Hyperbolic Trigonometric Functions

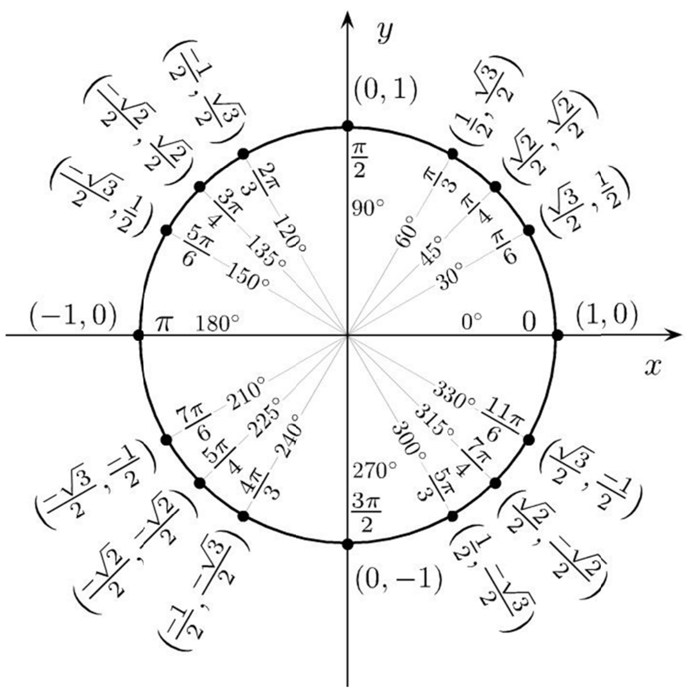
Let us quickly recall the analytic definitions of the trigonometric functions of a real variable – that is the definition that did not use the right triangles. You have seen the definition of the six trigonometric functions in terms of a right triangle:

$$\sin(A) = \frac{opposite}{hypotenuse} \qquad \cos(A) = \frac{adjacent}{hypotenuse} \qquad \tan(A) = \frac{opposite}{adjacent}$$

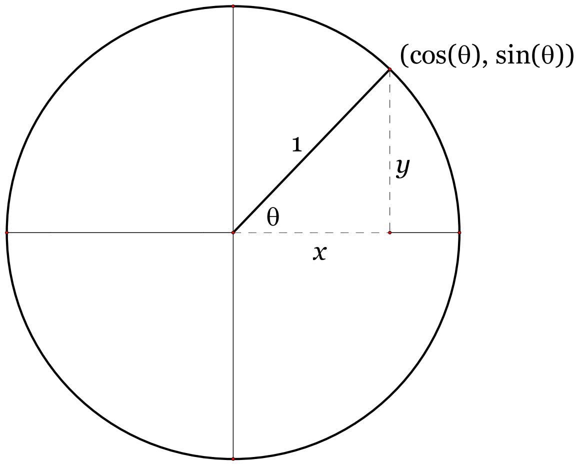
We can also define these functions in terms of the unit circle, the circle of radius one centered at the origin. This does not necessarily help us calculate the sine and cosine of an angle, because it takes us no further than the right triangle definitions; indeed it relies on right triangles for most angles. The unit circle definition does, however, allow us to define the trigonometric functions for all positive and negative real numbers, not just for values between 0 and π/2 radians. From the Pythagorean Theorem the equation for the unit circle is:

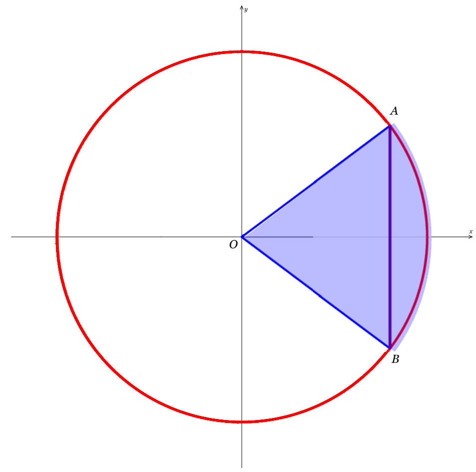
$$x^2 + y^2 = 1$$

In the figure below, some common angles, measured in radians, are given. Measurements in the counter clockwise direction are positive angles and measurements in the clockwise direction are negative angles.

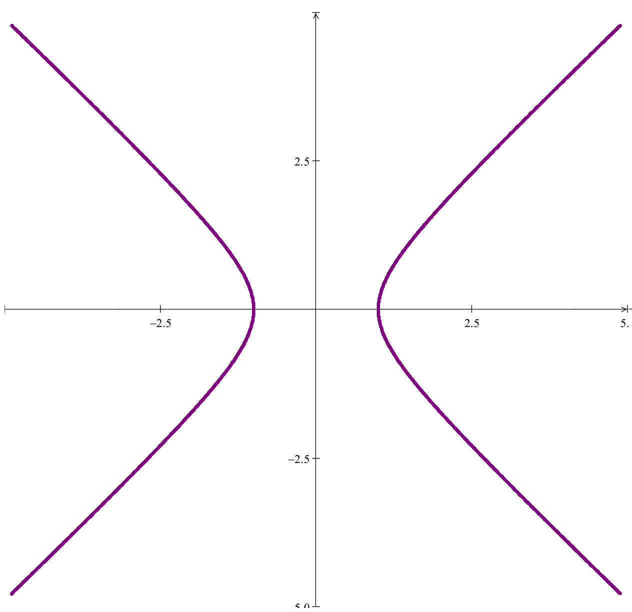


Let a line through the origin, making an angle of θ with the positive half of the *x*-axis intersect the unit circle. Then the point of intersection of this line with the unit circle is $(\cos\theta, \sin\theta)$. In the triangle below, the radius is equal to the hypotenuse and has length 1, so we have $\sin\theta = \frac{y}{1}$ and $\cos\theta = \frac{x}{1}$. The unit circle can be thought of as a way of looking at an infinite number of triangles by varying the lengths of their legs but keeping the lengths of their hypotenuses equal to 1.

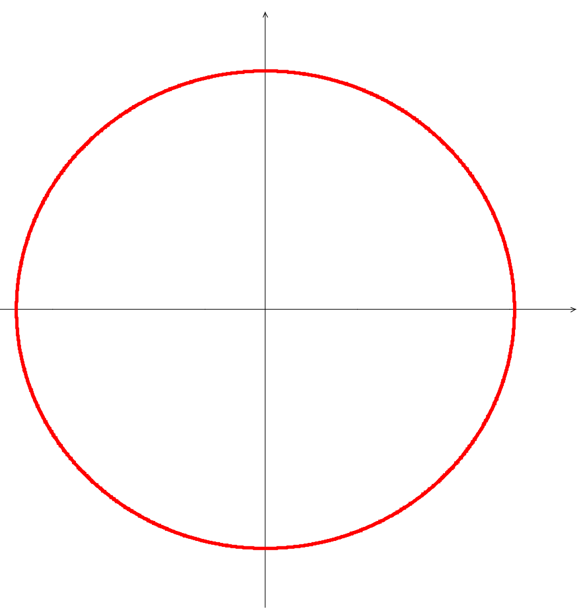




We have used the unit circle to define a class of functions that are very useful. We use the unit circle to define the *circular functions* – our standard trigonometric functions. The usual process is to measure the central angle in radians and define the sine and cosine of that angle to be the x- and y-coordinates of the point at which the ray defining the angle intersects the unit circle. This reinforces the definition of these functions in terms of ratios of sides of a right triangle, in that the use of rectangular coordinates gives us appropriate right triangles so that the definitions agree. If the unit circle is $x^2 + y^2 = 1$, then the unit hyperbola is defined to be the graph of $x^2 - y^2 = 1$.



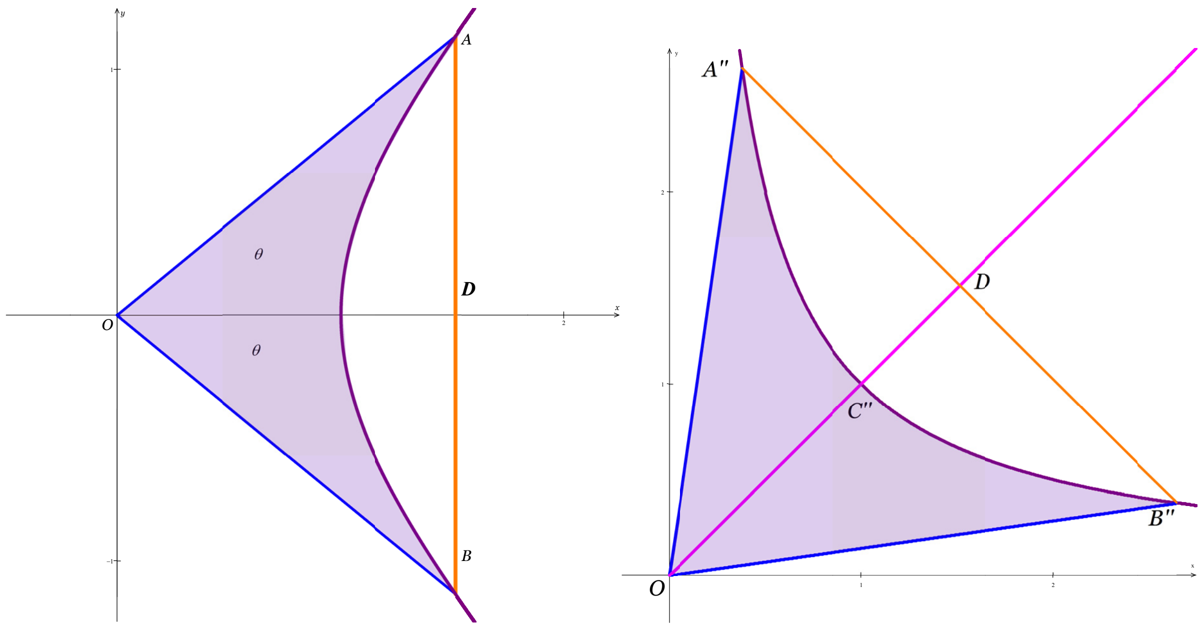
Unit hyperbola: $x^2 - y^2 = 1$



Unit circle: $x^2 + y^2 = 1$

In the unit circle we have $AC = \sin\theta$, $AB = 2\sin\theta$, $OC = \cos\theta$. The area of a sector is $\frac{1}{2}r^2\theta$, where the central angle θ is measured in radians. The area of the sector with $r = 1$ and central angle 2θ is θ. Therefore, the sine and cosine of θ can be defined with θ representing either an angle or the area of a sector.

Now, let's consider the unit hyperbola, $x^2 - y^2 = 1$, instead of the unit circle, $x^2 + y^2 = 1$. We will use the hyperbola to define functions that are reminiscent of the circular functions, since and cosine. Consider the unit hyperbola in Figure 2. Define the shaded sector to have area θ, where θ now represents an area rather than an angle. Define $\overline{A'C'}$ to be the hyperbolic sine of θ ($\overline{A'C'} = \sinh\theta$), $\overline{O'C'}$ to be the hyperbolic cosine of θ ($\overline{O'C'} = \cosh\theta$), and $\overline{A'B'} = 2\sinh\theta$. Now we want to find a specific formula – either geometric or algebraic – that will represent $\sinh\theta$ and $\cosh\theta$.



First rotate the hyperbola $x^2 - y^2 = 1$ by $\pi/4$ radians in a counterclockwise direction so that it becomes the graph of a function of one variable x: $y = 1/(2x)$. We now want to know the coordinates of the images of A', B', and C' under this rotation. O' goes to itself, but call the images of the other points A'', B'', and C''.

The point B'' has coordinates $(x_o, 1/(2x_o))$. The line OC'' is the line $y=x$. Let D be the point where $y = x$ intersects this hyperbola, so that $D = (\sqrt{2}/2, \sqrt{2}/2)$. Now, we have the following from the rotation:

- $\overleftrightarrow{B''C''}$ is perpendicular to $\overleftrightarrow{OC''}$;
- $\overline{OC''} = \overline{OC'}$;
- $\overline{B''C''} = \overline{B'C'}$;
- $\overline{B''C''} = \frac{1}{2}\overline{A'B'} = \sinh\theta$;
- $\overline{OC''} = \cosh\theta$.

The area of the region bounded by $\overline{OD}$, the curve $y = 1/(2x)$ and $\overline{OB''}$ can be found as follows:

$$\text{area} = \left[\frac{1}{2}\times\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}\right] + \int_{\sqrt{2}/2}^{x_o}\frac{dx}{2x} - \left[\frac{1}{2}\times x_o\times\frac{1}{2x_o}\right] = \frac{1}{2}\ln x\Big|_{\sqrt{2}/2}^{x_o} = \frac{1}{2}\ln\left(\sqrt{2}x_o\right).$$

Now from above we know that this area is $\frac{1}{2}\theta$. Thus,

$$\frac{1}{2}\theta=\frac{1}{2}\ln\left(\sqrt{2}x_0\right)$$
$$\theta=\ln\left(\sqrt{2}x_0\right)$$
$$e^{\theta}=\sqrt{2}x_0$$

Thus, the value of x_o can be expressed in terms of the area θ,

$$x_0=\frac{e^{\theta}}{\sqrt{2}}.$$

Now, we want to find the coordinates of C''. Since $\overleftrightarrow{B''C''}\perp\overleftrightarrow{OC''}$, then the slope of $B''C''$ has slope -1, with the equation

$$y-\frac{1}{2x_o}=-1(x-x_o).$$

At the point C'', $y=x$ so substituting we get

$$x=\frac{x_o}{2}+\frac{1}{4x_o}=y.$$

So, the length

$$\overline{B''C''}=\left[\left(x_0-\left[\frac{x_o}{2}+\frac{1}{4x_o}\right]\right)^2+\left(\frac{1}{2x_o}-\left[\frac{x_o}{2}+\frac{1}{4x_o}\right]\right)^2\right]^{1/2}$$
$$=\sqrt{2}\left(\frac{x_0}{2}-\frac{1}{4x_o}\right),\qquad \left(\frac{x_0}{2}-\frac{1}{4x_o}\right)>0 \text{ whenever } x_o>\sqrt{2}/2.$$

Now, $x_o=e^{\theta}/\sqrt{2}$ and $\overline{B''C''}=\sinh\theta$. Therefore,

$$\sinh\theta=\sqrt{2}\left(\frac{e^{\theta}}{2\sqrt{2}}-\frac{\sqrt{2}}{4e^{\theta}}\right)=\frac{e^{\theta}-e^{-\theta}}{2}.$$

Similarly,

$$\overline{OC''}=\left[\left(\frac{x_o}{2}+\frac{1}{4x_o}\right)^2+\left(\frac{x_o}{2}+\frac{1}{4x_o}\right)^2\right]^{1/2}$$
$$\cosh\theta=\sqrt{2}\left(\frac{x_o}{2}+\frac{1}{4x_o}\right)$$
$$=\sqrt{2}\left(\frac{e^{\theta}}{2\sqrt{2}}+\frac{\sqrt{2}}{4e^{\theta}}\right)=\frac{e^{\theta}+e^{-\theta}}{2}.$$

What do the graphs of these functions look like? We could plot them on the calculator, but because of their pattern, we want to try some algebra first.

Note that

$$\sinh^2(x)=\left(\frac{e^x-e^{-x}}{2}\right)^2=\frac{e^{2x}-2+e^{-2x}}{4}$$

and

$$\cosh^2(x)=\left(\frac{e^x+e^{-x}}{2}\right)^2=\frac{e^{2x}+2+e^{-2x}}{4},$$

so that

$$\cosh^2(x)-\sinh^2(x)=\frac{e^{2x}+2+e^{-2x}}{4}-\frac{e^{2x}-2+e^{-2x}}{4}=\frac{2}{4}+\frac{2}{4}=1$$

That is, we get that these functions satisfy the equation

$$u^2-v^2=1.$$

We also have seen the basic hyperbolic trigonometric identity:

$$\cosh^2(x)-\sinh^2(x)=1$$

Once we have the hyperbolic sine and hyperbolic cosine defined, then we define the other four functions as:

$$\tanh(x)=\frac{\sinh(x)}{\cosh(x)}=\frac{e^x-e^{-x}}{e^x+e^{-x}}$$

$$\coth(x)=\frac{\cosh(x)}{\sinh(x)}=\frac{e^x+e^{-x}}{e^x-e^{-x}},\ x\neq 0$$

$$\operatorname{sech}(x)=\frac{1}{\cosh(x)}=\frac{2}{e^x+e^{-x}}$$

$$\operatorname{csch}(x)=\frac{1}{\sinh(x)}=\frac{2}{e^x-e^{-x}},\ x\neq 0$$

Based on these definitions and the basic hyperbolic trigonometric identity, we find a large number of hyperbolic trigonometric identities that are analogous to the usual trigonometric identities

Trigonometric Identity	Hyperbolic Trigonometric Identity
$\cos^2 x+\sin^2 x=1$	$\cosh^2 x-\sinh^2 x=1$
$1+\tan^2 x=\sec^2 x$	$1-\tanh^2 x=\operatorname{sech}^2 x$
$\sin(x\pm y)=\sin x\cos y\pm\cos x\sin y$	$\sinh(x\pm y)=\sinh x\cosh y\pm\cosh x\sinh y$
$\cos(x\pm y)=\cos x\cos y\mp\sin x\sin y$	$\cosh(x\pm y)=\cosh x\cosh y\pm\sinh x\sinh y$
$\tan(x+y)=\dfrac{\tan x+\tan y}{1+\tan x\tan y}$	$\tanh(x+y)=\dfrac{\tanh x+\tanh y}{1+\tanh x\tanh y}$
$\sin^2\dfrac{x}{2}=\dfrac{1-\cos x}{2}$	$\sinh^2\dfrac{x}{2}=\dfrac{\cosh x-1}{2}$
$\cos^2\dfrac{x}{2}=\dfrac{1+\cos x}{2}$	$\cosh^2\dfrac{x}{2}=\dfrac{\cosh x+1}{2}$
$\tan^2\dfrac{x}{2}=\dfrac{1-\cos x}{1+\cos x}$	$\tanh^2\dfrac{x}{2}=\dfrac{\cosh x-1}{\cosh x+1}$

$\tan\frac{x}{2}=\frac{\sin x}{1+\cos x}$	$\tanh\frac{x}{2}=\frac{\sinh x}{\cosh x+1}$
$\tan\frac{x}{2}=\frac{1-\cos x}{\sin x}$	$\tanh\frac{x}{2}=\frac{\cosh x-1}{\sinh x}$
$\sin x=2\sin\frac{x}{2}\cos\frac{x}{2}$	$\sinh x=2\sinh\frac{x}{2}\cosh\frac{x}{2}$
$\sin x\pm\sin y=2\sin\frac{1}{2}(x\pm y)\cos\frac{1}{2}(x\mp y)$	$\sinh x\pm\sinh y=2\sinh\frac{1}{2}(x\pm y)\cosh\frac{1}{2}(x\mp y)$
$\cos x+\cos y=2\cos\frac{1}{2}(x+y)\cos\frac{1}{2}(x-y)$	$\cosh x+\cosh y=2\cosh\frac{1}{2}(x+y)\cosh\frac{1}{2}(x-y)$
$\cos x-\cos y=-2\sin\frac{1}{2}(x+y)\sin\frac{1}{2}(x-y)$	$\cosh x-\cosh y=2\sinh\frac{1}{2}(x+y)\sinh\frac{1}{2}(x-y)$

Certain Values Hyperbolic Trigonometric Functions

Just as with the circular functions and certain basic angles for which we want to know the values, there are certain base values of the hyperbolic functions that we might want to know:

$$\begin{array}{ll} \sinh 0=0 & \coth 0=\textit{undefined} \\ \cosh 0=1 & \operatorname{sech} 0=1 \\ \tanh 0=0 & \operatorname{csch} 0=\textit{undefined} \end{array}$$

This is a good start. Are there other values of which we should be aware? Well, we might expect that it will involve logarithms. For example,

$$\begin{array}{l|l} \sinh(\ln 2)=\frac{e^{\ln 2}-e^{-\ln 2}}{2}=\frac{2-1/2}{2}=\frac{3}{4} & \coth(\ln 2)=\frac{5}{3} \\ \cosh(\ln 2)=\frac{e^{\ln 2}+e^{-\ln 2}}{2}=\frac{2+1/2}{2}=\frac{5}{4} & \operatorname{sech}(\ln 2)=\frac{4}{5} \\ \tanh(\ln 2)=\frac{\sinh(\ln 2)}{\cosh(\ln 2)}=\frac{3}{5} & \operatorname{csch}(\ln 2)=\frac{4}{3} \end{array}$$

What other properties do they seem to have in common? We should consider at least three more things: inverse functions and derivatives and graphs.

Inverse Hyperbolic Trigonometric Functions

Since the hyperbolic trigonometric functions are defined in terms of exponentials, we might expect that the inverse hyperbolic functions might involve logarithms. Let us first consider the inverse function to the hyperbolic sine: arcsinh(x).

By the definition of an inverse function, $y=\operatorname{arcsinh}(x)$ means that $x=\sinh(y)$. Thus,

$$x=\frac{e^y-e^{-y}}{2}$$
$$e^y-e^{-y}=2x$$
$$\left(e^y-e^{-y}\right)e^y=2xe^y$$

$$e^{2y} - 2xe^{y} - 1 = 0$$

Let $u = e^y$, then this equation becomes

$$u^2 - 2xu - 1 = 0$$

$$u = \frac{2x \pm \sqrt{4x^2 + 4}}{2} = x \pm \sqrt{x^2 + 1}$$

$$e^y = x + \sqrt{x^2 + 1}$$

$y = \ln(x + \sqrt{x^2 + 1})$, and it must be the positive square root because $e^y > 0$.

$$\operatorname{arcsinh}(x) = \ln(x + \sqrt{x^2 + 1})$$

We should not find this too surprising. We would expect the others to be similar. Doing similar work, we find that:

$$\operatorname{arccosh}(x) = \ln(x + \sqrt{x^2 - 1})$$

$$\operatorname{arctanh}(x) = \ln\left(\sqrt{\frac{1+x}{1-x}}\right) = \frac{1}{2}\ln(1+x) - \frac{1}{2}\ln(1-x)$$

$$\operatorname{arccoth}(x) = \ln\left(\sqrt{\frac{x+1}{x-1}}\right) = \frac{1}{2}\ln(x+1) - \frac{1}{2}\ln(x-1)$$

$$\operatorname{arcsech}(x) = \ln\left(\frac{1 + \sqrt{1 - x^2}}{x}\right)$$

$$\operatorname{arccsch}(x) = \ln\left(\frac{1 + \sqrt{1 + x^2}}{x}\right)$$

Wow! No, you don't have to memorize all of this!!

Derivatives of Hyperbolic Trigonometric Functions

Just like everything else, we would expect the derivatives of the hyperbolic trigonometric functions to take an analogous route to those of the regular trigonometric functions.

We can easily find the derivatives since they are defined in terms of the exponential function.

$$\frac{d}{dx}\sinh(x) = \frac{d}{dx}\frac{e^x - e^{-x}}{2} = \frac{1}{2}\frac{d}{dx}(e^x - e^{-x}) = \frac{1}{2}(e^x - (-e^{-x})) = \frac{1}{2}(e^x + e^{-x}) = \cosh(x)$$

$$\frac{d}{dx}\cosh(x) = \frac{d}{dx}\frac{e^x + e^{-x}}{2} = \frac{1}{2}\frac{d}{dx}(e^x + e^{-x}) = \frac{1}{2}(e^x + (-e^{-x})) = \frac{1}{2}(e^x - e^{-x}) = \sinh(x)$$

Thus we have found that these functions have a nice derivative periodicity – and we do not have to take negative signs into account:

$$\frac{d}{dx}\sinh(x) = \cosh(x)$$

$$\frac{d}{dx}\cosh(x)=\sinh(x)$$

From these, we can compute the other derivatives – expecting analogous results.

$$\frac{d}{dx}\tanh(x)=\frac{d}{dx}\frac{\sinh(x)}{\cosh(x)}=\frac{\cosh x\cosh x-\sinh x\sinh x}{(\cosh x)^2}=\frac{1}{(\cosh x)^2}=\operatorname{sech}^2(x)$$

$$\frac{d}{dx}\coth(x)=\frac{d}{dx}\frac{\cosh(x)}{\sinh(x)}=\frac{\sinh x\sinh x-\cosh x\cosh x}{(\sinh x)^2}=\frac{-1}{(\sinh x)^2}=-\operatorname{csch}^2(x)$$

$$\frac{d}{dx}\operatorname{sech}(x)=\frac{d}{dx}\frac{1}{\cosh(x)}=\frac{-\sinh x}{(\cosh x)^2}=-\operatorname{sech}(x)\tanh(x)$$

$$\frac{d}{dx}\operatorname{csch}(x)=\frac{d}{dx}\frac{1}{\sinh(x)}=\frac{-\cosh x}{(\sinh x)^2}=-\operatorname{csch}(x)\coth(x)$$

These are pretty close to what we expected.

Graphs of Hyperbolic Trigonometric Functions

We wait until now to look at the graphs so that we can use the derivative to help us. First, let us look at cosh(x).

$\cosh(x)=\dfrac{e^x+e^{-x}}{2}>0$ since the numerator is always positive.

Since $\dfrac{d}{dx}\sinh(x)=\cosh(x)$, and $\cosh(x)>0$ for all x, we see that the hyperbolic sine function is always increasing. We also note that it has no critical points, since its derivative is always defined and is never 0. Now, looking at the graph it is not too surprising to find that it looks like the figure to the right.

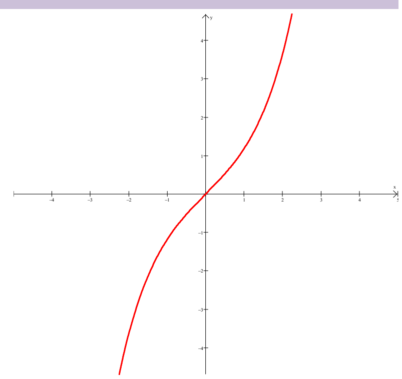


Figure 1: $y=\sinh(x)$

Now that we know what the hyperbolic sine looks like, we can analyze the hyperbolic cosine. Since its derivative is 0 at $x=0$, we know that it has a critical point there. Since the second derivative is always positive this critical point must be a local minimum. It is not hard to show that it is a global minimum. The graph looks like the figure on the left.

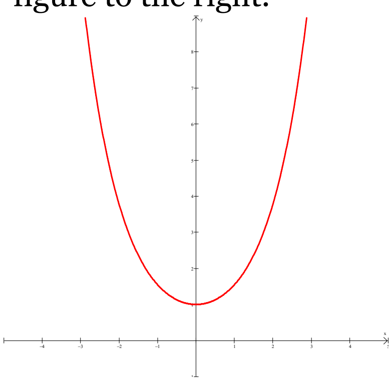
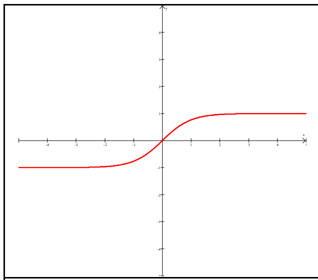
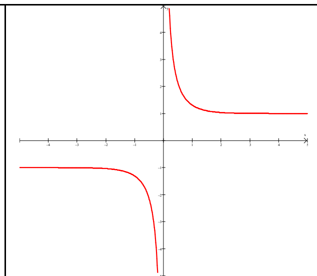
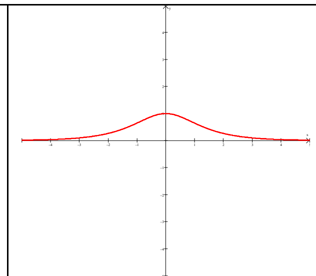
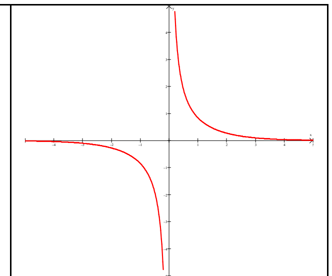


Figure 2: $y=\cosh(x)$

This may look somewhat like a parabola to you, but it actually grows faster than any parabola. The interesting thing is that it does describe a physical setting. If you put two pegs at the same height some distance apart and let a rope hang between the two pegs so that the ends just hang over the pegs (not tied to them), then this rope takes on a shape called a catenary. The catenary is a hyperbolic cosine shape. Examples are electrical wires hanging between power poles.

			
$y=\tanh(x)$	$y=\coth(x)$	$y=\operatorname{sech}(x)$	$y=\operatorname{csch}(x)$

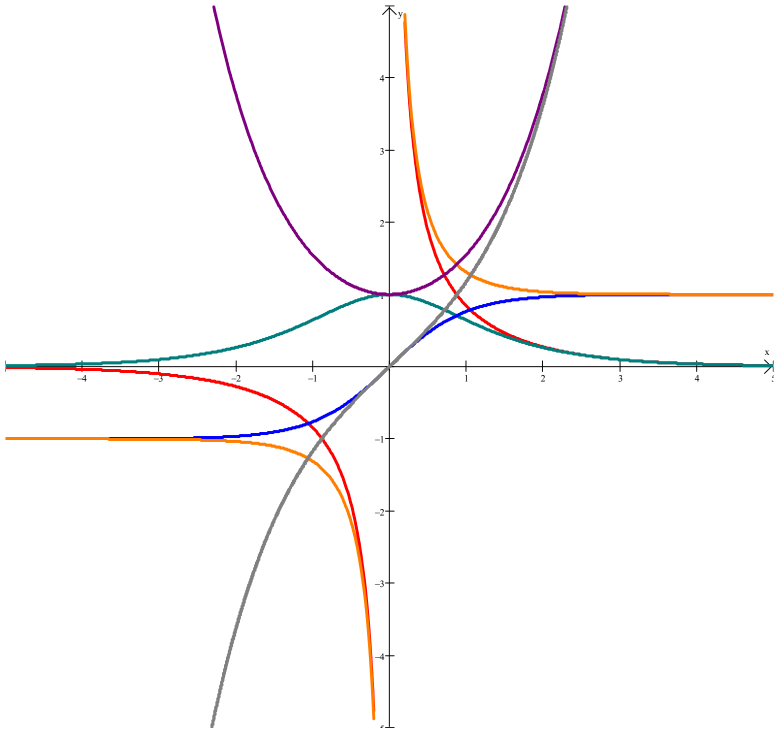


Figure 3: A Web of Hyperbolic Trigonometric Functions

Power Series Expansions for Hyperbolic Trigonometric Functions

Since the exponential function has a power series expansion $e^x=\sum_{k=0}^{\infty}\frac{x^k}{k!}$ and since the hyperbolic functions are defined in terms of the exponential function, we find that the power series expansions for the hyperbolic functions are:

$$\begin{aligned}
\cosh x &= \frac{e^x+e^{-x}}{2}\\
&= \frac{1}{2}\sum_{n=0}^{\infty}\frac{x^n}{n!}+\frac{1}{2}\sum_{n=0}^{\infty}\frac{(-x)^n}{n!}\\
&= \frac{1}{2}\sum_{n=0}^{\infty}\frac{x^n}{n!}+\frac{1}{2}\sum_{k=0}^{\infty}\frac{x^{2k}}{(2k)!}-\frac{1}{2}\sum_{k=0}^{\infty}\frac{x^{2k+1}}{(2k+1)!}\\
\cosh x &= \sum_{k=0}^{\infty}\frac{x^{2k}}{(2k)!}
\end{aligned}$$

Likewise the hyperbolic sine function has a power series expansion

$$\sinh x = \sum_{k=0}^{\infty} \frac{x^{2k+1}}{(2k+1)!}$$

Now, these look vaguely familiar – the more vague the longer it has been since you studied power series. As a reminder let me point out that the Maclaurin series for the circular functions are:

$$\sin x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!}$$

$$\cos x = \sum_{k=0}^{\infty} \frac{(-1)^k x^{2k}}{(2k)!}$$

So, the Maclaurin series for the hyperbolic functions and those for the circular functions are very similar.

Now, there is even more of a relationship between these two – using complex numbers. Recall that we let $i^2 = -1$. In the power series expansion for $\cos x$, let's replace x by ix.

$$\cos(ix) = \sum_{n=0}^{\infty} \frac{(-1)^n (ix)^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n i^{2n} x^{2n}}{(2n)!} = \sum_{n=0}^{\infty} \frac{x^{2n}}{(2n)!} = \cosh(x)$$

$$\sin(ix) = \sum_{n=0}^{\infty} \frac{(-1)^n (ix)^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{(-1)^n i^{2n+1} x^{2n+1}}{(2n+1)!} = \sum_{n=0}^{\infty} \frac{ix^{2n+1}}{(2n+1)!} = i\sinh(x)$$

Therefore, we see that we now have a relationship between the circular and the hyperbolic functions as follows:

$$\cosh(x) = \cos(ix) = \cos\left(\frac{x}{i}\right)$$

$$\sinh(x) = -i\sin(ix) = i\sin\left(\frac{x}{i}\right)$$

Uses for Hyperbolic Functions

1. Antiderivatives

These inverse hyperbolic trigonometric functions often appear in antiderivative formulas instead of logarithms. As an example, we get

$$\int \frac{dx}{\sqrt{x^2 - 9}} = \ln\left|x + \sqrt{x^2 - 9}\right| + C$$

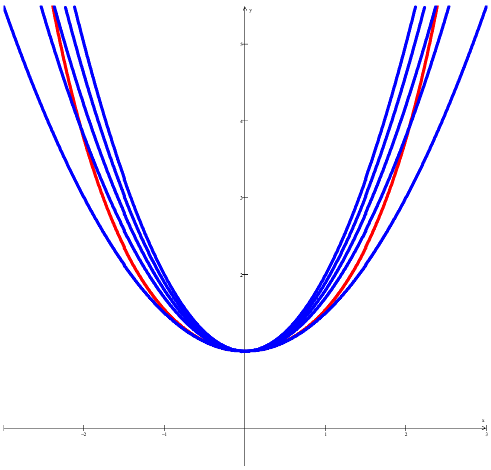
using the substitution $x = 3 \sec u$. On the other hand we get

$$\int \frac{dx}{\sqrt{x^2 - 9}} = \cosh^{-1}\left(\frac{x}{3}\right) + C$$

using the substitution $x = 3 \cosh u$. This second integral is actually easier to compute symbolically – if you remember to use hyperbolic functions.

2. Applications

What shape does a chain take when hanging freely between two pegs? This does not mean that it is fastened at the two pegs, but is, in fact, free to move at the pegs. For the most part we would think that this shape is the shape of a parabola.

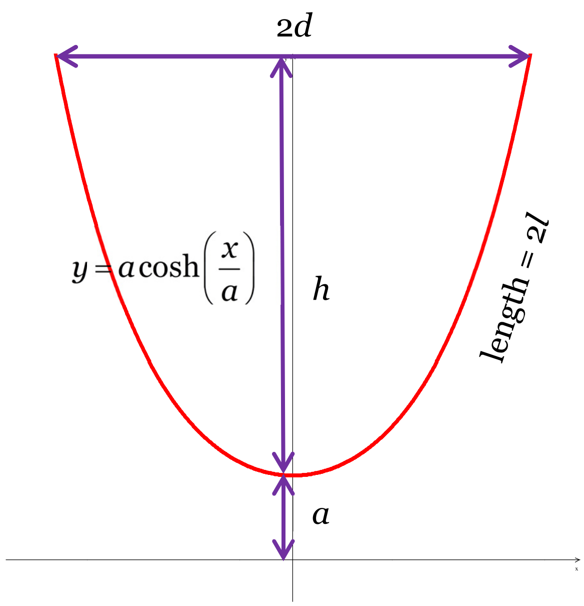


In the figure to the left, the freely hanging cable is the red curve and the blue curves are parabolas. The parabolas appear to be "too pointy." Galileo discusses this question in two places in his works. While in the first writing he says that the hanging cable *resembles* a parabola, in a later visit to this problem, he indicated that the hanging cable is approximated by parabolas. Joachim Jungius (1587–1657) proved that this curve is not a parabola and it was published posthumously in 1669.

Robert Hooke, English scientist and architect, applied the catenary to the construction of arches. He discovered it while involved in the rebuilding of St Paul's Cathedral. In 1671, Hooke announced to the Royal Society that he had solved the problem of the optimal shape of an arch, and in 1675 published an encrypted solution as a Latin anagram in an appendix to his *Description of Helioscopes*. He wrote that he had found *a true mathematical and mechanical form of all manner of Arches for Building*. He didn't publish the solution of this anagram in his lifetime, but in 1705 his executor provided the solution as *Ut pendet continuum flexile, sic stabit contiguum rigidum inversum*, meaning "*As hangs a flexible cable so, inverted, stand the touching pieces of an arch.*"

In 1691 Gottfried Leibniz, Christiaan Huygens, and Johann Bernoulli derived the equation in response to a challenge by Jakob Bernoulli.

This curve is known as the catenary curve, the funicular curve, and the velar curve.



2(b). Suspension Bridges

Cables on a suspension bridge are catenaries before the road bed is attached. Once the road bed is attached, the shape becomes a parabola. A true hanging bridge though takes the shape of a catenary.


2(c). Sails and Velar Curves

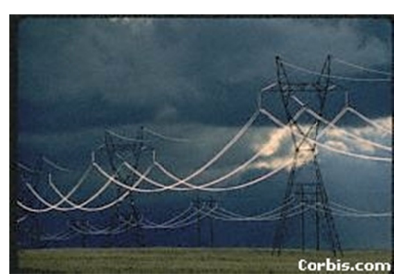
At this time, sailing ships provided all of the commerce and the common defense. One of the problems with sails was that they tore and they might be inefficient. The country with the fastest ships would have an advantage. Bernoulli studied this problem and showed that the profile of a rectangular sail attached to 2 horizontal bars, swollen by a wind blowing perpendicular to the bar took on the cross-section of a catenary. Bernoulli's work spoke of the *velar* curve, for the sail that was filled by the wind.

Now, this is not just of interest to historians and those wishing to relive sailing ships. Consider the problem that one has in backpacking. You need some shelter, but it needs to be light. You want something that will hold up to wind and rain. Experience shows that rectangular tarps tear and blow away relatively easily. Enter the new design: the *catenary tarp*:

> To help our members answer, a catenary cut tarp (or "cat" cut for short) is a tarp with the natural "sag" that gravity imposes in a line or chain suspended between two points, cut into the fabric along a seam. This results in a shape the opposite of an arch shape. It's done mainly to reduce flapping in wind, although the loss of that little bit of material also makes the tarp very slightly lighter in weight than a flat cut tarp.

2(d). Other Uses

Other examples where these hyperbolic functions are used are:



- Anchor rope for anchoring marine vessels– shape is mostly a catenary
- Sail design for racing sloops
- Waves propagating through a narrow canal
- Power lines