Day 17/18/19 Guided Notes

Name: **Key**   Date: **7/15**   1

OBJECTIVE: We will be able to···

1. Comprehend that solving systems of equations involves finding values of the variables that satisfy both equations simultaneously

2. Coordinate this understanding by solving equations with variables on each side

3. Recognize these solutions as points of intersection in systems of two linear equations.

Day 17: Representing 2 Separate Relationships as Systems of Equations

What are Systems of Equations?

- A set of **2 or more** linear equations working together

Ex: I have 2 different kinds of bamboo planted at the same time. On which day would both plants be the same height?

- Plant A starts at 6 ft tall and grows $\frac{1}{4}$ foot each day
- Plant B starts at 3 ft tall and grows $\frac{1}{2}$ foot each day

- How could this be represented algebraically?

Algebraically Representation: **x = # of days**

Plant A:

$$6 + \frac{1}{4}x$$

Plant B:

$$3 + \frac{1}{2}x$$ **Practice Problems: Situations and Systems**

Consider the following scenarios. Write a system of expressions (without solving) and interpret what the solution to the system would tell you about the situation.

1. Sam and Alex start working on their homework at the same time. Sam starts with 5 problems already done and completes 4 problems each hour. Alex starts with 8 problems already done and completes 3 problems each hour.

   Equation: initial # of problems + problems solved per hour

   Sam: $5 + 4x$
   Alex: $8 + 3x$

   $x =$ har(s)

2. A town's water tank has 1000 liters of water and 200 liters is added every day. Another tank starts with 3000 liters but is used up at a rate of 100 liters every day.

   Equation 1: (initial liters of water) + liters add per day
   Equation 2: (initial liters of water) − liters used per day

   TANK A: $1{,}000 + 200d$
   TANK B: $3{,}000 - 100d$

   $d = $ # of days

3. Jordan is looking to join fitness classes. Fitness Studio A charges a base fee of $25 plus $7 per class. Fitness Studio B charges a base fee of $20 plus $8 per class.

   Equation: $ base fee + $ charge per class

   A: $\$25 + 7C$

   B: $\$20 + 8C$

   $C = $ # of classes

4. Warren and Danielle are both making paintings at the same time. Warren made 10 Paintings yesterday, and is completing 1 painting every hour today. Danielle made 4 paintings yesterday, but is making 5 paintings every hour today.

Equation: (initial # of paintings) + # of paintings per hour

$$W: 10 + 1h$$

$$D: 4 + 5h$$

$$h = \text{\# of hours}$$

5. Sarah is comparing two cell phone plans. Plan A charges a flat fee of $20 per month plus $0.10 per text message. Plan B charges a flat fee of $15 per month plus $0.15 per text message. Write a system of expressions to represent the total monthly cost for each plan based on the number of text messages sent.

Monthly Equation: $ flat month rate + $ rate per text message

monthly: Plan A: $20 + .10m

Plan B: $15 + .15m

↳ m represents # of text messages used monthly.

Student Solution (class discussion):

↳ Solution: Plan A: $20x + 0.10m

Plan B: $15x + 0.15m

* m = message(s)
* x = # of months

* This only works if you are solving for the annual or <u>total</u> amount paid over-time because it has a variable (x) to substitute # of months with the rate per text message
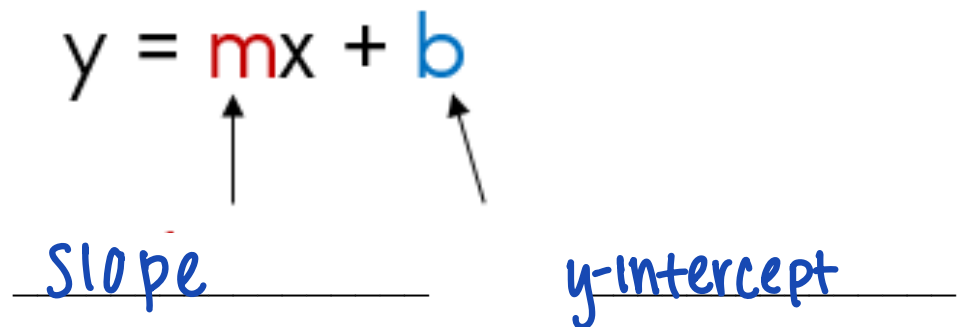
Day 18: Graphically representing Systems of Equations Method 1: Table

Video Notes:

✳ Student complete on their own for Khan Academy video.

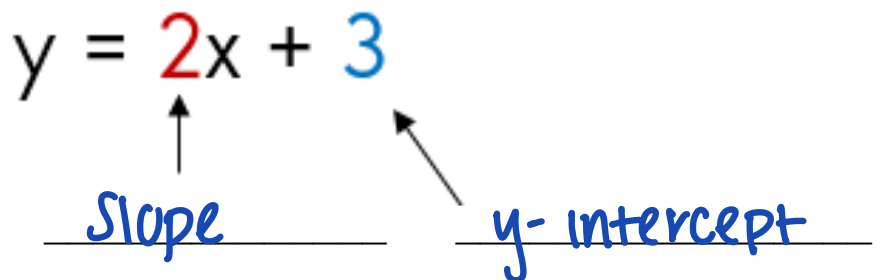### Slope-Intercept Form of Linear Equation

$$y = mx + b$$

Slope _____   y-intercept _____

Example:

$$y = 2x + 3$$

Slope _____   y- intercept _____

✳ y-intercept can be found on a graph by looking at the point that crosses the y-axis.

Example 2: Let's review the bamboo problem from yesterday: What is the system of equations created for Plant A and Plant B?

Plant A: $y = \frac{1}{4}x + 6$ **Plant B:** $y = \frac{1}{2}x + 3$

Fill out the table below for x and y values for each Plant based on the equation:

Plant A

x	y
0	6
1	$6.25 = 6\frac{1}{4}$
2	6.5
4	7
6	7.5
8	8
10	8.5
12	9

Plant B

x	y
0	3
1	3.5
2	4
4	5
6	6
8	7
10	8
12	9

Question: At what ~~week~~ day would they both be the same height? At what height?

At 12 days both plants will be 9ft tall.

Flip back to Practice Problems on page 2.

✱ Work will differ based on problem selected.

1. Write down the system of equations you created for the scenario below using slope intercept form:

→ Problem #3 (Jordan)

Equation 1:

A: $25 + 7C

$y = 7c + 25$

Equation 2:

B: $20 + 8C

$y = 8c + 20$

2. Create a table for each equation (there should be 2 separate tables like Plant A and B from page 4):

$y = 7c + 25$	
X	y
0	25
2	39
4	53
6	67

$y = 8c + 20$	
X	y
0	20
2	36
4	52
6	68

X = 5 ⇒ both equal 60

3. Explain your solution for the system of equations in context of the problem and provide your solution in coordinate form.

At 5 months, Jordan will pay $60 at both studios. However, after 5 months Plan B will have a slightly higher monthy rate than plan A.

Day 19: Graphing Systems of Equations

➔ To graph each linear equation we have in slope-intercept form, we need to find the
_____**slope**_____ and the ___**y-intercept**___.

- **y-intercept**: where the line __**intersects**__ the y-axis which is written as
  ___**(0, y)**___ in coordinate form **where x is always 0**

recall : (x, y)

- **Slope:** is represented as ____**rise**____ over ____**run**____, it is often written in
  ____**fraction**____ form mathematically as: **(units up or down between two selected points which depends on increasing or decreasing function)**
  ✻ run is the units shifted vertically to reach another point after finding the rise.

Note: slope can be calculated algebraically:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

↳ where we select two distinct points on a line graph and substitute the coordinate:
Point 1: (x_1, y_1)
Point 2: (x_2, y_2)

Intro Example of Applying the Skills:

1. Consider the following Graph:

   a. Find the intercepts of the given graph

   ↳ **y-intercept at the point (0, 1)**

   ↳ **x-intercept at the point (-0.5, 0)**

point 2 (1, 3)
y-intercept
point 1 (0, 1)
x-intercept where y-value is always 0 and crosses x-axis.

   b. Find the slope of the graph

   Method 1 on graph:

   slope = $\frac{rise}{run}$ = $\frac{2 \text{ units up}}{1 \text{ unit to right}}$ = $\frac{2}{1}$ = **2**

   Method 2:

   $m = \frac{(y_2 - y_1)}{(x_2 - x_1)} = \frac{3-1}{1-0} = \frac{2}{1} = \boxed{2}$ ← same value
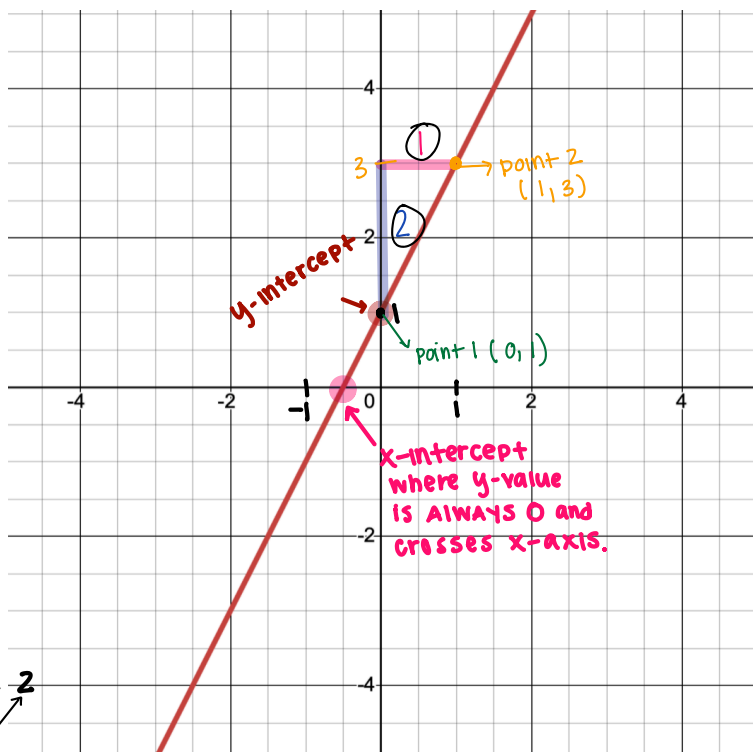
   Point 1: $(0, 1)$ Point 2: $(1, 3)$

   c. Write the linear equation in slope intercept form that represents the given graph.

   Slope Intercept Form: $y = mx + b$ ← y-intercept
                                        ↑
                                      slope

$$\boxed{y = 2x + 1}$$

Second Example:
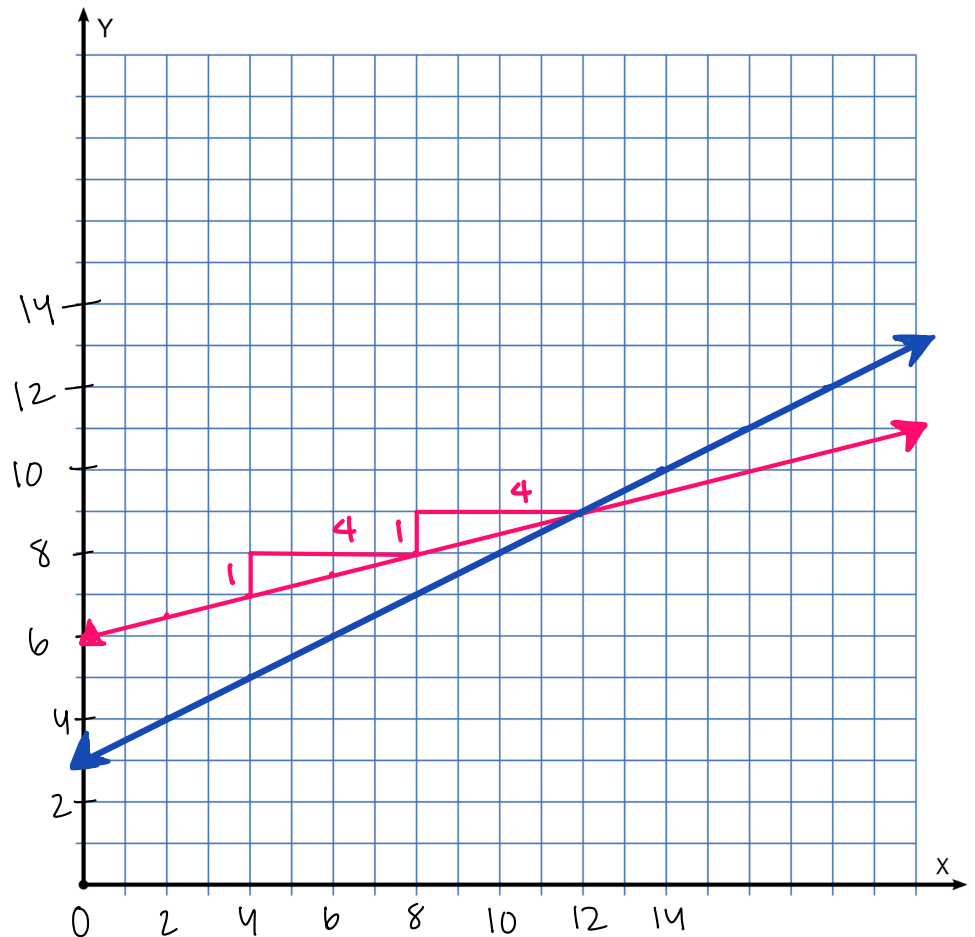
Let's try graph the bamboo problem onto the graph below:

Equation 1:

$$y = \frac{1}{4}x + 6$$

Equation 2:

$$y = \frac{1}{2}x + 3$$

X	y	X	y
0	6	0	3
2	6.5	2	4
4	7	4	5
6	7.5	6	6
8	8	8	7
10	8.5	10	8
12	9	12	9

The graph tells me that...

1. There is a __point__ of __intersection__ at the coordinate: __(12, 9)__ .

   ➤ Solving system of equations means finding the __solution__ to the equations that make both __equations__ equal and __true__ .

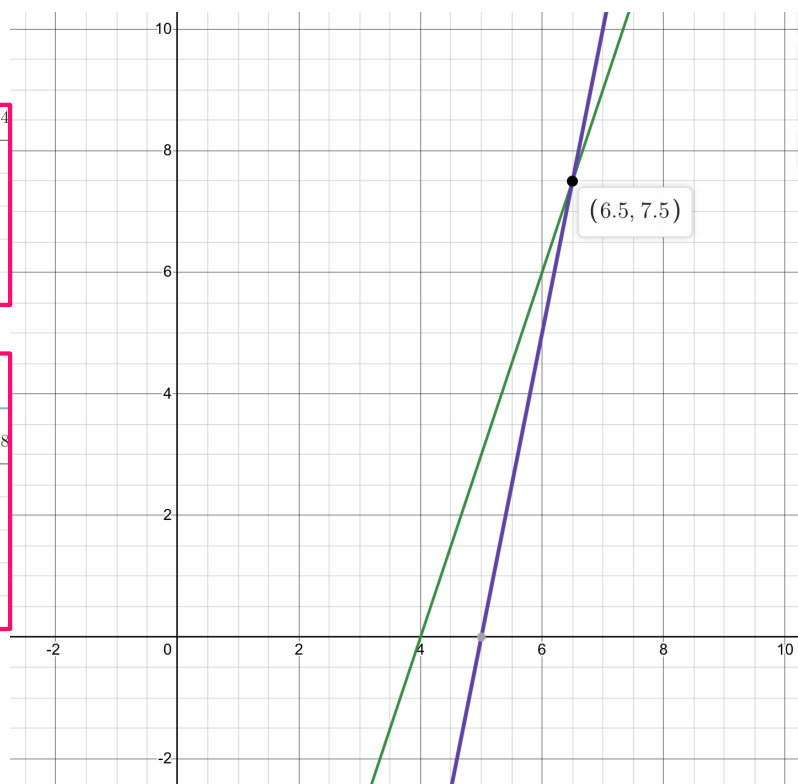## PRACTICE PROBLEMS

1. Consider the two following equations:

   I. $y = 3x + 2 - 14$ → $y = 3x - 12$

   II. $y = 5x - 7 - 18$ → $y = 5x - 25$

   a. Graph the system of equations onto the graph

   b. Label the axes

   c. Does there exist a point of intersection? If so, state the coordinate point notation and respond to what does this tell about the situation?

x	$3x + 2 - 14$
-2	-18
-1	-15
0	-12
1	-9
2	-6

$= 5x - 7 - 18$

x	$5x - 7 - 18$
-2	-35
-1	-30
0	-25
1	-20
2	-15

$(6.5, 7.5)$

Equation 1

- Slope: __3__

- y-intercept: __$(0, -12)$__

Equation 2:

- Slope: __5__

- y-intercept: __$(0, -25)$__

This tells me that....

both equations are equal to each other and there is a point of intersection at coordinate: $(6.5, 7.5)$
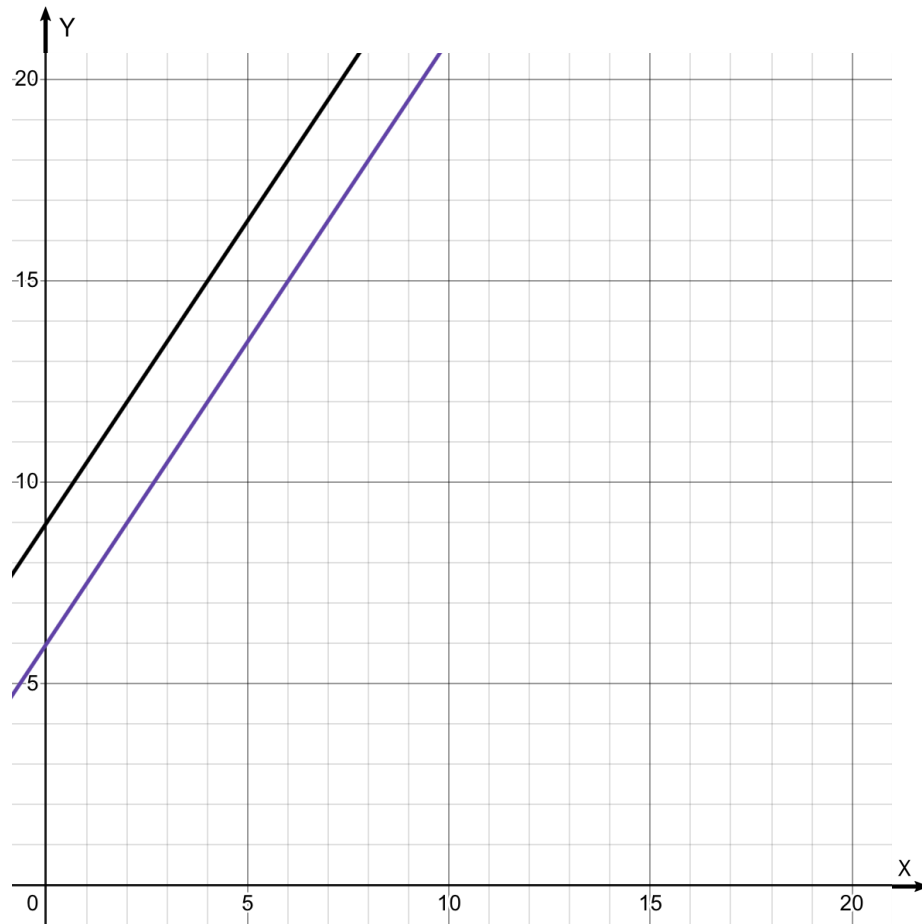
2. A stack of *n* small cups have a height, *h*, in centimeters represented by $h = 1.5n + 6.$ A stack of *n* large cups have a height, *h*, represented by $h = 1.5h + 9.$

   a. Graph each of the equations for each of the cups on the same Cartesian plane.

   b. Label your axis

   c. At what value of cups will the heights be the same? (provide the coordinate form solution)

         ↳ The two following lines will never intersect and remain parallel so there is no solution to when both cups will have the same height.