MODULE 1

Coplanar System of Forces & Centroids

Coplanar system of forces:

Force has a key role in learning Engineering Mechanics. Different types of arrangements of forces on the body constitutes System of forces which aids in understanding concepts like Resultant, Moment, couple, equilibrium etc.

Pre-requisite:

Knowledge of fundamentals of physics (forces) and mathematical formulation learnt at higher secondary level of education (trigonometry).

Objectives:

- **Coplanar system of forces:**

1) Understanding various systems of forces
2) Resultant of two forces by Parallelogram law of forces
3) Resultant of Three of more forces by Method of resolution
4) Moment and Couple
5) General Force system and locating its resultant by Varignon's Theorem

- **Centre of gravity and Centroid:**

1) The centroid location is very important as it helps in the location and placing of forces such that the body remains balanced.
2) Given a lamina of several standard shapes or any composite object, the position of centroid of the complete composite body can be calculated.
3) This helps in simpler calculations and finding the effect forces exerted on them.

1.6 Abbreviations:

- **Centre of gravity and Centroid:**
  C.G = Centre of gravity

1.7 Notations :

<u>**Coplanar system of forces:**</u> R = Resultant. P, Q = Two forces acting at a point. $\sum$ Fx = Summation of all horizontal components of forces. $\sum$ Fy = Summation of all vertical components of forces. $\sum$ M = Summation of Moments of all forces taken about a point. θ = Angle between two forces P and Q for Parallelogram Law of forces. α = Angle made by the resultant with the horizontal force. d = Perpendicular distance between line of action of force and point about which moment is required to be taken. x = Perpendicular distance between $\sum$ Fy i.e. R and point about which moment is required to be taken for vertical parallel force system.	<u>**Centre of gravity and Centroid**</u> x = x-co-ordinate of the C.O.G. y = y-co-ordinate of the C.O.G A = area of given lamina

y = Perpendicular distance between $\sum$ Fx i.e. R and point about which moment is required to be taken for horizontal parallel force system.	

8. Formulae:

- **Coplanar system of forces:**

1. Parallelogram law of forces,

- Magnitude: $R^2 = P^2 + Q^2 + 2PQ\cos\theta$.
- Direction : $\tan^{-1} = (Q \sin\theta) / (P + Q\cos\theta)$

2) Lami's Theorem, $(P/\sin\alpha) = (Q/\sin\beta) = (R/\sin\gamma)$.

3) Resolution of forces,

- Magnitude:

$\sum$ Fx = Forces along X-direction , $\sum$ Fy = Forces along Y-direction

$$R = \sqrt{(\sum F_X)^2 + (\sum F_Y)^2}$$

- Direction : $\theta = \tan^{-1}[(Fy)/(Fx)]$

4) Moment: $M_o = F \times d$ (Force X Perpendicular distance)

- **Centre of gravity and Centroid:**

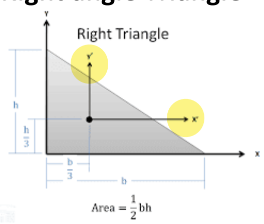
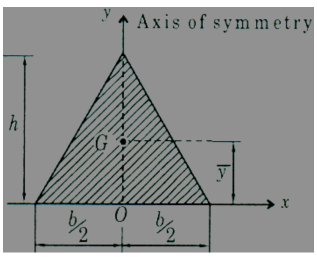
1. For areas,

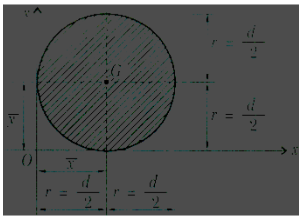
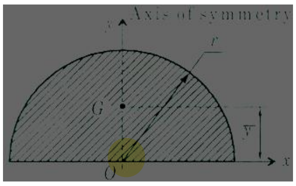
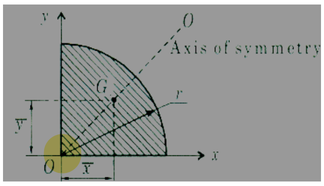
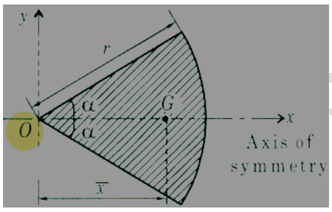
$X = (\sum A_iX_i / \sum A_i)$, $Y = (\sum A_iY_i / \sum A_i)$

2. For wires and rods,

$X = (\sum l_iX_i / \sum l_i)$, $Y = (\sum l_iY_i / \sum l_i)$

Centroid of Laminas

Plane Figure	Area	x	Y
1. Right angle Triangle 	**½ bd**	**b/3**	**h/3**
2. Symmetrical Triangle 	**½ bh**	**-**	**h/3**

3. Circle 	**πr^2**	**r**	**r**
4. Semi-Circle 	**$\pi r^2/2$**	**0**	**$4r/3\pi$**
5. Quarter Circle 	**$\pi r^2/4$**	**$4r/3\pi$**	**$4r/3\pi$**
6) Sector 	**$r^2\alpha$**	**$2r\sin\alpha / 3\alpha$**	**0**

9. Definitions:

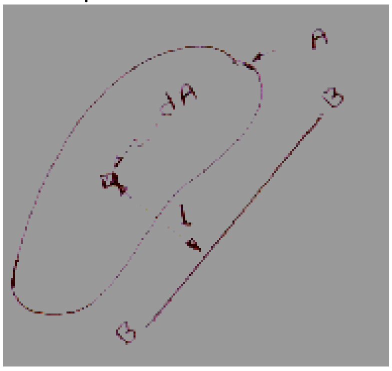
- **Coplanar system of forces**

1. **Statics:**
   It is the branch of mechanics which deals with the particles and bodies at rest (or moving with constant velocity).
2. **Dynamics:**
   It is the branch of mechanics which deals with the particles and bodies which are in motion.
3. **Kinetics:**
   It is the study of motion of particles and bodies with references to masses and effects of forces.
4. **Kinematics:**
   It is the study of motion of particles and bodies without any references to masses and effects of forces (cause of motion).
5. **Force:**
   An external agency which produces or tends to produce, destroys or tends to destroy the motion.
6. **Resultant:**
   A single force producing the same effect that as produced by number of forces when acting together.

7. **Moment:**
Turning effect produced by a force.

8. **Couple:**
Two unlike parallel, non-collinear forces having same magnitude form a couple.

9. **Composition of forces:**
The process of addition of forces is called as composition of forces.



10. **Resolution of forces:**
It is the procedure of splitting up a single force into number of components without changing the effects of the same.

- **Centre of gravity and Centroid:**

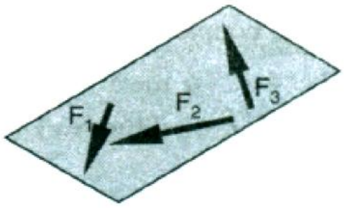
1. **Centroid:**
It is defined as geometrical centre of a body (e.g. center of a rectangle, center of triangle etc.).
2. **Centre of mass:**
It is the point where the entire mass may be supposed to be concentrated.
3. **Center of Gravity:**
It is defined as the point of intersection of all the gravity axes of the body.

1.10 Theory:

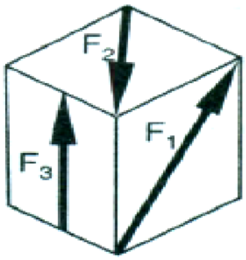
1) **Force:** It is defined as an external agency which produces or tends to produce, destroys or tends to destroy the motion.
It is characterized by magnitude, direction, sense and point of application
It is a vector quantity and S.I unit is Newton (N).
1 Newton force is defined as force required to produce unit acceleration on unit mass.
Therefore, 1 Kg = 9.81 N

2. **System of forces:**
There are mainly seven types of system of forces:

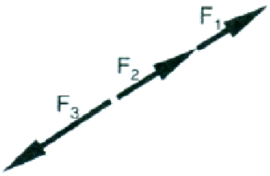
- **Co-planer forces:**
  The forces which are acting in the same plane are known as co-planer forces.



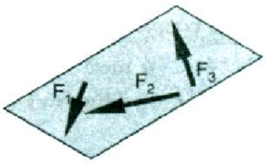
- **Non-Coplaner forces:**
  The force system in which the forces acting in the different planes is called as non coplaner forces.



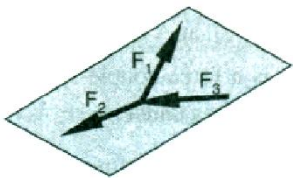
- **Collinear forces:**
  The forces which are acting along the same straight line are called as collinear forces.



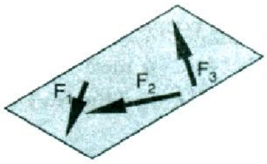
- **Non-collinear forces:**
  The forces which are not acting along the straight line are called as Non-collinear forces.



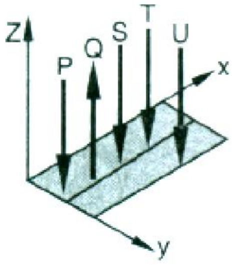
- **Concurrent forces:**
  The forces which are passing through a common point are called concurrent forces.



- **Non-concurrent forces:**
  The forces which are not passing through a common point are called as non-concurrent forces.



- **Parallel forces:**
  The forces whose lines of action are parallel to each other are known as parallel forces.



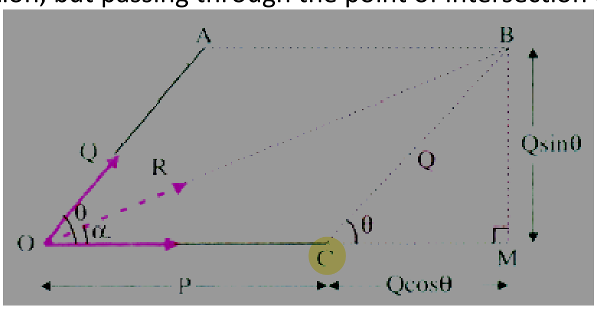
- **Like parallel forces:**
  The forces which are parallel to each other and having same direction are called as like parallel forces.
  In above fig. PSTU are examples of like forces.
- **Unlike parallel forces:**
  The forces which are parallel to each other and having different direction are called as unlike parallel forces.
  In above fig. PQS are examples of unlike forces.

3. Resultant:

A single force producing the same effect that as produced by number of forces when acting together. It is denoted by 'R'

Methods of composition: (to find R)-

- **Resultant of two concurrent forces: (Law of parallelogram of forces):**
  It states that "if two forces simultaneously acting at a point be represented in magnitude and direction by two adjacent sides of a parallelogram, the diagonal will represent resultant in magnitude and direction, but passing through the point of intersection of two forces"



Consider two forces P and Q acting at a point represented by two sides OA and OC of a parallelogram OABC.

Let θ be the angle between two forces P and Q , α be the angle between P and R
Draw perpendicular BM and produce QC
In triangle CMB, BM= Q sinθ and CM = Q cosθ

- Magnitude of R:
  In triangle OMB, $OB^2 = OM^2 + BM^2$

$OB^2 = (OC + CM)^2 + BM^2$

$R^2 = (P + Q\cos\theta)^2 + (Q\sin\theta)^2$

$R^2 = (P^2 + 2PQ\cos\theta + Q^2\cos^2\theta) + (Q^2\sin^2\theta)$

$R^2 = P^2 + 2PQ\cos\theta + Q^2(\sin^2\theta + \cos^2\theta)$

$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

- Direction of R:

  In triangle OMB, $\tan\alpha = BM/OM$

  $\tan\alpha = BM/(OC + CM)$

  $\tan\alpha = Q\sin\theta/(P + Q\cos\theta)$

  If $\theta = 90^\circ$

  $R = \sqrt{P^2 + Q^2}$, $\tan\alpha = Q/P$

- **Resultant of two or more forces: (Method of resolution):**

  When two or more co-planer concurrent or non-concurrent forces acting on a body the resultant can be found out by using resolution procedure.

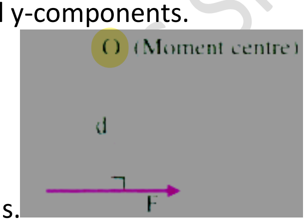
  Magnitude of resultant, $R = \sqrt{(\sum F_X)^2 + (\sum F_Y)^2}$

  Direction, $\tan\theta = (\sum F_y / \sum F_x)$

  Where, $\sum F_x$ = Algebraic sum of all x-components.

  $\sum F_y$ = Algebraic sum of all y-components.

  θ = Angle of 'R' with x-axis.



4. **Moment:**

   It is the turning effect produced by a force. Moment of a force about any point is the product of magnitude of the force and perpendicular distance about that point. The point about which moment is taken is called as moment center.

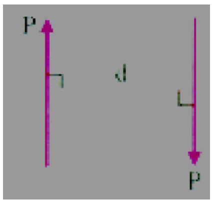
   Moment about O, $M_o = F \times d$

   S.I unit : N-m

- While taking moment of any force do not observe direction of force but observe direction of rotation.
- If any force is passing through the moment centre, the moment of that force is zero because for the case perpendicular distance would become zero.

5. **Couple:**

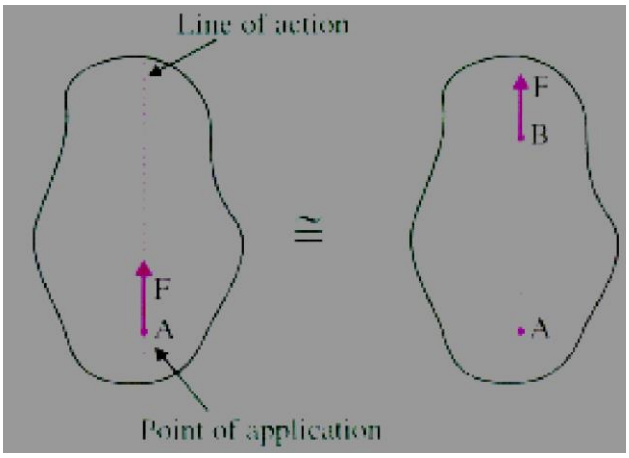
   Two unlike parallel, non-collinear forces having same magnitude form a couple. The distance between two forces is known as arm or lever of the couple.

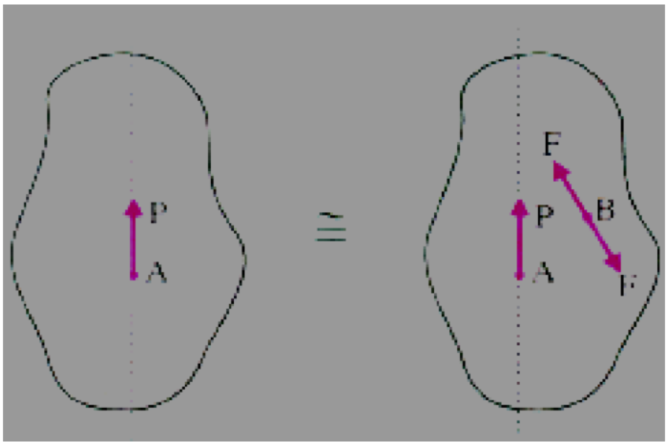


- **Properties of couple:**
- Two unlike parallel, non-collinear forces of same magnitude form a couple.
- The resultant of a couple is always zero.
- The moment of a couple is the product of one of the forces and lever arm of the couple. Therefore, M= F X d.
- A couple cannot be balanced by a single force.
- It can be balanced only by another couple of opposite nature.
- The moment of couple is independent of the moment centre.

6. Principle of transmissibility:

The point of application of a force can be transmitted anywhere along its line of action, but within the body.





It is only applicable to rigid bodies. The principle is neither applicable from the point of view of internal resistances nor internal forces developed in the body nor to deformable bodies under any circumstances.

7. **Principle of superposition:**

The effect of a force on a body remains unaltered if we add or subtract any system which is in equilibrium.

8. **Varignon's theorem:**

Statement : The sum of the moment of all the forces about a point is equal to the moment of their resultant about the same point. Mathematically this can be represented as $\sum M_O^F = M_O^R$

Proof : Consider a force F acting at a point A and having component F_1 and F_2 in any two directions.

Let us chose any point O, lying in the plane of the forces, as a moment center. Attach at A two rectangular axes such that the y-axis is along the line AO and the x-axis is perpendicular to it, as shown in *Figure 1.12*.

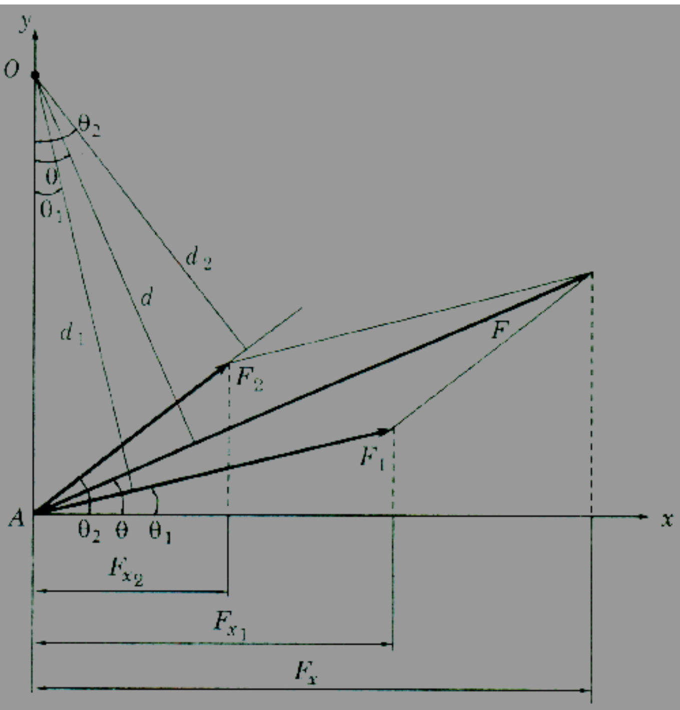


Fig. 1.12

Moment of the force F about O,

$$F \times d = F \times OA\cos\theta = OA \times F\cos\theta$$

$$F \times d = OA \times F_x \text{ .... } (1)$$

Moment of the force F_1 about O,

$$F_1 \times d_1 = F_1 \times OA\cos\theta_1 = OA \times F_1\cos\theta_1$$

$$F_1 \times d_1 = OA \times F_{x1} \text{ .... } (2)$$

Moment of the force F_2 about O,

$$F_2 \times d_2 = F_2 \times OA\cos\theta_2 = OA \times F_2\cos\theta_2$$

$$F_2 \times d_2 = OA \times F_{x2} \text{ .... } (3)$$

Adding (2) and (3)

$$F_1 \times d_1 + F_2 \times d_2 = OA \times (F_{x1} + F_{x2}) \text{ .... } (4)$$

$F_1 \times d_1 + F_2 d_2 = OA \times F_x$ .... [$\therefore$ sum of the x-components of the forces F_1 and F_2 = x-components of the resultant force F $\therefore F_x = F_{x1} + F_{x2}$]

$F_1 \times d_1 + F_2 \times d_2 = F \times d$ .... From (1) and (4)

i.e. $\sum M_O^F = M_O^R$

Centre of gravity and centroid:

1) Procedure to solve problems of centroid of a given respective figure.

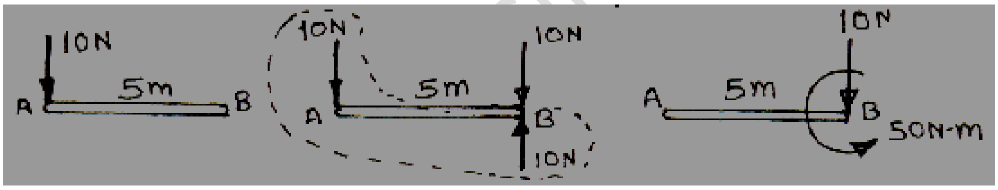
<u>Procedure</u>:

- From a given composite figure, consider each figure separately in the form of triangle, circle, semicircle, etc.
- Specify the reference axis as x-axis and y-axis, if not specified.
- Determine the area of each figure as A_1, A_2, A_3, A_4, etc. and find the addition of all areas considering the shape to be subtracted.
- Determine x_1, x_2, x_3, x_4, etc i.e. distance between centroid of the figure and references y-axis.
- Similarly for y_1, y_2, y_3, y_4, etc i.e. distance between centroid of the figure and references x-axis.
- Adding the product of area and distance ($A_i x_i$) for plane figure where as for hollow figure required figure is to be added and remaining part is to be deducted.
- By using formula, $x = (A_1 x_1 + A_2 x_2 + A_3 x_3 + A_4 x_4) / (A_1 + A_2 + A_3 + A_4)$
  $y = (A_1 y_1 + A_2 y_2 + A_3 y_3 + A_4 y_4) / (A_1 + A_2 + A_3 + A_4)$
  we can determine co-ordinates of centroid with respect to the reference axis.

1.11 Short Answer Questions:

- **Coplanar system of forces:**

1. What do you mean by resolution of a force into a force and a couple?



Ans: If line of action of a force is changed it gets resolved into force and a couple. Force component is a Force of same magnitude and direction but a point of application is along new line of action parallel to that of original force and couple component is a couple of moment equal to moment of original force about new point of application.

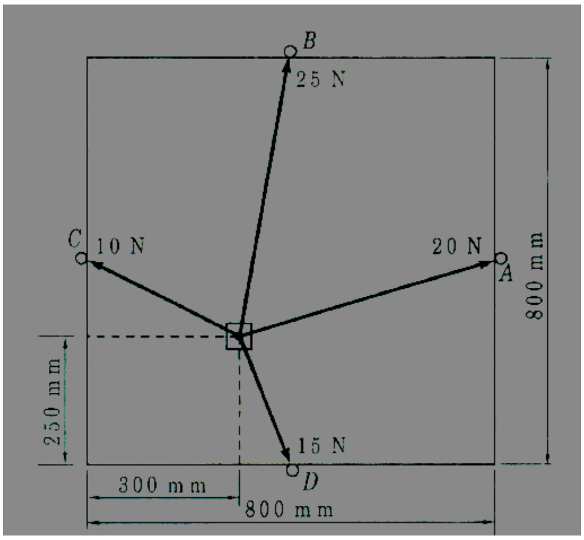
2. State conditions of equilibrium of concurrent system of forces.

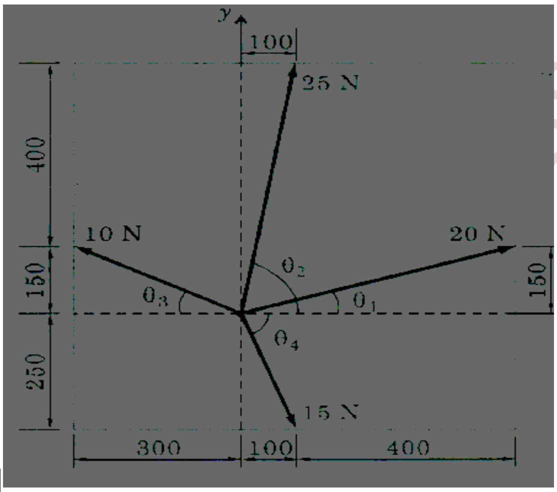
Ans: Set 1 $\quad \Sigma F_x = 0 \quad \Sigma F_y = 0$

Set 2 $\quad \Sigma F_x = 0 \quad \Sigma M_A = 0$ where A is not on X – axis.

Set 3 $\quad \Sigma M_A = 0 \quad \Sigma M_B = 0$ where A and B are non collinear with concurrence.

4. The striker of carrom board lying on the board is being pulled by four players as shown in fig. The players are sitting exactly at the centre of the four sides. Find the resultant forces in magnitude & direction.





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Solutions: tan θ_1= [AG/OG

θ_1= $\tan^{-1}$[AG/OG]

= $\tan^{-1}$[150/500]

= 16.7°

tanα = [EB/OE]

α = $\tan^{-1}$[EB/OE]

= $\tan^{-1}$[100/550]

= 10.3°

θ_2= (90° - α)

= (90°-10.3°)

$= 79.7^\circ$

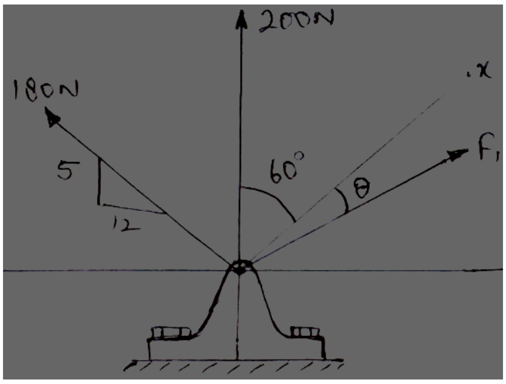
$\tan \theta_3 = [CH/OH]$

$\theta_3 = \tan^{-1}[CH/OH]$

$= \tan^{-1}[150/300]$

$= 26.56^\circ$

7. Three forces act on the bracket. Determine the magnitude and direction of θ of F_1 so that the resultant force is directed along the positive X-axis and has a magnitude of 800N.



Solutions: $\tan\alpha = (5/4)$

$\alpha = \tan^{-1}(5/4)$

$\alpha = 22.62^\circ$

$\sum F_x = R_x(\rightarrow)$ +ve

$-180 \cos 22.62^\circ + F_1 \cos(30^\circ - \theta) = 800 \cos 30^\circ$

$F_1 \cos(30^\circ - \theta) = 858.9$ — (I)

$\sum F_y = R_y(\uparrow)$ +ve

$180 \sin 22.62^\circ + F_1 \sin(30^\circ - \theta) + 200 = 800 \sin 30^\circ$

$F_1 \sin(30^\circ - \theta) = 130.8$ — (II)

$\{F_1 \sin(30^\circ - \theta) / F_1 \cos(30^\circ - \theta)\} = \{130.8 / 858.9\}$

$\tan(30^\circ - \theta) = 0.1523$

$(30^\circ - \theta) = \tan^{-1}(0.1523)$

$(30^\circ - \theta) = 8.66$

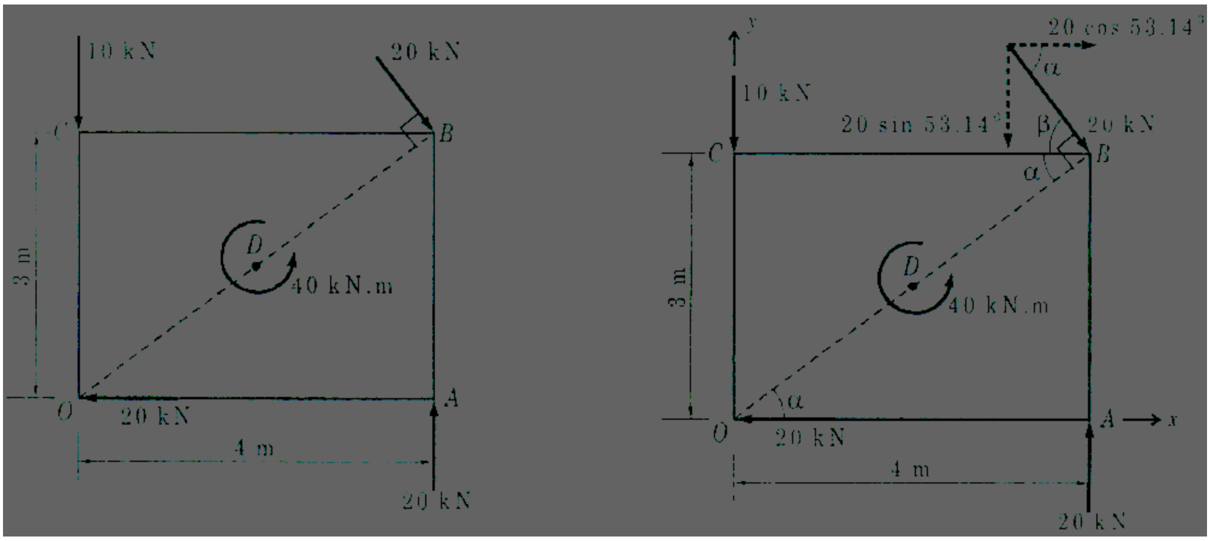
$\theta = 21.34^\circ$

Substitute value of θ in equation (I)

$F_1 \cos(30^\circ - \theta) = 858.9$

$F_1 = 868.8N$

10) Find the resultant of the force system acting on a body OABC shown in the fig. Find the resultant from O. Also find the points where the resultant will cut the X & Y axis.(Dec - 2010)



<u>Solutions</u>:

tan α = (3/4)

α = $\tan^{-1}$ (3/4)

α = 36.86°

$\sum F_x (\rightarrow)$ +ve

$\sum F_x$= 20 cos53.13º – 20

$\sum F_x$= -8 KN

= 8 KN($\leftarrow$)

$\sum F_y (\uparrow)$ +ve

$\sum F_y$ = -10 - 20 sin53.13 + 20

$\sum F_y$ = - 6KN

= 6KN ($\downarrow$)

R = $\sqrt{\sum F_X{}^2 + \sum F_Y{}^2}$

= $\sqrt{8^2 + 6^2}$

=10KN

tanθ = ($\sum F_y$ / $\sum F_x$)

θ = $\tan^{-1}$ (6/8)

= 36.86°

To locate the position of resultant
Using Varignon's theorems,

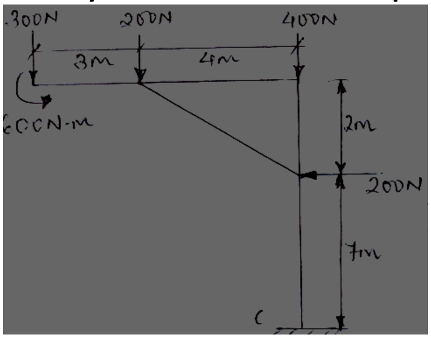
$\sum M_o^F = M_o^R$ (   ) +ve

(20 x 4) + 40 - (20 x 4 sin53.13°) - (20 x 3 cos53.13°) = - 10 x d

d = - 2m

= 2m

11) Replace the loading on the frame by a force and moment at point A.



<u>Solution</u>: $\sum F_x = 0(\rightarrow)$ +ve

$-200 = 0$

$\sum F_x = 200N\ (\leftarrow)$

$\sum F_y = 0(\uparrow)$ +ve

$-300 - 200 - 400 = 0$

$\sum F_y = -900N$

$= 900N\ (\downarrow)$

$$R = \sqrt{\sum {F_X}^2 + \sum {F_Y}^2}$$

$$= \sqrt{200^2 + 900^2}$$

$= 921.9N$

$\tan\theta = (\sum F_y / \sum F_x)$

$\theta = \tan^{-1}(900/200)$

$\theta = 77.5°$

Moment about point A,
Using Varignon's theorem,

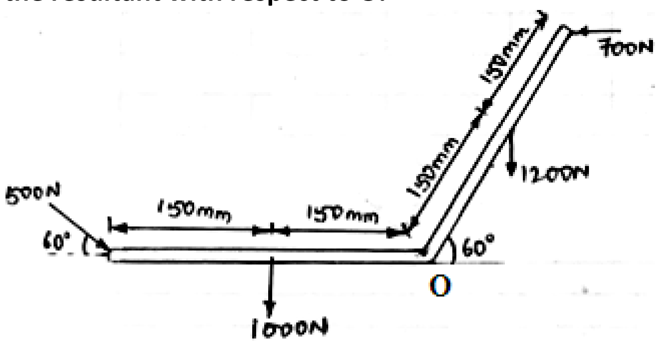
$\sum M_A = R \times d$ ( ) +ve

$600 - (200 \times 3) - (400 \times 7) - (200 \times 2) = -921.9 \times d$

$-3200 = -921.9 \times d$

$d = 3.47m.$

12) A system of forces acting on a bell crank as shown. Determine the magnitude, direction and point of application of the resultant with respect to O.



Solution: $\sum F_x = 0 (\rightarrow)$ +ve

$500 \cos 60^\circ - 700 = 0$

$\sum F_x = -450N$

$= 450N (\leftarrow)$

$\sum F_y = 0 (\uparrow)$ +ve

$-500 \sin 60^\circ - 1000 - 1200 = 0$

$\sum F_y = -2633N$

$= 2633N (\downarrow)$

$$R = \sqrt{\sum F_X^2 + \sum F_Y^2}$$

$$= \sqrt{450^2 + 2633^2}$$

$= 2671.2N$

$\tan\theta = (\sum F_y / \sum F_x)$

$\theta = \tan^{-1} (2633/450)$

$\theta = 80.3^\circ$

Moment about point O,

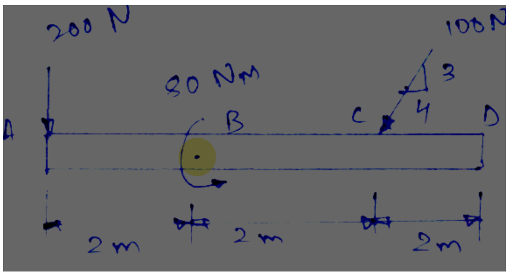
Using Varignon's theorem,

$\sum M_o^F = M_o^R$ ( ) +ve

$(500 \sin 60^\circ \times 300)+(1000 \times 150)+(700 \times 300 \cos 30^\circ)-(1200 \times 150 \cos 60^\circ) = 2671.2 \times d$

$d = 139.2mm.$

13) Resolve the system of forces shown in fig. into a force and couple at point A.



Solutions: $\tan\theta = (3/4)$

$\theta = \tan^{-1} (3/4)$

$\theta = 36.87^\circ$

$\sum F_x = 0(\rightarrow)$ +ve

$-100 \cos 36.87^\circ = 0$

$\sum F_x = -80N$

$= 80N (\leftarrow)$

$\sum F_y = 0(\uparrow)$ +ve

$-200 - 100 \sin 36.87^\circ = 0$

$\sum F_y = -260N$

$= 260N (\downarrow)$

$R = \sqrt{\sum {F_X}^2 + \sum {F_Y}^2}$

$= \sqrt{80^2 + 260^2}$

$= 272.03N$

$\tan\theta = (\sum F_y / \sum F_x)$

$\theta = \tan^{-1}(260/80)$

$\theta = 72.89^\circ$

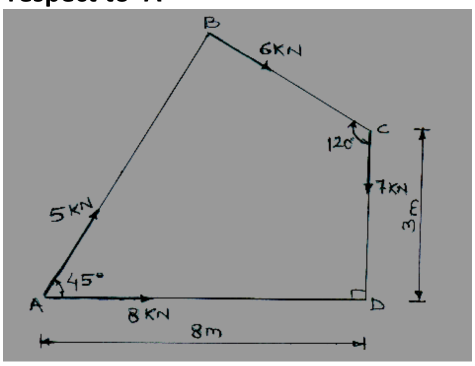
Using Varignon's theorem,

$\sum M_A^F = M_A^R$ ( ) +ve

$80 - (100 \sin 36.87^\circ \times 4) = 2671.2 \times d$

$d = 0.59m$

14) Determine completely the resultant of the four coplanar forces shown in the fig. Locate the line of action of the resultant with respect to 'A'



Solutions: Four coplanar forces shown in the fig. at a respective angle

$\sum F_x (\rightarrow)$ +ve

$\sum F_x = 5 \cos 45^\circ + 8 + 6 \cos 30^\circ$

$\sum F_x = 16.73KN (\rightarrow)$

$\sum F_y (\uparrow)$ +ve

$\sum F_y = 5 \sin 45^\circ - 7 - 6 \sin 30^\circ$

$\sum F_y = -6.46KN$

$\sum F_y = 6.46KN (\downarrow)$

$$R = \sqrt{\sum F_X{}^2 + \sum F_Y{}^2}$$

$$= \sqrt{16.73^2 + 6.46^2}$$

$= 17.93KN$

$\tan\theta = (\sum F_y / \sum F_x)$

$\theta = \tan^{-1}(6.46/16.73)$

$\theta = 21.1^\circ$

Moment about point A,

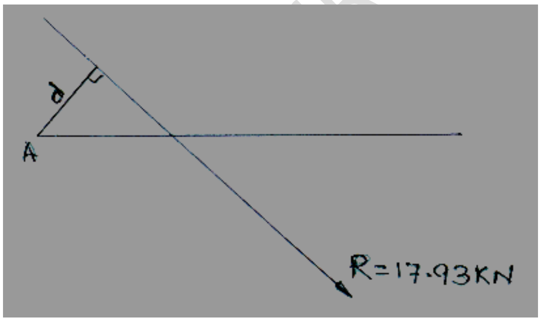
Using Varignon’s theorem,

$\sum M_A^F = M_A^R$ ( ) +ve

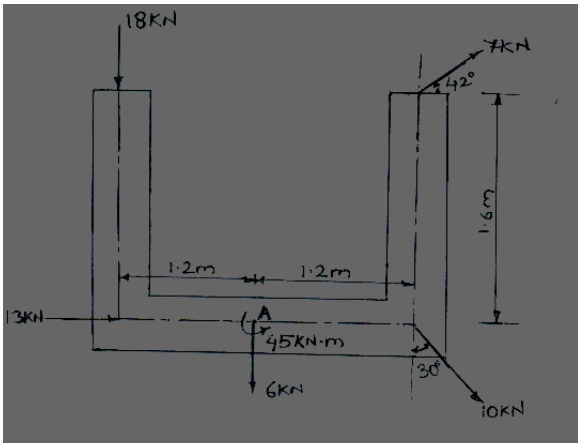
$(7 \times 8) + (6 \times 6 \sin 30^\circ) - (4.15 \times 6 \cos 30^\circ) = 17.93 \times d$

d = 5.33m.

Representation of Line of action of Resultant wrt A



15) Determine the resultant of general coplanar force system shown in fig.



<u>Solutions</u>: $\sum F_x (\rightarrow)$ +ve

$\sum F_x = 7 \cos 42^\circ + 13 + 10 \sin 30^\circ$

$\sum F_x = 23.2KN (\rightarrow)$

$\sum F_y (\uparrow)$ +ve

$\sum F_y = -18 - 6 - 10\cos30^\circ + 7\sin42^\circ$

$\sum F_y = -27.98KN$

$\sum F_y = 27.98KN (\downarrow)$

R = $\sqrt{\sum {F_X}^2 + \sum {F_Y}^2}$

= $\sqrt{23.2^2 + 27.98^2}$

= 36.3KN

$\tan\theta = (\sum F_y / \sum F_x)$

$\theta = \tan^{-1}(27.98/23.2)$

$\theta = 50.33^\circ$

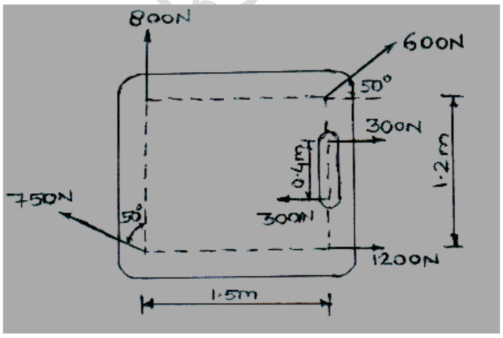
Moment about point A,
Using Varignon's theorem,

$\sum M_A^F = M_A^R$ ( ) +ve

$(18 \times 1.2) - (1.6 \times 7\cos42^\circ) + (1.2 \times 7\sin42^\circ) = 36.3 \times d$

d = 1.474m.

16) Determine completely the resultant of the system of a coplanar forces as shown in the fig.



Solutions: $\sum F_x (\rightarrow)$ +ve

$\sum F_x = 600\cos50^\circ + 1200 - 750\sin50^\circ + 300 - 300$

$\sum F_x = 1011.14N (\rightarrow)$

$\sum F_y (\uparrow)$ +ve

$\sum F_y = 800 + 750\cos50^\circ + 600\sin50^\circ$

$\sum F_y = 1741.7N (\uparrow)$

R = $\sqrt{\sum {F_X}^2 + \sum {F_Y}^2}$

= $\sqrt{1011.14^2 + 1741.7^2}$

= 2013.9N

$\tan\theta = (\sum F_y / \sum F_x)$

$\theta = \tan^{-1} (1741.7/1011.14)$

$\theta = 59.86^\circ$

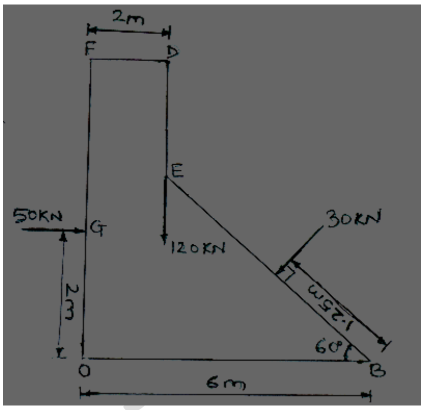
Moment about point A,
Using Varignon's theorem,

$\sum M_A^F = M_A^R$ ( ) +ve

$-(600 \cos 50^\circ \times 1.2) + (1.5 \times 600 \sin 50^\circ) - (0.4 \times 300) = 2013.9 \times d$

d = 0.06m.

17) The forces acting on 1 m length of a dam as shown in fig. Determine the resultant force acting on the dam. Calculate the point of intersection of the resultant with the base.



Solutions: $\sin 60^\circ$ = (Opposite / Hypoténuse)

$\sin 60^\circ = (AC / 1.25)$

AC = 1.08m

$\tan 60^\circ$ = (Opposite / Adjacent)

1.73 = (1.08 / AB)

AB = 0.624m

OB = OA + AB

OA = OB - AB

= 5.376

$\sum F_x (\rightarrow)$ +ve

$\sum F_x = 50 - 30 \cos 30^\circ$

$\sum F_x = 24.02KN (\rightarrow)$

$\sum F_y (\uparrow)$ +ve

$\sum F_y = -120 - 30 \sin 30^\circ$

$\sum F_y = -135KN$

$\sum F_y = 135KN\ (\downarrow)$

$R = \sqrt{\sum F_X^{\ 2} + \sum F_Y^{\ 2}}$

$= \sqrt{24.02^2 + 135^2}$

$= 137.12KN$

$\tan\theta = (\sum F_y / \sum F_x)$

$\theta = \tan^{-1}(135/24.02)$

$\theta = 79.91^\circ$

Moment about point O,
Using Varignon's theorem,

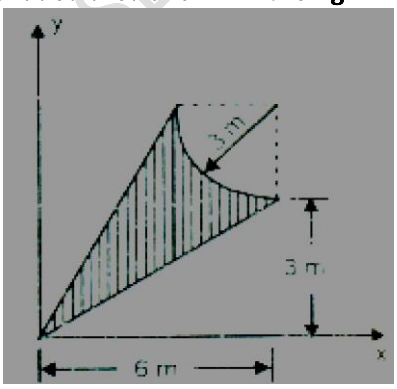
$\sum M_A^F = M_A^R\ (\ \ )$ +ve

$-(30 \cos 30^\circ \times 1.08) + (5.376 \times 30 \sin 30^\circ) + (102 \times 2) + (50 \times 2) = 137.12 \times d$

d = 2.9m

- **Centre of gravity and Centroid:**

2. Find the centroid of the shaded area shown in the fig.



Solution:

Component	Area A_i (m^2)	Co ordinates X_{Gi} (m)	$A_i X_{Gi}$ (m^3)
Triangle 012	½ X b X h = ½ X 3 X 6 = 9 m^2	2b/3 = 2[3]/3 = 2 m	18 m^3

Rectangle 1234	l X b = 3 X 6 = 18 m^2	3+ (3/2) = 3 + 1.5 = 4.5 m	81 m^3
Triangle 035	½ X b X h = ½ X 6 X 3 = - 9 m^2	2b/3 = 2[6]/3 = 4 m	- 36 m^3
Quarter Circle 145	π r^2/4 = π $[3]^2$/4 = - 7.07m^2	4.727 m	- 33.42m^3
	$\sum A_i$ = 10.93 m^2		$\sum A_i X_{Gi}$ mm^3 = 29.58 m^3

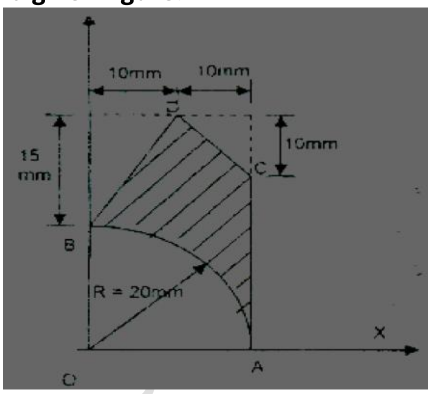
Using, X = ($\sum A_i X_{Gi}$) / ($\sum A_i$)

X = (29.58) / (10.93)

= 2.706 m

Y = X = 2.706 m (Area is symmetrical about 45° line through origin)

5) Find centroid of plane area of a given figure.



Solution: To find centroid of plane area of a given fig.

Component	Area 'A$_i$' (m^2)	Co-ordinates 'X_{Gi}' (m)	Co-ordinates 'Y_{Gi}' (m)	'$A_i X_{Gi}$' (m^3)	'$A_i Y_{Gi}$' (m^3)
Rectangle 'OAFE'	l X b = 20 X 35 = 700 m^2	l/2 = 20/2 = 10 m	b/2 = 35/2 = 17.5 m	7000 m^3	12250 m^3
Quarter Circle 'OAB'	π r^2 / 4 = π $[20]^2$ / 4 = - 314.16 m^2	4r/3π =4[20]/3π = 8.49 m	4r/3π =4[20]/3π = 8.49 m	- 2667.22 m^3	- 2667.22 m^3
Triangle 'BED'	½ X b X h = ½ X 10 X 15 = - 75 m^2	h/3 = 10/3 = 3.33 m	20 + 2r/3π =20+2[15]/3π = 30 m	- 249.75m^3	- 2250 m^3
Triangle 'CFD'	½ X b X h = ½ X 10 X 10 = - 50 m^2	10 + (2b/3) =10+(2[10]/3) = 16.67 m	25 + (2h/3) =25+(2[10]/3) =31.67m	- 833.5 m^3	- 1583.5m^3

	$\sum A_i$ = 260.84m^2			$\sum A_i X_{Gi}$ = 3249.53m^3	$\sum A_i Y_{Gi}$ = 5749.28m^3

Using, $X = (\sum A_i X_{Gi}) / (\sum A_i)$

$X = (3249.53) / (260.84)$

$= 12.45$ m

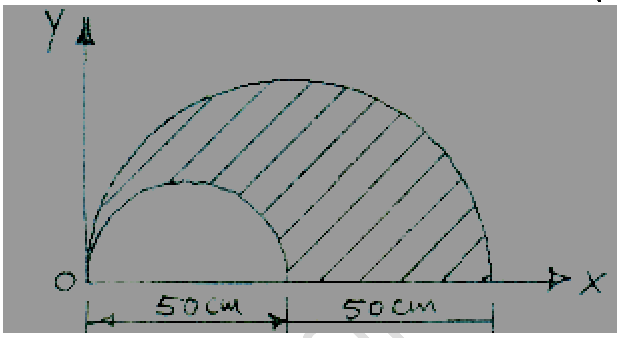
$Y = (\sum A_i Y_{Gi}) / (\sum A_i)$

$Y = (5749.28) / (260.84)$

$= 22.04$ m

Centroid [x,y] = [12.45, 22.04].

6) Find the centroid of shaded area of the semicircle of diameter 100cm. (May 2011)



Solution:

Component	Area ‘A_i’ (m^2)	Co-ordinates ‘X_{Gi}’ (m)	Co-ordinates ‘Y_{Gi}’ (m)	‘$A_i X_{Gi}$’ (m^3)	‘$A_i Y_{Gi}$’ (m^3)
Semicircle	$\pi r^2 / 2$ $= \pi [50]^2 / 2$ = 3926.99 cm^2	100/2 = 50 cm	$4r/3\pi$ = 4[50]/3Π = 21.22 cm	196349.5 cm^3	83330.73 cm^3
Semicircle	$\pi r^2 / 2$ $= \pi [25]^2 / 2$ = - 981.75 cm^2	50/2 = 25 cm	$4r/3\pi$ $=4[25]/3\pi$ = 10.61cm	- 24543.75 cm^3	- 10416.36 cm^3
	$\sum A_i$ = 2945.24cm^2			$\sum A_i X_{Gi}$ = 171805.75 cm^3	$\sum A_i Y_{Gi}$ = 72914.37 cm^3

Using, $X = (\sum A_i X_{Gi}) / (\sum A_i)$

$X = (171805.75) / (2945.24)$

$= 58.33$ cm

$Y = (\sum A_i Y_{Gi}) / (\sum A_i)$

$Y = (72914.37) / (2945.24)$

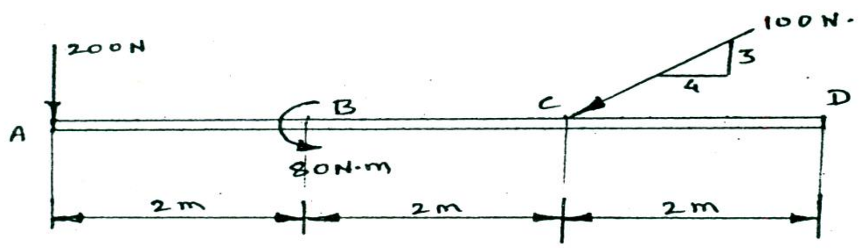
$= 24.76$ cm

Centroid [x,y] = [58.33, 24.76].

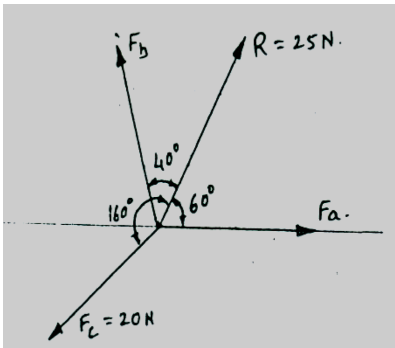
1.14 University Problems:

- **Coplanar system of forces:**

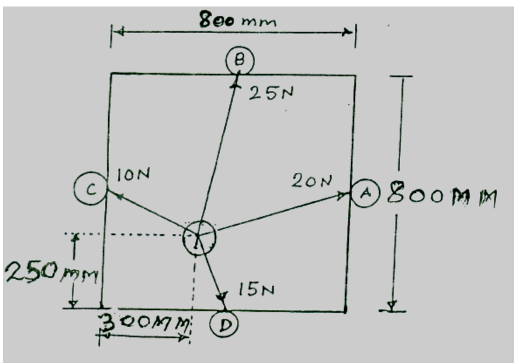
1. **Resolve the system of forces shown in figure into a force and couple at point 'A'. (Dec'07) [05 M]**



2. A force R = 25 N has components F_a, F_b & F_c as shown in figure. If F_c = 20 N. Find F_a & F_b (Dec'07) [05 M]

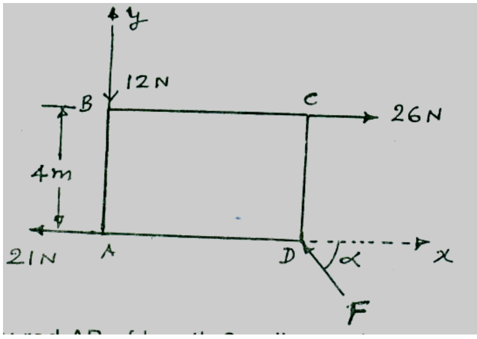


3. The striker of carom board laying on the board is being pulled by four players as shown in the fig. The players are sitting exactly at the centre of the four sides. Find the resultant forces in magnitude and direction. (May'08) (10 M)



4. Forces act on the plate ABCD as shown in the fig. The distance AB is 4m. Given that the plate is in equilibrium find

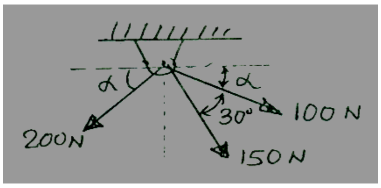
(i) force F, (ii) angle and (iii) the distance AD (May'08) (05 M)



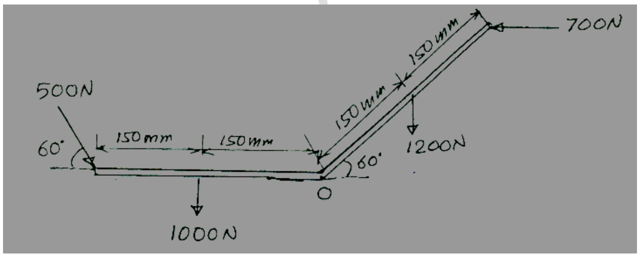
5. **For the system shown, determine**

(i)The required value of if resultant of three forces is to be vertical

(ii) The corresponding magnitude of resultant **(Dec'08) [05 M]**

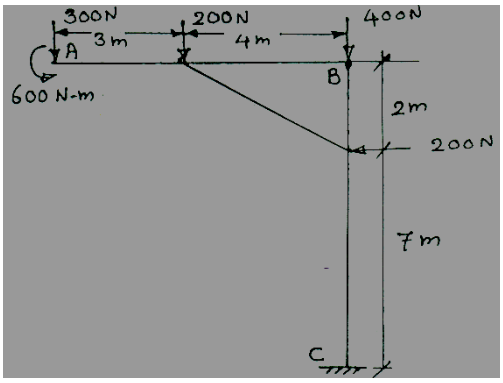


6. A system of forces acting on a bell crank as shown. Determine the magnitude, direction and the point of application of the resultant w.r.t. 'O'**(Dec'08) [10 M]**



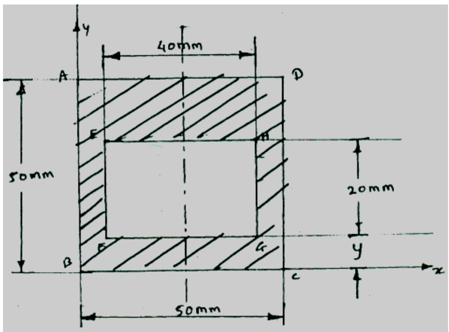
7. Replace the loading on the frame by a force and moment at point A.

(May'09) [05 M]



- **Centre of gravity and Centroid:**

1) Find the distance Y so that the C.G of the given area in fig. has co-ordinates 25,20)**(May'07) (05 M)**

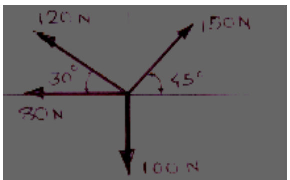


Practice Problems: (Based on University Patterns):

- **Coplanar system of forces:**

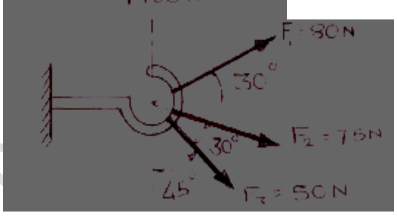
  Find the resultant system of four concurrent forces as shown in the fig. analytically.

  [Ans: R=100.7 N, α = 143.68°]



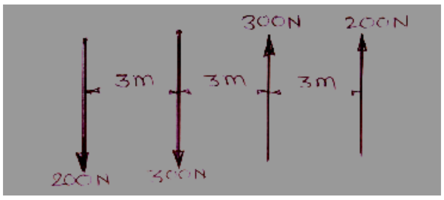
2 Determine the magnitude and direction of the resultant of the forces shown in the fig.

[Ans.: 178 N, 355°]



3 Determine the resultant of the vertical force system shown in fig.

[Ans : M=4500 N-m]



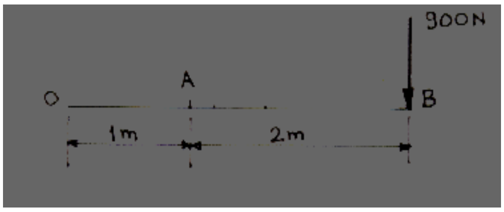
4 Resolve the force f equal to 900N acting at B , as shown in the fig. into

i.parallel components at A and 0,

i.A couple and force at O.

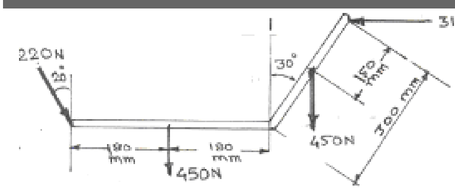
[Ans. F_A= 2700 N (↓) F_o = 1800 N (↓),

F =900 N (↓) M=2700 N-m]



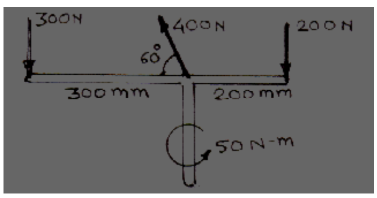
5 Determine the resultant of the forces acting on the bell crank.

[Ans. R =1131.35 N, θ =78.02°]



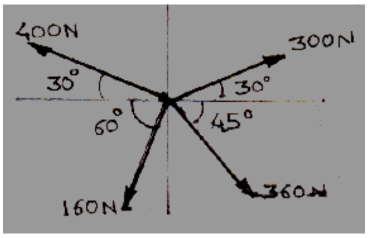
6 A bracket is subjected to a co-planer force system as shown in fig. Determine the magnitude and line of action of the resultant.

[Ans. R = 252.18 N, θ = 37.52°, x = 350 m]



7 Determine the equilibrant of the co-planer concurrent forces shown in figure.

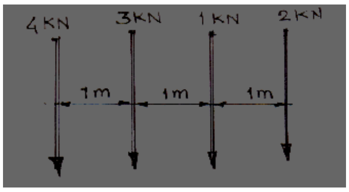
[Ans. R=97.95 N E= 97.95 N, θ =26.11°]



8

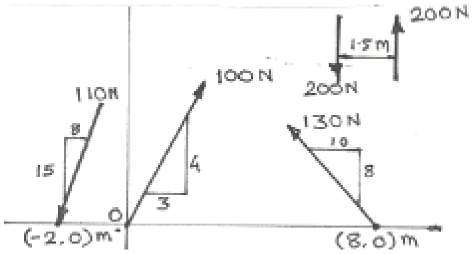
Four like parallel forces as shown in fig act on a body. Find their resultant.

[Ans. R= 10 kN ($\downarrow$), d = 1.1m to the right of 4 kN force.]

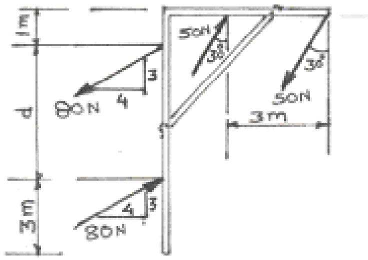


9 Find the resultant of the system of coplanar forces shown in fig.

[Ans. R =113.25 N θ = 34.51°, x = 17.83 m.]

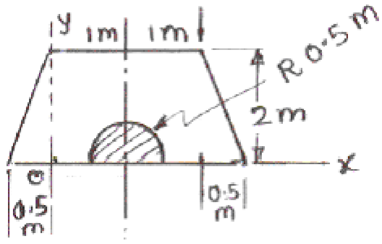


1 Two couples act on the frame. If the resultant couple moment is to be zero determine the distance d between the 80N couple forces. [Ans. d =2 m]

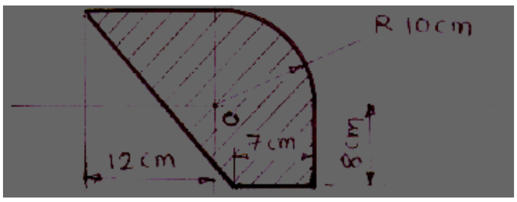


Centre of gravity and Centroid:

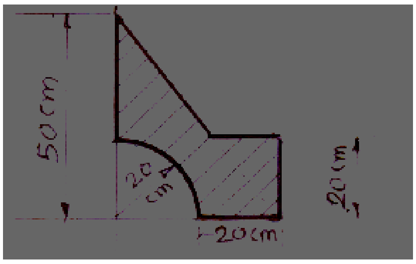
1. Locate the centroid for the cross section shown in fig. Shaded portion is removed.
[Ans.: CG(1,1)]



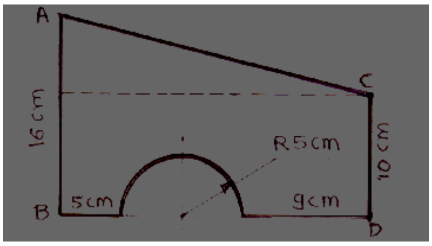
4. Determine the position of the centroid of the plane-shaded area shown in figure.
[Ans.1.130,1.671]



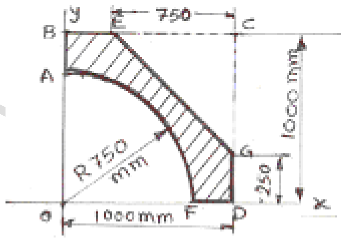
5. Determine the co-ordinate of centroid of the shaded portion shown in figure.
[Ans.19.168, 19.0738]



9. A plane lamina is hung freely from point D. Find the angle made by BD with the vertical.
[Ans.: ϴ =29.62°]



10. Determine the CG of the shaded area. FOA is a part of circle of radius R =750 mm.
[Ans.: x =y =535.8 mm]



12. Find the Centroid of the following lamina: