1.15 Nodal Analysis (Node Voltage Method)

This method is based on Kirchhoff's current law (KCL). Normally, this analysis is carried out to determine voltages of different nodes with respective to reference node. However, after determination of node voltage, currents in all branches can be determined. This method is useful where number of loops is large and hence, mesh analysis becomes lengthy. Nodal analysis also has advantage that a minimum number of equations need to be written to determine the unknown quantities.

Following steps are to be taken while solving a problem by nodal analysis. Consider the circuit of Fig. 1.198.

Step I: Mark all nodes. Every junction of the network where three or more branches meet is regarded as a node. In a circuit of Fig. 1.198, there are four nodes (marked by bold points). But lower two nodes are same, and by joining them, we get only three nodes as shown in Fig. 1.199.

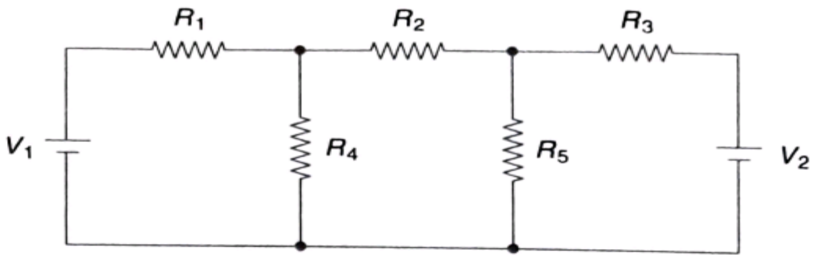

Fig. 1.198 Identification of nodes

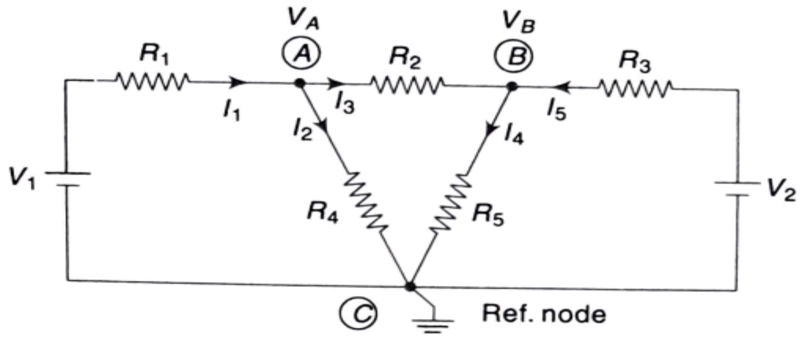

Fig. 1.199 Assigning the branch currents and unknown node voltages

Step II: Select one of the nodes as reference node. Normally, for convenience, choose that node as reference where maximum elements are connected or maximum branches are meeting. Obviously, node C is selected as reference node. Reference node is also called zero potential node or datum/ground node.

Step III: Assign the unknown potentials of all nodes with respect to the reference node. For example, at nodes A and B, let the potentials are V_A and V_B with respect to the reference node.

Step IV: At each node (excluding reference node), assume the unknown currents and mark their directions (choose the current directions arbitrarily).

Step V: Apply the KCL at each node and write the equations in terms of node voltages. By solving the equations, determine the node voltages. From node voltages, current in any branch can be determined.

Example 1.56 By node voltage method, find the current through 15 Ω resistor in the circuit of Fig. 1.200.

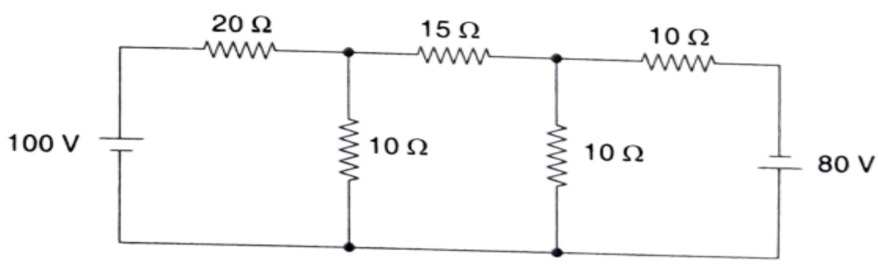

Fig. 1.200

Solution

Marking the different nodes and assigning the unknown currents, we obtain the following circuit:

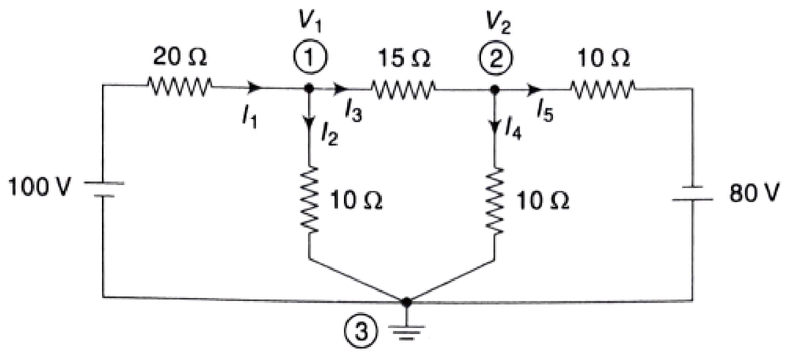

Fig. 1.201

Applying KCL at node 1,

$$I_1 = I_2 + I_3$$

or $$\frac{100 - V_1}{20} = \frac{V_1 - 0}{10} + \frac{V_1 - V_2}{15}$$

or $$13V_1 - 4V_2 = 300 \quad \text{(i)}$$

Applying KCL at node 2,

$$I_3 = I_4 + I_5$$

or $$\frac{V_1 - V_2}{15} = \frac{V_2 - 0}{10} + \frac{V_2 - (-80)}{10}$$

or $$V_1 - 4V_2 = 120 \quad \text{(ii)}$$

From Eqs (i) and (ii), $V_1 = 15$ V, $V_2 = -26.25$ V

Hence, $I_{15\Omega} = I_3 = \dfrac{V_1 - V_2}{15} = 2.75 \text{ A } (\rightarrow)$

Example 1.57 By node voltage method, find the currents I_1, I_2, and I_3 in the circuit of Fig. 1.202.

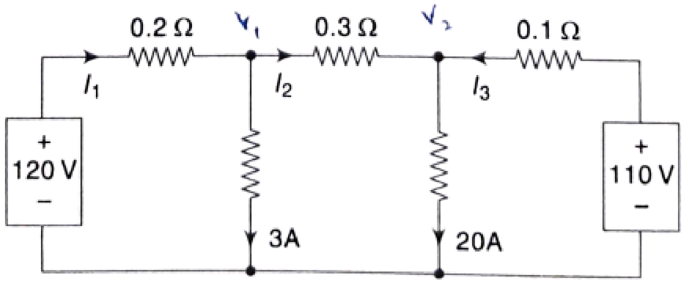

Fig. 1.202

Solution

Marking the different nodes, we get Fig. 1.203.

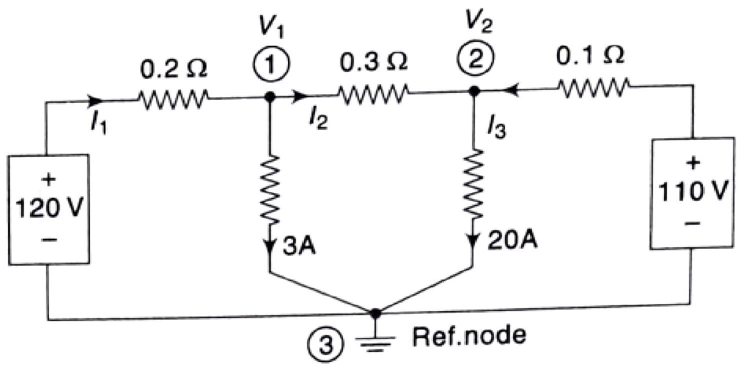

Fig. 1.203

Applying KCL at node 1,

$$I_1 = 3 + I_2$$

or $$\frac{120 - V_1}{0.2} = 3 + \frac{V_1 - V_2}{0.3}$$

or $$5V_1 - 2V_2 = 358.2 \tag{i}$$

Applying KCL at node 2,

$$I_2 + I_3 = 20$$

or $$\frac{V_1 - V_2}{0.3} + \frac{110 - V_2}{0.1} = 20$$

or $$V_1 - 4V_2 = -324 \tag{ii}$$

From Eqs (i) and (ii),

$$V_1 = 115.6 \text{ V}$$

$$V_2 = 109.9 \text{ V}$$

So, $$I_1 = \frac{120 - V_1}{0.2} = 22 \text{ A}, \quad I_2 = \frac{V_1 - V_2}{0.3} = 19 \text{ A}, \quad I_3 = \frac{110 - V_2}{0.1} = 1 \text{ A}$$

Example 1.58 By nodal analysis, determine the voltages at nodes A and B in the circuit of Fig. 1.204.

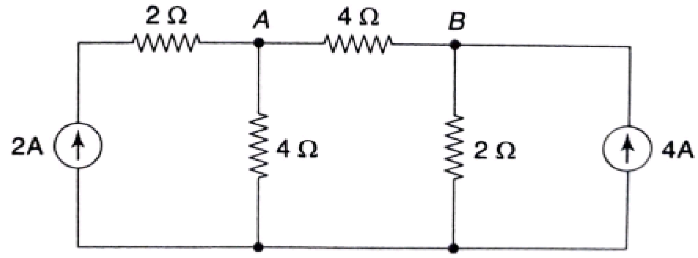

Fig. 1.204

Solution

Assigning the unknown currents, we get the circuit as shown in Fig. 1.205.

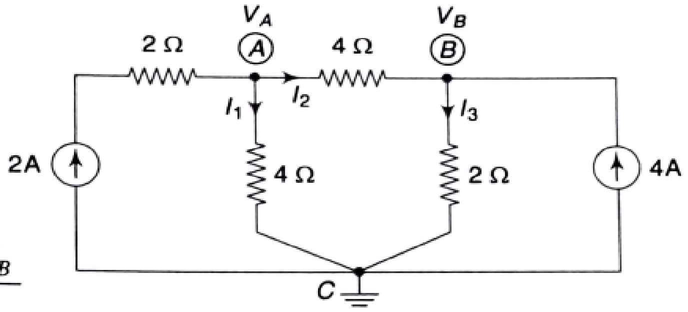

Fig. 1.205

Applying KCL at node A,

$$2 = I_1 + I_2$$

or $$2 = \frac{V_A - 0}{4} + \frac{V_A - V_B}{4}$$

or $$2V_A - V_B = 8 \qquad \text{(i)}$$

Applying KCL at node B,

$$I_2 + 4 = I_3$$

or $$\frac{V_A - V_B}{4} + 4 = \frac{V_B - 0}{2}$$

or $$V_A - 3V_B = -16 \qquad \text{(ii)}$$

From Eqs (i) and (ii),

$$V_A = 8 \text{ V}$$
$$V_B = 8 \text{ V}$$

Example 1.59 Find the current through 2 Ω and 3 Ω resistances using nodal analysis in the circuit of Fig. 1.206.

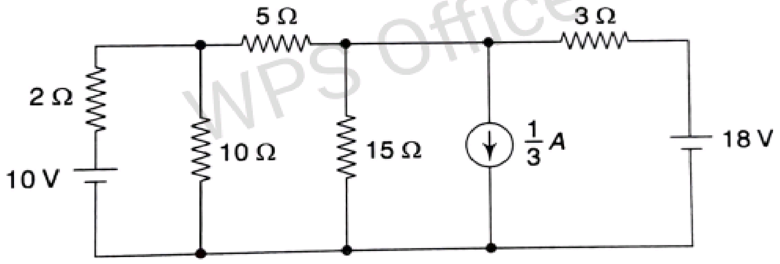

Fig. 1.206

Solution

By joining the same nodes and assigning the unknown currents, the given circuit is redrawn in Fig. 1.207.

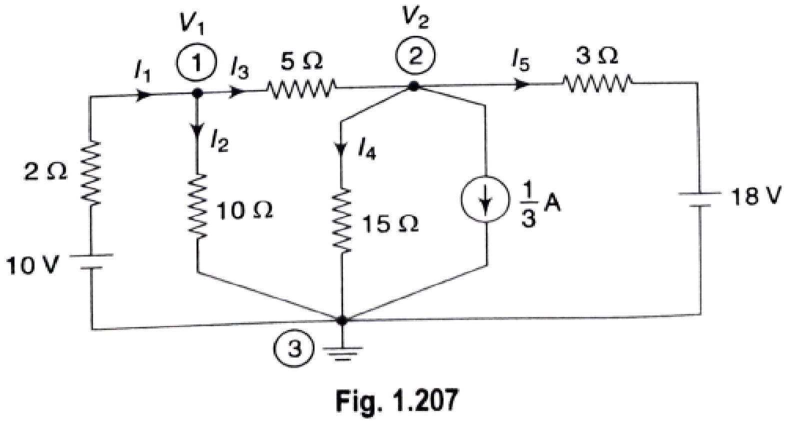

Fig. 1.207

Applying KCL at node 1,

$$I_1 = I_2 + I_3$$

or $$\frac{10 - V_1}{2} = \frac{V_1 - 0}{10} + \frac{V_1 - V_2}{5}$$

or $$8V_1 - 2V_2 = 50 \quad \text{(i)}$$

Applying KCL at node 2,

or $$I_3 = I_4 + I_5 + \frac{1}{3}$$

or $$\frac{V_1 - V_2}{5} = \frac{V_2 - 0}{15} + \frac{V_2 - 18}{3} + \frac{1}{3}$$

or $$3V_1 - 9V_2 = -85 \quad \text{(ii)}$$

From Eqs (i) and (ii),

$$V_1 = 9.394 \text{ V}$$
$$V_2 = 12.576 \text{ V}$$

Now, $I_{2\Omega} = \dfrac{10 - V_1}{2} = 0.303 \text{ A}(\uparrow)$, $\quad I_{3\Omega} = \dfrac{V_2 - 18}{3} = -1.81 \text{ A} (\rightarrow)$

Example 1.60 Find the currents in the various resistors of the circuit shown in Fig. 1.208.

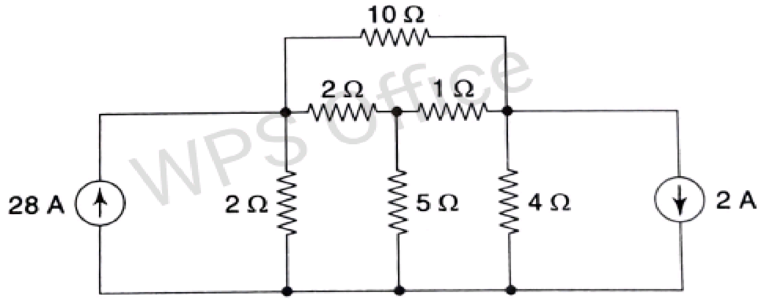

Fig. 1.208

Solution

By joining the same nodes and assigning the unknown currents, the given circuit is redrawn in Fig. 1.209.

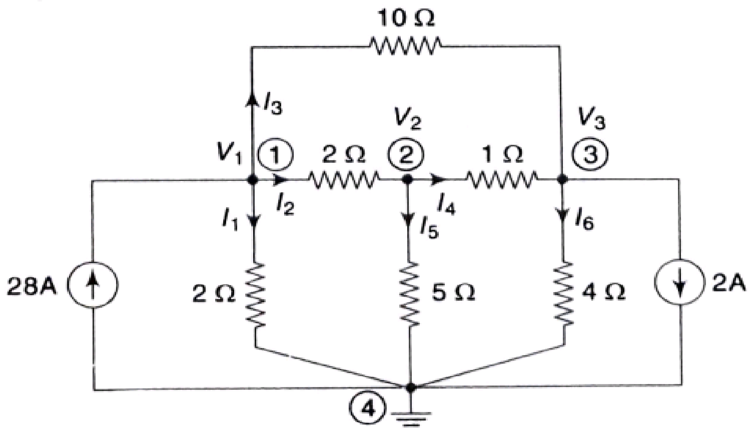

Fig. 1.209

Applying KCL at node 1,

$$28 = I_1 + I_2 + I_3$$

or $$28 = \frac{V_1 - 0}{2} + \frac{V_1 - V_2}{2} + \frac{V_1 - V_3}{10}$$

or $$11V_1 - 5V_2 - V_3 = 280 \quad \text{(i)}$$

Applying KCL at node 2,

$$I_2 = I_4 + I_5$$

or $$\frac{V_1 - V_2}{2} = \frac{V_2 - V_3}{1} + \frac{V_2 - 0}{5}$$

or $$5V_1 - 17V_2 + 10V_3 = 0 \quad \text{(ii)}$$

Applying KCL at node 3,

$$I_3 + I_4 = I_6 + 2$$

or $$\frac{V_1 - V_3}{10} + \frac{V_2 - V_3}{1} = \frac{V_3 - 0}{4} + 2$$

or $$V_1 + 10V_2 - 13.5V_3 = 20 \quad \text{(iii)}$$

The values of V_1, V_2, and V_3 may be found by solving the above three simultaneous equations or by the method of determinants as given below:
Putting the above three equations in matrix form, we have

$$\begin{bmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} 280 \\ 0 \\ 20 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 11 & -5 & -1 \\ 5 & -17 & 10 \\ 1 & 10 & -13.5 \end{vmatrix} = 970, \qquad \Delta_1 = \begin{vmatrix} 280 & -5 & -1 \\ 0 & -17 & 10 \\ 20 & 10 & -13.5 \end{vmatrix} = 34{,}920$$

$$\Delta_2 = \begin{vmatrix} 11 & 280 & -1 \\ 5 & 0 & 10 \\ 1 & 20 & -13.5 \end{vmatrix} = 19{,}400, \qquad \Delta_3 = \begin{vmatrix} 11 & -5 & 280 \\ 5 & -17 & 0 \\ 1 & 10 & 20 \end{vmatrix} = 15{,}520$$

By Cramer's rule,

$$V_1 = \frac{\Delta_1}{\Delta} = \frac{34{,}920}{970} = 36 \text{ V}, \qquad V_2 = \frac{\Delta_2}{\Delta} = \frac{19{,}400}{970} = 20 \text{ V},$$

$$V_3 = \frac{\Delta_3}{\Delta} = \frac{15{,}520}{970} = 16 \text{ V}$$

The various currents shown in Fig. 1.209 can be calculated as

$$I_1 = \frac{V_1 - 0}{2} = \frac{36 - 0}{2} = 18 \text{ A}$$

$$I_2 = \frac{V_1 - V_2}{2} = \frac{36 - 20}{2} = 8 \text{ A}$$

$$I_3 = \frac{V_1 - V_3}{10} = \frac{36 - 16}{10} = 2\text{ A}$$

$$I_4 = \frac{V_2 - V_3}{1} = \frac{20 - 16}{1} = 4\text{ A}$$

$$I_5 = \frac{V_2 - 0}{5} = \frac{20 - 0}{5} = 4\text{ A}$$

$$I_6 = \frac{V_3 - 0}{4} = \frac{16 - 0}{4} = 4\text{ A}$$

Example 1.61 Find the current I by using node-voltage analysis for the circuit given in Fig. 1.210.

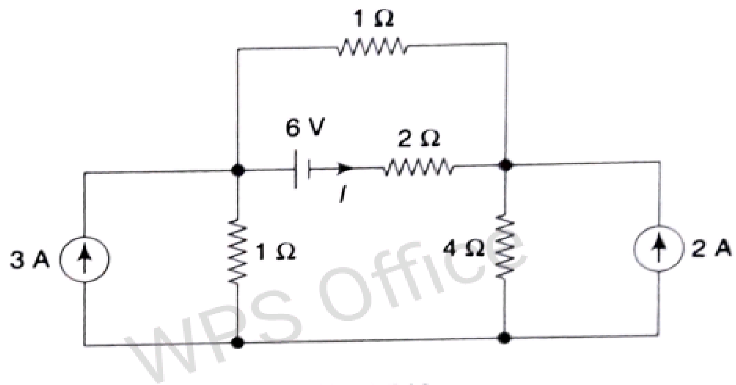

Fig. 1.210

Solution

Marking the different nodes and assuming the unknown currents, we obtain the following circuit:

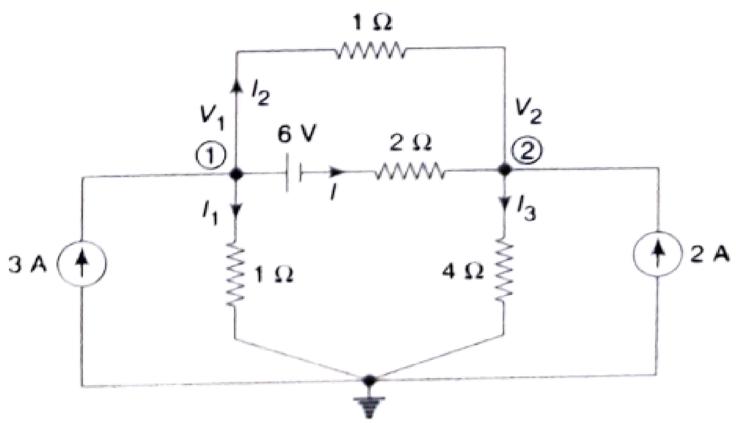

Fig. 1.211

Applying KCL at node 1,

$$3 = I_1 + I + I_2$$

or $$3 = \frac{V_1 - 0}{1} + \frac{(V_1 - 6) - V_2}{2} + \frac{V_1 - V_2}{1}$$

or $$5V_1 - 3V_2 = 12 \qquad \text{(i)}$$

Applying KCL at node 2,

$$I_2 + I + 2 = I_3$$

or $$\frac{V_1 - V_2}{1} + \frac{(V_1 - 6) - V_2}{2} + 2 = \frac{V_2 - 0}{4}$$

or $$6V_1 - 7V_2 = 4 \qquad \text{(ii)}$$

From Eqs (i) and (ii),

$$V_1 = 4.2353 \text{ V}$$
$$V_2 = 3.0588 \text{ V}$$

Now, $I = \dfrac{(V_1 - 6) - V_2}{2} = \dfrac{(4.2353 - 6) - 3.0588}{2} = -2.412 \text{ A}$

Hint In Fig. 1.211, the expression of current I in terms of node voltages is determined as follows:

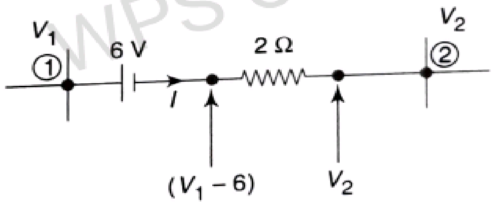

Fig. 1.212

By Ohm's law,

$$I = \frac{\text{Potential difference across } 2\,\Omega \text{ resistor}}{\text{Resistance}}$$

The current I flows through the 2 Ω resistor from node 1 to node 2 (i.e., from right end to left end of 2 Ω resistor)

So, potential difference across 2 Ω resistor

= Potential at right end of 2 Ω resistor – Potential at left end of 2 Ω resistor

$= (V_1 - 6) - V_2$

So,

$$I = \frac{(V_1 - 6) - V_2}{2}$$

Example 1.62 Find the current through 4 Ω and 3 Ω resistances using nodal analysis in the circuit of Fig. 1.213.

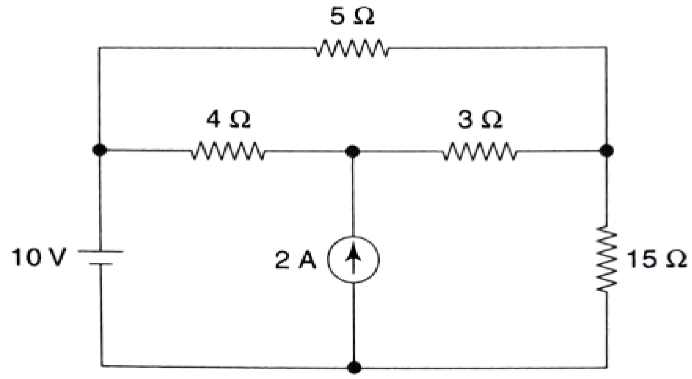

Fig. 1.213

Solution

Marking the different nodes, we get Fig. 1.214.

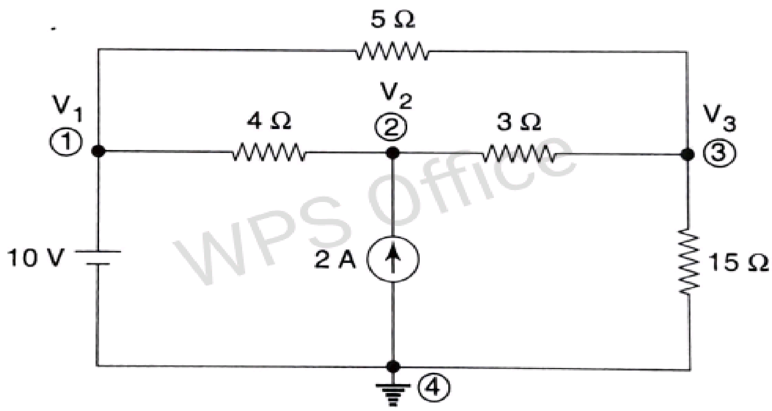

Fig. 1.214

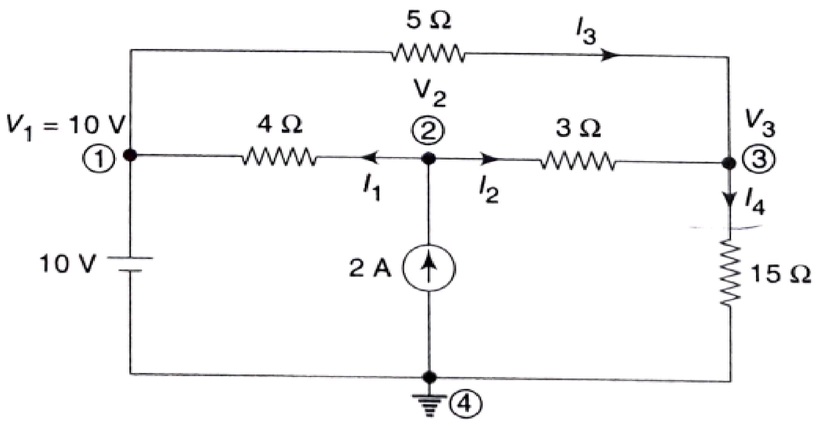

Fig. 1.215

As voltage source of 10 V is directly connected between the reference node (node 4) and non-reference node (node 1) with positive polarity towards node 1, the voltage at node 1 is 10 V with respect to reference node.

Thus $V_1 = 10$ (i)

Hint: If voltage source is connected between the reference node and non-reference node, then set a voltage at non-reference node equal to the voltage source. In this case, the solution becomes simple as application of KCL at non-reference node is not required.

Now assigning the unknown currents at node 2 and node 3, we get Fig. 1.215. Note that as voltage at node 1 is known, application of KCL at this node is not required.

Applying KCL at node 2,

$$2 = I_1 + I_2$$

or $$2 = \frac{V_2 - 10}{4} + \frac{V_2 - V_3}{3}$$

or $$7V_2 - 4V_3 = 54 \qquad \text{(i)}$$

Applying KCL at node 3,

$$I_3 + I_2 = I_4$$

or $$\frac{10 - V_3}{5} + \frac{V_2 - V_3}{3} = \frac{V_3 - 0}{15}$$

or $$5V_2 - 9V_3 = -30 \qquad \text{(ii)}$$

From Eqs (i) and (ii)

$$V_2 = 14.09 \text{ V}$$

$$V_3 = 11.16 \text{ V}$$

So, $$I_{4\Omega} = \frac{V_2 - 10}{4} = \frac{11.16 - 10}{4} = 0.29 \text{ A}$$

$$I_{3\Omega} = \frac{V_2 - V_3}{3} = \frac{14.09 - 11.16}{3} = 0.98 \text{ A}$$

Example 1.63 Find the current through 6 Ω resistance using nodal analysis in the circuit of Fig 1.216.

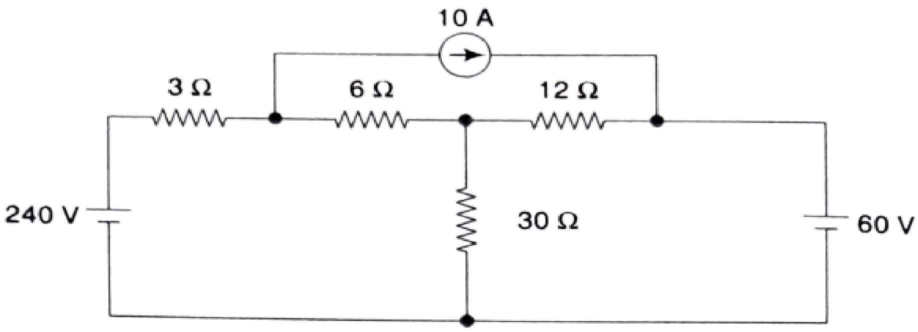

Fig. 1.216

Solution

Marking the different nodes and assigning the unknown currents, the given circuit is redrawn in Fig. 1.217.

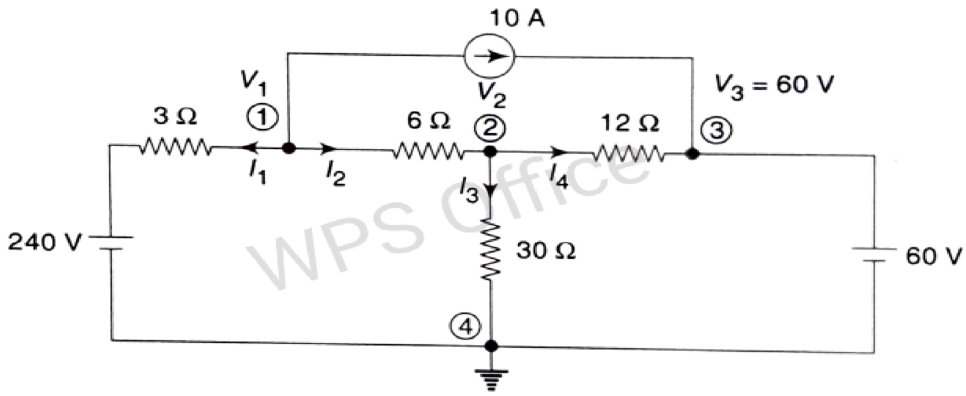

Fig. 1.217

A voltage source of 60 V is connected between node 3 and the reference node (node 4).

So, $V_3 = 60$

Applying KCL at node 1,

$$I_1 + I_2 + 10 = 0$$

or $$\frac{V_1 - 240}{3} + \frac{V_1 - V_2}{6} + 10 = 0$$

or $$3V_1 - V_2 = 420 \tag{i}$$

Applying KCL at node 2,

$$I_2 = I_3 + I_4$$

or $$\frac{V_1 - V_2}{6} = \frac{V_2 - 0}{30} + \frac{V_2 - 60}{12}$$

or $$60V_1 - 102V_2 = -1800 \qquad \text{(ii)}$$

From Eqs (i) and (ii)

$V_1 = 181.46$ V

$V_2 = 124.39$ V

So, $$I_{6\,\Omega} = \frac{V_1 - V_2}{6} = \frac{181.46 - 124.39}{6} = 9.51 \text{ A}$$