Failure of Reputation for Privacy

Yunfei (Jesse) Yao*

(University of California, Berkeley)

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Abstract

As consumers become increasingly concerned about their privacy, firms can benefit from committing not to sell consumer data. However, the holdup problem prevents them from doing so in a static setting. This paper studies whether the reputation consideration of the firm can serve as a commitment device in a long-run game when consumers have imperfect monitoring technology. We find that a patient enough monopoly can commit because its reputation will be permanently destroyed if consumers observe the data sale. The persistent punishment provides the monopoly a strong incentive not to deviate. In contrast, reputation may fail to serve as a commitment device when there are multiple firms. The penalty for selling data is smaller as consumers cannot know which exact firm sold the data. Also, other firms can hurt the reputation of a particular firm even if it does not sell data. We find some sufficient conditions under which the incentive to deviate is so strong that firms lose the commitment power. Reputation failure in the presence of multiple firms persists when we consider endogenous or asymmetric monitoring.

1 Introduction

The information market emerges in the digital era. The business of collecting and selling consumer data is estimated to be worth around $200 billion.[1] Firms use detailed information about individuals to offer a personalized product, price discriminate, show targeted ads, etc. Aware of the costs of revealing information, consumers are becoming increasingly concerned about their privacy. People started to raise concerns about their privacy even in the 1990s. About 0.01% of the US population opted out of the database of Lotus MarketPlace.[2] But most people at that time were either not aware of the privacy issues or did not care much about it. A recent survey by KPMG in 2021[3] among the US general population found that 86% of consumers viewed data privacy as a growing concern. One of the reasons people worry about the firm collecting their data is that they do not know how the firm will use it. According to the same survey, 40% of the consumers do not trust the firm to use their data ethically. Taylor (2004) shows that the firm can be better off by not protecting consumer privacy (selling customer data) if consumers are naive and unaware of it. However, selling data can backfire if consumers are sophisticated and expect the firm to sell their data. A large body of literature has documented that commitment benefits the firm. As a result, companies pay increasing attention to privacy. Apple, for instance, invested heavily in the operating systems to protect consumer privacy and spent lots of resources advertising their progress in privacy protection. In this particular setting, the firm desires the ability to commit to protecting consumer privacy by not selling consumer data. However, the non-verifiable nature of digital data makes it hard for the firm to commit. This paper looks at one possible solution - building trust by reputation.

The main contribution of the paper is to characterize some sufficient conditions such that reputation considerations cannot serve as a commitment device for privacy, even if firms are arbitrarily patient. When the firm is a monopoly and is patient enough, reputation enables it to commit never to sell the data. It achieves the Stackelberg payoff in all but a finite number of periods. However, when there are multiple firms, reputation may fail to enable any firms to commit. The intuition is that one firm's reputation depends on other firms' actions. Selling data by one firm has a negative externality on other firms. Firms do not take it into account in equilibrium. So, the benefit of not selling data is lower, as other firms' behavior may still hurt the firm's reputation. Anticipating such externality, consumers penalize each

[1] https://www.latimes.com/business/story/2019-11-05/column-data-brokers
[2] https://www.forbes.com/sites/forbestechcouncil/2020/12/14/the-rising-concern-around-consumer-data-and-privacy/?sh=73c76330487e
[3] https://advisory.kpmg.us/content/dam/advisory/en/pdfs/2021/corporate-data-responsibility-bridging-the-consumer-trust-gap.pdf

firm less when observing data sales. In addition, the likelihood of the deviation being pivotal reduces in the number of firms. Therefore, the cost of selling data is lower. So, the firm has more incentive to deviate. When the number of firms is large, or the monitoring technology is good, the incentive for the firm is so strong that no firm could commit never to sell the data. This reputation failure result hurts all the firms.

We consider long-lived firms interacting with short-lived consumers repeatedly in two markets. In the product market, the consumer decides how much information to reveal. Each firm infers consumer preferences based on the revealed information and offers a personalized product and price. The consumer then makes the purchase decision. By revealing more information, she[4] gets a better recommendation. However, the firm will charge a higher price when it collects more information from the consumer, knowing that she has a higher expected valuation for the product. So, the consumer faces a tradeoff between better product fit and lower price. In the information market, the firm could sell consumer data to third parties (e.g., data intermediaries). Consumers may suffer disutility from the sale of their data. For example, they may experience scam emails/calls or account hacking. If consumers reveal more information, they will be more vulnerable to data sales. Therefore, the consumer's decision of how much information to reveal in the product market depends on her belief about the firm's behavior in the information market. If the consumer thinks the firm will sell her data, she will reveal no information to minimize the cost of privacy loss. If she trusts the firm not to sell her data, she will reveal some information to get a better product recommendation. The Stackelberg action of the firm is not to sell data. But the decision of whether to sell data or not is made after the consumer reveals the information. Hence, the holdup problem prevents the firm from doing so in a static setting.

This paper studies whether the reputation consideration of the firm can serve as a commitment device in a long-run game when consumers have imperfect monitoring technology. Reputation can be a commitment device for a patient enough monopoly but may fail to be one when there are multiple firms, even when firms are arbitrarily patient. In particular, we have a reputation failure result when the number of firms is large, or the difference between the payoff with and without commitment is small, the likelihood of selling data being pivotal is low, and the privacy loss of the consumer is high. The intuition is that the monopoly will never restore its reputation by deviating from selling the data and being caught. The high and permanent reputation cost provides a strong incentive for the monopoly to commit to privacy. In contrast, when there are multiple firms, consumers do not know which firm exactly sold the data, even if they observe data sales. Therefore, the penalty for selling data is lower, and a firm's reputation may be hurt even if it did not sell data. The low and

[4] We refer to the consumer as "she" throughout the paper.

temporary reputation cost provides a strong incentive for the firm to deviate.

We consider several extensions to the main model. Consumers can voluntarily incur efforts to monitor firms better. We find that endogenous monitoring helps a monopoly build up a reputation faster, benefiting both the rational firm and consumers. However, it does not provide enough incentives for multiple firms to commit not to sell data. Also, we consider asymmetric monitoring. The monopoly case implies that rational firms can commit without noise. In contrast, any noise from other firms will break down the commitment power. This fragility result shows that the possibility rather than the level of interaction of firms' behavior in the reputation-building process is critical to the reputation failure.

1.1 Literature Review

This paper contributes to the literature on the economics of privacy (see Acquisti et al. for a survey). Goldfarb and Tucker (2012) and Lin (2019) document the existence of substantial privacy concerns of the consumer. In a static framework, Ichihashi (2020) shows that sellers prefer to commit to the price of the good for the buyers to reveal more information. We investigate when such commitment is feasible without an external commitment device. Recent papers have paid much attention to the economic impact of regulations such as GDPR, CCPA, and AdChoices, which seek to protect consumer's privacy and give them more control over their data (Athey et al. 2017, Ke and Sudhir 2020, Goldberg et al. 2019, Goldfarb and Tucker 2011, Johnson et al. 2020, Johnson et al. 2021). There are two reasons why reputation is essential despite various regulations. First, the main focus of those regulations is to give consumers more control over the usage of their data rather than to provide the firm with commitment power. Second, the transparency and verifiability nature of data transactions raises concerns about the credibility of such policy. Even if firms do not sell data in the presence of such regulation, consumers may still not reveal enough information to the firm. Protecting consumer privacy will not benefit the firm if it fails to obtain consumers' trust about how firms handle their data. Absent the information market and the possibility of selling data, Chen and Iyer (2002) study competing firms' incentives to collect data. They find that firms may voluntarily collect less information about consumers to mitigate the price competition. Closely related to our paper, Jullien et al. (2020) study a website's incentive to sell consumer information in a two-period model. Unlike in our paper, the website in their paper does not try to change consumers' beliefs about its type. Instead, the website wants to affect consumer's behavior on the vulnerability of bad experiences due to data sales.

This paper is also related to the literature on reputation. The idea of modeling reputation by incomplete information comes from Kreps et al. 1982, Kreps and Wilson (1982), and Milgrom and Roberts (1982). Fudenberg and Levine (1989) show that a patient long-run player will commit to the Stackelberg action in the presence of a behavioral type and perfect monitoring. Reputation serves as a commitment device and selects away bad equilibria for the long-lived player. In contrast, Ely and Välimäki (2003) and Morris (2001) show that reputation concerns may hurt the firm under imperfect monitoring. Substantively, the paper most closely related to us is Phelan (2006), which studies a problem where the government builds a reputation for trust. The reputation shock is non-permanent despite perfect monitoring because the government's type can change over time. Tirole (1996) studies the economics of collective reputation. Similar to our paper, individual reputation and incentive depend not only on one's past behavior but also on other players' because of the noisy signal. The inability to build a reputation relies on the different arrival times of the players. In our paper, reputation failure is driven by the externality of one firm's behavior on the other one's reputation rather than the arrival time. Despite the long development of this literature, people have not paid much attention to the reputation for privacy. This paper shows that reputation may fail to help the firm commit when their reputations depend on each other's behavior, and there is a bad type who does not care about consumer privacy. It connects the bad reputation and collective reputation literature.

The remainder of the paper is organized as follows. Section 2 presents the main model. Section 3 shows the ability of the monopoly to commit. Section 4 characterizes some sufficient conditions under which reputation fails to serve as a commitment device when there are multiple firms. The next two sections consider several extensions to the main model. Section 5 studies endogenous monitoring. Section 6 studies asymmetric monitoring. Section 7 concludes.

2 Model

Time is infinite, $t = 0, 1, 2, ...$ and the discount factor is δ. There are N long-lived firms and a short-lived consumer at each period. The consumer interact with all the firms. The firm's payoff is $(1-\delta)\sum_{t=0}^{+\infty} \delta^t u_t$, where u_t is the stage payoff at time t. There is a product market and an information market.

2.1 Product Market

Consumers have different horizontal preferences and are located uniformly on a circle with a circumference of 1. When a consumer visits the firm in the product market, she chooses how much information to reveal. If the consumer locating at $x \sim [0,1)$ reveals $\eta \in [0,1]$ proportion of information, the firm gets a noisy signal $l \sim U[x-(1-\eta)/2 \mod 1, x+(1-\eta)/2 \mod 1]$ about the consumer's location,[5] as illustrated by Figure 1. Based on the signal, the firm offers a personalized product and sets the price p. The consumer then makes the purchase decision. Denote the distance between the product's location, y, and the consumer's location, x, by $d = |x-y|$. We have that $d \sim U[0,(1-\eta)/2]$. The baseline valuation of the product is v, and the disutility from the mismatch of the recommended product and the consumer's horizontal taste is td. Therefore, the consumer gets $v - td - p$ if she buys and 0 if she does not buy. We assume that there is enough horizontal differentiation, $t > v$.

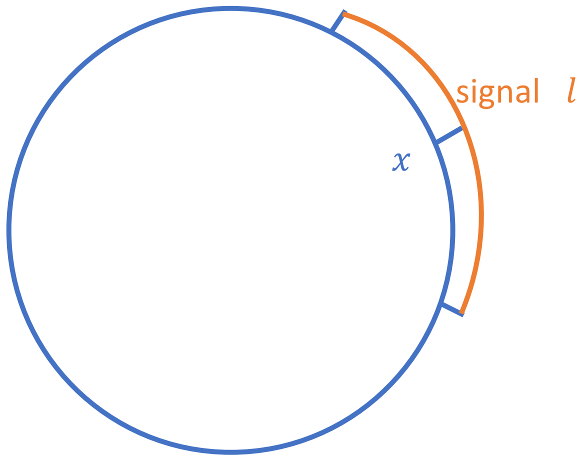


Figure 1: Consumer's location x and the signal l

Given the product recommendation and price, the consumer purchases if and only if the expected payoff is positive, $v - td - p \geq 0$. Thus, the firm's problem is:

$$\max_p p \cdot \mathbb{P}[v - td - p \geq 0] = p\left[\frac{2(v-p)}{(1-\eta)t} \wedge 1\right]$$

Therefore, the optimal price is:

$$p^*(\eta) = \begin{cases} \frac{v}{2},\ if\ \eta \leq 1 - \frac{v}{t} \\ v - \frac{(1-\eta)t}{2},\ if\ \eta > 1 - \frac{v}{t} \end{cases}$$

When the consumer reveals a lot of information, the firm accurately knows her preference. The optimal price makes the consumer located farthest away from the recommended product indifferent between purchasing or not. Therefore, she always makes the purchase. When the consumer reveals less information, the firm gets a noisier signal about her preference. The

[5] This implies that the firm will offer a product located at l given the signal.

profit from each purchase will be too low if the firm wants the consumer always to buy the product. Therefore, only consumers with a high enough valuation for the recommended product purchases it under the optimal price. If the recommended product locates far away from the consumer, the consumer will not buy it.

2.2 Information Market

The firm could sell consumer data in the information market to third parties (e.g., data intermediaries). For each consumer, the firm has the data directly revealed by her and the behavioral data of whether she makes the purchase given the product and price offered.[6] The firm gets $D(\eta)$ by selling the data. We assume that $D(\eta)$ increases in η to reflect that more accurate information is more valuable. The consumer might experience a scam or account hack if the firm sells data. Consumers are more vulnerable to such undesired activities when they reveal more information. So, we assume that the expected privacy cost of the consumer is ηu_b.[7] Consumers could imperfectly monitor the behavior of the firm in the information market. If a firm sold data in the previous period, the consumer detects it with probability q. The consumer will receive a signal $s = y$ if they caught any of the sales and $s = n$ if they did not detect any sales.[8]

Denote the probability of the firm selling the data by μ_s, by picking the privacy level η, the consumer's expected ex-ante payoff is:

$$U_0(\eta) = \begin{cases} -\mu_s \eta u_b + \frac{v^2}{4(1-\eta)t}, \ if \ \eta \le 1 - \frac{v}{t} \\ -\mu_s \eta u_b + \frac{(1-\eta)t}{4}, \ if \ \eta > 1 - \frac{v}{t} \end{cases}$$

In the product market, revealing more information has two opposite consequences. On the one hand, the firm could offer a better-matched product, which benefits the consumer. On the other hand, the firm will charge a higher price, knowing that the consumer has a higher expected valuation.[9] Price discrimination hurts the consumer. In the extreme case, if the firm perfectly knows the consumer's preference, it will extract all the consumer surplus. Therefore, the consumer never reveals all the information. The firm can only recommend a random product if the consumer does not reveal anything. The poor match

[6] Firms can infer consumers' willingness to pay from the data they reveal directly (Bergemann et al. 2020), or from the behavioral data of the consumer (Shen and Villas-Boas 2018, Taylor 2004, Villas-Boas 1999, 2004).

[7] Changing it to $\eta u_b + k$ won't change the result qualitatively, and we choose this form for simplicity.

[8] The assumption that consumers cannot distinguish which firm sold the data gives the sharpest illustration of the main idea. We extend it later to give consumers a better sense of which firm sold the data.

[9] For more discussions about this kind of holdup problem, see Villas-Boas (2009) and Wernerfelt (1994).

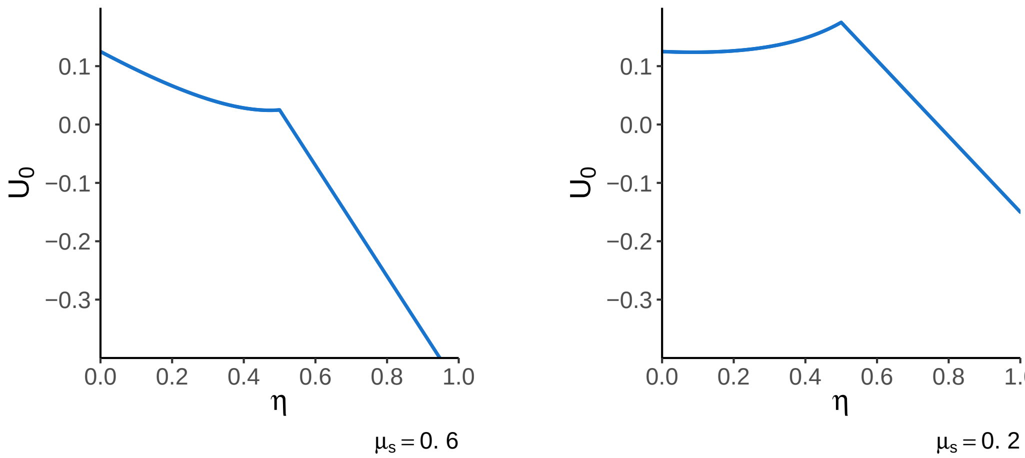


Figure 2: Ex-ante consumer payoff as a function of η for $v = 1$, $t = 2$, $u_b = 0.75$, and $\mu_s = 0.6$ (left) or 0.2 (right).

also hurts the consumer. So, it is optimal for her to reveal the information partially. However, the consumer also needs to consider the effect of information revelation in the information market. Disclosing more information to the firm always hurts the consumer there, as the consumer is more vulnerable when the firm sells her data. As Figure 2 illustrates, revealing too much information is never optimal for the consumer. The firm can charge a high price because the product recommendation is very accurate with lots of information about the consumer. In addition, the privacy loss from data sales in the information market is also high. Consumers may, however, prefer revealing a moderate amount of information to revealing nothing. By revealing some information, consumers benefit from a better recommendation in the product market but suffer a privacy cost if the firm sells it in the information market. If the firm's likelihood of selling the data is high, the high expected privacy loss in the information market outweighs the gain from the better match in the product market. Thus the consumer reveals no information. On the contrary, If the firm's likelihood of selling data is low, the consumer partially reveals her preference for a better recommendation. We have the following result.

Proposition 1. *The optimal amount of information to reveal is* $\eta^* = \begin{cases} 1 - v/t, \ if \ \mu_s \leq \widehat{\mu} \\ 0, \ if \ \mu_s > \widehat{\mu} \end{cases}$, *where* $\widehat{\mu} = \frac{v}{4u_b}$.[10]

[10] For the problem to be interesting, we assume that the threshold $\widehat{\mu} \in (0, 1)$. Also, consumers are indifferent between $\eta = 1 - v/t$ and 0. We assume they choose $\eta = 1 - v/t$, which does not affect any analyses.

Corollary 1. *The firm's profit in the product market is* $\Pi^* = \begin{cases} v/2, \ if \ \mu_s \leq \widehat{\mu} \\ v^2/2t, \ if \ \mu_s > \widehat{\mu} \end{cases}$.

If the firm could commit not to sell consumer data, the consumer will pick $\eta = 1 - v/t$, which gives the firm a stage payoff of $v/2$. If, instead, the firm always sells consumer data, the consumer will not reveal any information by picking $\eta = 0$. The firm obtains a stage payoff of $v^2/2t + D(0)$. When the following assumption holds, the firm will prefer to commit to protecting consumer privacy and never sell the data. The benefit from the increased profit from the product market outweighs the cost of not selling consumer data.

2.3 Reputation

There are two types of firms. A behavioral type (type B) always sells the consumer data. A rational type (type R) maximizes the expected sum of discounted utilities. The firm's reputation is consumers' belief about the probability that the firm is type B. The common priors on each firm being type B are $\mu_0 \in (0, 1)$. Consumers update the belief of the firm's type by Baye's rule. Denote the belief about firm i's type at time t by $\mu_{i,t}$. Reputation for privacy in this paper refers to the reputation for protecting consumer privacy in the information market. Figure 3 illustrates the timing of the game.

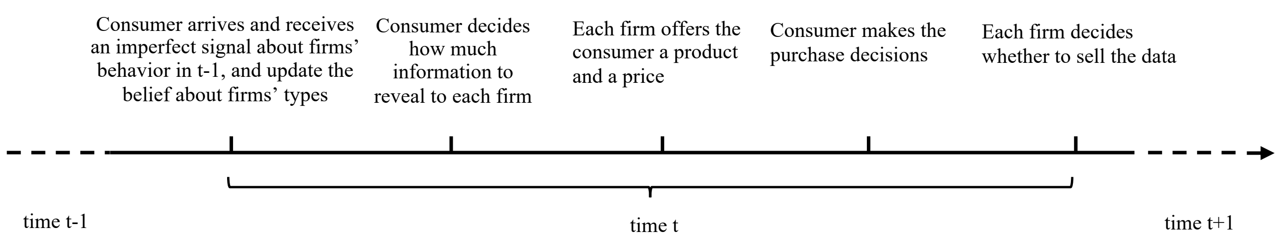


Figure 3: Timing of the Game

2.4 Solution Concept

We focus on whether a Markov Perfect Equilibrium (MPE)[11] exists where rational firms could commit to never selling the data. To make the problem interesting, we assume that rational firms prefer commitment, $v/2 > v^2/2t + D(0)$. MPE requires that firms' and

[11] Technically, the solution concept we use is the Markov Perfect Bayesian Equilibrium since there is incomplete information about the firm's type. However, the reputation literature usually uses the notion MPE.

consumers' strategies are measurable with respect to some payoff relevant states. It is widely used in the reputation literature as the belief $\vec{\mu}_t = (\mu_{1,t}, \mu_{2,t}, ..., \mu_{N,t})$ is a natural state variable.[12]

3 Reputation as a Commitment Device for Monopoly

3.1 Stage Game

We first analyze the property of the stage game of a single firm. If the firm can commit to any action in the information market (e.g., by moving first), the firm takes the *Stackelberg action* and obtains the *Stackelberg payoff.*

Definition 1. *Suppose player 1 chooses action $a \in A$ and player 2 chooses action $b \in B$. Player $i \in \{1, 2\}$'s stage-game payoff is $u_i(a, b)$. $BR_2(a) \subset B$ is player 2's best response correspondence to a. Then, player 1's Stackelberg action is* $\arg\max_{a\in A}[\min_{b\in BR_2(a)} u_1(a, b)]$*, and player 1's Stackelberg payoff is* $\max_{a\in A}[\min_{b\in BR_2(a)} u_1(a, b)]$.

One can see that the Stackelberg action for the firm is not to sell the data. The consumer will reveal $\eta = 1 - v/t$ proportion of information, and the firm gets the Stackelberg payoff of $v/2$. If the consumer acts first and minimizes the firm's payoff, one can see that the consumer reveals no information, and the firm sells data. The firm gets the *minmax payoff* of $v^2/2t + D(0)$, which is the payoff the firm can guarantee regardless of the consumer's action.

Definition 2. *Suppose player 1 chooses action $a \in A$ and player 2 chooses action $b \in B$. Player $i \in \{1, 2\}$'s stage-game payoff is $u_i(a, b)$. Then, player 1's minmax payoff is* $\min_{\beta\in\Delta(B)}[\max_{a\in A} u_1(a, b)]$.

However, since the firm decides whether to sell data after the consumer reveals the information, it always sells the data in a static game. We will see that reputation considerations enable the monopoly to commit to the Stackelberg action.

3.2 Belief Updating

We first derive the belief updating of the consumers about a monopoly's type, assuming that the rational type never sells the data. We could derive the belief updating when the

[12] In the symmetric equilibrium where every firm always has the same reputation, we can use $\mu_t = \mu_{i,t}$ as the state variable.

rational firm uses other strategies by similar methods. Consider the consumer's belief about the monopoly's type after observing a signal s. If $s = y$, the consumer knows that the firm sold the data in the previous period. So, the belief that the firm is a bad type will be one forever. The firm suffers a permanent reputation shock. If $s = n$, either the firm is the rational type and did not sell the data, or the firm is the bad type, but the consumer did not observe the data sales. The firm is more likely to be the rational type, but the consumer does not know it for sure. So, the belief of firm 1 being bad type decreases from μ_t to $\mu_{t+1} > 0$. Formally, the belief updating is as follows.

Proposition 2. *Suppose the rational type never sells the data in equilibrium.* $\mu_{t+1} = \begin{cases} \frac{1-q}{1-q\mu_t}\mu_t, \ if \ s = n \\ 1, \ if \ s = y \end{cases}$. *After receiving signal n for k consecutive periods, the belief becomes* $\mu_{t+k} = \frac{(1-q)^k}{(1-q)^k\mu_t + 1 - \mu_t}\mu_t$, *which approaches 0 as $k \to +\infty$.*

So, if the monopoly keeps not selling consumer data, the consumer's belief will keep decreasing. After enough time, the consumer is almost certain that the firm is not the bad type.

3.3 Equilibrium

Suppose consumers expect the rational firm never to sell the data in equilibrium. In that case, a signal n will destroy the firm's reputation by making the consumer believe that the firm is the bad type in all of the current and future periods. Then, the firm is stuck with the minmax payoff. By deviating, the firm risks being detected by the consumer with a positive probability. The persistent punishment gives the firm a strong incentive not to sell the data for short-term benefit. As a result, regardless of the monitoring technology or the price of data in the information market, the monopoly can always achieve commitment by reputation, as long as it is patient enough.

Proposition 3. *There exists a $\widehat{\delta} < 1$ such that for any $\delta > \widehat{\delta}$, there exists a MPE where the rational firm never sells consmer data and the consumer always reveals $\eta = 1 - v/t$ proportion of information after a finite period of time.*

By protecting consumer privacy, the rational firm keeps reducing the consumer's belief that it is a bad type. When the belief is below a threshold, the consumer is willing to reveal some information that benefits the firm. A patient firm does not want to deviate, as the consumer may observe the deviation and believe that the firm is the bad type. She will never reveal any information. So, the firm suffers from lower revenue in the product market forever. This severe punishment provides a strong incentive for the firm to trade the short-term benefit of selling data in the information market for the long-term benefit of earning a higher profit in the product market.

4 Reputation Failure with Multiple Firms

When there is more than one firm, reputation may fail to serve as a commitment device for privacy. The difference comes from the interaction of firms' behavior in the reputation-building process.

4.1 Belief Updaing

When there are multiple firms ($N \geq 2$), the belief updating is qualitatively different from the monopoly case. Consider the consumer's belief about firm 1's type after observing a signal s, assuming that a rational firm never sells the data. If $s = y$, the consumer knows that at least one firm sold the data in the previous period but is not sure whether firm 1 sold it. So, the belief of firm 1 being bad type increases but is still lower than 1, unlike the monopoly case. The reputation shock is temporary, and firm 1 can rebuild the reputation. Conditional on other firms' behavior, the likelihood that $s = n$ if firm 1 is the rational type and did not sell the data is higher than if firm 1 is the bad type, but consumers did not observe the sale of data. Consequently, firm 1 is more likely to be the rational type, but the consumer is uncertain. So, the belief of firm 1 being a bad type decreases but is still positive. Formally, the belief updating is as follows.

Proposition 4. *Suppose the rational type never sells the data in equilibrium.* $\mu_{t+1} = \begin{cases} \frac{1-q}{1-q\mu_t}\mu_t, \ if \ \ s=n \\ \frac{1-(1-q)(1-q\mu_t)^{N-1}}{1-(1-q\mu_t)^N}\mu_t, \ if \ \ s=y \end{cases}$. μ_{t+1} *does not depend on the number of firms* N *if* $s = n$ *and decreases in* N *if* $s = y$.

Even if firm 1 sold the data and the consumer observes a signal y, she knows at least one firm sold the data but did not know the exact firm. Therefore, she penalizes firm 1 less than she will do in the monopoly case. Even if the consumer obtains a signal y, she does not know for sure that firm 1 is the bad type and the updated belief is lower than 1. Therefore, the reputation cost is temporary and the consumer's belief will decrease if she receives signal n in the future. When there are more firms, the signal's noise is larger, and the consumer has less idea about which firm sold the data. Therefore, the belief increase upon getting signal y will be smaller. If firm 1 did not sell the data and the consumer observes a signal n, the

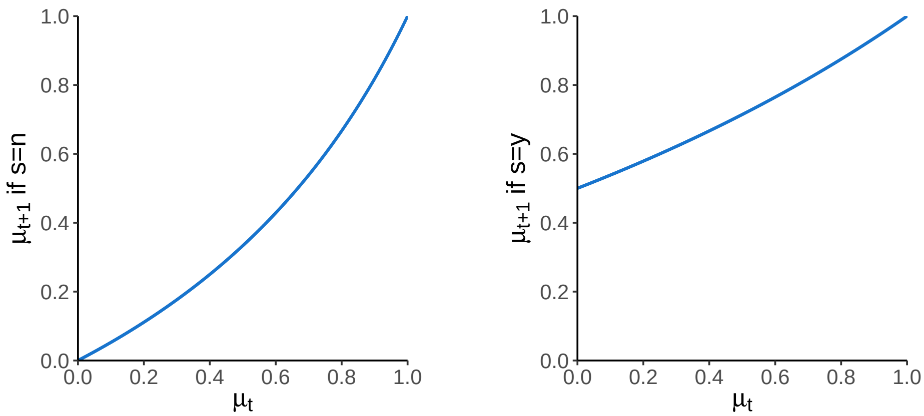


Figure 4: Belief Updating as a function of μ_t for $q = 0.5$ and $N = 2$.

belief reduction does not depend on the number of firms. So, the firm is penalized less for selling the data but not rewarded more for not doing so. In addition, the realization of the signal depends little on a single firm's action when there are many firms. So, the likelihood that firm 1's action is pivotal decreases in the number of firms. Figure 4 illustrates the belief updating when there are two firms.

The above forces imply that the firm has more incentive to sell the data when the number of firms increases.

4.2 Fixed Discount Factor

We first fix the discount factor and look at the effect of the number of firms on reputation building.

Proposition 5. *For any $\delta \in (0, 1), \exists N_\delta$ s.t. $\forall N \geq N_\delta$, firms always sell data and consumers reveal nothing in the unique MPE. Consumer's belief about each firm's type is always μ_0.*

No matter how patient firms are, they cannot build a reputation for privacy. The intuition for the failure of reputation as a commitment device is the following. On the one hand, the consumer has a noisy signal about which firm sold the data. Even if the firm deviates and the consumer observes it, the penalty for that particular firm is less than that for the monopoly firm. Moreover, it decreases in the number of firms. On the other hand, even if all the firms do not deviate, other firms may be the bad type and sell the data. So, it is less likely that the sale of the data is pivotal as the number of firms increases. Both forces give the firm more incentive to deviate and sell the data. So, it becomes harder to commit when the number of firms increases. Eventually, the firm loses all the commitment power and sells data every period.

Anticipating the firm always to sell data, consumers do not reveal anything. The belief of each firm's type remains the same over time, and there is no reputation building. The monopoly can get Stackelberg payoffs in all but a finite number of periods under substantial punishment for selling data. In contrast, each firm can only get the minmax payoff under weaker punishment when there are multiple firms, even if there is no competition.

4.3 Fixed Number of Firms

In this section, we study whether it is possible to achieve commitment by reputation when the number of firms is fixed. The previous section shows that it is harder to commit by reputation when there are more firms. So, we look at the case of two firms. If reputation can not help duopoly commit, we will also have reputation failure when there are more firms.

Proposition 6. *Suppose there are two firms. There does not exist any MPE in which any rational firms could commit to never sell the data, even when $\delta \to 1$, if the following conditions hold:*

$$(1-q)(v/2 - v^2/2t) < D(0) \tag{1}$$

$$q(1-q)v/2 < D(0) \tag{2}$$

$$v < 2u_b \tag{3}$$

Even if firms are very patient, reputation may not suffice to be a commitment device for privacy. This proposition identifies sufficient conditions under which firms cannot commit even if they are (almost) perfectly patient. The role of each condition is the following. Condition (1) requires that the commitment payoff is not much higher than the one without commitment. Condition (2) requires that the likelihood of selling data being pivotal is low. Because of the imperfect monitoring technology, consumers may get a signal y if a firm did not sell data and n if a firm sold data. If the signal's noise is very high, the consumer is likely to get a signal y even if a firm did not sell data because the other firm sold the data. If it is very low, the consumer is likely to get a signal n even if a firm sold data because of the poor monitoring. Condition (3) requires that the privacy loss of the consumer is high enough such that consumers will not reveal information if the firm is equally likely to sell data or not. According to the belief update formula in proposition 4, a single signal y will increase the belief above 1/2. Thus, consumers will reveal nothing to the firm when they get a signal y. Even if the firm does not sell data and reduces the belief, a single signal y

in the future periods will make the consumer reveal no information. The fast depreciation of reputation makes building it less attractive. Equivalently, rational firms have a stronger incentive to sell data. When all these three conditions hold,[13] selling data does not hurt the reputation much, a better reputation increases firm's stage payoff slightly, and good reputation is highly non-persistent. Firms have little incentive to build a reputation. As a result, reputation considerations provide no commitment power to rational firms.

4.4 Managerial Implications

Even though commitment may be desirable for the firm, it may not be possible without strict external regulations. A monopoly can always build a reputation for caring about consumer privacy by not selling data. After a finite period, consumers will reward it by sharing more information. The monopoly can enjoy a high profit by recommending better-fit products and charging a premium. However, when there are multiple firms in the market, it may not be in the firm's best interest to protect consumer privacy. Even if a firm never sells consumer data, it may not be able to build a reputation for privacy. So, it loses the revenue from selling consumer information while does not have any (or enough) gain. Since firms benefit from committing never to sell consumer data, they need to think about other ways of achieving the commitment. Our model shows that the key to the commitment power is the tradeoff between the short-term benefit of increased revenue in the information market and the long-term benefit of the profit in the product market. A potential solution is to improve the recommendation algorithm so that the firm has a higher marginal benefit from the information collected from consumers. It will have a stronger incentive to maintain a good reputation and profit from the product market. The other solution is to invest in better monitoring technology to make it easier for consumers to identify what specific firm sells the data. Lastly, the firm can offer some compensation to consumers if the signal is y. It will face an additional penalty for selling consumer data. Therefore, the "free lunch" in the information market is more costly for the firm.

5 Endogenous Monitoring

The monitoring technology is exogenous in the main model. In reality, consumers observe some data sales without any effort. They know that the phone number has been sold if they get a scam call. If they get a pre-approved credit card with their name in the mail, they know

[13] For example, these conditions will hold if the privacy cost for consumers is high and the monitoring technology has low noise.

that some firms have sold their address and credit history. However, as consumers become more concerned about privacy issues, they may endogenously invest in better monitoring firms' use of their data by incurring more effort or purchasing security apps. We consider this possibility and allow for endogenous monitoring in this section.

The setup is the same as before, except that the consumer can incur costs to obtain an extra signal s' about the data sale after observing the costless signal s. By incurring an effort $h \in [0, \bar{h}]$ ($\bar{h} < 1$), the consumer obtains a signal s_h. If a firm sold data in the previous period, the consumer detects it with probability h. The consumer would receive a signal $s_h = y$ if they caught any sales and $s_h = n$ if they did not detect any sales. We make the following assumption on the cost $c(h)$.

Assumption 1. $c(\cdot) \in \mathcal{C}^2(\mathbb{R}_+), c(0) = 0, c'(h) > 0, c''(h) > 0, \lim_{h \to \bar{h}} c'(h) = +\infty.$

We assume that it is costless not to incur any effort, the marginal cost of the consumer increases at an increasing rate when the precision of monitoring improves, and it is very costly to monitor the data sales very precisely.

5.1 Monopoly

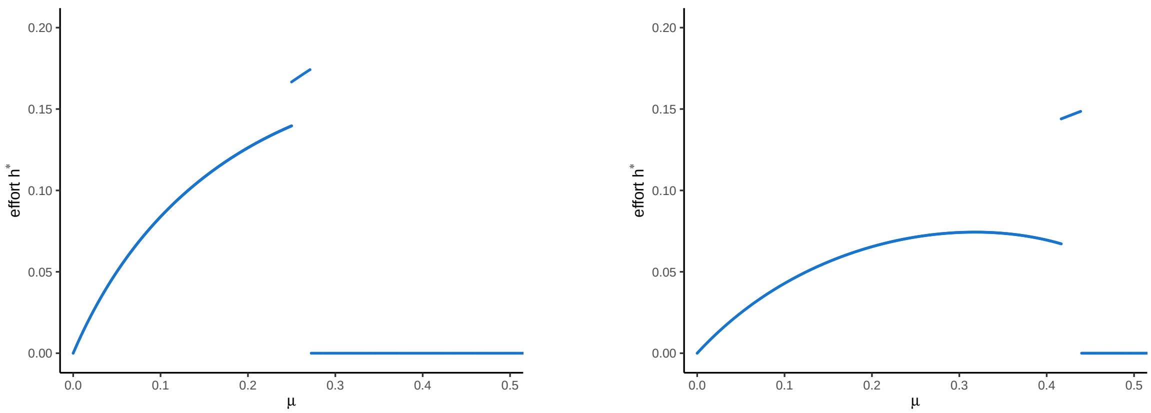
Let μ be the consumer's belief after observing signal s. The consumer can incur effort h to gain an additional signal s_h. We consider the MPE in which the rational firm never sells data. Suppose there exists such an equilibrium. There are two cases.

5.1.1 $\mu > \widehat{\mu}$

Without an extra signal, the consumer will reveal nothing according to Proposition 1. If the consumer obtains a costly signal, she must take a different action under some circumstances. Otherwise, she will be better off by not incurring any costs. Therefore, the belief must be below $\widehat{\mu}$ if the consumer incurs effort h and receives a signal $s_h = n$. When the belief μ is too high, the updated belief will be above $\widehat{\mu}$ regardless of the effort. So, the consumer will not incur an effort to get an extra signal. When the belief μ is close to $\widehat{\mu}$, the belief will be below $\widehat{\mu}$ if the consumer incurs enough efforts and receive signal n. The consumer benefits from costly monitoring by being more likely to identify the rational type.

5.1.2 $\mu \leq \widehat{\mu}$

Without an extra signal, the consumer will reveal $1 - \eta/t$ amount of information according to Proposition 1. If the consumer seeks an extra signal and receives $s_h = y$, she knows that the firm is a bad type and does not reveal information. If $s_h = n$, the belief decreases,



(a) $v = 1, t = 3, u_b = 1, \bar{h} = 0.5, k = 0.1$ (b) $v = 1, t = 3, u_b = 0.6, \bar{h} = 0.5, k = 1$

Figure 5: The optimal monitoring effort, where $c(h) = \frac{kh^2}{\bar{h}-h}$.

and the consumer reveals some information. The consumer benefits from costly monitoring by being more likely to identify the bad type.

Examining both cases, we have the following result.

Proposition 7. *There exists a $\widehat{\delta} < 1$ such that for any $\delta > \widehat{\delta}$, there exists a MPE where the rational firm never sells consmer data. In such equilibrium, there exists a $\widehat{\widehat{\mu}} > \widehat{\mu}$ such that the consumer incurs efforts in monitoring if and only if $\mu \leq \widehat{\widehat{\mu}}$. The monitoring effort strictly increases in μ for $\mu > \widehat{\mu}$. It vanishes as μ approaches zero.*

Figure 5 illustrates the monitoring effort as a function of the belief. As we can see, the consumer does not incur any monitoring costs when the belief is far above the threshold belief of revealing information, $\widehat{\mu}$. When the consumer strongly believes that the firm is rational, she also incurs little costs because the likelihood of detecting the data sale is very low. She will reveal the same amount of information without an extra signal. Hence, costly monitoring provides little benefits to her. In contrast, additional monitoring will be valuable for the consumer when the belief is slightly above $\widehat{\mu}$. Since the consumer is quite uncertain about the firm's type, her expected payoff is low. If she reveals nothing and the firm rational, she gives up the opportunity of receiving better product recommendations. If she reveals some information and the firm is bad, she suffers a high privacy loss. By getting another signal, the consumer becomes more certain about the firm's type. A y signal convinces her that the firm will sell her data. So, she reveals nothing. A n signal makes her more confident that the firm will not sell her data. So, she reveals some information. Consequently, the consumer incurs a relatively high effort in this case.

The consumer starts revealing information at a higher belief when they can voluntarily monitor the firm's behavior. So, the rational firm builds up the reputation and achieves the Stackelberg payoff faster under endogenous monitoring. The consumer makes better decisions with an extra signal. Both players are better off.

5.2 Multiple Firms

From the monopoly case, we can see that the ability of consumers to gain additional signals makes it easier to build a reputation. In the presence of multiple firms, we have the reputation failure results when the monitoring is exogenous. One natural question is whether endogenous monitoring suffices to reverse the negative results. The following result shows that it is not enough to restore reputation building.

Proposition 8. *For any $\alpha > 0$ and $\delta \in (0, 1), \exists N_\delta$ s.t. $\forall N \geq N_\delta$, firms always sell data and consumers reveal nothing in the unique MPE. Consumer's belief about each firm's type is always μ_0.*

Even though endogenous monitoring can help firms build up reputation faster when they do not sell data, it also hurts their reputation more frequently when some firms sell data. With the possibility of a bad type who always sells data, rational firms are tempted to sell data as well because their reputation is affected by other firms. When the number of firms increases, it becomes harder for rational types to commit. Eventually, the firm loses all the commitment power and sells data every period.

6 Asymmetric Monitoring

In the main model, the monitoring technology of the consumer is symmetric. The consumer observes whether some firms sold the data without further information about which firm is more likely to sell it. We now consider an asymmetric monitoring technology.

If firm 1 sells the data in the previous period, the consumer detects it with probability q. If firm $i \neq 1$ sells the data in the previous period, the consumer detects it with probability αq $(0 < \alpha < 1)$. The consumer will receive a signal $s = y$ if they caught any sales and $s = n$ if they did not detect any. The consumer has less noise about whether firm 1 sold the data. To get some intuition, notice that $\alpha = 1$ corresponds to the monitoring technology in the main model. If $\alpha = 0$ instead, the consumer knows that firm 1 sold the data in the previous period. The result about the monopoly, Proposition 3, implies that a sufficiently patient firm 1 could commit privacy by reputation, no matter how large the total number of firms is. When α is close to zero, the monitoring technology is close to the monopoly case. The following result shows that any noise from other firms leads to reputation failure.

Proposition 9. *For any $\alpha > 0$ and $\delta \in (0, 1), \exists N_\delta$ s.t. $\forall N \geq N_\delta$, firms always sell data and consumers reveal nothing in the unique MPE. Consumer's belief about each firm's type is always μ_0.*

When $\alpha = 0$, the result in the monopoly case implies that rational firms can commit to never selling data. However, any noise from other firms will break down the commitment power. This fragility result shows that the possibility rather than the level of interaction of firms' behavior in the reputation-building process is critical to the reputation failure.

7 Conclusion

This paper studies whether reputation consideration can serve as a commitment device for privacy. We show that it depends on the market structure. For a patient enough monopoly, reputation enables it to commit to the Stackelberg action of not selling consumers' data. However, reputation may fail to help the firm commit when there are multiple firms. We characterize some sufficient conditions in which firms cannot commit not to sell the data even if they are very patient. Consumers know the monopoly sold data when they observe it. So, the firm will never restore its reputation by selling data and being caught. The high and permanent reputation cost provides a strong incentive for the monopoly to commit to privacy. In contrast, consumers can never know the specific firm selling the data when there are multiple firms. Therefore, the penalty for data sale is lower, and a firm's reputation may be hurt even if it does not sell data. The minor and temporary reputation cost provides a strong incentive for the firm to deviate.

Reputation failure in the presence of multiple firms persists when we consider several extensions. Endogenous monitoring helps a monopoly build up a reputation faster, benefiting both the rational firm and consumers. However, it does not provide enough incentives for multiple firms to commit not to sell data. Also, we consider asymmetric monitoring. The monopoly case implies that rational firms can commit without noise. In contrast, any noise from other firms will break down the commitment power. This fragility result shows that the possibility rather than the level of interaction of firms' behavior in the reputation-building process is critical to the reputation failure.

There are a couple of limitations to the current work. Consumers can reveal an arbitrary amount of information in the product market. However, the firm sometimes restricts the communication space. So, the consumer can only choose from a menu of the amounts of information to disclose. It will be interesting to study the optimal design of the menu and how much advantage the firm could gain by offering such a contract. Also, the consumer's privacy loss from data sales is exogenous in this paper. Engonenizing the privacy cost in a game theoretic model can provide further insights. We leave them for future research.

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Appendix

Proof of Proposition 1. The consumer's expected ex-ante payoff by choosing to reveal η proportion of information is:

$$U_0(\eta) = \begin{cases} -\mu_s \eta u_b + \frac{v^2}{4(1-\eta)t}, \; if \;\eta \leq 1 - \frac{v}{t} \\ -\mu_s \eta u_b + \frac{(1-\eta)t}{4}, \; if \;\eta > 1 - \frac{v}{t} \end{cases}$$

$U_0(\eta)$ decreases in η for $\eta > 1 - \frac{v}{t}$, so the consumer will not reveal more than $1 - \frac{v}{t}$ proportion of information. Consider $\eta \in [0, 1 - \frac{v}{t}]$.

$$\frac{dU_0(\eta)}{d\eta} = -\mu_s u_b + \frac{v^2}{4t(1-\eta)^2}$$

, which increases in η. So, the optimal η is either 0 or $1 - \frac{v}{t}$. $U_0(1 - \frac{v}{t}) \geq U_0(0) \Leftrightarrow \mu_s \leq \widehat{\mu}$, where $\widehat{\mu} = \frac{v}{4u_b}$. □

Proof of Proposition 2. Since the rational type does not sell the data, a signal $s = y$ implies the firm is the behavioral type. So, $\mu_{t+1} = 1$. Now consider $s = n$. By Baye's rule,

$$\begin{aligned} \mathbb{P}[type\ B|s = n] &= \frac{\mathbb{P}[s = n|type\ B]\mathbb{P}[type\ B]}{\mathbb{P}[s = n|type\ B]\mathbb{P}[type\ B] + \mathbb{P}[s = n|type\ R]\mathbb{P}[type\ R]} \\ &= \frac{(1-q)\mu_t}{(1-q)\mu_t + 1 \cdot (1-\mu_t)} \\ &= \frac{1-q}{1-q\mu_t}\mu_t \end{aligned}$$

By induction, we have $\mu_{t+1} = \frac{(1-q)\mu_t}{(1-q)\mu_t+1-\mu_t}$ after receiving signal n once. Suppose $\mu_{t+k} = \frac{(1-q)^k}{(1-q)^k\mu_t+1-\mu_t}\mu_t$ after receiving signal n for k consecutive periods. After receiving signal n for $k+1$ consecutive periods, we have $\mu_{t+k+1} = \frac{(1-q)\mu_{t+k}}{(1-q)\mu_{t+k}+1-\mu_{t+k}} = \frac{(1-q)^{k+1}}{(1-q)^{k+1}\mu_t+1-\mu_t}\mu_t$. So, it shows that $\mu_{t+k} = \frac{(1-q)^k}{(1-q)^k\mu_t+1-\mu_t}\mu_t$ after receiving signal n for k consecutive periods.

One can see that $\frac{(1-q)^k}{(1-q)^k\mu_t+1-\mu_t}\mu_t$ approaches 0 as $k \to +\infty$. □

Proof of Proposition 3. Let $\widehat{k} = \left\lceil \frac{ln\frac{v(1-\mu_0)}{(4u_b-v)\mu_0}}{ln(1-q)} \right\rceil$. We first show that the consumer reveals $\eta = 1 - v/t$ proportion of information after $\widehat{k}$ periods, if the firm never sells the data in equilibrium. By Proposition 2, the belief after not selling data for k consecutive periods is $\mu_k = \frac{(1-q)^k}{(1-q)^k\mu_0+1-\mu_0}\mu_0$. By Proposition 1, consumer reveals $\eta = 1 - v/t$ proportion of information if and only if $\mu_k \leq \widehat{\mu} \Leftrightarrow k \geq \frac{ln\frac{v(1-\mu_0)}{(4u_b-v)\mu_0}}{ln(1-q)}$.

We now show that the rational firm has no incentive to deviate to selling data at any time. The game is continuous at infinity because of discounting. So, we can use the single-deviation property. Suppose the firm deviates once at period t when the belief is μ_t. There are two cases.

1. $\mu \leq \widehat{\mu}$

   The value function of the equilibrium strategy (never sell data) is:

   $V(\mu_t) = (1-\delta)\frac{v}{2}\frac{1}{1-\delta} = \frac{v}{2}$

   The value function of deviating once at period t is (assuming the firm sells data when the belief is 1, which maximizes the payoff):

   $\tilde{V}(\mu_t) = (1-\delta)[\frac{v}{2} + D(1-\frac{v}{t}) + \delta(q\frac{\frac{v^2}{2t}+D(0)}{1-\delta} + (1-q)\frac{\frac{v}{2}}{1-\delta})]$

   The rational firm will not deviate if $V(\mu_t) > \tilde{V}(\mu_t) \Leftrightarrow \frac{\delta}{1-\delta} > \frac{D(1-\frac{v}{t})}{q(v/2-v^2/2t-D(0))}$. One can see that $\exists \delta_1 \in (0,1)$ s.t. the inequality holds for any $\delta \geq \delta_1$.

2. $\mu > \widehat{\mu}$

   The value function of the equilibrium strategy (never sell data) is:

   $V(\mu_t) = (1-\delta)[\sum_{k=0}^{\widehat{k}-1}\delta^k\frac{v^2}{2t} + \sum_{k=\widehat{k}}^{+\infty}\delta^k\frac{v}{2}]$

   The value function of deviating once at period t is (assuming the firm sells data when the belief is 1, which maximizes the payoff):

   $\tilde{V}(\mu_t) = (1-\delta)[\frac{v^2}{2t} + D(0) + \delta(q\frac{\frac{v^2}{2t}+D(0)}{1-\delta} + (1-q)[\sum_{k=0}^{\widehat{k}-2}\delta^k\frac{v^2}{2t} + \sum_{k=\widehat{k}-1}^{+\infty}\delta^k\frac{v}{2}])]$

   The rational firm will not deviate if $V(\mu_t) > \tilde{V}(\mu_t) \Leftrightarrow \frac{\delta^{\widehat{k}}}{(1-\delta)[1-(1-q)\delta]} > \frac{D(0)}{q(v/2-v^2/2t)}$. One can see that $\exists \delta_2 \in (0,1)$ s.t. the inequality holds for any $\delta \geq \delta_2$.

Let $\widehat{\delta} = \max\{\delta_1, \delta_2\}$. One can see that $\widehat{\delta} < 1$ and for any $\delta > \widehat{\delta}$, the firm never sells consmer data, $\eta = 0$ in the first $\widehat{k}$ periods, and $\eta = 1 - v/t$ after $\widehat{k}$ periods is a MPE. □

Proof of Proposition 4. By Baye's rule, for a given firm,

$$
\begin{aligned}
&\mathbb{P}[type\ B|s=n] \\
=&\frac{\mathbb{P}[s=n|type\ B]\mathbb{P}[type\ B]}{\mathbb{P}[s=n|type\ B]\mathbb{P}[type\ B] + \mathbb{P}[s=n|type\ R]\mathbb{P}[type\ R]} \\
=&\frac{(1-q)[1\cdot(1-\mu_t)+(1-q)\mu_t]^{N-1}\mu_t}{(1-q)[1\cdot(1-\mu_t)+(1-q)\mu_t]^{N-1}\mu_t + 1\cdot[1\cdot(1-\mu_t)+(1-q)\mu_t]^{N-1}(1-\mu_t)} \\
=&\frac{1-q}{1-q\mu_t}\mu_t, \text{ which does not depend on } N.
\end{aligned}
$$

$$
\begin{aligned}
&\mathbb{P}[type\ B|s=y] \\
=&\frac{\mathbb{P}[s=y|type\ B]\mathbb{P}[type\ B]}{\mathbb{P}[s=y|type\ B]\mathbb{P}[type\ B]+\mathbb{P}[s=y|type\ R]\mathbb{P}[type\ R]} \\
=&\frac{\left[1-(1-q)[1\cdot(1-\mu_t)+(1-q)\mu_t]^{N-1}\right]\mu_t}{\left[1-(1-q)[1\cdot(1-\mu_t)+(1-q)\mu_t]^{N-1}\right]\mu_t+\left[1-1\cdot[1\cdot(1-\mu_t)+(1-q)\mu_t]^{N-1}\right](1-\mu_t)} \\
=&\frac{1-(1-q)(1-q\mu_t)^{N-1}}{1-(1-q\mu_t)^N}\mu_t,\ \text{which decreases in } N \text{ by checking the derivative.}
\end{aligned}
$$

□

Proof of Proposition 5. Fix $\delta \in (0,1)$. Suppose $\forall N_\delta,\ \exists N \geq N_\delta$ s.t. there exists a MPE in which a rational firm (label it by firm 1 WLOG) does not sell the data at $t=0$. Denote the equilibrium strategy of all the firms by σ and the value function of firm 1 by $V_1(\cdot)$. The prior belief is $\vec{\mu}_0 = (\mu_0, \mu_0, ..., \mu_0)$. Denote the posterior belief upon observing signal y (n) by $\vec{\mu}^y$ $(\vec{\mu}^n)$ when the initial belief is $\vec{\mu}$ and the equilibrium strategy is σ.

Suppose $\mu_0 > \widehat{\mu}$.

$$V_1(\vec{\mu_0}) = (1-\delta)\frac{v^2}{2t} + \delta\left[\mathbb{P}(s=y|\sigma)V_1(\vec{\mu}_0^y) + \mathbb{P}(s=n|\sigma)V_1(\vec{\mu}_0^n)\right]$$

, where $\begin{cases}\mathbb{P}(s=n|\sigma) \leq (1-q\mu_0)^{N-1} \\ \mathbb{P}(s=y|\sigma) = 1-\mathbb{P}(s=n|\sigma) \geq 1-(1-q\mu_0)^{N-1}\end{cases}$

The upper bound of the probability of signal n, $\mathbb{P}(s=n|\sigma)$, is obtained when no rational firm sells data under σ given belief $\vec{\mu}_0$. The value function of firm 1 if it deviates once in the first period (denote the strategy by σ') is:

$$V_{1,dev}(\vec{\mu}_0) = (1-\delta)\left(\frac{v^2}{2t} + D(0)\right) + \delta\left[\mathbb{P}(s=y|\sigma')V_1(\vec{\mu}_0^y) + \mathbb{P}(s=n|\sigma')V_1(\vec{\mu}_0^n)\right]$$

, where $\begin{cases}\mathbb{P}(s=n|\sigma') = (1-q)\mathbb{P}(s=n|\sigma) \\ \mathbb{P}(s=y|\sigma') = 1-\mathbb{P}(s=n|\sigma')\end{cases}$ Therefore, we have:

$$V_{1,dev}(\mu_0) - V_1(\mu_0) = (1-\delta)D(0) - \delta\left[V_1(\mu_0^n) - V_1(\mu_0^y)\right]q\mathbb{P}(s=n|\sigma)$$

Since $V_1(\cdot) \in [\frac{v^2}{2t}, \frac{v}{2} + D(1-\frac{v}{t})]$, $V_1(\mu_0^n) - V_1(\mu_0^y) \leq \frac{v}{2} + D(1-\frac{v}{t}) - \frac{v^2}{2t}$, which is a constant. $\mathbb{P}(s=n|\sigma) \leq (1-q\mu_0)^{N-1} \to 0$ $(N \to +\infty)$. Hence, given δ, $\exists N_\delta$ s.t. $\forall N \geq N_\delta$, $V_{1,dev}(\mu_0) - V_1(\mu_0) > 0$. But we assume that firm 1 does not sell the data at $t=0$. A contradiction.

In sum, firms always sell data and consumers reveal nothing at $t = 0$ in equilibrium. Anticipating that, the consumer's posterior belief about each firm's type is μ_0. This repeats in every period. Therefore, for any $\delta \in (0,1), \exists N_\delta$ $s.t.$ $\forall N \geq N_\delta$, firms always sell data and consumers reveal nothing in the unique MPE. Consumer's belief about the firm's type is always μ_0.

Suppose $\mu_0 \leq \widehat{\mu}$. The first period payoff will be $\frac{v}{2}$ in equilibrium and $\frac{v}{2} + D(1 - \frac{v}{t})$ if the firm deviates. All the remaining proof is the same as above. $\square$

Proof of Proposition 6. Consider firm 1 WLOG. We first list the updated belief after one signal:

$$\begin{cases} \mu^y = \mathbb{P}(\text{firm 1 is bad type}|s = y,\ \text{initial belief is } \mu) = \frac{1-(1-q)(1-q\mu)}{1-(1-q\mu)^2}\mu \\ \mu^n = \mathbb{P}(\text{firm 1 is bad type}|s = n,\ \text{initial belief is } \mu) = \frac{1-q}{1-q\mu}\mu \end{cases}$$

Both μ^y and μ^n increase in μ. $\mu^y \geq 1/2,\ \forall \mu$.

Suppose there exists an equilibrium in which rational firms never sells the data. Then consumers have identical beliefs for both firms. Denote the corresponding value function by $V(\cdot)$. Consider a belief $\mu > \widehat{\mu}$.

$$V(\mu) = (1-\delta)\frac{v^2}{2t} + \delta\left[q\mu V(\mu^y) + (1-q\mu)V(\mu^n)\right]$$

The value function of deviating once in the current period is:

$$V_{dev}(\mu) = (1-\delta)\left(\frac{v^2}{2t} + D(0)\right) + \delta\left[q[1+(1-q)\mu]V(\mu^y) + (1-q)(1-q\mu)V(\mu^n)\right]$$

$$V_{dev}(\mu) - V(\mu) = (1-\delta)D(0) - \delta\left[V(\mu^n) - V(\mu^y)\right]q(1-q\mu) \tag{4}$$

$v < 2u_b \Rightarrow \widehat{\mu} < 1/2$. $\mu^y \geq 1/2,\ \forall \mu$ implies that consumer will reveal no information after one signal y, which gives the rational firm a stage equilibrium payoff of $\frac{v^2}{2t}$. If the signal is n and $\mu^n \leq \widehat{\mu}$, rational firm gets a stage payoff of $v/2$; If the signal is n and $\mu^n > \widehat{\mu}$, rational firm gets a stage payoff of $\frac{v^2}{2t}$. So, we gets an upper bound of $V(\mu^n)$ by assuming that the belief is always no greater than $\widehat{\mu}$:

$$\begin{aligned} V(\mu^n) &\leq (1-\delta)\left[\frac{v}{2} + \sum_{k=1}^{+\infty}\delta^k[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}]\right] \\ &= (1-\delta)\frac{v}{2} + \delta[q\mu\frac{v^2}{2t} + (1-q\mu)\frac{v}{2}] \end{aligned}$$

Always selling consumer data gives a lower bound on the value function:

$$V(\mu^y) \geq (1-\delta)\sum_{k=0}^{+\infty}\delta^k[\frac{v^2}{2t}+D(0)] = \frac{v^2}{2t}+D(0)$$

Hence, we have:

$$V(\mu^n)-V(\mu^y) \leq (1-\delta)\frac{v}{2}+\delta[q\mu\frac{v^2}{2t}+(1-q\mu)\frac{v}{2}]-[\frac{v^2}{2t}+D(0)]$$

Plug it back to (4), we have:

$$\begin{aligned}&V_{dev}(\mu)-V(\mu)\\ \geq&(1-\delta)[D(0)-\delta q(1-q\mu)\frac{v}{2}]-\delta q(1-q\mu)\left[\delta[q\mu\frac{v^2}{2t}+(1-q\mu)\frac{v}{2}]-[\frac{v^2}{2t}+D(0)]\right]\end{aligned} \tag{5}$$

With a strictly positive probability, the signal will be y for k consecutive periods, $\forall k$. Denote the belief after k consecutive signal y by μ^{y^k}. One can see that $\mu^y \in (\mu,1), \forall \mu \in (0,1)$. So, μ^{y^k} strictly increases in k and is bounded by 1. Thus, $\{\mu^{y^k}\}_{k=1}^{+\infty}$ has a limit. Denote the limit by $\mu^{y^{+\infty}}$. We have $(\mu^{y^{+\infty}})^y = \mu^{y^{+\infty}} \Rightarrow \mu^{y^{+\infty}} = 1$. So, μ^{y^k} could be arbitrarily close to 1 with a strictly positive probability. If $(1-q)(v/2-v^2/2t) < D(0)$, for large enough δ and μ, we have $\delta[q\mu\frac{v^2}{2t}+(1-q\mu)\frac{v}{2}]-[\frac{v^2}{2t}+D(0)] < 0$. If $q(1-q)v/2 < D(0)$, for large enough δ and μ, we have $D(0)-\delta q(1-q\mu)\frac{v}{2} > 0$. Together, we get that $(1-\delta)[D(0)-\delta q(1-q\mu)\frac{v}{2}]-\delta q(1-q\mu)\left[\delta[q\mu\frac{v^2}{2t}+(1-q\mu)\frac{v}{2}]-[\frac{v^2}{2t}+D(0)]\right] > 0, \overset{(5)}{\Rightarrow} V_{dev}(\mu)-V(\mu) > 0$. Therefore, rational firm will sell the data when the belief is μ and the discount factor is high enough. A contradiction. $\square$

Proof of Proposition 7. Suppose there exists such an equilibrium.

(1) $\mu \leq \widehat{\mu}$

Without any monitoring effort, the consumer does not get an additional signal. So, the consumer reveal $1-v/t$ amount of information according to Proposition 1. The expected consumer surplus is:

$$CS(0,\mu) := -\mu u_b(1-v/t)+v/4$$

By incuring effort h, the consumer receives an extra signal s_h. The updated belief will be:

$$\begin{cases}\frac{1-h}{1-h\mu}\mu, & if\ \ s_h = n \text{ (with probability } 1-\mu h)\\ 1, & if\ \ s_h = y \text{(with probability } \mu h)\end{cases}$$

The expected consumer surplus is:[14]

$$CS(h,\mu) := -c(h) - \frac{1-h}{1-h\mu}\mu u_b(1-\frac{v}{t})(1-\mu h) + \frac{v^2}{4t}\mu h + \frac{v}{4}(1-\mu h),\ \ h \in [0,\bar{h}]$$

The difference of the expected consumer surplus between incurring monitoring effort h and no effort is:

$$\Delta CS(h,\mu) := CS(h,\mu) - CS(0,\mu) = -c(h) + \mu\frac{v}{4t}(t-v)h\left[-1 + \frac{4u_b(1-\mu)}{v(1-\mu h)}\right],\ \ h \in [0,\bar{h}] \tag{6}$$

The consumer incurs a strictly monitoring effort if and only if $\Delta CS(h,\mu) > 0$ for some $h \in (0,\bar{h}]$. Notice that $\Delta CS(0,\mu) = 0$. So, a sufficient condition for the consumer to incur a strictly monitoring effort is $\frac{\partial \Delta CS(h,\mu)}{\partial h}|_{h=0} > 0$.

$$\begin{aligned}
\frac{\partial \Delta CS(h,\mu)}{\partial h} &= -\, c'(h) + \frac{\mu v(t-v)}{4t}\left[-1 + \frac{4u_b(1-\mu)}{v(1-\mu h)^2}\right] \\
\frac{\partial \Delta CS(h,\mu)}{\partial h}|_{h=0} > 0 \Leftrightarrow &- 1 + \frac{4u_b(1-\mu)}{v} > 0 \\
\Leftrightarrow &\mu < 1 - \widehat{\mu}
\end{aligned}$$

If $v < 2u_b$, $\widehat{\mu} < 1/2 \Rightarrow \mu \leq \widehat{\mu} < 1 - \widehat{\mu}$, $\forall \mu \leq \widehat{\mu}$. So, the consumer always incurs a strictly positive monitoring effort when $\mu < \widehat{\mu}$.

Equation (6) implies that $h^*(\mu) \to 0$ as $\mu \to 0$, since $\Delta CS(h^*(\mu),\mu) \geq 0$

(2) $\mu > \widehat{\mu}$

Without any monitoring effort, the consumer does not get an additional signal. So, the consumer reveal nothing according to Proposition 1. The expected consumer surplus is:

$$\widetilde{CS}(0,\mu) := v^2/4t$$

By incuring effort h, the consumer receives an extra signal s_h. The updated belief will be:

$$\begin{cases} \frac{1-h}{1-h\mu}\mu, & if\ s_h = n \text{ (with probability } 1-\mu h) \\ 1, & if\ s_h = y \text{(with probability } \mu h) \end{cases}$$

If μ is high enough such that $\frac{1-\bar{h}}{1-\bar{h}\mu}\mu > \widehat{\mu}$, the consumer will not reveal anything regardless

[14] Technically, the domin is $h \in (0,\bar{h}]$. But, one can check that the expression holds for $h = 0$ as well.

of the signal realization. So, there is no gain from an additional signal and the consumer will not acquire an extra signal. For the consumer to incur costly monitoring, she must reveal some information ($\eta = 1 - v/t$) if $s_h = n$.[15] The expected consumer surplus is the same as the first case:

$$\widetilde{CS}(h,\mu) = CS(h,\mu) = -c(h) - \frac{1-h}{1-h\mu}\mu u_b(1-\frac{v}{t})(1-\mu h) + \frac{v^2}{4t}\mu h + \frac{v}{4}(1-\mu h),\ h \in (0,\bar{h}]$$

The difference of the expected consumer surplus between incurring monitoring effort h and no effort is:

$$\Delta\widetilde{CS}(h,\mu) := \widetilde{CS}(h,\mu) - \widetilde{CS}(0,\mu) = -c(h) + (1-\mu h)\frac{v(t-v)}{4t}\left[1 - \frac{4u_b(1-h)\mu}{v(1-\mu h)}\right],\ h \in (0,\bar{h}] \tag{7}$$

The consumer incurs a strictly monitoring effort if and only if $\Delta\widetilde{CS}(h,\mu) > 0$ for some $h \in (0,\bar{h}]$.

$$\Delta\frac{\partial\widetilde{CS}(h,\mu)}{\partial h} = -\,c'(h) + \frac{\mu v(t-v)}{4t}\left(\frac{4u_b}{v} - 1\right) \tag{8}$$

Since $c'(0) = 0$, $c(\cdot)$ is convex, and $\lim_{h\to\bar{h}} c'(h) = +\infty$, we have $\max_{0<h\leq\bar{h}} \Delta\widetilde{CS}(h,\mu) = \Delta\widetilde{CS}(\widehat{h}(\mu),\mu)$, where $\widehat{h}(\mu)(>0)$ is determined by the first order condition:

$$c'(\widehat{h}(\mu)) = \frac{\mu v(t-v)}{4t}\left(\frac{4u_b}{v} - 1\right) \tag{9}$$

The consumer's optimal effort is either 0 or $\widehat{h}(\mu)$. This leads to the following lemma.

Lemma 1. *Suppose the belief is $\mu > \widehat{\mu}$. The consumer incurs monitoring effort $\widehat{h}(\mu)$ if and only if $\Delta\widetilde{CS}(\widehat{h}(\mu),\mu) > 0$.*

The next lemma characterize the optimal effort of the consumer.

Lemma 2. *There exists a $\widehat{\widehat{\mu}} > \widehat{\mu}$ such that the consumer incurs efforts in monitoring if and only if $\mu \leq \widehat{\widehat{\mu}}$. For $\mu \in [\widehat{\mu}, \widehat{\widehat{\mu}}]$, the optimal effort $h^*(\mu)$ strictly increases in μ.*

Proof. We first show that the optimal effort follows a cutoff strategy. Suppose the consumer incurs monitoring effort $\widehat{h}(\mu_1) > 0$ when the belief is $\mu_1 > \widehat{\mu}$. $\Delta\widetilde{CS}(h)$ and $\widehat{h}$ depend on μ. Lemma 1 implies that $\Delta\widetilde{CS}(\widehat{h}(\mu_1),\mu_1) > 0$. $\forall \mu_2 \in (\widehat{\mu},\mu_1)$. Since

[15] This may be worse than revealing nothing for the consumer. But the latter is always dominated by not incurring monitoring costs.

$\Delta\widetilde{CS}(h,\mu)$ strictly decreases in μ, we have that $\Delta\widetilde{CS}(\widehat{h}(\mu_2),\mu_2) \geq \widetilde{CS}(\widehat{h}(\mu_1),\mu_2) > \Delta\widetilde{CS}(\widehat{h}(\mu_1),\mu_1) > 0$, where the first inequality is by the optimality of $\widehat{h}(\mu_2)$ for $\widetilde{CS}(h,\mu_2),\ h \in (0,\bar{h}]$. Therefore, there exists a $\widehat{\widehat{\mu}} \geq \widehat{\mu}$ such that the consumer incurs efforts in monitoring if and only if $\mu \leq \widehat{\widehat{\mu}}$. For $\mu \in [\widehat{\mu}, \widehat{\widehat{\mu}}]$, the optimal effort $h^*(\mu) = \widehat{h}(\mu)$. According to equation (9), $\widehat{h}(\mu)$ strictly increases in μ.

We now show that $\widehat{\widehat{\mu}} > \widehat{\mu}$.This can be shown by consider $\mu = \widehat{\mu} + \varepsilon, h = \sqrt{\varepsilon}$. Taking Taylor expansion in the expression for $\Delta\widetilde{CS}(h,\mu)$ and let $\varepsilon \to 0$ gives the result.

So, we finish the proof of the lemma. □

Consumer optimality has been shown in the above analyses. Noticing that the benefit for not selling data and the penalty for selling data are higher under endogenous monitoring. One can see that the rational firm has no incentive to deviate when it is patient enough by similar arguments as the proof of Proposition 3.

□

Proof of Proposition 8. The proof of Proposition 5 applies to this case as well. □

Proof of Proposition 9. Fix $\delta \in (0,1)$. Suppose $\forall N_\delta,\ \exists N \geq N_\delta$ s.t. there exists a MPE in which a rational firm j does not sell the data at $t = 0$. Denote the equilibrium strategy of all the firms by σ and the value function of firm i by $V_i(\cdot)$. The prior belief is $\vec{\mu}_0 = (\mu_0, \mu_0, ..., \mu_0)$. Denote the posterior belief upon observing signal y (n) by $\vec{\mu}^y$ $(\vec{\mu}^n)$ when the initial belief is $\vec{\mu}$ and the equilibrium strategy is σ.

Suppose $\mu_0 > \widehat{\mu}$. There are two possibilities:

(1) $j = 1$

$$V_1(\vec{\mu}_0) = (1-\delta)\frac{v^2}{2t} + \delta\left[\mathbb{P}(s=y|\sigma)V_1(\vec{\mu}_0^y) + \mathbb{P}(s=n|\sigma)V_1(\vec{\mu}_0^n)\right]$$

, where $\begin{cases} \mathbb{P}(s=n|\sigma) \leq (1-\alpha q\mu_0)^{N-1} \\ \mathbb{P}(s=y|\sigma) = 1-\mathbb{P}(s=n|\sigma) \geq 1-(1-\alpha q\mu_0)^{N-1} \end{cases}$

The upper bound of the probability of signal n, $\mathbb{P}(s=n|\sigma)$, is obtained when no rational firm sells data under σ given belief $\vec{\mu}_0$. The value function of firm 1 if it deviates once in the first period (denote the strategy by σ') is:

$$V_{1,dev}(\vec{\mu}_0) = (1-\delta)\left(\frac{v^2}{2t} + D(0)\right) + \delta\left[\mathbb{P}(s=y|\sigma')V_1(\vec{\mu}_0^y) + \mathbb{P}(s=n|\sigma')V_1(\vec{\mu}_0^n)\right]$$

, where $\begin{cases} \mathbb{P}(s=n|\sigma') = (1-q)\mathbb{P}(s=n|\sigma) \\ \mathbb{P}(s=y|\sigma') = 1-\mathbb{P}(s=n|\sigma') \end{cases}$ Therefore, we have:

$$V_{1,dev}(\mu_0) - V_1(\mu_0) = (1-\delta)D(0) - \delta\left[V_1(\mu_0^n) - V_1(\mu_0^y)\right] q\mathbb{P}(s=n|\sigma)$$

Since $V_1(\cdot) \in [\frac{v^2}{2t}, \frac{v}{2} + D(1-\frac{v}{t})]$, $V_1(\mu_0^n) - V_1(\mu_0^y) \leq \frac{v}{2} + D(1-\frac{v}{t}) - \frac{v^2}{2t}$, which is a constant. $\mathbb{P}(s=n|\sigma) \leq (1-\alpha q\mu_0)^{N-1} \to 0$ $(N \to +\infty)$. Hence, given δ, $\exists N_\delta$ s.t. $\forall N \geq N_\delta$, $V_{1,dev}(\mu_0) - V_1(\mu_0) > 0$. But we assume that firm 1 does not sell the data at $t=0$. A contradiction.

(2) $j \neq 1$

$$V_j(\vec{\mu_0}) = (1-\delta)\frac{v^2}{2t} + \delta\left[\mathbb{P}(s=y|\sigma)V_j(\vec{\mu}_0^y) + \mathbb{P}(s=n|\sigma)V_j(\vec{\mu}_0^n)\right]$$

, where $\begin{cases} \mathbb{P}(s=n|\sigma) \leq (1-q\mu_0)(1-\alpha q\mu_0)^{N-2} \\ \mathbb{P}(s=y|\sigma) = 1-\mathbb{P}(s=n|\sigma) \geq 1-(1-q\mu_0)(1-\alpha q\mu_0)^{N-2} \end{cases}$

The upper bound of the probability of signal n, $\mathbb{P}(s=n|\sigma)$, is obtained when no rational firm sells data under σ given belief $\vec{\mu}_0$. The value function of firm j if it deviates once in the first period (denote the strategy by σ') is:

$$V_{j,dev}(\vec{\mu}_0) = (1-\delta)\left(\frac{v^2}{2t} + D(0)\right) + \delta\left[\mathbb{P}(s=y|\sigma')V_j(\vec{\mu}_0^y) + \mathbb{P}(s=n|\sigma')V_j(\vec{\mu}_0^n)\right]$$

, where $\begin{cases} \mathbb{P}(s=n|\sigma') = (1-\alpha q)\mathbb{P}(s=n|\sigma) \\ \mathbb{P}(s=y|\sigma') = 1-\mathbb{P}(s=n|\sigma') \end{cases}$ Therefore, we have:

$$V_{j,dev}(\mu_0) - V_j(\mu_0) = (1-\delta)D(0) - \delta\left[V_j(\mu_0^n) - V_j(\mu_0^y)\right] \alpha q\mathbb{P}(s=n|\sigma)$$

Since $V_j(\cdot) \in [\frac{v^2}{2t}, \frac{v}{2} + D(1-\frac{v}{t})]$, $V_j(\mu_0^n) - V_j(\mu_0^y) \leq \frac{v}{2} + D(1-\frac{v}{t}) - \frac{v^2}{2t}$, which is a constant. $\mathbb{P}(s=n|\sigma) \leq (1-q\mu_0)(1-\alpha q\mu_0)^{N-2} \to 0$ $(N \to +\infty)$. Hence, given δ, $\exists N_\delta$ s.t. $\forall N \geq N_\delta$, $V_{j,dev}(\mu_0) - V_j(\mu_0) > 0$. But we assume that firm j does not sell the data at $t=0$. A contradiction.

In sum, firms always sell data and consumers reveal nothing at $t=0$ in equilibrium. Anticipating that, the consumer's posterior belief about each firm's type is μ_0. This repeats in every period. Therefore, for any $\delta \in (0,1), \exists N_\delta$ $s.t.$ $\forall N \geq N_\delta$, firms always sell data and consumers reveal nothing in the unique MPE. Consumer's belief about the firm's type is always μ_0.

Suppose $\mu_0 \leq \widehat{\mu}$. The first period payoff will be $\frac{v}{2}$ in equilibrium and $\frac{v}{2} + D(1 - \frac{v}{t})$ if the firm deviates. All the remaining proof is the same as above. $\square$