MODULE 4
KINEMATICS OF PARTICLES

Objectives:

- To learn motions with uniform velocity-acceleration as well as variable acceleration using Newton's Laws of Motion
- To learn the velocities and accelerations of the body moving along the curved path
- To understand the graphical solution to find the displacement, velocity, acceleration and time of the particle' motion through motion curves
- To learn about the parabolic motion of the particle i.e. motion under the effect of gravity along a parabolic path.
- Understanding concept of dependent motion

4.6. Key Notations:

Rectilinear Motion u = initial velocity v = final velocity a = acceleration t = time s = displacement	• Projectile Motion 1. U = initial velocity 2. R = horizontal range 3. H_{MAX} = Maximum Height 4. T= time of flight 5. α = Angle of Projection • Curvilinear motion 1. a_n = normal acceleration 2. a_t = tangential acceleration 3. a = total acceleration 4. ρ = radius of curvature

4.7. Formulae :

- Rectilinear Motion:

  Equations of motion:

➢ For Uniform Acceleration Motion:

$$1. v = u + at$$

$$2. s = ut + \frac{1}{2}at^2$$

$$3. v^2 = u^2 + 2as$$

➢ For Uniform Motion:

$S = u\,t$

➢ Displacement in n^{th} second:

$s^{nth} = u + \frac{a}{2}(2n - 1)$

- **Motion under gravity:**

As 'g' is always downward, the kinematic equations can be formulated as

$$1. v = u - gt$$

$$2. s = ut - \frac{1}{2}gt^2$$

$$3. v^2 = u^2 - 2gs$$

where ‘s’ is the vertical distance between point of projection and second point which may or may not be a point of landing.

Sign conventions:

1. All upward quantities as positive
2. All downward quantities as negative
3. If motion of the particle is upward, consider ‘s’ as positive
4. If motion of the particle is downward, consider ‘s’ as negative

- **Curvilinear Motion**

1. For particles moving along the curve with uniform tangential acceleration

$$v = u + a_t t$$

$$s = ut + \frac{1}{2}a_t t^2$$

$$v^2 = u^2 + 2a_t s$$

$$v = \frac{ds}{dt}$$

Tangential Acceleration, $a_t = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$

Normal Acceleration, $a_n = \frac{v^2}{\rho}$

Total Acceleration: $a = \sqrt{a_t^2 + a_n^2}$

2. **Radius of Curvature:**

When a particle is moving along a curve y = f(x), the radius of curvature at a point is given by

$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|}$$

Also $\rho = \frac{[v_x{}^2 + v_y{}^2]^{3/2}}{v_x a_y - a_x v_y}$

4.8. Key Definitions:

1. **Dynamics:** It is the branch of mechanics which deals with the particles and bodies which are in motion.

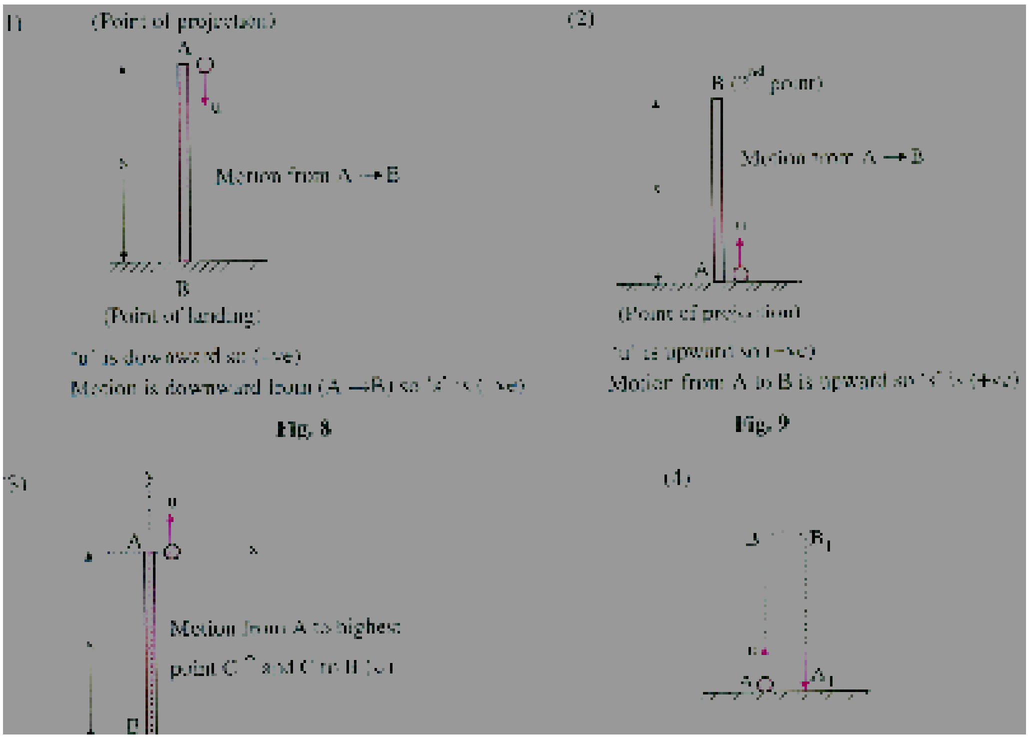
2. **Kinematics**: It is the branch of dynamics which deals with the motion of particles and bodies without reference to the forces which cause the motion as well as mass of particle or body. It is the study of geometry of motion.
3. **Particle**: In a particular problem if the dimensions of the body under consideration are not relevant in solving that problem, then we consider the body as a particle. i.e its entire mass is assumed to be concentrated at a point.
4. **Rectilinear motion**: The motion of the particle in the straight line. E.g. car traveling along a straight road, a stone thrown straight up in the air, etc.
5. **Motion under Gravity:** Motion under gravitational field: it may be controlled (eg. Rising helicopter, balloon, etc.) or motion under the action of gravity (eg. Freely falling body)
6. **Curvilinear Motion**: It is the motion in which the particle travels along a curved path .E.g. car traveling along a curved road, a stone thrown at an angle with the horizontal, etc.
7. **Projectile motion:** In projectile motion the particle moves in space along vertical and horizontal directions simultaneously. For projectile motion the particle is projected in space at a certain angle with horizontal.
8. **Absolute motion:** The motion of the particle with respect to the fixed frame of reference is called absolute motion of a particle.
9. **Relative motion:** The motion of a particle relative to a set of axes which are moving is called as relative motion.
10. **Dependent motion:** When motion of one particle depends upon the motion of other particle or several particles, the motion is called as dependent motion.

4.9. Theory:

- **Rectilinear Motion:**

  If the particle is moving along straight path then it is called as rectilinear motion.
  e.g. A train moving on single track, a stone released from top of tower etc.
- **Motion under gravity:**

  The vertical motion of a particle under the influence of constant gravitational acceleration 'g' is known as motion under gravity.
  Referring to the sign conventions following examples will illustrate the details:



- **Rectilinear motion with variable acceleration:**

  Sometimes a particle moves along a straight line with acceleration which varies w.r.t. one or more variables x, v and t.

  According to the variation of acceleration as a function of variable the motion can be classified as 1) a = f(t) 2) a = f(x) 3) a = f(v)

- **Displacement in the n^th second: (With uniform acceleration)**

  Consider a particle moving along a straight line with uniform acceleration 'a'.

  The displacement of the particle in n seconds is

$$s_n = un + \frac{1}{2}an^2$$

And, the displacement of particle in (n-1) seconds is

$$s_{(n-1)} = u(n-1) + \frac{1}{2}a(n-1)^2$$

So displacement of particle in n^{th} second,

$$s^{nth} = s_n - s_{n-1} = [u\,n + \tfrac{1}{2}\,a\,n^2] - [u\,(n-1) + \tfrac{1}{2}\,a\,(n-1)^2$$

$$= u\,n + \frac{a\,n^2}{2} - [u\,n - u + \frac{a}{2}(\,n^2 - 2\,n + 1)]$$

$$= u\,n + \frac{a\,n^2}{2} - u\,n + u - \frac{a}{2}n^2 + an + \frac{a}{2}$$

$$\text{So, } s^{nth} = u + \frac{a}{2}(2\,n - 1)$$

If the particle moves with uniform velocity and acceleration then equations of motion can be used, but if the velocity and/or varies w.r.t time or displacement, then the equations of displacement, velocity and acceleration will be derivate or integrated correspondingly.

- **Motion Curves:**

This is the graphical approach to motion of particle and particularly useful when acceleration, velocity and position are not analytical functions of ‘t’.
Following are the main types of motion curves:

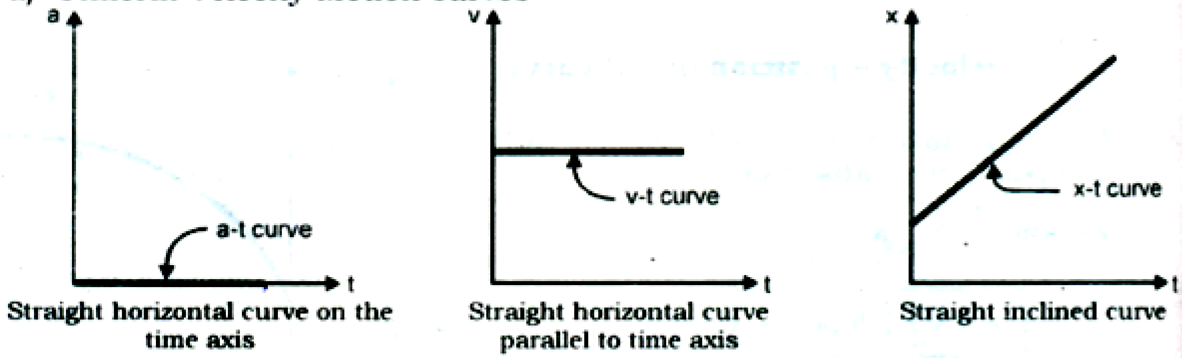
1) a – t diagram 2) v – t diagram

3) x – t diagram 4) v – x diagram

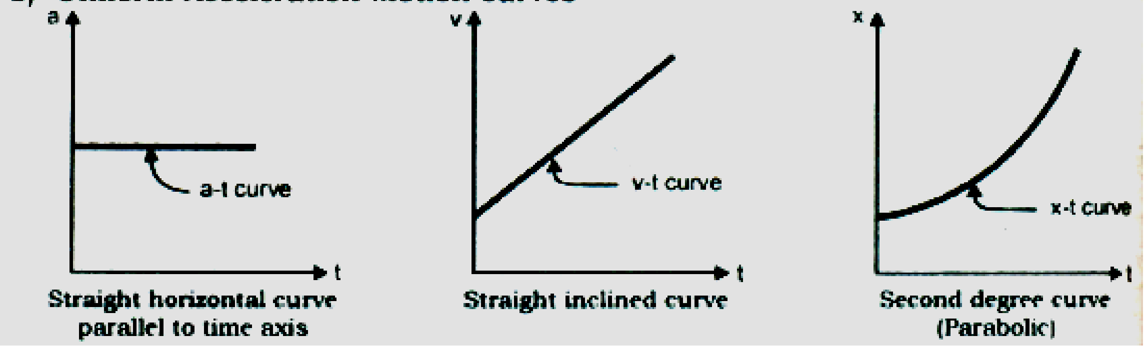
No.	Motion Curve	Use	Formula
1	x - t	Slope of x-t curve gives velocity	V = (slope of x-t curve)
2	v - t	Slope of v-t curve gives acceleration Area underv-t curve gives change in position and hence the new position.	A = (Slope of v-t curve) $X_f = X_i$ + (area under v-t curve)
3	a - t	Area under a-t curve gives change in velocity and hence the new velocity Area under a-t curve also helps in finding the particle’s position	$V_f = V_i$ + (Area under a-t curve) $X_f = X_i + V_i$ x t + (area under a-t curve) $(t – t_G)$

Key concepts in motion diagrams:

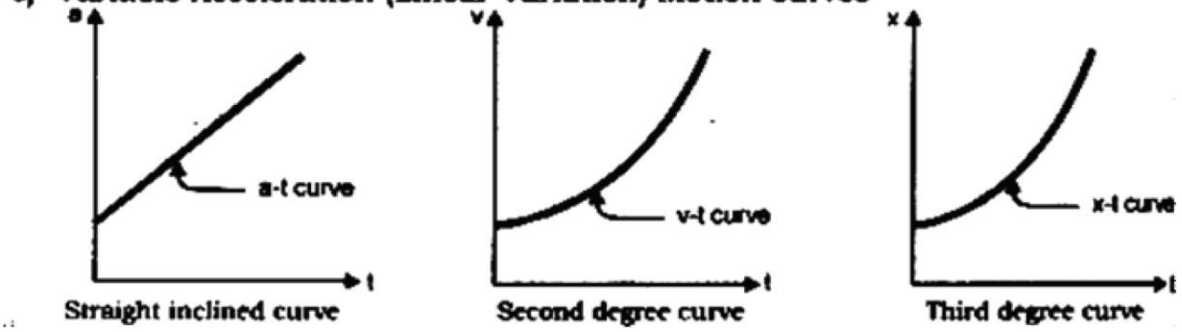
a) Uniform Velocity Motion curves



b) Uniform Acceleration Motion curves



c) Variable Acceleration (Linear Variation) Motion curves



- **Curvilinear Motion**

Coordinate systems in Curvilinear motion:

1) Rectangular coordinates system
2) Normal and tangential coordinate system (path variables)

➢ **RECTANGULAR COORDINATES**

In this system, for plane curve we use x and y coordinates and for space curve we use x, y and z coordinates.

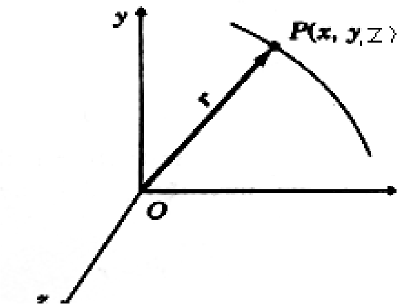
Position Vector: Location of the particle P situated at point x, y and z is defined by position vector, $\underline{r}$.

$\underline{r}$ = x i + y j + z k (for space curve)

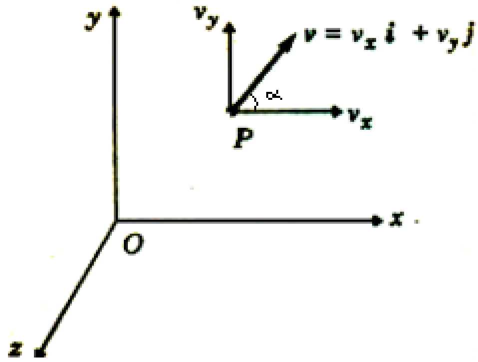
$\underline{r}$ = x i + y j (for plane curve)

The magnitude of position vector is $|r| = r = \sqrt{x^2 + y^2 + z^2}$ and its direction is

$$cos\ cos\ \alpha\ = \frac{x}{r},\ cos\ cos\ \beta\ = \frac{y}{r}\quad cos\ cos\ \gamma\ = \frac{z}{r}$$



Velocity: The velocity of the particle can be expressed as



$$v = \frac{dr}{dt} = \frac{d}{dt}\ xi + yj$$

$$= \frac{dx}{dt}i + \frac{dy}{dt}j$$

$$= v_x i + v_y j$$

The magnitude of velocity is given by $|v| = v = \sqrt{v_x^2 + v_y^2}$

And its direction is given by $tan\ tan\ \alpha\ = \frac{v_y}{v_x}$

Acceleration: Acceleration of the particle can be expressed as

$$a = \frac{dv}{dt} = \frac{d}{dt}v_x i + v_y j$$

$$= \frac{dv_x}{dt}i + \frac{dv_y}{dt}j$$

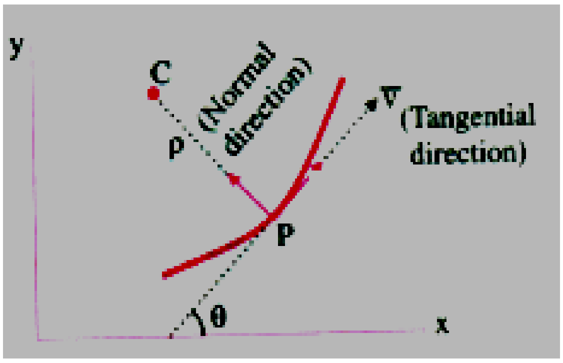
$$= a_x i + a_y j$$

The magnitude of acceleration 'a' is given by $|a| = a = \sqrt{a_x^2 + a_y^2}$ and its direction is given by

$$tan\ tan\ \beta\ = \frac{a_y}{a_x}$$

NORMAL AND TANGENTIAL CO-ORDINATE SYSTEM:

Consider a particle moving along a curve and is located at point P at any instant t. As velocity is always tangential to the path, the direction along tangent drawn at that point is 'tangential direction' and the direction which is perpendicular to velocity vector and directed towards the centre of curvature is known as normal direction.



If motion of particle is described with these two directions, the co-ordinate system is called normal and tangential co-ordinate system.

Velocity components: Since velocity of the particle is always tangential to the path, the normal component of velocity must be zero and the tangential component is equal to the velocity itself.

So $v_n = 0$ and $v_t = v$

Acceleration components:

Tangential Accelerations, $a_t = \frac{dv}{dt} = \frac{dv}{ds}\frac{ds}{dt} = v\frac{dv}{ds}$

Normal Acceleration, $a_n = \frac{v^2}{\rho}$

Total Acceleration: $a = \sqrt{a_t^2 + a_n^2}$

Direction of total acceleration, $tan\ tan\ \alpha\ = \frac{a_t}{a_n}$

Key Concepts:

1) The normal acceleration a_n is always directed towards the centre of the curvature.
2) The tangential acceleration a_t will be in positive t-direction if speed is increasing and in the negative t-direction if speed is decreasing.
3) At an inflection point i.e the top most point on the curvature, the normal acceleration a_n = 0 because at that point radius of curvature (ρ)becomes infinite.
4) a_t reflects change in speed of the particle while normal acceleration a_n reflects change in direction of motion of the particle.
5) If particle is moving with constant speed a_t = 0.

Radius of Curvature:

When a particle is moving along a curve y = f(x), the radius of curvature at a point is given by

$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\left|\frac{d^2y}{dx^2}\right|} \quad \text{......... (I)}$$

The above equation can be written in x,y coordinates as follows

We can write, $\frac{dy}{dx} = \frac{dy}{dt}\frac{dt}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{v_y}{v_x}$ ……… (II)

By product of derivatives $\frac{d^2y}{dx^2} = \frac{v_x\frac{d}{dt}v_y - v_y\frac{d}{dt}v_x}{v_x^2} = \frac{v_x a_y - v_y a_x}{v_x^2}$ …………(III)

Substituting equations (II) and (III) in equation (I)

$$\rho = \frac{\left[1+\left(\frac{v_y}{v_x}\right)^2\right]^{3/2}}{\frac{v_x a_y - v_y a_x}{v_x^2}}$$

$$\text{So } \rho = \frac{[v_x^2+v_y^2]^{3/2}}{v_x a_y - v_y a_x}$$

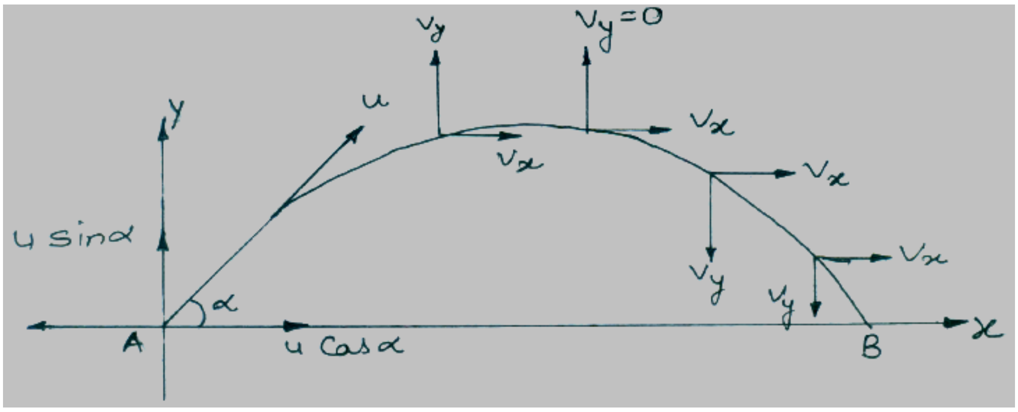
- **Projectile Motion:**

  When a particle is projected in space, its motion is a combination of horizontal and vertical motion (rectangular co-ordinates). The motion of such a particle is called as projectile motion.

  In projectile motion wind resistance, curvature and rotation of earth which affects the actual path will be neglected. Also assume 'g' as constant.

  There are two rectangular components of acceleration.

$$a_x = 0 \quad \text{and} \quad a_y = -g$$



If we observe motion in x-direction, we see that acceleration in x-direction, $a_x = 0$ so the motion in x-direction is uniform motion. While acceleration in y-direction is gravitational acceleration 'g' so the motion in y-direction is motion under gravity.

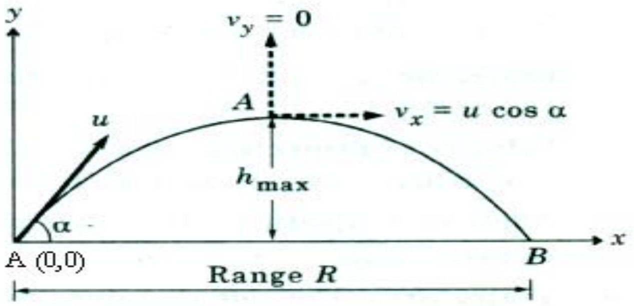
➢ **Derivation of time of flight, horizontal range and maximum height attained by a projectile on horizontal plane:**

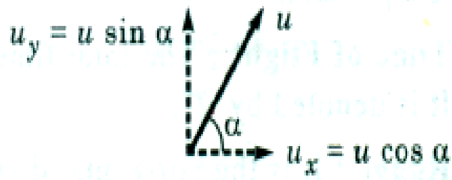
  Consider a particle projected from point 'A' and lands at point 'B' both on HP.

  Let, u = Initial velocity of projection

  α = Angle of projection

  t = Total time of flight





Since air resistance is to be neglected, x motion is uniform motion and y motion is motion under gravity.

1) Time of flight:

Consider y-motion from A→B (MUG)

$$S_y = U_y t - \frac{1}{2} g t^2$$

$$0 = (u \, sin \, sin \, \alpha \,) t - \frac{1}{2} g t^2$$

$t = \frac{2usinsin \, \alpha}{g}$ Expression for the time of flight of the projectile

2) Horizontal Range:

Consider x-motion from A→B (UM)
S = velocity x time
$S_X = v_X \, . \, t$
So R = u cos . t
Substituting value of $t = \frac{2usinsin \, \alpha}{g}$ in above equation

$$R = u \, cos \, cos \, \alpha \frac{2u \, sin \, sin \, \alpha}{g} = \frac{u^2}{g} (2 \, sin \, sin \, \alpha \;\; cos \, cos \, \alpha \,)$$

$R = \frac{u^2 sinsin \, 2\alpha}{g}$ Expression for horizontal range

3) Maximum Range:

For the range to be maximum,

$$\frac{d}{d\alpha} (sin \, sin \, 2\alpha \,) = 0 \qquad \therefore cos \, cos \, 2\alpha = 0$$

$$\therefore 2\alpha = 90 \qquad \therefore \alpha = 45°$$

∴ for maximum range angle of projection should be 45°

$\therefore R_{max} = \frac{u^2}{g}$ Expression for maximum range on horizontal range

For maximum range α = 45^0 only when there is no restriction in vertical direction. For a situation involving such restriction always first find the allowable angle of projection and corresponding maximum possible range.

4) Maximum Height:

Consider y-motion from A→C (MUG)

$$V_y^2 = U_y^2 - 2gS_y$$

$$0 = (u \sin \sin \alpha)^2 - 2gh$$

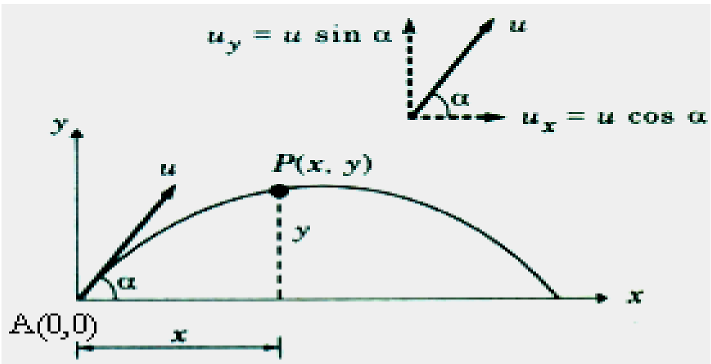
$\therefore \quad H = \frac{u^2 \sin^2 \alpha}{2g}$ Expression for maximum height attained

The above sets of formulae are applicable only when point of projection and point of landing are at the same level on HP. If the two points at different levels use x-motion and y-motion for analysis.

- **Derivation of the equation for the path of the projectile: (Equation of trajectory):**

  Let the particle be projected from A(0,0) with initial velocity 'u' and angle of projection 'α'. Let P (x, y) be some point on the path of projectile at time t_1

  Consider horizontal and vertical motion of projectile



Let, after a time 't_1' particle be reached at point P(x, y)

Consider x-motion from A→P (UM)

$\therefore x = u \cos \cos \alpha \, t_1$

$\therefore t_1 = \frac{x}{u \cos \cos \alpha}$ (I)

Consider y-motion from A to P (MUG)

$\therefore \quad S_y = U_y t - \frac{1}{2} g t^2$

$y = u \sin \sin \alpha \, t_1 - \frac{1}{2} g t_1^2$ (II)

We know that the path equation is independent of time, we substitute value of time $t_1 = \frac{x}{u \sin \sin \alpha}$ in equ. (II)

We get, $y = u \sin \sin \alpha \frac{x}{u \cos \cos \alpha} - \frac{1}{2} g \left(\frac{x^2}{u^2 \cos^2 \alpha} \right)$

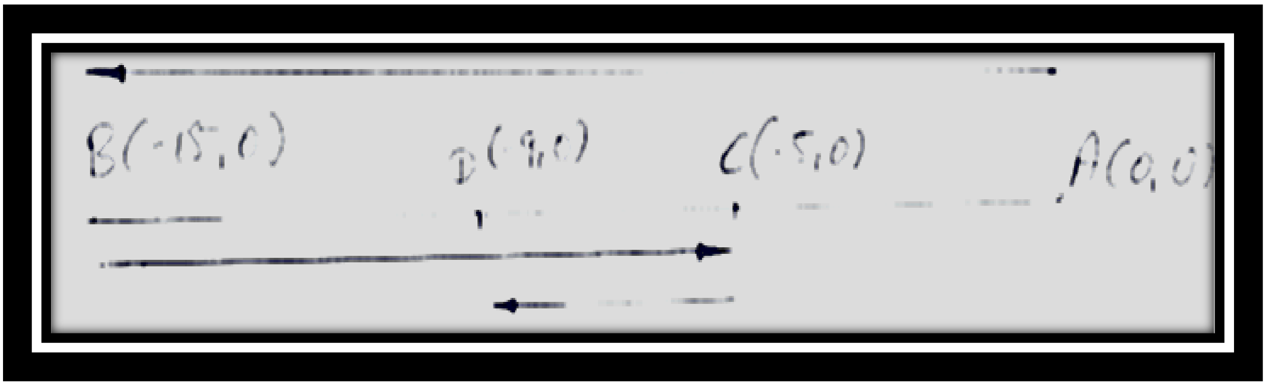
$y = x \tan \tan \alpha - \frac{g x^2}{2u^2 \cos^2 \alpha}$ …… Expression of trajectory

4.

Problems:

Rectilinear Motion with constant/variable velocity and acceleration

1. The path of travel of the particle is represented by following diagram .Determine displacement and distance traveled



Path of travel: 1) A to B 2) B to C 3) C to D

Soln: starting point A(0,0)

Displacement : final point – initial point

point	Displacement (-> +ve) W.R.T origin 'A'	distance
A	0	0
B	-15	15
C	-5	25
D	-9	29

Point of reversal-> B, C

As the direction changes v=0 at B, C

2) A car covers a distance of 100 m in 6 seconds and it takes another 5 seconds to cover next 120 m. Find the initial velocity of the car and uniform acceleration of the car.

Given : In $t = 6$ sec $\quad s = 100$ m

In $t = (6 + 5) = 11$ sec $\quad s = 100 + 120 = 220$ m

Let u = Initial velocity of car

a = Uniform acceleration of car

Now, using equation of motion

$$s = ut + \frac{1}{2}at^2$$

$$100 = u \times 6 + \frac{1}{2} \times a \times 6^2 \qquad \therefore \; 100 = 6u + 18a \qquad \text{... (i)}$$

$$220 = u \times 11 + \frac{1}{2} \times a \times 11^2 \qquad \therefore \; 220 = 11u + 60.5a \qquad \text{... (ii)}$$

Solving equation (i) and (ii), we get

Initial velocity, $u = 12.666$ m/s ... ***Ans.***

Uniform acceleration, $a = 1.333$ m/s^2 ... ***Ans.***

3) The position of the particle which moves along a straight line, is defined by the relation

$X = t^3 - 6t^2 - 15t + 40$

Where x is expressed in m and t in sec. Determine

i. the time at which the velocity will be 0
ii. the position and the distance traveled by the particle at that time
iii. the acceleration of the particle at that time
iv. the distance traveled by the particle from t = 4 s to t = 6s

Soln:

Equation of the position $X = t^3 - 6t^2 - 15t + 40$

time at which the velocity will be 0

for velocity diff. the above equation w.r.t. time 't'

$$v = \frac{dx}{dt} = 3t^2 - 12t - 15$$

$$3t^2 - 12t - 15 = 0$$

hence t = 5 or t = -1 (invalid)

thus velocity will be 0 at t = 5sec

position of the particle: $X_{t=5} = 5^3 - 6(5)^2 - 15 \times 5 + 40 = -60\,m$

acceleration of the particle for acceleration of the particle diff velocity equation w.r.t. t

$$a = \frac{dx}{dt} = 6t - 12$$

$$a_{t=5} = 30 - 12 = 18\,m/sec^2$$

distance traveled by the particle from t = 4 s to t = 6s

now we know that the velocity of the particle becomes 0 at t = 5s,

which implies that the direction of the particle changes at t = 5s

hence calculating the distance from t = 4 s to t = 5s & from t = 4 s to t = 6s

displacement $[x_{t=4}\ to\ x_{t=5}] = -60 - [-52] = -8\,m$

distance travelled from t = 4 s to t = 5s = $|displacement|$ = 8m (since the direction remains same)

displacement $[x_{t=5}\ to\ x_{t=6}] = -50 - [-60] = 10\,m$

distance travelled from t = 5 s to t = 6s = $|displacement|$ = 10m (since the direction remains same)

therefore total distance traveled = 8 + 10 = 18m

3. The velocity of the particle is defined as $v = t^3 - 5t^2 + 3t + 4$ where v is in m/s and t is in seconds. Assuming initial displacement of the particle to be 2 m, find (a) initial velocity (b) initial acceleration (c) time interval at which acceleration will be zero (d) displacement in first 4 seconds (e) displacement in 6th second.

Soln: Given relation $v = t^3 - 5t^2 + 3t + 4$ ......(i)

(a) To find initial velocity Put t = 0 in equation (i) to get $v = v_{initial} = 4$ m/s

(b) To find initial acceleration Differentiate equation (i) w.r.t. 't' and put t = 0 to get

$\therefore \frac{dv}{dt} = a = 3t^2 - 10t + 3$ .... (ii)

At $t = 0, a = a_{initial} = 3m/s^2$ Ans

(c) To find time interval at which acceleration will be zero Put a = 0 in equation (ii) to get

$a = 0 = 3t^2 - 10t + 3$

$\therefore t = \frac{10 \pm \sqrt{(-10)^2 - 4 \times 3 \times 3}}{2 \times 3} = \frac{10 \pm 8}{6}$

$\therefore t = \frac{1}{3}\ sec$ **and** $t = 3sec$

Thus acceleration is 0 at t = 1/3 s and at t = 3 s.

(d) To find displacement at the end of 4 second we have $v = t^3 - 5t^2 + 3t + 4$ **but** $v = \frac{dx}{dt}$

$\therefore \frac{dx}{dt} = t^3 - 5t^2 + 3t + 4$

$dx = (t^3 - 5t^2 + 3t + 4)dt$

$x = \frac{t^4}{4} - \frac{5t^3}{3} + \frac{3t^2}{2} + 4t + c$

To find c, put t = 0, x = 2 m ... {Given condition} thus c = 2

General expression for position is given by

$$x = \frac{t^4}{4} - \frac{5t^3}{3} + \frac{3t^2}{2} + 4t + 2 \quad \text{.... (iii)}$$

Put t = 4 s in equation (iii), to get displacement

$\therefore x_{t=4s} = \frac{t^4}{4} - \frac{5 \times 4^3}{3} + \frac{3 \times 4^2}{2} + 4 \times 4 + 2$

$\therefore x = -0.67\ m$ Ans

(e) To find displacement in 6th second

Displacement in 6th second, x_6^{th} = Displacement in 6 seconds - Displacement in 5 seconds

x_6^{th} = $x_6 - x_5$ [use equation (iii)]

$\therefore\ x_6 = \frac{6^4}{4} - \frac{5\times6^3}{3} + \frac{3\times6^2}{2} + 4\times6 + 2 = 44\ m$

$x_5 = \frac{5^4}{4} - \frac{5\times5^3}{3} + \frac{3\times5^2}{2} + 4\times5 + 2 = 7.416\ m$`

Displacement in 6th second = 44 – 7.416 = 36.854 m

4) The velocity of the body is defined by the expression $v = (6 - 0.03x)m/s$ where x is in metres. If x = 0 at t = 0 determine:

a) distance traveled by the body when it comes to rest

b) acceleration at t = 0

c) time when x = 100m

Soln

put v = 0 in the given relation

0 = 6 - 0.03 x

∴ x = 200m

b) to find acceleration at t = 0

v = (6 – 0.03 x) diff. w.r.t. x

dv/dt = -.03 dx/dt

a = -0.03v = 0.03(6 – 0.03 x)

at t = 0 x = 0

a = -0.03(6 - .03 * 0)

= -0.18 m/s^2

c)to find time t when x = 100m

v = 6 – 0.03x

dx/dt = 6 – 0.03x

$$\frac{dx}{6-0.03x} = dt$$

integrating both sides

$$\int_{x=0}^{x=100} \frac{dx}{6-0.03x} = \int_{t=0}^{t=t} dt$$

$$\frac{-1}{0.03}[\log_e(6-0.03x)]_0^{100} = [t]_0^t$$

Thus t = 23.105s

5) A particle has a straight line motion given by the equation $x = (t^3 - 2t^2 - 4)m$ where t is in seconds. What is the change in displacement when velocity changes from 4 m/s to 32 m/s.

Soln:

Given relation $x = (t^3 - 2t^2 - 4)m$

$v_1 = 4m/s$ at $t = t_1$

$v_2 = 32m/s$ at $t = t_2$

Differentiating x w.r.t. t

$$\therefore v = \frac{dx}{dt} = 3t^2 - 4t$$

$$\text{Now, } v_1 = 4 = 3t_1^2 - 4t_1$$

$$\text{or } 3t_1^2 - 4t_1 - 4 = 0$$

$$t_1 = \frac{4 + \sqrt{16 + 4 \times 3 \times 4}}{2 \times 3} = 2\text{ s}$$

$$\text{and } v_2 = 32 = 3t_2^2 - 4t_2$$

$$\text{or } 3t_2^2 - 4t_2 - 32 = 0$$

$$t_2 = \frac{4 + \sqrt{16 + 4 \times 3 \times 32}}{2 \times 3} = 4\text{ s}$$

Change in displacement,

$$\Delta x = x_2 - x_1$$

$$= (t_2^3 - t_1^3) - 2(t_2^2 - t_1^2) - (4 - 4) = (4^3 - 2^3) - 2(4^2 - 2^2) - 0$$

$$\Delta x = 32\text{ m} \quad \ldots \textit{Ans.}$$

Motion under Gravity

1. A stone is thrown vertically upwards and returns to the ground in 6 sec. How high does it go?

Soln: Let the stone projected with an initial velocity V = 0 travel h metres to the peak.
If it takes 6 sec to return back, implies that 3 sec were spent in going up and 3 sec in coming down.

Motion of stone (Ground to peak)

M.U.G $\uparrow$ + ve
$u = v_0$ m/s
$v = 0$
$s = h$ metres
$g = -9.81\ m/s^2$
$t = 3$ sec

$v = u + at$
$0 = v_0 - 9.81 \times 3$
$v_0 = 29.43$ m/sec

$v^2 = u^2 + 2as$
$0 = (29.43)^2 + 2 \times -9.81 \times h$
$h = 44.145$ m

2. From the top of a tower 100m high, a stone was dropped down at the same time, another stone was thrown up from the foot of the tower with a velocity of 30m/s. when and where the 2 stones will cross each other? Find the velocity of each stone at the time of crossing

Soln:

Stone 1

$U_1 = 0$ x_1 downwards $a_1 = -9.81m/s^2$

Stone 2

$U_2 = 30m/s$ upwards x_2 upwards

$a_2 = -\ 9.81m/s^2$

Using $x = ut + ½ at^2$

$-x_1 = 0 – 9.81/2\ t^2$

$x_2 = 30t – 9.81/2\ t^2$

Now we know that mod $x_x + x_2 = 100$

implies $x_1 = 100 – x_2$

$-(100 – x_2) = -9.81/2\ t^2$

$100 – x_2 = 9.81/2\ t^2$

$100\ [\ 30t – ½ * 9.81\ t^2] = 9.81/2\ t^2$

hence $t = 3.33s$

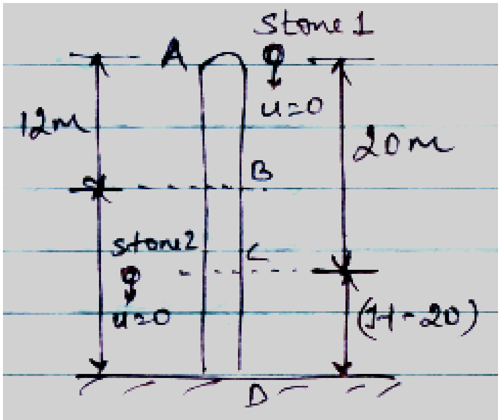
$X_{1\ t=3.33} = 9.81 * 3.33^2 = 54.3m$

$X_{1\ t=3.33} = 100 – 54.3 = 45.7m$

$V_1 = u + at = 0 – 9.81 * 3.33 = 32.7m/s$ downwards

$V_2 = u + at = 30 – 9.81 * 3.33 = 2.7m/s$ downwards

3.A stone has fallen a distance of 12m after being dropped from the top of tower. Another stone is dropped from a point 20m below the top of the tower. If both stones reach the ground together, find the height of the tower.



Soln. the stone 1 is dropped from top of tower, when it reaches 12m below the top (at point B) its velocity is given by

$V_B^2 = U_A^2 – 2gs = 0 – 2 \times 9.8 \times (-12)$

$V_B = 15.34m/s$ downwards

Let t be the time taken by the stone 2 to reach ground (C to D)

Therefore time taken by stone 1 from B to D must be t sec.

For stone 1 (B to D)

Use $s = ut – ½ gt^2$

$-(H - 12) = -15.34t - ½\ 9.8\ t^2$

$H = 15.34t + 4.9\ t^2 + 12$...........(i)

For stone 2 (C to D)

Use $s = ut - ½\ gt^2$

$-(H - 20) = 0 - ½\ 9.8\ t^2$ $H = 20 + 4.9t^2$...................(ii)

Equating I and ii we get

$20 + 4.9t^2 = 15.34t + 4.9t^2 + 12$ $t = 0.52s$

Substituting in ii

$H = 20 + 4.9\ (0.52)^2$ $H = 21.33m$

4. **In a flood relief area, a helicopter going up with a constant velocity, first batch of food packet is released which takes 4s to reach the ground. No sooner than this batch reaches the ground, second batch of food packet is released, which takes 5s to reach the ground. From what height the first batch of packet was released and what is the velocity with which the helicopter is going up?**

Soln. Let u be the constant velocity of the helicopter

For first food packet $s_1 = ut - ½\ gt^2$

$-s_1 = (u \times 4) - ½ \times 9.8 \times 4^2$

$s_1 = 78.48 - 4u$.......(i)

for second food packet

$-s_2 = (u \times 5) - ½ \times 9.8 \times 5^2$

$S_2 = 122.62 - 5u$......(ii)

for helicopter (h – s) = velocity x time (as velocity is constant)

$s_2 - s_1 = u \times 4$...(iii)

substituting the values of s and s_2 from i and ii

$122.62 - 5u - (78.48 - 4u) = 4u$

$5u = 44.14$ **$u = 8.83$ m/s**

By equation (i) $s_1 = 43.16$ m

Thus first batch of packets is released from 43.16m.

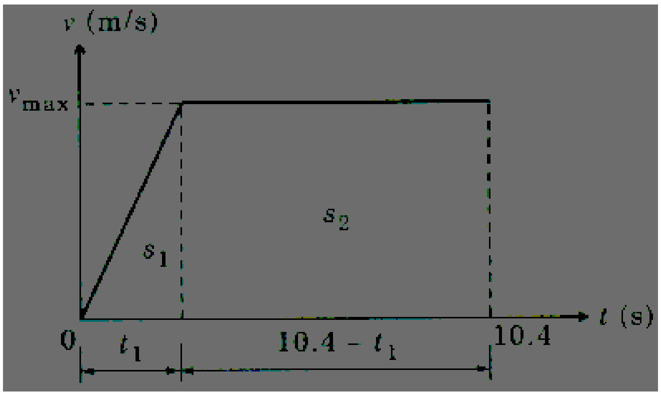
5) Motion Curves

1. **In an Asian games event of 100 m run, an athlete accelerates uniformly from the start to his maximum velocity in a distance of 4 m and runs the remaining distance with that velocity. If the athlete completes the race in 10.4 s, determine (a) his initial acceleration (b) his maximum velocity.**

Soln

Representing the given data, we get v-t graph as shown in figure Ex.47.

Distance travelled = Area under v-t graph



For

$$0 \le t \le t_1$$
$$A_1 = \frac{1}{2} \times v_{max} \times t_1 = 4$$
$$\therefore \; t_1 = \frac{8}{v_{max}} \qquad .....(i)$$

For

$$t_1 \le t \le 10.4$$
$$A_2 = (10.4 - t_1)v_{max} = 96 \qquad .....(ii)$$

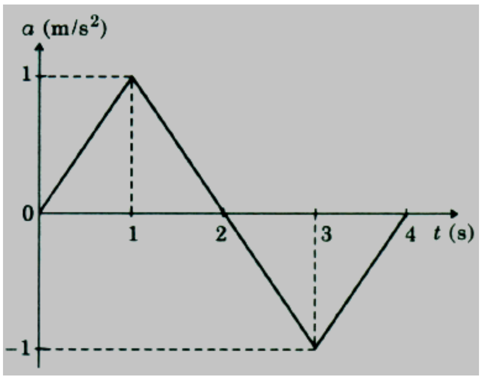
Put value of t from equation (i) in equation (ii) to get

$$\left(10.4 - \frac{8}{v_{max}}\right) \times v_{max} = 96$$
$$(10.4v_{max} - 8) = 96$$
$$\therefore \; v_{max} = 10 \text{ m/s.}$$
$$t_1 = \frac{8}{v_{max}} = \frac{8}{10} = 0.8 \text{ s}$$

Initial acceleration = Slope of v-t diagram from 0 to 0.8 s

$$\therefore \; a = \frac{dv}{dt} = \frac{v_{max} - v_0}{t_1 - 0} = \frac{10 - 0}{0.8} = 12.5 \text{ m/s}^2$$

2. For the a-t diagram of particle shown in the fig. draw v-t & x-t diag. Also calculate velocity at the end of 3 sec. & distance traveled in 4 sec.



Assume particle starts from rest.

Soln: Initial condition (assumed)

At t=0, $x_0 = 0$, $v_0 = 0$

Area under a-t diagram = change in velocity (Δv)

For 0<= t <= 1 sec area A = ½ X 1 X 1 = Δv

$\Delta v = v_1 - v_0$ $0.5 = v_1 - 0$ $v_1 = 0.5$ m/s

for 1<= t <=2 area A = ½ X 1 X 1 = Δv

$\Delta v = v_2 - v_1$ $0.5 = v_2 - 0.5$ $v_2 = 1$ m/s

for 2<= t <=3 area A = -½ X 1 X 1 = Δv

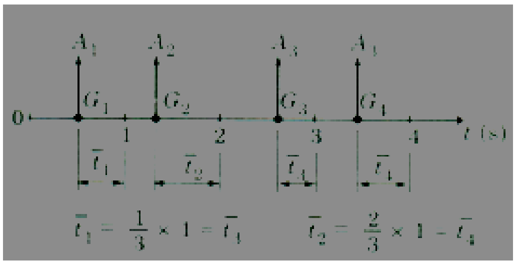
$\Delta v = v_3 - v_2$ $-0.5 = v_3 - 1$ $v_3 = 0.5$ m/s

for 3<= t <=4 area A = -½ X 1 X 1 = Δv

$\Delta v = v_4 - v_3$ $-0.5 = v_4 - 0.5$ $v_4 = 0$ m/s

to find the position of the particle from a-t diagram we use

$x_t = x_0 + v_0 t + At$



For 0<= t <= 1 sec $x_1 = x_0 + v_0 t + A_1 t_1$

$X_1 = 0 + 0 + 0.5 * 1/3 * 1 = 0.166$m

For 0<= t <= 2 sec $x_2 = x_1 + v_1 t_2 + A_2 t_2$

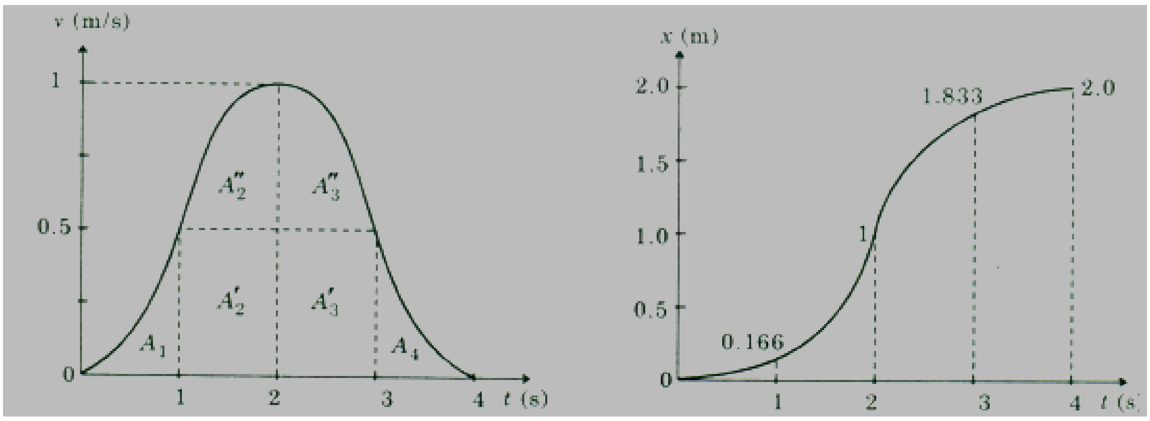
$X_1 = 0.166 + 0.5*1 + 0.5 * 2/3 * 1 = 1$m

For 0<= t <= 3sec $x_3 = x_2 + v_2 t_3 + A_3 t_3$

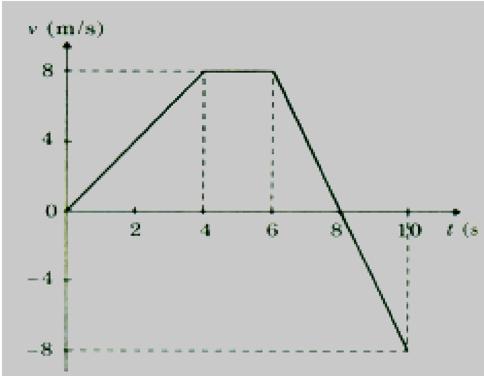
$X_3 = 1 + 1*1 + 0.5 * v_3 = 1.833$m

For 0<= t <= 4 sec $x_4 = x_3 + v_3 t_4 + A_4 t_4$

$X_1 = 1.833 + 0.5*1 + (-0.5 * 2/3) = 2$m



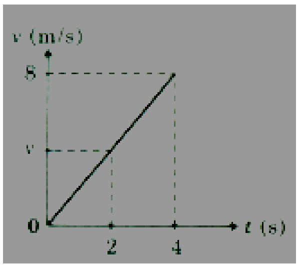
3. **For a particle performing rectilinear motion v-t diagram is shown in the figure. Draw x-t diagram for the motion if at t = 2s x = 20m**



What is the displacement during 6s to 10s?

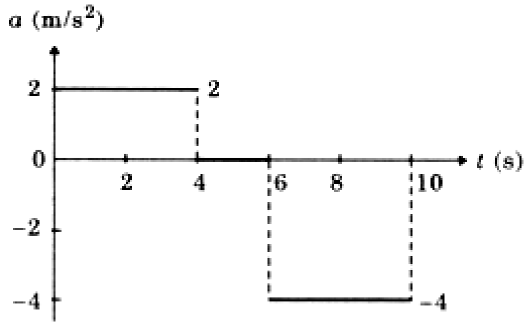
Soln

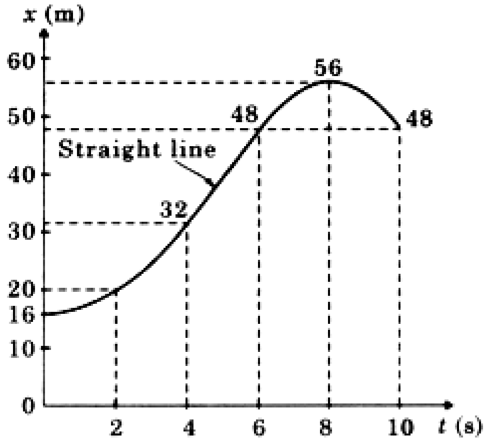
For a-t diagram, slope of v-t diagram is the acceleration



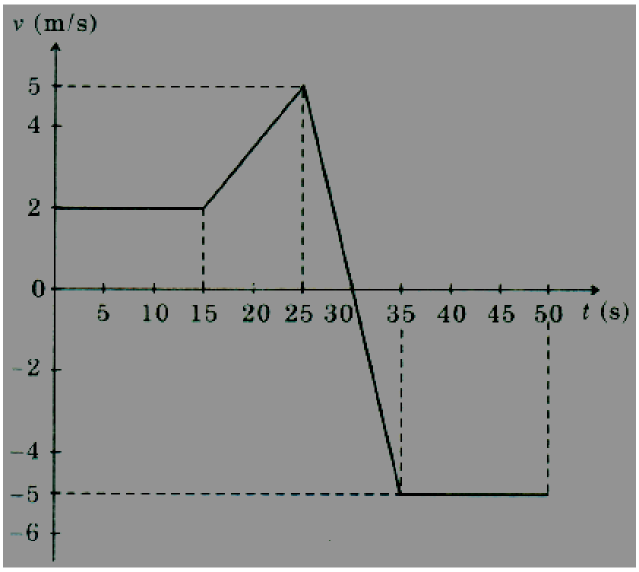
For t lying between 0 and 4 sec. $a = dv/dt = (v_4 – v_0)/ (4 – 0) = 2m/s^2$
For t lying between 4 and 6 sec. $a = dv/dt = (v_6 – v_4)/ (6 – 4) = 0$
For t lying between 6 and 10 sec. $a = dv/dt = (v_{10} – v_6)/ (10 – 6) = -4m/s^2$
For x-t diagram

From the given condition at t = 2s, $x = x_{20}$ we calculate x_0 at t = 0.
We know that area under v-t diagram gives change in position of the particle. By comparing the following 2 triangles from v-t diagram we get
$8/4 = v/2$ v = 4m/s at t = 2s
For t lying between 0 and 2s area $A = ½ * 2 * 4 = x_2 – x_0$
$4 = 20 – x_0$ $x_0 = 16m$
Thus particle starts from position $x_0 = 16m$ from the origin
Calculating the position of the particle at different intervals
For t lying between 0 and 4s area $A_1 = ½ * 4 * 8 = x_4 – x_0$
$16 = x_4 - 16$ $x_4 = 32m$
For t lying between 4 and 6s area $A_2 = 2 * 8 = x_6 – x_4$
$16 = x_6 - 32$ $x_6 = 48m$
For t lying between 6 and 8s area $A_3 = ½ * 2 * 8 = x_8 – x_6$
$8 = x_8 - 48$ $x_8 = 56m$
For t lying between 8 and 10s area $A_4 = -½ * 2 * 8 = x_{10} – x_8$
$-8 = x_{10} - 56$ $x_{10} = 48m$
Now displacement during 6-10s = $A_3 + A_4 = 8 – 8 = 0$
Thus the total distance travelled = mod A_1 + mod A_2 + mod A_3 + mod A_4 = 16 + 16 + 8 + 8 = 48m





4. The v - t diagram for a particle moving along straight line is shown in figure. Knowing that x = -10m at t = 0 (a) Plot x-t and a-t diagram for 0 < t < 50 s. (b) Determine the maximum value of position coordinate and the value of t for which the particle is at a distance of 55 m from the origin.



Soln Given initial condition, at t=0,Xo = -10 m, Vo = 2 m/s

For x-t diagram Area under v-t diagram : v-t diagram Change in position (Δx)

For $0 \le t \le 15$ s	$A_1 = 15 \times 2 = x_{15} - x_0$	$\Rightarrow 30 = x_{15} - (-10)$	$\therefore x_{15} = 20$ m
For $15 \le t \le 25$ s	$A_2 = \left(\frac{2+5}{2}\right) \times 10 = x_{25} - x_{15}$	$\Rightarrow 35 = x_{25} - 20$	$\therefore x_{25} = 55$ m
For $25 \le t \le 30$ s	$A_3 = \frac{1}{2} \times 5 \times 5 = x_{30} - x_{25}$	$\Rightarrow 12.5 = x_{30} - 55$	$\therefore x_{30} = 67.5$ m
For $30 \le t \le 35$ s	$A_4 = -\frac{1}{2} \times 5 \times 5 = x_{35} - x_{30}$	$\Rightarrow -12.5 = x_{35} - 67.5$	$\therefore x_{35} = 55$ m
For $35 \le t \le 50$ s	$A_5 = -15 \times 5 = x_{50} - x_{35}$	$\Rightarrow -75 = x_{50} - 55$	$\therefore x_{50} = -20$ m

Maximum position coordinate is x = 67.5 m at t = 30 s

At t = 25 sand t = 35 s the particle is at a distance of 55 m

(b) For a-t diagram Slope of v-t diagram = Acceleration

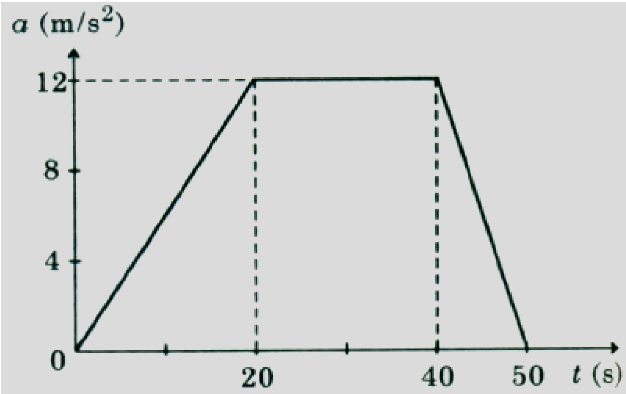
$$\text{For } 0 \le t \le 15 \text{ s} \quad a = \frac{dv}{dt} = \frac{v_{15} - v_0}{15 - 0} = \frac{2 - 2}{15} = 0$$

$$\text{For } 15 \le t \le 25 \text{ s} \quad a = \frac{dv}{dt} = \frac{v_{25} - v_{15}}{25 - 15} = \frac{5 - 2}{10} = 0.3 \text{ m/s}^2$$

$$\text{For } 25 \le t \le 35 \text{ s} \quad a = \frac{dv}{dt} = \frac{v_{35} - v_{25}}{35 - 25} = \frac{-5 - 5}{10} = -1 \text{ m/s}^2$$

$$\text{For } 35 \le t \le 50 \text{ s} \quad a = \frac{dv}{dt} = \frac{v_{50} - v_{35}}{50 - 35} = \frac{-5 - (-5)}{15} = 0$$

5. Figure shows a plot of a v/s t for a particle moving along x-axis. What is the speed and distance covered by the particle after 50 s? Find also the maximum speed and time at which the speed attained by the particle. Draw v-t and x-t diagram.



Soln

Initial condition(assumed), at t = 0, $x_0 = 0$, Vo = 0 Area under a-t diagram = Change in velocity (Δv)

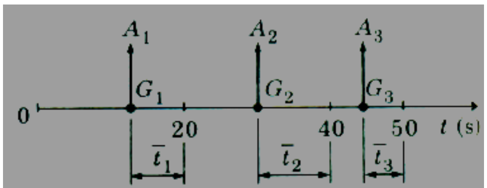
$$\text{For } 0 \le t \le 20 \text{ s} \quad A_1 = \frac{1}{2} \times 20 \times 12 = 120 = v_{20} - v_0 = v_{20} - 0 \quad \therefore v_{20} = 120 \text{ m/s}$$

$$\text{For } 20 \le t \le 40 \text{ s} \quad A_2 = 20 \times 12 = 240 = v_{40} - v_{20} = v_{40} - 120 \quad \therefore v_{40} = 360 \text{ m/s}$$

$$\text{For } 40 \le t \le 50 \text{ s} \quad A_3 = \frac{1}{2} \times 10 \times 12 = 60 = v_{50} - v_{40} = v_{50} - 360 \quad \therefore v_{50} = 420 \text{ m/s}$$

To find position of the particle, we use

$$x_t = x_0 + v_0 t + A\,\bar{t}$$



$$\bar{t}_1 = \frac{1}{3} \times 20$$

$$\bar{t}_2 = \frac{1}{2} \times 20$$

$$\bar{t}_3 = \frac{2}{3} \times 10$$

For $0 \le t \le 20$ s, $x_{20} = x_0 + v_0 \times t_{(20-0)} + A_1 \bar{t}_1$

$$= 0 + 0 + 120 \times \frac{1}{3} \times 20$$

$$= 800 \text{ m}$$

For $0 \le t \le 40$ s, $x_{40} = x_{20} + v_{20} \times t_{(40-20)} + A_2 \bar{t}_2$

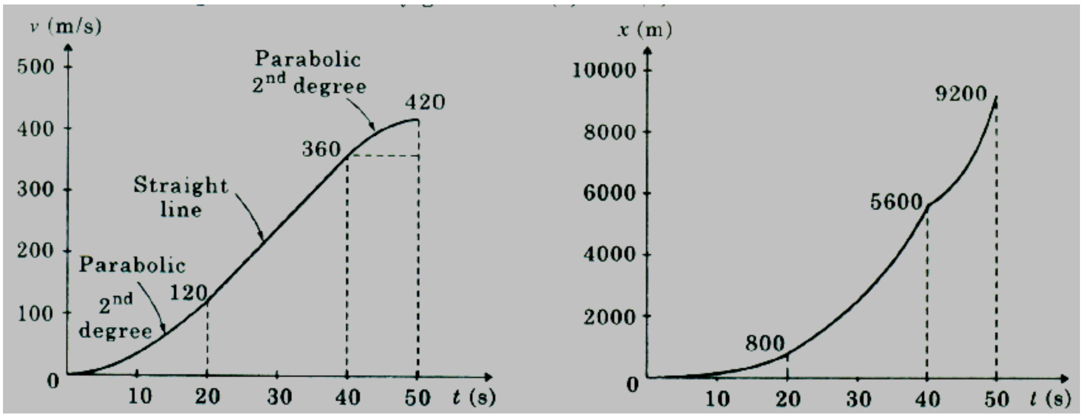
$$= 800 + 120 \times 20 + 240 \times \frac{1}{2} \times 20$$

$$= 5600 \text{ m}$$

For $0 \le t \le 50$ s, $x_{50} = x_{40} + v_{40} \times t_{(50-40)} + A_3 \bar{t}_3$

$$= 5600 + 360 \times 10 + 60 \times \frac{2}{3} \times 10 = 9600 \text{ m}$$

Plotting the v-t and x-t diagram as shown in the figure



Alternative solution for x-t diagram. (using v-t diagram) Area under v-t diagram = Change in position (Δx)

For $0 < t < 20$ s, $A_1 = \frac{a \times b}{n+1}$. Here $a = 20$, $b = 120$ and $n = 2$ (Degree)

$\therefore A_1 = \frac{20 \times 120}{2+1} = 800 = x_{20} - x_0 = x_{20} - 0$ $\therefore x_{20} = 800$ m

For $20 \le t \le 40$ s, $A_2 = \left(\frac{120 + 360}{2}\right) \times 20 = 4800 = x_{40} - x_{20} = x_{40} - 800 \therefore x_{40} = 5600$ m

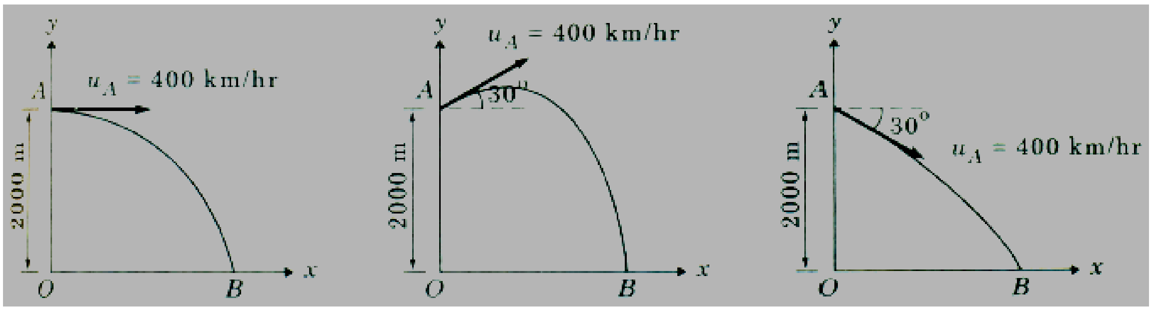
For $40 \le t \le 50$ s, $A_3 = 10 \times 360 + \frac{nab}{n+1}$. Here $n = 2$, $a = 10$, $b = 60$

$$= 3600 + \left(\frac{2 \times 10 \times 60}{2+1}\right) = 4000 = x_{50} - x_{40} = x_{50} - 5600$$

$\therefore x_{50} = 9600$ m

5) Projectile Motion

1. **An aeroplane flying at an altitude of 2000 m accidently loses a rivet. Determine thelocation of the rivet on the ground for the following cases.**
   (i) aeroplane is moving with a horizontal velocity of 400 km/hr. (ii) aeroplane is moving with a velocity of 400 km/hr inclined at an angle of 30° in the upward direction. iii)aeroplane is moving with a velocity of 400 km/hr inclined at an angle of 30° in the ndownward direction.



Soln: Given y = -2000 m; u_A = 400 x 5/18 = 111.11 m/s

Case (i) Aeroplane is moving horizontally. Rivet will start off horizontally with the same velocity of aeroplane.

Initial horizontal velocity of rivet, u_{AX} = 111.11 m/s

Initial vert.cal velocity of rivet, u_{AY} = 0

For vertical motion:

$$y = u_{ay}t + \frac{1}{2}gt^2$$

$$-200 = 0 + \frac{1}{2} \times (-9.81) \times t^2$$

$$\therefore t = 20.1928\, sec$$

For horizontal motion:

$$x = u_{Ax}t$$

$$x = 111.11 \times 20.1928$$

$$\therefore x = OB = 2243.6368 \text{ m}$$

Alternate: We can use equation of path of projectile to solve this problem.

For this u_A = 111.11 m/s; α. = 0, y = -2000 m

Now

$$y = x \tan \alpha + \frac{\frac{1}{2} g x^2}{u^2 \cos^2 \alpha}$$

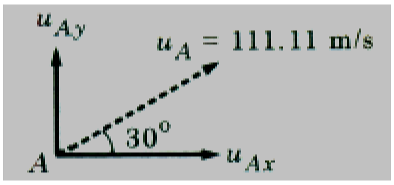
$$-2000 = x \tan 0^\circ + \frac{\frac{1}{2} \times (-9.81)\, x^2}{(111.11)^2 \cos^2 0^\circ}$$

$$\therefore x = OB = 2243.6368 \text{ m}$$

Case (ii) Aeroplane is moving at an upward inclination of 30° with horizontal. Falling rivet will have horizontal and vertical component of velocity at point A.

u_{AX} = 111.11 cos 30° = 96.224 m/s

u_{AY}= 111.11 sin 30° 55.555 m/s



For vertical motion:

$$y = u_{Ay}\,t + \frac{1}{2}gt^2$$

$$-2000 = 55.555 \times t + \frac{1}{2} \times (-9.81) \times t^2$$

$$4.905t^2 - 55.555t - 2000 = 0.$$

$$t = 26.635 \text{ s}$$

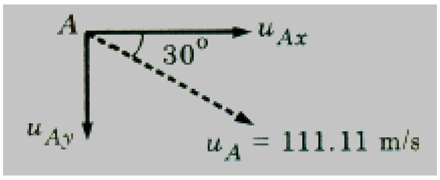
For horizontal motion:

$$x = u_{Ax}\,t$$

$$x = 96.224 \times 26.635$$

$$\therefore \quad x = OB = 2562.926 \text{ m}$$

Case (iii) Aeroplane is moving downward at an inclination of 30° with horizontal. Falling rivet will have horizontal and vertical component of velocity at point A.



U_{AX} = 111.11 cos 30° = 96.224 m/s

U_{AY} = -111.11 sin 30° = -55.555 m/s

For vertical motion:

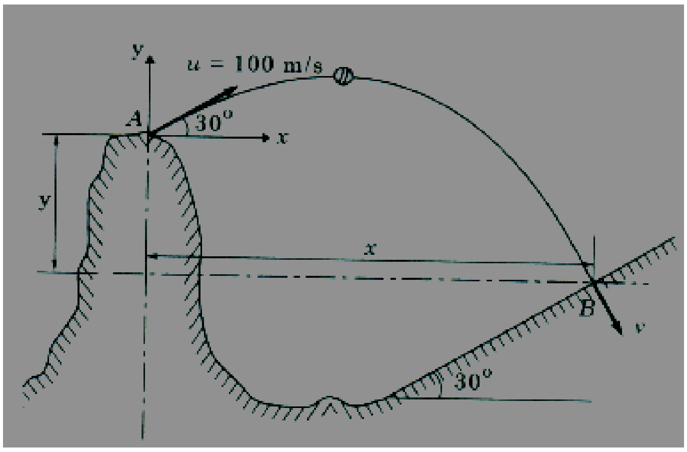
$$y = u_{Ay}\,t + \frac{1}{2}gt^2$$

$$-2000 = -55.555t + \frac{1}{2} \times (-9.81) \times t^2$$

$$4.905t^2 + 55.555t - 2000 = 0.$$

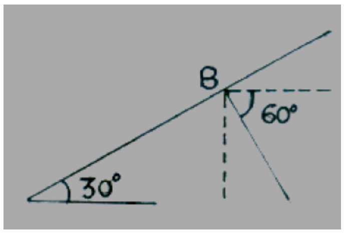
$$t = 15.3087 \text{ s}$$

2. **A ball is thrown upward from a high cliff with a velocity of 100 m/s at an angle of elevation of 30 degrees iwth -horizontal. The ball strikes the inclined ground at right angles. If inclination of ground is 30 degrees as hown, determine (i) velocity with which the ball strikes the ground, (ii) the time at which the ball strikes the ground, (iii) coordinates (x, y) of point of**



Let v be the velocity of striking ball at B, since ball strikes the incline at right angle the components of velocity at B are

$v_x = v\ cos\ cos\ 60° \ = 0.5\ v$ and $v_y = -v\ sin\ sin\ 60° \ = -.866\ v$



At point of projection A

x-component of velocity $u_x = u\ cos\ cos\ 30° \ = 100\ cos\ cos\ 30° \ = 86.6025\ m/s$

y-component of velocity $u_y = u\ sin\ sin\ 30° \ = 100\ sin\ sin\ 30° \ = 50\ m/s$

for projectile motion, x-component of velocity remains constant at point A and B.

$\therefore \quad u_x = v_x$

86.6025=0.5 v

$\therefore v = 173.025\ m/s$ this is velocity when ball strikes the ground…

$\therefore$ y component of velocity at B, $v_y = -0.866v = \ -0.866 \times 173.025$

$$v_y = -150\ m/s$$

Using equations of motion in the vertical direction

$v_y = u_y + gt$ here $v_y = -150 m/s, \qquad u_y = 50\ m/s$

$$-150 = 50 - 9.81 \times t$$

$\therefore t = 20.387\ s =$ time taken to hit the incline………. Ans

For motion in horizontal and vertical direction, we use

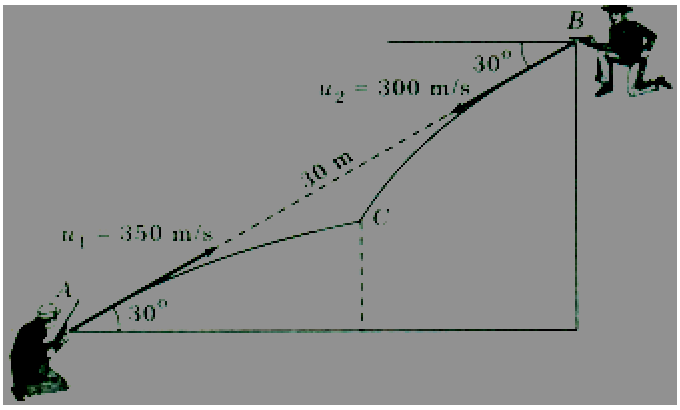
$x = u_y \times t$ and $y = u_y t + \frac{1}{2} g t^2$

$x = 85.5025 \times 20.387 \quad y = 50 \times 20.387 + \frac{1}{2} \times (-9.81) \times (20.387)^2$

$x = 1765.565\, m \qquad y = -1019.314\, m$

Coordinates of (x,y) is as $(1765.565\,, -1019.314\,)$

3. 2 guns are pointed at each other, one, upwards at an angle of 30 degrees and the other at the same angle of depression, the muffle being 30m apart. If the charges leave the guns with the velocity of 350m/s and 300m/s respectively, find when and where will they meet



Soln:

let the charges fired from A and B meet at point C at a distance of 'x' from A

or $(30cos30° - x$ from B, after time t

let y be the distance travelled by A in the vertical direction considering horizontal motion

for A

$x = 360cos30° \times t$

for B

$(30cos30° - x) = 300cos30° \times t$

adding the above equations

$x + (30cos30° - x) = 350cos30° \times t + 300cos30° \times t \qquad x = 0.046sec$

put t in equation for A

x = 350 cos 30 * 0.046 = 13.94m

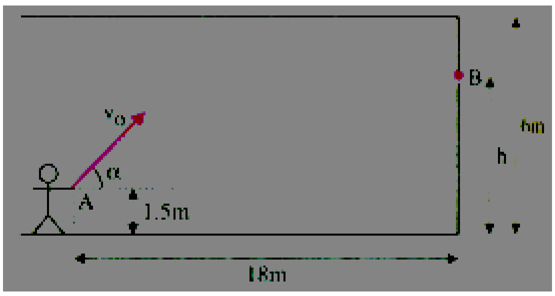
considering vertical motion of A

y = u_y t + ½ gt^2 = 350 sin30 * 0.046 + ½ (- 9.81) * 0.046^2 = 8.04m

both the charges meet after time t = 0.046s

point of meeting [x, y] = [13.94, 8.04]m

3. A player throws a ball with an initial velocity u 16m/s from point A located 1.5m above the floor, if h = 3.5m, determine the angle α for which the ball will strike the wall at point B



Soln:

For projectile from A to B we have x = 18m y = 3.5 – 1.5 = 2m u = 16m

By equation of path of projectile

$y = x \tan \alpha - g x^2/(2 * u^2 \cos^2\alpha)$

$2 = 18 \tan \alpha - 9.81 * 18^2 * \sec^2 \alpha / (2 * 16^2)$

$2 = 18 \tan \alpha - 6.21 (1 + \tan^2\alpha)$

Solving the above quadratic equation

Tan α = 2.33 tan α = 0.567

α = 66.77 degrees α = 29.55 degrees

Now checking the possible angle of projection for maximum height to not exceed (6 – 1.5) = 4.5m

$H_{max} = u^2 \sin^2\alpha / (2 * 9.81)$

$4.5 = 16^2 \sin^2\alpha / (2 * 9.81)$ α = 35.97degrees

Thus for given arrangement the angle of projection should not exceed 35.97 degrees

So the permissible angle to hit the point B is 29.55 degrees

6) Curvilinear Motion

1) The speed of the racing car is increasing at a constant rate from 72 kmph to 144kmph over a distance of 200 m along a curve of radius 250 m. Determine the magnitude of total acceleration after it has travelled 120 m.

Solution: Given data:

Initial velocity, u= 72 x5/18 = 20 m/s

Final velocity, v = 144 x 5/18= 40 m/s

Distance covered, s = 200 m

Radius of curvature, p = 250 m

To find: Acceleration, a = ? when s = 120 m

Car is moving with uniform tangential acceleration. Hence following equations of motion can be applied

$v^2 = u^2 + 2\, a_t s$

$(40)^2 = (20)^2 + 2\, a_t * 200$

$\therefore\ a_t = 3 \text{ m/s}^2$

To find speed of the vehicle when s = 120 m, use again

$v^2 = u^2 + 2\, a_t s$

$(v)^2 = (20)^2 + 2 * 3 * 120$

v = 33.4664 m/s

Normal acceleration, $a_N = v^2/ p = (33.4664)^2 / (250)$

$= 4.48 \text{ m/s}^2$

Acceleration,

$$a = \sqrt{a_n^2 + a_t^2} = \sqrt{(4.48)^2 + (3)^2}$$

$a = 5.392 \text{ m/s}^2$

$\theta = \left(\frac{a_t}{a_n}\right) = \left(\frac{3}{4.48}\right) = 33.81°$ Ans

2) A car is travelling along a circular curve that has a radius of curvature of 50 m. If the speed of the car is 16 m/s and increasing uniformly at the rate of 8 m/s ', determine the magnitude of its acceleration at this instant.

Soln

Given data:

Radius of curvature, ρ = 50 m Speed of car, v = 16 m/s Tangential acceleration, a_t = 8 m/s^2

Magnitude of acceleration a = ?

$$\text{Normal acceleration, } a_N = \frac{v^2}{\rho} = \frac{(16)^2}{50} = 5.12 \text{ m/s}^2$$

$$\text{Acceleration, } a = \sqrt{a_t^2 + a_N^2} = \sqrt{(8)^2 + (5.12)^2} = 9.5 \text{ m/s}^2$$

$$\theta = \tan^{-1}\left(\frac{a_t}{a_n}\right) = \tan^{-1}\left(\frac{8}{5.12}\right) = 57.38^\circ$$

3) A car starts from rest at t = 0 along a circular track of radius 200 m. The rate of i~crease in speed of the car is uniform. At the end of 90 s the speed of the car is 36 kmph. Find the tangential and normal components of acceleration at t = 30 s.

Soln

Given data: Initial velocity, u = 0, radius of curvature ρ = 200m. Final velocity v = 36 kmph = 10 m/s

Time of travel, t = 90 s

In this problem, car is moving with a uniform tangential acceleration. Hence tangential acceleration at any time remain constant.

Using the equation of motion

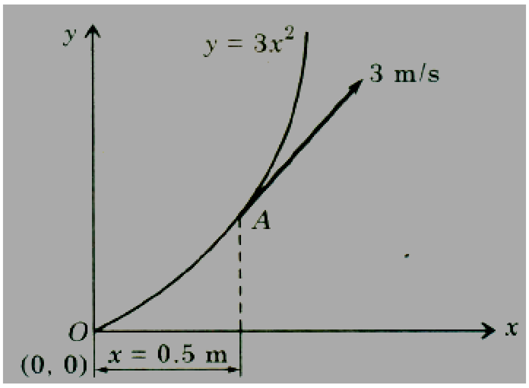
$$v = u + a_t \times t$$
$$10 = 0 + a_t \times 90$$
$$\therefore a_t = 0.1111 \text{ m/s}^2$$

To find speed when *t* 30 s. Use again

$$v = u + a_t \times t$$
$$v = 0 + 0.1111 \times 30$$
$$\therefore v = 3.333 \text{ m/s}$$

Normal Acceleration, $a_n = v^2/\rho = (3.333)^2/200 = 0.056$ m/s^2

4) A particle moves with a constant speed of 3 m/s along the path shown in figure. What is the resultant acceleration at a position on the path where x = 0.5 m? Also represent the acceleration in vector form.



Soln Given Data

Speed of the particle, v = 3 m/s (constant) equation of the curve y = $3x^2$

$$a = (a_n^2 + a_t^2)^{1/2}$$

body is moving with constant speed, hence $a_t = 0$

normal acceleration at A, $a_n = \frac{v^2}{\rho} = \frac{9}{\rho}$

$$y = 3x^2$$

$$\therefore \frac{dy}{dx} = 6x. \text{ At point } A, \left(\frac{dy}{dx}\right)_{x=0.5} = 6 \times 0.5 = 3$$

$$\frac{d^2y}{dx^2} = 6 \text{ [Constant]}$$

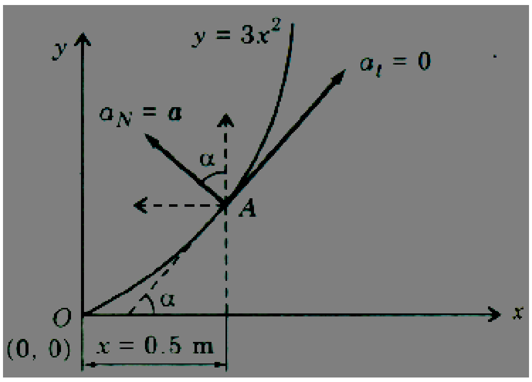
$$\rho = \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \qquad \therefore \rho_{\text{at } x = 0.5} = \frac{\left[1 + (3)^2\right]^{3/2}}{6}$$

$$\therefore \rho_{\text{at } x = 0.5} = 5.27 \text{ m.}$$

$a = a_n$= 9/5.27 = 1.708 m/s^2

To represent acceleration in vector form

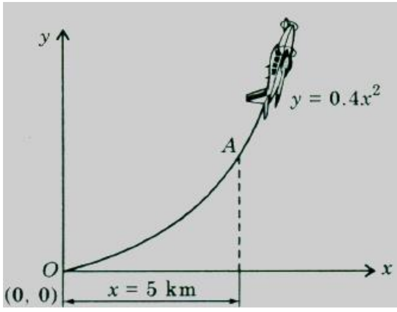
As shown in figure, tangential acceleration is tangent to the curve at point A and normal acceleration is towards the centre of curvature and is perpendicular to the direction of tangential acceleration. From the figure



$$\tan\alpha = \left(\frac{dy}{dx}\right)_{x=0.5} = 3 \qquad \therefore\ \alpha = 71.565^\circ$$

$$a = a_x\,i + a_y\,j$$
$$a = (-a_N \sin\alpha)\,i + (a_N\cos\alpha)\,j$$
$$a = (-1.708\sin 71.565^\circ)\,i + (1.708\cos 71.565^\circ)\,j$$
$$a = -1.62\,i + 0.54\,j$$

5) A Jet plane travels along the parabolic path as shown in *figure Ex.BO.* When it is at point *A,* it has a speed of 200 m/s which is increasing at the rate of 0.8 m/s2. Determine the magnitude of the acceleration of the plane when it is at *A.*



Soln Given data:

Equation of parabolic path, $y = 0.4\,x^2$

Speed of the Jet plane at A, v = 200 m/s

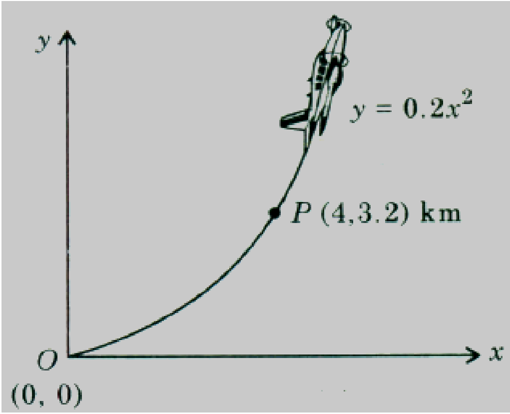
Tangential acceleration at A, $a_t = 0.8\ m/s^2$

$$a = \sqrt{a_t^2 + a_N^2} = \sqrt{(0.8)^2 + a_N^2}$$
$$= \sqrt{0.64 + \left(\frac{v^2}{\rho}\right)^2} = \sqrt{0.64 + \frac{(200)^4}{\rho^2}} \quad \ldots (i)$$

$$y = 0.4x^2$$
$$\therefore\ \frac{dy}{dx} = 0.8x \qquad \therefore\ \left(\frac{dy}{dx}\right)_{x=5\ km} = 0.8\times 5 = 4$$
$$\frac{d^2y}{dx^2} = 0.8\ \text{[Constant]}$$
$$\rho = \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}}$$
$$\therefore\ \rho_{x=0.5\ km} = \frac{\left[1+(4)^2\right]^{3/2}}{0.8} = 87.616\ km = 87.616\times 10^3\ m$$

$$a = \sqrt{0.64 + \frac{(200)^4}{(87.616\times 10^3)^2}} = 0.921\ m/s^2$$

6) An airplane travels along a curved path. At point *P* it has a speed of 360 kmph and it is increasing at the rate of 0.5 m/s", Determine at *P* (i) the magnitude of total acceleration. (ii) the angle made by the acceleration vector with positive x-axis.



Soln Speed of airplane at point P, v = 360 kmph = 100 m/s
Tangential acceleration at point P, $a_t = 0.5\ m/s^2$
Equation of path of airplane, $y = 0.2\ x^2$

$$\boldsymbol{a} = \boldsymbol{a}_t + \boldsymbol{a}_N \quad \therefore\ |\boldsymbol{a}| = \sqrt{a_t^2 + a_N^2}$$

$$a = \sqrt{(0.5)^2 + \left(\frac{v^2}{\rho}\right)^2} = \sqrt{0.25 + \frac{(100)^4}{\rho^2}} \quad \ldots (i)$$

$$y = 0.2\ x^2$$

$$\therefore\ \frac{dy}{dx} = 0.4\ x \quad \therefore\ \left(\frac{dy}{dx}\right)_{x=4} = 0.4 \times 4 = 1.6 = \tan\alpha$$

$$\therefore\ \alpha = 58^\circ$$

$$\frac{d^2y}{dx^2} = 0.4\ [\text{Constant}]$$

$$\rho = \left| \frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} \right| = \left| \frac{\left[1 + (1.6)^2\right]^{3/2}}{0.4} \right| = 16.793\ \text{km}$$

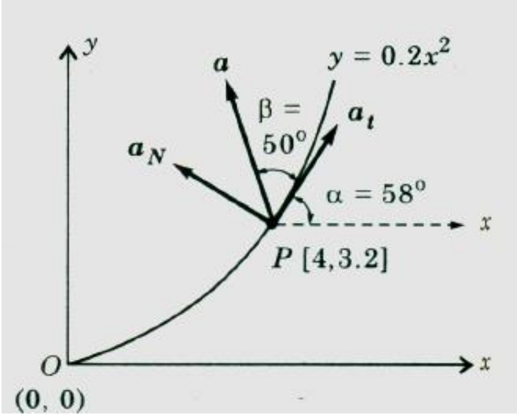
$$\therefore\ \rho = 16793\ \text{m.}$$

$$a = \sqrt{0.25 + \frac{(100)^4}{(16793)^2}} = 0.778\ \text{m/s}^2$$

Normal acceleration, $a_n = v^2/\rho = 1000^2/16793 = 0.596\ m/s^2$

$$tan\ tan\ \beta = \frac{a_n}{a_t} = \frac{0.596}{0.5}$$

$$\therefore \quad \beta = 50°$$



Vector diagram at point P is as shown in figure

Magnitude of total acceleration a = 0.778 m/s^2

Angle made by acceleration vector

$\theta = \alpha + \beta = 58° + 50° = 108°$ with positive x

Problems

Rectilinear Motion with constant/variable velocity, acceleration

1. A motorist is travelling at 90 kmph, when he observes a traffic light 250 m ahead of him turns red. The traffic light is timed to stay red for 12 sec. if the motorist wishes to pass the light without stopping, just as it turns green. Determine i) The required uniform deceleration of the motor.
   ii) the speed of the motor as it passes the traffic light. [Ans: a= -0.6944 m/s^2 , v= 60kmph]
2. In traveling a distance of 3 km between points A and 0, a car is driven at 100 km/hour from A to B for t seconds. If breaks are applied for 4 seconds between B and C to give a car uniform deceleration from 100 km/hour, to 60 km/hour and it takes t seconds to move from C to 0 with a uniform speed of 60 km/hour. Find the value of t.[Ans.: 65.5 sec]
3. The position of a particle which moves along a straight line is defined by the relation $x = t^3 - 6t^2 - 15t + 40$ where x is expressed in m and t in seconds. Determine
   (a) the time at which the velocity will be zero
   (b) the position and distance traveled by the particle at that time
   (c) the acceleration of the particle at the time
   (d) the distance traveled by the particle from t =4 see to t =6 sec.
   [Ans.: t = 5 sec x_s = -60m, 100m, 18m/s^2, 18m]
4. A particle moves along a horizontal straight line such that its velocity is given by v = ($3t^2$ - 6t) m/s. Where t is the time in seconds. In its initially located at the origin ‘0' determine the distance traveled during the time interval t = 0 to t = 3.5 sec. Find average velocity and the average speed of the particle during this time interval.
   [Ans.: s_r= 14.1m, v_{av}=1.75m/s (→), $v_{speed\ av}$=4.03 m/s]
5. The motion of a particle is defined by a relation v = $4t^2$ - 3t -1, where v is in m/s and 't' is in sec. If the displacement x = -4m at t = 0. Determine the displacement and acceleration when t = 3 sec. Find also the time where the velocity becomes zero and the distance traveled by the particle during that time.
   [Ans: x = 15.5 m, a = 21 m/sec^2, t = 1, and distance = -1.17 m.]
6. The acceleration of a particle moving along a straight line is given by a =1 - (X/70), where a is in m/s^2 and x is in meters. The particle starts from rest. Find the maximum velocity.
   [Ans. V_{max}= 8. 37 m/s]

7. Acceleration of a particle is defined by the relation a =100 sin (□t/2) where t is expressed in mm/s^2 and t in seconds. Knowing that at t =0, x =0 and v =0 in the usual notations determine
   (i) maximum velocity of the particle.
   (ii) Position of the particle at t =4 sec.
   [Ans. 127.32 mm/s, 254. 65 mm]
8. A particle moving in positive X direction has an acceleration a = (100 - 4 v^2) m/s^2 where v is in m/sec. Determine-
   (i) the time interval and displacement of particle when speed changes from 1 m/sec to 3 r/sec.
   (ii) The speed of the particle at t = 0.05 sec.
   [Ans.: (i) 0.0245 sec; 0.05 m]
9. A sphere is fired downward into a medium with an initial speed of 27 m/s. If It experiences a deceleration a = -6 t m/s^2, (Where t is in seconds) determine the distance traveled before it comes to rest.

   [Ans : 54 m]

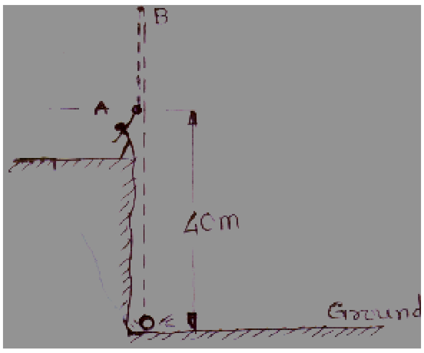
10. In college games, for 100 m event an athlete accelerates uniformly from the start to his maximum velocity in a distance of 4 m and rums the remaining distance with that velocity. If the athlete finishes the race in 10.4 sec, determine i) his initial acceleration ii) his maximum velocity.

[Ans: Vmax.= 10 m/s , a = 12 m/s^2]

- **Motion Under Gravity**

1. A boy tosses a ball in the vertical direction off the side of a cliff as shown in the figure. If the initial velocity of the ball is 15 m/s upward, and the ball is released 40m from the bottom of the cliff, determine the maximum height so reached by the ball and the speed of the ball just before it hits the ground. During the entire time the ball is in motion. It is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.

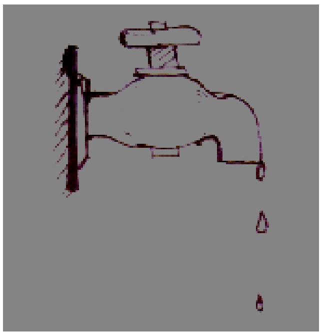
   [Ans.: s_B = 51.5 m V_o = 31.8 m/s (↓)]



2. The depth of a well up to water surface in it is 'H' meters. A stone is dropped into the well from the ground. After 3 seconds the sound of the splash is heard at the ground. If the velocity of sound is 330 m/s, find the value of 'H'
   [Ans. H = 40. 59 m]

3. From the top of a tower, 100m high, a stone was dropped down at the same time, another stone was thrown up from the foot of the tower with a velocity of 30 m/s. When and where the two stone will cross each other? Find the velocity of each stone at the time of crossing.
   [Ans. t = 3.33 s, h_1 = 54.5m, h_2 = 45.5 m, 32.7 m/s (↓), 2.7 m/s (↓)]

4. Water drips from a faucet at the rate of 5 drops per second as shown in figure. Determine the vertical separation between two consecutive drops after the lower drop has attained a velocity of 3 m/s.
   [Ans. : 0.4 m]

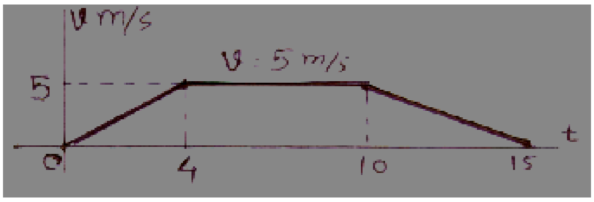


5. In a flood relief area a helicopter going vertically up with a constant velocity drops first batch of food pockets, which takes 4 seconds to reach the ground. No sooner this batch reaches the ground, second batch of food packets are released and this batch takes 5 seconds to reach the ground. From what height was the first batch is released? Also determine the velocity with which the helicopter is moving up? [Ans. : u = 8.83 m/s, h = 43.16m]
6. A balloon starts moving upwards from the ground with constant acceleration of 1.6 m/s^2. Four seconds later, a stone is thrown upwards from the same point:-
   (i) What velocity should be imparted to the stone so that it just touches the ascending balloon?
   (ii) At what height will the stone touch the balloon?
   [Ans.: (i) V_s = 23.57 m/sec. (ii) h = 24.02 m.]
7. A ball is tossed with an initial velocity of 10 m/s directed vertically upwards from a window located 20 m above the ground. Knowing that the acceleration of the ball is constant and equal to 9.81 m/s^2 downward, determine
   (a) the velocity 'V' and elevation y of the ball above the ground at any time 't'
   (b) the highest elevation reached by the ball and corresponding value of 't'
   (c) the time when the ball will hit the ground and corresponding velocity.
   Draw v-t and y-t curves.[Ans. t = 1.019 sec, y = 25.1 m, t = 3.28 sec, u = 22.2 m/s (↓)]
8. A ball is thrown vertically upward from the 12m levels in an elevator shaft, with an initial velocity of 18m/s. At the same instant an open platform elevator passes the 5m level, moving upward with a constant velocity of 2 m/s. Determine
   (a) when and where the ball will hit the elevator
   (b) the relative velocity of the ball with respect to the elevator when the ball hits the elevator.
   [Ans. t = 3.6 sec, elevation from ground = 12.3 m $V_{B/E}$ = 19.81 m/s]
9. A particle falling under gravity falls 30 m in a certain second. Find the time required to cover the next 30 m. [Ans : 0.775 secs]
10. A particle falls from rest and in the last second of its motion it passes 70 m. Find the height from which it fell and the time of Its fall.[Ans : 286.1 m]
11. A vehicle moves at a uniform velocity of 54 km/h for the first 75 seconds. Then it accelerates uniformly at 2 m/s^2 and attains a maximum velocity of 180 km/h. It now moves further with this uniform velocity for

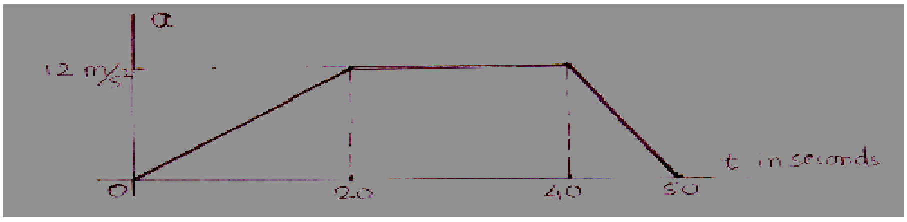
the next 4 minutes and then moves with uniform retardation and comes to rest in 25 seconds. Find the total time of travel and the total distance covered.[Ans: t = 357.5 secs, S =14.32 km]

- **Motion Curves**

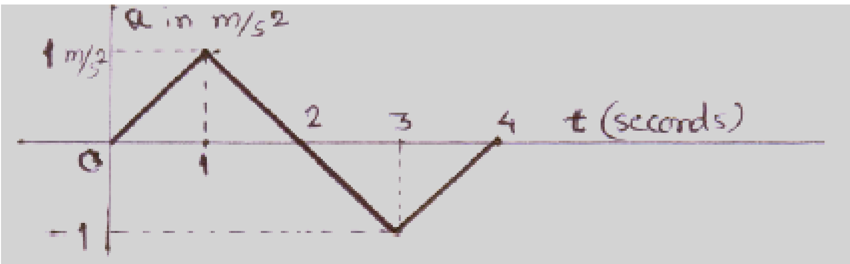
1. A motorcycle starts from rest at s = 0 and travels along a straight road with the speed shown by the v-t graph. Determine the total distance the motorcycle travels until it stops when t = 15 sec. Also plot the a-t and s-t graphs.[Ans.: s = 52.5 m]



2. Figure shows a plot of a-t for a particle moving along x-axis. What is the speed and distance covered by the particle after 50 seconds? Find also the maximum speed and the time at which the speed is attained by the particle. [Ans: s = distance = 9600m, V = V_{max} = 420 m/s]



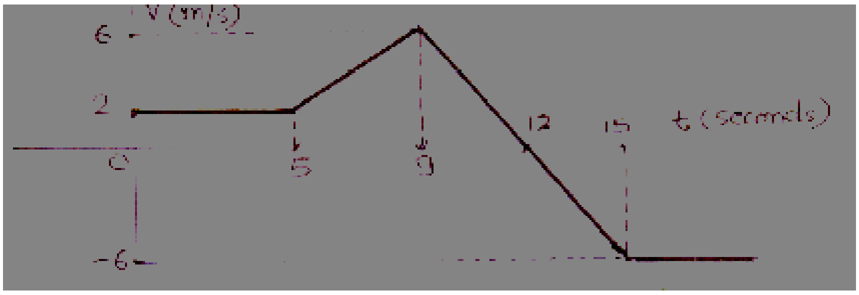
3. For the acceleration-time diagram of a particle shown in figure calculate the velocity at the end of 3 seconds and distance traveled in 4 sec.[Ans. V_3 = 0.5 m/s, displacement = 2m]



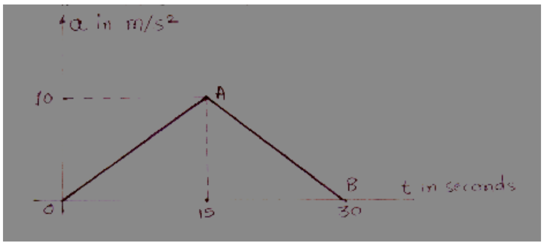
4. A particle moves in a straight line with the velocity shown in the figure. Knowing that x = -8m at t = 0, draw the a-t and x-t curve for 0 < t < 20 sec and determine

(a) the maximum value of the position co-ordinate of the particle

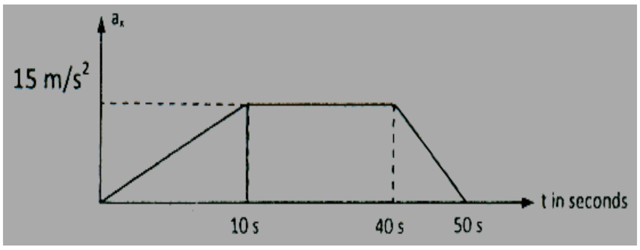
(b) the value of 't' for which the particle is at a distance of 18 m from the origin. [Ans. (a) S_{max} = 27 m (b) t = 9 sec and 15 sec]



5. The a-t diagram for a particle is shown in figure. Plot the v-t and the s-t diagrams. Find the maximum speed attained and the maximum distance covered. The particle starts from rest from the origin in a straight line.[Ans: V_A =1.5 m/s, V_B = 150 m/s, S_A = 375 m, S_B = 2250 m.]



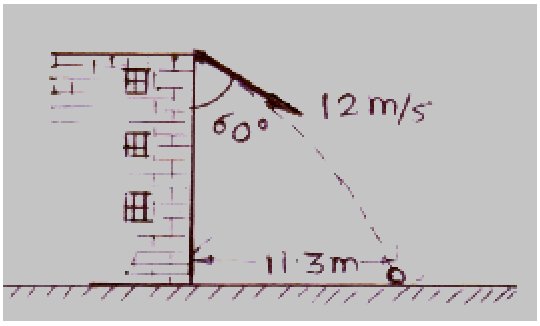
6. Figure shows a plot of a_x versus time for a particle moving along x-axis. What is the speed and distance covered by the particle after 50 sec 600m/s 15000 m.



7. In Asian Games of 100m event an athlete accelerates uniformly from the start to his maximum velocity in a distance of 4 m and runs the remaining distance with that velocity. If the athlete finishes the race in 10.4 seconds, determine
   (i) his initial acceleration,
   (ii) his maximum velocity.
   [Ans. : α =12.5 m/s^2, v = 10 m/s]

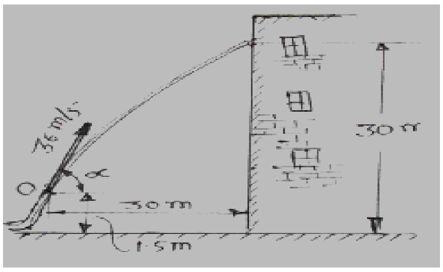
- **Projectile Motion**

1. A shot is fired with a velocity of 40 meters per second from a point 20 meters in front of a vertical wall 10 meters high. Find the angle of projection to the horizontal to enable the shot just to clear the top of the wall. [Ans.:α = 36.13°]
2. A gun fires a projectile with velocity 300m/s.Find the angle of inclination so that it strikes a target at horizontal distance of 4000 m from gun and 200 m above it.[Ans.: 76.6°]
3. A mortar fires a projectile across a level field so that the range 'r' is maximum and to equal to 1,000 meters. Find the time of flight.[Ans.: 14.14 sec]
4. A missile thrown at 30° to horizontal falls 10 m short of target, and goes 20 m beyond the target when thrown at 40° to horizontal. Determine correct angle of projection if velocity remains the same in all the cases.[Ans.: 32.49°]
5. A ball thrown with a speed of 12 m/sec at an angle of 60° with a building strikes the ground 11.3 m horizontally from the foot of the building as shown. Determine the height of the building.[Ans.: 12.42 m]



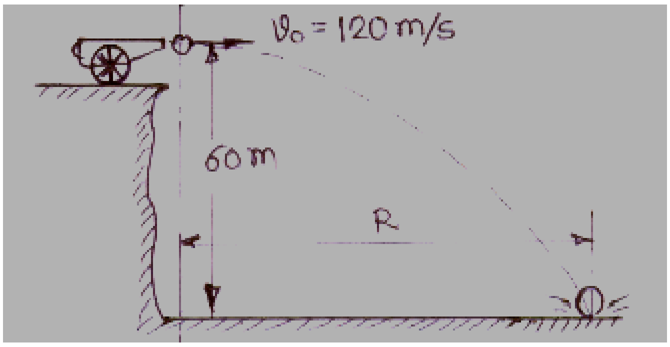
6. A ball is thrown by a player with an initial velocity of 15 m/s from a point 1.5 m above ground. If the ceiling is 6 m high, determine the highest point on the wall at which the ball strikes the wall, 18 m away.[Ans.: h = 4.2 m]

7. A stone is thrown in a vertical plane with an initial velocity or 30 m/sec at an angle of 60° above the horizontal. Compute the radius of curvature of its path at the position when it is 15 m horizontally away from its initial position.[Ans.: R = 70.193 m]
8. A fire nozzle located at A discharges water initial velocity v =36 m/s. Knowing that the stream of water strikes the building at a height h = 30 meters above the ground determine the angle α made by the nozzle with the horizontal.[Ans.: α = 83.42° or 7.385°.]

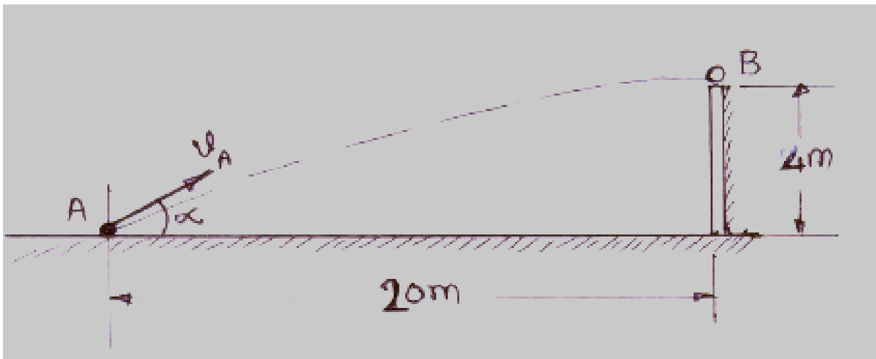


9. A bullet is fired at an angle of elevation of 30° with respect to the horizontal to strike a target, which is 50 m below the horizontal. The initial velocity of the bullet is 80 m/s. Calculate: -
(i)The maximum height to which the bullet will rise.
(ii) The time required hitting the target.
[Ans.: (i) h_{max} = 80m (ii) t = 10.1sec.

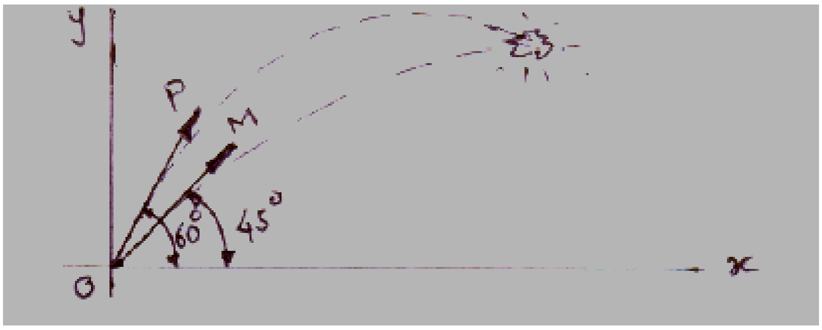
10. A cannon ball is fired from point A with a horizontal muzzle velocity of 120 m/s, as shown in figure. If the cannon is located at an elevation of 60 m above the ground, determine the time for cannon ball to strike the ground and the range 'R'.[Ans=3.5 sec R= 420 m]



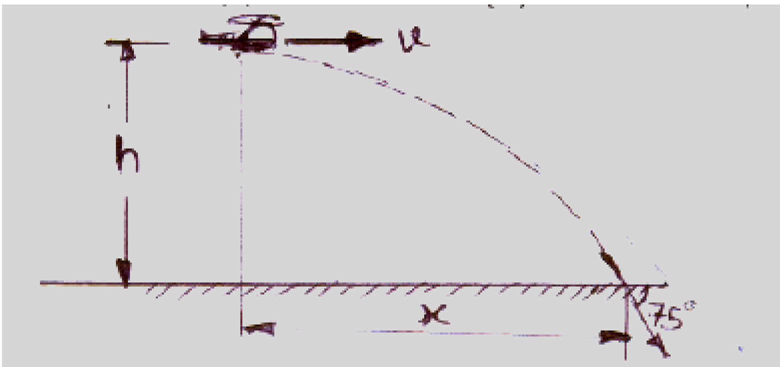
11. When a ball is kicked from 'A' as shown in fig. it just clears the top of the wall at 'B' as it reaches the maximum height. Knowing that the distance form 'A' to the wall is 20 m and the wall is 4m high, determine the initial speed at which the ball was kicked. Neglect the size of the ball.[Ans. 23.9 m/s]



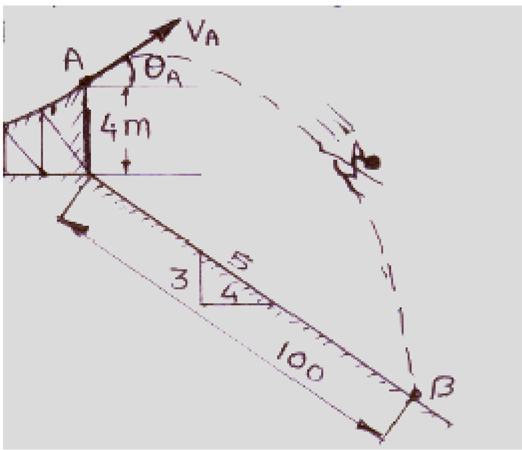
12. A projectile P is fired at a muzzle velocity of 200 m/s at an angle of elevation of 60°. After some time a missile M is fired at muzzle velocity of 2000 m/s and at an angle of elevation of 45°, form the same point, to destroy the projectile P. Find:
    (I) Height
    (ii) Horizontal distance and
    (iii) Time with respect to P firing at which the destruction takes place.
    [Ans.: (i) h = 1494.4m; (ii) x =1499.9m; (iii) Time lag = 14 sec.]



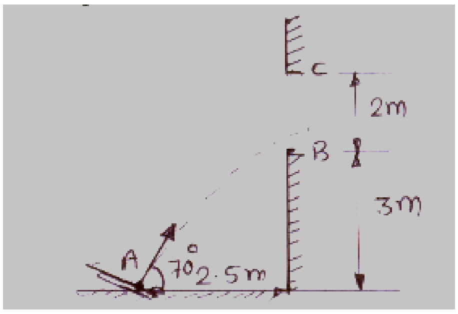
13. A box released from a helicopter moving horizontally with constant velocity 'u' from a certain height 'h' from the ground takes 5 seconds to reach the ground hitting at an angle of 75°as shown in figure. Determine (i) the horizontal distance 'x' (ii) the height 'h' and (iii) the velocity 'u'.
    [Ans.: (i) x = 65.72 m (ii) h =122.63 m (iii) u =13.143 m/s →



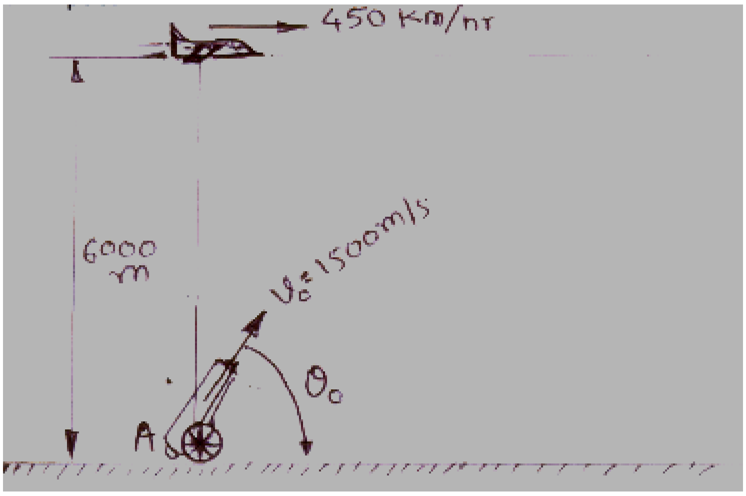
14. A projectile is aimed at an object on the horizontal plane through the point of projection and falls 10m short when the angle of projection is 15°, while it overshoots the object by 20m when the angle of projection is 45°. Determine the angle of projection to hit the object correctly. Derive the formulae you use.[Ans. 20.9°]



15. A ball is dropped on to a pad at 'A' and rebound with a velocity V_o at an angle 70°with the horizontal. Determine the range of values of V_o for which the ball will enter the opening BC.
[Ans.: $8.23 \leq V_o \leq 11.842$ m/s]



16. An antiaircraft gun fires a shell as a plane passes directly over the position of the gun, at an altitude of 6000 m the muzzle velocity of the shell is 1500 m/s. Knowing that the plane is flying horizontally at 450 km/h. determine : (a) the required firing angle if the shell is to hit the plane, (b) the velocity and acceleration of the shell relative to the plane at the time of impact.

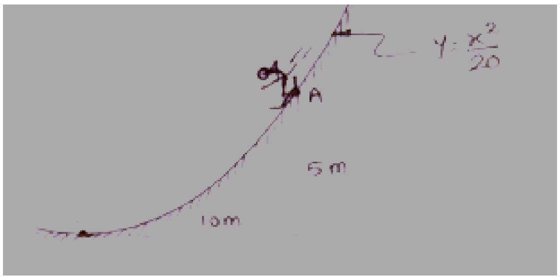


[Ans : θ= 85.22°, $V_{M/P}$ = 1455.54 m/s (↑), $a_{M/P}$ = 9.81 m/s² (↓)]

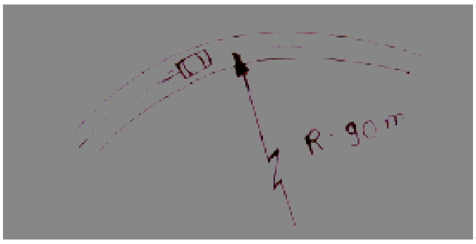
- **Curvilinear Motion**

1. The position of a particle is given by $r = 2t^2\, i + (4/t^2)\, j$ m where t is in seconds. Determine when t =1 sec (i) the magnitude of normal and tangential components of acceleration of the particle and (ii) the radius of curvature of the path.[Ans.: (i) a_t = 19.68 m/s²,a_n = 14.32 m/s²]
2. A particle moves in the x - y plane with velocity components $V_x = 8t - 2$ and $V_y = 2$. If it passes through the point (x,y) = (14,4) at t = 2 seconds determine, the equation of the path traced by the particle. Find also the resultant acceleration at t = 2 seconds.[Ans: Assuming at $\Theta = 0$, $\omega = 0$; v = 0.3 m/s, a = 0.335 m/s², α = 26.56°]
3. A rocket follows the path such that its acceleration is given by a = (4 i + t j) m/s² . At r = 0 it starts from rest. At t = 10 seconds, determine, (i) speed of the rocket. (ii) radius of curvature of its path.[Ans.: (i) V_x= 40 m/s, V_y= 50 m/s, (ii) R = 1312.6 m]

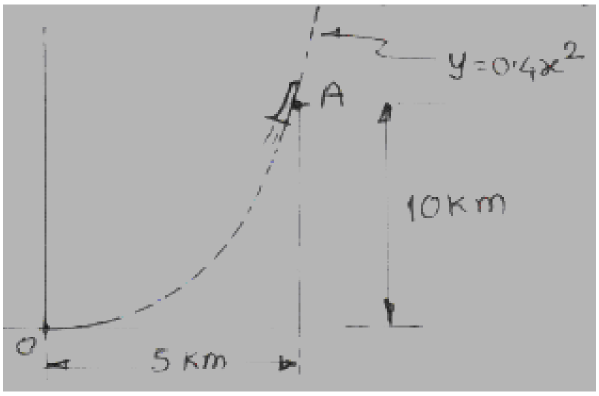
4. A particle moves along a hyperbolic path $(x^2/16) - y^2 = 28$. If the x - component of velocity is $V_x = 4$ m/s and remains constant, determine the magnitudes of particle's velocity and acceleration when it is at point (32m,6m) [Ans.: V = 4.216 m/sec. a = - 0.129 m/s^2]
5. A particle at the position (4,6,3) at start, is accelerated at $a = 4ti - 10t^2 j$ m/s^2. Determine the acceleration, velocity and the displacement after 2 seconds.[Ans.: a =40.792 m/s^2 V = 27.84 m/sec. S = 14.67 m.]
6. A skier travels with a constant speed at 6 m/s along the parabolic path $y = x^2/20$. Determine his velocity and acceleration at the instant he arrives at 'A'. Neglect the size of the skier.[Ans. V_A = 6 m/s, 45°, a_A = 1.27 m/s², 135°]



7. A car 'C' travels around the horizontal circular track that has radius of 90m if the car increases its speed at a constant rate of 2 m/s^2 starting from rest. Determine the time needed for it to reach an acceleration of 2.5 m/s². What is its speed at this instant?[Ans : 11.61 m/s, 5.809 sec]

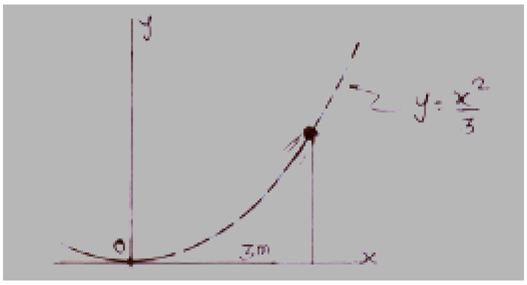


8. The jet plane travels along the vertical parabolic path. When it is at point A it has a speed of 200 m/s, which is increasing at the rate of 0.8 m/s², determine the magnitude of the plane's acceleration when it is at point A. [Ans. 0.521 m/s^2]

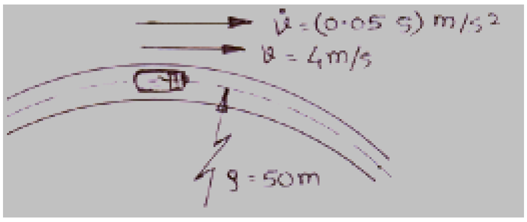


9. A motorist is traveling on a curved section of a highway of radius 750 m at the speed of 108 km/h. The motorist suddenly applies the break causing the automobile to slow down at a constant rate. Knowing that after 8 sec the speed has been reduced to 72 km/h. Determine the acceleration of the automobile immediately after the breaks have been applied.[Ans : 1.732 m/s²]
10. A car travels along a depression in a road, the equation of depression being $x^2 = 200y$. The speed of the car is constant and being equal to 72 km/h. Find the acceleration when the car is at the deepest point in the depression. What is the radius of curvature at the depression at the point?
[Ans: a = 4 m/s², ρ = 100 m]

11. A car starts from rest at t = 0 along a circular track of radius 200m. The rate of increase in the speed of the car is uniform. At the end of 60 seconds the speed of the car is 24 km/hr. Find the tangential and normal components of acceleration at t = 30 sec.
[Ans. $a_t = 0.11$ m/s², $a_n = 0.054$ m/s²]
12. A point moves along the path $y = (1/3)x^2$ with a constant speed of 8 m/s. what are the x and y components of the velocity when x = 3m? What is the acceleration at the point when x = 3m.
[Ans: x = 3.578 m/s, y = 7.155 m/s]



13. The truck travels in a circular path having a radius of 50 m at a speed of 4 m/s. For a short distance from s = 0, its speed is increased by a = (0.05 s) m/s², where s is in metres. Determine its speed and the magnitude of its acceleration when it has moved s = 10m. [Ans.: a = 0.653 m/s²]



University Questions:

- **Rectilinear Motion**

1. The velocity of the particle travelling in a straight line is given by $v = 6t - 3t^2$ m/s, where t is in seconds. If s = 0 when t = 0, determine the particle's deceleration and position when t= = 3s. How far has the particle travelled during the 3 second time interval and what is its average speed? (Dec '10) (05 marks)

2. The motion of a particle moving in a straight line is given by the expression s = t3 – 3t2 + 2t + 5 where's' is the displacement in metres and't' is time in seconds. Find:(i) velocity and acceleration after 4 seconds (ii) maximum or minimum velocity and corresponding displacement and (iii) time at which velocity is zero (May'09) (05 marks)

3. A point is moving with uniform acceleration. In the 11th and 15th second from the commencement, it moves through 7.2m and 9.6m respectively. Find its initial velocity and the acceleration with which it moves (May'08) (05 marks)

4. The acceleration of the particle is defined by the relation $a = 25 - 3x^2$ mm/s². The particle starts with no initial velocity at the position x = 0. Determine (a) the velocity when x = 2mm (b) the position when velocity

is again 0 (c) the position where the velocity is maximum and the corresponding maximum velocity. (Dec '10) (08 marks)

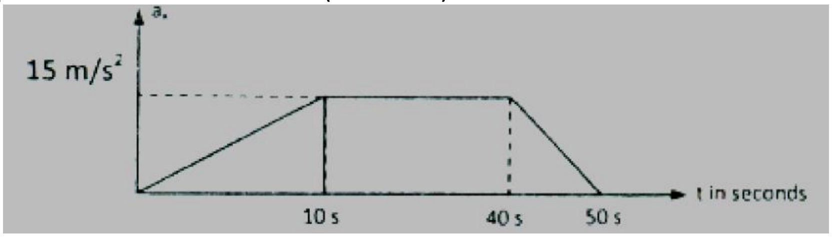
- **Motion Under Gravity**

1. At a certain instant a body of mass 15 kg is falling freely under gravity was found to be falling at a speed of 25m/s. What force will stop the body in 2 seconds? (May'08) (05 marks)

2. A stone released from the top of the tower, during the last second of its motion it covers 1/4th of the height of the tower. Find the height of the tower. (Dec'08) (05 marks)

- **Motion Curves**

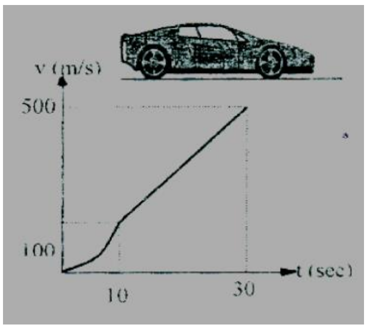
1. Figure shows a plot of ax versus time for a particle moving along x-axis. What is the speed and distance covered by the particle after 50 seconds? (05 marks)



2. The car starts from rest and travels along a straight track such that it accelerates at a constant rate of 10 seconds and then decelerates at a constant rate. Draw the v-t and s-t graphs and determine the time t needed to stop the car. How far has the car travelled? (May'09) (05 marks)

3. The race car starts from rest and travels along a straight road until it reaches a speed of 42 m/s in 50 seconds as shown by v-t graph. Determine the distance traveled by race car in 50 seconds. Draw x-t and a-t graph (May'08) (10 marks)

4. A car moves along a straight line such that its velocity is described by the graph shown in figure. For the first ten seconds the velocity variation is parabolic and between ten seconds and thirty seconds the variation is linear. Construct the s-t and a-t graphs for the time period 0 ≤ t ≤ 30s. (Dec'10) (12 marks)



- **Projectile Motion**

1. By what percentage the range of projectile is increased if initial velocity is increased by 5%?
(May'10)

2. A ball rebounds at A and strikes the incline plane at point B at a distance 76 m as shown in fig. If the ball rises to a maximum height h = 19 m above the point of projection. Compute the initial velocity and angle of projection (May'09)

3. An airplane is flying in horizontal direction with a velocity of 540 kmph and at a height of 2200m. When it is vertically above the point A on the ground, a body is dropped from it. The body strikes the ground at pinot B. Calculate the distance AB (ignore air resistance). Also find the velocity at B and time taken to reach B. (Dec'10) (08 marks)

4. The water sprinkler positioned at the base of a hill releases a stream of water with a velocity of 6m/s as shown. Determine the point B(x,y) where the water particles strike the ground on the hill. Assume that the hill is defined by the equation $y = 0.2\ x^2$m, and neglect the size of the sprinkler.
(May'10) (08 marks)

5. A boy throws a ball so that it may just clear a wall 3.6 m high. The boy is at a distance of 4.8 m from the wall. The ball was found to hit the ground at a distance of 3.6 m on the other side of wall. Find the least velocity with which the ball can be thrown. (Dec'08) (10 marks)

6. An object is projected so that it just clears two obstacles each 7.5 m high which are situated 50 m from each other. If the time of passing between the two obstacles is 2.5 seconds, calculate the complete range of projection and initial velocity of projection (May'08) (10 marks)

- **Curvilinear Motion**

1. The position of the charged particle moving in horizontal plane is measured electronically. This information is fed into a computer which employs a curve fitting techniques to generate analytical expression for its position given by. Where is in meters and t is in seconds. For t = 1 sec, find (i) the acceleration of the particle in rectangular components (ii) its normal and tangential acceleration and (iii) the radius of curvature of the path (May'08) (10 marks)

2. A rocket follows a path such that its acceleration is given by a = (4i + tj) m/s2 at r = 0, it starts from rest. At t = 10 sec. Determine a)Speed of the rocket b)Radius of curvature of its path c)Magnitude of normal and tangential components of acceleration (Dec'08) (10 marks)

3. If x = 1-t and y = t2, where x and y are in metres and't' is in seconds, find x and y components of velocity and acceleration. Also write equation of the path. (May'09) (05 marks)

4. A particle moves in a plane with constant acceleration a = 4i m/s^2. At t = 0 the velocity of the particle was v_0 = I + 1.732 j m/s. Find the velocity of the particle at t = 1s. (May'10) (05 marks)