## *AC Circuit Containing Resistance Only (Resistive Circuit)*

Consider a circuit containing a pure resistance of $R$ Ω connected across the alternating voltage source (see Fig. 2.44).

Let the alternating voltage be given by the equation

$$\boxed{v = V_m \sin \omega t} \tag{2.3}$$

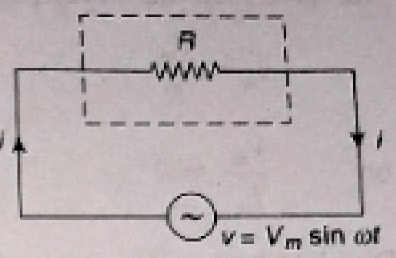


Fig. 2.44 Resistive AC circuit

As a result of this voltage, an alternating current $i$ will flow in the circuit. According to Ohm's law, we can find the equation of current $i$ as

$$i = \frac{v}{R}$$

Substituting the value of $v$, we get

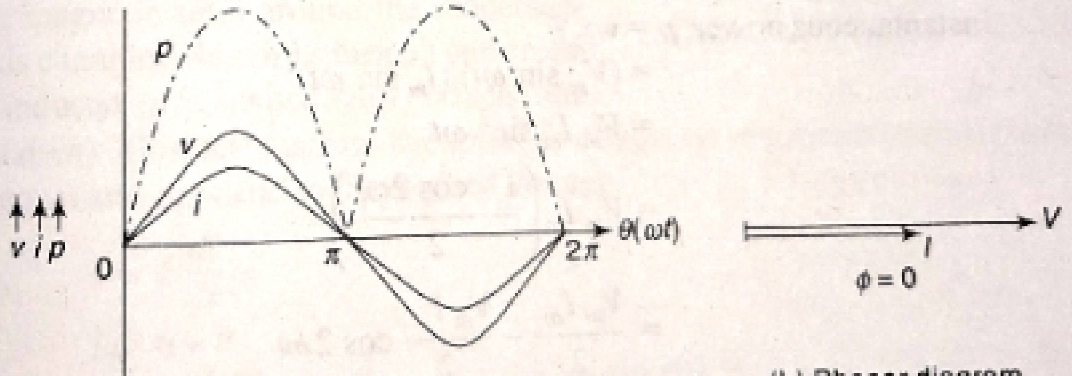
$$i = \frac{V_m}{R} \sin \omega t \tag{2.4}$$

The value of $i$ will be maximum (i.e., $I_m$) when $\sin \omega t = 1$.

So, $$I_m = \frac{V_m}{R}$$

Thus, Eq. (2.4) becomes $\boxed{i = I_m \sin \omega t}$ (2.5)

***Phase angle*** From Eqs (2.3) and (2.5), it is clear that the applied voltage and the circuit current are in phase with each other. Therefore, nature of the circuit is resistive. The waveforms of voltage and current and the corresponding phasor diagram are shown in Figs 2.45(a) and (b).



(a) Waveforms

(b) Phasor diagram

Fig. 2.45 Voltage and current in phase with each other

In the phasor diagram, the phasors are drawn in phase and there is no phase difference. The angle between voltage across the circuit and current through the circuit is known as phase angle of the circuit. So, in this case

Phase angle of the circuit, $\phi = 0$

Hence, power factor of the circuit, pf = $\cos \phi = 1$ (unity)

***Opposition to current (Z)*** In general, opposition offered by circuit elements to the current flow is called impedance ($Z$). The impedance of the circuit can be calculated as

$$\frac{V}{I} = Z \qquad \text{(by Ohm's law)}$$

We have seen that in resistive circuit,

$$I_m = \frac{V_m}{R}$$

or $$\frac{V_m}{I_m} = R$$

Dividing both numerator and denominator by $\sqrt{2}$, we get

$$\frac{\frac{V_m}{\sqrt{2}}}{\frac{I_m}{\sqrt{2}}} = R$$

or $$\frac{V}{I} = R$$

Thus, $Z = R\ \Omega$

***Power (P)*** In any circuit, electric power consumed at any instant is the product of voltage and current at that instant, i.e.,

$$p = v \times i$$

In the above equation, voltage and current are varying sinusoidally. Therefore, power is also varying w.r.t. time.

$$\begin{aligned}\text{Instantaneous power, } p &= v \times i \\ &= (V_m \sin \omega t)(I_m \sin \omega t) \\ &= V_m I_m \sin^2 \omega t \\ &= V_m I_m \left(\frac{1 - \cos 2\omega t}{2}\right) \\ &= \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t\end{aligned}$$

Since power is a scalar quantity, average power over a complete cycle is to be considered. So, we have

$$\text{Power consumed, } P = \frac{1}{2\pi} \int_0^{2\pi} p \, d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2} - \frac{V_m I_m}{2}\cos 2\omega t\right] d\omega t$$

$$= \frac{1}{2\pi}\int_0^{2\pi}\frac{V_m I_m}{2}\, d\omega t - \frac{1}{2\pi}\int_0^{2\pi}\frac{V_m I_m}{2}\cos 2\omega t\, d\omega t$$

$$= \frac{V_m I_m}{2\pi \times 2}[\omega t]_0^{2\pi} - \frac{V_m I_m}{2\pi \times 2}[\sin 2\omega t]_0^{2\pi}$$

$$= \frac{V_m I_m}{2\pi \times 2}[2\pi - 0] - \frac{V_m I_m}{2\pi \times 2}[0 - 0]$$

$$= \frac{V_m I_m}{2} - 0$$

$$= \frac{V_m}{\sqrt{2}} \times \frac{I_m}{\sqrt{2}}$$

So, $P = VI$

where $V$ = rms value of the applied voltage

$I$ = rms value of the circuit current

### AC Circuit Containing Pure Inductance Only (Pure Inductive Circuit)

Consider a circuit containing a pure inductance of $L$ henry as shown in Fig. 2.46.

An alternating voltage is applied to a pure inductance. Let the equation of the applied voltage be

$$\boxed{v = V_m \sin \omega t} \qquad (2.6)$$

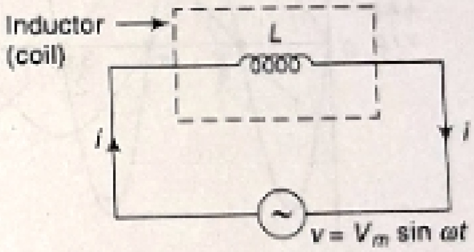



Fig. 2.46 Pure inductive circuit

As a result of this voltage, an alternating current '$i$' flows through the inductance $L$. The alternating current sets up an alternating magnetic field around the inductance. This changing flux links the coil and an emf is induced in it, called self- induced emf ($=Ldi/dt$). This emf opposes the applied voltage. At any instant, self-induced emf is equal and opposite to the applied voltage.

So, $v = L\dfrac{di}{dt}$

or $Ldi = vdt$

or $Ldi = V_m \sin \omega t\, dt \qquad (\because v = V_m \sin \omega t)$

or $di = \dfrac{V_m}{L}\sin \omega t\, dt$

Integrating both sides, we get

$$i = \frac{V_m}{L}\int \sin \omega t\, dt$$

$$i = \frac{V_m}{\omega L}(-\cos \omega t)$$

$$i = \frac{V_m}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right) \quad (2.7)$$

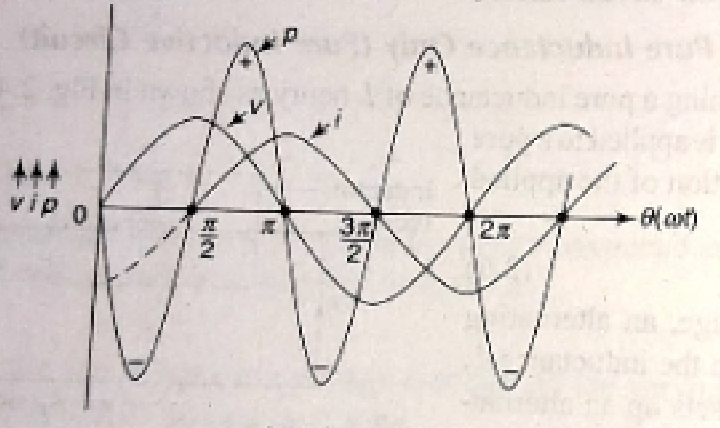
The value of $i$ will be maximum (i.e., $I_m$) when $\sin\left(\omega t - \frac{\pi}{2}\right) = 1$.

So, $\quad I_m = \dfrac{V_m}{\omega L}$

Substituting the value of $\dfrac{V_m}{\omega L} = I_m$ in Eq. (2.7), we get

$$\boxed{i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)} \quad (2.8)$$

***Phase angle and power factor*** From Eqs (2.6) and (2.8), it is clear that the current lags behind the voltage by $\pi/2$ rad or 90°. Hence, the nature of the circuit is pure inductive. The waveforms of voltage and current and the corresponding phasor diagram are shown in Figs 2.47(a) and (b).



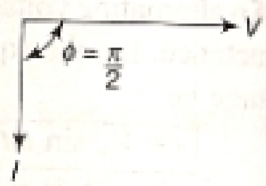




**Fig. 2.47** Current lagging behind voltage by 90°

Thus,

Phase angle of the circuit, $\phi = 90°$

Hence, power factor of the circuit, pf = $\cos \phi = \cos 90° = 0$ lagging

*Note:* Power factor has magnitude and nature. Magnitude of the power factor equals to $\cos \phi$. The nature of the power factor is same as nature of current. In this case, circuit current lags to the applied voltage, therefore nature of the power factor is lagging.

***Opposition to current (Z)*** We have seen that

$$I_m = \frac{V_m}{\omega L}$$

or $\dfrac{V_m}{I_m} = \omega L$

Dividing both numerator and denominator by $\sqrt{2}$,

$$\frac{\frac{V_m}{\sqrt{2}}}{\frac{I_m}{\sqrt{2}}} = \omega L$$

So, $\dfrac{V}{I} = \omega L$

or $Z = \omega L \ \Omega$

Thus, opposition offered by inductance to current flow is $\omega L$. This quantity $\omega L$ is called inductive reactance $X_L$ of the coil.

So, $X_L = \omega L \ \Omega$

or $X_L = 2\pi f L \ \Omega$

*Note:* There are two reactive elements: inductor and capacitor. The opposition offered by the reactive element to current flow is called reactance (denoted by $X$). Thus, the opposition offered by inductor to current flow is called inductive reactance (denoted by $X_L$), and the opposition offered by capacitor to current flow is called capacitive reactance (denoted by $X_C$).

***Power*** Instantaneous power,

$$p = vi$$

$$= V_m \sin \omega t \times I_m \sin\left(\omega t - \frac{\pi}{2}\right)$$

$$= -V_m I_m \sin \omega t \cos \omega t$$

$$= -\frac{V_m I_m}{2} \sin 2\omega t$$

Since power is a scalar quantity, average power over a complete cycle is to be considered.

So, Average power, $P$ = Average of $p$ over one cycle

or $$P = \frac{1}{2\pi}\int_0^{2\pi} p \, d\omega t$$

or $$P = \frac{1}{2\pi}\int_0^{2\pi} -\frac{V_m I_m}{2} \sin 2\omega t \, d\omega t$$

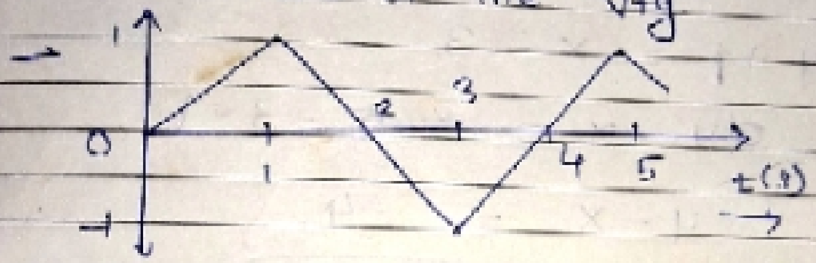
or $P = 0$

Hence, power absorbed in pure inductance is zero.

Figure 2.47(a) shows the power curve. During the first 90° of the cycle, the voltage is +ve and the current is –ve. Therefore, the power supplied is negative. It

Problems -

1) Through a coil of inductance 1 henry a current of the waveform shown in fig(a) is flowing. Sketch the waveform of the vtg across the inductance and calculate the r.m.s value of the vtg




$\frac{y_2 - y_1}{x_2 - x_1} = \frac{1-0}{1-0}$

$m = 1$

$(y - y_1) = m(x - x_1)$

$y = 1x$

$i = t$

→ The instantaneous current i(t) is given by.

i) $0 < t < 1$    Slop eqⁿ = $(y - y_1) = m(x - x_1)$

$(y-1) = 1(x-1)$

$(0,0), (1,1)$    $m = \frac{y_2 - y_1}{x_2 - x_1}$    $y = x$    $\boxed{i = t}$ A

$\frac{0-1}{0-1} = m$

$m = 1$

time = x
i = y    $\boxed{i = t}$    $\boxed{i = t}$

$\frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-1}{3-1} = \frac{-2}{2} = -1$

(2) $1 < t < 3$    $(1, 1)$    $(3, -1)$

$(y - 1) = \frac{-1-1}{3-1}(x - 1)$

$y - 1 = \frac{-2}{2}(x - 1)$

$y = 2 - x$    $i = 2 - t$    $y - 1 = -x + 1$    $i = 2 - x$    $\boxed{(i = 2 - t)A}$

$x + y = 2$

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| 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| 27 | 28 | 29 | 30 | | | |

Important Notes

(3) $3 < t < 4$ $\quad (3,-1) \quad (4,0)$

$\quad\quad\quad\quad\quad\quad\quad\quad x_1\, y_1 \quad\quad x_2\, y_2$

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0-(-1)}{4-3} = 1/1 = 1$$

$$(y - y_1) = m(x - x_1)$$

$$y - (-1) = 1(x-3)$$

$$y + 1 = 1x - 3$$

$$y + 1 = x - 3$$

$$y - x = \cancel{-3-1} -3 - 1$$

$$y - x = -4$$

$$\boxed{i = t - 4\ A}$$

The corresponding Vtgs are,

$L = 1H$

(1) $V_1 = L\frac{di}{dt} = \cancel{1\times 1 = 1V}\ L\times\frac{d}{dt}(t) = 1\times 1 = 1V$

(2) $V_1 = L\frac{di}{dt} = L\frac{d}{dt}(2-t) = 1\times -1 = -1V$

(3) $V_1 = L\frac{di}{dt} = L\frac{d}{dt}(t-4) = 1\times 1 = 1V$

Important Notes

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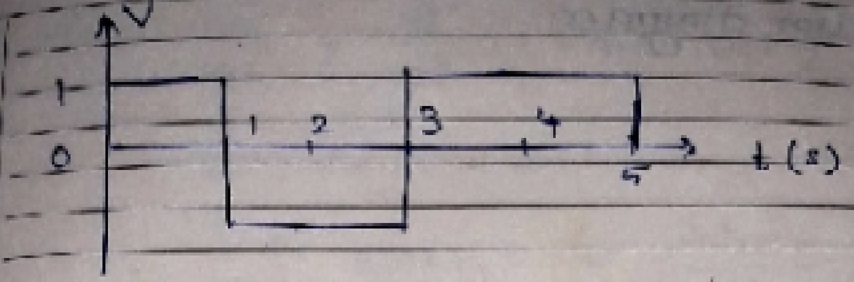



2) A 60 Hz voltage of 230 V, effective value is impressed on an inductance of 0.265 H

(i) write the time equation for the voltage and the resulting current. let the zero axis of the voltage wave be at t = 0

(ii) show the voltage and current on a phasor diagram.

(iii) Find the maximum energy stored in the inductance.

→ $V_{max} = \sqrt{2}V = \sqrt{2} \times 230 V$ , $f = 60 Hz$

$\omega = 2\pi f = 2\pi \times 60 = 377$ rad/sec.

$X_L = \omega L = 2\pi f L = 377 \times 0.265 = 100 \Omega$

(i) The time eq$^n$ for voltage is

$v(t) = 230\sqrt{2} \sin 377 t$

$I_{max} = V_{max} / X_L = 230\sqrt{2}/100 = 2.3\sqrt{2}$

$\phi = 90°$ (lag)

∴ Current eq$^n$ $i(t) = 2.3\sqrt{2} \sin(377t - \pi/2)$

$i(t) = 2.3\sqrt{2} \cos 377 t$

(iii) $E_{max} = \frac{1}{2} L I^2_{max} = \frac{1}{2} \times 0.265 \times (2.3\sqrt{2})^2$

$= 1.4 J$

### AC Circuit Containing Pure Capacitance Only (Pure Capacitive Circuit)

Consider an alternating voltage applied to a capacitor of capacitance $C$ F as shown in Fig. 2.48. Let the equation of the applied alternating voltage be

$$\boxed{v = V_m \sin \omega t} \qquad (2.9)$$

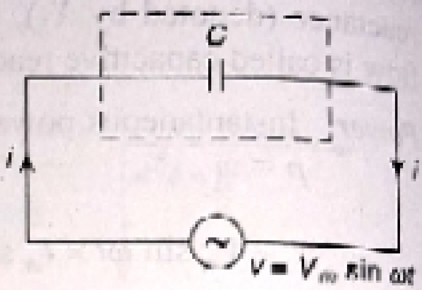



Fig. 2.48 Pure capacitive circuit

As a result of this alternating voltage, alternating current will flow through the circuit. Let at any instant, $i$ is the current and $q$ is the charge on the plates.

Charge on capacitor, $q = Cv$

So, Circuit current, $i = \dfrac{dq}{dt}$

$$= \frac{dCv}{dt}$$

$$= \frac{d}{dt}(C\,V_m \sin \omega t)$$

$$= \omega C\,V_m \cos \omega t$$

or $$i = \omega C V_m \sin\left(\omega t + \frac{\pi}{2}\right) \qquad (2.10)$$

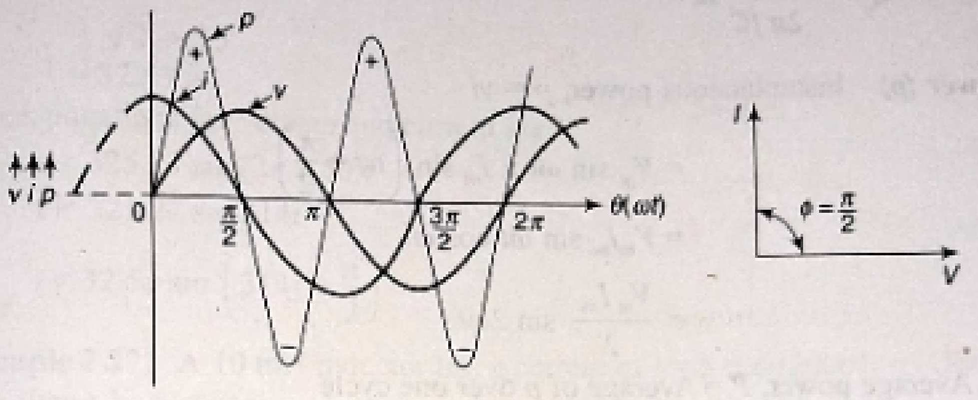
The value of $i$ will be maximum (i.e., $I_m$) when $\sin\left(\omega t + \dfrac{\pi}{2}\right)$ is unity.

So, $I_m = \omega C V_m$

Substituting the value $\omega C V_m = I_m$ in Eq. (2.10), we get

$$\boxed{i = I_m \sin\left(\omega t + \frac{\pi}{2}\right)} \tag{2.11}$$

***Phase angle and power factor*** From Eqs (2.9) and (2.11), it is clear that the current leads the voltage by $\pi/2$ rad or 90°. Hence, in a pure capacitance, the current leads the voltage by 90°. The nature of the circuit is said to be pure capacitive. The waveforms of voltage and current and the corresponding phasor diagram are shown in Figs 2.49(a) and (b).




(a) Waveforms (b) Phasor diagram

Fig. 2.49 Current leads voltage by 90°

Thus, Phase angle of the circuit, $\phi = 90°$

So, power factor of the circuit, pf = $\cos\phi = \cos 90° = 0$ leading

***Opposition to current (Z)*** We have seen that

$$I_m = \omega C V_m$$

or $$\frac{V_m}{I_m} = \frac{1}{\omega C}$$

Dividing both numerator and denominator by $\sqrt{2}$,

$$\frac{\frac{V_m}{\sqrt{2}}}{\frac{I_m}{\sqrt{2}}} = \frac{1}{\omega C}$$

So, $$\frac{V}{I} = \frac{1}{\omega C}$$

or $$Z = \frac{1}{\omega C}\ \Omega$$

Thus, opposition offered by capacitance to current flow is $\frac{1}{\omega C}$. This quantity $\frac{1}{\omega C}$ is called capacitive reactance $X_C$ of the capacitor.

So, $\quad X_C = \frac{1}{\omega C}\ \Omega$

So, $\quad X_C = \frac{1}{2\pi fC}\ \Omega$

***Power (p)*** Instantaneous power, $p = vi$

$$= V_m \sin \omega t \times I_m \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$= V_m I_m \sin \omega t \cos \omega t$$

$$= \frac{V_m I_m}{2} \sin 2\omega t$$

So, Average power, $P$ = Average of $p$ over one cycle

or $\quad P = \frac{1}{2\pi}\int_0^{2\pi} p\, d\omega t$

or $\quad P = \frac{1}{2\pi}\int_0^{2\pi}\left[\frac{V_m I_m}{2}\sin 2\omega t\right] d\omega t$

or $\quad P = 0$

Hence, power absorbed in a pure capacitance is zero.

Figure 2.49(a) shows the power curve. The power curve is similar to that for a pure inductor because now the current leads the voltage by 90°. It is clear that positive power is equal to the negative power over one cycle. Hence, the net power absorbed in a capacitor is zero. When voltage rises across the plates of a capacitor, energy is required to build up electrostatic field between the plates of the capacitor. This energy is supplied from the source and is stored in the capacitor in the form of electrostatic field energy. As the voltage falls, the collapsing electrostatic field returns the stored energy to the source. Since the power supplied is equal to the power returned (positive areas being equal to the negative areas) over one cycle, the net power absorbed in a capacitor is zero.

---

**Example 2.26** A 318 μF capacitor is connected across a 230 V, 50 Hz system. Determine (i) capacitive reactance, (ii) rms value of current, and (iii) equations for voltage and current.

***Solution***

(i) Capacitive reactance, $X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 318 \times 10^{-6}} = 10\ \Omega$

(ii) The rms value of current, $I = \frac{V}{Z} = \frac{V}{X_C} = \frac{230}{10} = 23$ A

(iii) $V_m = \sqrt{2} \times V$

$= \sqrt{2} \times 230$

$= 325.27$ V

and $I_m = \sqrt{2} \times I$

$= \sqrt{2} \times 23$

$= 32.53$ A

Hence, equations for voltage and current are:

$v = 325.27 \sin (2\pi \times 50)t$

$v = 325.27 \sin 314t$

$i = 32.53 \sin \left(314t + \frac{\pi}{2}\right)$

**Example 2.27** A 10 mH inductor has a current of $i = 5 \cos (2000t)$ A. Obtain the voltage $V_L$ across it.

***Solution***

Given $L = 10 \times 10^{-3}$ H

and $i = 5 \cos (2000t)$

Converting the current equation into standard sinusoidal form,

$$i = 5 \sin \left(2000t + \frac{\pi}{2}\right) \qquad \text{(i)}$$

From Eq. (i), $\omega = 2000$ rad/sec and $\phi = \frac{\pi}{2}$ rad $= 90°$

The rms value of current, $I = \frac{5}{\sqrt{2}} = 3.54$ A

Now, $X_L = \omega L = 2000 \times 10 \times 10^{-3} = 20\ \Omega$

So, $V_L = IX_L = 3.54 \times 20 = 70.8$ V

*Alternative method:*

The equation of the voltage across the inductor ($v_L$) can be calculated as

$$v_L = L\frac{di}{dt}$$

$$= 10 \times 10^{-3} \frac{d}{dt} [5 \cos (2000t)]$$

$$= -10 \times 10^{-3} \times 5 \times 2000 \sin 2000t$$

or $\quad v_L = 100 \sin(2000t + 180°)$

Comparing with standard sinusoidal form,

$$V_m = 100 \text{ V}$$

So, $\quad V_L = \frac{100}{\sqrt{2}} = 70.71 \text{ V}$

**Example 2.28** A voltage of 150 V, 50 Hz is applied to a coil of negligible resistance and inductance 0.2 H. Write the time equation for voltage and current.

***Solution***

Given: $V = 150$ V

So, $\quad V_{max} = \sqrt{2}\,V = \sqrt{2} \times 150 = 212.13 \text{ V}$

Supply frequency, $f = 50$ Hz

Inductance, $L = 0.2$ H

Inductive reactance, $X_L = 2\pi fL = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$

In a pure inductive coil,

if applied voltage, $v = V_m \sin \omega t$,

then $\quad i = I_m \sin\left(\omega t - \frac{\pi}{2}\right)$

The rms value of current, $I = \frac{V}{X_L} = \frac{150}{62.83} = 2.39 \text{ A}$

So, $\quad I_m = \sqrt{2}\,I = \sqrt{2} \times 2.39 = 3.38 \text{ A}$

Now, $\quad v = 212.13 \sin \omega t$

or $\quad v = 212.13 \sin 2\pi ft$

Hence, $\quad v = 212.13 \sin 314t$

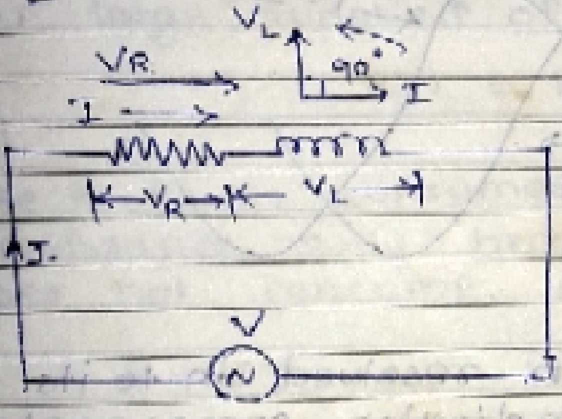
and $\quad i = 3.38 \sin\left(314t - \frac{\pi}{2}\right)$

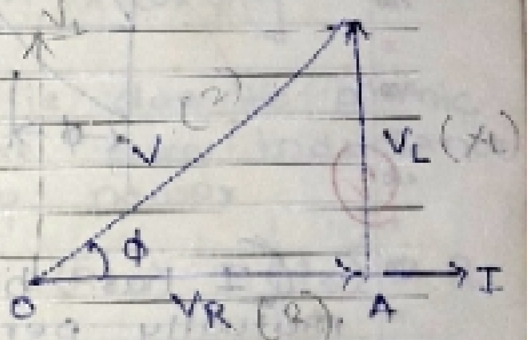
# A.C. through Resistance & Inductance (R-L)

A pure resistance R and a pure inductance L are connected in series.

V – r.m.s value of the applied V/g
I – " " " current

$V_R = IR$ Voltage drop across R (in phase with I)
$V_L = I \cdot X_L$ " " coil (ahead of I by 90°)




$$\vec{V} = \sqrt{(V_R^2 + V_L^2)} = \sqrt{(IR)^2 + (I \cdot X_L)^2} = I\sqrt{R^2 + X_L^2}$$

$$I = \frac{V}{\sqrt{R^2 + X_L^2}} \quad \cdots \quad \left(Z = \sqrt{R^2 + X_L^2}\right)$$

$Z = R + jX_L$ $\quad Z = Z\angle\phi$

Applied voltage leads the current I by an angle $\phi$

$$\tan\phi = \frac{V_L}{V_R} = \frac{I \cdot X_L}{I \cdot R} = \frac{X_L}{R} = \frac{\omega L}{R}$$

$$\boxed{\phi = \tan^{-1}(X_L/R)}$$

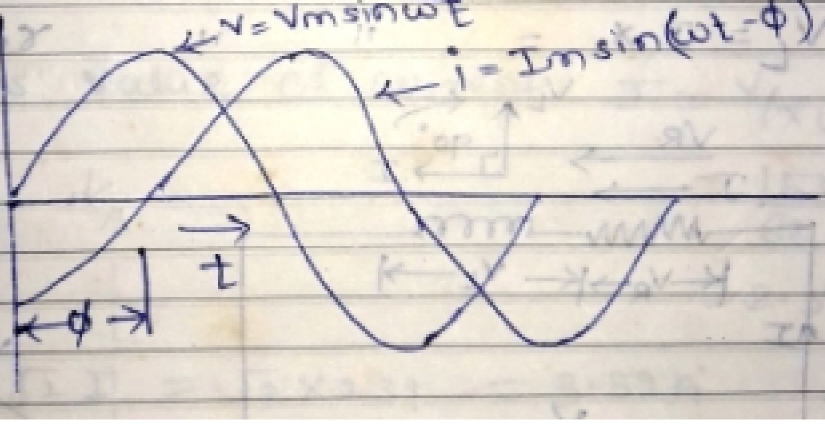
If the applied voltage v is given by

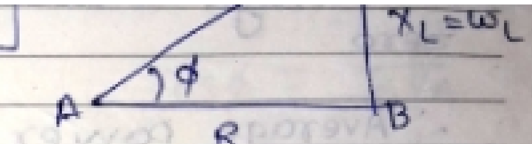
$$\boxed{v = V_m \sin \omega t}$$

The current eqⁿ is,

$$\boxed{i = I_m \sin(\omega t - \phi)}$$

where, $\boxed{I_m = V_m / Z}$






## Instantaneous Power

$$= vi = V_m \sin \omega t \times I_m \sin(\omega t - \phi)$$

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| 20 | 21 | 22 | 23 | 24 | 25 | 26 |
| 27 | 28 | 29 | 30 | | | |

$$= V_m I_m \sin \omega t \, \sin(\omega t - \phi)$$

$$\left[ \sin \alpha \sin \beta = \frac{1}{2}\left[\cos(\alpha + \beta) - \cos(\alpha - \beta)\right] \right]$$

① Power factor - (cosine of angle of lead or lag)

According to Impedance triangle.

Ratio $= \frac{R}{Z} = \frac{\text{resistance}}{\text{Impedance}}$

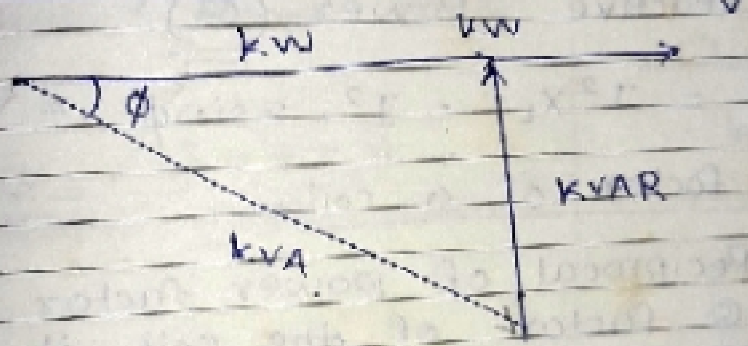
$PF = R/Z$

Ratio $= \frac{\text{true Power}}{\text{Apparent Power}} = \frac{\text{Watts}}{\text{Volt Amperes.}}$

According to Power triangle

$PF = \frac{P}{S} = \frac{W}{VA}$

* Active & Reactive components of circuit current I -




→ Active component is that which is in phase with the applied voltage V i.e. $I\cos\phi$ ie kW.

$\cos\phi = \frac{kW}{kVA}$

$kW = kVA \cos\phi$

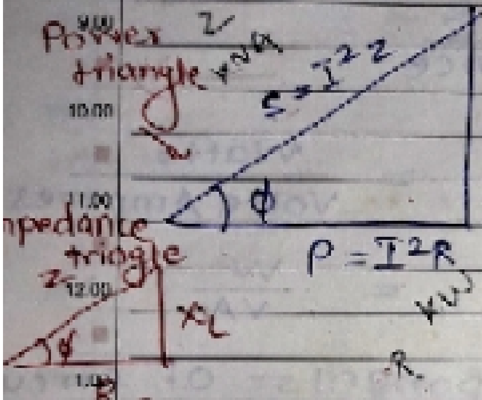
→ Reactive component is that which in quadrature with V ie. $I\sin\phi$ ie kVAR.

$\sin\phi = \frac{kVAR}{kVA}$

$kVAR = kVA \sin\phi$

$kVA = \sqrt{kW^2 + kVAR^2}$

## * Active, reactive & Apperent Power -




(i) Apperent Power

$S = VI = (IZ) \cdot I$

$= I^2 Z$

(ii) Active Power (P or W)

$P = I^2 R \cdot VI \cos\phi$

$\frac{V}{Z} \cdot IR = VI \frac{R}{Z}$

$= VI \cos\phi$

$\cos\phi = R/Z \quad \sin\phi = X_L/Z$

(iii) Reactive Power (Q)

$Q = I^2 X_L = I^2 \cdot Z\sin\phi = VI\sin\phi$

$= V/_Z I Z \sin\phi \quad = VI\sin\phi$

## * Q factor of a coil -

Reciprocal of power factor is called the Q factor of the coil it is also known as quality factor of the coil

$$\boxed{Q \text{ factor} = \frac{1}{\text{Power factor}} = \frac{1}{\cos\phi} = \frac{Z}{R}}$$

## Problems

① (i) In a series circuit containing pure resistance & pure inductance, the current and the voltage are expressed as,

$i(t) = 5 \sin(314t + 2\pi/3)$

$v(t) = 15 \sin(314t + 5\pi/6)$

(a) what is the impedance of the ckt?
(b) what is the value of the resistance,
(c) what is the inductance in henrys?
(d) what is the average power drawn by the ckt?
(e) what is the power factor?

→ Phase angle of the current

$= 2\pi/3 = \frac{2 \times 180}{3} = 120^\circ$

Phase angle of the voltage

$= 5\pi/6 = \frac{5 \times 180^\circ}{6} = 150^\circ$

$Z = V_m / I_m = 3\Omega$

Current lags behind Voltage by $30^\circ$.

$\omega = 2\pi f \qquad 314 = 2\pi f \qquad f = 50Hz$

$R/Z = \cos 30 = 0.866$

Important Notes

$R = 2.6\Omega \qquad X_L/Z = \sin 30^\circ = 0.5$

$X_L = 1.5\Omega$

(1) $2\pi fL = 1.5$

$L = 4.18\, mH$

$P = I^2 R = (5/\sqrt{2})^2 \times 2.6$

$\boxed{P = 32.5\, W}$

[Convert Polar & see $Z\angle -\phi$]

~~$pf = \cos 30° = 0.866 (lag)$~~

2) The potential difference measured across a coil is 4.5V, when it carries a direct current of 9A. The same coil when carrying an alternating current of 9A at 25Hz, the potential difference is 24V. Find the current, Power & Power factor when it is supplied by 50V, 50 Hz supply.

→ Let R be the d.c. resistance & L be the inductance of the coil.

$R = V/I = 4.5/9 = 0.5\, \Omega$

With a.c. current of 25 Hz, $Z = V/I = 24/9$

$= 2.66\, \Omega$

$X_L = \sqrt{Z^2 - R^2} = \sqrt{2.66^2 - 0.5^2}$

$= 2.62\, \Omega$

$X_L = 2\pi f L \qquad L = 0.0167 \text{ Henry}$

At 50 Hz, $2\pi \times 25 \times 2 \times L$

$2\pi \times f L$

$X_L = 2.62 \times 2 = 5.24\,\Omega$

$Z = \sqrt{0.5^2 + 5.24^2} = 5.26\,\Omega$

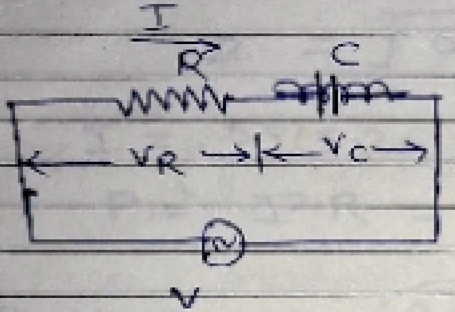
$I = 50/5.26 = 9.5\,A$

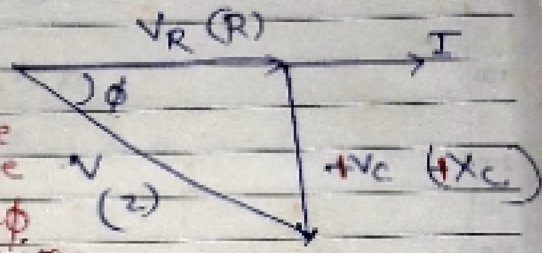
$P = I^2 R = 9.5^2 \times 0.5 = 45\,W$

## A.C. through Resistance & capacitance –

$V_R = IR$ = drop across R in phase with I

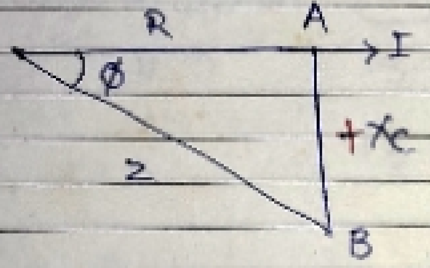
$V_C = I X_C$ = drop across capacitor which is lagging I by $\pi/2$




Phase angle $= -\phi$

$Z = R - jX_C$




$$V = \sqrt{V_R^2 + (+V_C)^2}$$

$$= \sqrt{(IR)^2 + (+IX_C)^2}$$

$$= I\sqrt{R^2 + X_C^2}$$

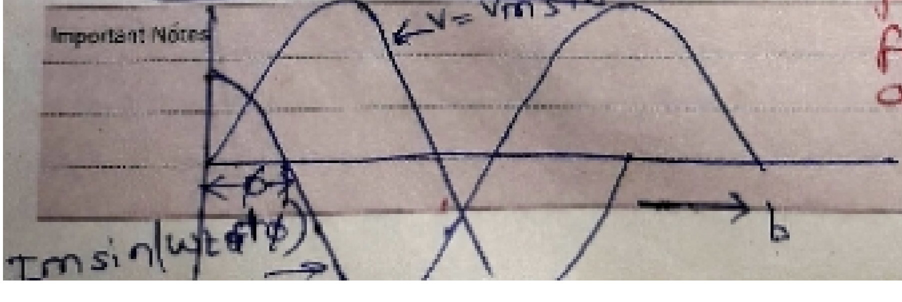
$$I = \frac{V}{\sqrt{R^2 + X_C^2}} = \frac{V}{Z} \qquad Z = \sqrt{R^2 + X_C^2}$$

I leads V by angle φ

$Z = R - jX_C$

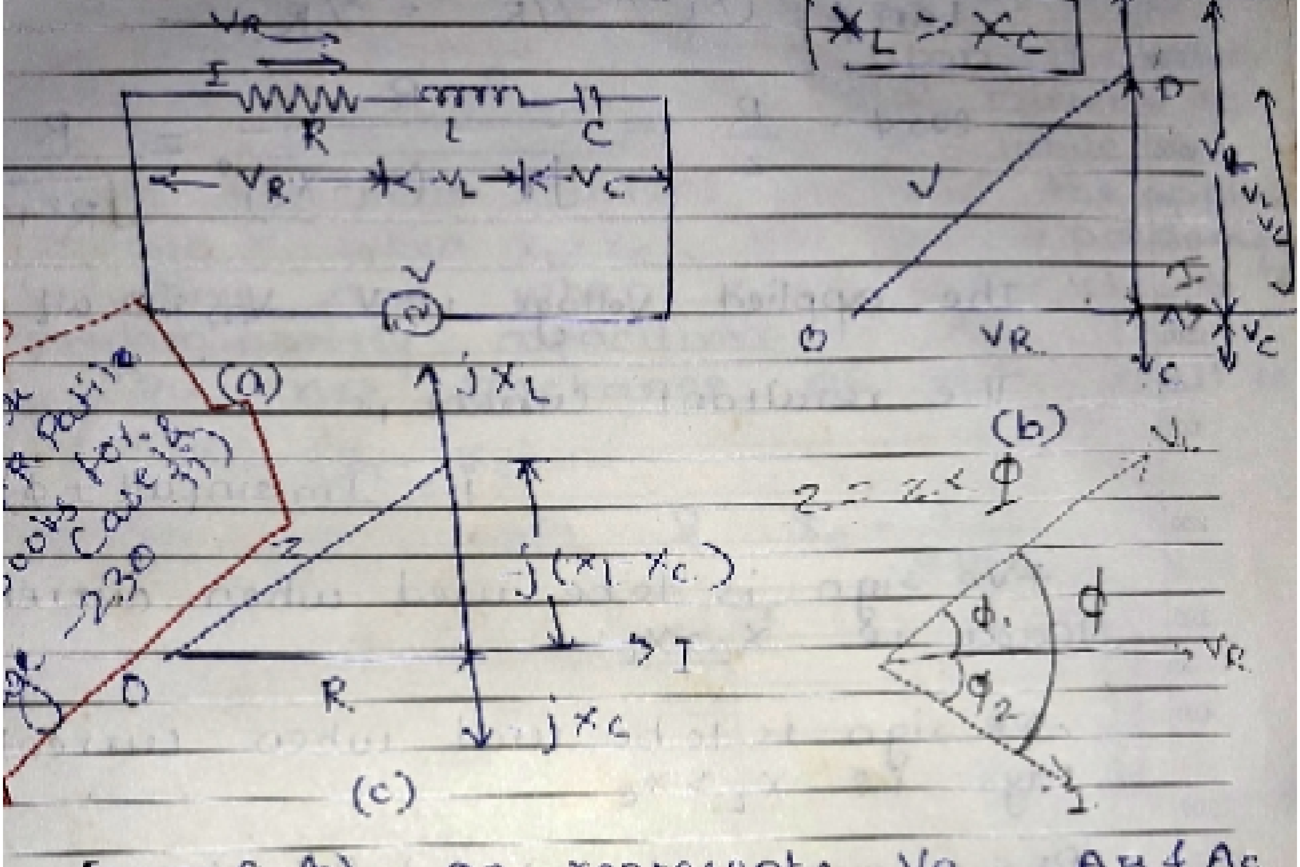
$\bar{Z} = Z\angle -\phi$

$$\boxed{\tan\phi = +X_C/R}$$




Power & Power factors are same like RL

RLC in series -




From fig (b) OA represents $V_R$, AB & AC represent the inductive and capacitive drop respectively. Here $V_L$ & $V_C$ are 180° out of phase with each other i.e. they are in direct opposition to each other.

$OD^2 = OA^2 + AD^2$ $\qquad$ $OD = \sqrt{OA^2 + AD^2}$

$$V = \sqrt{(IR)^2 + (IX_L - IX_C)^2}$$

$$= I\sqrt{R^2 + (X_L - X_C)^2}$$

$$I = \frac{V}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{V}{\sqrt{R^2 + X^2}} =$$

$\tan\phi = (X_L - X_C)/R = X/R$

$$\cos\phi = \frac{R}{Z} = \frac{R}{\sqrt{R^2 + (X_L - X_C)^2}} = \frac{R}{\sqrt{R^2 + X^2}}$$

$\therefore$ The applied voltage is $V = V_m \sin \omega t$

The resultant current is

$$i = I_m \sin(\omega t \pm \phi)$$

+ve sign is to be used when current leads i.e $X_C > X_L$

-ve sign is to be used when current lags i.e $X_L > X_C$

Phase angle $\phi = \tan^{-1}[X_L - X_C / R]$

$Z = Z \angle \tan^{-1}[(X_L - X_C)/R] = Z \angle \tan^{-1}(X/R)$

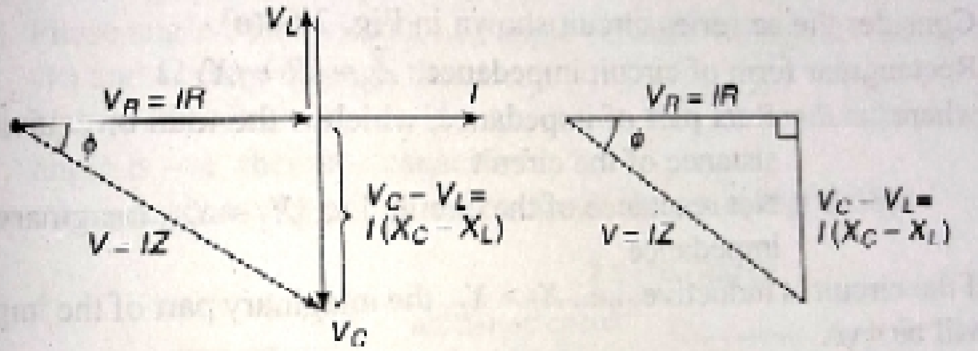
if $V = V \angle 0$ then $I = V/Z$.

### Case (ii) $X_C > X_L$

If $X_C > X_L$, then $V_C > V_L$. The inductive effect gets neutralized and the circuit behaves like a *R-C* circuit.

***Phasor diagram*** Taking current as reference phasor, the phasor diagram of the circuit can be drawn as shown in Fig. 2.66(a).

From circuit diagram, $\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$




(a) Phasor diagram (b) Voltage triangle

Fig. 2.66

It is clear from the phasor diagram that the current $I$ leads the applied voltage $V$ by $\phi°$ ($\phi° < 90°$). Therefore, nature of the circuit is capacitive.

Phase angle of the circuit, $\phi = \tan^{-1}\dfrac{V_C - V_L}{V_R} = \tan^{-1}\dfrac{I(X_C - X_L)}{IR}$

$$= \tan^{-1}\left(\frac{X_C - X_L}{R}\right)$$

So, power factor of the circuit, $\text{pf} = \cos\phi = \frac{V_R}{V} = \frac{IR}{IZ} = \frac{R}{Z}$ leading

***Impedance (Z)***

The voltage triangle of the circuit is shown in Fig. 2.66(b). Dividing each of voltage phasor by $I$, we get the impedance triangle as shown in Fig. 2.67. From the impedance triangle, circuit impedance,

$$Z = \sqrt{R^2 + (X_C - X_L)^2}\ \Omega$$

The impedance can be expressed in rectangular and polar forms as

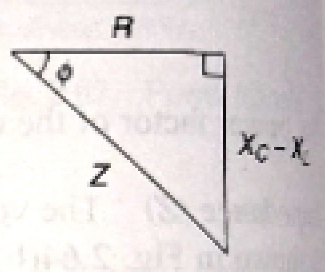



Fig. 2.67 Impendance triangle (when $X_c > X_L$)

$$\bar{Z} = [R - j(X_C - X_L)]\ \Omega$$

and $$\bar{Z} = (Z \angle -\phi)\ \Omega$$

**Example 2.29** A coil having a resistance of 7 Ω and an inductance of 31.8 mH is connected to a 230 V, 50 Hz supply. Calculate (i) circuit current, (ii) phase angle, (iii) power factor, and (iv) power consumed.

***Solution***

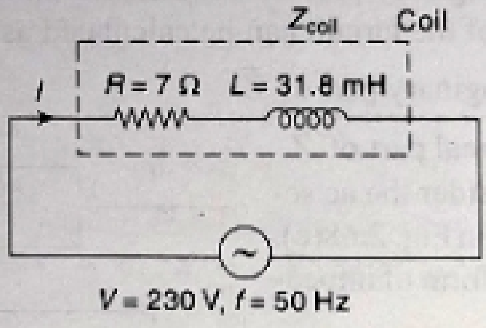



**Fig. 2.69**

We have Inductive reactance, $X_L = 2\pi f L = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\ \Omega$

Impedance of coil, $Z_{coil} = \sqrt{R^2 + X_L^2} = \sqrt{(7)^2 + (10)^2} = 12.21\ \Omega$

(i) By Ohm's law,

Circuit current, $I = \dfrac{V}{Z_{coil}} = \dfrac{230}{12.21} = 18.84$ A

(ii) Phase angle, $\phi = \tan^{-1}\dfrac{X_L}{R} = \tan^{-1}\dfrac{10}{7} = 55°$

(iii) Power factor, PF = $\cos\phi = \cos 55° = 0.574$ lagging
(As circuit is inductive, pf is lagging.)

(iv) Power consumed, $P = VI\cos\phi$
$= 230 \times 18.84 \times 0.574 = 2487.26$ W

*Alternative method (by using phasor algebra)*

Taking applied voltage as reference,

$\bar{V} = (230 \angle 0)$ V

$\bar{Z} = (7 + j10)\ \Omega$

So, $\bar{Z} = (12.21 \angle 55)\ \Omega$

(i) By Ohm's law,

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{(230\angle 0)}{(12.21\angle 55)} = (18.84 \angle -55)\text{ A}$$

(ii) From polar form of impedance,
phase angle, $\phi = 55°$

(iii) Power factor, pf = $\cos\phi = \cos 55° = 0.574$ lagging

(iv) Power consumed, $P = VI\cos\phi = 230 \times 18.84 \times 0.574 = 2487.26$ W

**Example 2.30** For a circuit in Fig. 2.70, determine the (i) circuit impedance (ii) circuit current (iii) power factor (iv) active power (v) reactive power (vi) apparent power.

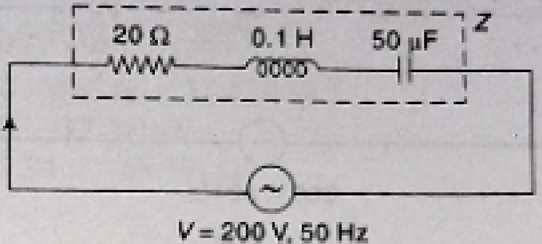



Fig. 2.70

***Solution***

Given: $V = 200$ V, $f = 50$ Hz, $R = 20\ \Omega$

$L = 0.1\ \text{H} \Rightarrow X_L = 2\pi f L = 31.42\ \Omega$

$$C = 50 \times 10^{-6}\ \text{F} \Rightarrow X_C = \frac{1}{2\pi f C} = 63.66\ \Omega$$

Net reactance, $X = X_C - X_L = 32.24\ \Omega$

As $X_C > X_L$, circuit is capacitive.

Taking applied voltage as reference,

$$\bar{V} = (200 \angle 0)\ \text{V}$$

$$\bar{Z} = (20 - j32.24)\ \Omega$$

or $$\bar{Z} = (37.94 \angle -58.19)\ \Omega$$

(i) From polar form of impedance, $Z = 37.94\ \Omega$

(ii) By Ohm's law,

$$\text{Circuit current, } \bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{(200\angle 0)}{(37.94\angle -58.19)} = (5.27 \angle 58.19)\ \text{A}$$

(iii) Power factor, pf $= \cos\phi$
$= \cos(-58.19°)$
$= 0.527$ leading

[As circuit is capacitive ($X_C > X_L$), pf is leading]

(iv) Active power, $P = VI\cos\phi$
$= 200 \times 5.27 \times \cos(-58.19°)$
$= 555.46$ W

(v) Reactive power, $Q = VI\sin\phi$
$= 200 \times 5.27 \times \sin(-58.19°)$
$= -895.69$ VAR

(vi) Apparent power, $S = VI$

$$= 200 \times 5.27$$
$$= 1054 \text{ VA}$$

**Example 2.31** For a circuit shown in Fig. 2.71, determine (i) circuit current, (ii) voltage drop $V_1$, and (iii) voltage drop $V_2$.

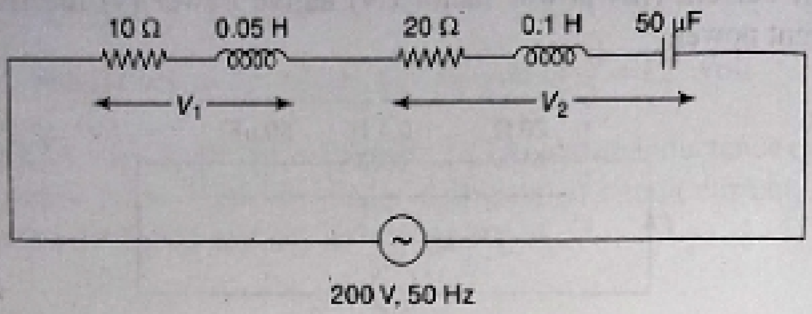



Fig. 2.71

***Solution***

Let us redraw then the given circuit and assume the different unknown quantities. We then get the circuit as shown in Fig. 2.72.

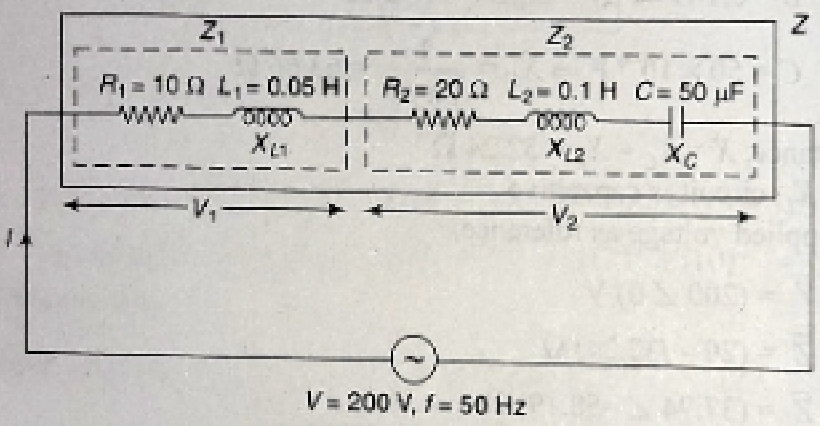



Fig. 2.72

Taking applied voltage as reference,

$\bar{V} = (200 \angle 0)$ V, $\quad R = R_1 + R_2 = 10 + 20 = 30\ \Omega$

$L_1 = 0.05 \text{ H} \Rightarrow X_{L_1} = 2\pi f L_1 = 15.71\ \Omega$

$L_2 = 0.1 \text{ H} \Rightarrow X_{L_2} = 2\pi f L_2 = 31.42\ \Omega$

So, $\quad X_L = X_{L_1} + X_{L_2} = 47.13\ \Omega$

$$C = 50 \times 10^{-6} \text{ F} \Rightarrow X_C = \frac{1}{2\pi f C} = 63.66\ \Omega$$

Net reactance, $X = X_C - X_L = 16.53\ \Omega$

As $X_C > X_L$, circuit is capacitive.

Total impedance of the circuit,

$\bar{Z} = (30 - j16.53)\ \Omega$

$\bar{Z} = (34.25 \angle -28.85)\ \Omega$

By Ohm's law,

Circuit current, $\bar{I} = \dfrac{\bar{V}}{\bar{Z}} = \dfrac{200 \angle 0}{(34.25 \angle -28.85)} = (5.84 \angle 28.85)$ A

From circuit diagram,

Impedance, $\bar{Z}_1 = (R_1 + j X_{L_1})\ \Omega$

$= (10 + j15.71)\ \Omega$

$= (18.62 \angle 57.52)\ \Omega$

Impedance, $\bar{Z}_2 = [R_2 - j(X_C - X_{L_2})]\ \Omega$

$= [20 - j(63.66 - 31.42)]\ \Omega$

$= (20 - j32.24)\ \Omega$

$= (37.94 \angle -58.19)\ \Omega$

By Ohm's law,

$$\bar{V}_1 = \bar{I}\bar{Z}_1 = (5.84 \angle 28.85)(18.62 \angle 57.52) = (108.74 \angle 86.37) \text{ V}$$

and $$\bar{V}_2 = \bar{I}\bar{Z}_2 = (5.84 \angle 28.85)(37.94 \angle -58.19) = (221.57 \angle -29.34) \text{ V}$$

**Example 2.32** An rms voltage of $(100 \angle 0)$ V is applied to series combination of $Z_1$ and $Z_2$, where $\bar{Z}_1 = (20 \angle 30°)\Omega$. The effective voltage drop across $Z_1$ is known to be $(40 \angle -30°)$ V. Find the reactive component of $Z_2$.

***Solution***

The conditions of the example are shown in Fig. 2.73.

Given:

$\bar{V} = (100 \angle 0)$ V

$\bar{Z}_1 = (20 \angle 30)\ \Omega$

$\bar{V}_1 = (40 \angle -30)\ \Omega$

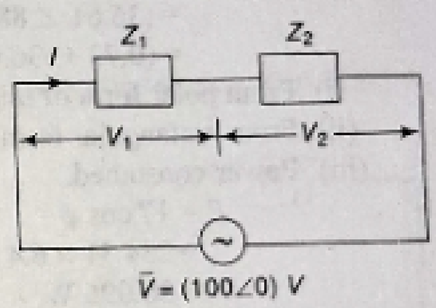



Fig. 2.73

*Required:* Reactive component of $Z_2$

For reactive component (imaginary part) of $Z_2$, the rectangular form of $Z_2$ is required. By using Ohm's law, $\bar{Z}_2$ can be calculated as

$$\bar{Z}_2 = \frac{\bar{V}_2}{\bar{I}}$$

Thus, for calculation of $\bar{Z}_2$, $\bar{V}_2$ and $\bar{I}$ are required.

$\bar{V}_2 = \bar{V} - \bar{V}_1 = (100 \angle 0) - (40 \angle -30)$

$= (100 + j0) - (34.64 - j20)$

$= (65.36 + j20)$ V

$= (68.35 \angle 17.01)$ V

By Ohm's law,

Circuit current, $\bar{I} = \dfrac{\bar{V}_1}{\bar{Z}_1} = \dfrac{(40 \angle -30)}{(20 \angle 30)} = (2 \angle -60)$ A

By Ohm's law,

$$\bar{Z}_2 = \frac{\bar{V}_2}{\bar{I}} = \frac{(68.35 \angle 17.01)}{(2 \angle -60)}$$
$$= (34.175 \angle 77.01)\ \Omega$$
$$= (7.68 + j33.3)\ \Omega$$

Hence, reactive component of $Z_2 = 33.3\ \Omega$

**Example 2.33** A voltage $\bar{V} = (150 + j180)$V is applied across an impedance and the current is found to be $\bar{I} = (5 - j4)$A. Determine (i) scalar impedance, (ii) reactance, and (iii) power consumed.

***Solution***

Given: $\bar{V} = (150 + j180)$ V
$= (234.31 \angle 50.19)$ V
$\bar{I} = (5 - j4)$ A
$= (6.4 \angle -38.66)$ A

Circuit impedance,

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{(234.31 \angle 50.19)}{(6.4 \angle -38.66)}$$
$$= (36.61 \angle 88.85)\ \Omega$$
$$= (0.73 + j36.6)\ \Omega$$

(i) From polar form of impedance, scalar impedance, $Z = 36.61\ \Omega$

(ii) From rectangular form of impedance, reactance $= 36.6\ \Omega$

(iii) Power consumed,

$$P = VI \cos \phi$$
$$= 234.31 \times 6.4 \times \cos 88.85°$$
$$= 30.096 \text{ W}$$

**Example 2.34** The voltage applied to a series circuit consisting of two pure elements is given by $v = 180 \sin \omega t$ and the resulting current is given by $i = 2.5 \sin(\omega t - 45)$. Find the average power taken by the circuit and values of the elements.

***Solution***

The conditions of the example are shown in Fig. 2.74.

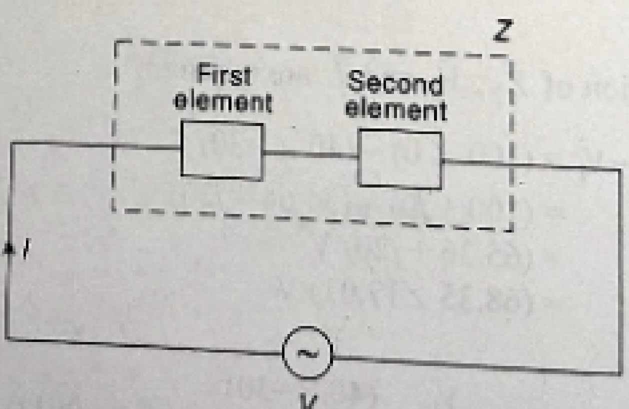



**Fig. 2.74**

Given:

$$v = 180 \sin \omega t$$

So, $\bar{V} = (127.28 \angle 0)$ V

$i = 2.5 \sin (\omega t - 45)$

Also given:

So, $\bar{I} = (1.77 \angle -45)$ A

By Ohm's law, circuit impedance can be calculated as

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{(127.28 \angle 0)}{(1.77 \angle -45)}$$

$$= (71.91 \angle 45)\ \Omega$$

$$= (50.85 + j50.85)\ \Omega$$

Power, $P = VI \cos \phi$

$$= 127.28 \times 1.77 \times \cos 45°$$

$$= 159.3 \text{ W}$$

From rectangular form of impedance, values of the circuit elements are:

$$R = 50.85\ \Omega,\ X_L = 50.85\ \Omega$$

**Example 2.35** An emf of 50 V, 50 Hz is applied to an impedance $\bar{Z}_1 = (12.5 + j21)\ \Omega$. An impedance $Z_2$ is added in series with $Z_1$, the current becomes half of the original, and leads it by 14.2°. Determine $Z_2$.

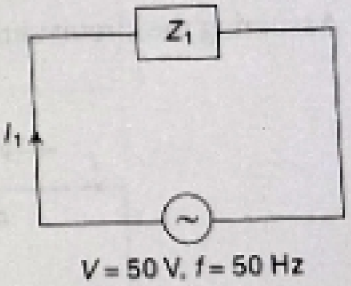
***Solution***

***Case (i)*** When an emf of 50 V, 50 Hz is applied to an impedance $\bar{Z}_1 = (12.5 + j21)\ \Omega$

Let the circuit current is $I_1$A (see Fig. 2.75).

Taking applied voltage as reference,

$\bar{V} = (50 \angle 0)$ V




V = 50 V, f = 50 Hz

Fig. 2.75

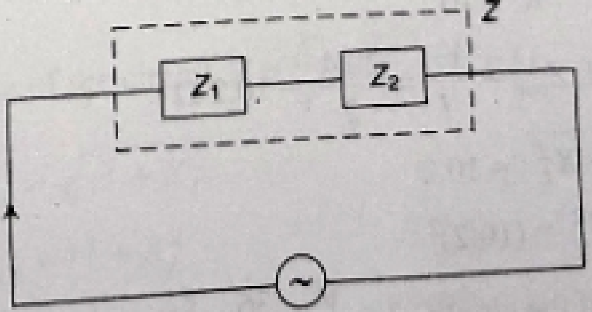
$$\text{Circuit current, } I_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{(50 \angle 0)}{(12.5 + j21)}$$

$$= \frac{(50 \angle 0)}{(24.44 \angle 59.24)}$$

$$= (2.046 \angle -59.24) \text{ A}$$

***Case (ii)*** When an impedance $Z_2$ is added in series with $Z_1$, the current becomes half of the original, and leads it by 14.2°.

Let the circuit current is $I_2A$ (see Fig. 2.76).




V = 50 V, f = 50 Hz

Fig. 2.76

As $I_2$ is half of $I_1$ and leads it by 14.2°,

$$\bar{I}_2 = (1.023 \angle -45.04) \text{ A}$$

Now, total circuit impedance can be calculated as

$$\bar{Z} = \frac{\bar{V}}{\bar{I}_2} = \frac{(50 \angle 0)}{(1.023 \angle -45.04)} = (48.88 \angle 45.04)\ \Omega$$

From the circuit diagram,

$$\bar{Z} = \bar{Z}_1 + \bar{Z}_2$$

So, $\bar{Z}_2 = \bar{Z} - \bar{Z}_1$

$$= (48.88 \angle 45.04) - (12.5 + j21)$$
$$= (34.54 + j34.59) - (12.5 + j21)$$
$$= (22.04 + j13.59)\ \Omega$$
$$= (25.89 \angle 31.66)\ \Omega$$

**Example 2.36** Find $r$ and $L$ in the circuit shown in Fig. 2.77.

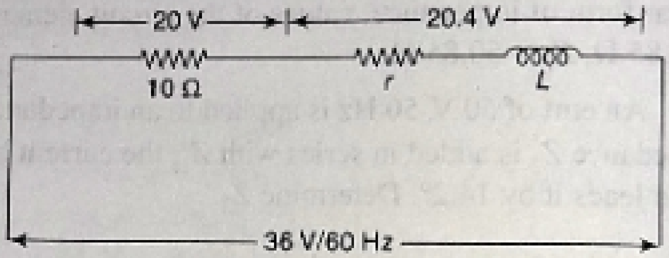



Fig. 2.77

***Solution***

Assuming the circuit current, various voltages and impedances, we get Fig. 2.78.

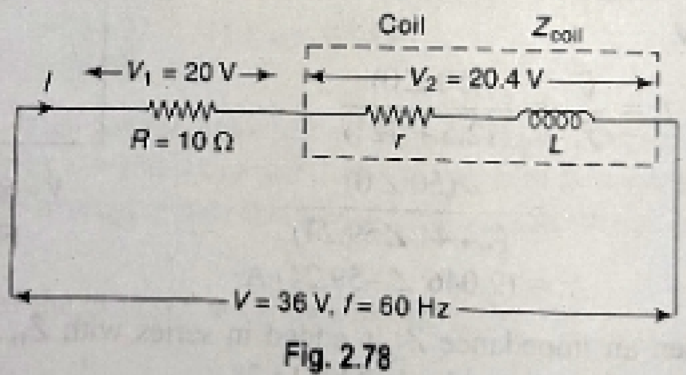



Fig. 2.78

Circuit current, $I = \frac{V_1}{R} = \frac{20}{10} = 2 \text{ A}$

Impedance of coil, $Z_{coil} = \frac{V_2}{I} = \frac{20.4}{2} = 10.2\ \Omega$

We have $\sqrt{r^2 + X_L^2} = 10.2$

or $r^2 + X_L^2 = (10.2)^2$

Total impedance of the circuit, $Z = \frac{V}{I} = \frac{36}{2} = 18\ \Omega$

So, $\sqrt{(R+r)^2 + X_L^2} = 18$

or $\sqrt{(10+r)^2 + X_L^2} = 18$

or $(10+r)^2 + X_L^2 = (18)^2$ (ii)

From Eqs (i) and (ii),

$r = 6\ \Omega$

and $X_L = 8.25\ \Omega$

So, $2\pi f L = 8.25\ \Omega$

or $2\pi \times 60 \times L = 8.25$

or $L = 0.0219$ H

**Example 2.37** A current of 5 A flows through a non-inductive resistance in series with a choking coil supplied at 250 V, 50 c/sec. If the voltage across the resistance is 125 V and across the coil 200 V, calculate

(i) Impedance, reactance, and resistance of the coil
(ii) Power absorbed by the coil
(iii) Power factor of the coil

Also draw phasor diagram.

***Solution***

The conditions of the example are shown in Fig. 2.79.

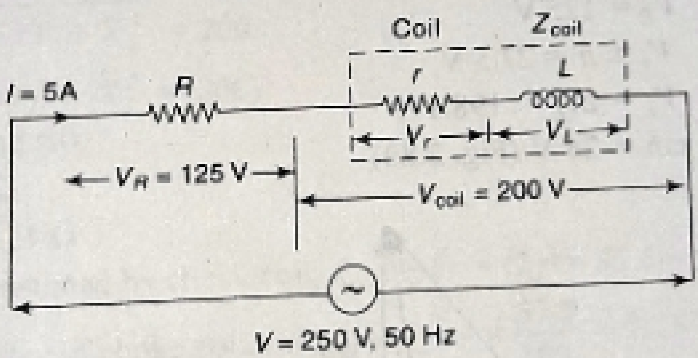



**Fig. 2.79**

(i) Resistance, $R = \dfrac{V_R}{I} = \dfrac{125}{5} = 25\ \Omega$

Impedance of the coil, $Z_{coil} = \dfrac{V_{coil}}{I} = \dfrac{200}{5} = 40\ \Omega$

Total impedance of the circuit, $Z = \dfrac{V}{I} = \dfrac{250}{5} = 50\ \Omega$

We have $Z_{coil} = \sqrt{r^2 + X_L^2}$

or $Z_{coil}^2 = r^2 + X_L^2$

or $(40)^2 = r^2 + X_L^2$ (i)

Also $\quad Z = \sqrt{(R+r)^2 + X_L^2}$

or $\quad Z^2 = (R+r)^2 + X_L^2$

or $\quad (50)^2 = (25+r)^2 + X_L^2 \qquad$ (ii)

From Eqs (i) and (ii),

$r = 5.5\ \Omega$

$X_L = 39.62\ \Omega$

(ii) Power absorbed by the coil, $P_{coil} = I^2 r = (5)^2 \times 5.5 = 137.5$ W

(iii) $(pf)_{coil} = \dfrac{r}{Z_{coil}}$

$= \dfrac{5.5}{40}$

$= 0.137$ lagging

Phasor diagram

- Take '$I$' as reference phasor.
- Equation of the circuit, $\bar{V} = \bar{V}_R + \bar{V}_{coil}$

So, $\quad \bar{V} = \bar{V}_R + \bar{V}_r + \bar{V}_L$

where $\quad V_R = 125$ V

$V_r = Ir = 27.5$ V

$V_L = IX_L = 198.1$ V

- Scale: 1 cm = 25 V (Fig. 2.80)

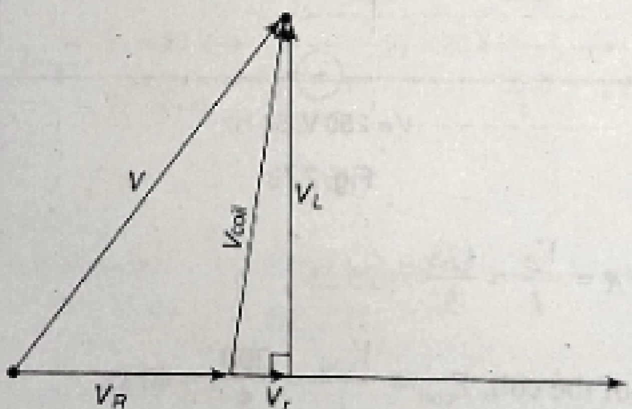



Fig. 2.80

**Example 2.38** A 100 Ω resistor is connected in series with a choke coil. When a 400 V, 50 Hz supply is applied to this combination, the voltages across the resistance and the choke coil are 200 V and 300 V respectively. Find the power consumed by the choke coil. Also calculate the power factor of choke coil and power factor of the circuit.

***Solution***

The conditions in the example are shown in Fig. 2.81.

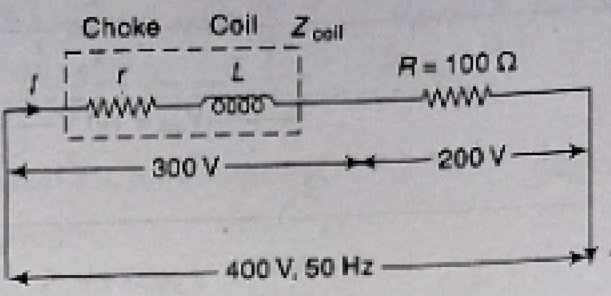



**Fig. 2.81**

Circuit current, $I = \dfrac{200}{100} = 2$ A

Impedance of choke coil, $Z_{coil} = \dfrac{300}{2} = 150\ \Omega$

So, $\sqrt{r^2 + X_L^2} = 150$

or $r^2 + X_L^2 = (150)^2$ (i)

Total impedance, $Z = \dfrac{400}{2} = 200\ \Omega$

So, $\sqrt{(R+r)^2 + X_L^2} = 200$

or $(100 + r)^2 + X_L^2 = (200)^2$ (ii)

From Eqs (i) and (ii),

$r = 37.5\ \Omega$

$X_L = 145.24\ \Omega$

Power consumed by choke coil, $P_{coil} = I^2 r = (2)^2 \times 37.5 = 150$ W

Power factor of choke coil, $(\text{pf})_{coil} = \dfrac{r}{Z_{coil}} = \dfrac{37.5}{150} = 0.25$ lagging

Power factor of the circuit, $\text{pf} = \dfrac{R+r}{Z} = \dfrac{100+37.5}{200} = 0.687$ lagging

**Example 2.39** When a resistance and coil in series are connected to a 240 V supply, current of 3 A flows lagging 37° behind the supply voltage, while the voltage across the coil is 171 V. Find the resistance of resistor, resistance, and reactance of the coil.

***Solution***

The given conditions are shown in Fig. 2.82.

As current of 3 A flows lagging 37° behind the supply voltage, circuit is inductive, and the phase angle, $\phi = 37°$.

Impedance of the coil, $Z_{coil} = \dfrac{V_{coil}}{I} = \dfrac{171}{3} = 57\ \Omega$

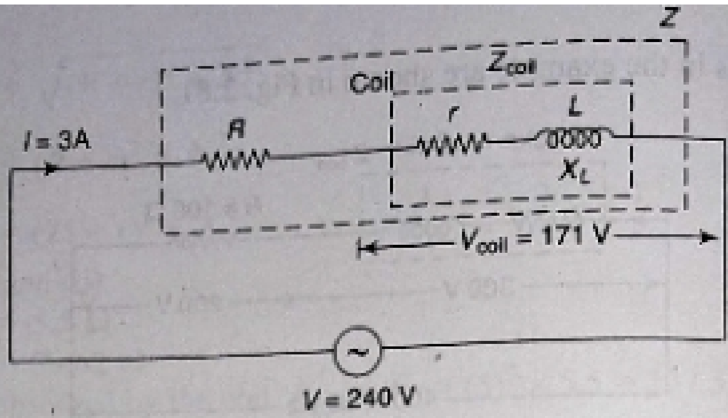



Fig. 2.82

Total impedance of the circuit, $Z = \frac{V}{I} = \frac{240}{3} = 80\ \Omega$

As pf of the circuit can be expressed as

$$(\text{pf})_{ckt} = \frac{R + r}{Z}$$

or $\quad \cos \phi = \frac{R + r}{Z}$

or $\quad R + r = \cos \phi \times Z$

$= \cos 37 \times 80$

$= 63.89\ \Omega$

We have $\quad Z = \sqrt{(R + r)^2 + X_L^2}$

or $\quad Z^2 = (R + r)^2 + X_L^2$

or $\quad 80^2 = (63.89)^2 + X_L^2$

or $\quad X_L = 48.15\ \Omega$

We have $\quad Z_{coil} = \sqrt{r^2 + X_L^2}$

or $\quad Z_{coil}^2 = r^2 + X_L^2$

or $\quad (57)^2 = r^2 + (48.15)^2$

So, $\quad r = 30.51\ \Omega$

As $\quad R + r = 63.89,$

$R + 30.51 = 63.89$

or $\quad R = 33.38\ \Omega$

**Example 2.40** Voltage and current in an ac circuit are given as

$$v = 200 \sin (377t) \text{ and } i = 8 \sin \left(377t - \frac{\pi}{6}\right)$$

Determine the active, reactive and apparent power drawn by the circuit.

***Solution***

The standard sinusoidal forms are given as

$$v = 200 \sin 377t$$

$$i = 8 \sin\left(377t - \frac{\pi}{6}\right)$$

The rms values are

$$V = \frac{200}{\sqrt{2}} = 141.42 \text{ V}, \quad I = \frac{8}{\sqrt{2}} = 5.66 \text{ A}$$

From the above equations, the current lags behind the voltage by $\frac{\pi}{6}$ rad or 30°,

i.e., phase angle, $\phi = 30°$.

Active power, $P = VI \cos \phi$
$= 141.42 \times 5.66 \times \cos 30$
$= 693.19$ W

Reactive power, $Q = VI \sin \phi$
$= 141.42 \times 5.66 \times \sin 30$
$= 400.22$ VAR

Apparent power, $S = VI$
$= 141.42 \times 5.66$
$= 800.44$ VA

**Example 2.41** A coil of 0.6 pf is in series with a 100 μF capacitor and is connected to a 50 Hz supply. The potential difference across the coil is equal to the potential difference across the capacitor. Find inductance and resistance of the coil.

***Solution***

The given conditions are shown in Fig. 2.83.

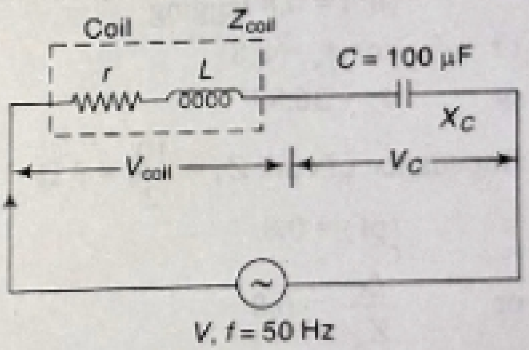



**Fig. 2.83**

$$C = 100 \times 10^{-6} \text{ F}$$

So, $$X_C = \frac{1}{2\pi f C} = 31.83 \ \Omega$$

Potential difference across the coil = Potential difference across the capacitor

or $V_{coil} = V_C$

or $IZ_{coil} = IX_C$

or $Z_{coil} = X_C$

or $Z_{coil} = 31.83 \ \Omega$

Now, $(\text{pf})_{coil} = 0.6$

or $$\frac{r}{Z_{coil}} = 0.6$$

or $r = 0.6 \times Z_{coil} = 0.6 \times 31.83 = 19.098 \ \Omega$

Thus, $Z_{coil} = \sqrt{r^2 + X_L^2}$

or $31.83 = \sqrt{(19.098)^2 + X_L^2}$

or $\quad X_L = 25.46\ \Omega$

or $\quad 2\pi fL = 25.46$

or $\quad 2\pi \times 50 \times L = 25.46$

or $\quad L = 0.081$ H

**Example 2.42** A coil $A$ takes 2 A at power factor 0.8 lagging with an applied voltage of 10 V. A second coil $B$ takes 2 A with power factor of 0.7 lagging with an applied voltage of 5 V. What voltage will be required to produce a total current of 2 A with coils $A$ and $B$ in series? Find power factor in this case.

***Solution***

The given conditions are shown in Fig. 2.84.

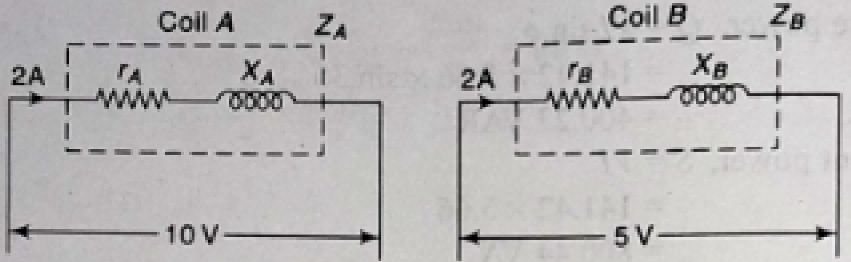



Fig. 2.84

Let phase angle of coil $A$ is $\phi_A$ and phase angle of coil $B$ is $\phi_B$.

*For coil A:*

$$(\text{pf}) = 0.8 \text{ lagging}$$

or $\quad \cos \phi_A = 0.8$

or $\quad \phi_A = 36.87°$

Impedance of coil, $Z_A = \dfrac{10}{2} = 5\ \Omega$

$$(\text{pf}) = 0.8$$

or $\quad \dfrac{r_A}{Z_A} = 0.8$

or $\quad r_A = 0.8 \times Z_A = 0.8 \times 5 = 4\ \Omega$

Now, $\quad Z_A = \sqrt{r_A^2 + X_A^2}$

or $\quad 5 = \sqrt{(4)^2 + (X_A)^2}$

So, $\quad X_A = 3\ \Omega$

*For coil B:*

$$\text{pf} = 0.7$$

or $\quad \cos \phi_B = 0.7$

or $\quad \phi_B = 45.57°$

Impedance of coil, $Z_B = \dfrac{5}{2} = 2.5\ \Omega$

$$\text{pf} = 0.7$$

$$\frac{r_B}{Z_B} = 0.7$$

or

$$r_B = 0.7 \times Z_B = 0.7 \times 2.5 = 1.75\ \Omega$$

So,

$$Z_B = \sqrt{r_B^2 + X_B^2}$$

Now,

$$2.5 = \sqrt{(1.75)^2 + (X_B)^2}$$

or

$$X_B = 1.79\ \Omega$$

So,

*When coils A and B are connected in series (Fig. 2.85):*

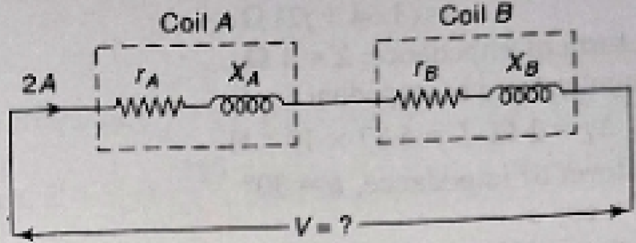



**Fig. 2.85**

Total impedance of the circuit,

$$Z = \sqrt{(r_A + r_B)^2 + (X_A + X_B)^2}$$

or

$$Z = \sqrt{(4 + 1.75)^2 + (3 + 1.79)^2}$$

or

$$Z = 7.48\ \Omega$$

Voltage, $V = IZ = 2 \times 7.48 = 14.96$ V

$$(\text{pf})_{\text{ckt}} = \frac{r_A + r_B}{Z}$$

$$= \frac{4 + 1.75}{7.48}$$

$$= 0.77 \text{ lagging}$$

**Example 2.43** In a series circuit containing resistance and inductance, the current and voltage are expressed as $i(t) = 5 \sin\left(314t + \frac{2\pi}{3}\right)$ and $v(t) = 20 \sin\left(314t + \frac{5\pi}{6}\right)$. (i) What is the impedance of the circuit? (ii) What are the values of resistance, inductance, and power factor? (iii) What is the average power drawn by the circuit?

***Solution***

Given:

$$i(t) = 5 \sin\left(314t + \frac{2\pi}{3}\right)$$

$$v(t) = 20 \sin\left(314t + \frac{5\pi}{6}\right)$$

Converting the above standard sinusoidal forms into polar forms:

$$\bar{I} = (3.54 \angle 120°)\ \text{A}$$

$$\bar{V} = (14.14 \angle 150°)\ \text{A}$$

By Ohm's law,

$$\text{Circuit impedance, } \bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{(14.14 \angle 150)}{(3.54 \angle 120)}$$

$$= (4 \angle 30)\ \Omega$$

$$= (3.64 + j2)\ \Omega$$

(a) From polar form of impedance, $Z = 4\ \Omega$

(b) From rectangular form of impedance,

$R = 3.64\ \Omega$, $X_L = 2\ \Omega$, $L = 6.37 \times 10^{-3}$ H

From polar form of impedance, $\phi = 30°$

So, $\text{pf} = \cos\phi$

$= \cos 30°$

$= 0.866$ lagging

(c) Average power, $P = VI \cos\phi$

$= 14.14 \times 3.54 \times \cos 30°$

$= 43.35$ W

**Example 2.44** A capacitor of 35 μF is connected in series with a variable resistor. The circuit is connected across 50 Hz mains. Find the value of resistor for a condition when the voltage across the capacitor is half the supply voltage.

***Solution***

The given conditions are shown in Fig. 2.86.

We have $C = 35 \times 10^{-6}$ F

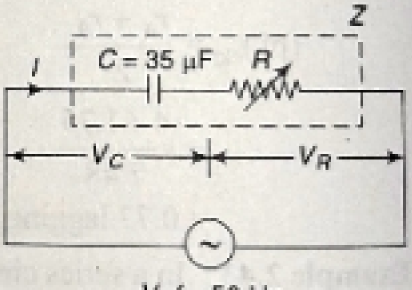



**Fig. 2.86**

So, $X_C = \dfrac{1}{2\pi fC} = 90.946\ \Omega$

Also $V_C = \dfrac{1}{2}V$

So, $IX_C = \dfrac{1}{2}IZ$

or $Z = 2X_C$

or $Z = 2 \times 90.946$

or $Z = 181.89\ \Omega$

Now, $Z = \sqrt{R^2 + X_C^2}$

or $(181.89) = \sqrt{R^2 + (90.946)^2}$

So, $R = 157.5\ \Omega$

**Example 2.45** A resistance and a capacitance in series, connected across 250 V supply draws 5 A current at a frequency of 50 Hz. When frequency is increased to 60 Hz, it draws a current of 5.8 A. Find the values of $R$ and $C$ and also power drawn in the second case.

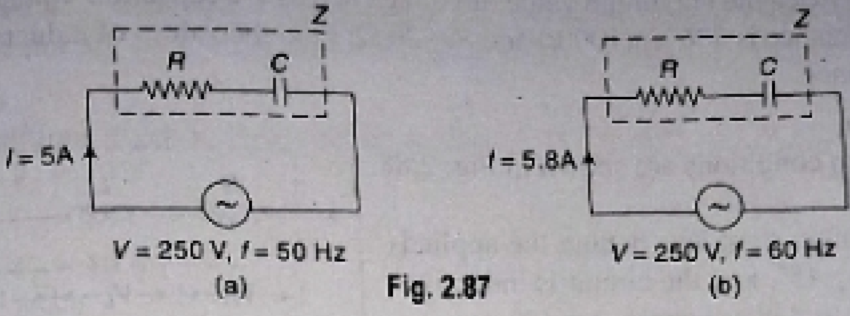


Fig. 2.87

***Solution***

***Case (i)*** When frequency, $f = 50$ Hz, $I = 5$ A [see Fig. 2.87(a)]

Circuit impedance, $Z = \dfrac{V}{I} = \dfrac{250}{5} = 50\ \Omega$

So, $Z = \sqrt{R^2 + X_C^2}$

or $Z^2 = R^2 + \left(\dfrac{1}{2\pi fC}\right)^2$

or $(50)^2 = R^2 + \left(\dfrac{1}{2\pi \times 50 \times C}\right)^2$ (i)

***Case (ii)*** When frequency, $f = 60$ Hz, $I = 5.8$ A [see Fig. 2.87(b)]

Circuit impedance, $Z = \dfrac{V}{I} = \dfrac{250}{5.8} = 43.1\ \Omega$

So, $Z = \sqrt{R^2 + X_C^2}$

or $Z^2 = R^2 + \left(\dfrac{1}{2\pi fC}\right)^2$

or $(43.1)^2 = R^2 + \left(\dfrac{1}{2\pi \times 60 \times C}\right)^2$ (ii)

From Eqs (i) and (ii),

$C = 69.42 \times 10^{-6}$ F

or $C = 69.42\ \mu$F

$R = 19.94\ \Omega$

Power absorbed in the second case,

$P = I^2R$

or $P = (5.8)^2 \times 19.94$

or $P = 670.78$ W

**Example 2.46** An $R$-$L$-$C$ series circuit has a current that lags behind the applied voltage by 45°. The voltage across the inductance has a maximum value equal to twice the maximum value of voltage across the capacitor. Voltage across the inductance is 300 sin(1000$t$) and $R$ = 20 Ω. Find the values of inductance and capacitance.

***Solution***

The given conditions are shown in Fig. 2.88.

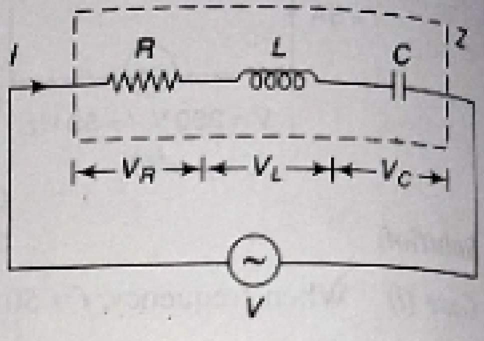


Fig. 2.88

Given:

The circuit current lags behind the applied voltage by 45°, i.e., the circuit is inductive ($X_L > X_C$) and phase angle, $\phi = 45°$.

$$v_L = 300 \sin (1000t)$$

So, $\omega = 1000$ rad/sec

$$R = 20\ \Omega$$

$$V_{L\max} = 2\, V_{C\max}$$

Required: $L$ and $C$

We have $V_{L\max} = 2\, V_{C\max}$

or $\sqrt{2}\, V_L = 2(\sqrt{2} V_C)$

So, $V_L = 2V_C$

or $IX_L = 2IX_C$

Hence, $X_L = 2X_C$

Now, $\text{pf} = \cos \phi = \dfrac{R}{Z}$

So, $\cos 45° = \dfrac{20}{Z}$

or $Z = \dfrac{20}{\cos 45°} = 28.28\ \Omega$

Circuit impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2}$

or $28.28 = \sqrt{(20)^2 + (2X_C - X_C)^2}$ $\quad (\because X_L = 2X_C)$

So, $X_C = 20\ \Omega$

As $X_L = 2\, X_C$,

$X_L = 40\ \Omega$

So, $\omega L = 40$

or $1000 \times L = 40$

or $L = 0.04$ H

As $X_C = 20\ \Omega$,

$$\frac{1}{\omega C} = 20$$

or $\dfrac{1}{1000 \times C} = 20$

or $\quad C = 50 \times 10^{-6}$ F
$\quad = 50\ \mu$F

**Example 2.47** In an *R-L-C* series circuit, the voltage across the resistor, inductor, and capacitor are 10 V, 15 V, and 10 V respectively. What is the supply voltage?

***Solution***

The conditions given in the example are shown in Fig. 2.89.

Given: $V_R = 10$ V
$V_L = 15$ V
$V_C = 10$ V
$V = ?$

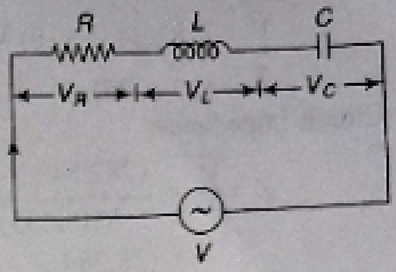



Fig. 2.89

*Method (i)* *From phasor diagram (not to scale):*

Take circuit current as reference.

Equation of the circuit, $\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$

From phasor diagram,

$$V = \sqrt{V_R^2 + (V_L - V_C)^2}$$

or $\quad V = \sqrt{(10)^2 + (15-10)^2}$

So, $\quad V = 11.18$ V

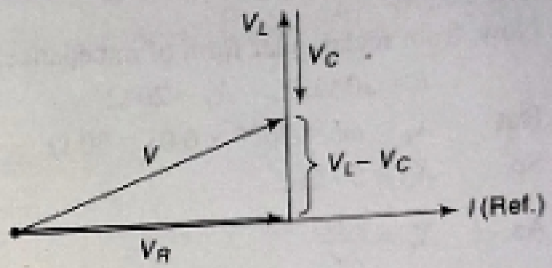



Fig. 2.90

*Method (ii)* *By phasor algebra:*

$\bar{V}_R = (10 \angle 0)$
$\bar{V}_L = (15 \angle 90)$
$\bar{V}_C = (10 \angle -90)$

So, $\quad \bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C$
$\quad = (10 \angle 0) + (15 \angle 90) + (10 \angle -90)$

or $\quad \bar{V} = (10 + j0) + (0 + j15) + (0 - j10)$

or $\quad \bar{V} = (10 + j5)$ V

or $\quad \bar{V} = (11.18 \angle 26.565)$ V

**Example 2.48** A resistance *R* and inductance $L = 0.01$ H and a capacitance *C* are connected in series. When a voltage $v = 400 \sin(3000t - 10°)$ V is applied to the series combination, the current flowing is $10\sqrt{2} \cos(3000t - 55°)$ A. Find *R* and *C*.

***Solution***

The conditions given in the example are shown in Fig. 2.91.

Given:

$v = 400 \sin(3000t - 10)$
$i = 10\sqrt{2} \cos(3000t - 55)$

Converting the given quantities into standard sinusoidal forms, we get

$$v = 400 \sin(3000t - 10)$$

$$i = 10\sqrt{2} \sin(3000t + 35)$$

Converting the standard sinusoidal forms into polar forms, we get

$$\bar{V} = (282.84 \angle -10) \text{ V}$$

$$\bar{I} = (10 \angle 35) \text{ A}$$

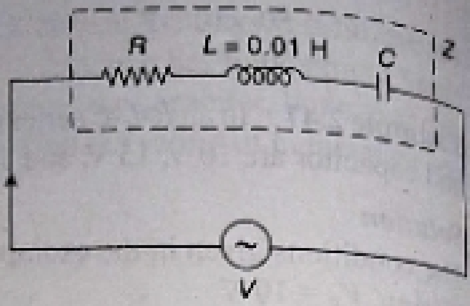



Fig. 2.91

Circuit impedance,

$$\bar{Z} = \frac{\bar{V}}{\bar{I}} = \frac{(282.84 \angle -10)}{(10 \angle 35)}$$

$$= (28.284 \angle -45) \ \Omega$$

$$= (20 - j20) \ \Omega$$

It is given that the current leads the applied voltage. Therefore, circuit is capacitive, i.e., $X_C > X_L$.

Now, from rectangular form of impedance,

$$R = 20 \ \Omega, X_C - X_L = 20 \ \Omega$$

But $\quad X_L = \omega L = 3000 \times 0.01 = 30 \ \Omega$

So, $\quad X_C = 50 \ \Omega$

As $\quad X_C = \dfrac{1}{\omega C},$

or $\quad 50 = \dfrac{1}{3000 \times C}$

or $\quad C = 6.66 \times 10^{-6}$ F

$\quad = 6.66 \ \mu$F

*Alternative method:*

Given: $\quad v = 400 \sin(3000t - 10)$

So, $\quad V = 282.84$ V

Also $\quad i = 10\sqrt{2} \cos(3000t - 55)$

or $\quad i = 10\sqrt{2} \sin(3000t + 35)$

So, $\quad I = 10$ A

The current leads the applied voltage by 45°.

So, phase angle, $\phi = 45°$ and $X_C > X_L$

Circuit impedance, $Z = \dfrac{V}{I} = \dfrac{282.84}{10} = 28.284 \ \Omega$

$$\text{pf} = \cos\phi = \frac{R}{Z}$$

or $\quad \cos 45° = \dfrac{R}{28.284}$

So, $R = \cos 45° \times 28.284 = 0.707 \times 28.284 = 20 \ \Omega$

Now,

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

$$28.284 = \sqrt{(20)^2 + (X_C - X_L)^2}$$

or $\quad X_C - X_L = 20$

or $\quad X_L = \omega L = 3000 \times 0.01 = 30\ \Omega$

But $\quad X_C = 50$

So,

$$\frac{1}{\omega C} = 50$$

or

$$\frac{1}{3000 \times C} = 50$$

or

Hence, $\quad C = 6.66 \times 10^{-6}$ F

$= 6.66\ \mu$F

**Example 2.49** For the circuit shown in Fig. 2.92, determnine the (i) supply frequency ($f$), (ii) coil resistance ($r$), and (iii) supply voltage ($V$).

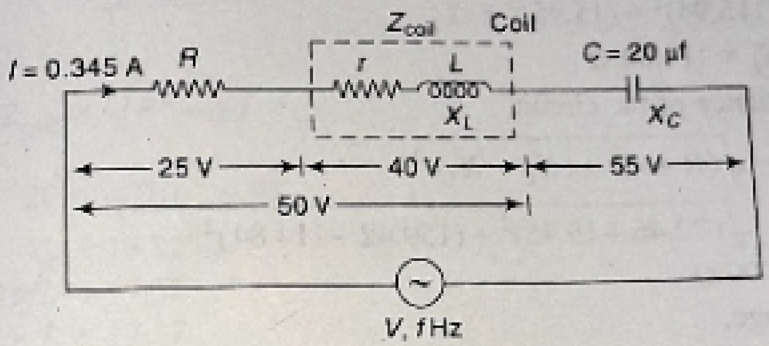



Fig. 2.92

***Solution***

Given: $\quad V_R = 25$ V

$V_{coil} = 40$ V

$V_C = 55$ V

$V_{R+coil} = 50$ V

$I = 0.345$ A

$C = 20\ \mu$F

We have $\quad V_C = IX_C$

So, $\quad 55 = 0.345 \times X_C$

or $\quad X_C = 159.42\ \Omega$

or

$$\frac{1}{2\pi f C} = 159.42$$

or

$$\frac{1}{2\pi \times f \times 20 \times 10^{-6}} = 159.42$$

or $\quad f = 50$ Hz

$$R = \frac{V_R}{I} = \frac{25}{0.345} = 72.46\ \Omega$$

$$Z_{coil} = \frac{V_{coil}}{I} = \frac{40}{0.345} = 115.94\ \Omega$$

$$Z_{R+coil} = \frac{V_{R+coil}}{I} = \frac{50}{0.345} = 144.93\ \Omega$$

Now, $\quad Z_{R+coil} = \sqrt{(R+r)^2 + X_L^2}$

or $\quad (144.93)^2 = (R+r)^2 + X_L^2$

or $\quad (144.93)^2 = R^2 + 2Rr + r^2 + X_L^2$

or $\quad (144.93)^2 = R^2 + 2Rr + Z_{coil}^2 \qquad (\because r^2 + X_L^2 = Z_{coil}^2)$

or $\quad (144.93)^2 = (72.46)^2 + 2 \times 72.46 \times r + (115.94)^2$

So, $\quad r = 15.95\ \Omega$

$$Z_{coil}^2 = r^2 + X_L^2$$

or $\quad (115.94)^2 = (15.95)^2 + X_L^2$

or $\quad X_L = 114.84\ \Omega$

Total impedance of the circuit,

$$Z = \sqrt{(R+r)^2 + (X_C - X_L)^2}$$

$$= \sqrt{(72.46 + 15.95)^2 + (159.42 - 114.84)^2}$$

$$= 99\ \Omega$$

Supply voltage,

$$V = IZ$$

$$= 0.345 \times 99 = 35.046\ \text{V}$$

**Example 2.50** A coil connected to a 100 V dc supply draws 10 A current and the same coil when connected a 100 V ac voltage of frequency 50 Hz draws 5 A current. Calculate the parameters of the coil and power factor.

***Solution***

***Case (i)*** When the coil is connected to 100 V dc supply:

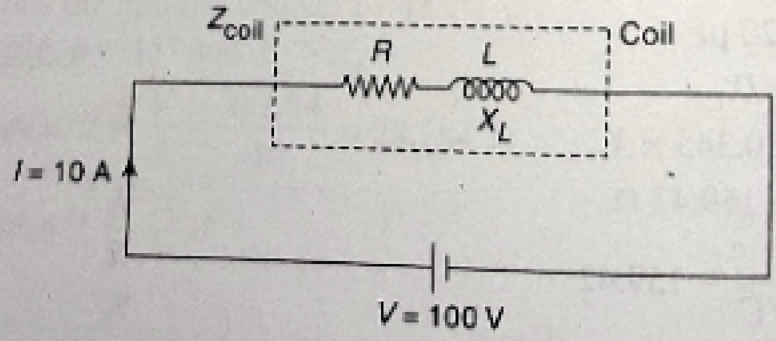



Fig. 2.93

When the coil is connected across dc supply, the inductive reactance ($X_L$) is zero.

So, $$R = \frac{V}{I} = \frac{100}{10} = 10\ \Omega$$

*Case (ii)* When the coil is connected to 100 V ac supply of frequency 50 Hz:

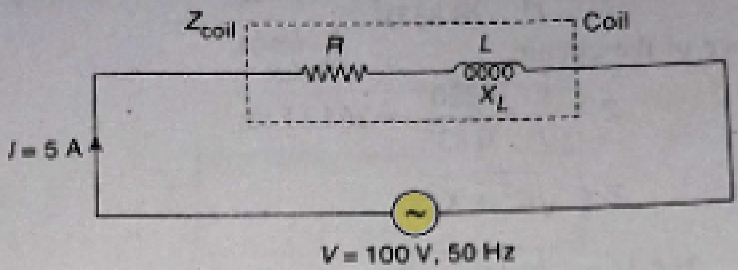



**Fig. 2.94**

When the coil is connected across ac supply, the reactance comes into existance.

So, $$Z_{coil} = \frac{V}{I} = \frac{100}{5} = 20\ \Omega$$

Now,

$$Z_{coil} = \sqrt{R^2 + X_L^2}$$

$$\therefore \quad 20 = \sqrt{(10)^2 + X_L^2}$$

$$\therefore \quad X_L = 17.32\ \Omega$$

So, $$2\pi f L = 17.32$$

$$2\pi \times 50 \times L = 17.32$$

$$L = 55.13 \times 10^{-3}\ \text{H}$$

or $$L = 55.13\ \text{mH}$$

Now, power factor of the coil:

$$\cos\phi = \frac{R}{Z} = \frac{10}{20} = 0.5 \text{ lagging}$$

**Example 2.51** A 120 V, 100 W lamp is to be connected to a 220V, 50 Hz ac supply. What value of pure inductance should be connected in series in order to run the lamp on the rated voltage?

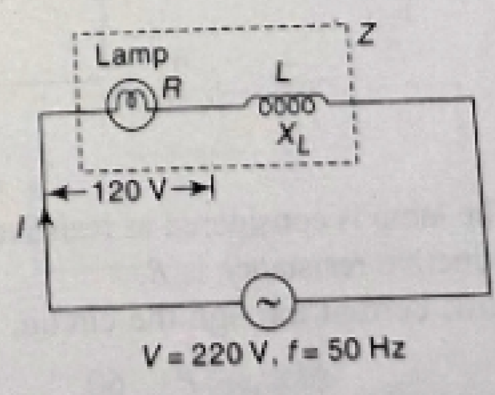



**Fig. 2.95**

***Solution***

The conditions of the example are shown in Fig. 2.95.

The value of $L$ is required so that the lamp is run on the rated voltage, i.e. 120 V. The lamp is considered as resistive load. Let the resistance of lamp is $R$ Ω.

Now, current drawn by the lamp,

$$I = \frac{P}{V} = \frac{100}{120} = 0.833 \text{ A}$$

Resistance of lamp,

$$R = \frac{P}{I^2} = \frac{100}{(0.833)^2} = 144\ \Omega$$

So, impedance of the circuit,

$$Z = \frac{V}{I} = \frac{220}{0.833} = 264.11\ \Omega$$

We have, $\quad Z = \sqrt{R^2 + X_L^2}$

or $\quad 264.11 = \sqrt{(144)^2 + X_L^2}$

or $\quad X_L = 221.4\ \Omega$

or $\quad 2\pi fL = 221.4$

$$2\pi \times 50 \times L = 221.4$$

$$L = 0.705 \text{ H}$$

**Example 2.52** A 120 V, 60 W metal filament lamp is to be operated on 220 V, 50 Hz supply mains. Calculate the value of

(i) Non-inductive resistance

(ii) Pure-inductance

that would be required in order to run the lamp on correct voltage. Which method is preferable and why?

***Solution***

Lamp ratings: 120 V, 60 W

Supply voltage: 220V, 50 Hz

***(i) Value of non-inductive resistance (R):***

Let a non-inductive resistance ($R$) be connected in series with the lamp, so that it runs on correct voltage (Fig. 2.96).

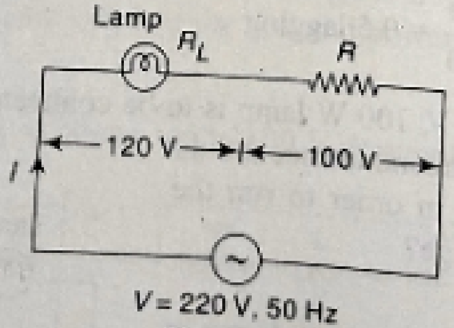



**Fig. 2.96**

The lamp is considered as resistive load having resistance $R_L$. The value of non-inductive resistance is $R$.

Now, current through the circuit,

$$I = \frac{P}{V} = \frac{60}{120} = 0.5 \text{ A}$$

So, $R = \frac{100}{0.5} = 200\ \Omega$

***(ii) Value of pure-inductance:***

Let pure-inductance $L$ H is connected in series with the lamp so that it runs on correct voltage (Fig. 2.97).

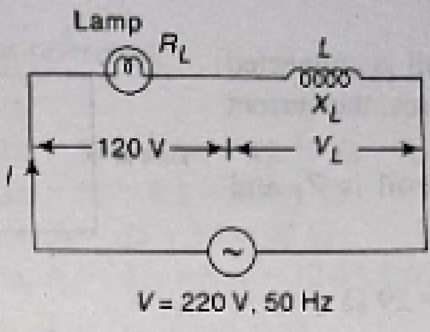



**Fig. 2.97**

From the circuit diagram,

$$V = \sqrt{(120)^2 + V_L^2}$$

$$220 = \sqrt{(120)^2 + V_L^2}$$

$\therefore\ V_L = 184.39$ V

Circuit current, $I = 0.5$ A

So, $X_L = \frac{V_L}{I} = \frac{184.39}{0.5} = 368.78\ \Omega$

i.e., $2\pi f L = 368.78$

$\therefore\ L = \frac{368.78}{2\pi f} = \frac{368.78}{2\pi \times 50} = 1.1739$ H

The method of connecting a pure inductance in series with the lamp is preferable because a pure inductance does not consume any power.

**Example 2.53** When a coil is connected to a 114 V, 60 Hz source, the current is 3 A. The current rises to 4 A when 116 V, 25 Hz source is connected to the same coil. Determine the value of the resistance and inductance of the coil.

***Solution***

***Case (i)*** When the coil is connected across 114 V, 60 Hz source, the current is 3 A [Fig. 2.98].

Let the impedance of the coil is $Z_1$ and reactance is $X_{L1}$.

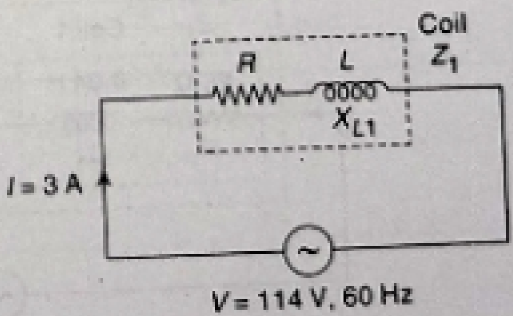



**Fig. 2.98**

So, $Z_1 = \frac{V}{I} = \frac{114}{3} = 38\ \Omega$

We have,

$$Z_1 = \sqrt{R^2 + X_{L1}^2}$$

$$Z_1^2 = R^2 + (2\pi f L)^2$$

$$(38)^2 = R^2 + (2\pi \times 60 \times L)^2 \qquad \text{(i)}$$

*Case (ii)* When the coil is connected across 116 V, 25 Hz source, the current is 4 A. (Fig. 2.99)

Let the impedance of coil is $Z_2$ and reactance is $X_{L2}$.

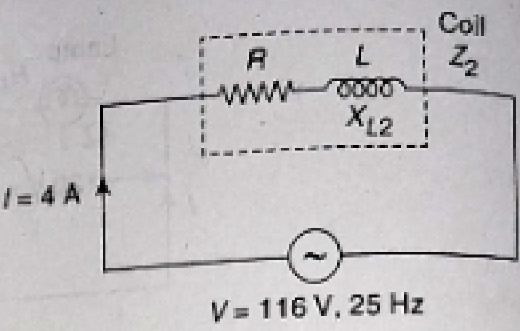



Fig. 2.99

So, $Z_2 = \dfrac{V}{I} = \dfrac{116}{4} = 29\ \Omega$

We have,

$$Z_2 = \sqrt{R^2 + X_{L2}^2}$$

$$Z_2^2 = R^2 + (2\pi f L)^2$$

$$(29)^2 = R^2 + (2\pi \times 25 \times L)^2 \qquad \text{(ii)}$$

Eq. (i) – Eq. (ii),

$$(38)^2 - (29)^2 = R^2 + (2\pi \times 60 \times L)^2 - R^2 - (2\pi \times 25 \times L)^2$$

$$L = 0.07165 \text{ H}$$

or $\quad L = 71.65$ mH

So, $\quad R = 26.71\ \Omega$

**Example 2.54** An inductive coil having inductance of 0.04 H and resistance of 25 Ω has been connected in series with another inductive coil of inductance 0.2 H and resistance 15 Ω. The whole circuit has been energized from 230 V, 50 Hz mains. Calculate power dissipation of each coil and power factor of the whole circuit.

***Solution***

The given conditions are shown in Fig. 2.100.

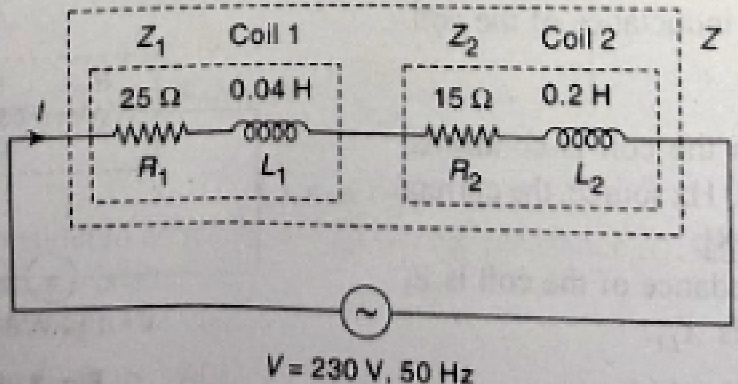



Fig. 2.100

Let the two coils are coil 1 and coil 2.

Given: $R_1 = 25\ \Omega$
$L_1 = 0.04$ H
$R_2 = 15\ \Omega$
$L_2 = 0.2$ H
$V = 230$ V
$f = 50$ Hz

Taking applied voltage as reference,

$$\overline{V} = (230\ \angle 0)\ \text{V}$$

Now, $X_{L1} = 2\pi f L_1 = 2\pi \times 50 \times 0.04 = 12.57\ \Omega$
$X_{L2} = 2\pi f L_2 = 2\pi \times 50 \times 0.2 = 62.83\ \Omega$

Total resistance, $R = R_1 + R_2 = 25 + 15 = 40\ \Omega$

Total inductive reactance, $X_L = X_{L1} + X_{L2} = 12.57 + 62.83 = 75.4\ \Omega$

So, impedance of circuit, $\overline{Z} = (40 + j75.4)\ \Omega$

$$\overline{Z} = (85.35\ \angle 62.05)\ \Omega$$

Circuit current, $\overline{I} = \dfrac{\overline{V}}{\overline{Z}}$

$$\therefore \quad \overline{I} = \frac{(230\ \angle 0)}{(85.35\ \angle 62.05)\text{A}}$$

$$\therefore \quad \overline{I} = (2.695\ \angle -62.05)\ \text{A}$$

Now, power dissipation in coil1,

$$P_1 = I^2 \times R_1 = (2.695)^2 \times 25 = 181.57\ \text{W}$$

Power dissipation in coil 2,

$$P_2 = I^2 \times R_2 = (2.695)^2 \times 15 = 108.95\ \text{W}$$

Now, phase angle of circuit, $\phi = 62.05$

$\therefore$ Power factor of circuit,

pf = cos 62.05 = 0.4687 lagging.

**Example 2.55** A load consisting of a capacitor in series with a resistor has an impedance of 50 Ω and pf 0.707 leading. The load is connected in series with 40 Ω resistor across ac supply and the resulting current is 3 A. Determine the supply voltage and the overall phase angle.

***Solution***

The given conditions are shown in Fig. 2.101.

Now, $Z_L = 50\ \Omega$ (given)

Since, pf = 0.707 leading, $\phi = 45°$

So, $\overline{Z}_L = (50\ \angle -45)\ \Omega$

$\therefore \quad \overline{Z}_L = (35.36 - j35.36)\ \Omega$

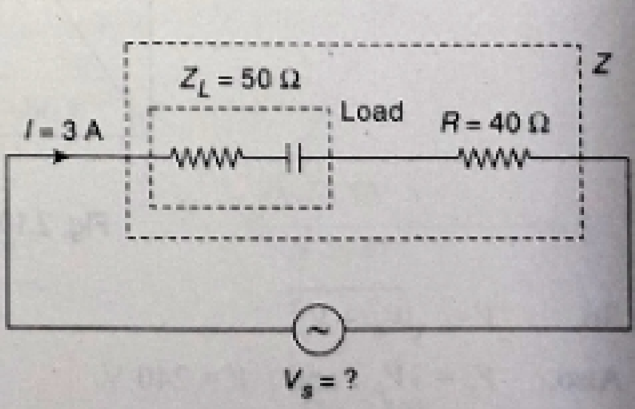



Fig. 2.101

Thus, resistance of load = 35.36 Ω
Capacitive reactance of load = 35.36 Ω
Now, total impedance of the circuit,

$$\overline{Z} = [(35.36 + 40) - j35.36]\ \Omega$$

$$\therefore \quad \overline{Z} = (75.36 - j35.36)\ \Omega$$

$$\therefore \quad \overline{Z} = (83.24 \angle -25.14)\ \Omega$$

Hence, the source voltage is given by

$$V_s = IZ = 3 \times 83.24 = 249.72\ \text{V}$$

Overall phase angle, $\phi = 25.14°$

**Example 2.56** For a circuit given in Fig. 2.102, $V_x = 3V_y$ and $V_x$ and $V_y$ are in quadrature. Determine the values of (i) $R$ and $C$ and (ii) phase relationship between $V$, $V_x$, $V_y$, and $I$.

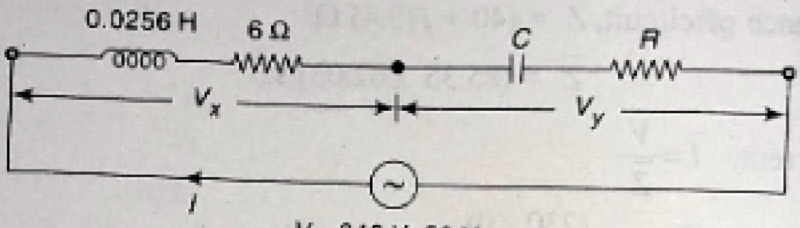



Fig. 2.102

***Solution***

Given :

$V_x = 3V_y$, $V = 240$ V, $f = 50$ Hz, $V_x$ and $V_y$ are in quadrature.

***Calculation of $V_x$ and $V_y$ :***

$V_x$ and $V_y$, are in quadrature as shown in Fig. 2.103. The resultant voltage is $V$.

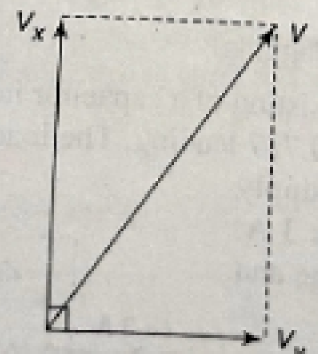



Fig. 2.103

So, $\quad V = \sqrt{V_x^2 + V_y^2}$

Also, $\quad V_x = 3V_y$ and $V = 240$ V

$\therefore \quad 240 = \sqrt{(3V_y)^2 + V_y^2}$

$V_y = 75.89$ V

∴ $V_x = 3V_y = 227.67$ V

So,

***Calculation of current I, $Z_x$, and $Z_y$:***

Marking the different impedances in Fig. 2.102, we get the circuit as follows,

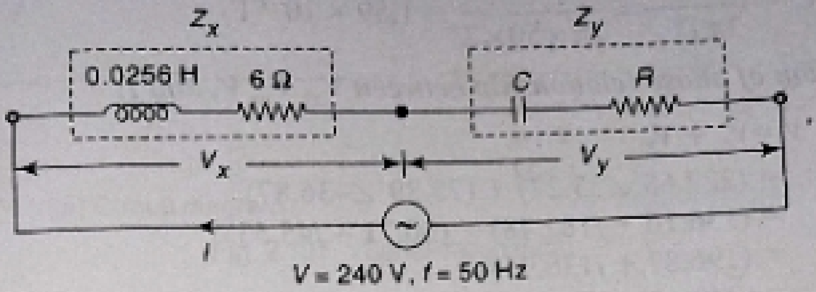



**Fig. 2.104**

Now, $\bar{Z}_x = [6 + j(2\pi \times 50 \times 0.0256)]\ \Omega$

$\bar{Z}_x = (6 + j8.04)\ \Omega$

$\bar{Z}_x = (10.03\ \angle 53.27)\ \Omega$

i.e., $Z_x = 10.03\ \Omega$

So, circuit current, $I = \dfrac{V_x}{Z_x} = \dfrac{227.67}{10.03} = 22.7$ A

Hence, $Z_y = \dfrac{V_y}{I} = \dfrac{75.89}{22.7} = 3.34\ \Omega$

***Calculation of R and C:***

Let the circuit current $I$ is a reference quantity.

∴ $\bar{I} = (22.7\ \angle 0)$ A

So, $\bar{V}_x = \bar{I}\, Z_x$

$= (22.7\ \angle 0)\ (10.03\ \angle 53.27)$

$= (227.68\ \angle 53.27)$ V

Since $V_x$ and $V_y$ are in quadrature and $V_y$ is voltage across a capacitive circuit, we assume that, $V_y$ lags $V_x$ by 90° as shown in Fig. 2.105.

So, phase angle of $V_y = 53.13 - 90 = -36.87°$

So, $\bar{V}_y = (75.89\ \angle -36.87)$ V

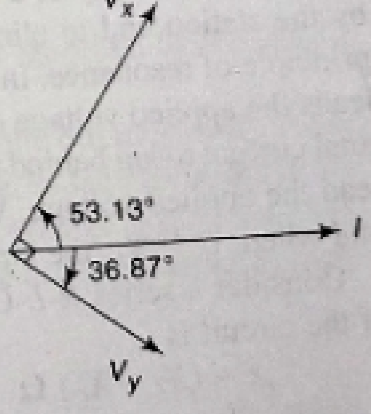



**Fig. 2.105**

Now, $\bar{Z}_y = \dfrac{\bar{V}_y}{\bar{I}} = \dfrac{(75.89\angle -36.87)}{(22.7\angle 0)}$

$\bar{Z}_y = (3.34\ \angle -36.87)\ \Omega$

So, $\bar{Z}_y = (2.67 - j2)\ \Omega$

Hence, $R = 2.67\ \Omega$; $X_C = 2\ \Omega$

As, $$X_C = \frac{1}{2\pi fC}$$

$$\therefore \quad C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 2} = 1.59 \times 10^{-3}\ \text{F}$$

***Calculation of phase relationship between $V_x$, $V_y$, $V$, and $I$:***

$$\bar{V} = \bar{V}_x + \bar{V}_y$$
$$= (227.68\ \angle 53.27) + (75.89\ \angle -36.87)$$
$$= (136.16 + j182.48) + (60.71 - j45.53)$$
$$= (196.87 + j136.95)$$

So, $$= (240\ \angle 34.82)\ \text{V}$$

Figure 2.106 shows the complete phasor diagram showing the relation between $V_x$, $V_y$, $V$, and $I$.

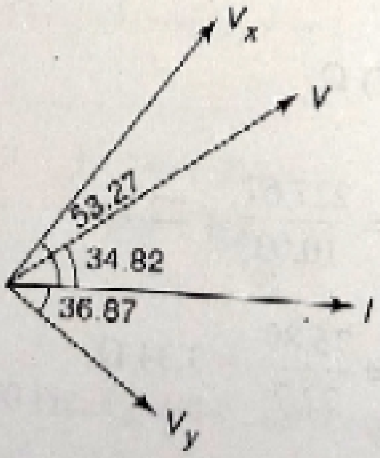



Fig. 2.106

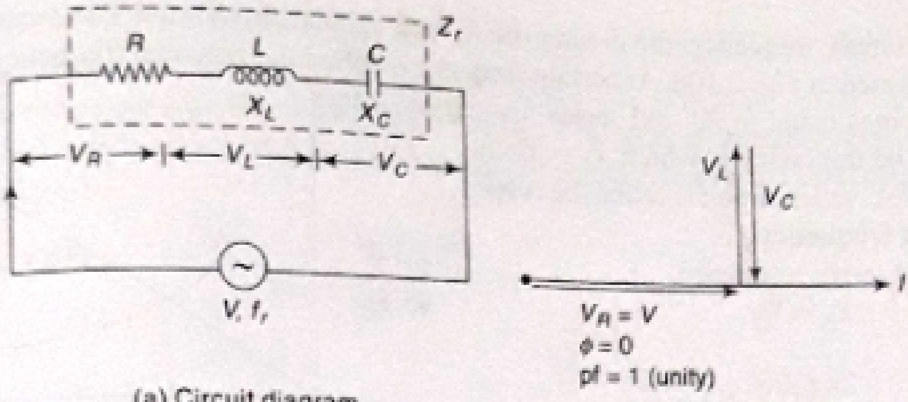
### 2.2.5 Series Resonance

In many electrical circuits, resonance is very important phenomenon. The study of resonance is very useful, particularly in the area of communications. For example, the ability of a radio receiver to select a certain frequency transmitted by the station and to eliminate frequencies from other stations is based on the principle of resonance. In a series *R-L-C* circuit, the current either lags behind or leads the applied voltage depending upon the values of $X_L$ and $X_C$. $X_L$ causes the total current to lag behind the applied voltage, while $X_C$ causes the total current to lead the applied voltage. When $X_L > X_C$, the circuit is predominantly inductive, and when $X_C > X_L$, the circuit is predominantly capacitive.

Consider a series *R-L-C* circuit as shown in Fig. 2.107(a). The net reactance of the circuit is

$$X = (X_L - X_C)\ \Omega$$

where $X_L = 2\pi f L\ \Omega$ and $X_C = \dfrac{1}{2\pi f C}\ \Omega$




(a) Circuit diagram

(b) Phasor diagram

**Fig. 2.107** Series resonance

The inductive reactance is directly proportional to the supply frequency, whereas the capacitive reactance is inversely proportional to the supply frequency. At a certain supply frequency, called resonance frequency ($f_r$), the inductive reactance becomes equal to the capacitive reactance, and the net reactance ($X$) becomes zero. Therefore, impedance ($Z$) of the circuit becomes purely resistive (i.e., $Z = R$). In other words, the whole circuit behaves as a purely resistive circuit and the current remains in phase with the applied voltage (pf = 1). This condition is said to be the condition for electrical resonance. Figure 2.107(b) shows the phasor diagram at resonance.

A circuit containing reactive elements ($L$ and $C$) is resonant when the circuit power factor is unity, i.e., the applied voltage and the circuit current are in phase. If such condition occurs in a series circuit, it is termed as **series resonance**.

For a series resonance, the circuit power factor must be unity, which is possible only if the net reactance of the circuit is zero, i.e., $X_L - X_C = 0$ or $X_L = X_C$.

### *Effects of series resonance*

1. The net reactance of the circuit is zero. Therefore, impedance of the circuit is minimum and is equal to the resistance of the circuit, i.e., $Z_r = R\ \Omega$.
2. The current in the circuit is maximum as it is limited by the resistance of the circuit alone. So, $I_r = \dfrac{V}{Z_r} = \dfrac{V}{R}$
3. As the current is at its maximum value, the power absorbed by the circuit will also be at its maximum value.
4. Since at series resonance, the current flowing in the circuit is very large, the voltage drop across $L$ and $C$ are also very large. In fact, these drops are much greater than the applied voltage. However, voltage drop across $L$-$C$ combination as a whole will be zero because these drops are equal in magnitude but 180° out of phase with each other.

### *Resonant frequency ($f_r$)*

The series resonance (i.e., $X_L = X_C$) can be achieved by changing the supply frequency because $X_L$ ($=2\pi fL$) and $X_C$ ($=1/2\pi fC$) are frequency dependent. Higher the supply frequency, the greater the $X_L$ and smaller the $X_C$ and vice versa. This indicated in Fig. 2.108. At certain frequency, called the resonant frequency $f_r$, $X_L$ becomes equal to $X_C$ and series resonance occurs.

The frequency at which $X_L = X_C$ in a *R-L-C* series circuit is called the **resonant frequency** $f_r$.

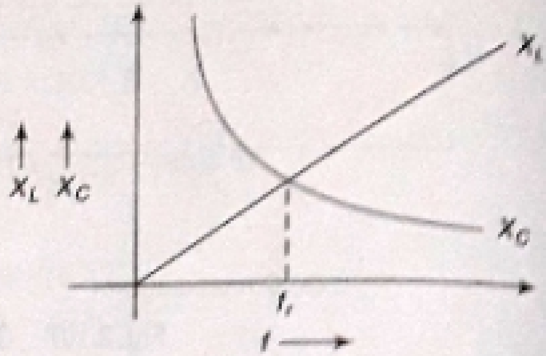



Fig. 2.108 Variation of reactances with frequency

At series resonance,

$$X_L = X_C$$

or $$2\pi f_r L = \frac{1}{2\pi f_r C}$$

So, $$f_r^2 = \frac{1}{4\pi^2 LC}$$

or $$f_r = \frac{1}{2\pi\sqrt{LC}} \text{ hertz} \tag{2.12}$$

or $$2\pi f_r = \frac{1}{\sqrt{LC}}$$

Hence, $$\omega_r = \frac{1}{\sqrt{LC}} \text{ rad/sec} \tag{2.13}$$

***Resonance curve***

The curve between current and frequency is known as **resonance curve.** Figure 2.109 shows the resonance curve of the typical *R-L-C* series circuit. Note that current reaches at its maximum value at the resonant frequency ($f_r$), falling off rapidly on either side at that point. It is because if the frequency is below $f_r$, $X_C > X_L$ and the net reactance is no longer zero. If the frequency is above $f_r$, then $X_L > X_C$ and the net reactance is again not zero. In both the cases, the circuit impedance will be more than the impedance $Z_r$ (=$R$) at resonance.

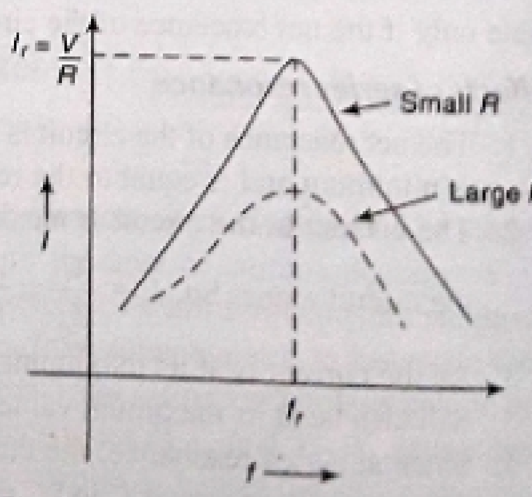



Fig. 2.109 Variation of circuit current with frequency

The result is that the magnitude of the circuit current decreases rapidly as the frequency changes from the resonant frequency.

The shape of the resonance curve depends upon the value of resistance ($R$). The smaller the resistance, the greater the current at resonance and sharper the curve. On the other hand, the greater the resistance, the lower the resonant peak and flatter the curve.

### Q-factor of series resonance circuit

At series resonance, the voltage drops across $L$ or $C$ are very large. In fact, these drops are much greater than the applied voltage. This voltage magnification produced by resonance is termed as $Q$-factor of the series resonant circuit ($Q$ stands for quality), i.e.,

$$Q\text{-factor} = \frac{\text{Voltage across } L \text{ or } C}{\text{Applied voltage}} \tag{2.14}$$

$$= \frac{V_L \text{ or } V_C}{V}$$

$$= \frac{V_L}{V_R} \qquad (\because \text{At resonance, } V = V_R)$$

$$= \frac{I_r X_L}{I_r R}$$

$$= \frac{X_L}{R}$$

So, $$Q\text{-factor} = \frac{2\pi f_r L}{R} \tag{2.15}$$

We know that $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

Substituting the value of $f_r$ in Eq. (2.15), we get

$$Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{2.16}$$

### Bandwidth of series resonance circuit

**Bandwidth** (BW) of a series resonance circuit is defined as the range of frequency over which circuit current is equal to or greater than 70.7% of maximum current (i.e., $I_r$, current at resonance).

The current in series $R$-$L$-$C$ circuit changes with frequency. Referring to the resonance curve in Fig. 2.110, it is clear that for any frequency lying between $f_1$ and $f_2$, the circuit current is equal to or greater than 70.7% of maximum current (i.e., $I_r = V/R$). Therefore, $(f_2 - f_1)$ is the bandwidth of the circuit, i.e.,

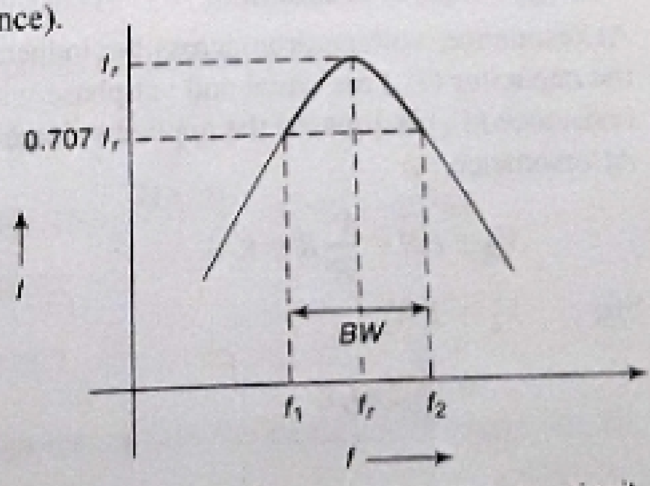


Fig. 2.110 Bandwidth of series resonance circuit

Bandwidth, $\text{BW} = (f_2 - f_1)$ Hz

or $\text{BW} = (\omega_2 - \omega_1)$ rad/sec (2.17)

It can be proved that, for a series resonance circuit,

$$BW = \frac{R}{2\pi L} \text{ Hz} \tag{2.18}$$

or $$BW = \frac{R}{L} \text{ rad/sec}$$

Note that $f_1$ and $f_2$ are the limiting frequencies at which current is exactly equal to 70.7% of the maximum value. The frequency $f_1$ is called lower cut-off frequency and the frequency $f_2$ is called upper cut-off frequency. The resonant frequency is sufficiently centred w.r.t. the two cut-off frequencies ($f_1$ and $f_2$). Then

$$f_2 = f_r + \frac{BW}{2} \tag{2.19}$$

$$f_1 = f_r - \frac{BW}{2}$$

In case of angular frequency ($\omega$), we have

$$\omega_2 = \omega_r + \frac{BW}{2}$$

$$\omega_1 = \omega_r - \frac{BW}{2} \tag{2.20}$$

***Relation between Q-factor and bandwidth (BW)***

We know that

$$Q\text{-factor} = \frac{2\pi f_r L}{R}$$

or $$Q\text{-factor} = \frac{2\pi L}{R} f_r$$

or $$Q\text{-factor} = \frac{f_r}{BW} \qquad \left(\because \frac{2\pi L}{R} = \frac{1}{BW}\right)$$

or $$f_r = Q\text{-factor} \times BW \tag{2.21}$$

***Voltage drops at resonance***

At resonance, voltage drop across the inductance ($V_L$) and voltage drop across the capacitor ($V_C$) are equal and antiphase with each other. The drop across the resistance ($V_R$) is equal to the applied voltage ($V$).

At resonance,

$$V_R = I_r R = \frac{V}{R} R = V \tag{2.22}$$

So, $$V_L = I_r X_L$$

$$= \frac{V}{R} 2\pi f_r L$$

$$= \frac{V}{R} 2\pi \frac{1}{2\pi\sqrt{LC}} L \qquad \left(\because f_r = \frac{1}{2\pi\sqrt{LC}}\right)$$

$$= \frac{V}{R}\sqrt{\frac{L}{C}} \tag{2.23}$$

Also $\quad V_C = I_r X_C$

$$= \frac{V}{R} \cdot \frac{1}{2\pi f_r C}$$

$$= \frac{V}{R} \cdot \frac{1}{2\pi \frac{1}{2\pi\sqrt{LC}} C} \qquad \left(\because f_r = \frac{1}{2\pi\sqrt{LC}}\right)$$

$$= \frac{V}{R}\sqrt{\frac{L}{C}} \tag{2.24}$$

---

**Example 2.57** A series *R-L-C* circuit has the following parameter values: $R = 10\ \Omega$, $L = 0.014$ H, $C = 100\ \mu$F.

Compute the following:

(a) Resonance frequency in rad/sec
(b) Quality factor of the circuit
(c) Bandwidth
(d) Lower and upper frequency points of the bandwidth
(e) Maximum value of the voltage appearing across the capacitor if the voltage $v = 1\sin 1000t$ is applied to the *R-L-C* circuit.

***Solution***

(a) $\omega_r = \dfrac{1}{\sqrt{LC}} = \dfrac{1}{\sqrt{0.014 \times 100 \times 10^{-6}}} = 845.15$ rad/sec

(b) $Q = \dfrac{1}{R}\sqrt{\dfrac{L}{C}} = \dfrac{1}{10}\sqrt{\dfrac{0.014}{100 \times 10^{-6}}} = 1.183$

(c) $\text{BW} = \dfrac{R}{L} = \dfrac{10}{0.014} = 714.29$ rad/sec

(d) Lower and upper frequency points of the bandwidth

$$\omega_1 = \omega_r - \frac{\text{BW}}{2} = 845.15 - \frac{714.29}{2} = 488 \text{ rad/sec}$$

$$\omega_2 = \omega_r + \frac{\text{BW}}{2} = 845.15 + \frac{714.29}{2} = 1202.3 \text{ rad/sec}$$

(e) Applied voltage, $v_1 = 1 \sin 1000t$

So, $\quad V = \dfrac{V_m}{\sqrt{2}} = \dfrac{1}{\sqrt{2}} = 0.707$

At resonance, voltage that appears across the capacitor is maximum and given by

$$V_C = \frac{V}{R}\sqrt{\frac{L}{C}} = \frac{0.707}{10}\sqrt{\frac{0.014}{100 \times 10^{-6}}} = 0.837 \text{ V}$$

**Example 2.58** A voltage of $v = 10 \sin \omega t$ is applied to R-L-C circuit. At the resonance frequency of the circuit, the maximum voltage across the capacitor is found to be 500 V. Moreover, the bandwidth is known to be 400 rad/sec, and impedance at resonance is 100 Ω.

(i) Find the resonant frequency.
(ii) Compute the upper and lower limits of the bandwidth.
(iii) Determine the value of $L$ and $C$ for this circuit.

***Solution***

A voltage of $v = 10 \sin \omega t$ is applied to $R$-$L$-$C$ circuit as shown in Fig. 2.111.

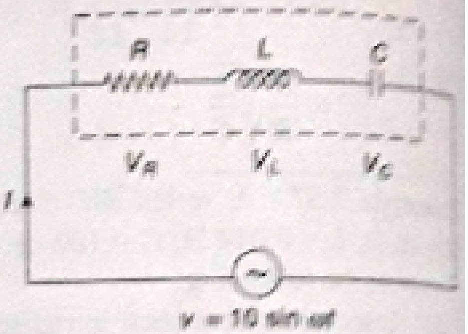



Fig. 2.111

Given: $v = 10 \sin \omega t$

RMS value of the applied voltage,

$$V = \frac{10}{\sqrt{2}} = 7.071 \text{ V}$$

At resonance, $V_C = 500$ V

Bandwidth, BW = 400 rad/sec

or $$\text{BW} = \frac{400}{2\pi} = 63.66 \text{ Hz}$$

Impedance at resonance, $Z_r = R = 100\ \Omega$

(i) $$Q\text{-factor} = \frac{V_C}{V} = \frac{500}{7.071} = 70.71$$

Resonance frequency, $f_r = \text{BW} \times Q\text{-factor}$
$= 63.66 \times 70.71$
$= 4501.4$ Hz

(ii) Lower limit of the bandwidth,

$$f_1 = f_r - \frac{\text{BW}}{2} = 4501.4 - \frac{63.66}{2} = 4469.57 \text{ Hz}$$

Now, upper limit of the bandwidth

$$f_2 = f_r + \frac{\text{BW}}{2} = 4501.4 + \frac{63.66}{2} = 4533.23 \text{ Hz}$$

(iii) $$\text{BW} = \frac{R}{2\pi L} \text{ Hz}$$

or $$63.66 = \frac{100}{2\pi \times L}$$

or $$L = 0.25 \text{ H}$$

Now, $$f_r = \frac{1}{2\pi\sqrt{LC}}$$

So, $$4501.4 = \frac{1}{2\pi\sqrt{0.25 \times C}}$$

or $$C = 5 \times 10^{-9} \text{ F}$$

or $$C = 5 \text{ nF}$$

**Example 2.60** A coil of resistance 2 Ω and inductance of 0.01 H is connected in series with a capacitor across 230 V mains. What must be the capacitance, in order that maximum current occurs at a frequency of 50 Hz? Find also the current and the voltage across the capacitor.

***Solution***

A coil is connected in series with a capacitance. The resultant circuit is a series *R-L-C* circuit (see Fig. 2.113).

The value of capacitance is required. So, at 50 Hz, maximum current flows in the circuit (i.e., resonance occurs in the circuit). Thus, resonant frequency, $f_r = 50$ Hz.

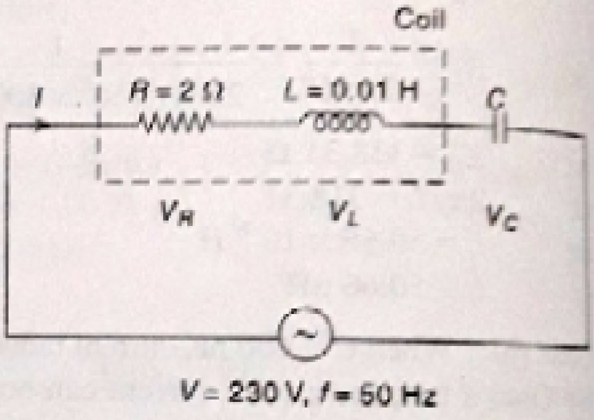



Fig. 2.113

Now, $$f_r = \frac{1}{2\pi\sqrt{LC}}$$

or $$50 = \frac{1}{2\pi\sqrt{0.01 \times C}}$$

or $$C = 1.013 \times 10^{-3}\ \text{F}$$

At resonance circuit current, $I_r = \dfrac{V}{R} = \dfrac{230}{2} = 115$ A

At resonance, voltage across the capacitor can be calculated as

$$V_C = \frac{V}{R}\sqrt{\frac{L}{C}} = \frac{230}{2}\sqrt{\frac{0.01}{1.013 \times 10^{-3}}} = 361.32\ \text{V}$$

**Example 2.61** A coil of 10 Ω resistance and 0.1 H inductance is connected in series with a capacitor of 200 μF capacitance. Calculate the current, the coil voltage, and the capacitor voltage. The supply is 230 V at 50 Hz. At what frequency will the circuit resonate? Calculate the voltages at resonant frequency across the coil and the capacitor. For this, assume that the supply voltage is 230 V of variable frequency.

*Solution*

*Case (i)* A coil is connected in series with a capacitor across a supply of 230 V at 50 Hz (see Fig. 2.114).

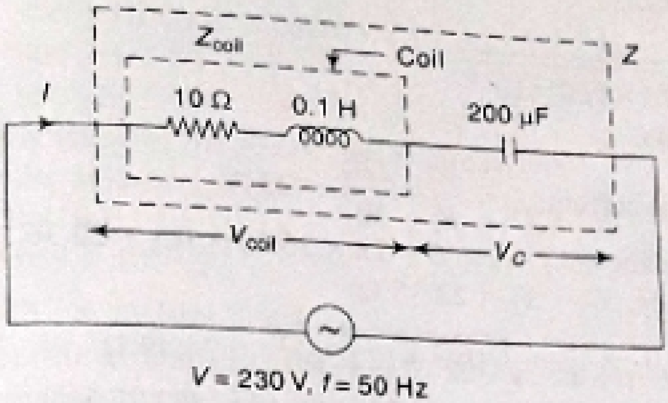



**Fig. 2.114**

The resulting circuit is *R-L-C* series circuit.

So, $X_L = 2\pi f L = 2\pi \times 50 \times 0.1 = 31.42\ \Omega$

Also, $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{2\pi \times 50 \times 200 \times 10^{-6}} = 15.92\ \Omega$

Hence, circuit impedance, $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(10)^2 + (31.42 - 15.92)^2}$
$= 18.45\ \Omega$

By Ohm's law,

Circuit current, $I = \dfrac{V}{Z} = \dfrac{230}{18.45} = 12.47\ \text{A}$

Impedance of the coil, $Z_{coil} = \sqrt{R^2 + X_L^2} = \sqrt{(10)^2 + (31.42)^2} = 32.97\ \Omega$

Voltage across the coil, $V_{coil} = IZ_{coil} = 12.47 \times 32.97 = 411.14\ \text{V}$

Capacitor voltage, $V_C = IX_C = 12.47 \times 15.92 = 198.52\ \text{V}$

*Case (ii)* The supply is 230 V of variable frequency (circuit remains same) as shown in Fig. 2.115.

Let the frequency of the circuit is $f_r$.

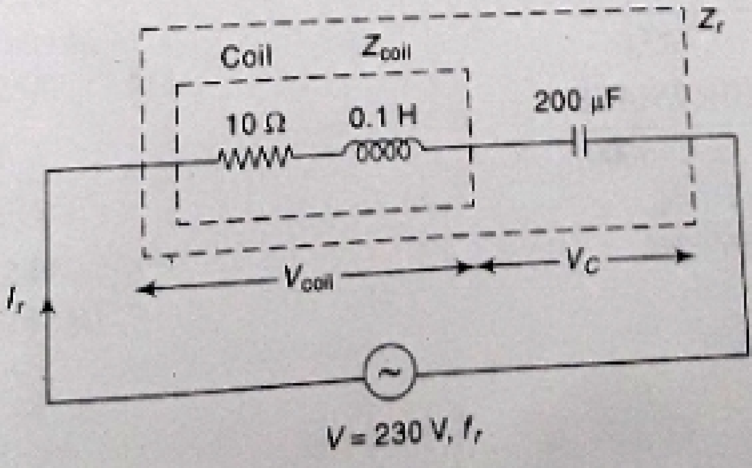



**Fig. 2.115**

The resonant frequency ($f_r$) is required. So,

$$f_r = \frac{1}{2\pi\sqrt{LC}}$$
$$= \frac{1}{2\pi\sqrt{0.1\times 200\times 10^{-6}}}$$
$$= 35.59 \text{ Hz}$$

At resonance, circuit current, $I_r = \frac{V}{R} = \frac{230}{10} = 23$ A

Now, inductive reactance, $X_L = 2\pi fL = 2\pi\times 35.59\times 0.1 = 22.36\ \Omega$

At resonant frequency, $X_C = X_L = 22.36\ \Omega$

Impedance of the coil, $Z_{coil} = \sqrt{(10)^2 + (22.36)^2} = 24.49\ \Omega$

Voltage across the coil, $V_{coil} = I_r Z_{coil} = 23\times 24.49 = 563.27$ V

Capacitor voltage, $V_C = I_r X_C = 23\times 22.36 = 514.28$ V

**Example 2.62** Determine the parameter of an *R-L-C* series circuit that will resonate at 10,000 Hz, has a bandwidth of 1000 Hz and draws 15.3 W from a 200 V supply, operating at the resonant frequency of the circuit.

***Solution***

At resonance, the voltage $V_R$ (voltage drop across resistance) is equal to the applied voltage, i.e., $V_R = 200$ V.

So, $$P = \frac{V_R^2}{R}$$

or $$15.3 = \frac{(200)^2}{R}$$

or $$R = 2614.38\ \Omega$$

Quality factor of the circuit, $Q\text{-factor} = \frac{f_r}{\text{BW}} = \frac{10000}{1000} = 10$

But $$Q\text{-factor} = \frac{2\pi f_r L}{R}$$

So, $$L = \frac{Q\text{-factor}\times R}{2\pi f_r}$$
$$= \frac{10\times 2614.38}{2\pi\times 10000}$$
$$= 0.416 \text{ H}$$
$$= 416 \text{ mH}$$

At resonance,

$$X_L = X_C$$

or $$2\pi f_r L = \frac{1}{2\pi f_r C}$$

So, $C = \dfrac{1}{4\pi^2 f_r^2 L}$

$$= \frac{1}{4\pi^2 \times (10000)^2 \times 0.416}$$
$$= 609 \times 10^{-12} \text{ F}$$
$$= 609 \text{ pF}$$

Thus, the parameters of the *R-L-C* series circuit are:

$R = 2614.38\ \Omega$

$L = 0.416$ H

$C = 609 \times 10^{-12}$ F

**Example 2.63** A resistor and a capacitor are in series with a variable inductor (pure). When the circuit is connected to 220 V, 50 Hz supply, the maximum current obtainable by varying the inductance is 0.314 A. The voltage across capacitance is then 800 V. Find the circuit constants.

***Solution***

The conditions of the example are shown in Fig. 2.116.

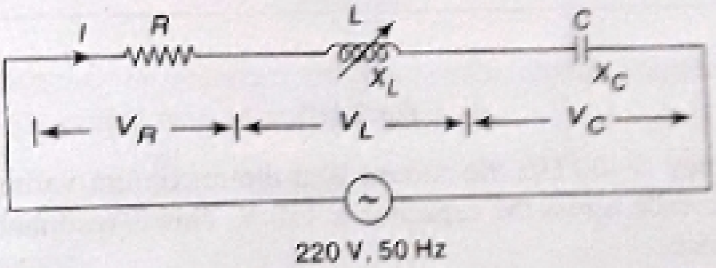



**Fig. 2.116**

The maximum current of a series *RLC* circuit is at resonance.

$\therefore \quad f_r = 50$ Hz and the circuit is resonated by varying inductor *L*.

$$\therefore \quad Z_r = \frac{V}{I_r} = \frac{220}{0.314} = 700.64\ \Omega$$

At resonance, $Z_r = R = 700.64\ \Omega$

Now, $V_C = I_r X_C$

$800 = 0.314 X_C$

$\therefore \quad X_C = 2547.77\ \Omega$

$$\therefore \quad \frac{1}{2\pi \times 50 \times C} = 2547.77$$

$\therefore \quad C = 1.249 \times 10^{-6}$ F

$\therefore \quad C = 1.249\ \mu$F

Now, $f_r = \dfrac{1}{2\pi\sqrt{LC}}$

$$\therefore \quad 50 = \frac{1}{2\pi\sqrt{L \times 1.249 \times 10^{-6}}}$$

$$\therefore \quad L = 8.11\ \text{H}$$

**Example 2.64** A 20 Ω resistor is connected in series with an inductor and a capacitor, across a variable frequency, 25 V supply. When the frequency is 400 Hz, the current is at its maximum value of 0.5 A and the potential difference across the capacitor is 150 V. Calculate the resistance and inductance of the inductor.

***Solution***

The conditions of the example are shown in Fig. 2.117.

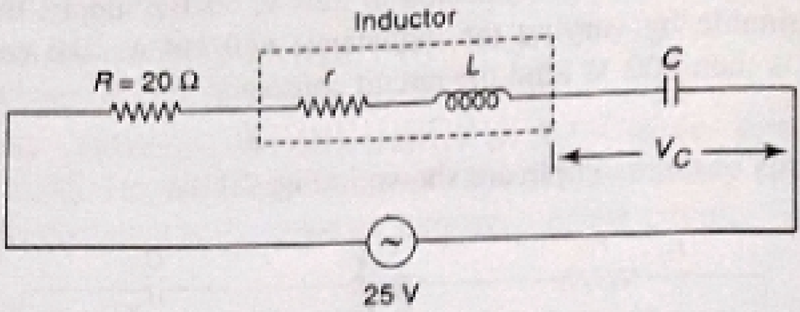



**Fig. 2.117**

When frequency is 400 Hz, the current is at the maximum value of 0.5 A and potential difference across the capacitor is 150 V. This is resonance condition. So, at resonance,

$$I_r = \frac{V}{(R + r)}$$

$$\therefore \quad 0.5 = \frac{25}{(R + r)}$$

$$\therefore \quad (R + r) = 50\ \Omega$$

$$\therefore \quad (20 + r) = 50$$

$$\therefore \quad r = 30\ \Omega$$

Now, $V_C = I_r X_C$

$$150 = 0.5 X_C$$

$$\therefore \quad X_C = 300\ \Omega$$

Now, at resonance,

$$X_L = X_C$$

$$\therefore \quad 2\pi \times 400 \times L = 300$$

$$\therefore \quad L = 119.37\ \text{mH}$$

## 2.3 AC Parallel Circuits

An ac circuit that has number of branches connected in parallel such that the voltage across them is same, is called an **ac parallel circuit**. The current in any one branch depends upon the impedance of that branch. The total line current supplied to the circuit is the phasor sum of branch currents. Parallel circuits are used more often in practice as all lighting and power circuits are constant voltage circuits and the loads/equipments are connected in parallel.

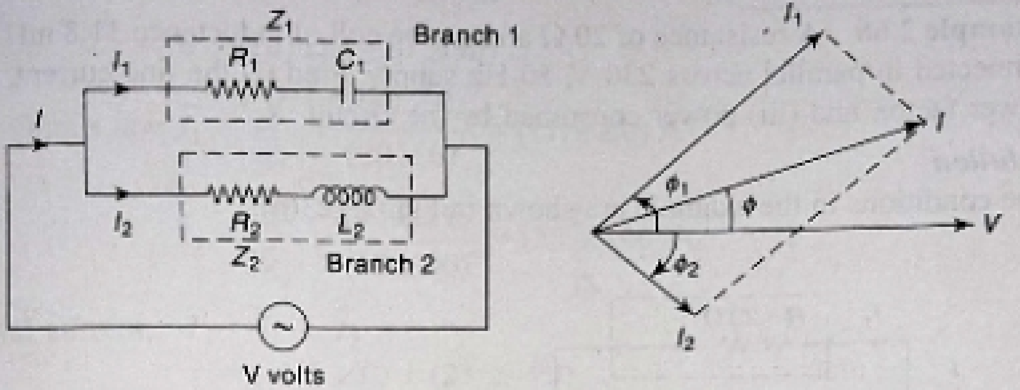
While analysing an ac parallel circuit, two important points must be kept in mind. First, a parallel circuit, in fact, consists of two or more series circuits connected in parallel. Therefore, each branch of the circuit can be analysed separately as a series circuit and then the effect of the separate branches can be combined. Secondly, alternating voltages and currents are phasor quantities. This implies that both magnitudes and phase angles must be taken account while carrying out circuit calculations. There are three methods of solving ac parallel circuits, namely;

1. By phasor diagram
2. By phasor algebra
3. Admittance method

The use of a particular method will depend upon the conditions of the problem. However, in general, that method should be used which yields quick results.

### 2.3.1 Phasor diagram

In this method, we find the magnitude and phase angle of each branch current. We then draw the phasor diagram taking voltage as reference phasor. The circuit or line current is the phasor sum of the branch currents and can be determined by parallelogram law. Consider a parallel circuit as shown in Fig. 2.122(a), consisting of two branches and connected to an alternating voltage of $V$ volts.




(a) Circuit diagram (b) Phasor diagram

**Fig. 2.122** Analysis of parallel circuit by phasor diagram

**Branch 1** $Z_1 = \sqrt{R_1^2 + X_{C1}^2}\ ; I_1 = \dfrac{V}{Z_1}; \phi_1 = \tan^{-1}\dfrac{X_{C1}}{R_1}$

The current $I_1$ in branch 1 leads the applied voltage $V$ by $\phi_1^\circ$.

**Branch 2** $Z_2 = \sqrt{R_2^2 + X_{L2}^2}\ ; I_2 = \dfrac{V}{Z_2}; \phi_2 = \tan^{-1}\dfrac{X_{L2}}{R_2}$

The current $I_2$ in branch 2 lags behind the applied voltage $V$ by $\phi_2^\circ$.

The phasor diagram is shown in Fig. 2.122(b). The line current $I$ is the phasor sum of $I_1$ and $I_2$.

The phasor diagram method is suitable only when the parallel circuit is simple and contains two branches. However, if the parallel circuit is complex having more than two branches, this method becomes very inconvenient. In such cases, use of phasor algebra is recommended to solve parallel-circuit problems.

### 2.3.2 Phasor Algebra

In this method, voltages, currents and impedances are expressed in the complex form, i.e., either in the rectangular or polar form. Since complex form includes both magnitude and phase angle, the solution of parallel circuit problems can be obtained mathematically by using the rules of phasor algebra. This eliminates the need of phasor diagram. Referring back to the parallel circuit shown in Fig. 2.122(a), we have

$$\bar{V} = (V - j\,0) = (V \angle 0)$$

$$\bar{Z}_1 = (R_1 - j\,X_{C1}) = (Z_1 \angle -\phi_1)$$

$$\bar{Z}_2 = (R_2 + jX_{L2}) = (Z_2\angle\phi_2)$$

Branch current, $\bar{I}_1 = \dfrac{\bar{V}}{\bar{Z}_1}$

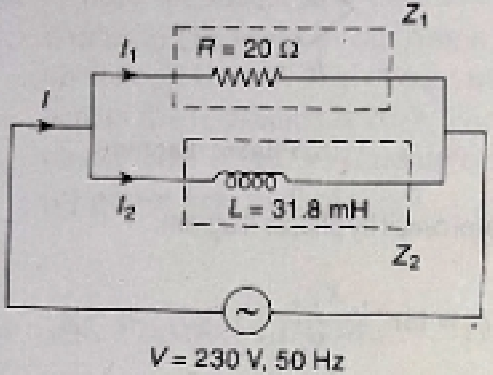
Branch current, $\bar{I}_2 = \dfrac{\bar{V}}{\bar{Z}_2}$

Total circuit current, $\bar{I} = \bar{I}_1 + \bar{I}_2$

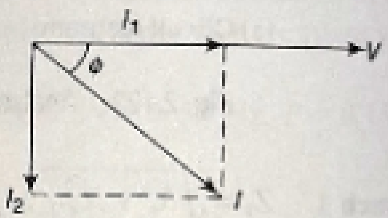
**Example 2.66** A resistance of 20 Ω and a pure coil of inductance 31.8 mH are connected in parallel across 230 V, 50 Hz supply. Find (i) the line current, (ii) power factor, and (iii) power consumed by the circuit.

***Solution***

The conditions in the example are shown in Fig. 2.123(a).




(a) Circuit diagram




(b) Phasor diagram

**Fig. 2.123**

Given: $V = 230$ V

$f = 50$ Hz

$L = 31.8 \times 10^{-3}$ H $\Rightarrow$ $X_L = 2\pi fL = 2\pi \times 50 \times 31.8 \times 10^{-3} = 10\ \Omega$

Branch current, $I_1 = \dfrac{V}{R} = \dfrac{230}{20} = 11.5$ A

The current $I_1$ is in phase with the applied voltage.

Branch current, $I_2 = \dfrac{V}{X_L} = \dfrac{230}{10} = 23$ A

The current $I_2$ lags behind the applied voltage by 90°.

So, line current, $I = \sqrt{I_1^2 + I_2^2}$

$= \sqrt{(11.5)^2 + (23)^2}$

$= 25.71$ A

From phasor diagram,

$$\text{pf} = \cos\phi = \frac{I_1}{I} = \frac{11.5}{25.71} = 0.447 \text{ lag}$$

Power consumed, $P = VI\cos\phi$

$$= 230 \times 25.71 \times 0.447$$
$$= 2643 \text{ W}$$

*Alternative method:*

Taking $V$ as reference phasor,

$$\bar{V} = (230 \angle 0) \text{ V}$$
$$\bar{Z}_1 = (20 + j0)\ \Omega = (20 \angle 0)\ \Omega$$
$$\bar{Z}_2 = (0 + j10)\ \Omega = (10 \angle 90)\ \Omega$$

By Ohm's law, $\bar{I}_1 = \dfrac{\bar{V}}{\bar{Z}_1} = \dfrac{(230 \angle 0)}{(20 \angle 0)} = (11.5 \angle 0)$ A

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{(230 \angle 0)}{(10 \angle 90)} = (23 \angle -90) \text{ A}$$

Total current, $\bar{I} = \bar{I}_1 + \bar{I}_2$

$$= (11.5 \angle 0) + (23 \angle -90)$$
$$= (11.5 + j0) + (0 - j23)$$
$$= (11.5 - j23) \text{ A}$$
$$= (25.71 \angle -63.43) \text{ A}$$

So, pf $= \cos\phi$

$$= \cos 63.43$$
$$= 0.447 \text{ lagging}$$

Power, $P = VI\cos\phi$

$$= 230 \times 25.71 \times 0.447$$
$$= 2643 \text{ W}$$

**Example 2.67** For a circuit shown in Fig. 2.124, determine:

(i) Total impedance of the circuit and total current

(ii) Branch current $I_1$ and $I_2$

(iii) Power factor of each branch and total power factor

(iv) Power consumed by each branch

***Solution***

Assuming the different impedances, we get Fig. 2.125.

Taking applied voltage as reference phasor,

$$\bar{V} = (230 \angle 0)$$
$$L = 10 \times 10^{-3} \text{ H};\ X_L = 2\pi f L = 2\pi \times 50 \times 10 \times 10^{-3} = 3.14\ \Omega$$
$$C = 50 \times 10^{-6} \text{ F};\ X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 50 \times 500 \times 10^{-6}} = 6.37\ \Omega$$
$$\bar{Z}_1 = (10 + j3.14)\ \Omega = (10.48 \angle 17.43)\ \Omega$$
$$\bar{Z}_2 = (10 - j6.37)\ \Omega = (11.86 \angle -32.5)\ \Omega$$

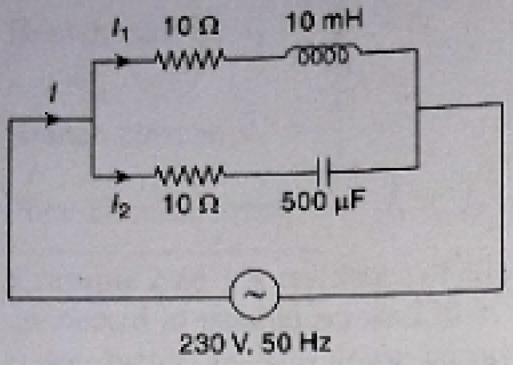


Fig. 2.124

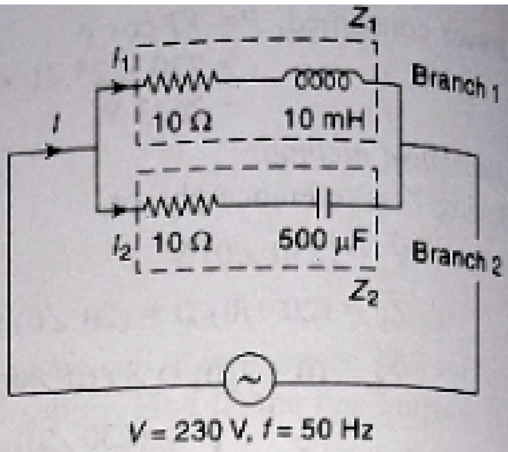


Fig. 2.125

(i) Total impedance of the circuit,

$$\bar{Z} = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$= \frac{(10.48 \angle 17.43)(11.86 \angle -32.5)}{(10 + j3.14) + (10 - j6.37)}$$

$$= \frac{(124.29 \angle -15.07)}{(20 - j3.23)}$$

$$= \frac{(124.29 \angle -15.07)}{(20.26 \angle -9.17)}$$

$$= (6.13 \angle -5.9)\ \Omega$$

Total current, $I = \frac{\bar{V}}{\bar{Z}} = \frac{(230 \angle 0)}{(6.13 \angle -5.9)} = (37.52 \angle 5.9)$ A

(ii) Branch current, $\bar{I}_1 = \frac{\bar{V}}{\bar{Z}_1} = \frac{(230 \angle 0)}{(10.48 \angle 17.43)} = (21.95 \angle -17.43)$ A

$$\bar{I}_2 = \frac{\bar{V}}{\bar{Z}_2} = \frac{(230 \angle 0)}{(11.86 \angle -32.5)} = (19.39 \angle 32.5)\ \text{A}$$

(iii) Power factor of branch 1,

$(\text{pf})_1 = \cos \phi_1$

where $\phi_1$ is the phase angle of branch 1. From polar form of impedance $Z_1$, $\phi_1 = 17.43$. Branch 1 is inductive.

So, $(\text{pf})_1 = \cos 17.43$

$= 0.954$ lagging

Power factor of branch 2,

$$(pf)_2 = \cos \phi_2$$

where $\phi_2$ is the phase angle of branch 2. From polar form of impedance $Z_2$, $\phi_2 = -32.5$. Branch 2 is capacitive.

So, $(pf)_2 = \cos(-32.5)$

$= 0.843$ leading

Total power factor.

$$pf = \cos \phi$$

where $\phi$ is the phase angle total circuit. From polar form of impedance $Z$, $\phi = -5.9$. As phase angle is negative total circuit is capacitive.

So, $pf = \cos(-5.9)$

$= 0.995$ leading

(iv) Power consumed by branch 1,

$$P_1 = I_1^2 R = (21.95)^2 \times 10 = 4.818 \text{ kW}$$

Power consumed by branch 2,

$$P_2 = I_2^2 R = (19.39)^2 \times 10 = 3.76 \text{ kW}$$

**Example 2.70** Two circuits $A$ and $B$ are connected in parallel across a 200 V, 50 Hz mains. Circuit $A$ consists of a resistance of 10 Ω and an inductance of 0.12 H connected in series. Circuit $B$ consists of a resistance of 20 Ω in series with a capacitor of 40 μF. Calculate (i) current in each branch and (ii) power factor. Draw phasor diagram.

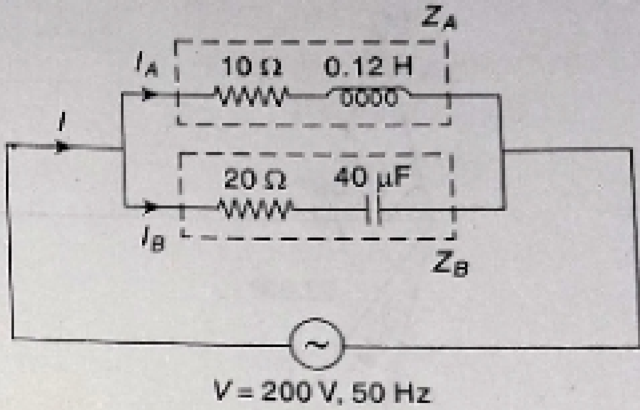


Fig. 2.128

***Solution***

The conditions in the example are shown in Fig. 2.128. Assuming the various impedances, currents and voltage, we have,

$$L = 0.12 \text{ H};\ X_L = 2\pi fL = 2\pi \times 50 \times 0.12 = 37.7\ \Omega$$

$$C = 40 \times 10^{-6} \text{ F};\ X_C = \frac{1}{2\pi FC} = \frac{1}{2\pi\times 50\times 40\times 10^{-6}} = 79.58\ \Omega$$

$$\overline{Z}_A = (10 + j37.7)\ \Omega = (39\ \angle 75.14)\ \Omega$$

$$\overline{Z}_B = (20 - j79.58)\ \Omega = (82.05\ \angle{-75.89})\ \Omega$$

Total or equivalent impedance of the circuit,

$$Z_{eq} = \frac{\overline{Z}_A\,\overline{Z}_B}{\overline{Z}_A + \overline{Z}_B} = \frac{(39\angle 75.14)(82.05\angle{-75.89})}{(10+j37.7)(20-j79.58)}$$

$$= \frac{(3200\angle{-0.75})}{(30-j41.88)} = \frac{(3200\angle{-0.75})}{(51.52\angle{-54.38})}$$

$$= (62.11\ \angle 53.63)\ \Omega$$

Taking applied voltage as reference, we have

$$\overline{V} = (200\ \angle 0)\ \text{V}$$

(i) Branch current, $\overline{I}_A = \dfrac{\overline{V}}{\overline{Z}_A} = \dfrac{(200 \angle 0)}{(39 \angle 75.14)} = (5.13 \angle -75.14)$ A

Branch current, $\overline{I}_B = \dfrac{\overline{V}}{\overline{Z}_B} = \dfrac{(200 \angle 0)}{(82.05 \angle -75.89)} = (2.44 \angle 75.89)$ A

(ii) pf = cos 53.63
= 0.59 lagging

*Phasor diagram (Fig. 2.129):*

Taking applied voltage as reference phasor.

From circuit diagram, $\overline{I} = \overline{I}_A + \overline{I}_B$

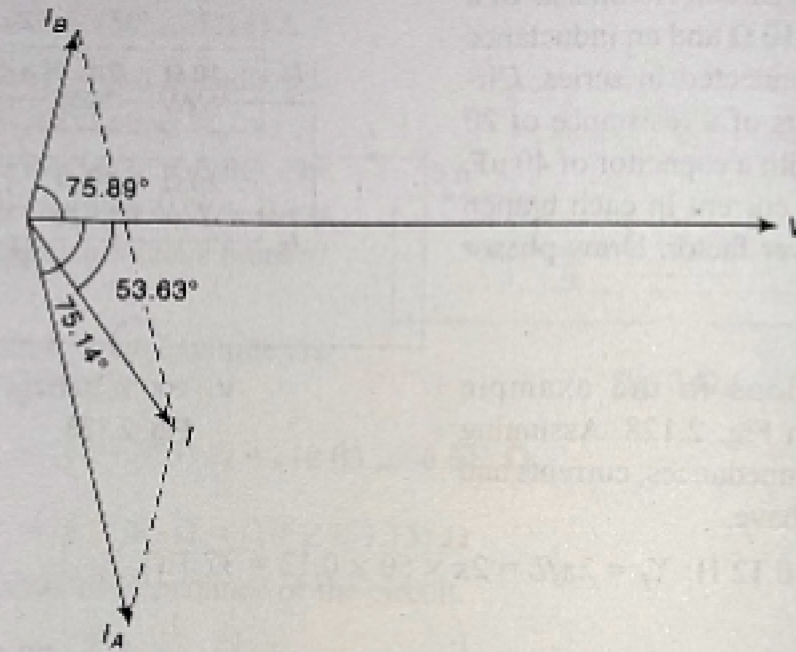



Fig. 2.129

**Example 2.71** Two impedances (20 ∠–45) Ω and (30 ∠30) Ω are connected in series across a certain ac supply and the resulting current is drawn to be 10 A. If the supply voltage remains unchanged, calculate the supply current, when the two impedances are connected in parallel.

***Solution***

Let $\overline{Z}_1 = (20 \angle -45)\ \Omega = (14.14 - j14.14)\ \Omega$

$\overline{Z}_2 = (30 \angle 30)\ \Omega = (26 + j15)\ \Omega$

*When impedances are in series, current is 10 A (Fig. 2.130):*

Total impedances,

$\overline{Z}_{eq} = \overline{Z}_1 + \overline{Z}_2$

$= (14.14 - j14.14) + (26 + j15)$

$= (40.14 + j0.86)\ \Omega$

$= (40.15 \angle 1.23)\ \Omega$

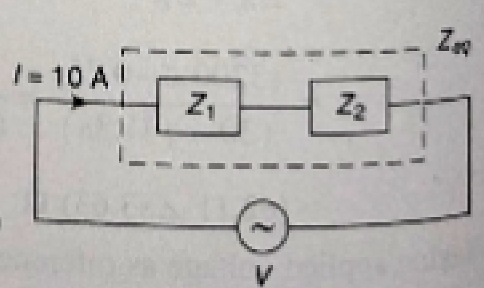



Fig. 2.130

Now, taking '$I$' as reference, $\bar{I} = (10 \angle 0)$

$\therefore$ Supplied voltage $= \bar{I}\bar{Z}_{eq}$

$$= (10 \angle 0)(40.15 \angle 1.23)$$
$$= (401.5 \angle 1.23) \text{ V}$$

*When impedances are in parallel across the same supply (Fig. 2.131):*
Total impedances,

$$\bar{Z}_{eq} = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$= \frac{(20\angle{-45})(30\angle 30)}{(14.14 - j14.14) + (26 + j15)}$$

$$= \frac{(600\angle{-15})}{(40.14 + j0.86)}$$

$$= \frac{(600\angle{-15})}{(40.15\angle 1.23)}$$

$$= (14.94\angle{-16.23})\,\Omega$$

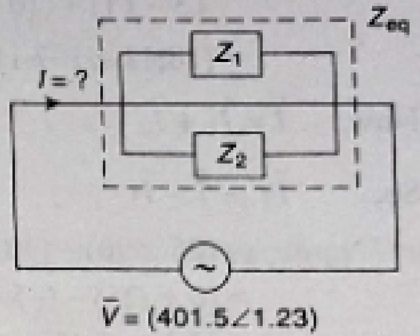



Fig. 2.131

Hence, Supply current, $I = \frac{\bar{V}}{\bar{Z}_{eq}}$

$$= \frac{(401.5\angle 1.23)}{(14.94\angle{-16.23})}$$

$$= (26.87 \angle 17.46) \text{ A}$$

**Example 2.72** Find $I_1$ and $I_2$ in the circuit shown in Fig. 2.132.

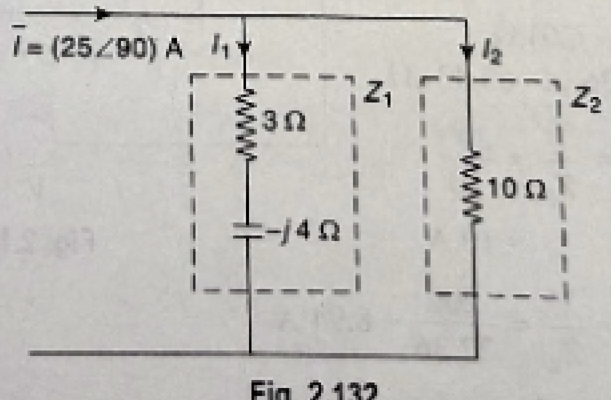



Fig. 2.132

***Solution***

$$\bar{I} = (25 \angle 90) \text{ A}$$

$$\bar{Z}_1 = (3 - j4)\,\Omega = (5 \angle{-53.13})\,\Omega$$

$$\bar{Z}_2 = (10 + j0)\,\Omega = (10 \angle 0)\,\Omega$$

By current division rule,

$$\bar{I}_1 = \frac{\bar{I} \times \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$$

$$= \frac{(25\angle 90)(10\angle 0)}{(3-j4)+(10+j0)} = \frac{(250\angle 90)}{(13-j4)} = \frac{(250\angle 90)}{(13.6\angle -17.1)}$$

$$= (18.38\ \angle 107.1)\ \text{A}$$

Now, $\bar{I} = \bar{I}_1 + \bar{I}_2$

So, $\bar{I}_2 = \bar{I} - \bar{I}_1$

$$= (25\ \angle 90) - (18.38\ \angle 107.1)$$
$$= (0 + j25) - (-5.4 + j17.57)$$
$$= (5.4 + j7.43)$$
$$= (9.19\ \angle 53.99)\ \text{A}$$

**Example 2.73** A voltage of $(200\ \angle 53.13)$ V is applied across two impedances in parallel. The values of the impedances are $(12 + j16)\ \Omega$ and $(10 - j20)\ \Omega$. Determine kVA, kVAR, and kW in each branch and power factor of the whole circuit.

***Solution***

The conditions in the example are shown in Fig. 2.133.

Given:

$$\bar{V} = (200\ \angle\ 53.13)\ \text{V}$$

Let $\bar{Z}_1 = (12 + j16)\ \Omega$
$= (20\ \angle\ 53.13)\ \Omega$

$\bar{Z}_2 = (10 - j20)\ \Omega$
$= (22.36\ \angle -63.43)\ \Omega$

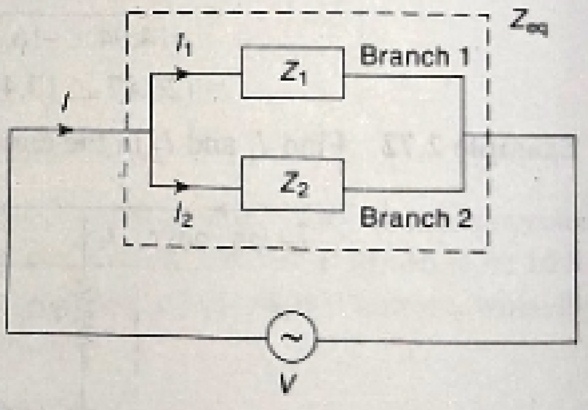



Fig. 2.133

Branch current, $I_1 = \dfrac{V}{Z_1} = \dfrac{200}{20}$

$$= 10\ \text{A}$$

Branch current, $I_2 = \dfrac{V}{Z_2} = \dfrac{200}{22.36} = 8.94\ \text{A}$

For branch 1 (i.e., impedance $Z_1$):

$$\text{kW} = \frac{VI_1 \cos\phi}{1000} = \frac{200 \times 10 \times \cos 53.13^\circ}{1000} = 1.2$$

$$\text{kVAR} = \frac{VI_1 \sin\phi}{1000} = \frac{200 \times 10 \times \sin 53.13^\circ}{1000} = 1.6$$

$$kVA = \frac{VI_1}{1000} = \frac{200 \times 10}{1000} = 2$$

For branch 2 (i.e., impedance $Z_2$):

$$kW = \frac{VI_2 \cos\phi}{1000} = \frac{200 \times 8.94 \times \cos(-63.43)}{1000} = 0.8$$

$$kVAR = \frac{VI_2 \sin\phi}{1000} = \frac{200 \times 8.94 \times \sin(-63.43)}{1000} = -1.6$$

$$kVA = \frac{VI_2}{1000} = \frac{200 \times 8.94}{1000} = 1.788$$

For power factor of the circuit, the phase angle ($\phi$) of the circuit is required. The phase angle ($\phi$) of the circuit can be calculated as:

Total impedance, $\bar{Z}_{eq} = \dfrac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2}$

$$= \frac{(20 \angle 53.13)(22.36 \angle -63.43)}{(12 + j16) + (10 - j20)}$$

$$= \frac{(447.2 \angle -10.3)}{(22 - j4)}$$

$$= \frac{(447.2 \angle -10.3)}{(22.36 \angle -10.3)}$$

$$= (20 \angle 0)\ \Omega$$

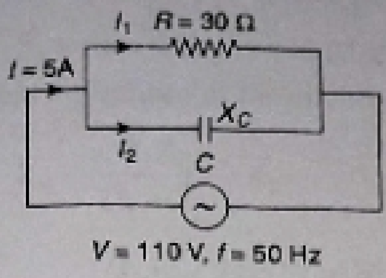
Thus, from polar form of impedance,

phase angle, $\phi = 0$

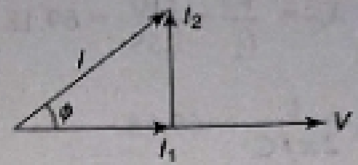
So, $pf = \cos\phi = \cos 0 = 1$ (unity)

**Example 2.75** A resistor of 30 $\Omega$ and capacitor of unknown value are connected in parallel across a 110 V, 50 Hz, 1 – $\phi$ supply. The combination draws a current of 5 A from the supply. Find the value of unknown capacitance of the capacitor. This combination is again connected across a 110 V supply of unknown frequency. It is observed that the total current drawn from the mains falls to 4 A. Determine the frequency of the supply. Draw the relevant diagrams.

*Solution*
The conditions in the example are shown in Fig. 2.136(a). The phasor diagram is shown in Fig. 2.136(b).



(a) Circuit diagram



(b) Phasor diagram

**Fig. 2.136**

From circuit diagram, by Ohm's law

Branch current, $I_1 = \frac{V}{R} = \frac{110}{30} = 3.67$ A

From phasor diagram,

$$I = \sqrt{I_1^2 + I_2^2}$$

or $\quad 5 = \sqrt{(3.67)^2 + I_2^2}$

or $\quad I_2 = 3.4$ A

Now, $\quad V = I_2 X_C$

or $\quad 110 = 3.4 \times X_C$

or $\quad X_C = 32.35\ \Omega$

Now, $\quad \frac{1}{2\pi f C} = 32.35$

or $\quad C = \frac{1}{2\pi \times 50 \times 32.35}$

or $\quad C = 98.39 \times 10^{-6}$ F
$\quad = 98.39\ \mu$F

***Case (ii)*** Now the same circuit is connected across a 110 V supply of unknown frequency. The circuit current is 4 A (see Fig. 2.137). From case (i), we know the capacitance, i.e., $C = 98.39$ μF.

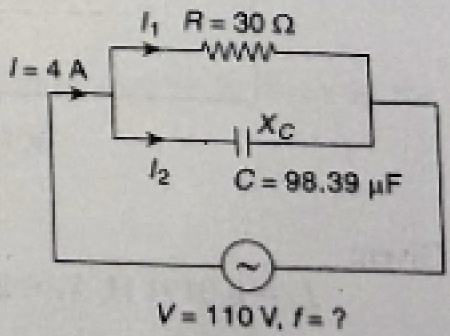


**Fig. 2.137**

From circuit diagram, by Ohm's law,

Branch current, $I_1 = \frac{V}{R} = \frac{110}{30} = 3.67$ A

As $\quad I = \sqrt{I_1^2 + I_2^2}$,

$$4 = \sqrt{(3.67)^2 + I_2^2}$$

So, $\quad I_2 = 1.59$ A

Hence, $X_C = \dfrac{V}{I_2} = \dfrac{110}{1.59} = 69.18\ \Omega$

So, $\quad \dfrac{1}{2\pi fC} = 69.18$

or $\quad f = \dfrac{1}{2\pi C \times 69.18} = \dfrac{1}{2\pi \times 98.39 \times 10^{-6} \times 69.18} = 23.38$ Hz

**Example 2.76** Determine the equivalent impedance of the circuit shown in Fig. 2.138.

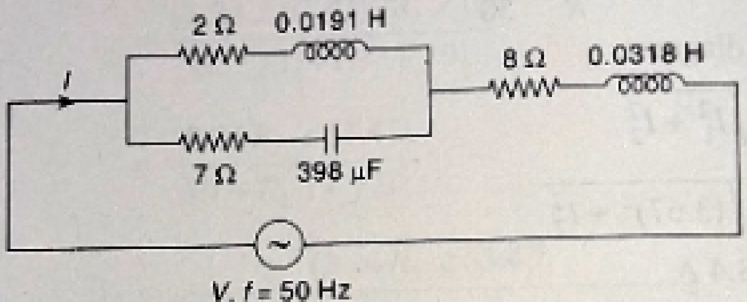



Fig. 2.138

***Solution***

By marking the various impedances in Fig. 2.138, we get Fig. 2.139.

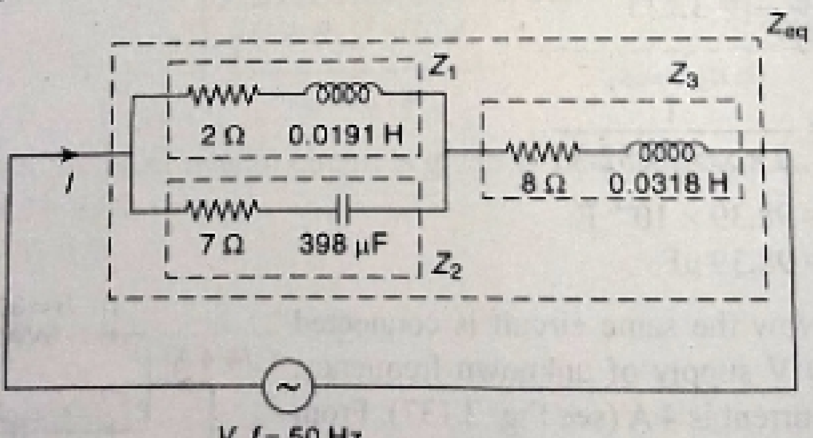



Fig. 2.139

Given:

$$L_1 = 0.0191 \text{ H}, X_L = 2\pi f L_1 = 2\pi \times 50 \times 0.0191 = 6\ \Omega$$

$$C = 398 \times 10^{-6} \text{ F}, X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 398 \times 10^{-6}} = 8\ \Omega$$

$L_3 = 0.0318$ H; $X_L = 2\pi f L_3 = 2\pi \times 50 \times 0.0318 = 10\ \Omega$

$\bar{Z}_1 = (2 + j6)\ \Omega = (6.32\ \angle 71.57)\ \Omega$

$\bar{Z}_2 = (7 - j8)\ \Omega = (10.63\ \angle -48.81)\ \Omega$

$\bar{Z}_3 = (8 + j10)\ \Omega = (12.81\ \angle 51.34)\ \Omega$

Equivalent impedance of the circuit,

$$\bar{Z}_{eq} = \frac{\bar{Z}_1 \bar{Z}_2}{\bar{Z}_1 + \bar{Z}_2} + \bar{Z}_3$$

$$= \frac{(6.32\ \angle 71.57)(10.63\ \angle -48.81)}{(2 + j6) + (7 - j8)} + (8 + j10)$$

$$= \frac{(67.18\ \angle 22.76)}{(9 - j2)} + (8 + j10)$$

$$= \frac{(67.18\ \angle 22.76)}{(9.22\ \angle -12.53)} + (8 + j10)$$

$$= (7.29\ \angle 35.29) + (8 + j10)$$
$$= (5.95 + j4.21) + (8 + j10)$$
$$= (13.95 + j14.21)\ \Omega$$
$$= (19.91\ \angle 45.53)\ \Omega$$

From polar form of equivalent impedance,

phase angle, $\phi = 31.32$

(as $\phi$ is positive, circuit is inductive)

So, $\quad pf = \cos 31.32$

$= 0.854$ lagging

**Example 2.78** In a series-parallel circuit, two parallel branches $A$ and $B$ are in series with $C$. The impedances are $\bar{Z}_A = (10 + j8)\ \Omega$, $\bar{Z}_B = (9 - j6)\ \Omega$, and $\bar{Z}_C = (3 + j2)\ \Omega$. If the voltage across $Z_C$ is $(100\ \angle 0)$ V, determine the values of $I_A$ and $I_B$.

***Solution***

Various voltages, impedances and current given in the example are marked in the circuit of Fig. 2.142.

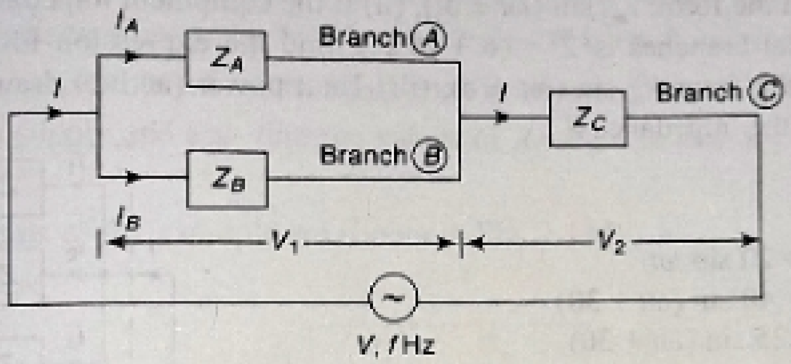



Fig. 2.142

Given:

$$\bar{Z}_A = (10 + j8)\ \Omega = (12.81\ \angle 38.66)\ \Omega$$

$$\bar{Z}_B = (9 - j6)\ \Omega = (10.82\ \angle -33.69)\ \Omega$$

$$\bar{Z}_C = (3 + j2)\ \Omega = (3.61\ \angle 33.69)\ \Omega$$

$$\bar{V}_2 = (100\ \angle\ 0)\ \text{V}$$

Circuit current, $\bar{I} = \dfrac{\bar{V}_2}{\bar{Z}_C} = \dfrac{(100\ \angle 0)}{(3.61\ \angle 33.69)} = (27.7\ \angle -33.69)$ A

By current division rule,

Branch current, $\bar{I}_A = \dfrac{\bar{I}\ \bar{Z}_B}{\bar{Z}_A + \bar{Z}_B}$

$$= \frac{(27.7\ \angle -33.69)(10.82\ \angle -33.69)}{(10 + j8) + (9 - j6)}$$

$$= \frac{(299.71\ \angle -67.38)}{(19 + j2)}$$

$$= \frac{(299.71\angle -67.38)}{(19.1\angle 6)}$$
$$= (15.69\angle -73.38)\text{ A}$$

Branch current, $\bar{I}_B = \bar{I} - \bar{I}_A$

$$= (27.7\angle -33.69) - (15.69\angle -73.38)$$
$$= (23.05 - j15.37) - (4.49 - j15.03)$$
$$= (18.56 - j0.34)\text{ A}$$
$$= (18.56\angle -1.05)\text{ A}$$

**Example 2.79** Three impedances $Z_1$, $Z_2$ and $Z_3$ are connected in parallel. A sinusoidal voltage $V_m \sin(\omega t \pm \phi)$ is applied to it. The current drawn by these impedances are $i_1 = 20 \sin \omega t$, $i_2 = 40 \sin(\omega t + 30)$, $i_3 = I_{m3} \sin(\omega t \pm \phi_3)$ and the total current drawn from the supply is $i = 25\sin(\omega t + 30)$ amp. Calculate (i) the current $i_3$ in the form: $I_{m3} \sin(\omega t \pm \phi_3)$; (ii) if the equipment impedance of these three parallel branches is $\bar{Z} = (6 + j8\ \Omega)$, find the expression for the supply voltage in the form $V_m \sin(\omega t \pm \phi)$; (iii) Total power (active) drawn from the mains; (iv) the impedance $Z_1$.

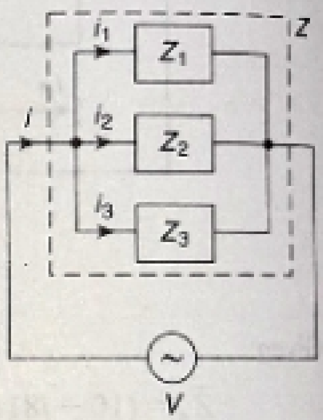



Fig. 2.143

***Solution***

Given:

$$i_1 = 20 \sin \omega t$$
$$i_2 = 40 \sin(\omega t + 30)$$
$$i = 25 \sin(\omega t + 30)$$
$$\bar{Z} = (6 + j8)\ \Omega = (10\angle 53.13)\ \Omega$$

Converting the standard sinusoidal form into polar forms,

$$\bar{I}_1 = (14.14\angle 0)\text{ A}$$
$$\bar{I}_2 = (28.28\angle 30)\text{ A}$$
$$I = (17.68\angle 30)\text{ A}$$

(i) $\bar{I} = \bar{I}_1 + \bar{I}_2 + \bar{I}_3$

So, $\bar{I}_3 = \bar{I} - (\bar{I}_1 + \bar{I}_2)$

$$= (17.68\angle 30) - [(14.14\angle 0) + (28.28\angle 30)]$$
$$= (15.31 + j8.84) - [(14.14 + j0) + (24.49 + j14.14)]$$
$$= (15.31 + j8.84) - (38.63 + j14.14)$$
$$= (-23.32 - j5.3)\text{ A}$$
$$= (23.91\angle -167.2)\text{ A}$$

Converting the polar form into standard sinusoidal form,

$$i_3 = 33.81 \sin(\omega t - 167.2)$$

(ii) Supply voltage, $\bar{V} = \bar{I}\bar{Z}$

$$= (17.68\angle 30)(10\angle 53.13)$$
$$= (176.8\angle 83.13)\text{ V}$$

Converting the polar form into standard sinusoidal form

$$v = 250 \sin(\omega t + 83.13)$$

(iii) Active power, $P = VI \cos \phi$

$$= 176.8 \times 17.68 \times \cos 53.13$$
$$= 1875 \text{ W}$$

(iv) Impedance, $Z_1 = \dfrac{\bar{V}}{\bar{I}_1} = \dfrac{(176.8 \angle 83.13)}{(14.14 \angle 0)}$

$$= (12.5 \angle 83.13)\ \Omega$$

**Example 2.80** Two impedances $(R_1 - jX_{C1})$ and $(R_2 + jX_{L2})$ are connected in parallel across supply voltage, $v = 100\sqrt{2}\sin(314t)$. The current flowing through the two impedances are given by $i_1 = 10\sqrt{2}\sin(314t + \pi/4)$ and $i_2 = 10\sqrt{2}\sin(314t + \pi/4)$, respectively. Find the equation of instantaneous value of total current drawn from supply and also find the values of $R_1$, $R_2$, $X_{c1}$, and $X_{L2}$.

***Solution***

The conditions of the example are shown in Fig. 2.144.

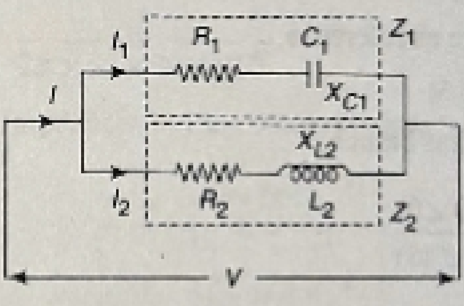



**Fig. 2.144**

Equation of branch current $i_1$ is given as:

$$i_1 = 10\sqrt{2}\sin(314t + \pi/4)$$

So, $I_1 = \dfrac{I_m}{\sqrt{2}} = \dfrac{10\sqrt{2}}{\sqrt{2}} = 10 \text{ A}$

So, $\bar{I}_1 = (10 \angle 45°)$ A (Polar form)

$\bar{I}_1 = (7.071 + j7.071)$ A (Rectangular form)

Equation of branch current $i_2$ is given as:

$$i_2 = 10\sqrt{2}\sin(314t + \pi/4)$$

So, $I_2 = \frac{I_m}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} = 10$ A

So, $\overline{I}_2 = (10\angle{-45°})$ A (Polar form)

$\overline{I}_2 = (7.071 - j7.071)$ A (Rectangular form)

Now, total current of the circuit:

$$\overline{I} = \overline{I}_1 + \overline{I}_2$$

$\therefore$ $\overline{I} = (7.071 + j7.071) + (7.071 - j7.071)$

$\therefore$ $\overline{I} = (14.142 + j0)$ (Rectangular form)

$\therefore$ $\overline{I} = (14.142 \angle 0)$ (Polar form)

Now, equation of instantaneous value of total current:

$$I_m = \sqrt{2}I = \sqrt{2} \times 14.142 = 20 \text{ A}$$

$\therefore$ $i = 20 \sin(314t)$

From equation of voltage,

$$V = \frac{V_m}{\sqrt{2}} = \frac{100\sqrt{2}}{\sqrt{2}} = 100 \text{ V}$$

Taking applied voltage as reference,

$$\overline{V} = (100\angle 0) \text{ V}$$

Now, impedance of first branch:

$$\overline{Z}_1 = \frac{\overline{V}}{\overline{I}_1} = \frac{(100\angle 0)}{(10\angle 45)}$$

$\therefore$ $\overline{Z}_1 = (10\angle{-45})\ \Omega$ (Polar form)

$\therefore$ $\overline{Z}_1 = (7.071 - j7.071)\ \Omega$ (Rectangular form)

So, $R_1 = 7.071\ \Omega$ and $X_{C1} = 7.071\ \Omega$

Now, impedance of second branch:

$$\overline{Z}_2 = \frac{\overline{V}}{\overline{I}_2} = \frac{(100\angle 0)}{(10\angle -45)}$$

$\therefore$ $\overline{Z}_2 = (10\angle 45)\ \Omega$ (Polar form)

$\therefore$ $\overline{Z}_2 = (7.071 + j7.071)$ (Rectangular form)

So, $R_2 = 7.071\ \Omega$ and $X_{L2} = 7.071\ \Omega$

**Example 2.81** A parallel circuit of 25 Ω resistor, 64 mH inductor, and 80 μF capacitor is connected across: 110 V, 50 Hz, single phase supply, as shown in Fig. 2.145. Calculate the current in individual element, the total current drawn from the supply, and the overall power factor of the circuit. Draw a neat phasor diagram.

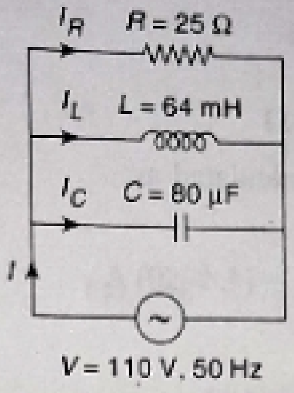



Fig. 2.145

***Solution***

Inductive reactance,

$$X_L = 2\pi fL = 2\pi \times 50 \times 64 \times 10^{-3} = 20.106\ \Omega$$

Capacitive reactance,

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 80 \times 10^{-6}} = 39.79\ \Omega$$

Marking the different impdances in the circuit, we get,

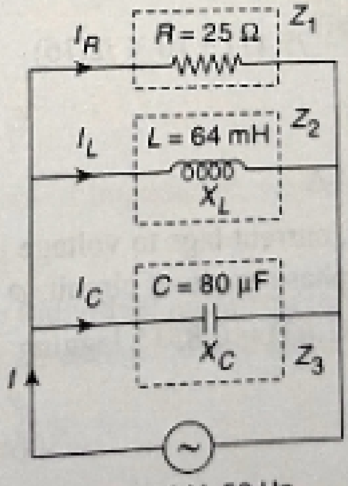



Fig. 2.146

Taking applied voltage as reference,

$$\overline{V} = (110\ \angle 0)\ \text{V}$$

Now, $\overline{Z}_1 = (25\ \angle 0)\ \Omega$

$$\overline{Z}_2 = (0 + j20.106\ )\ \Omega$$

So, $\overline{Z}_2 = (20.106\ \angle 90)\ \Omega$

$$\overline{Z}_3 = (0 - j39.79)\ \Omega$$

$$\overline{Z}_3 = (39.79\ \angle{-90})\ \Omega$$

So, branch currents can be calculated as:

$$\overline{I}_R = \frac{\overline{V}}{\overline{Z}_1} = \frac{(110\ \angle 0)}{(25\ \angle 0)} = (4.4\ \angle 0)\ \text{A}$$

$$\overline{I}_L = \frac{\overline{V}}{\overline{Z}_2} = \frac{(110\ \angle 0)}{(20.106\ \angle 90)} = (5.47\ \angle{-90})\ \text{A}$$

$$\overline{I}_C = \frac{\overline{V}}{\overline{Z}_3} = \frac{(110\ \angle 0)}{(39.79\ \angle -90)} = (2.76\ \angle 90)\ \text{A}$$

Now, total current drawn from supply,

$$\overline{I} = \overline{I}_R + \overline{I}_L + \overline{I}_C$$

$$= (4.4\ \angle 0) + (5.47\ \angle{-90}) + (2.76\ \angle 90)$$

$$= (4.4 + j0) + (0 - j5.47) + (0 + j2.76)$$

$$= (4.4 - j2.71)\ \text{A} \qquad \text{(Rectangular form)}$$

$$= (5.17\ \angle{-31.63})\ \text{A} \qquad \text{(Polar form)}$$

From polar form of $V$ and $I$, current lags to voltage by 31.63°. Thus, the overall circuit is inductive and the phase angle of circuit, $\phi = 31.63$.

$\therefore$ pf = cos $\phi$ = cos (31.63) = 0.8515 lagging

*Phasor diagram*:

We have, $\overline{I}_R = (4.4\ \angle 0)$ A

$\overline{I}_L = (5.47\ \angle{-90})$ A

$\overline{I}_C = (2.76\ \angle 90)$ A

$I = (5.17\ \angle{-31.63})$ A

Taking scale, 1 cm = 1 A; and taking $V$ as reference, we get,

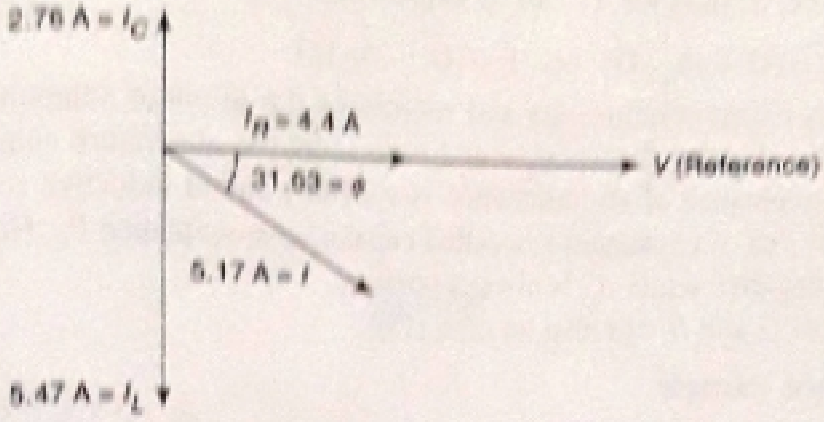



Fig. 2.147

### 2.3.3 Admittance (Y)

The admittance is defined as the reciprocal of impedance, i.e.,

$$\text{Admittance, } Y = \frac{1}{Z} = \frac{I}{V} \quad \text{or} \quad \bar{Y} = \frac{1}{\bar{Z}} = \frac{\bar{I}}{\bar{V}}$$

The unit of admittance is mho (ohm spelt backward) and its symbol is ℧. The admittance of the circuit may be considered as a measure of the ease with which a circuit can conduct alternating current.

Thus, a circuit with higher value of admittance will have a higher value of current.

Consider a parallel circuit as shown in Fig. 2.148.

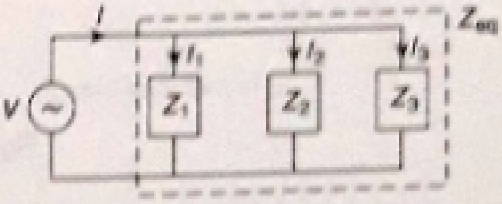



Fig. 2.148 Parallel circuit

By law of parallel circuit,

$$\frac{1}{\bar{Z}_{eq}} = \frac{1}{\bar{Z}_1} + \frac{1}{\bar{Z}_2} + \frac{1}{\bar{Z}_3}$$

Since admittance is reciprocal of impedance, we have

$$\bar{Y}_{eq} = \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3$$

where $Y_1$, $Y_2$, and $Y_3$ are the individual admittances of the parallel branches and $Y_{eq}$ is the total or equivalent admittance of the circuit.

$$\text{So, Line current, } \bar{I} = \frac{\bar{V}}{\bar{Z}_{eq}} = \bar{V}\,\bar{Y}_{eq}$$

***Components of admittance***

The impedance of the circuit can be expressed in the complex form as $\bar{Z} = (R + jX_L)\ \Omega$ or $\bar{Z} = (R - jX_C)\ \Omega$ depending upon the nature of reactance, where $R$ is the resistance or in phase component of $Z$ while $X_L$ or $X_C$ is the reactive or quadrature component of $Z$. The reciprocal of impedance (i.e., admittance) will also have a complex form.

Therefore, admittance $\bar{Y}$ can be expressed as

$$\bar{Y} = (G - jB_L)\,℧ \quad \text{or} \quad \bar{Y} = (G + jB_C)\,℧$$

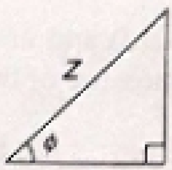
where $G$ is called conductance and represents the in-phase component of $\bar{Y}$, while $B$ is called the susceptance and represents the quadrature component of $\bar{Y}$. The susceptance of an inductance is specially called inductive susceptance $B_L$ whereas that of capacitance is called capacitive susceptance $B_C$. Note that $B_L$ is always negative while $B_C$ is always positive.

The units of $G$ and $B$ will also be mho (℧).

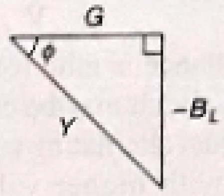
### *Admittance triangle*

Since admittance has in-phase component (i.e., $G$) as well as quadrature component (i.e., $B_L$ or $B_C$), it can be represented by a right-angled triangle, called admittance triangle.

(i) For an inductive circuit, the impedance and admittance triangle are shown in Fig. 2.149.



(a) Impedance triangle



(b) Admittance triangle

**Fig. 2.149**

Note that admittance angle is equal to the impedance angle but is negative.

Conductance, $G = Y\cos\phi = \dfrac{1}{Z} \times \dfrac{R}{Z}$

So, $G = \dfrac{R}{Z^2} = \dfrac{R}{R^2 + X_L^2}\,℧$

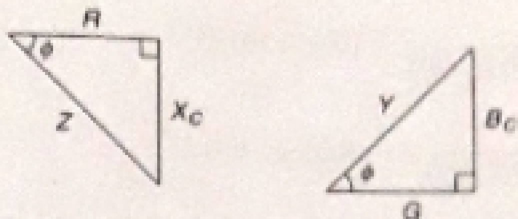
Susceptance, $B_L = Y\sin\phi = \dfrac{1}{Z} \times \dfrac{X_L}{Z}$

So, $B_L = \dfrac{X_L}{Z^2} = \dfrac{X_L}{R^2 + X_L^2}\,℧$

(ii) For a capacitive circuit, the impedance and admittance triangle are shown in Fig. 2.150.

Conductance, $G = Y\cos\phi = \dfrac{1}{Z} \times \dfrac{R}{Z}$

So, $G = \dfrac{R}{Z^2} = \dfrac{R}{R^2 + X_C^2}\,℧$




(a) Impedance triangle (b) Admittance triangle

**Fig. 2.150**

Susceptance, $B_C = Y \sin \phi = \frac{1}{Z} \cdot \frac{X_C}{Z}$

So, $$B_C = \frac{X_C}{Z^2} = \frac{X_C}{R^2 + X_C^2}\ ℧$$

**Example 2.82** Calculate the admittance ($\bar{Y}$) and draw the admittance triangle of the circuit shown in Fig. 2.151.

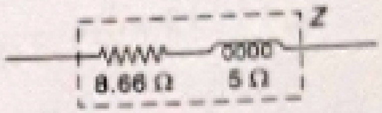



**Fig. 2.151**

***Solution***

Circuit impedance, $\bar{Z} = (8.66 + j5)\ \Omega$
$= (10 \angle 30)\ \Omega$

So, Circuit admittance, $\bar{Y} = \frac{1}{\bar{Z}}$

$$= \frac{1}{(10 \angle 30)}$$
$$= (0.1 \angle -30)\ ℧$$
$$= (0.0866 - j0.05)\ ℧$$

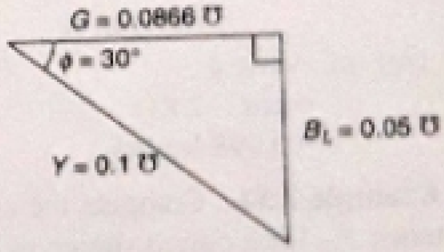



**Fig. 2.152**

From polar form of admittance, $Y = 0.1$ ℧ and $\phi = -30°$.

From rectangular form of admittance,

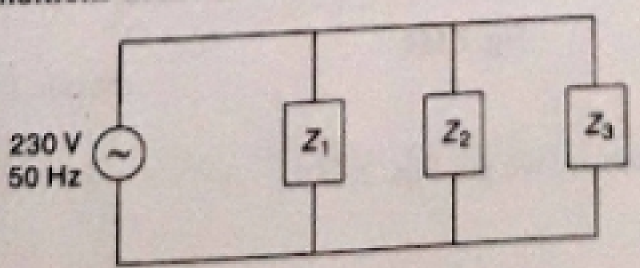
$G = 0.0866$ ℧ and $B_L = 0.05$ ℧

The admittance triangle is shown in Fig. 2.152.

**Example 2.83** Three loads are placed across 230 V, 50 Hz supply. The loads are $(10\angle -30)\ \Omega$; $(20\angle 60)\ \Omega$ and $(40\angle 0)\ \Omega$. Determine (i) admittance, (ii) equivalent impedance, (iii) power consumed, and (iv) power factor.

***Solution***

The conditions in the example are shown in Fig. 2.153.




Let
$\bar{Z}_1 = (10 \angle -30)\Omega$
$\bar{Z}_2 = (20 \angle 60)\Omega$
$\bar{Z}_3 = (40 \angle 0)\Omega$

**Fig. 2.153**

(i) $\bar{Y}_1 = \dfrac{1}{\bar{Z}_1} = \dfrac{1}{(10\angle -30)} = (0.1\ \angle 30)\ \mho$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{(20\angle 60)} = (0.05\ \angle -60)\ \mho$$

$$\bar{Y}_3 = \frac{1}{\bar{Z}_3} = \frac{1}{(40\angle 0)} = (0.025\ \angle 0)\ \mho$$

The total admittance of the circuit,

$$\begin{aligned}\bar{Y} &= \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3\\ &= (0.1\ \angle 30) + (0.05\ \angle -60) + (0.025\ \angle 0)\\ &= (0.0866 + j0.05) + (0.025 - j0.0433) + (0.025 + j0)\\ &= (0.1366 + j0.0067)\ \mho\\ &= (0.137\ \angle 2.81)\ \mho\end{aligned}$$

(ii) Equivalent impedance of the circuit,

$$\bar{Z} = \frac{1}{\bar{Y}} = \frac{1}{(0.137\ \angle 2.81)} = (7.3\ \angle -2.81)\ \Omega$$

(iii) Total current, $I = \dfrac{V}{Z} = \dfrac{230}{7.3} = 31.51$ A

Power consumed by the circuit,

$$\begin{aligned}P &= VI\cos\phi\\ &= 230 \times 31.51 \times \cos(-2.81)\\ &= 7238\ \text{W}\end{aligned}$$

(iv) $$\begin{aligned}\text{pf} &= \cos\phi\\ &= \cos(-2.81)\\ &= 0.998\ \text{leading}\end{aligned}$$

**Example 2.84** Compute the equivalent impedance $Z_{eq}$ and equivalent admittance $Y_{eq}$ for a circuit shown in Fig. 2.154. Also calculate the total current.

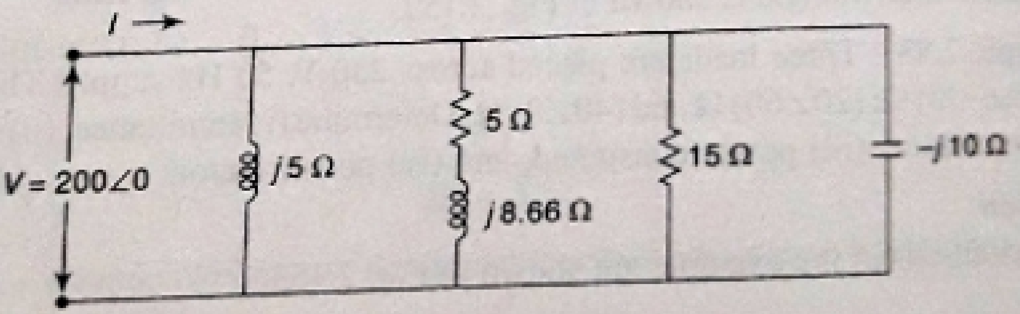



Fig. 2.154

*Solution*

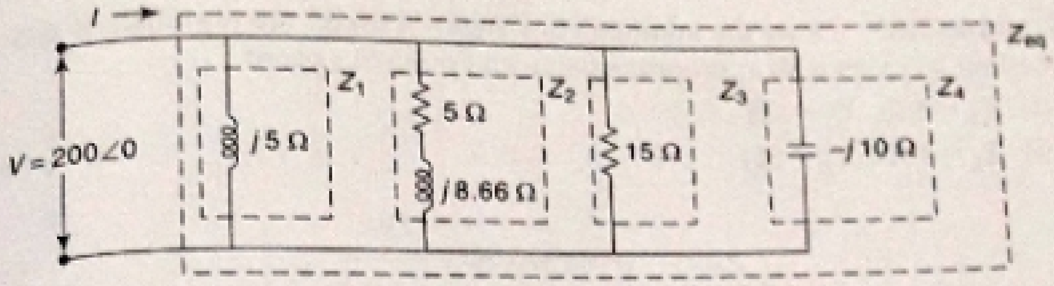



Fig. 2.155

Given: $\bar{Z}_1 = (0 + j5)\ \Omega = (5\ \angle 90)\ \Omega$

$$\bar{Z}_2 = (5 + j8.66)\ \Omega = (10\ \angle 60)\ \Omega$$

$$\bar{Z}_3 = (15 + j0)\ \Omega = (15\ \angle 0)\ \Omega$$

$$\bar{Z}_4 = (0 - j10)\ \Omega = (10\ \angle -90)\ \Omega$$

The individual admittances of the parallel branches are:

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{(5\angle 90)} = (0.2\ \angle -90)\ ℧$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{(10\angle 60)} = (0.1\ \angle -60)\ ℧$$

$$\bar{Y}_3 = \frac{1}{\bar{Z}_3} = \frac{1}{(15\angle 0)} = (0.0667\ \angle 0)\ ℧$$

$$\bar{Y}_4 = \frac{1}{\bar{Z}_4} = \frac{1}{(10\angle -90)} = (0.1\ \angle 90)\ ℧$$

The equivalent admittance of the circuit,

$$\begin{aligned}
\bar{Y}_{eq} &= \bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 + \bar{Y}_4 \\
&= (0.2\ \angle -90) + (0.1\ \angle -60) + (0.0677\ \angle 0) + (0.1\ \angle 90) \\
&= (0 - j0.2) + (0.05 - j0.0866) + (0.0677 + j0) + (0 + j0.1) \\
&= (0.1177 - j0.1866)\ ℧ \\
&= (0.22\ \angle -57.76)\ ℧
\end{aligned}$$

The equivalent impedance of the circuit,

$$\bar{Z}_{eq} = \frac{1}{\bar{Y}_{eq}} = \frac{1}{(0.22\ \angle -57.76)} = (4.545\ \angle\ 57.76)\ \Omega$$

The total current,

$$\bar{I} = \frac{\bar{V}}{\bar{Z}_{eq}} = \bar{V}\,\bar{Y}_{eq} = (200\ \angle\ 0)\ (0.22\ \angle -57.76) = (44\ \angle -57.76)\ \text{A}$$

**Example 2.85** Draw the admittance triangle between the terminals $A$ and $B$, labelling its sides with appropriate value and units in case of

(i) $X_L = 4\ \Omega, X_C = 8\Omega$

(ii) $X_L = 10\Omega, X_C = 5\Omega$

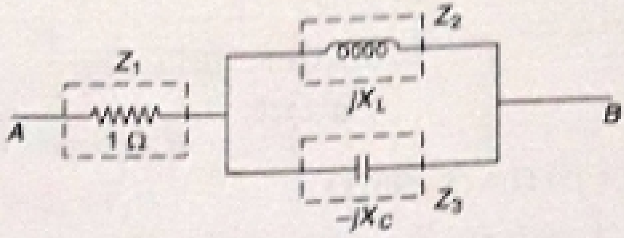



Fig. 2.156

***Solution***

***Case (i)*** When $X_L = 4\ \Omega$ and $X_C = 8\ \Omega$

$$\bar{Z}_1 = (1 + j0)\ \Omega = (1\ \angle 0)\ \Omega$$

$$\bar{Z}_2 = (0 + j4)\ \Omega = (4\ \angle 90)\ \Omega$$

$$\bar{Z}_3 = (0 - j8)\ \Omega = (8\ \angle -90)\ \Omega$$

$$\bar{Z}_{eq} = \bar{Z}_1 + \frac{\bar{Z}_2 \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3}$$

$$= (1 + j0) + \frac{(4\ \angle 90)(8\ \angle -90)}{(0 + j4) + (0 - j8)}$$

$$= (1 + j0) + \frac{(32\ \angle 0)}{(0 - j4)}$$

$$= (1 + j0) + \frac{(32\ \angle 0)}{(4\ \angle -90)}$$

$$= (1 + j0) + (8\ \angle 90)$$

$$= (1 + j0) + (0 + j8)$$

$$= (1 + j8)\ \Omega$$

$$= (8.062\ \angle 82.87)\ \Omega$$

Equivalent admittance, $\bar{Y} = \dfrac{1}{\bar{Z}_{eq}} = \dfrac{1}{(8.062\ \angle 82.87)} = (0.124\ \angle -82.87)\ ℧$

$$= (0.0154 - j0.123)\ ℧$$

The admittance triangle is shown in Fig. 2.157.

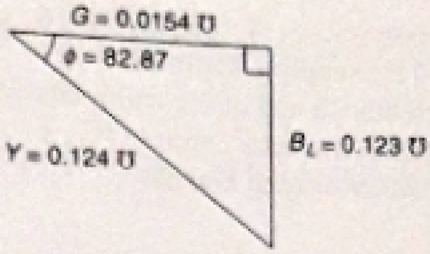



**Fig. 2.157**

*Case (ii)* When $X_L = 10\ \Omega$ and $X_C = 5\ \Omega$

We have $\bar{Z}_1 = (1 + j0)\ \Omega = (1\ \angle 0)\ \Omega$

$$\bar{Z}_2 = (0 + j10)\ \Omega = (10\ \angle 90)\ \Omega$$

$$\bar{Z}_3 = (0 - j5)\ \Omega = (5\ \angle -90)\ \Omega$$

$$\bar{Z}_{eq} = \bar{Z}_1 + \frac{\bar{Z}_2 \bar{Z}_3}{\bar{Z}_2 + \bar{Z}_3}$$

$$= (1 + j0) + \frac{(10\ \angle 90)(5\ \angle -90)}{(0 + j10) + (0 - j5)}$$

$$= (1 + j0) + \frac{(50\ \angle 0)}{(0 + j5)}$$

$$= (1 + j0) + \frac{(50\ \angle 0)}{(5\ \angle 90)}$$

$$= (1 + j0) + (10\ \angle -90)$$
$$= (1 + j0) + (0 - j10)$$
$$= (1 - j10)\ \Omega$$
$$= (10.05\ \angle -84.29)\ \Omega$$

Equivalent admittance, $\bar{Y}_{eq} = \dfrac{1}{\bar{Z}_{eq}} = \dfrac{1}{(10.05\ \angle -84.29)} = (0.0995\ \angle 84.29)$ ℧

$$= (0.009899 + j0.099)\ ℧$$

The admittance triangle is shown in Fig. 2.158.

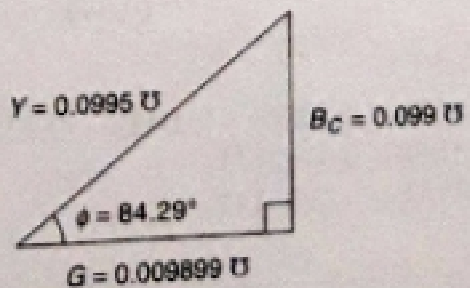



**Fig. 2.158**

**Example 2.86** Two impedances $(14 + j5)\ \Omega$ and $(18 + j10)\ \Omega$ are connected in parallel across 200 V, 50 Hz supply. Determine:

(i) admittance of each branch and of the entire circuit
(ii) current in each branch and total current
(iii) power and power factor of each branch
(iv) total power factor

Also draw phasor diagram.

***Solution***

The conditions in the example are shown in Fig. 2.159.

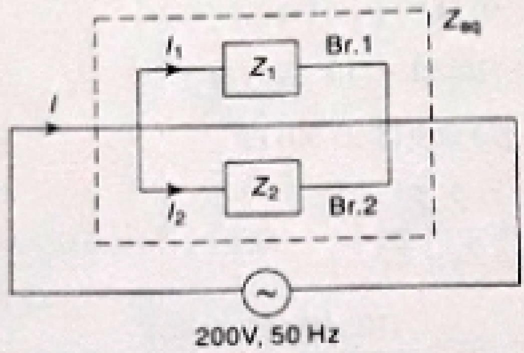



Fig. 2.159

Let $\quad \overline{Z}_1 = (14 + j5)\ \Omega = (14.87\ \angle 19.65)\ \Omega$

$$\overline{Z}_2 = (18 + j10)\ \Omega = (20.59\ \angle 29.05)\ \Omega$$

Taking $V$ as reference,

$$\overline{V} = (200\angle 0)\ \text{V}$$

(i) Admittance of branch 1,

$$\overline{Y}_1 = \frac{1}{\overline{Z}_1} = \frac{1}{(14.87\angle 19.65)} = (0.0672\ \angle -19.65)\ \mho$$
$$= (0.0633 - j0.0226)\ \mho$$

Admittance of branch 2,

$$\overline{Y}_2 = \frac{1}{\overline{Z}_2} = \frac{1}{(20.59\angle 29.05)} = (0.0486\ \angle -29.05)\ \mho$$
$$= (0.0425 - j0.0236)\ \mho$$

Now, $\quad \overline{Y}_{eq} = \overline{Y}_1 + \overline{Y}_2$

$$= (0.0633 - j0.0226) + (0.0425 - j0.0236)$$
$$= (0.1058 - 0.0462)\ \mho$$
$$= (0.1154\ \angle -23.59)\ \mho$$

Also $\quad \overline{Z}_{eq} = \frac{1}{\overline{Y}_{eq}} = \frac{1}{(0.1154\angle -23.59)} = (8.67\ \angle 23.59)\ \Omega$

(ii) Branch current, $\bar{I}_1 = \dfrac{\bar{V}}{\bar{Z}_1}$ or $\bar{I}_1 = \bar{V}\,\bar{Y}_1$

So, $\bar{I}_1 = \dfrac{(200\angle 0)}{(14.87\angle 19.65)} = (13.45\angle -19.65)$ A

Branch current, $\bar{I}_2 = \dfrac{\bar{V}}{\bar{Z}_2}$ or $\bar{I}_2 = \bar{V}\,\bar{Y}_2$

So, $\bar{I}_2 = \dfrac{(200\angle 0)}{(20.59\angle 29.05)} = (9.71\angle -29.05)$ A

Total current, $\bar{I} = \dfrac{\bar{V}}{\bar{Z}_{eq}}$ or $\bar{I} = \bar{Y}\,\bar{V}$ or $\bar{I} = \bar{I}_1 + \bar{I}_2$

So, $\bar{I} = \dfrac{(200\angle 0)}{(8.67\angle 23.59)} = (23.07\angle -23.59)$ A

(iii) Power consumed by branch 1,

$P_1 = I_1^2 R_1 = (13.45)^2 \times 14 = 2.533$ kW

Power consumed by branch 2,

$P_2 = I_2^2 R_2 = (9.71)^2 \times 18 = 1.697$ kW

Power factor of branch 1, $(\text{pf})_1 = \cos\phi_1 = \cos(19.65) = 0.942$ lagging

Power factor of branch 2, $(\text{pf})_2 = \cos\phi_2 = \cos(29.05) = 0.874$ lagging

(iv) Total power factor, $(\text{pf}) = \cos\phi = \cos(23.59) = 0.9164$ lagging

(v) Phasor diagram:

We know that, $\bar{I}_1 = (13.45\angle -19.65)$ A

$\bar{I}_2 = (9.71\angle -29.05)$ A

Scale: 1 cm = 2 A

Taking $V$ as reference phasor,

$\bar{I} = \bar{I}_1 + \bar{I}_2$

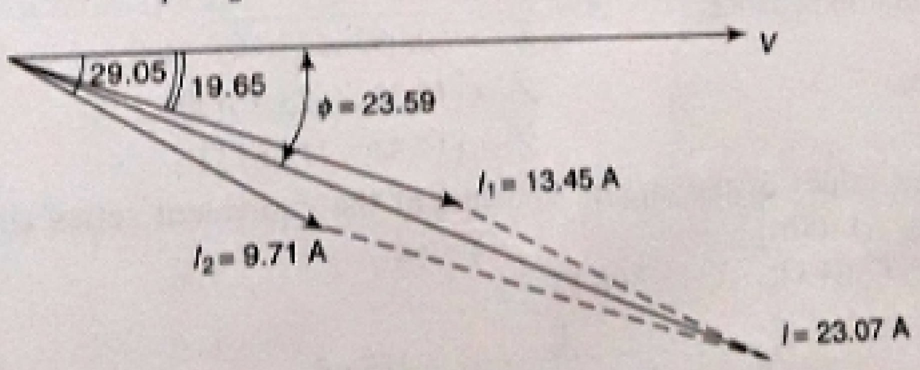



Fig. 2.160

**Example 2.87** Two parallel circuits comprise respectively (a) a coil of resistance 20 Ω and inductance 0.08 H (b) a capacitor of capacitance 200 μF in series with e resistance of 10 Ω. If the circuit is connected to 230 V, 50 Hz supply, determine

(i) supply current and power factor
(ii) equivalent series circuit
(iii) the value and nature of reactance to be connected in series with the circuit to bring the overall power factor to unity.

***Solution***

The conditions in the example are shown in Fig. 2.140(a).

$$L = 0.08 \text{ H}; X_L = 2\pi fL = 2\pi \times 50 \times 0.08 = 25.13\ \Omega$$

$$C = 200 \times 10^{-6} \text{ F}; X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi \times 50 \times 200 \times 10^{-6}} = 15.92\ \Omega$$

$$\bar{Z}_1 = (20 + j25.13)\ \Omega = (32.12 \angle 51.49)\ \Omega$$

$$\bar{Z}_2 = (10 - j15.92)\ \Omega = (18.8 \angle -57.87)\ \Omega$$

$$\bar{Y}_1 = \frac{1}{\bar{Z}_1} = \frac{1}{(32.12 \angle 51.49)} = (0.0311 \angle -51.49)\ ℧$$

$$\bar{Y}_2 = \frac{1}{\bar{Z}_2} = \frac{1}{(18.8 \angle -57.87)} = (0.0532 \angle 57.87)\ ℧$$

So, Total admittance,

$$\begin{aligned}\bar{Y} &= \bar{Y}_1 + \bar{Y}_2\\ &= (0.0311 \angle -51.49) + (0.0532 \angle 57.87)\\ &= (0.0194 - j0.0243) + (0.0283 + j0.045)\\ &= (0.0477 + j0.0207)\ ℧\\ &= (0.052 + \angle 23.46)\ ℧\end{aligned}$$

(i) Now, supply current, $\bar{I} = \bar{V}\bar{Y} = (230 \angle 0)(0.052 \angle 23.46)$
$= (11.96 \angle 23.46)$ A

Power factor, pf $= \cos \phi$
$= \cos 23.46$
$= 0.917$ lead

(ii) Total impedance, $\bar{Z} = \frac{1}{\bar{Y}} = \frac{1}{(0.052 \angle 23.46)}$

So, $\bar{Z} = (19.23 \angle -23.46)\ \Omega$

or $\bar{Z} = (17.64 - j7.66)\ \Omega$

The values of the circuit elements of the equivalent series circuit [see Fig. 2.140(b)] are:

$R = 17.64\ \Omega;\quad X_C = 7.66\ \Omega$

So, $C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi \times 50 \times 7.66} = 415\ \mu\text{F}$

(iii) In order that the circuit power factor is unity, the reactance of the circuit should be zero. To do so, we would connect an inductive reactance $X_L = 7.66\ \Omega$ in series with the circuit.

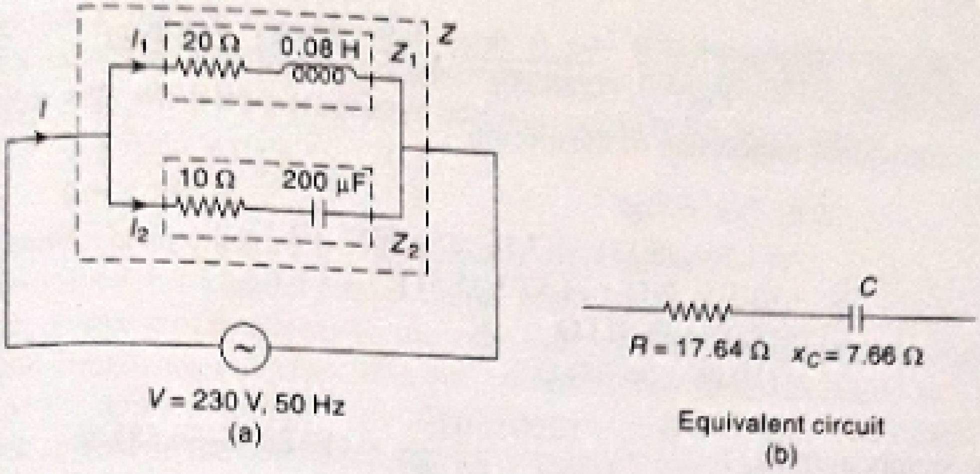



Fig. 2.161

**Example 2.88** Applying admittance method to series-parallel circuit shown in Fig. 2.162. Find: (i) total impedance, (ii) supply current, and (iii) pf of entire circuit.

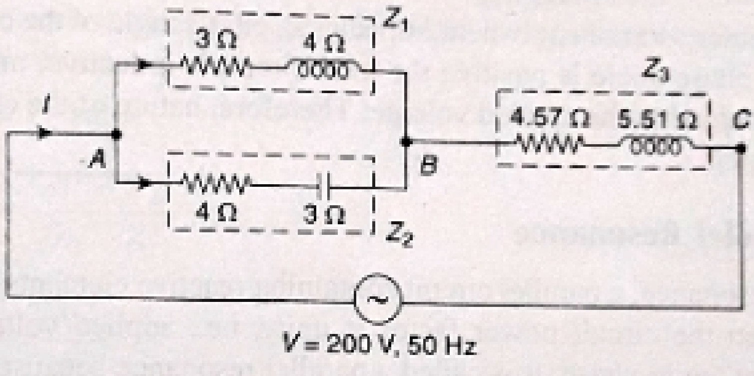



Fig. 2.162

***Solution***

Given: $\overline{Z}_1 = (3 + j4)\ \Omega = (5\ \angle 53.13)\ \Omega$

$\overline{Z}_2 = (4 - j3)\ \Omega = (5\ \angle -36.87)\ \Omega$

$\overline{Z}_3 = (4.57 + j5.51)\ \Omega = (7.16\ \angle\ 50.33)\ \Omega$

(i) It is a series–parallel circuit. We will first calculate the equivalent impedance of the parallel combination, i.e., $Z_{AB}$.

$$\overline{Y}_1 = \frac{1}{\overline{Z}_1} = \frac{1}{(5\angle 53.13)} = (0.2\ \angle -53.13)\ ℧$$

$$\overline{Y}_2 = \frac{1}{\overline{Z}_2} = \frac{1}{(5\angle -36.87)} = (0.2\ \angle 36.87)\ ℧$$

So, $\overline{Y}_{AB} = \overline{Y}_1 + \overline{Y}_2$

$$= (0.2\ \angle -53.13) + (0.2\ \angle 36.87)$$
$$= (0.12 - j0.16) + (0.16 + j0.12)$$
$$= (0.28 - j0.04)\ ℧$$
$$= (0.2828\ \angle -8.13)\ ℧$$

Now, $\overline{Z}_{AB} = \frac{1}{\overline{Y}_{AB}} = \frac{1}{(0.2828 \angle -8.13)} = (3.54 \angle 8.13)\ \Omega$

Equivalent impedance of the circuit,

$$\overline{Z} = \overline{Z}_{AB} + \overline{Z}_{BC}$$
$$= (3.54 \angle 8.13) + (7.16 \angle 50.33)$$
$$= (3.5 + j0.5) + (4.57 + j5.51)$$
$$= (8.07 + j6.01)\ \Omega$$
$$= (10.06 \angle 36.68)\ \Omega$$

(ii) Supply current, $\overline{I} = \frac{\overline{V}}{\overline{Z}} = \frac{(200 \angle 0)}{(10.06 \angle 36.68)} = (19.88 \angle -36.68)$ A

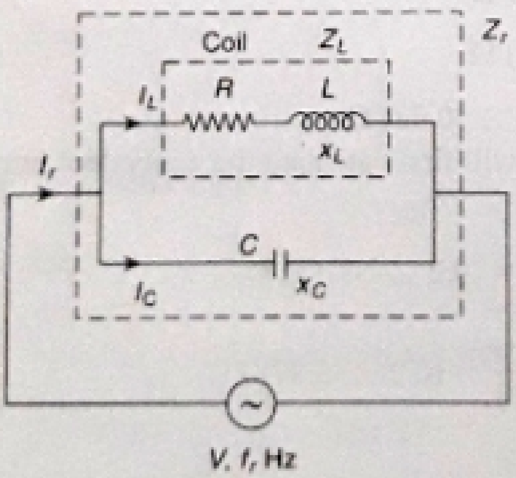
(iii) Power factor of the entire circuit,

$$\text{pf} = \cos \phi$$
$$= \cos 36.68$$
$$= 0.802 \text{ lagging}$$

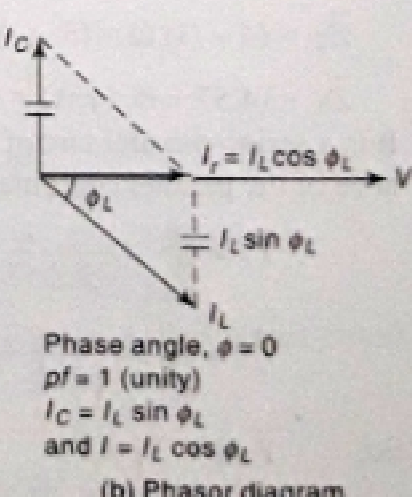
*Note:* From polar form of equivalent impedance, phase angle of the circuit, i.e., $\phi$ = 36.68. As phase angle is positive the total circuit is inductive, means supply current lags behind to the applied voltage. Therefore, nature of the circuit power factor is lagging.

### 2.3.4 Parallel Resonance

Like series resonance, a parallel circuit containing reactive elements ($L$ and $C$) is resonant when the circuit power factor is unity, i.e., applied voltage and the supply current are in phase. It is called a parallel resonance because it concerns a parallel circuit. The most common resonant parallel circuit is an inductor (coil) in parallel with a pure capacitor $C$ as shown in Fig. 2.163(a). The phasor diagram of this parallel circuit is shown in Fig. 2.163(b). The coil current $I_L$ has two



(a) Circuit diagram



(b) Phasor diagram (when $I_C = I_L \sin \phi_L$)

Fig. 2.163 Parallel resonance

components, viz active component $I_L \cos\phi_L$ and reactive component $I_L \sin\phi_L$. This parallel circuit will resonant when the circuit power factor is unity. This is possible only when the net reactive component of the circuit is zero,

i.e., $\quad I_C - I_L \sin\phi_L = 0$

or $\quad I_C = I_L \sin\phi_L$

or Reactive component of $I_C$ = Reactive component of $I_L$

Resonance in a parallel circuit can be achieved by changing the supply frequency. Increase of frequency increase the reactance of the coil and consequently the coil impedance increases. The coil current $I_L$, therefore, decreases and lags behind the applied voltage $V$ by a progressively greater angle. The capacitive branch current on the other hand, increases although it will always lead $V$ by 90°. At some frequency $f_r$ (called resonant frequency), $I_C$ becomes equal to $I_L \sin\phi_L$ and resonance occurs.

### *(i) Resonance/resonant frequency ($f_r$)*

The supply frequency at which parallel resonance occurs (i.e., reactive component of circuit current becomes zero) is called the resonant frequency.

At parallel resonance,

$$I_C = I_L \sin\phi_L$$

$$\frac{V}{X_C} = \frac{V}{Z_L} \cdot \frac{X_L}{Z_L}$$

$$\frac{1}{X_C} = \frac{X_L}{Z_L^2}$$

$$Z_L^2 = \frac{L}{C} \tag{2.25}$$

$$R^2 + X_L^2 = \frac{L}{C}$$

$$R^2 + (2\pi f_r L)^2 = \frac{L}{C}$$

$$(2\pi f_r L)^2 = \frac{L}{C} - R^2$$

$$f_r = \frac{1}{2\pi L}\sqrt{\frac{L}{C} - R^2}$$

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \tag{2.26}$$

If $R$ is very very small,

$$f_r = \frac{1}{2\pi\sqrt{LC}} \tag{2.27}$$

***(ii) Impedance at resonance ($Z_r$)***

At parallel resonance,

$$I = I_L \cos \phi_L$$

or $$\frac{V}{Z_r} = \frac{V}{Z_L} \cdot \frac{R}{Z_L}$$

or $$\frac{1}{Z_r} = \frac{R}{Z_L^2}$$

or $$Z_r = \frac{Z_L^2}{R}$$

From Eq. (2.25), $Z_L^2 = \dfrac{L}{C}$

So, Circuit impedance, $$Z_r = \frac{L}{CR}\,\Omega \tag{2.28}$$

Thus, at parallel resonance, the circuit impedance is equal to $L/CR$. This is known as equivalent or dynamic impedance of the parallel circuit at resonance.

$Z_r$ is the pure resistance because in the expression of $Z_r$, frequency term is not present. Also $Z_r$ is very high because the ratio $L/C$ is very large at parallel resonance.

As supply frequency changes, value of circuit impedance also varies. If we plot impedance–frequency graph for a parallel circuit shown in Fig. 2.163(a), the shape of the curve will be as shown in Fig. 2.164. Note that impedance of the circuit is maximum at resonance. As the frequency changes from resonance, the circuit impedance decreases very rapidly.

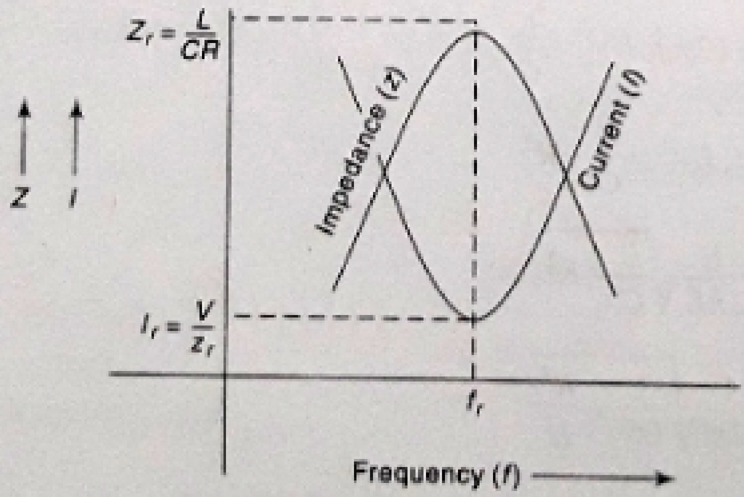



Fig. 2.164 Variation of circuit impedance and current with frequency

### (iii) Circuit current at resonance ($I_r$)

At parallel resonance, the circuit impedance ($Z_r$) becomes maximum and the circuit current ($I_r$) is at minimum value. The current–frequency curve of the parallel circuit is shown in Fig. 2.143. Note that the value of line current is minimum at resonance.

At parallel resonance, the impedance of each branch (i.e., $X_C$ and $Z_L$) is relatively small compared with circuit impedance ($Z_r$). Therefore, current flowing through the capacitor and coil are much greater than the line current $I_r$. But these currents are approximately 180° out of phase, therefore resultant current $I_r$ is extremely small.

### (iv) Q-factor of parallel resonance circuit

At parallel resonance, the current circulating between the two branches is many times greater than the line current. The current amplification produced by the resonance is termed as $Q$-factor of the parallel resonant circuit, i.e.,

$$Q\text{-factor} = \frac{I_L \text{ or } I_C}{I_r} \quad (\because I_L \equiv I_C) \tag{2.29}$$

Let $$Q\text{-factor} = \frac{I_C}{I_r}$$

Now, $$I_C = \frac{V}{X_C} = 2\pi f_r CV \quad \text{and} \quad I_r = \frac{V}{(L/CR)}$$

So, $$Q\text{-factor} = \frac{2\pi f_r CV}{\dfrac{V}{\left(\dfrac{L}{CR}\right)}}$$

$$= \frac{2\pi f_r L}{R} \tag{2.30}$$

Now, $$f_r = \frac{1}{2\pi\sqrt{LC}}$$

By substituting the value of $f_r$ in Eq. (2.30), we get

$$Q\text{-factor} = \frac{1}{R}\sqrt{\frac{L}{C}} \tag{2.31}$$

## 2.3.5 Comparison of Series and Parallel Resonant Circuits

| Particular | Series circuit | Parallel circuit |
|---|---|---|
| 1. Impedance at resonance | Minimum ($Z_r = R$) | Maximum ($Z_r = L/CR$) |
| 2. Current at resonance | Maximum $\left(I_r = \frac{V}{R}\right)$ | Minimum $\left(I_r = \frac{V}{Z_r}\right)$ |
| 3. Resonant frequency | $f_r = \frac{1}{2\pi\sqrt{LC}}$ Hz | $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$ Hz |
| 4. $Q$-Factor | $= \frac{V_L \text{ or } V_C}{V}$ | $= \frac{I_L \text{ or } I_C}{I}$ |
| 5. It magnifies | Voltage | Current |
| 6. When $f < f_r$ | Circuit is capacitive. | Circuit is inductive. |
| 7. When $f > f_r$ | Circuit is inductive. | Circuit is capacitive. |

**Example 2.89** A parallel circuit consists of a 2.5 μF capacitor and a coil whose resistance and inductance are 15 Ω and 260 mH, respectively. Determine (i) the resonant frequency, (ii) $Q$-factor of the circuit at resonance, and (iii) dynamic impedance of the circuit.

***Solution***

(i) Resonant frequency ($f_r$),

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{260 \times 10^{-3} \times 2.5 \times 10^{-6}} - \frac{(15)^2}{(260 \times 10^{-3})^2}}$$

$$= 197.19 \text{ Hz}$$

(ii) $Q\text{-factor} = \dfrac{2\pi f_r L}{R} = \dfrac{2\pi \times 197.19 \times 260 \times 10^{-3}}{15} = 21.48$

(iii) Dynamic impedance ($Z_r$),

$$Z_r = \frac{L}{CR} = \frac{260 \times 10^{-3}}{2.5 \times 10^{-6} \times 15} = 6933.33\ \Omega$$

**Example 2.90** An inductive coil of resistance 20 Ω and inductance 0.2 H is connected in parallel with 200 μF capacitor with variable frequency, 230 V supply. Find the resonant frequency at which the total current taken from the supply is in phase with supply voltage. Also find the value of this current. Draw the phasor diagram.

***Solution***

The conditions in the example are shown in Fig. 2.165.

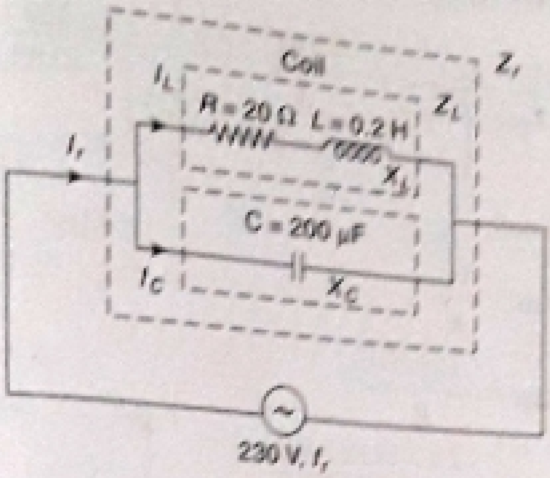



Fig. 2.165

Resonant frequency,

$$f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \quad \text{Hz}$$

$$= \frac{1}{2\pi}\sqrt{\frac{1}{0.2 \times 200 \times 10^{-6}} - \frac{20^2}{(0.2)^2}}$$

$$= \frac{1}{2\pi}\sqrt{25000 - 10000}$$

$$= \frac{1}{2\pi}\sqrt{15000}$$

$$= 19.49 \text{ Hz}$$

Dynamic impedance of the circuit,

$$Z_r = \frac{L}{CR}$$

$$= \frac{0.2}{200 \times 10^{-6} \times 20}$$

$$= 50\ \Omega$$

Circuit current at resonance,

$$I_r = \frac{V}{Z_r} = \frac{230}{50} = 4.6 \text{ A}$$

*Phasor diagram (see Fig. 2.145):*
For phasor diagram, we need to calculate the values of branch currents ($I_L$ and $I_C$) and phase angle of the coil ($\phi_L$).

Now, $$I_L = \frac{V}{Z_L} = \frac{230}{\sqrt{(20)^2 + (2\pi \times 19.49 \times 0.2)^2}} = \frac{230}{31.62} = 7.274 \text{ A}$$

$$I_C = \frac{V}{X_C} = \frac{230}{\left(\frac{1}{2\pi \times 19.49 \times 200 \times 10^{-6}}\right)} = \frac{230}{40.83} = 5.63 \text{ A}$$

Phase angle of the coil,

$$\phi_L = \tan^{-1}\frac{X_L}{R}$$

$$= \tan^{-1}\frac{(2\pi f, L)}{R}$$

$$= \tan^{-1}\frac{(2\pi \times 19.49 \times 0.2)}{20}$$

$$= \tan^{-1}\frac{24.49}{20}$$

So, $\phi_L = 50.76$

Take $V$ as reference phasor.

We know that $\bar{I}_r = \bar{I}_L + \bar{I}_C$.

Now, $I_L = 7.274$ A

$I_C = 5.63$ A

$I_r = 4.6$ A

$\phi_L = 50.76°$

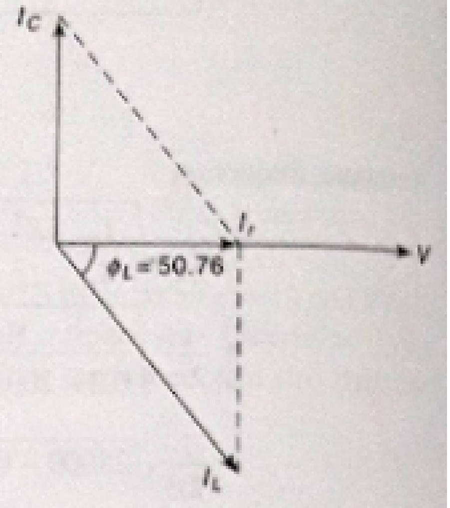


Fig. 2.166

Taking scale 1 cm = 1 A, the phasor diagram can be drawn as shown in Fig. 2.166.

**Example 2.91** A coil takes a current of 1 A at 0.3 pf when connected to a 100 V, 50 Hz supply. Determine the value of the capacitance, which when connected in parallel with the coil, will reduce the line current to a minimum. Calculate the impedance of the parallel circuit at 50 Hz.

***Solution***

When a coil is connected to 100 V, 50 Hz supply, it takes a current of 1 A at 0.3 pf (see Fig. 2.167).

$(\text{pf})_{\text{coil}} = 0.3$

or $\cos\phi_L = 0.3$

or $\phi_L = 72.542$ ($\phi_L$ is the phase angle of the coil)

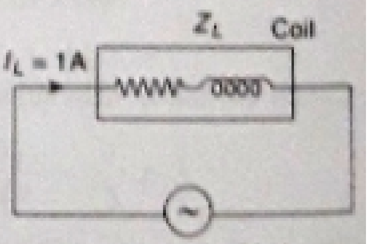


Fig. 2.167

Now, capacitor is connected across the coil (see Fig. 2.168). We need a value of $C$, so that the line current reduces to a minimum value, i.e., parallel resonance occurs at $f = 50$ Hz.

At resonance, $I_C = I_L \sin\phi_L$

So, $I_C = 1 \times \sin(72.542)$

or $I_C = 0.954$ A

Now, $X_C = \frac{V}{I_C} = \frac{100}{0.954} = 104.82\ \Omega$

So, $$\frac{1}{2\pi f_r C} = 104.82$$

or $$\frac{1}{2\pi \times 50 \times C} = 104.82$$

or $$C = 30.37 \times 10^{-6}\ \text{F}$$

or $$= 30.37\ \mu\text{F}$$

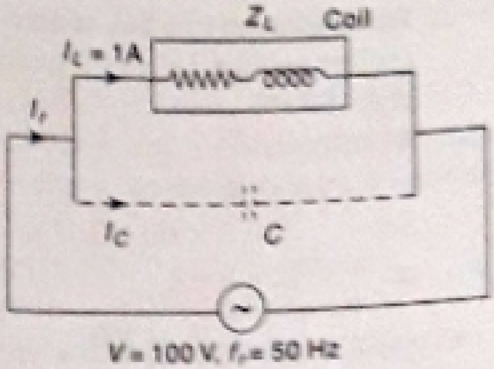



Fig. 2.168

At resonance,

$$I_r = I_L \cos \phi_L$$

$$I_r = 1 \times \cos 72.542$$

or $$I_r = 0.3\ \text{A}$$

or Dynamic impedance of the circuit,

$$Z_r = \frac{V}{I_r} = \frac{100}{0.3} = 333.33\ \Omega$$

**Example 2.92** A circuit has $X_L = 20\ \Omega$ at 50 Hz, its resistance being 15 Ω. For an applied voltage of 200 V at 50 Hz, calculate (i) the pf, (ii) the current, (iii) the value of shunting capacitance to bring the resultant current into phase with the applied voltage, and (iv) the resultant current in case (iii).

***Solution***

Impedance of the circuit, $Z_L = \sqrt{(15)^2 + (20)^2}$

$= 25\ \Omega$

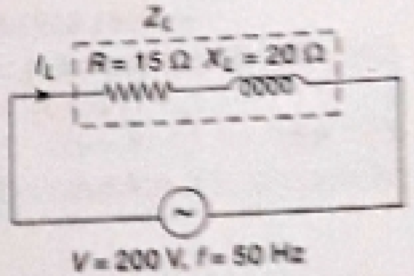



Fig. 2.169

(i) $(\text{pf})_{\text{coil}} = \cos \phi_L = \dfrac{R}{Z_L} = \dfrac{15}{25}$

$= 0.6$ lagging

So, $\phi_L = \cos^{-1} 0.6 = 53.15$

(ii) Current, $I_L = \dfrac{V}{Z_L} = \dfrac{200}{25} = 8$ A

(iii) Now, capacitance is connected in parallel with the above circuit (see Fig. 2.170).

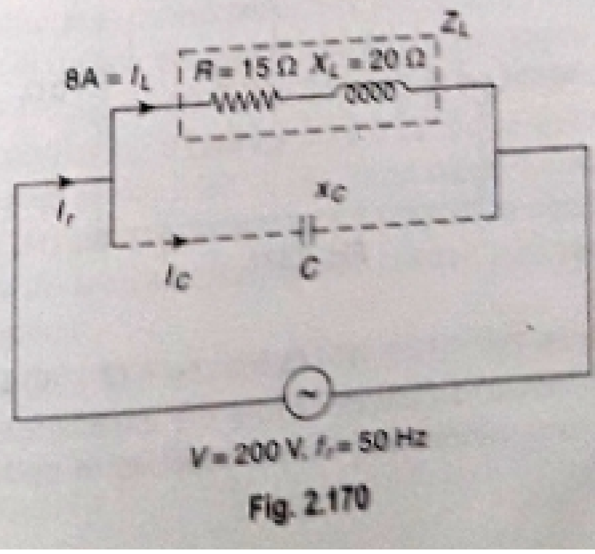



Fig. 2.170

We need a value of $C$, so that resulting current comes in phase with the applied voltage, i.e., pf = 1 (unity). In other words, parallel resonance occurs at $f = 50$ Hz.

At resonance,

$$I_C = I_L \sin \phi_L$$

So, $$I_C = 8 \times \sin (53.13)$$

or $$I_C = 6.4 \text{ A}$$

Now, $$X_C = \frac{V}{I_C} = \frac{200}{6.4} = 31.25 \ \Omega$$

So, $$\frac{1}{2\pi f_r C} = 31.25$$

or $$\frac{1}{2\pi \times 50 \times C} = 31.25$$

So, $$C = \frac{1}{2\pi \times 50 \times 31.25}$$

or $$C = 101.859 \times 10^{-6} \text{ F}$$

$$= 101.859 \ \mu\text{F}$$

(iv) The circuit current at resonance,

$$I_r = I_L \cos \phi_L$$

or $$I_r = 8 \times 0.6$$

or $$I_r = 4.8 \text{ A}$$