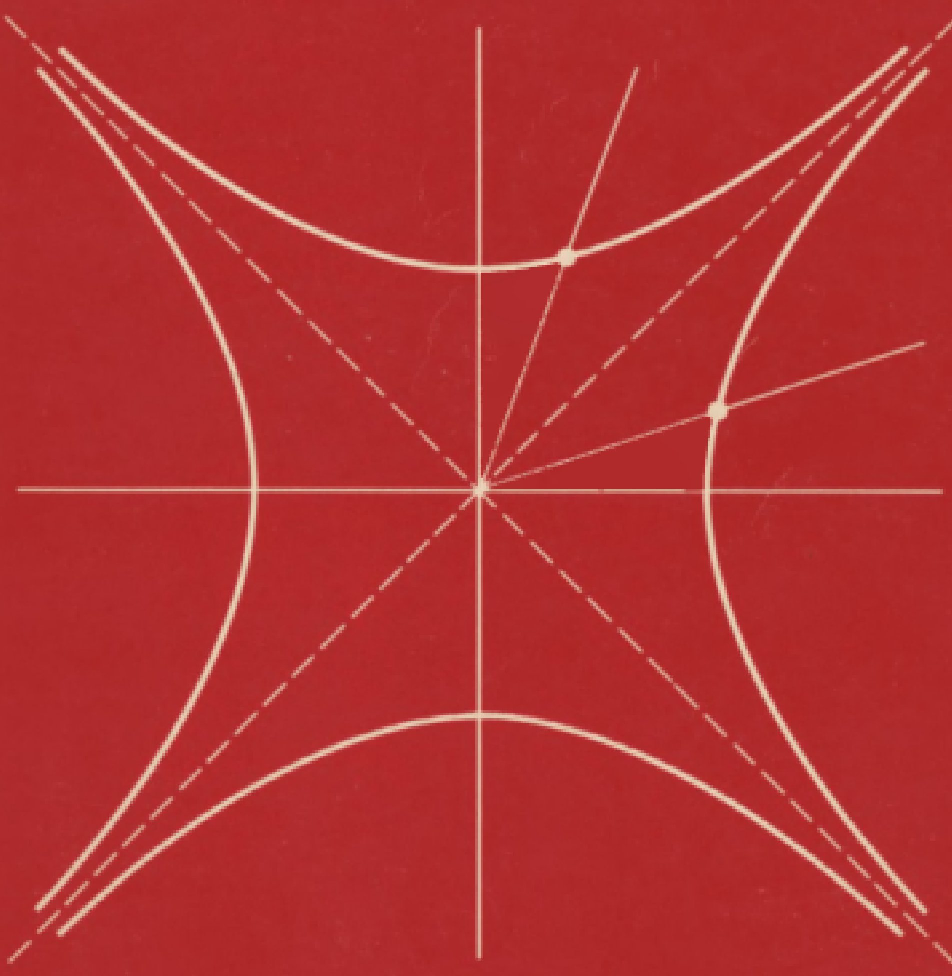
INTRODUCTIO
TO SPECIAL RELATIVIT

ROBERT RESNI



Introduction to Special Relativity

Robert Resnick
Professor of Physics
Rensselaer Polytechnic Institute

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To

Trudy Resnick

In loving memory

Preface

This book can be regarded as the first part of a text on "modern physics." Indeed, it is the basis for the treatment of relativity in such a text that I am now writing with Robert Eisberg. That text will be the third and concluding volume of a series on introductory physics, the first two volumes of which I have written with David Halliday.

However, the material in this *Introduction to Special Relativity* has a coherence of its own and can be used in many ways. In the two-year introductory physics course at Rensselaer, for example, these chapters build upon the background in electromagnetism and optics of the Halliday-Resnick text and precede the full development of quantum physics. Applications of relativity to certain areas, such as high-energy physics, are given when, as in our third volume, those areas are presented later. There are other ways to use these chapters, as well. For instance, they can be integrated easily with the classical material. The early chapters on the experimental background and kinematic aspects of relativity could follow immediately the development of Newtonian mechanics, as could much of the relativistic dynamics, whereas the electromagnetic aspects of relativity could follow the presentation of Maxwell's equations. Or, this book could replace the brief and sketchy treatment of the foundations of relativity characteristic of modern physics courses of the immediate past. Still other uses will suggest themselves to physics instructors.

A good deal of optional material is presented here not only because of its intrinsic interest but also to permit the instructor to vary the length and depth of his treatment. Thus, in separate appendices, there are supplementary topics on the geometrical representation of space-time, on the twin paradox, and on the principle of equivalence and general relativity. Also, in the body of the text, some material of an historical, an advanced, or a special nature is printed in reduced type for optional use. Similarly, the problems and thought questions, nearly 250 in number, span a wide range of content and level of difficulty so that the impact of the course can be altered significantly by the choice of which ones and how many are assigned. Many references are cited especially to encourage students to read widely in relativity. The writing is expansive, however, so that the book is self-contained. Pedagogic aids, such as summary tables and worked-out examples, are employed to help the student to learn on his own.

Writing this book has been a labor of love. Relativity has always been a favorite subject of mine, and Einstein was one of the heroes of my youth. Over two decades ago, Franco Rasetti impressed the beauty of the subject upon me in a course at The Johns Hopkins University. Also I was much influenced by the relativity treatments in the classic advanced texts of Peter Bergmann and of Wolfgang Panofsky and Melba Phillips. In revising my notes through successive drafts, classroom trials, and production, I have received constructive criticism or other valuable assistance from many individuals, especially Richard Albagli, Kenneth Brownstein, Benjamin Chi, Robert Eisberg, David Halliday, and Roland Lichtenstein. I am grateful to Mrs. Cassie Young for her skill and dedication in typing the many versions of the notes and to the publishers, John Wiley and Sons, Inc., for their outstanding cooperation. To my wife and daughters, whose forbearance over years of writing is nearly habitual, my deepest thanks. My release from some other duties during the preparation of the manuscript was made possible in part by a Ford Foundation grant to the Engineering School at Rensselaer for curricular development.

It is my earnest hope that this effort will make relativity accessible to beginning students and arouse in them some of the excitement that is physics.

Robert Resnick

Troy, New York
January 1968

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to

Special Relativity

Chapter One

The Experimental Background of the Theory of Special Relativity

1.1 *Introduction*

To send a signal through free space from one point to another as fast as possible, we use a beam of light or some other electromagnetic radiation such as a radio wave. *No faster method of signaling has ever been discovered.* This experimental fact suggests that the speed of light in free space, c $(= 3.00 \times 10^8$ m/sec),* is an appropriate limiting reference speed to which other speeds, such as the speeds of particles or of mechanical waves, can be compared.

In the macroscopic world of our ordinary experiences, the speed u of moving objects or mechanical waves with respect to any observer is always less than c. For example, an artificial satellite circling the earth may move at 18,000 mph with respect to the earth; here $u/c = 0.000027$. Sound waves in air at room temperature move at 332 m/sec through the air so that $u/c = 0.0000010$. It is in this ever-present, but limited, macroscopic environment that our ideas about space and time are first formulated and in which Newton developed his system of mechanics.

In the microscopic world it is readily possible to find particles whose speeds are quite close to that of light. For an electron accelerated through a 10-million-volt potential difference, a value reasonably easy to obtain, the speed u equals $0.9988c$. We cannot be certain without direct experimental test that Newtonian mechanics can be safely extrapolated from the ordinary region of low speeds $(u/c \ll 1)$ in which it was developed to this high-speed region $(u/c \to 1)$. Experiment shows, in fact, that Newtonian mechanics does *not* predict the correct answers when it is applied to such fast particles. Indeed, in Newtonian mechanics there is no limit in principle to the speed attainable by a particle, so that the speed of light c should play no special role at all. And yet, if the energy of the 10 Mev electron above is increased by a factor of four (to 40 Mev) experiment [1] shows that the speed is not doubled to $1.9976c$, as we might

*The presently accepted value of the speed of light is $2.997925 \pm 0.000003 \times 10^8$ m/sec expect from the Newtonian relation $K = \frac{1}{2}Mv^2$, but remains below c; it increases only from $0.9988c$ to $0.9999c$, a change of 0.11 percent. Or, if the 10 Mev electron moves at right angles to a magnetic field of 2.0 weber/m^2, the measured radius of curvature of its path is not 0.53 cm (as may be computed from the classical relation $r = m_e v/qB$) but, instead, 1.8 cm. Hence, no matter how well Newtonian mechanics may work at low speeds, it fails badly as $u/c \rightarrow 1$.

In 1905 Albert Einstein published his special theory of relativity. Although motivated by a desire to gain deeper insight into the nature of electromagnetism, Einstein, in his theory, extended and generalized Newtonian mechanics as well. He correctly predicted the results of mechanical experiments over the complete range of speeds from $u/c = 0$ to $u/c \rightarrow 1$. Newtonian mechanics was revealed to be an important special case of a more general theory. In developing this theory of relativity, Einstein critically examined the procedures used to measure length and time intervals. These procedures require the use of light signals and, in fact, an assumption about the way light is propagated is one of the two central hypotheses upon which the theory is based. His theory resulted in a completely new view of the nature of space and time.

The connection between mechanics and electromagnetism is not surprising because light, which (as we shall see) plays a basic role in making the fundamental space and time measurements that underlie mechanics, is an electromagnetic phenomenon. However, our low-speed Newtonian environment is so much a part of our daily life that almost everyone has some conceptual difficulty in understanding Einstein's ideas of space-time when he first studies them. Einstein may have put his finger on the difficulty when he said "Common sense is that layer of prejudices laid down in the mind prior to the age of eighteen." Indeed, it has been said that every great theory begins as a heresy and ends as a prejudice. The ideas of motion of Galileo and Newton may very well have passed through such a history already. More than a half-century of experimentation and application has removed special relativity theory from the heresy stage and put it on a sound conceptual and practical basis. Furthermore, we shall show that a careful analysis of the basic assumptions of Einstein and of Newton makes it clear that the assumptions of Einstein are really much more reasonable than those of Newton.

In the following pages, we shall develop the experimental basis for the ideas of special relativity theory. Because, in retrospect, we found that Newtonian mechanics fails when applied to high-speed particles, it seems wise to begin by examining the foundations of Newtonian mechanics. Perhaps, in this way, we can find clues as to how it might be generalized to yield correct results at high speeds while still maintaining its excellent agreement with experiment at low speeds.

1.2 *Galilean Transformations*

Let us begin by considering a physical *event*. An event is something that happens independently of the reference frame we might use to describe it. For concreteness, we can imagine the event to be a collision of two particles or the turning-on of a tiny light source. The event happens at a point in space and at an instant in time. We specify an event by four (space-time) measurements in a particular frame of reference, say the position numbers x, y, z and the time t. For example, the collision of two particles may occur at $x = 1$ m, $y = 4$ m, $z = 11$ m, and at time $t = 7$ sec in one frame of reference (e.g., a laboratory on earth) so that the four numbers (1, 4, 11, 7) specify the event in that reference frame. The same event observed from a different reference frame (e.g., an airplane flying overhead) would also be specified by four numbers, although the numbers may be different than those in the laboratory frame. Thus, if we are to describe events, our first step is to establish a frame of reference.

We define an *inertial system* as a frame of reference in which the law of inertia—Newton's first law—holds. In such a system, which we may also describe as an *unaccelerated* system, a body that is acted on by zero net external force will move with a constant velocity. Newton assumed that a frame of reference fixed with respect to the stars is an inertial system. A rocket ship drifting in outer space, without spinning and with its engines cut off, provides an ideal inertial system. Frames accelerating with respect to such a system are not inertial.

In practice, we can often neglect the small (acceleration) effects due to the rotation and the orbital motion of the earth and to solar motion.* Thus, we may regard any set of axes fixed on the earth as forming (ap-

*Situations in which these effects are noticeable are the Foucault pendulum experiment or the deflection from the vertical of a freely falling body. The order of magnitude of such effects is indicated by the result that in falling vertically 100 ft (1200 in.) a body at the Equator is deflected less than $\frac{1}{4}$ in. from the vertical proximately) an inertial coordinate system. Likewise, any set of axes moving at uniform velocity with respect to the earth, as in a train, ship, or airplane, will be (nearly) inertial because motion at uniform velocity does not introduce acceleration. However, a system of axes which accelerates with respect to the earth, such as one fixed to a spinning merry-go-round or to an accelerating car, is *not* an inertial system. A particle acted on by zero net external force will not move in a straight line with constant speed according to an observer in such noninertial systems.

The special theory of relativity, which we consider here, deals only with the description of events by observers in inertial reference frames. The objects whose motions we study may be accelerating with respect to such frames but the frames themselves are unaccelerated. The general theory of relativity, presented by Einstein in 1917, concerns itself with all frames of reference, including noninertial ones, and we shall discuss it briefly in Topical Appendix C.

Consider now an inertial frame S and another inertial frame S' which moves at a constant velocity $\mathbf{v}$ with respect to S, as shown in Fig. 1-1. For convenience, we choose the three sets of axes to be parallel and allow their relative motion to be along the common x, x' axis. We can easily generalize to arbitrary orientations and relative velocity of the frames later, but the physical principles involved are not affected by the particular simple choice we make at present. Note also that we can just as well

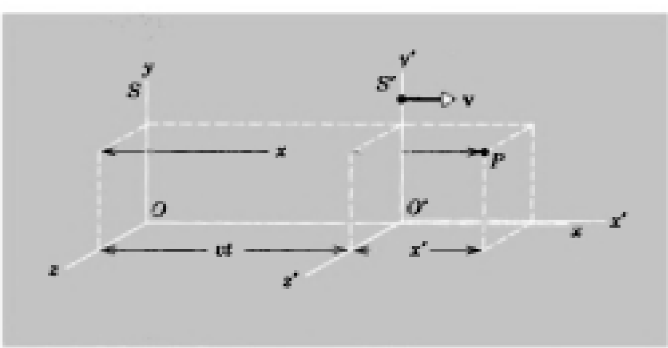

Fig. 1-1. Two inertial frames with a common x-x' axis and with the y-y' and z-z' axes parallel. As seen from frame S, frame S' is moving in the positive x-direction at speed v. Similarly, as seen from frame S', frame S is moving in the negative x'-direction at this same speed. Point P suggests an *event*, whose space-time coordinates may be measured by each observer. The origins O and O' coincide at time $t = 0$, $t' = 0$