)

The demand for type 0 content:

From $\theta = 0$ consumers: $\lambda P(v + \epsilon > 0) = \lambda(v + 1/2)$.

From $\theta = 1/2$ consumers: $\frac{1}{2}(1 - 2\lambda)P(\max\{v - 1/2 + \epsilon_0, v - 1/2 + \epsilon_1\} > 0) = \frac{(1-2\lambda)(2v-v^2)}{2}$.

From $\theta = 1$ consumers: 0.

Total demand: $\lambda(v + 1/2) + \frac{(1-2\lambda)(2v-v^2)}{2}$.

The demand for type 1 content is symmetric.

ii. (1/2, 1/2)

The demand for type 1/2 content:

From $\theta = 0/1$ consumers: $\frac{1}{2}\lambda P(\max\{v - 1/2 + \epsilon_{1/2}, v - 1/2 + \epsilon_{1/2'}\} > 0) = \frac{\lambda(2v-v^2)}{2}$.

From $\theta = 1/2$ consumers: $\frac{1}{2}(1-2\lambda)P(\max\{v+\epsilon_{1/2}, v+\epsilon_{1/2'}\} > 0) = \frac{(1-2\lambda)[1-(1/2-v)^2]}{2}$.

Total demand: $\lambda(2v - v^2) + \frac{(1-2\lambda)[1-(1/2-v)^2]}{2}$.

iii. (0,1/2)

The demand for type 0 content:

From $\theta = 0$ consumers: $\lambda P(v + \epsilon_0 > 0 \text{ and } v + \epsilon_0 > v - 1/2 + \epsilon_{1/2}) = \lambda(v/2 + 5/8)$.

From $\theta = 1/2$ consumers: $\frac{1}{2}\lambda P(v - 1/2 + \epsilon_0 > 0 \text{ and } v - 1/2 + \epsilon_0 > v + \epsilon_{1/2}) = \frac{(1-2\lambda)(v-v^2)}{2}$.

From $\theta = 1$ consumers: 0.

Total demand: $\lambda(v/2 + 5/8) + \frac{(1-2\lambda)(v-v^2)}{2}$.

The demand for type 1/2 content:

From $\theta = 0$ consumers: $\frac{\lambda(v-v^2)}{2}$.

From $\theta = 1/2$ consumers: $(1 - 2\lambda)(v/2 + 5/8)$.

From $\theta = 1$ consumers: $\lambda P(v - 1/2 + \epsilon_{1/2} > 0) = \lambda v$.

Total demand: $\frac{\lambda(v-v^2)}{2} + (1 - 2\lambda)(v/2 + 5/8) + \lambda v$.

Consider the equilibrium under behavioral advertising. Suppose $(0, 1/2)$ is an equilibrium. Firm 1 will deviate from $x = 0$ to $x = 1/2$ if and only if:

$$\begin{aligned} \lambda(2v - v^2) + \frac{(1 - 2\lambda)[1 - (1/2 - v)^2]}{2} &> \lambda(v/2 + 5/8) + \frac{(1 - 2\lambda)(v - v^2)}{2} \\ \Leftrightarrow \lambda < \lambda_2 &:= \frac{3}{11 - 12v + 8v^2} (< \frac{1}{3 - 2v}) \end{aligned}$$

Firm 2 will deviate from $x = 1/2$ to $x = 1$ if and only if:

$$\begin{aligned} \lambda(v + 1/2) + \frac{(1 - 2\lambda)(2v - v^2)}{2} &> \frac{\lambda(v - v^2)}{2} + (1 - 2\lambda)(v/2 + 5/8) + \lambda v \\ \Leftrightarrow \lambda > \lambda_1 &:= \frac{5 - 4v + 4v^2}{14 - 12v + 12v^2} \end{aligned}$$

One can see that λ_1 and λ_2 increases in v . One can also show that there exists $\underline{v} < 7/25$ such that $\lambda_1 < \lambda_2$ when $\underline{v} < v < 1/2$. Therefore, $(0, 1/2)$ will never be an equilibrium if $\underline{v} < v < 1/2$. We assume that $\underline{v} < v < 1/2$ in subsequent analyses.

Now suppose the equilibrium is $(1/2, 1/2)$. One can see that the firm will deviate to

0 or 1 if $\lambda > \lambda_2$. Similarly, if the equilibrium is (0,1), the firm will deviate to 1/2 if $\lambda < \lambda_1$.

In sum, the equilibrium is (1/2,1/2) if $\lambda < \lambda_1$, (1/2,1/2) or (0,1) if $\lambda_1 < \lambda < \lambda_2$, and (0,1) if $\lambda > \lambda_2$.

Now consider the equilibrium under contextual advertising.

We first characterize the equilibrium ad choices and profits for each firm.

A. (1/2,1/2)

By symmetry, $a = \mathbf{E}(\theta) = 1/2$. $\mathbf{Var}(\theta) = \frac{\lambda v(2-v)}{3+4v-4v^2+\lambda(4v^2-6)}$.

$$\begin{aligned}\pi(1/2, 1/2) &= D(1/2, 1/2) \cdot [1 - \gamma \mathbf{Var}(\theta)] \\ &= \lambda(2v - v^2) + \frac{(1 - 2\lambda)[1 - (1/2 - v)^2]}{2} - \frac{\gamma \lambda(2v - v^2)}{4}\end{aligned}$$

B. (0,1)

By symmetry, we only need to consider firm 1. Firm 2's ad choice will be $1 - a_1$, and firm 2's profit will be identical to firm 1's.

$$\mathbf{Var}(\theta) = \frac{1}{4} \frac{(1-2\lambda)(2v-v^2)/2}{(v+1/2)\lambda + (1-2\lambda)(2v-v^2)/2} \frac{(v+1/2)\lambda}{(v+1/2)\lambda + (1-2\lambda)(2v-v^2)/2}.$$

$$\begin{aligned}\pi(0, 1) &= D(0, 1) \cdot [1 - \gamma \mathbf{Var}(\theta)] \\ &= (v + 1/2)\lambda + (1 - 2\lambda)(2v - v^2)/2 - \gamma \frac{1}{4} \frac{\frac{1}{2}(1 - 2\lambda)(2v - v^2)(v + 1/2)\lambda}{(v + 1/2)\lambda + (1 - 2\lambda)(2v - v^2)/2}\end{aligned}$$

C. (0,1/2)

Consider firm 1 first. $\mathbf{Var}_1(\theta) = \frac{1}{4} \frac{(1-2\lambda)(v-v^2)/2}{(v/2+5/8)\lambda + (1-2\lambda)(v-v^2)/2} \frac{(v/2+5/8)\lambda}{(v/2+5/8)\lambda + (1-2\lambda)(v-v^2)/2}$.

$$\begin{aligned}\pi_1(0, 1/2) &= D_1(0, 1/2) \cdot [1 - \gamma \mathbf{Var}_1(\theta)] \\ &= \left(\frac{v}{2} + \frac{5}{8}\right)\lambda + \frac{(1 - 2\lambda)(v - v^2)}{2} - \gamma \frac{1}{4} \frac{\frac{1}{2}(1 - 2\lambda)(v - v^2)(v/2 + 5/8)\lambda}{(v/2 + 5/8)\lambda + (1 - 2\lambda)(v - v^2)/2}\end{aligned}$$

Consider firm 2 then. $\mathbf{Var}_2(\theta) = \frac{(1-2\lambda)(v/2+5/8)}{D_2(0,1/2)} \frac{1}{4} + \frac{\lambda v}{D_2(0,1/2)} \cdot 1 - \left[\frac{(1-2\lambda)(v/2+5/8)}{D_2(0,1/2)} \frac{1}{2} + \right.$

$$\frac{\lambda v}{D_2(0,1/2)} \cdot 1]^2.$$

$$\begin{aligned} & \pi_2(0, 1/2) \\ &= D_2(0, 1/2) \cdot [1 - \gamma \mathbf{Var}_2(\theta)] \\ &= \frac{1}{8} [5 + 4v - 2\lambda(5 - 2v + 2v^2)] [1 + \frac{\gamma \lambda v [-15 - 7v + 4v^2 + 6\lambda(5 - 3v + 4v^2)]}{[5 + 4v - 2\lambda(5 - 2v + 2v^2)]^2}] \end{aligned}$$

Suppose the equilibrium is $(0, 1/2)$. One can show that there exists a γ^d such that firm 1 will deviate from $x = 0$ to $x = 1/2$ if and only if:

$$\begin{aligned} & \pi(1/2, 1/2) > \pi_1(0, 1/2) \\ & \Leftrightarrow \gamma < \gamma^d \end{aligned} \tag{10}$$

Moreover, γ^d increases in v for $v \in (0, 1/2)$ and $\lambda \in (1/3, 1/2)$. Also, given that $1/3 < \lambda < 1/2$ and $0 < v < 1/2$, we have $\gamma^d > 0 \Leftrightarrow (3 - \sqrt{5})/4 < v < 1/2$ and $1/3 < \lambda < \lambda_2$. An immediate implication is that this deviation never happens if $\lambda \geq \lambda_2$.

One can show that there exists a $\gamma^{d'}$ such that firm 2 will deviate from $x = 1/2$ to $x = 1$ if and only if:³

$$\begin{aligned} & \pi(0, 1) > \pi_2(0, 1/2) \\ & \Leftrightarrow \gamma > \gamma^{d'} \end{aligned} \tag{11}$$

Moreover, given that $1/3 < \lambda < 1/2$ and $0 < v < 1/2$, we have $\gamma^{d'} \leq 0 \Leftrightarrow \lambda_1 \leq \lambda < 1/2$. An immediate implication is that deviation always happens if $\lambda \geq \lambda_1$. Therefore, $(0, 1/2)$ may only be an equilibrium if $\lambda < \lambda_1$. Now suppose $\lambda < \lambda_1$. In this case, we have shown that firm 1 will deviate if $\gamma < \gamma^d$ and firm 2 will deviate if $\gamma > \gamma^{d'}$.

³Firm 2 has a stronger incentive to deviate to $x = 1$ rather than $x = 0$ to soften competition. So, we only need to consider its deviation to $x = 1$.

Now suppose the equilibrium is $(1/2, 1/2)$. One can see that the firm will deviate to 0 or 1 if $\gamma > \gamma^d$, which may hold if $\lambda < \lambda_2$ and always holds if $\lambda > \lambda_2$. Similarly, if the equilibrium is $(0, 1)$, the firm will deviate to $1/2$ if $\gamma < \gamma^{d'}$, which may hold if $\lambda < \lambda_1$ and never holds if $\lambda > \lambda_1$.

Since $\gamma^m > \max\{\gamma^d, \gamma^{d'}\}$, $\forall v \in (\underline{v}, 1/2), \lambda \in (1/3, 1/2)$, Let us define $\gamma^{d''}$ by $\min\{\gamma^d, \gamma^{d'}\} \mathbf{1}_{[\lambda < \lambda_1]} + \gamma^d \mathbf{1}_{[\lambda_1 \leq \lambda < \lambda_2]}$. We have that the equilibrium under contextual advertising is more polarized than that under behavioral advertising if $\gamma > \gamma^{d''}$ and $1/3 < \lambda < \lambda_2$. Moreover, $\gamma^{d''} < \gamma^m$ for all $\lambda \in (1/3, \lambda_2)$.

2. $v \geq 1/2$:

In this case, the consumer always derives a non-negative utility from consuming the content if the content location perfectly matches the consumer type. In addition, the consumer may consume the content even if the content location is on the opposite end of the consumer's location. One can show that $\bar{v} \geq 5/6$. The analyses are similar to the previous case.

■