Mechanics Viva Solutions

1. System of Coplanar Forces

- Mechanics may be defined as that branch of physical science which is concerned with the study of resultant effect of action of forces on bodies, both in the state of rest and in motion

-Mechanics and its classification

- A force is the action of one body on another body. The action could be a 'push' or 'pull'. A force is exerted when there is an actual contact between the two bodies for e.g. a boy hitting a nail with a hammer. A force can also act even if there is no contact between the two bodies, such ad the magnetic or gravitational force. Force is a vector quantity and is completely defined by its magnitude, its direction and point of application.

-Definition of force

- Units of force :
  1. Newton (MKS)
  2. Dyne (CGS)
  3. Newton (SI)
- Characteristics of Forces :

  - Forces are due to an interaction of at least two objects.
  - It may change the state of motion of an object.
  - It may change the shape of an object.
  - Forces applied on an object in the same direction add to one another and the resultant is in the same direction.
  - When forces are applied on an object in the opposite direction then their resultant or net force is the difference between these opposing forces and its resulting direction is the same as that of larger force.
  - If the two forces acting on an object are equal in magnitude but opposite in direction, then the net force acting on the body is zero.

- It is a vector quantity hence they should be specified by giving its magnitude and the direction.
- If the magnitude or the Direction or both changes, then the effect of force also changes.
- If the body is stationary with respect to an observer, then the body is said to be at the rest with respect to the observer. The state of rest is considered to be a state of zero speed.
- If the body changes its position with respect to an observer, then the body is said to be in motion with respect to the observer.
- A change of state of motion means a change in either the speed of an object or its direction of motion or both. A force is necessary for bringing a change in the state of motion of an object.

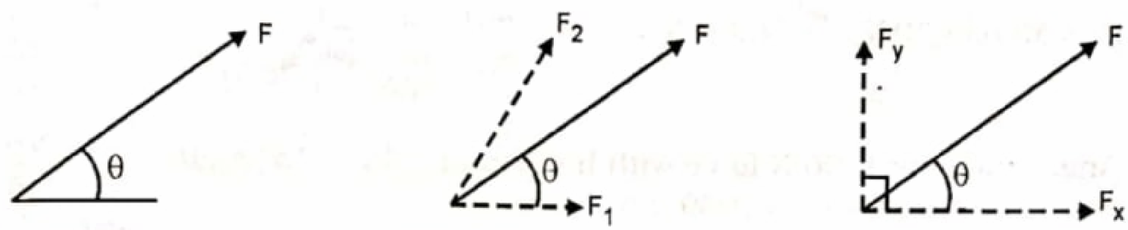
● Examples of Forces :

- Contact Forces :- A force which can be applied only when it is in contact with an object is called a contact force. All mechanical forces are contact forces e.g. Muscular force, Frictional force.
- Non-Contact Forces :- A force that can be applied without any contact with two bides is

called a non-contact force e.g. Magnetic force, Electrostatic force, Gravitational force.

-Characteristics and examples of force

- Resolution or resolving a force implies breaking the force into components, such that the components combined together would have the same effect as the original force. Fig. 2.3(a) shows a force F acting at an angle ‘θ’.



Force F
Fig. 2.3 (a)

Force F resolved into components F_1 and F_2
Fig. 2.3 (b)

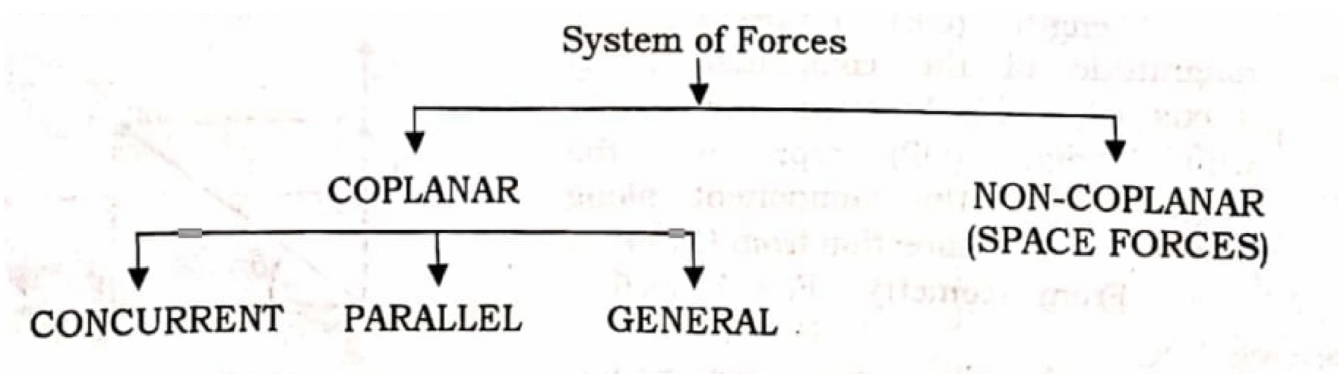
Force F resolved into components F_x and F_y
Fig. 2.3 (c)

This force can be resolved into components F_1 and F_2 as shown in Fig. 2.3(b) or into components F_x and F_y as shown in Fig.2.3(c). The components F_x and F_y of the force F as shown in Fig. 2.3(c) are known as the rectangular or perpendicular components of the force, since the two components are perpendicular to each other.

-Resolution of a force

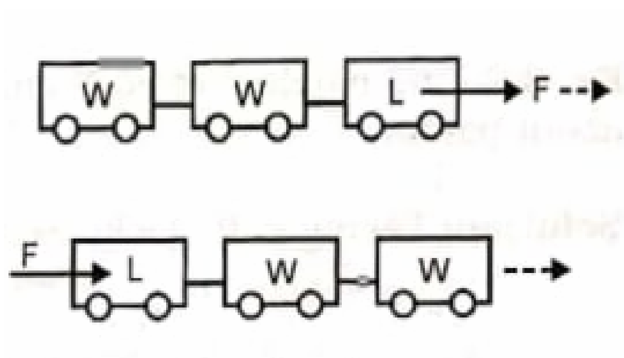
- ‘System of Forces’ tells us about how the forces are arranged. For example all the forces may lie on one plane or may lie on different planes. They may meet at a point, may be parallel to each other or may just neither be parallel, nor may meet at a point. ‘System’ defines this arrangement of forces.

-System of forces



-Classification of system of forces & Examples

- A practical example explaining Principle of Transmissibility is the case of a locomotive ‘L’ pulling the wagons ‘W’ to the right by exerting force F from the front. This force gets transmitted to all the wagons and they move forward. The same effect is observed if the locomotive pushes the wagons from behind. Again the force F is transmitted to all the wagons and they move forward.



-Principle of transmissibility

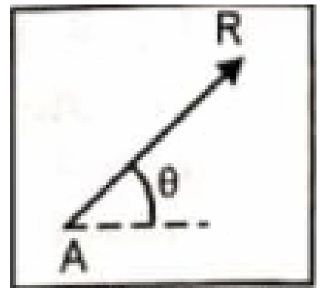
- Composition means to combine the forces acting in a system into a single force, which has the same effect as the number of forces acting together. Such a single force is known as resultant of the system.

-Composition of forces

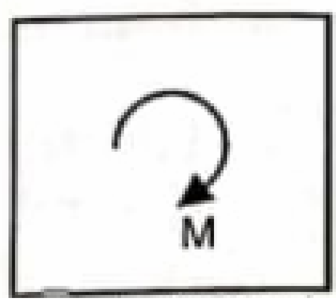
- A resultant force is a single force, that has the same effect on the body as produced by a system of forces acting on the body.

-Resultant of a force

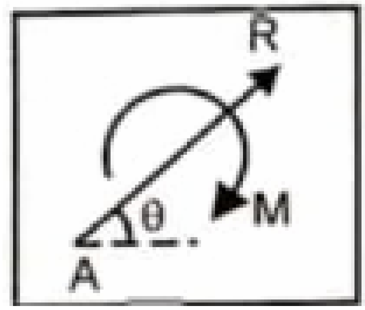
- Resultant of a force may either be a
  - Force : In a given force system consisting of number of forces, if on composition of these forces it results in a single force, we call such a resultant as 'Resultant-Force'.



  - Couple : In a force system if the resultant force is zero but the resultant moment is not zero such a system reduces into a couple. 'Resultant-Couple' is possible in a Parallel system or a General force system, but a concurrent force system does not result in a couple.

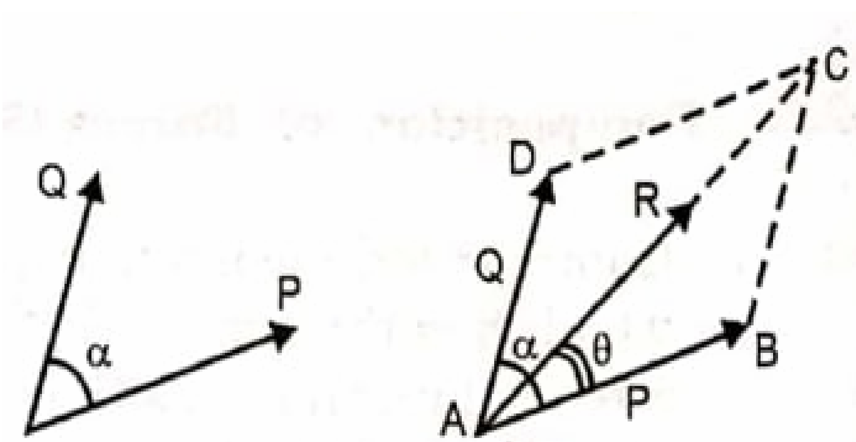


  - Force-Couple : A resultant force when shifted to a new parallel position without change in its direction and sense, introduces a couple in the system. Such a resultant consisting of a single force and a single couple is called a ‘Force-Couple’.



-Types of resultant

- Using Parallelogram Law of Forces, “If two forces acting simultaneously on a body at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram which passes through the point of intersection of the two sides representing the forces”.



Let P and Q be the two forces acting at a point and making an angle 'α' with each other as shown in figure above. The forces are drawn to scale in figure Such that AB and AD represent forces P & Q. Completing the parallelogram ABCD, the diagonal AC gives the magnitude and direction of the resultant R.

Mathematically,

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\alpha}$$

$$\tan\theta = \frac{Q\sin\alpha}{P + Q\cos\alpha}$$

-Resultant of concurrent system of Forces & Parallelogram Law of Forces

- When more than two forces act at a point, the use of method of resolution is made to avoid tedious repetition of parallelogram law of forces to successive forces. The following steps are adopted in the solution.

- o Step 1 : Resolve the inclined forces if any along the horizontal x direction and the vertical y direction.
- o Step 2 :
  1. Add up the horizontal forces to get ΣF_x . Use sign convention → +ve.
  2. Add up the vertical forces to get ΣF_y . Use sign convention ↑ +ve.
  3. The resultant force $R = \sqrt{(\Sigma F_x^2 + \Sigma F_y^2)}$
- o Step 3 : The direction of the resultant force is the angle θ made by it whith the x axis. To find angle θ, use :

$$\tan \theta = \Sigma F_y / \Sigma F_y$$

-Method of resolution

- Decide the quadrant of the resultant, depending on the signs of ΣF_x & ΣF_y.

If	Resultant lies in
$\sum F_x$ is + ve and $\sum F_y$ is + ve	1 st Quadrant
$\sum F_x$ is – ve and $\sum F_y$ is + ve	2 nd Quadrant
$\sum F_x$ is – ve and $\sum F_y$ is – ve	3 rd Quadrant
$\sum F_x$ is + ve and $\sum F_y$ is – ve	4 th Quadrant

-Determination of Quadrant of resultant

- The rotational effect of a force is known as the moment of the force. When we talk of the rotational effect. It has to be with respect to a point. The concerned point is known as the moment centre. The rotational effect of the same force will vary from one moment center to another and of course if the point (moment centre) lies on the line of action of the force, the moment of force about the point would be zero.The rotational effect or moment is measured as the product of the force and the perpendicular distance from the moment centre to the force. This perpendicular distance is known as the moment arm ‘d’. $\therefore M = F \times d$

-Moment of Force

- In mechanics, pair of equal parallel forces that are opposite in direction are known as couple. The turning effect, or moment, of a couple is measured by the product of the magnitude of either force and the perpendicular distance between the action lines of the force.

  Properties :
  - Resultant of a couple is always zero.
  - Moment of a couple about any point in the plane is constant.
  - A couple can be balanced by another couple of equal magnitude and opposite sense.
  - Two or more couples are said to be equal when they have same magnitude and same sense.
  - Number of couples can be replaced by a single couple, the moment of which is equal to algebraic sum of moments of all couples.

-Couple & its Properties

- Shifting of a force to a new position

- Algebraic sum of moment of all forces about any coplanar point = moment of their resultant about that point.
  i.e. $\Sigma M_A F = M_A R$

-Varignon's Theorem

- Steps for Determination of Resultant of Parallel Force System :-

  - $R = \Sigma F$ => Magnitude of resultant
  - $\Sigma M_A F = M_A R$ => Varignon's Theorem

- Steps for Determination of Resultant of General Force System : -

  - Find Resultant using the following method :

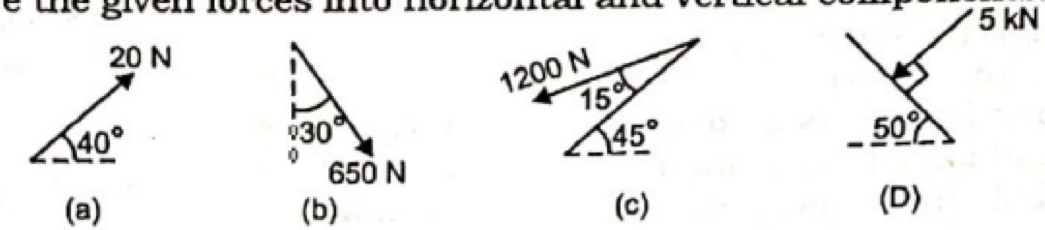
    Step 1 : Resolve the inclined forces if any along the horizontal x direction and the vertical y direction.

Step 2 :

1. Add up the horizontal forces to get ΣF_x .
   Use sign convention → +ve.
2. Add up the vertical forces to get ΣF_y .
   Use sign convention ↑ +ve.
3. The resultant force $R = \sqrt{(\Sigma F_x^2 + \Sigma F_y^2)}$

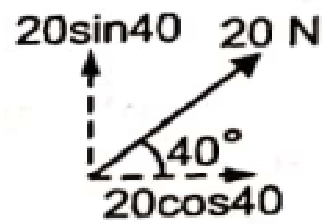
- The point of application of the resultant force is found out using Varignon's theorem. The resultant is initially assumed to act either to the right or left of the reference point at a perpendicular distance d.
  Varignon's theorem $\Sigma M_A F = M_A R$ is used. If a positive value of d is obtained, then the assumption made earlier is true. If a negative value of d is obtained, then the resultant lies on the opposite side to what was assumed.

Resolve the given forces into horizontal and vertical components.



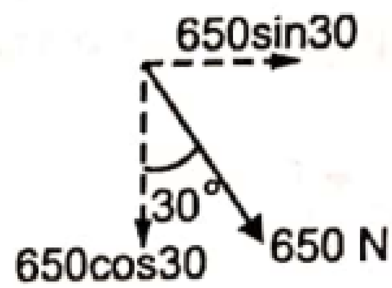
- (a) :- $F_x = 20\cos(40)\ N = 15.32\ N \rightarrow$

  $F_y = 20\sin(40)\ N = 12.86\ N \uparrow$



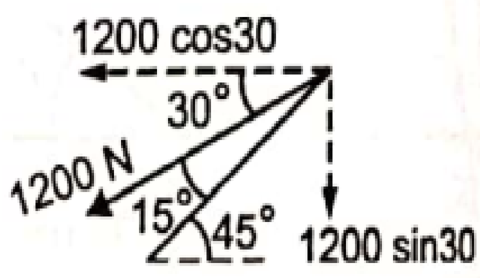
- (b) :- $F_x = 650\sin(30)\ N = 325\ N \rightarrow$

  $F_y = 650\cos(30)\ N = 562.9\ N \downarrow$

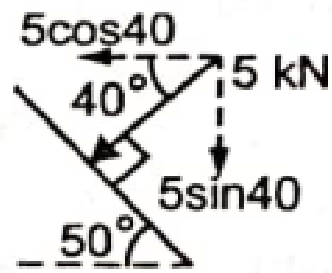


- (c) :- $F_x = 1200\cos(30)\ N = 1039.2\ N \leftarrow$

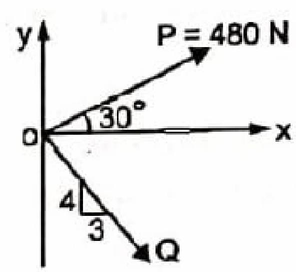
$F_y = 1200\sin(30)\ N = 600\ N \downarrow$



- (d) :- $F_x = 5\cos(40)\ N = 3.83\ N \leftarrow$
  $F_y = 5\sin(40)\ N = 3.21\ N \downarrow$



Two concurrent forces P and Q acts at O such that their resultant acts along x-axis. Determine the magnitude of Q and hence the resultant.



$\tan\theta = 3/4$

$\therefore \theta = \tan^{-1}(3/4) = 36.87°$

Since, Resultant acts along x-axis

$\therefore \ \Sigma F_y = 0$

$\Rightarrow 480\sin(30) – Q\cos(36.87) = 0$

$\Rightarrow Q = 480\sin(30)/\cos(36.87) = 0$

$\therefore \ Q = 300N$ at $-53.13°$

Thus, Resultant $R = 480\cos(30) + 300\sin(36.87)$

$R = 595.69N$

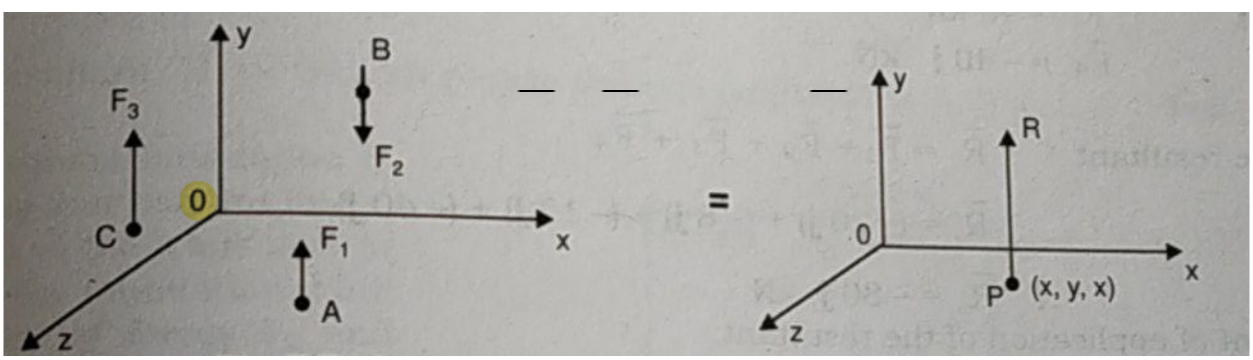
2. Space Force System

- Force in vector form :-

  For two points given (x_1,y_1,z_1) and (x_2,y_2,z_2) ,

$$\bar{F} = F\left[\frac{(x_2-x_1)i + (y_2-y_1)j + (z_2-z_1)k}{\sqrt{(x_2-x_1)^2+(y_2-y_1)^2+(z_2-z_1)^2}}\right]$$

- Represent a force in vector form when direction angles and magnitude of the force are given
- Moment vector of a force about a point in the space
- Position vector
- Resultant for concurrent force system
- The resultant of a parallel space force system is a single force R which acts parallel to the force system. The location of the resultant can be found out using Varignon's theorem.



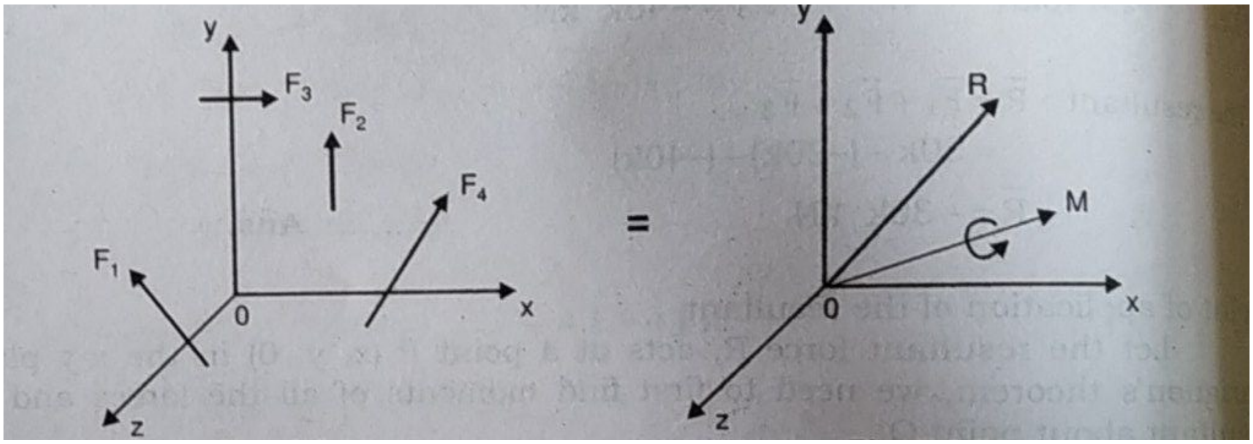
The single force $\bar{R} = \bar{F}_1 + \bar{F}_2 + \bar{F}_3$

The resultant acts at P. The coordinates (x,y,z) of the point P can be calculated by using Varignon's theorem, the moments for which can be taken about any convenient point like point O. The equation of Varignon's theorem for space forces is

$$\Sigma \overline{M}_o^F = \Sigma \overline{M}_o^R$$

-Resultant of Parallel Force System

- A general space force system is netiher a concurrent nor a parallel system. The resultant of such a system is a single force R and a moment M at any desired point. Since the resultant contains one force and one moment, it is also known as a *Force Couple System*.



The single force $\overline{R} = \overline{F}_1 + \overline{F}_2 + \overline{F}_3 + \overline{F}_4$

and

The single moment $\overline{M} = \overline{M}_o{}^{F1} + \overline{M}_o{}^{F2} + \overline{M}_o{}^{F3} + \overline{M}_o{}^{F4}$

-Resultant of General Force System

Centroid

- Centre of gravity is an imaginary point in a body of matter where, for convenience in certain calculations, the total weight of the body is thought to be concentrated

- Centroid is the centre of mass of an object of uniform density.

- Axis of symmetry is a line that divides an object into two equal halves, thereby creating a mirror-like reflection of either side of the object.
- When we take area positive or negative?

- When coordinates are considered as positive or negative?
- Explain the area addition or subtraction to get the shaded/required area.

- Area of a Sector :

$$A = \frac{\theta \pi r^2}{360} \quad \text{where } \theta \text{ is in degrees}$$

OR

$$A = \frac{\theta r^2}{2} \quad \text{where } \theta \text{ is in radians}$$

- Coordinate distance for a sector
- When to use ∝ in degree and radians

Equilibrium

- Equilibrium is stated as if the resultant of all the forces in a system is zero and the sum of moments of all the forces in the system about any coplanar point is also zero, then the force system is said to be in equilibrium.

- Conditions of equilibrium are :-

  $\Sigma F_x = 0$; $\Sigma F_y = 0$; $\Sigma M = 0$

- Condition of Equilibrium for concurrent force system

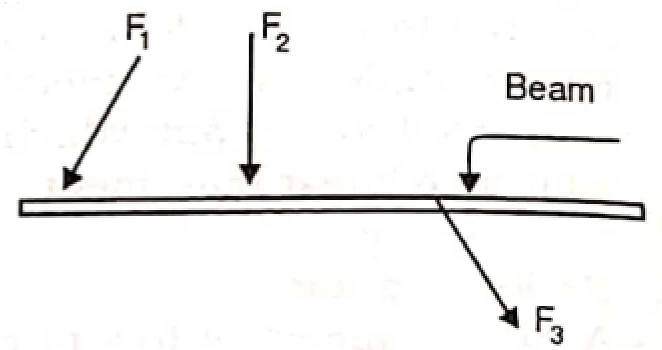
  $\Sigma F = 0$

- Condition of Equilibrium for parallel force system

  $\Sigma F = 0$

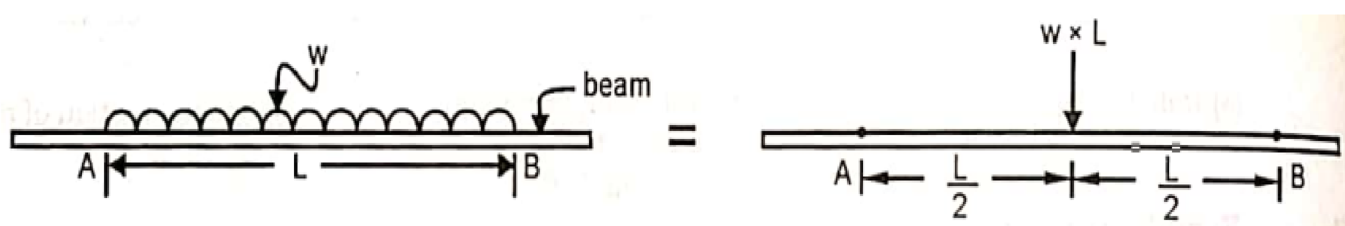
- A diagram formed by isolating the body from its surroundings and then showing all the forces acting on it is known as a Free Body Diagram (FBD).

- Types of Loads :-
  - Point load:-



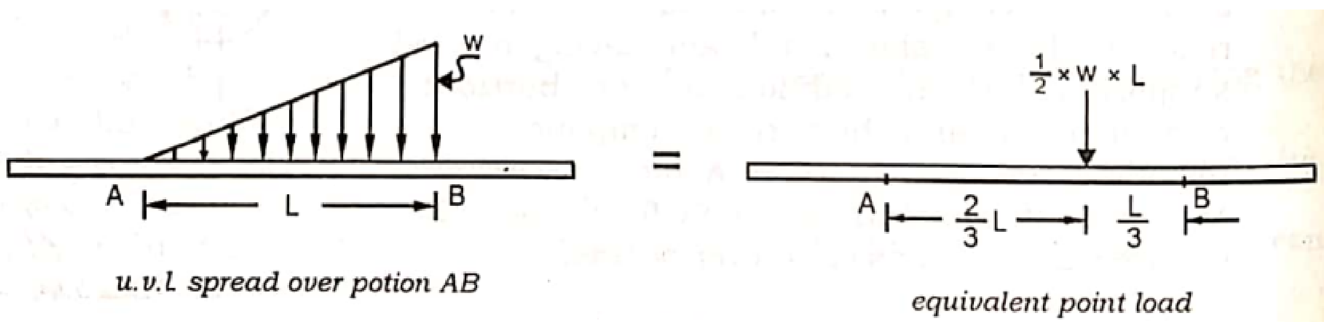
This load is concentrated at a point.

  - Uniformly distributed load :-
    In this loading the load of uniform intensity is spread over a length. UDL can be converted into an equivalent point load by multiplying the load intensity with the length. This equivalent point load would act at center of the spread.\\



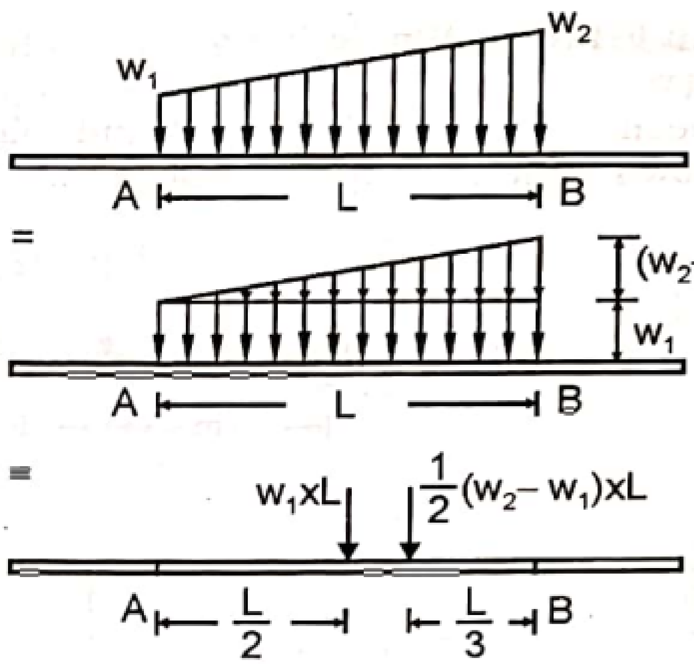
u.d.l spread over potion AB *equivalent point load*

o Uniformly varying load :-
In this loading, the load of uniformly varying intensity is spread over a length. UVL can be converted into an equivalent load which is equal to the area under the load diagram. The equivalent point load would act at the centroid of the load diagram.



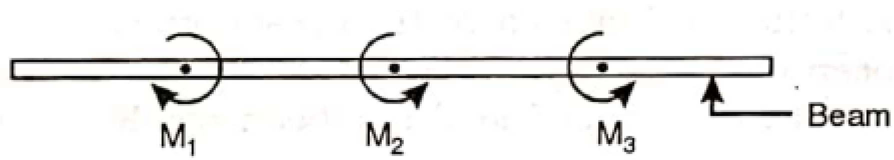
o Trapezoidal load :-
In this loading pattern, the load intensity varies uniformly from a lower intensity of w_1 N/m to a higher intensity w_2 N/m over a span of L meters. This loading is therefore a combination of a UDL of intensity and a UVL of intensity varying from zero to (w_2-w_1) N/m. The UDL portion is replaced by a point load of w_1 x L

acting at L/2 from A and the UVL portion is replaced by a point load of $(w_2 - w_1)/2$ acting at L/3 from B, as shown in figure.



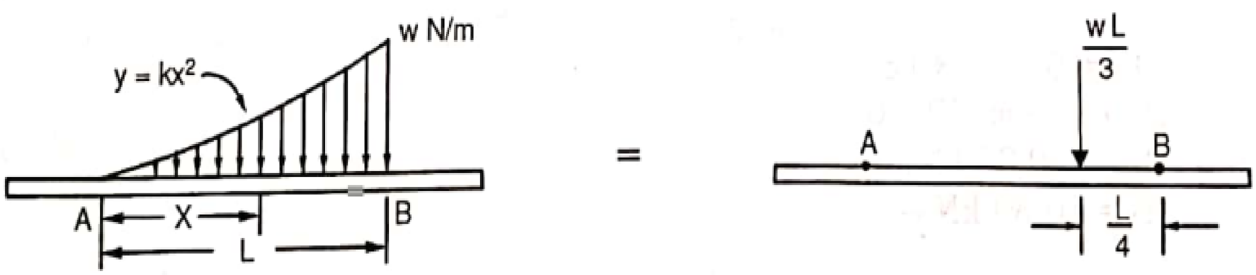
- Couple load :-

  A couple load acting on a body tends to cause rotation of the body. A couple load's location on the body is of no significance because couples are free vectors.



- Varying load :-

In this loading, the loading intensity varies as some relation. Figure shows a varying distributed load of parabolic nature. The equivalent point load is the are under the curve acting at the C.G of the area.



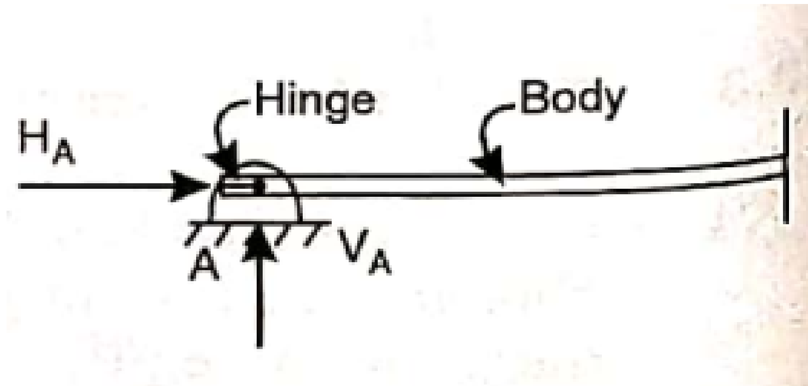
Varying load spread over portion AB

Equivalent point load

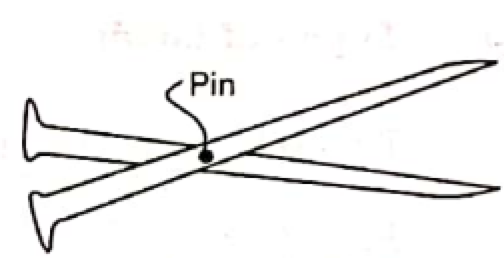
- Types of supports and their Reactions :-

  Whenever a body is supported, the support offers resistance, known as reaction.

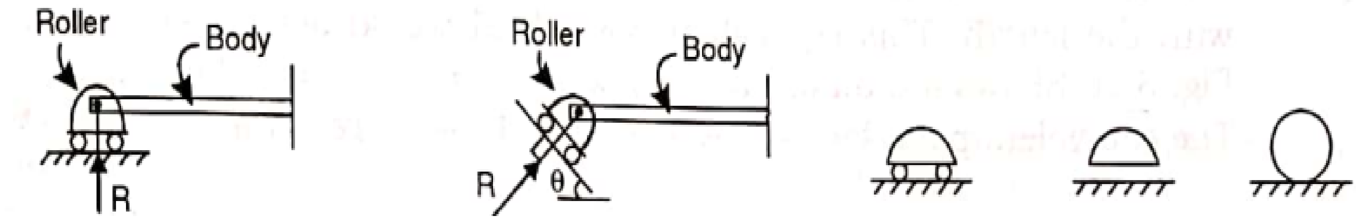
  - Hinge Support : A hinge allows free rotation of the body but does not allow the body to have any linear motion. It therefore offers a force reaction which can be split into horizontal and vertical components.



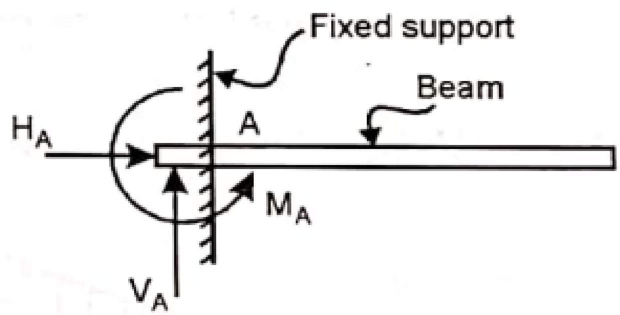
When two bodies are connected such that the connection allows rotation between them and behaves as a hinge then such a connection is referred to as an internal hinge or pin connection.



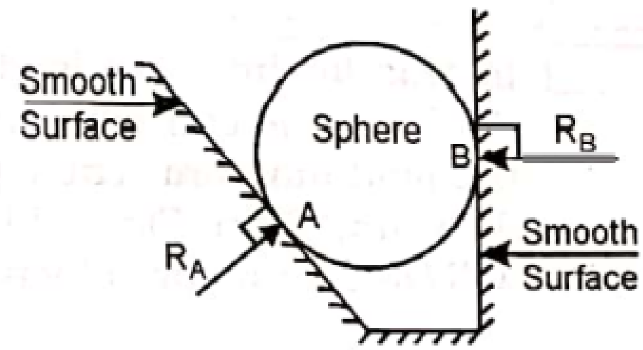
- Roller support : A roller support is free to roll on a surface on which it rests. It offers a force reaction in a direction normal to the surface on which the roller is supported.



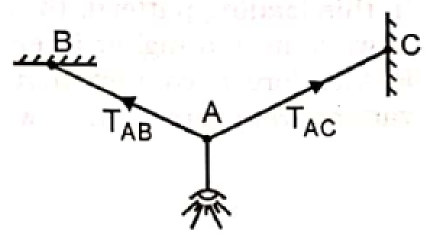
o Fixed support : A fixed support netiher allows any linear motion nor allows any rotation. It therefore offers a force reaction which can be split into a horizontal and a vertical component and also a moment reaction.



o Smooth surface support : A smooth surface offers a similar reaction as a roller support, i.e. a force reaction normal to the smooth surface. Each surface offers one force reaction, normal to the surface at contact points.



- o Rope/String/Cable support : It offers a pull force in a direction away from the body. This force is commonly referred to as the Tension



- Equilibrium of a Two force body.

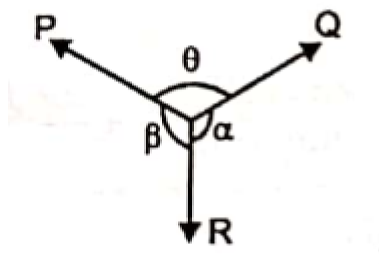
- Equilibrium of a Three force body.

  The concept of equilibrium of three force body states "If three coplanar forces act on a member and the member is in equilibrium, then the forces would be either concurrent or parallel."

- Lami’s Theorem :

  It deals with a particular case of equilibrium involving three forces only. It states “If three concurrent forces ac on a body keeping it in equilibrium, then each force is proportional to the sine of the angle between the other two forces”.

  For a system of three forces P, Q and R as shown ; Lami’s equation is as :



$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\theta}$$

- Conditions to use Lami’s theorem :

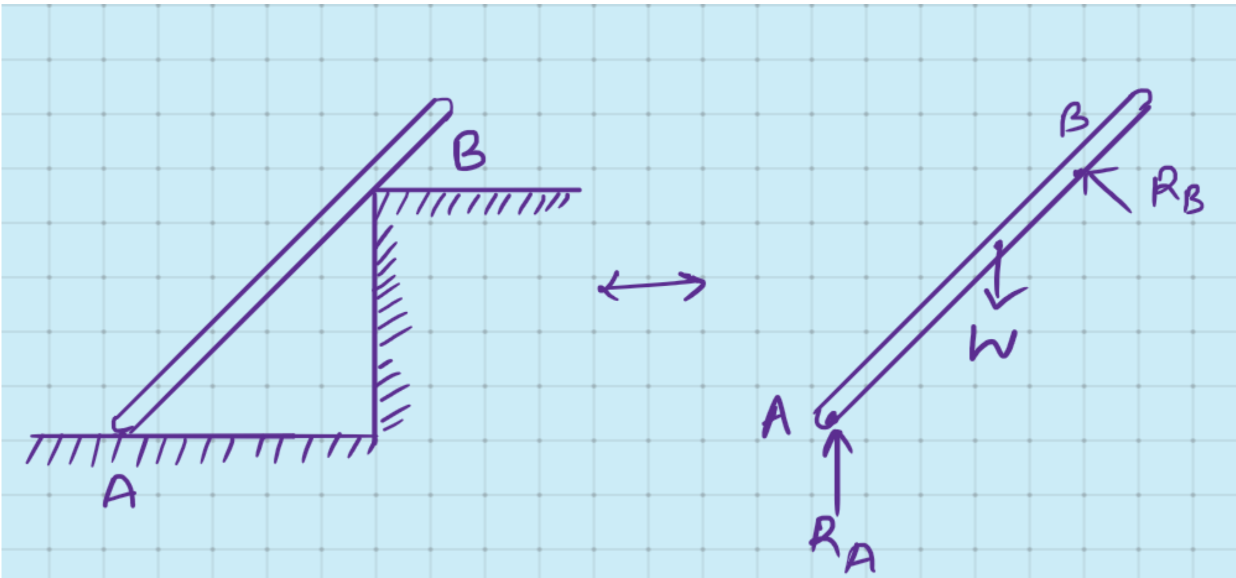
  The three concurrent forces should either act towards the point of concurrence or act away from it. If this is not the case, then using the principle of transmissibility they can be made in required form.

- Equilibrant Force :

  An unbalanced force system can be brought to equilibrium by adding an equilibrant force in the system. The equilibrant force has magnitude, direction and point of application as of the resultant of the system but has a sense opposite to that of the Resultant.

- FBD :-

1.



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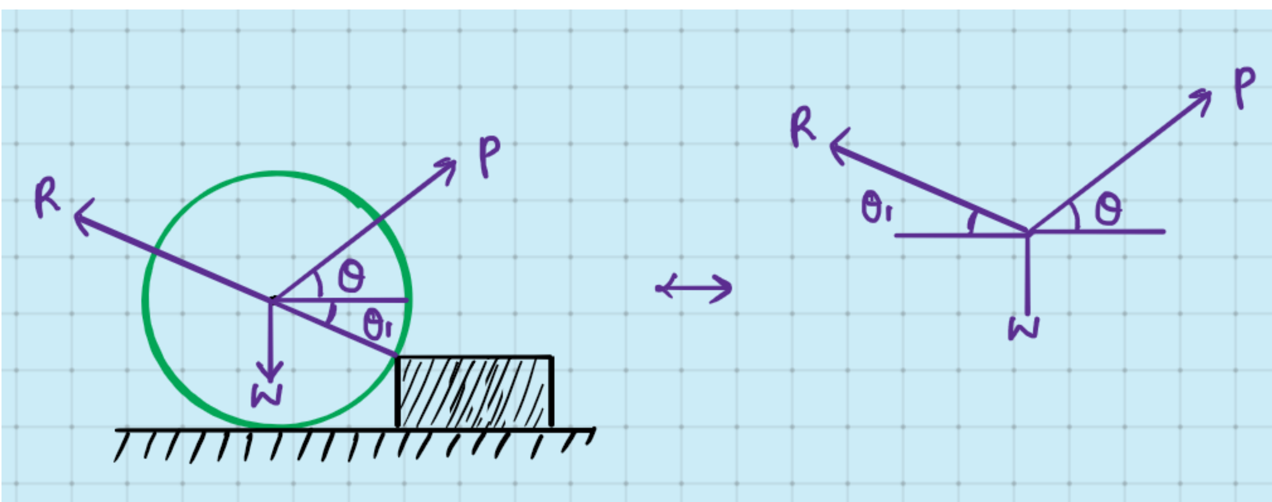

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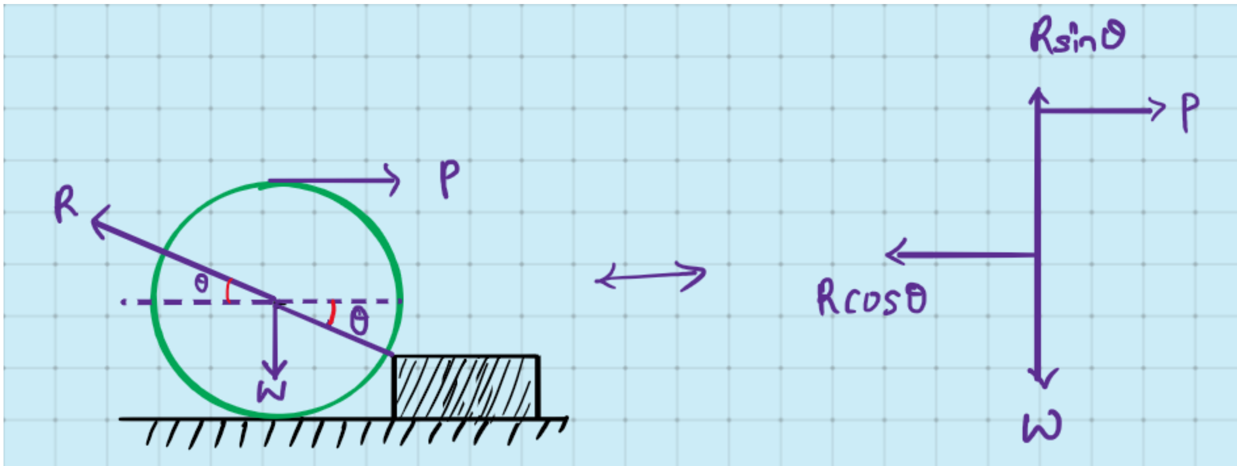
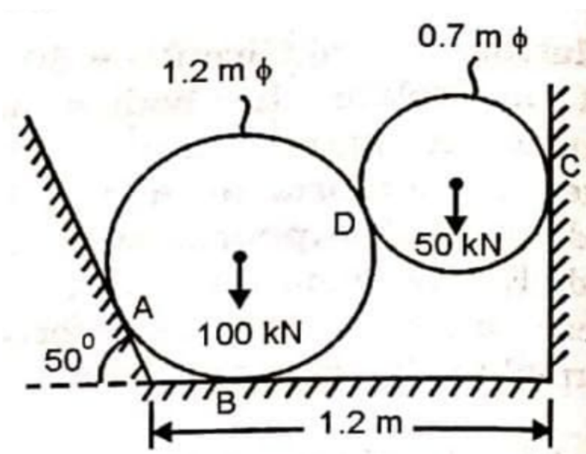
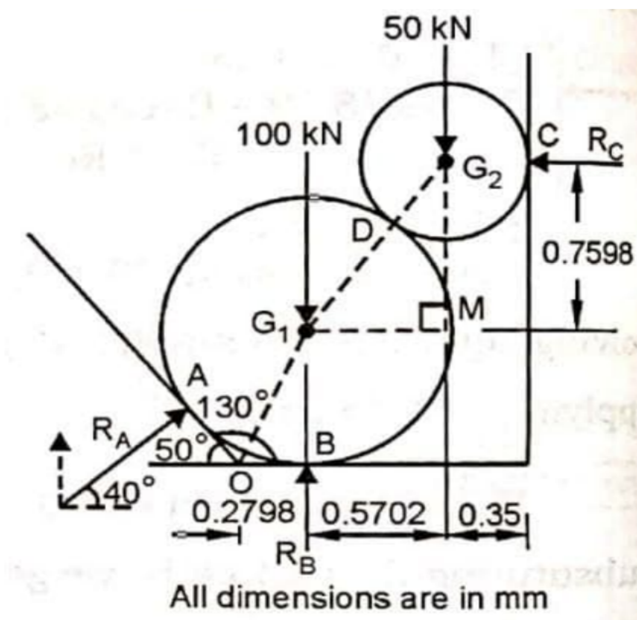
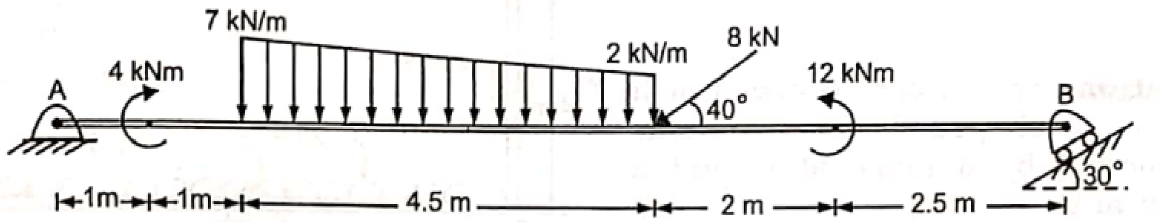

R
P
θ
θ
W
Rsinθ
P
Rcosθ
W

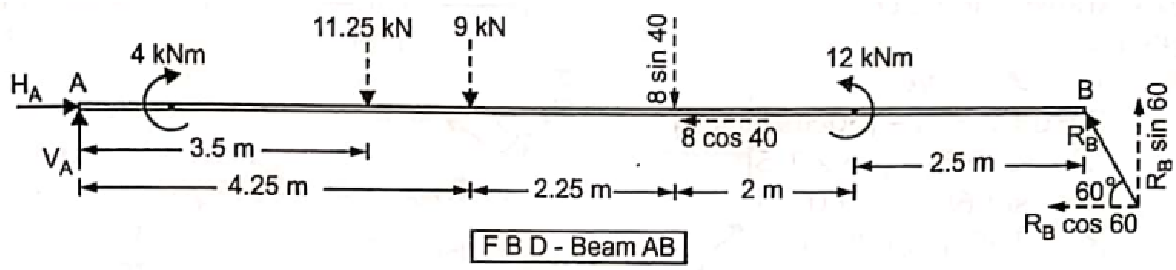
4.


0.7 m ϕ
1.2 m ϕ
C
D
50 kN
A
100 kN
50°
B
1.2 m

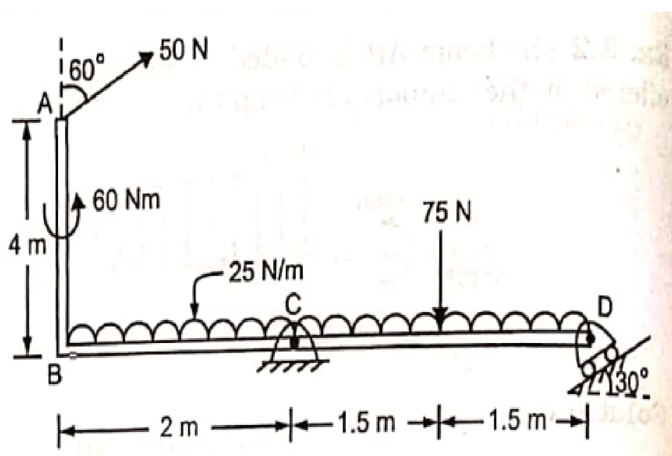

50 kN
100 kN
C
Rc
G2
D
0.7598
G1
M
A
130°
RA
50°
B
O
40°
0.2798
0.5702
0.35
RB
All dimensions are in mm

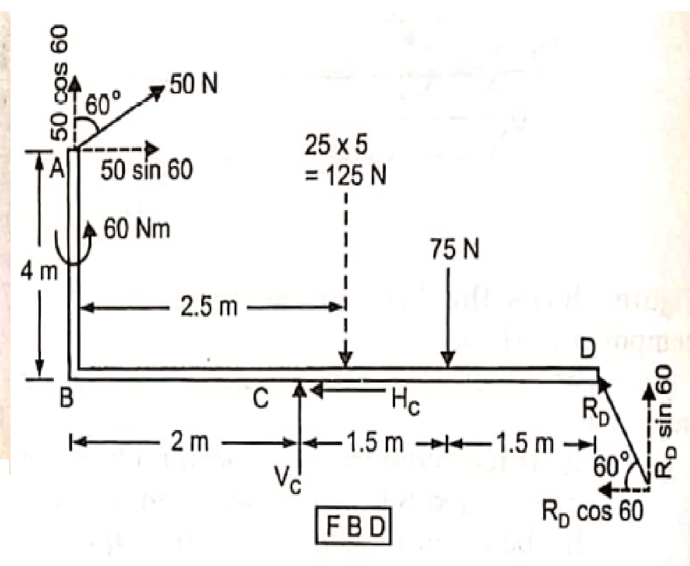
5.


7 kN/m
2 kN/m
8 kN
4 kNm
12 kNm
A
B
40°
30°
1m
1m
4.5 m
2 m
2.5 m


11.25 kN
9 kN
8 sin 40
4 kNm
12 kNm
H_A
A
B
V_A
8 cos 40
R_B
R_B sin 60
60°
R_B cos 60
3.5 m
4.25 m
2.25 m
2 m
2.5 m
F B D - Beam AB

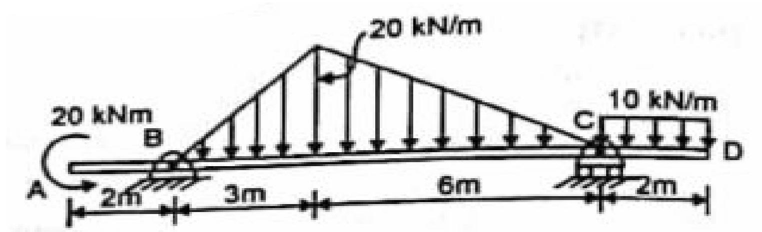
6.

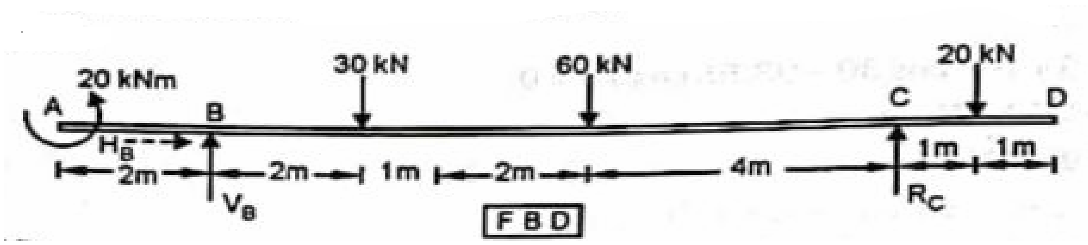

60°
50 N
A
60 Nm
4 m
75 N
25 N/m
C
D
B
30°
2 m
1.5 m
1.5 m


50 cos 60
60°
50 N
A
50 sin 60
25 x 5
= 125 N
60 Nm
4 m
75 N
2.5 m
D
B
C
H_C
R_D
R_D sin 60
2 m
1.5 m
1.5 m
60°
V_C
R_D cos 60
F B D

7.

8.





9.

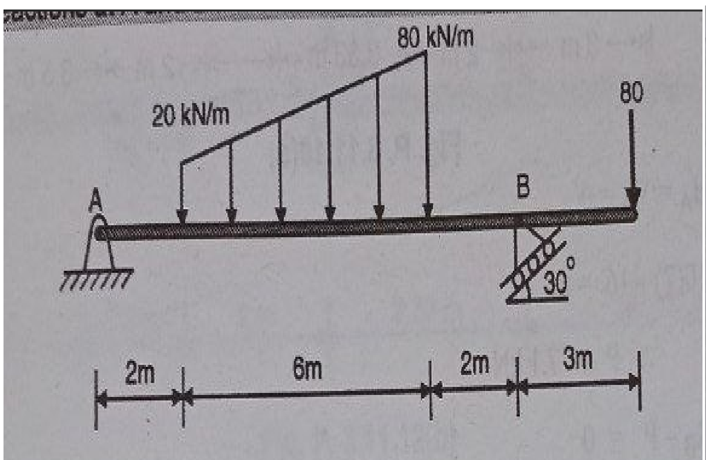


Fig. P. 3.11.9(a)

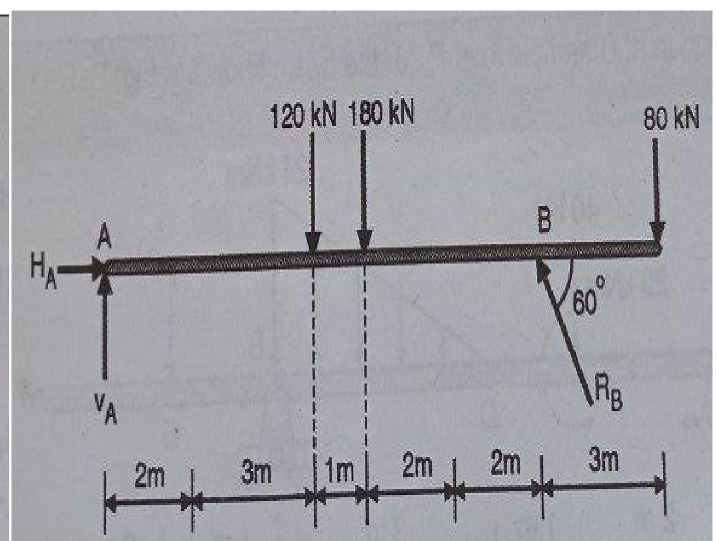


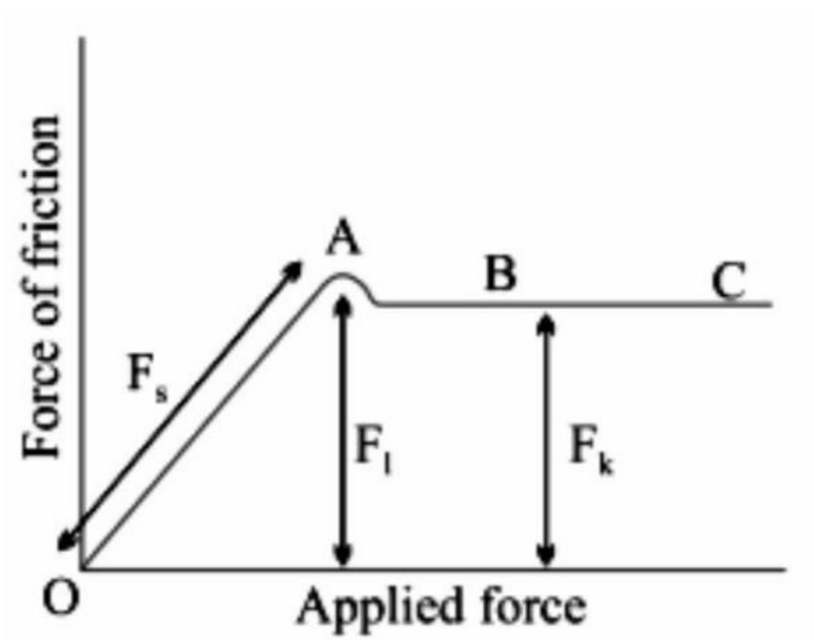
Fig. P. 3.11.9(b)

10.

- Friction is a force that resist the motion of the object that is in contact with another object or material. When objects don't move, the friction is called static. When they do move, the friction is called kinetic.

- Impending motion is the moment when the body is on the verge of slipping. Here, the static friction reaches its upper limit. For a given pair of surfaces in contact, the frictional force is given by the equation :

$$F = F_{max} = \mu_s N$$



- o Part OA = static friction (F_s).
- o At point A = limiting friction (F_1).
- o Beyond A, the force of friction is seen to decrease slightly. The portion BC = kinetic friction (F_k).
- o As the portion BC of the curve is parallel to x axis therefore kinetic friction does not change with the applied force.

- It has been experimentally found that the ratio of limiting friction F_{max} and the normal reaction N is a constant. This constant is referred to as *coefficient of static friction*, denoted as μ_s.

$$\mu_s = \frac{F_{max}}{N}$$

- Similarly, the ratio of kinetic frictional force and the normal reaction is known as *coefficient of kinetic friction*, denoted as μ_k.

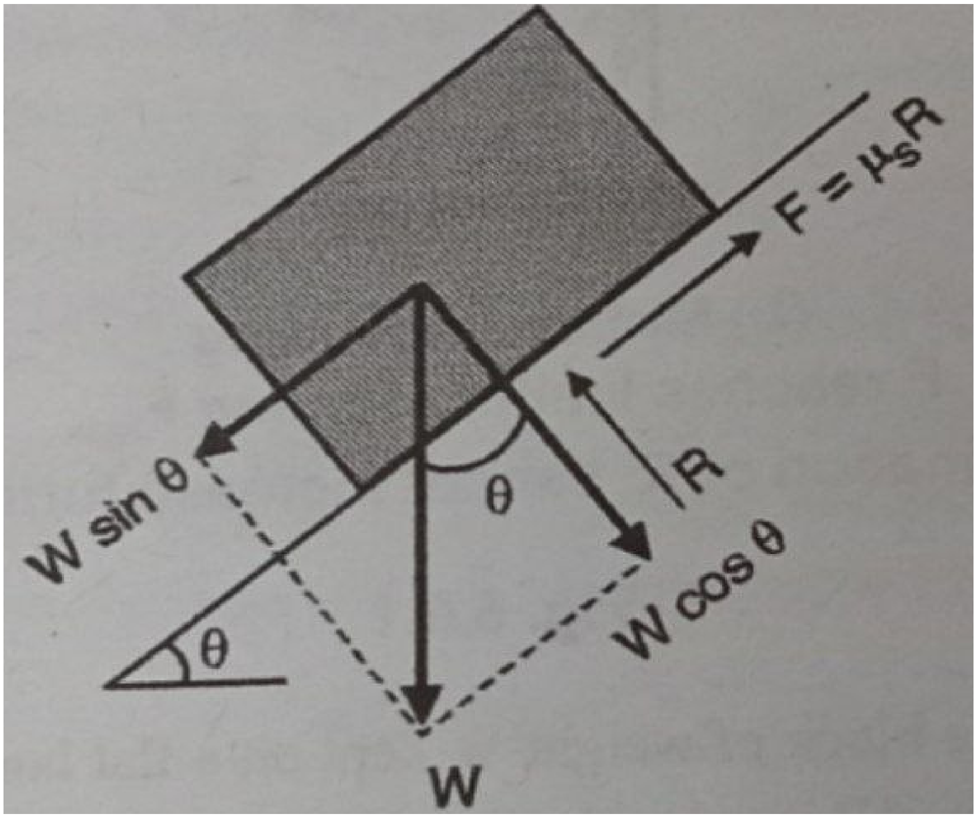
$$\mu_k = \frac{F_k}{N}$$

- Consider a block placed on a rough inclined plane. For small values of inclination, the frictional force would be sufficient to prevent the block from sliding down the slope. If the inclination of the plane is gradually increased, then at a particular angle θ, the block would have impending motion. This angle θ is called as the *angle of repose*. If the inclination is increased further the block would slide downward.

- Consider a block which has impending motion. However, it is still at rest and hence satisfies all equilibrium conditions,

Hence, $W \sin\theta = \mu_s R$ … (1)

$W \cos\theta = R$ … (2)



Dividing Equation (1) by (2) :-

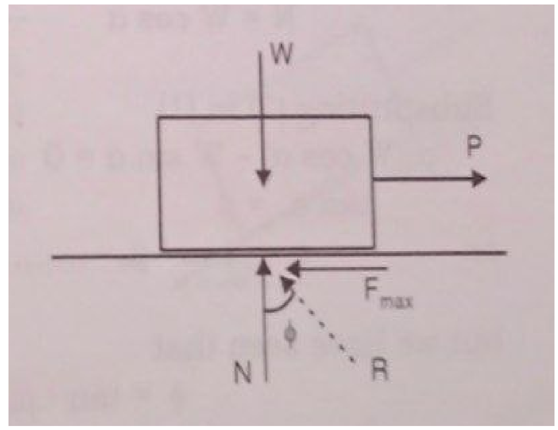
$\therefore \tan\theta = \mu_s$

$\therefore \quad \theta = \tan^{-1}(\mu_s)$

1. We can observe, that angle of repose θ is independent of weight of the body.
2. We observe that the numerical value of the 'Angle of repose' is equal to the 'Angle of friction'

   $\therefore \theta = \Phi$

- Let R be the resultant of F_{max} and N, making an angle Φ with the normal reaction. Here Φ is known as the angle of friction.



Here, $R = \sqrt{[F_{max}^2 + N^2]}$

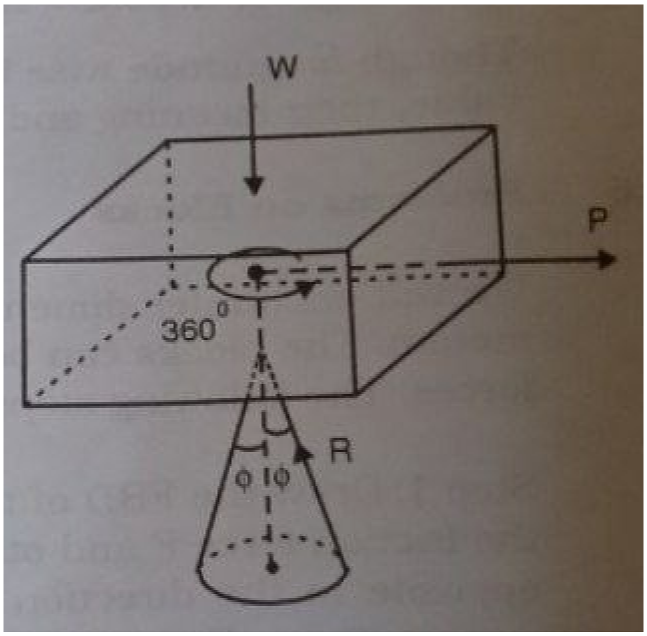
$\therefore R = \sqrt{[(\mu_s N)^2 + N^2]}$

Also, $\tan \Phi = \dfrac{F_{max}}{N} = \dfrac{\mu_s N}{N}$

$\therefore \tan \Phi = \mu_s$

$\therefore \quad \Phi = \tan^{-1}(\mu_s)$

- Figure shows a block of weight W on the verge of motion acted upon by force P. Let R be the resultant reaction at the contact surface acting at an angle of friction Φ. If the direction of force is changed by rotating it through 360 degrees, in a plane parallel to the contact surface, the force R also rotates and generates a right circular cone of semi-central angle equal to Φ. This right circular cone is known as the *cone of friction*.

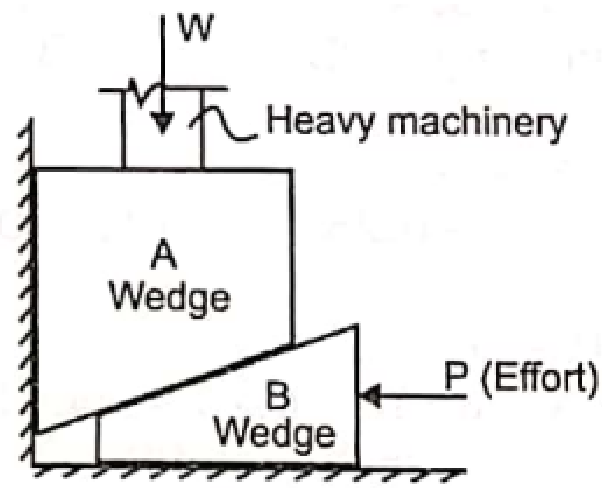


The significance of cone of friction is that for static body the resultant reaction force at the rough contact surface lies within the cone, while for a static body on the verge of motion, the reaction force lies on the surface of the cone of friction.

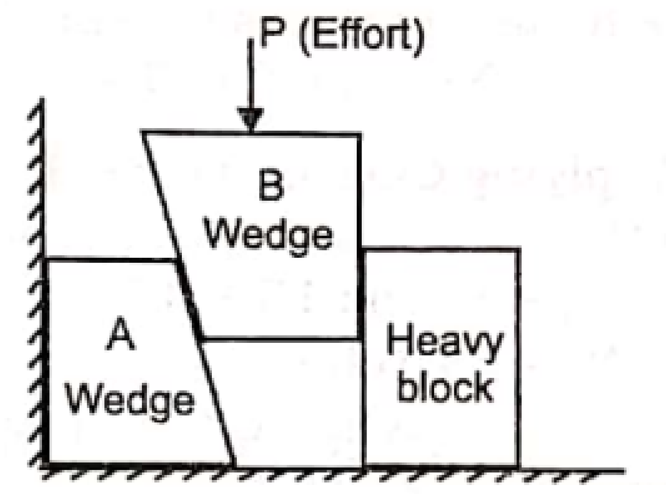
- Wedges are tapering shaped wooden or metal pieces. In combination with externally applied effort, they are used for giving a small movement to heavy blocks, machinery, pre-cast beam, columns etc. during erection or installation. They are also employed to keep heavy loads in equilibrium. Another advantage is that the effort can be applied in the convenient direction as desired.

Uses :-

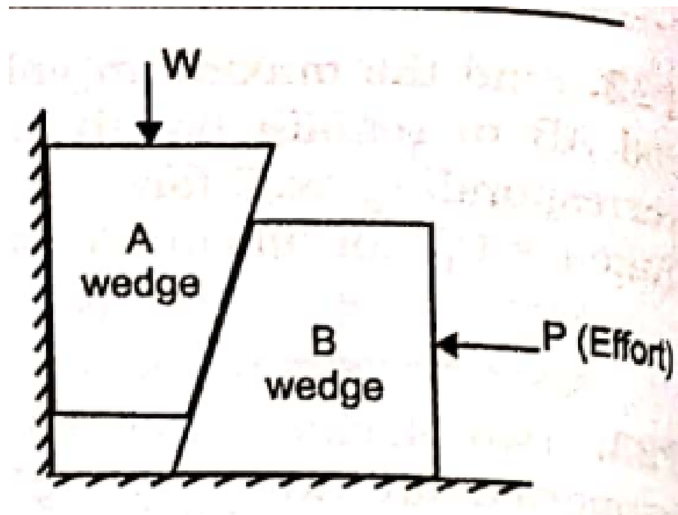
- To lift heavy load : In figure, two wedges and effort P in combination is used for imparting a small vertical movement to heavy machinery of weight W. Effort P is applied to the wedge B, which in turn raises wedge A causing the vertical movement of the load.



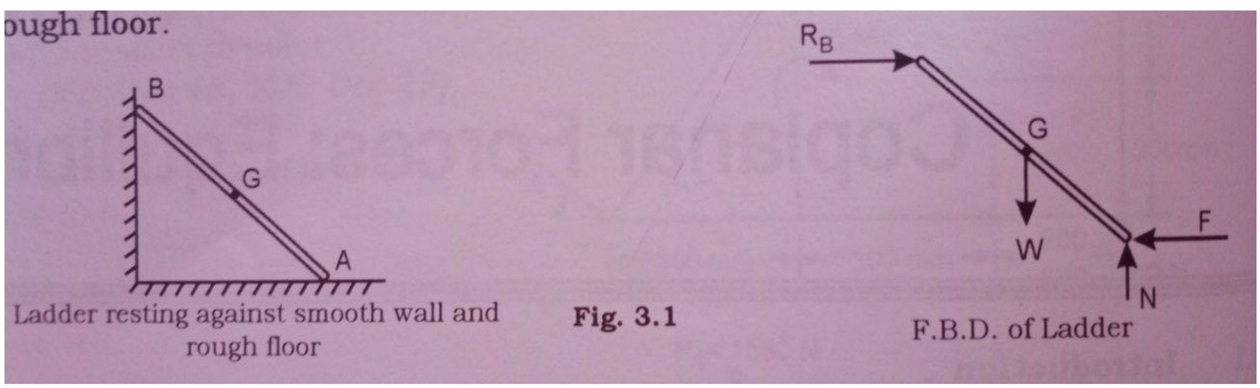
- To slide heavy load : In figure two wedges and effort P in combination is used to cause a small horizontal movement to the heavy block resting on the floor. Effort P is applied to wedge B, driving it down, thereby causing horizontal movement of the heavy block.



- To hold the system in equilibrium : Wedges may also be employed to prevent heavy loads from slipping. Figure shows a heavy load W kept on wedge A is prevented from slipping down by horizontal force P applied on wedge B. Another practical situation is truck drivers insert wooden wedges under the wheels when they park their trucks on slopes. This prevents the trucks from rolling down the slope.



- FBD of ladder :-



Ladder resting against smooth wall and rough floor

Fig. 3.1

F.B.D. of Ladder

Kinematics of a Particle

- Motion with variable equations and their applications to calculate different entities like displacement, velocity and acceleration?

- When a particle is thrown obliquely near the earth's surface, it moves along a curved path under constant acceleration that is directed towards the centre of the earth (we assume that the particle remains close to the surface of the earth). The path of such a particle is called a projectile and the motion is called projectile motion.

- The path described by a particle performing projectile motion is called trajectory.

- Equation of trajectory :-

$$y = x\tan\theta - \frac{gx}{2u^2\cos^2\theta}$$

OR

$$y = x\tan\theta(1 - x/R)$$

- Maximum height (H) :-

The maximum height of the projectile is when the projectile reaches zero vertical velocity

$$H = \frac{u^2\sin^2\theta}{2g}$$

- Time of flight (T) :-
  The time of flight of projectile motion is defined as the time from when the object is projected to the time it reaches the surface.

$$T = \frac{2u\sin\theta}{g}$$

- Range (R) :-
  Displacement in horizontal direction.

$$R = \frac{u^2\sin 2\theta}{g}$$

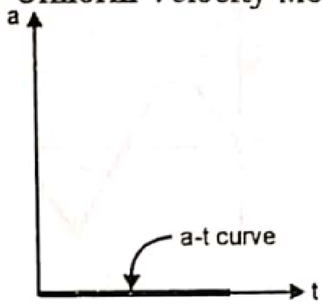
- Tangential and normal components of acceleration during projectile motion :-

$a_n = g\cos\theta$ $\qquad\qquad$ $a_t = g\sin\theta$

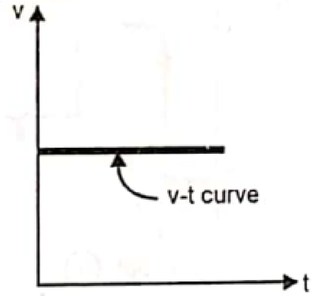
No.	Motion curve	Use	Graphical Formula
1	x – t	Slope of x – t curve gives velocity	v = (slope x – t curve)
2	v – t	a) Slope of v – t curve gives acceleration b) Area under v – t curve gives change in position and hence the new position.	a = (slope v – t curve) $x_f = x_i +$ [area under v – t curve]
3	a – t	a) Area under a – t curve gives change in velocity and hence the new velocity b) Area under a – t curve also helps in finding the particle's position	$v_f = v_i +$ [area under a – t curve] $x_f = x_i + v_i \times t +$ (area under a – t curve) $(t - t_G)$
4	v – x	Slope of v – x curve helps in finding the particle's acceleration	a = v × (slope v – x curve)

-Motion curves and applications

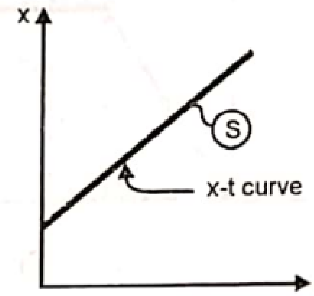
a) Uniform Velocity Motion curves



Straight horizontal curve on the time axis

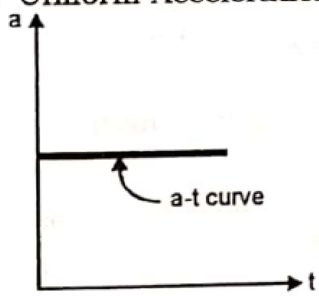


Straight horizontal curve parallel to time axis

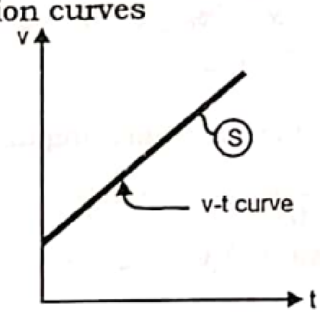


Straight inclined curve

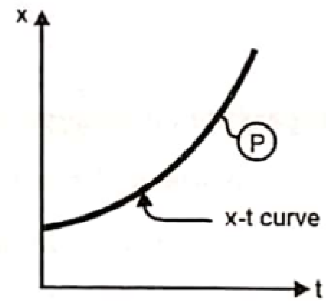
b) Uniform Acceleration Motion curves



Straight horizontal curve parallel to time axis

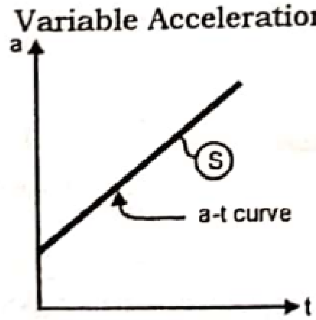


Straight inclined curve

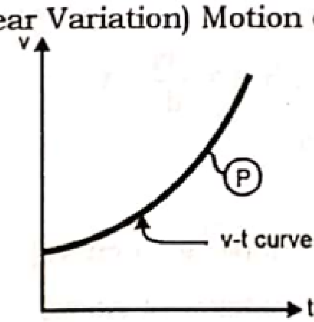


Second degree curve (Parabolic)

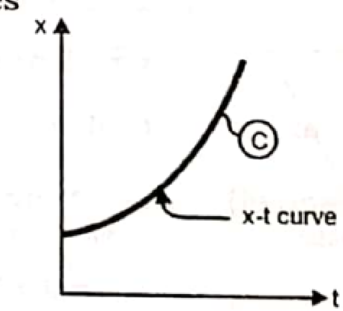
c) Variable Acceleration (Linear Variation) Motion curves



Straight inclined curve



Second degree curve (Parabolic)



Third degree curve (cubic)

- When the particle travels along a curved path, its motion is called curvilinear motion. If the particle moves in x-y plane, it is called 2 dimensional curvilinear motion, whereas if it moves in space, it is called 3 dimensional curvilinear motion.
  Examples :-
  A car travelling on a curved road
  A swinging pendulum
  A stone projected at an angle

- Curvilinear motion can be split into motion along x direction, y direction and z direction, which can be independently worked as three rectilinear motions along the x, y and z directions. For a curvilinear motion we therefor e write position, velocity and acceleration in the vector form as :-

$\bar{r} = x\bar{i} + y\bar{j} + z\bar{k}$ and magnitude $r = \sqrt{[x^2 + y^2 + z^2]}$

$\bar{v} = dr/dt = v_x\bar{i} + v_y\bar{j} + v_z\bar{k}$ and magnitude

$v = \sqrt{[v_x^2 + v_y^2 + v_z^2]}$

$\bar{a} = dv/dt = a_x\bar{i} + a_y\bar{j} + a_z\bar{k}$ and magnitude

$a = \sqrt{[a_x^2 + a_y^2 + a_z^2]}$

ICR

- Relation between angular and linear velocity :-
  V=RW or W=V/R

- The motion in which the body is undergoing transitional as well as rotational motion is known as general plane motion. E.g. Rolling wheel, Sliding rod, etc…

- ICR is the point around which the G.P body rotates at a given instant. This point keeps changing as the G.P body performs its motion.

- To find ICR :-
    - Locate a point on the G.P body whose magnitude, direction and sense of velocity is known. Mark the direction of velocity of these points
    - Draw perpendicular to the direction of velocities and extend them to intersect at a point. Call the point "I"

- Point I i.e. instantaneous center, is the center of rotation of the G.P body at the instant. Now treating the G.P body as a rotating body about I and using V=RW relation, the angular velocity of the G.P body can be found out.