1] Encode and Decode "THE PROFESSOR IS GOOD" using $\begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$ under i) modulo 26 & ii) modulo 27

Ans:- i]

Let the key matrix is $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

Consider the message "THE PROFESSOR IS GOOD"

$\therefore$ We have, TH EP RO FE SS OR IS GO OD
20,8 , 5,16 , 18,15 , 6,5 , 19,19 , 15,18 ,
9,19 , 7,15 , 15,4

... (If we use modulo 26, then we do not consider space as a character)

We get message Matrix B as,

$$B = \begin{bmatrix} 20 & 5 & 18 & 6 & 19 & 15 & 9 & 7 & 15 \\ 8 & 16 & 15 & 5 & 19 & 18 & 19 & 15 & 4 \end{bmatrix}$$

Consider the coded message C as,

$$C = AB = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 20 & 5 & 18 & 6 & 19 & 15 & 9 & 7 & 15 \\ 8 & 16 & 15 & 5 & 19 & 18 & 19 & 15 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 36 & 37 & 48 & 16 & 57 & 51 & 47 & 37 & 23 \\ 44 & 53 & 63 & 21 & 76 & 69 & 66 & 52 & 27 \end{bmatrix}$$

$$C = \begin{bmatrix} 10 & 11 & 22 & 16 & 5 & 25 & 21 & 11 & 23 \\ 18 & 1 & 11 & 21 & 24 & 17 & 14 & 26 & 1 \end{bmatrix} \pmod{26}$$

$C = 10, 18, 11, 1, 22, 11, 16, 21, 5, 24, 25, 17, 21, 14, 11, 26, 23, 1$

$C =$ JRKAVKPUEXYQUNKZWA

Now we consider the decoding part
Suppose the received message is

$C = 10, 18, 11, 1, 22, 11, 16, 21, 5, 24, 25, 17, 21, 14, 11, 26, 23, 1$

i.e $C = \begin{bmatrix} 10 & 11 & 22 & 16 & 5 & 25 & 21 & 11 & 23 \\ 18 & 1 & 11 & 21 & 24 & 17 & 14 & 26 & 1 \end{bmatrix}$

We have the key Matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

We calculate the inverse of matrix A as

$A^{-1} = \frac{1}{|A|} \cdot adj\, A$ $\qquad A^{-1} = (1 \times 3) - (2 \times 1) = 1$

$\therefore |A| = \frac{1}{1} \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$

Decoding of the message, let the message be B

$B = A^{-1} \times C = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 10 & 11 & 22 & 16 & 5 & 25 & 21 & 11 & 23 \\ 18 & 1 & 11 & 21 & 24 & 17 & 14 & 26 & 1 \end{bmatrix}$

$B = \begin{bmatrix} -6 & 31 & 44 & 6 & -33 & 41 & 35 & -19 & 67 \\ 8 & -10 & -11 & 5 & 19 & -8 & -7 & 15 & -22 \end{bmatrix}$

$$B = \begin{bmatrix} 20 & 5 & 18 & 6 & 19 & 15 & 9 & 7 & 15 \\ 8 & 16 & 15 & 5 & 19 & 18 & 19 & 15 & 4 \end{bmatrix} - (\text{mod } 26)$$

The message is

$B = 20, 8, 5, 16, 18, 15, 6, 5, 19, 19, 15, 18, 9, 19, 7, 15, 15, 4$

= TH EP RO FE SS OR IS GO OD

∴ "THE PROFESSOR IS GOOD"

ii] Let the key matrix is $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

Consider message "THE PROFESSOR IS GOOD"

We have, TH E_ PR OF ES SO R_ IS _G OO D_

20,8 , 5,0 , 16,18 , 15,6 , 5,19 , 19,15 , 18,0 , 9,19 , 0,7 , 15,15 , 4,0

We get message matrix B as,

$$B = \begin{bmatrix} 20 & 5 & 16 & 15 & 5 & 19 & 18 & 9 & 0 & 15 & 4 \\ 8 & 0 & 18 & 6 & 19 & 15 & 0 & 19 & 7 & 15 & 0 \end{bmatrix}$$

Consider the coded message C as,

$$C = AB = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} 20 & 5 & 16 & 15 & 5 & 19 & 18 & 9 & 0 & 15 & 4 \\ 8 & 0 & 18 & 6 & 19 & 15 & 0 & 19 & 7 & 15 & 0 \end{bmatrix}$$

$$C = \begin{bmatrix} 36 & 5 & 52 & 27 & 43 & 49 & 18 & 47 & 14 & 45 & 4 \\ 44 & 5 & 70 & 33 & 62 & 64 & 18 & 66 & 21 & 60 & 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 9 & 5 & 25 & 0 & 16 & 22 & 18 & 20 & 14 & 18 & 4 \\ 17 & 5 & 16 & 6 & 8 & 10 & 18 & 12 & 21 & 6 & 4 \end{bmatrix} \pmod{27}$$

$C = 9, 17, 5, 5, 25, 16, 0, 6, 16, 8, 22, 10, 18, 18, 20, 12, 14, 21,$
$18, 6, 4, 4$

$C =$ IQEEYP_FPHVJRRTLNURFDD

Now we consider the decoding part.
Suppose the received message is

$C = 9, 17, 5, 5, 25, 16, 0, 6, 16, 8, 22, 10, 18, 18, 20, 12, 14, 21,$
$18, 6, 4, 4$

i.e. $$C = \begin{bmatrix} 9 & 5 & 25 & 0 & 16 & 22 & 18 & 20 & 14 & 18 & 4 \\ 17 & 5 & 16 & 6 & 8 & 10 & 18 & 12 & 21 & 6 & 4 \end{bmatrix}$$

We have the key Matrix $A = \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix}$

We calculate the inverse of Matrix A as,

$$A^{-1} = \frac{1}{|A|} \cdot adj\, A$$

$$|A| = (1\times3) - (2\times1) = 1$$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}$$

Decoding the message, let the message be B

$$B = A^{-1} \times C = \begin{bmatrix} 3 & -2 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 9 & 5 & 25 & 0 & 16 & 22 & 18 & 20 & 14 & 18 & 4 \\ 17 & 5 & 16 & 6 & 8 & 10 & 18 & 12 & 21 & 6 & 4 \end{bmatrix}$$

$$B = \begin{bmatrix} -7 & 5 & 43 & -12 & 32 & 46 & 18 & 36 & 0 & 42 & 4 \\ 8 & 0 & -9 & 6 & -8 & -12 & 0 & -8 & 7 & -12 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 20 & 5 & 16 & 15 & 5 & 19 & 18 & 9 & 0 & 15 & 4 \\ 8 & 0 & 18 & 6 & 19 & 15 & 0 & 19 & 7 & 15 & 0 \end{bmatrix} \pmod{27}$$

The message is

B = 20, 8, 5, 0, 16, 18, 15, 6, 5, 19, 19, 15, 18, 0, 9, 19, 0, 7, 15, 15, 4, 0

= TH E_ PR OF ES SO R_ IS _G OO D_

$\therefore$ "THE PROFESSOR IS GOOD"

2] Using the key matrix $\begin{bmatrix} -3 & -3 & -4 \\ 0 & 1 & 4 \\ 4 & 3 & 4 \end{bmatrix}$ decode the following

i] DO NOT COPY ii] DO IT YOURSELF

Ans:- i] Let the key matrix is $A = \begin{bmatrix} -3 & -3 & -4 \\ 0 & 1 & 4 \\ 4 & 3 & 4 \end{bmatrix}$

Considering the message 'DO NOT COPY'

We have, DO_ NOT _CO PY_

4, 15, 0 14, 15, 20 0, 3, 15 16, 25, 0

We get message matrix B as,

$$B = \begin{bmatrix} 4 & 14 & 0 & 16 \\ 15 & 15 & 3 & 25 \\ 0 & 20 & 15 & 0 \end{bmatrix}$$

Consider the coded message C as,

$$C = AB = \begin{bmatrix} -3 & -3 & -4 \\ 0 & 1 & 4 \\ 4 & 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & 14 & 0 & 16 \\ 15 & 15 & 3 & 25 \\ 0 & 20 & 15 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -57 & -167 & -69 & -123 \\ 15 & 95 & 63 & 25 \\ 61 & 181 & 69 & 139 \end{bmatrix}$$

$$C = \begin{bmatrix} 24 & 22 & 12 & 12 \\ 15 & 14 & 9 & 25 \\ 7 & 19 & 15 & 4 \end{bmatrix} \text{(modulo 27)}$$

$C = 24, 15, 7, \ 22, 14, 19, \ 12, 9, 15, \ 12, 25, 4$

$C =$ XOGVNSLIOYD

Now we consider the decoding part.
Suppose received message is,

$C = 24, 15, 7, 22, 14, 19, 12, 9, 15, 12, 25, 4$

i.e. $C = \begin{bmatrix} 24 & 22 & 12 & 12 \\ 15 & 14 & 9 & 25 \\ 7 & 19 & 15 & 4 \end{bmatrix}$

We have the key Matrix $A = \begin{bmatrix} -3 & -3 & -4 \\ 0 & 1 & 4 \\ 4 & 3 & 4 \end{bmatrix}$

We calculate the inverse of matrix A as,

$$\therefore A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ -2 & 1/2 & -3/2 \\ 1/2 & 3/8 & 3/8 \end{bmatrix}$$

i.e $A^{-1} = \frac{1}{8} \begin{bmatrix} 8 & 0 & 8 \\ -16 & -4 & -12 \\ 4 & 3 & 3 \end{bmatrix}$

i.e $A^{-1} = 17 \begin{bmatrix} 8 & 0 & 8 \\ -16 & -4 & -12 \\ 4 & 3 & 3 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} 136 & 0 & 136 \\ -272 & -68 & -204 \\ 68 & 51 & 51 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 25 & 13 & 12 \\ 14 & 24 & 24 \end{bmatrix}$$

Decoding of message, Let the message be B

$$B = A^{-1} \times C = \begin{bmatrix} 1 & 0 & 1 \\ 25 & 13 & 12 \\ 14 & 24 & 24 \end{bmatrix} \begin{bmatrix} 24 & 22 & 12 & 12 \\ 15 & 14 & 9 & 25 \\ 7 & 19 & 15 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 31 & 41 & 27 & 16 \\ 879 & 960 & 597 & 673 \\ 864 & 1100 & 744 & 864 \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} 4 & 14 & 0 & 16 \\ 15 & 15 & 3 & 25 \\ 0 & 20 & 15 & 0 \end{bmatrix} \pmod{27}$$

The message is

B = 4, 15, 0, 14, 15, 20, 0, 3, 15, 16, 25, 0

= DO_NOT_COPY_

i.e 'DO NOT COPY'

ii) Let the key Matrix is $A = \begin{bmatrix} -3 & -3 & -4 \\ 0 & 1 & 4 \\ 4 & 3 & 4 \end{bmatrix}$

Consider message 'DO IT YOURSELF'

We have, DO_ IT_ YOU RSE LF_

4, 15, 0, 9, 20, 0, 25, 15, 21, 18, 19, 5, 12, 6, 0.

We get message matrix B as,

$$B = \begin{bmatrix} 4 & 9 & 25 & 18 & 12 \\ 15 & 20 & 15 & 19 & 6 \\ 0 & 0 & 21 & 5 & 0 \end{bmatrix}$$

Consider the coded message C as,

$$C = AB = \begin{bmatrix} -3 & -3 & -4 \\ 0 & 1 & 4 \\ 4 & 3 & 4 \end{bmatrix} \begin{bmatrix} 4 & 9 & 25 & 18 & 12 \\ 15 & 20 & 15 & 19 & 6 \\ 0 & 0 & 21 & 5 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -57 & -87 & -204 & -131 & -54 \\ 15 & 20 & 99 & 39 & 6 \\ 61 & 96 & 229 & 149 & 66 \end{bmatrix}$$

$$C = \begin{bmatrix} 24 & 21 & 12 & 4 & 0 \\ 15 & 20 & 18 & 12 & 6 \\ 7 & 15 & 13 & 14 & 12 \end{bmatrix} \pmod{27}$$

C = 24, 15, 7, 21, 20, 15, 12, 18, 13, 4, 12, 14, 0, 6, 12

C = XOGUTOLRMDLN_FL

Now we consider the decoding part
Suppose the received message is

$C = 24, 15, 7, 21, 20, 15, 12, 18, 13, 4, 12, 14, 0, 6, 12$

$$C = \begin{bmatrix} 24 & 21 & 12 & 4 & 0 \\ 15 & 20 & 18 & 12 & 6 \\ 7 & 15 & 13 & 14 & 12 \end{bmatrix}$$

We have Key matrix $A = \begin{bmatrix} -3 & -3 & -4 \\ 0 & 1 & 4 \\ 4 & 3 & 4 \end{bmatrix}$

we calculate the inverse of matrix A as,

$$\therefore A^{-1} = \begin{bmatrix} 1 & 0 & 1 \\ 25 & 13 & 12 \\ 14 & 24 & 24 \end{bmatrix}$$

Decoding of message
Let the message be B

We have, $B = A^{-1} \times C = \begin{bmatrix} 1 & 0 & 1 \\ 25 & 13 & 12 \\ 14 & 24 & 24 \end{bmatrix} \begin{bmatrix} 24 & 21 & 12 & 4 & 0 \\ 15 & 20 & 18 & 12 & 6 \\ 7 & 15 & 13 & 14 & 12 \end{bmatrix}$

$$B = \begin{bmatrix} 4 & 9 & 25 & 18 & 12 \\ 15 & 20 & 15 & 19 & 6 \\ 0 & 0 & 21 & 5 & 0 \end{bmatrix}$$

The message is

B = 4, 15, 0, 9, 20, 0, 25, 15, 21, 18, 19, 5, 12, 6, 0

= DO_ IT_ YOURSELF_

∴ DO IT YOURSELF

3] State and Prove Euler's theorem for function of 3 variables x, y, z.

Ans:- Let $u = f(x, y, z)$ is a homogeneous function of x, y, z with degree of homogeneity n then

$$f(tx, ty, tz) = t^n f(x, y, z)$$

Let $x = tx$, $y = ty$, $z = tz$

$$f(x, y, z) = t^n f(x, y, z)$$

Diff. w.r.t. t

$$\frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial t} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial t} + \frac{\partial f}{\partial z} \cdot \frac{\partial z}{\partial t} = n t^{n-1} f(x, y, z)$$

$$\frac{\partial f}{\partial x} \cdot x + \frac{\partial f}{\partial y} \cdot y + \frac{\partial f}{\partial z} \cdot z = n t^{n-1} f(x, y, z)$$

Consider $t = 1$ then $x = x$, $y = y$, $z = z$

$$x \cdot \frac{\partial f}{\partial x} + y \cdot \frac{\partial f}{\partial y} + z \cdot \frac{\partial f}{\partial z} = n f(x, y, z)$$

i.e $$x \cdot \frac{\partial u}{\partial x} + y \cdot \frac{\partial u}{\partial y} + z \cdot \frac{\partial u}{\partial z} = n u$$

Hence Proved

4] If $u = x^2 - y^2$, $v = 2xy$ where $x = r\cos\theta$
$y = r\sin\theta$

s:-

$$\frac{\partial u}{\partial r} = \frac{\partial u}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial r}$$

$$\frac{\partial u}{\partial x} = 2x, \quad \frac{\partial u}{\partial y} = -2y, \quad \frac{\partial v}{\partial x} = 2y, \quad \frac{\partial v}{\partial y} = 2x$$

$$\frac{\partial x}{\partial r} = \cos\theta, \quad \frac{\partial x}{\partial \theta} = -r\sin\theta, \quad \frac{\partial y}{\partial r} = \sin\theta, \quad \frac{\partial y}{\partial \theta} = r\cos\theta$$

from (1)

$$\frac{\partial u}{\partial r} = 2x\cos\theta + (-2y)\sin\theta = 2x\cos\theta - 2y\sin\theta$$

$$\frac{\partial u}{\partial \theta} = \frac{\partial u}{\partial x}\frac{\partial x}{\partial \theta} + \frac{\partial u}{\partial y}\cdot\frac{\partial y}{\partial \theta} = 2x(-r\sin\theta) + (-2y)(r\cos\theta)$$

$$= -2x.r\sin\theta - 2yr\cos\theta$$

$$\frac{\partial v}{\partial r} = \frac{\partial v}{\partial x}\cdot\frac{\partial x}{\partial r} + \frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial r} = 2y\cos\theta + 2x\sin\theta$$

$$\frac{\partial v}{\partial \theta} = \frac{\partial v}{\partial x}\cdot\frac{\partial x}{\partial \theta} + \frac{\partial v}{\partial y}\cdot\frac{\partial y}{\partial r} = 2y(-r\sin\theta) + 2xr\cos\theta$$
$$= 2xr\cos\theta - 2yr\sin\theta$$

By defination of Jacobian,

$$\frac{\partial(u,v)}{\partial(r,\theta)} = \begin{vmatrix} \frac{\partial u}{\partial r} & \frac{\partial u}{\partial \theta} \\ \frac{\partial v}{\partial r} & \frac{\partial v}{\partial \theta} \end{vmatrix} = \begin{vmatrix} 2x\cos\theta - 2y\sin\theta & -2xr\sin\theta - 2yr\cos\theta \\ 2y\cos\theta + 2x\sin\theta & 2xr\cos\theta - 2yr\sin\theta \end{vmatrix}$$

$$= r(2x\cos\theta - 2y\sin\theta)(2x\cos\theta - 2y\sin\theta) +$$
$$r(2x\sin\theta + 2y\cos\theta)(2y\cos\theta + 2x\sin\theta)$$

$$= r4x^2\cos^2\theta + r4x^2\sin^2\theta$$
$$+ r4y^2\sin^2\theta + r4y^2\cos^2\theta$$
$$- r8xy\sin\theta\cos\theta + r8xy\sin\theta\cos\theta$$

$$= 4x^2r + 4ry^2$$

$$= 4r(x^2+y^2)$$
$$= 4r(r^2\cos^2\theta + r^2\sin^2\theta)$$

$$= \underline{\underline{4r^3}} \quad (1)$$

$$\boxed{\therefore \frac{\partial(u,v)}{\partial(r,\theta)} = 4r^2}$$

Hence proved.

5] If $u = \frac{yz}{x}$, $v = \frac{xz}{y}$, $w = \frac{xy}{z}$ find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$

Ans:- $\frac{\partial u}{\partial x} = \frac{-yz}{x^2}$, $\frac{\partial u}{\partial y} = \frac{z}{x}$, $\frac{\partial u}{\partial z} = \frac{y}{x}$

$\frac{\partial v}{\partial x} = \frac{z}{y}$, $\frac{\partial v}{\partial y} = \frac{-xz}{y^2}$, $\frac{\partial v}{\partial z} = \frac{x}{y}$

$\frac{\partial w}{\partial x} = \frac{y}{z}$, $\frac{\partial w}{\partial y} = \frac{x}{z}$, $\frac{\partial w}{\partial z} = \frac{-xy}{z^2}$

By the defination of Jacobian,

$$\frac{\partial(u,v,w)}{\partial(x,y,z)} = \begin{vmatrix} \partial u/\partial x & \partial u/\partial y & \partial u/\partial z \\ \partial v/\partial x & \partial v/\partial y & \partial v/\partial z \\ \partial w/\partial x & \partial w/\partial y & \partial w/\partial z \end{vmatrix}$$

$$= \begin{vmatrix} -yz/x^2 & z/x & y/x \\ z/y & -xz/y^2 & x/y \\ y/z & x/z & -xy/z^2 \end{vmatrix}$$

$$= \frac{-yz}{x^2}\left[\left(\frac{-xz}{y^2}\right)\cdot\left(\frac{-xy}{z^2}\right) - \frac{x^2}{zy}\right] - \frac{z}{x}\left[\left(\frac{-xy}{z^2}\right)\left(\frac{z}{y}\right) - \left(\frac{x}{y}\right)\left(\frac{y}{z}\right)\right]$$

$$+ \frac{y}{x}\left[\left(\frac{z}{y}\right)\left(\frac{x}{z}\right) - \left(\frac{-xz}{y^2}\right)\left(\frac{y}{z}\right)\right]$$

$$= \frac{-yz}{x^2}\left[\frac{x^2}{zy} - \frac{x^2}{zy}\right] - \frac{z}{x}\left[\frac{-x}{z} - \frac{x}{z}\right] + \frac{y}{x}\left[\frac{x}{y} + \frac{x}{y}\right]$$

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$$= 0 + \frac{z}{x} \times \frac{2x}{z} + \frac{y}{x} \times \frac{2x}{y}$$

$$= \underline{\underline{4}}$$

$$\boxed{\therefore \frac{\partial(u, v, w)}{\partial(x, y, z)} = 4}$$

Sundaram

Teacher's Sign. :