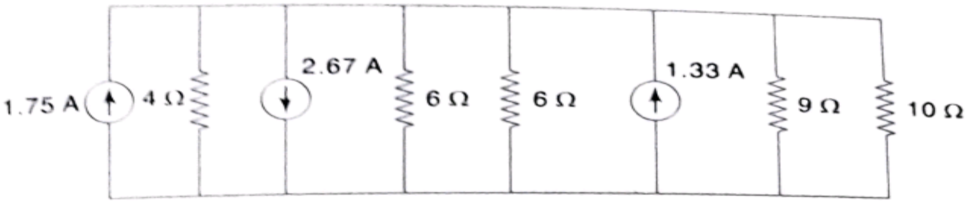

Fig. 1.109

In Fig. 1.109, all elements are in parallel. The resistors 4 Ω, 6 Ω, 6 Ω, and 9 Ω are in parallel. So, 4||6||6||9 = 1.44 Ω.

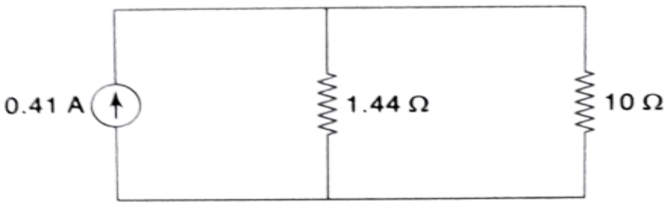

Fig. 1.110

Converting parallel combination of current source of 0.41 A and resistor of 1.44 Ω into equivalent series combination of voltage source and resistor, the circuit of Fig. 1.110 is further simplified as shown in Fig. 1.111. By Ohm's law, we get

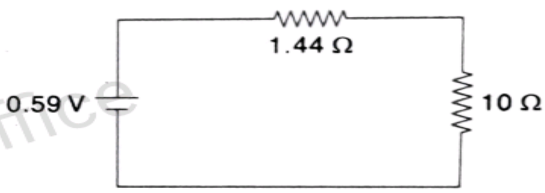

Fig. 1.111

$$I_{10\Omega} = \frac{0.59}{1.44 + 10} = 0.0516 \text{ A } (\downarrow)$$

1.12 Kirchhoff's Laws

Consider Fig. 1.112. While discussing Kirchhoff's laws and techniques, one often comes across the terms such as active element, passive element, node, junction, etc. These are discussed below.

Active element

An **active element** is one that supplies electrical energy to the circuit. Thus, in Fig. 1.112, V_1 and V_2 are the active elements because they supply energy to the circuit.

Passive element

A **passive element** is one that receives electrical energy, then either converts it into heat (resistance) or stores in electric field (capacitance) or magnetic field (inductance). In Fig. 1.112, there are three passive elements, namely R_1, R_2, and R_3. These passive elements (i.e., resistance in this case) receive energy from the active elements (i.e., V_1 and V_2) and convert it into heat.

Node

A **node** of network is an equipotential surface at which two or more circuit elements are joined. Thus, in Fig. 1.112, circuit elements R_1 and V_1 are joined at A and hence, A is the node. Similarly, B, C, and D are nodes.

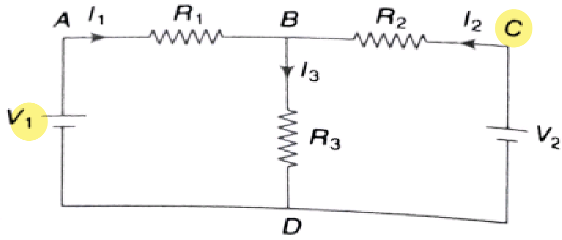

Fig. 1.112 Illustration of Kirchhoff's law

Junction

A **junction** is that point in a network where three or more circuit elements are joined. In Fig. 1.112, there are only two junction points, viz. B and D. That B is a junction is clear from the fact that three circuit elements R_1, R_2, and R_3 are joined at it. Similarly, point D is a junction because it joins three circuit elements R_3, V_1, and V_2. All the junctions are the nodes but all the nodes are not junctions.

Branch

A **branch** is the part of a network lying between two junction points. Thus, referring to Fig. 1.112, there are total of three branches, viz. BAD, BCD, and BD. The branch BAD consists of R_1 and V_1, the branch BCD consists of R_2 and V_2, and branch BD merely consists of R_3.

Loop

A **loop** is any closed path of a network. Thus, in Fig. 1.112, $ABDA$, $BCDB$, and $ABCDA$ are the loops.

Mesh

A **mesh** is the most elementary form of a loop and cannot be further divided into other loops. In Fig. 1.112, both loops $ABDA$ and $BCDB$ are meshes because they cannot be further divided into other loops. However, the loop $ABCDA$ cannot be called a mesh because it encloses two loops $ABDA$ and $BCDB$. All meshes are loops but all loops are not meshes.

1.12.1 Kirchhoff's Current Law (KCL)

This law relates to the currents at the junction points of a circuit and is stated below:

The algebraic sum of currents meeting at a junction or node in an electric circuit is zero.

Consider five conductors, carrying currents I_1, I_2, I_3, I_4, and I_5 meeting at node A as shown in Fig. 1.113. An algebraic sum is one in which the sign of the quantity is taken into account. If we take the signs of the currents flowing towards node A as positive, then currents flowing away from node A will be assigned negative sign. Thus, applying Kirchhoff's current law to node A in Fig. 1.113,

$$I_1 + (-I_2) + I_3 + (-I_4) + I_5 = 0$$

Hence, $I_1 - I_2 + I_3 - I_4 + I_5 = 0$

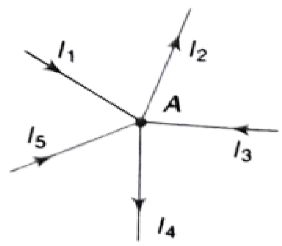

Fig. 1.113 Kirchhoff's current law

or $\quad I_1 + I_3 + I_5 = I_2 + I_4$

So, Incoming currents = Outgoing currents

Thus, the above law can also be stated as:

The sum of currents flowing towards any junction in an electric circuit is equal to the sum of currents flowing away from that junction.

1.12.2 Kirchhoff's Voltage Law (KVL)

This law relates to the electromotive forces and the voltage drops in a circuit and is stated below:

In any closed circuit or mesh, the algebraic sum of the electromotive forces and the voltage drops is equal to zero.

If we start from any point in a closed circuit and go back to that point, after going round the circuit, there is no increase or decrease in potential at that point. This means that the sum of electromotive forces of all the sources met on the way, and the voltage drops in the resistances must be zero.

1.12.3 Sign Conventions

While applying Kirchhoff's voltage law to a closed circuit, algebraic sums are considered. Therefore, it is very important to assign proper signs to emf's and voltage drops in the closed circuit. The follwing sign convention may be followed.

A rise in potential can be assumed positive while a fall in potential can be considered negative. The reverse is also possible and both conventions will give the same result.

(a) If we go from positive terminal of the battery or source to negative terminal, there is a fall in potential and so, the emf should be assigned negative sign. If we go from negative terminal of the battery or source to positive terminal, there is a rise in potential and so, the emf should be given positive sign.

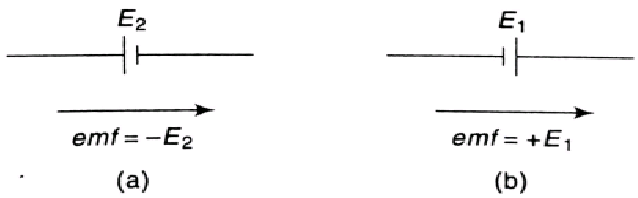

Fig. 1.114 Sign convention for emf

(b) When current flows through a resistor, there is a voltage drop across it. If we go through the resistance in the same direction as the current, there is a fall in the potential and so, the sign of this voltage drop is negative. If we go opposite to the direction of current flow, there is a rise in potential and hence, this voltage drop should be given positive sign.

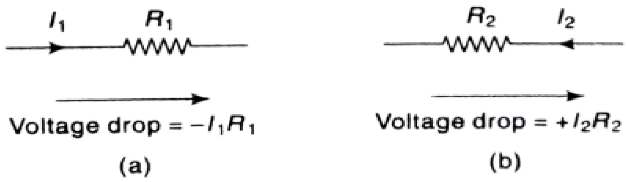

Fig. 1.115 Sign convention for voltage drop

Thus, applying the KVL to loop $ABDA$ of the circuit of Fig. 1.112, we get

$$V_1 - I_1R_1 - I_3R_3 = 0$$

Applying the KVL to loop $ABCDA$ of a circuit of Fig. 1.112, we get

$$V_1 - I_1R_1 + I_2R_2 - V_2 = 0$$

By using Kirchhoff's laws, we can calculate the unknown electrical quantities as discussed in the next sub-section.

1.12.4 Solving Circuit Problems by using Kirchhoff's Laws

1. Assume unknown currents in the given circuit and show their directions by arrows (see the note below).
2. By using KVL, write the equations for as many loops as the number of unknown currents.
3. In a solution if value of any unknown current comes out to be negative, it means that actual direction of the current is opposite to that of assumed direction.

Note: Procedure for assigning the unknown currents

1. Go to the junction and mark the current directions arbitrarily.
   For example, at junction B of a circuit shown in Fig. 1.117, three branches are meeting. The directions of three branch currents can be assigned as shown below:

(a) All three incoming (b) All three outgoing (c) Any one incoming, remaining two outgoing (d) Any two incoming, remaining one outgoing

2. Name (mark) these currents using KCL so as to minimize the number of unknowns, i.e., in the above case, out of three if any two currents are marked as I_1 and I_2, then the third must be marked in terms of I_1 and I_2 (not as I_3).

Example 1.31 By using Kirchhoff's laws, calculate the branch currents in the circuit shown in Fig. 1.116.

Solution

By marking the different nodes and assuming the unknown currents, we get the circuit as shown in Fig. 1.117.

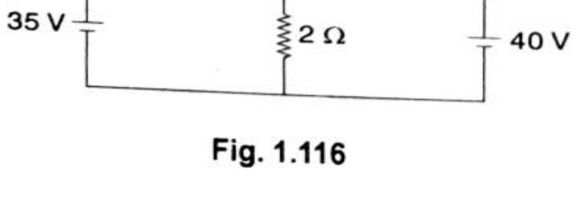

Fig. 1.116

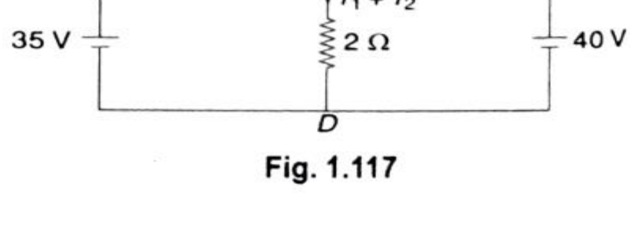

Fig. 1.117

Applying the KVL to loop $ABDA$,

$$-3I_1 - 2(I_1+I_2) + 35 = 0$$

or $$5I_1 + 2I_2 = 35 \qquad \text{(i)}$$

Applying the KVL to loop $ABCDA$,

$$-3I_1 + 4I_2 - 40 + 35 = 0$$

or $$3I_1 - 4I_2 = -5 \qquad \text{(ii)}$$

From Eqs (i) and (ii),

$$I_1 = 5\text{ A} \quad \text{and} \quad I_2 = 5\text{ A}$$

Thus, $I_{3\,\Omega} = 5$ A ($\rightarrow$), $I_{4\,\Omega} = 5$ A ($\leftarrow$), $I_{2\,\Omega} = 10$ A ($\downarrow$).

Example 1.32 By using Kirchhoff's laws, calculate the branch currents in the circuit shown in Fig. 1.118.

Solution

By marking the different nodes and assuming the unknown currents, we get the circuit as shown in Fig. 1.119.

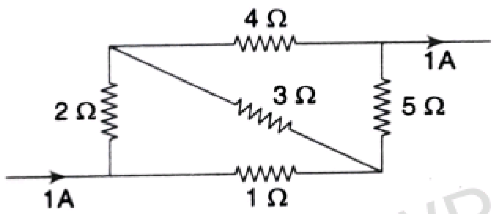

Fig. 1.118

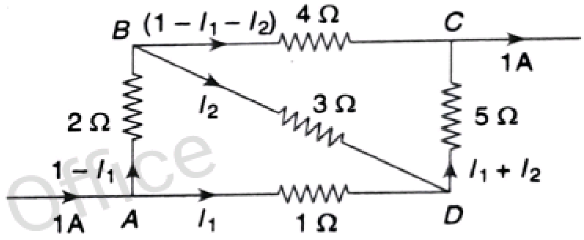

Fig. 1.119

Applying the KVL to loop $ABDA$,

$$-2(1 - I_1) - 3I_2 + I_1 = 0$$

or $$3I_1 - 3I_2 = 2 \qquad \text{(i)}$$

Applying the KVL to loop $BCDB$,

$$-4(1 - I_1 - I_2) + 5(I_1 + I_2) + 3I_2 = 0$$

or $$9I_1 + 12I_2 = 4 \qquad \text{(ii)}$$

From Eqs (i) and (ii),

$$I_1 = 0.571\text{ A} \quad \text{and} \quad I_2 = -0.0952\text{ A}$$

Branch currents are given by

$I_{1\Omega} = I_1 = 0.571$ A, from A to D

$I_{2\Omega} = 1 - I_1 = 0.429$ A, from A to B

$I_{3\Omega} = I_2 = -0.0952$ A, from B to D

$I_{4\Omega} = 1 - I_1 - I_2 = 0.524$ A, from B to C

$I_{5\Omega} = I_1 + I_2 = 0.475$ A, from D to C

Example 1.33 Determine the current supplied by the battery in the circuit shown in Fig. 1.120.

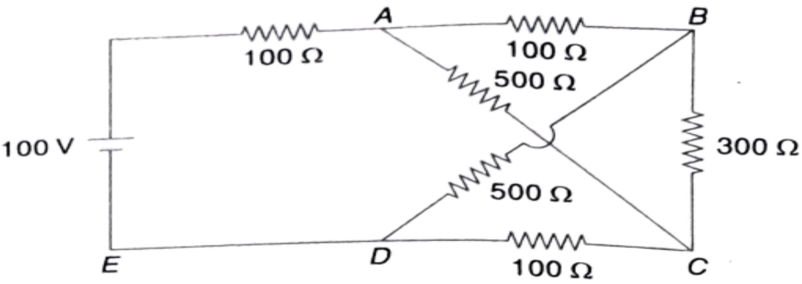

Fig. 1.120

Solution

Let the current distribution be as shown in Fig. 1.121.

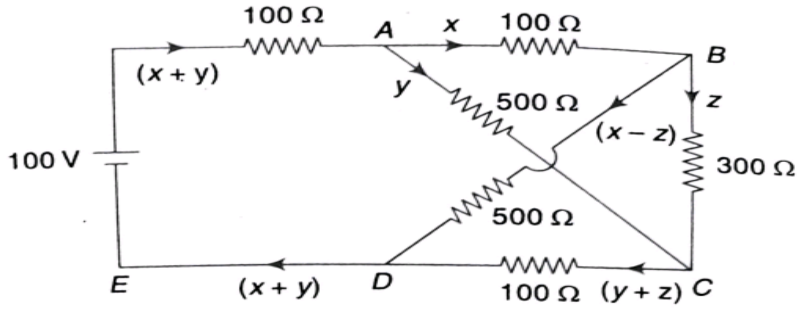

Fig. 1.121

Applying the KVL to loop $ABCA$, we have

$$-100x - 300z + 500y = 0$$

or $\quad x - 5y + 3z = 0 \qquad$ (i)

Similarly, applying the KVL to loop $BCDB$, we have

$$-300z - 100(y + z) + 500(x - z) = 0$$

or $\quad 5x - y - 9z = 0 \qquad$ (ii)

Similarly, applying the KVL to loop $ABDEA$, we have

$$-100x - 500(x - z) + 100 - 100(x + y) = 0$$

or $\quad 7x + y - 5z = 1 \qquad$ (iii)

The values of x, y and z may be found by solving the above three simultaneous equations or by the method of determinants as given below:

Putting the above three equations in the matrix form, we have

$$\begin{bmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 1 & -5 & 3 \\ 5 & -1 & -9 \\ 7 & 1 & -5 \end{vmatrix} = 240, \qquad \Delta_1 = \begin{vmatrix} 0 & -5 & 3 \\ 0 & -1 & -9 \\ 1 & 1 & -5 \end{vmatrix} = 48$$

$$\Delta_2 = \begin{vmatrix} 1 & 0 & 3 \\ 5 & 0 & -9 \\ 7 & 1 & -5 \end{vmatrix} = 24, \qquad \Delta_3 = \begin{vmatrix} 1 & -5 & 0 \\ 5 & -1 & 0 \\ 7 & 1 & 1 \end{vmatrix} = 24$$

This gives $x = \frac{\Delta_1}{\Delta} = \frac{48}{240} = \frac{1}{5}$ A, $y = \frac{\Delta_2}{\Delta} = \frac{24}{240} = \frac{1}{10}$ A, $z = \frac{\Delta_3}{\Delta} = \frac{24}{240} = \frac{1}{10}$ A

Hence, current supplied by the battery is $= x + y = \frac{1}{5} + \frac{1}{10} = \frac{3}{10}$ A

Example 1.34 Determine the current through 20 Ω resistor in the circuit shown in Fig. 1.122.

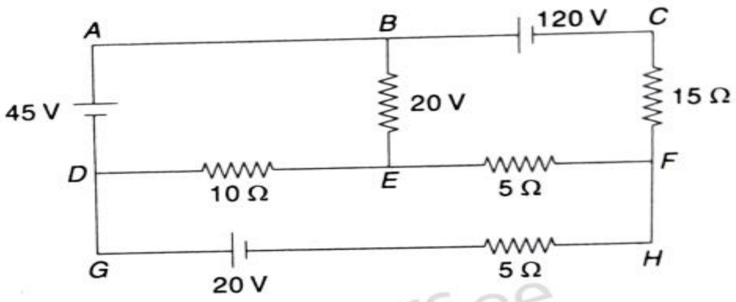

Fig. 1.122

Solution

Let the current distribution be as shown in Fig. 1.123.

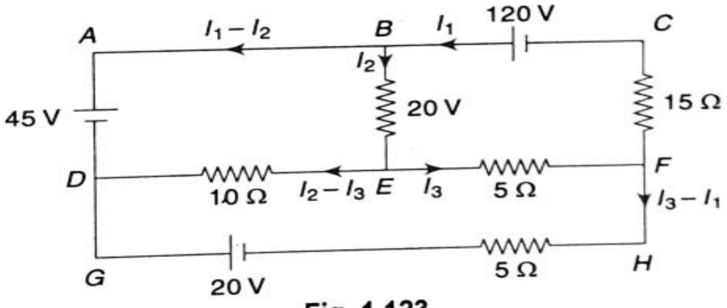

Fig. 1.123

Applying the KVL to loop *ABEDA*, we have

$$-20I_2 - 10(I_2 - I_3) + 45 = 0$$

or $\quad 30I_2 - 10I_3 = 45$

or $\quad 6I_2 - 2I_3 = 9 \qquad$ (i)

Applying the KVL to loop *BCFEB*, we have

$$-120 + 15I_1 + 5I_3 + 20I_2 = 0$$

or $\quad 15I_1 + 20I_2 + 5I_3 = 120$

or $\quad 3I_1 + 4I_2 + I_3 = 24 \qquad$ (ii)

Applying the KVL to loop *DEFHGD*, we have

$$10(I_2 - I_3) - 5I_3 - 5(I_3 - I_1) + 20 = 0$$

or $\quad 5I_1 + 10I_2 - 20I_3 = -20$

or $\quad I_1 + 2I_2 - 4I_3 = -4$ (iii

Current through 20 Ω is required. According to the current distribution as showr in Fig. 1.123, current I_2 flows through 20 Ω resistor. Current I_2 can be calculatec by the method of determinants as given below. Putting the above three equations in the matrix form, we have

$$\begin{bmatrix} 0 & 6 & -2 \\ 3 & 4 & 1 \\ 1 & 2 & -4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 24 \\ -4 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 0 & 6 & -2 \\ 3 & 4 & 1 \\ 1 & 2 & -4 \end{vmatrix} = 74, \qquad \Delta_2 = \begin{vmatrix} 0 & 9 & -2 \\ 3 & 24 & 1 \\ 1 & -4 & -4 \end{vmatrix} = 189$$

By Cramer's rule, $\quad I_2 = \dfrac{\Delta_2}{\Delta} = \dfrac{189}{74} = 2.554 \text{ A}$

Hence, current through 20 Ω resistor, $\quad I_{20\Omega} = 2.554 \text{ A } (\downarrow)$

Example 1.36 Calculate the power dissipation in each resistor in the network shown in Fig. 1.125.

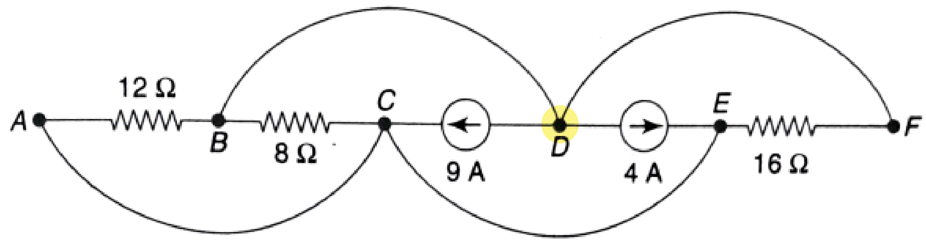

Fig. 1.125

Solution

In Fig. 1.125, the nodes A, C and E are same. Similarly the nodes B, D, and F are same. By joining the same nodes, the circuit simplifies as shown in Fig. 1.126.

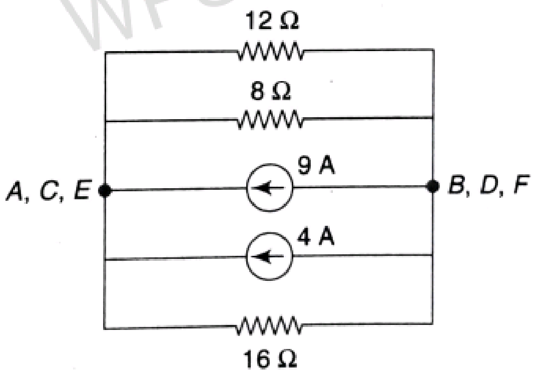

Fig. 1.126

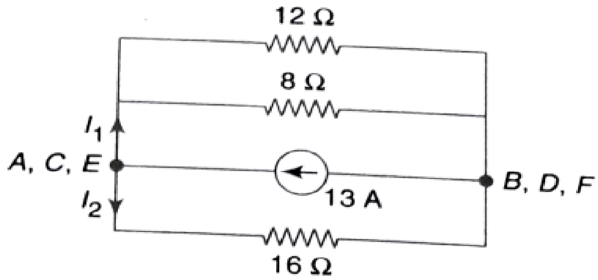

Fig. 1.127

In Fig. 1.127, 12 Ω and 8 Ω are in parallel. So $12\Omega \| 8\Omega = 4.8\ \Omega$.

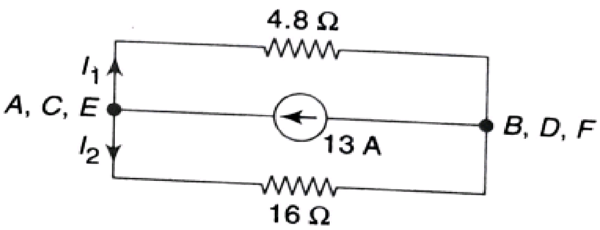

Fig. 1.128

In Fig. 1.128, let the total current 13 A divides as I_1 and I_2. By current division rule,

$$I_1 = \frac{13 \times 16}{16 + 4.8} = 10\text{ A}$$

$$I_2 = I_{16\,\Omega} = \frac{13 \times 4.8}{16 + 4.8} = 3\text{ A}$$

In Fig. 1.127, the current I_1 divides in two parallel resistors (i.e. 12 Ω and 8 Ω). So, by current division rule,

$$I_{8\,\Omega} = \frac{10 \times 12}{8 + 12} = 6\text{ A}$$

$$I_{12\,\Omega} = \frac{10 \times 8}{8 + 12} = 4\text{ A}$$

Now,

Power dissipation in 12 Ω, $P_{12\,\Omega} = (I_{12\,\Omega})^2 \times 12 = (4)^2 \times 12 = 192$ W

Power dissipation in 8 Ω, $P_{8\,\Omega} = (I_{8\,\Omega})^2 \times 8 = (6)^2 \times 8 = 288$ W

Power dissipation in 16 Ω, $P_{16\,\Omega} = (I_{16\,\Omega})^2 \times 16 = (3)^2 \times 16 = 144$ W

Example 1.37 Find the current in all branches of the network shown in Fig. 1.129.

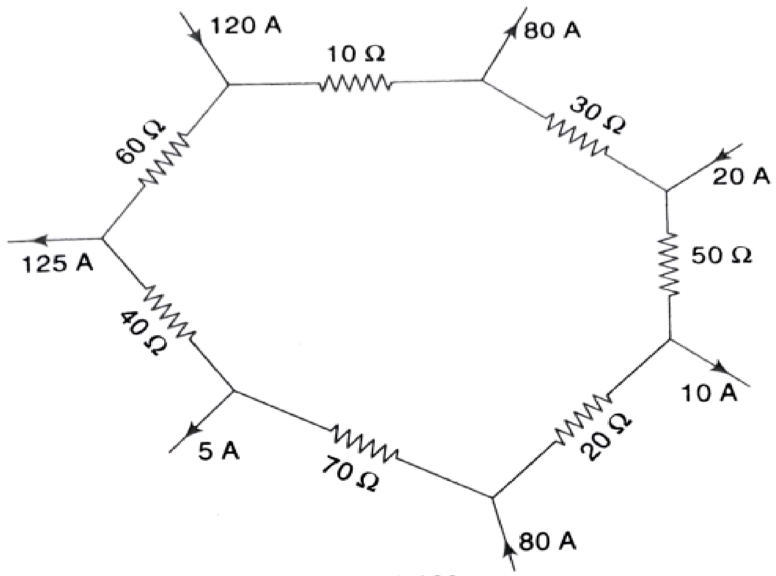

Fig. 1.129

Solution

By marking the different nodes and unknown currents, we get the circuit as follows:

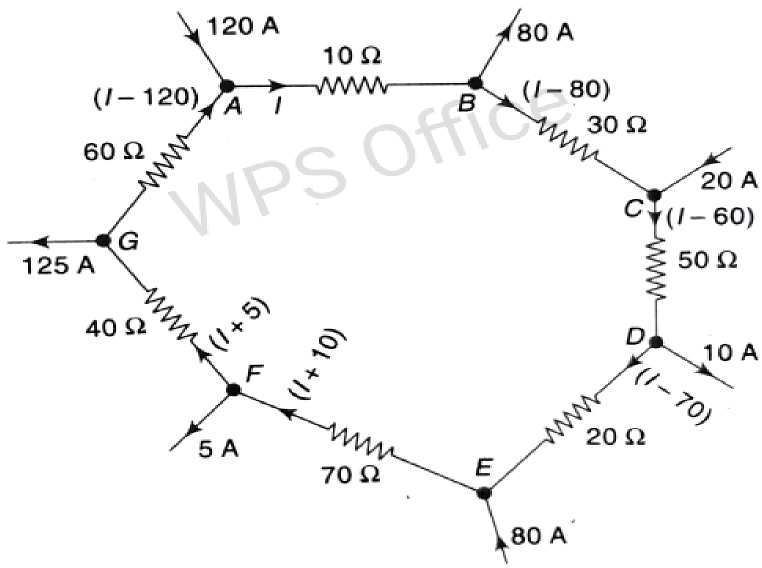

Fig. 1.130

Applying KVL to the loop, we get

$-30(I-80) -50(I-60) -20(I-70) -70(I+10) -40(I+5) -60(I-120) -10\,I = 0$

$-280I + 13100 = 0$

$\therefore \quad I = 46.786$ A

Thus $I_{10\,\Omega} = I = 46.786$ A from A to B

$I_{30\,\Omega} = (I-80) = -33.214$ A from B to C

$I_{50\,\Omega} = (I-60) = -13.214$ A from C to D

$I_{20\,\Omega} = (I-70) = -23.214$ A from D to E

$I_{70\,\Omega} = (I + 10) = 56.786$ A from E to F
$I_{40\,\Omega} = (I + 5) = 51.786$ A from F to G
$I_{60\,\Omega} = (I - 120) = -73.214$ A from G to A

Example 1.38 For the network shown in Fig. 1.131, determine (i) I_1, I_2, and I_3, (ii) resistance R, and (iii) value of emf E.

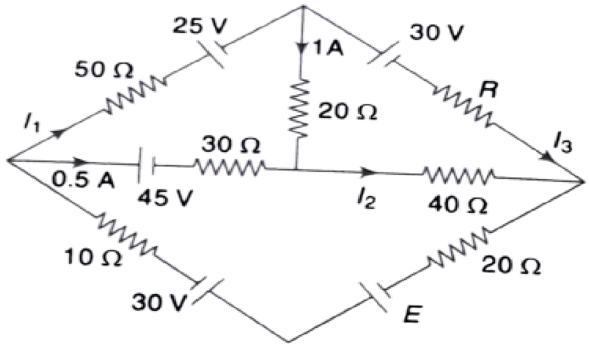

Fig. 1.131

Solution

Marking the different nodes, we obtain the circuit as shown in Fig. 1.132.

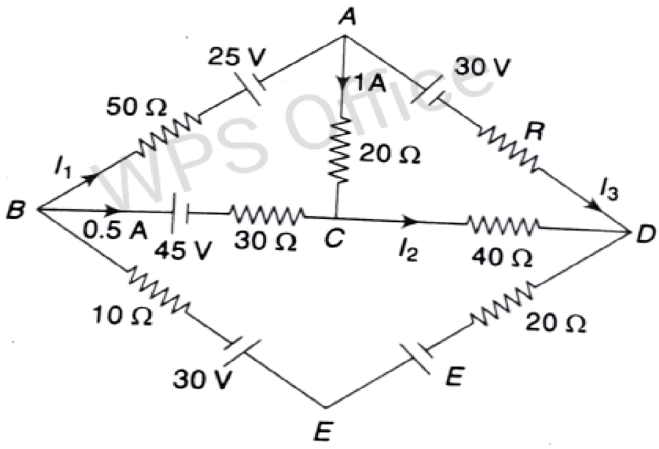

Fig. 1.132

(i) At node C, by applying KCL, we have

$0.5 + 1 = I_2 \quad \therefore \quad I_2 = 1.5$ A

Applying the KVL to loop $ACBA$, we have

$-20 + (30 \times 0.5) + 45 - (50 \times I_1) - 25 = 0$

$\therefore I_1 = 0.3$ A

At node A, by applying KCL, we have

$I_1 = 1 + I_3 \quad \therefore \quad I_3 = -0.7$ A

(ii) Applying the KVL to loop $ADCA$, we have

$-30 - I_3R + 40I_2 + (20 \times 1) = 0$

Substituting the value of I_2 and I_3, we get

$R = -71.43\ \Omega$

(iii) At node D, by applying KCL, we have

Current in branch $DEB = I_2 + I_3 = 0.8$ A

Applying the KVL to loop $DEBCD$, we have

$(-20 \times 0.8) + E - 30 - (10 \times 0.8) - 45 - (30 \times 0.5) - (40 \times 1.5) = 0$

or $E = 174$ V

Example 1.39 Find the potential at point a with respect to point b (V_{ab}) in the network shown in Fig. 1.133.

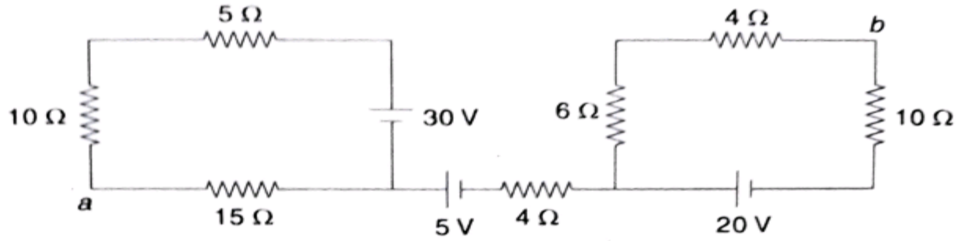

Fig. 1.133

Solution

Marking the different nodes and assuming the unknown currents, we get the following circuit:

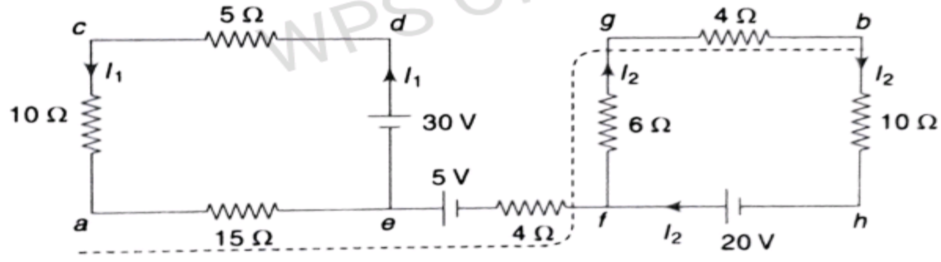

Fig. 1.134

Let us first calculate the unknown currents.

Applying the KVL to loop $acdea$, we have

$$10I_1 + 5I_1 - 30 + 15I_1 = 0$$

Hence, $I_1 = 1$ A

Applying the KVL to loop $fgbhf$, we have

$$-6I_2 - 4I_2 - 10I_2 + 20 = 0$$

Hence, $I_2 = 1$ A

As there is no returning path (closed path) for branch ef, no current flows through branch ef, i.e., $I_{ef} = 0$.

The potential at point a with respect to b (V_{ab}) can be calculated by travelling from point b to point a through any path and taking the algebraic sum of electromotive forces of all the sources and voltage drops across the resistances met on the way. Let the selected path is $bgfea$ (as shown by dotted line).

Thus, taking path *bgfea*, we have

$$V_{ab} = 4I_2 + 6I_2 + (4 \times 0) + 5 + 15I_1$$

Hence, $V_{ab} = 30$ V

Example 1.40 Find the potential at point *e* with respect to *c* (V_{ce}) in the network shown in Fig. 1.135.

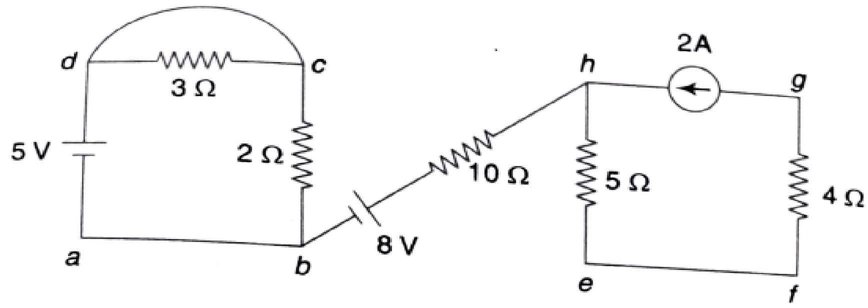

Fig. 1.135

Solution

Assuming the unknown currents, we get the following circuit:

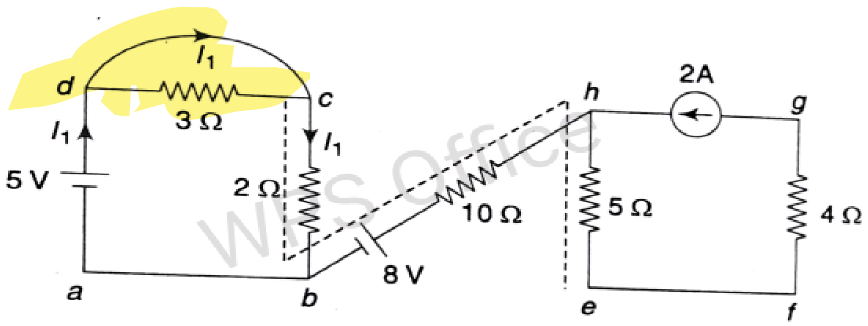

Fig. 1.136

Let us first calculate the unknown currents.

As there is no returning path (closed path) for branch *bh*, no current flows through branch *bh*, i.e., $I_{bh} = 0$.

The 3 Ω resistor gets short circuited. ∴ $I_{3\,\Omega} = 0$

Current through the loop *efgh* is 2A.

Current through the loop *abcd* can be calculated by applying KVL to loop *abcd*.

Applying KVL to loop *abcd*, we have

$$-2I_1 + 5 = 0$$

So, $I_1 = 2.5$ A

Let the selected path is *ehbc* (as shown by dotted line).

Thus, taking path *ehbc*, we have

$$V_{ce} = (5 \times 2) + (10 \times 0) - 8 + 2I_1$$

Hence, $V_{ce} = 7$ V

Example 1.43 Using source transformation, find V_A and V_B in the given circuit shown in Fig. 1.141.

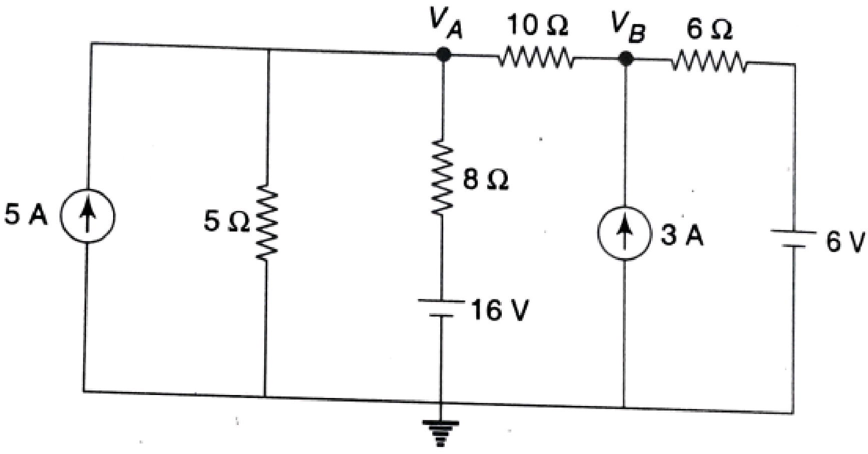

Fig. 1.141

Solution

The series combinations (of voltage source and resistor) are marked in the given circuit as shown in Fig. 1.142.

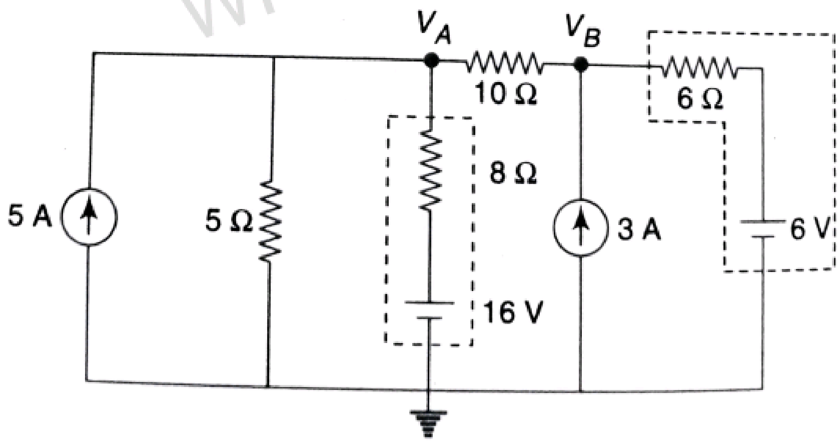

Fig. 1.142

By converting the series combinations into equivalent parallel combinations (of current source and resistor), we get the circuit as shown in Fig. 1.143.

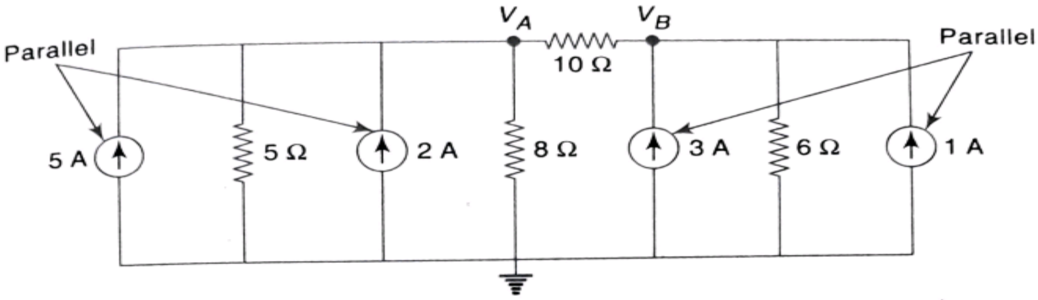

Fig. 1.143

By adding the parallel current sources, we get the circuit as shown in Fig. 1.144.

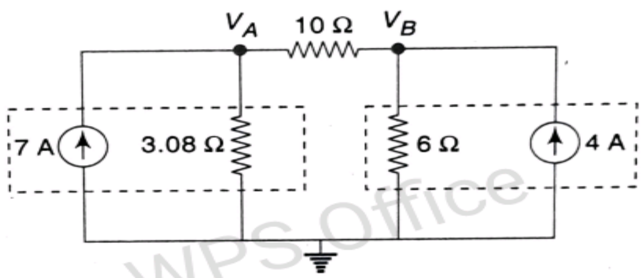

Fig. 1.144

The parallel combinations (of current source and resistor) are marked in Fig. 1.144. Replacing these parallel combinations into equivalent series combinations, we get the circuit as shown in Fig. 1.145.

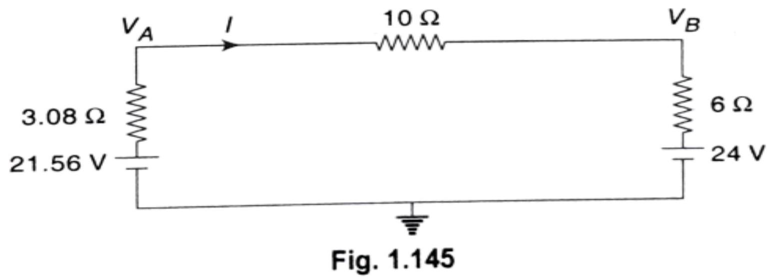

Fig. 1.145

Assuming the loop current I and applying KVL to the loop, we get

$$-10I - 6I - 24 + 21.56 - 3.08I = 0$$
$$-19.08I = 2.44$$
$$I = -0.128 \text{ A}$$

Referring Fig. 1.145,

$$V_A = 21.56 - 3.08I = 21.56 - 3.08\,(-0.128) = 21.95 \text{ V}$$
$$V_B = 24 + 6I = 24 + 6\,(-0.128) = 23.232 \text{ V}$$