

## De Broglie hypothesis of matter waves

In 1924, Louis de Broglie put forward the suggestion that matter has dual nature like radiation due to the reason that nature loves symmetry. Nature has two entities - matter and radiation. One of the entities - radiation shows a dual aspect, then the other entity, matter also should exhibit a dual nature (particle and wave aspect).

According to de Broglie hypothesis any moving particle is associated with a wave. The waves associated with particles are known as de Broglie waves or matter waves. The wavelength of the matterwaves associated with a particle is inversely proportional to magnitude of momentum of particle.

$$\boxed{\lambda = \frac{h}{mv}}$$

where  
 $m \rightarrow$  mass of particle  
 $v \rightarrow$  velocity of particle.  
 $h \rightarrow$  Planck's constant.

## De Broglie wavelength of matter waves

for a radiation of frequency  $\nu$ , energy related with photon is

$$E = h\nu \quad (\text{Planck's law})$$

$$E = mc^2 \quad (\text{Einstein's law})$$

$$h\nu = mc^2$$

$$h\nu = m(\underbrace{v}_{c})^2$$

$\therefore \lambda = \frac{h}{mc}$  is the wavelength of radiation (photon)

By analogy, the wavelength of matter wave is ( $c=v$ )

$$\boxed{\lambda = \frac{h}{mv} = \frac{h}{P}}$$

Other forms:

$$\lambda = \frac{h}{P} = \frac{h}{mv}$$

$$\lambda = \frac{h}{\sqrt{2mE}}$$

$$\lambda = \frac{h}{\sqrt{2mgV}}$$

$$\lambda = \frac{h}{\sqrt{3mkT}}$$

$$P^2 = 2mE$$

for electron

$$\lambda = \frac{\sqrt{150}}{\sqrt{V}} \text{ Å}^{\circ}$$
$$= \frac{12.27}{\sqrt{V}} \text{ Å}^{\circ}$$

when charge  $q$  is accelerated by a potential  $V$ ; ( $E = qV$ )

At thermal equm  $E = \frac{3}{2} kT$

where  $T \rightarrow$  Temperature

### Properties of matter waves.

1. Matter waves are produced by the motion of particles and are independent of charge. These waves are neither electromagnetic nor mechanical waves.
2. They can travel through vacuum and do not require any material medium for their propagation.
3.  $\lambda \propto \frac{1}{v}$ . The smaller the velocity of particle, the longer is the wavelength of matter waves associated with it.
4.  $\lambda \propto \frac{1}{m}$ . The lighter the particle, the longer is the wavelength of matter waves associated with it.
5. The velocity of matter waves depends on the velocity of material particle and is NOT a constant quantity.
6. The velocity of matter wave is greater than the velocity of light.
7. They exhibit diffraction phenomena as any other waves.

## Velocity of de Broglie wave

As per De Broglie concept, a material particle of mass 'm' moving with velocity 'v' is represented by a wave of wavelength  $\lambda = \frac{h}{mv}$ . Then velocity of propagation of wave is ( $v_p$ )

$$v_p = \lambda v$$

$$= \frac{E}{h} \frac{h}{P} = \frac{E}{P}$$

$$E = h\lambda \Rightarrow \lambda = \frac{E}{h}$$

$$\lambda = \frac{h}{P}$$

$$v_p = \frac{mc^2}{mv}$$

$$E = mc^2 \text{ & } P = mv$$

$$v_p = \frac{c^2}{v}$$

where  $\lambda$   $\rightarrow$  Wave velocity or phase velocity  
 $v \Rightarrow$  particle velocity

But speed of particle ( $v$ ) is always less than  $c$  (velocity of light)

$$\therefore v_p > v$$

$\therefore$  The phase velocity (wave velocity) of de Broglie wave associated with particle is always greater than 'c'.

### Phase Velocity,

The phase velocity also known as wave velocity is the velocity with which a definite phase of the wave propagates through a medium.

$$v_p = \lambda v$$

$$= \frac{\omega}{2\pi} \frac{2\pi}{K}$$

$$y = A \sin(\omega t - Kx)$$

$$\omega = 2\pi\nu \Rightarrow \nu = \frac{\omega}{2\pi}$$

$$K = \frac{2\pi}{\lambda} \Rightarrow \lambda = \frac{2\pi}{K}$$

$$v_p = \frac{\omega}{K}$$

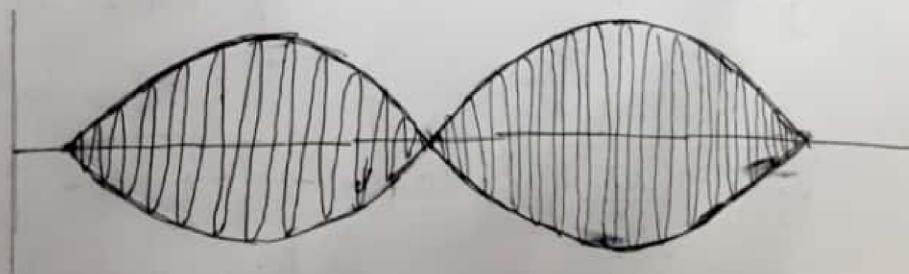
where  $\omega \rightarrow$  Angular frequency of wave  
 $K \rightarrow$  Propagation constant

It was unexpected result that the de Broglie wave associated with a particle travel faster than particle itself ( $v_p = \frac{c^2}{v}$ ), thus leaving the particle

far behind. The difficulty was resolved by Schrodinger by postulating that a material particle in motion is equivalent to a wavepacket rather than a single wave.

## • Wavepacket

Schrodinger postulated that a wavepacket represents a particle. A wavepacket consists of a group of harmonic waves, each having slightly different wavelength. The superposition of these harmonic waves differing slightly in frequency will produce a single wavepacket. The phases and amplitudes of waves in a packet are such that they undergo constructive interference only over a small region of space, where the particle can be located. Outside the region, they undergo destructive interference. The wavepacket propagates with a velocity  $v_g$  called group velocity. Each component wave forming the wavepacket propagates with a velocity called phase velocity  $v_p$ .



## Group Velocity

When a number of waves of slightly different wavelength travels in same direction, they form wave group or wavepackets. The velocity with which the wavegroup advances in the medium is known as group velocity  $v_g$

### Expression for Group velocity

Consider a wavegroup consists of two components of equal amplitude and slightly differing frequencies

Then the wave eqns are

$$y_1 = A \sin(\omega_1 t - k_1 x)$$

where  $k_1, k_2 \Rightarrow$  propagation constants

$$y_2 = A \sin(\omega_2 t - k_2 x)$$

Superposition of these two waves gives

$$y_1 + y_2 = A \sin(\omega_1 t - k_1 x) + A \sin(\omega_2 t - k_2 x)$$

[using relation  $\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$  where  $A = \omega_1 t - k_1 x$ ,  $B = \omega_2 t - k_2 x$ ]

$$y_1 + y_2 = 2A \sin\left[\frac{(\omega_1 + \omega_2)}{2}t - \frac{(k_1 + k_2)}{2}x\right] \cos\left[\frac{(\omega_1 - \omega_2)}{2}t - \frac{(k_1 - k_2)}{2}x\right]$$

$$y_1 + y_2 = 2A \underbrace{\sin\left(\omega t - kx\right)}_{(i)} \underbrace{\cos\left(\frac{\Delta\omega t}{2} - \frac{\Delta k x}{2}\right)}_{(ii)}$$

where  $\omega = \frac{\omega_1 + \omega_2}{2}$ ,  $K = \frac{k_1 + k_2}{2}$ ,  $\Delta\omega = \omega_1 - \omega_2$ ,  $\Delta k = k_1 - k_2$

∴ The resultant wave has two parts

(i) A wave of angular velocity  $\omega$  frequency  $\omega$  and propagation constant  $K$ , moving with a velocity

$$V_p = \frac{\omega}{K} = \lambda \quad \Rightarrow \text{Phase velocity}$$

(ii) A second wave of angular frequency  $\frac{\Delta\omega}{2}$  and propagation constant  $\frac{\Delta k}{2}$ , moving with velocity

$$v_g = \frac{\Delta\omega}{\Delta k}, \text{ when } \Delta\omega \text{ & } \Delta k \text{ are very small, we can write}$$

$$v_g = \frac{d\omega}{dk} \quad \Rightarrow \text{Group velocity}$$

This velocity is the group velocity of envelope of group of waves. Hence it is called group velocity.

The velocity of particle is equal to the group velocity of associated matter waves.

### Relation between phase velocity and Group velocity

$$\text{Group velocity } V_g = \frac{d\omega}{dk} \quad \text{if phase velocity } V_p = \frac{\omega}{k}$$

$$V_g = \frac{d}{dk} (k V_p) \quad \omega = k V_p$$

$$= k \frac{d V_p}{d k} + V_p = k \frac{d V_p}{d \lambda} \frac{d \lambda}{d k} + V_p \quad K = \frac{2\pi}{\lambda}$$

$$V_g = -\lambda \frac{d V_p}{d \lambda} + V_p \quad \frac{d k}{d \lambda} = -\frac{2\pi}{\lambda^2}$$

$$\frac{d k}{d \lambda} = -\frac{k}{\lambda}$$

$$V_g = V_p - \lambda \frac{d V_p}{d \lambda}$$

∴ Group velocity is generally less than phase velocity  
[In nondispersive medium  $V_g = V_p \quad \because V_p = \text{const} \text{ & } \frac{d V_p}{d \lambda} = 0$ ]

### Wave Function

Since microparticles exhibit wave properties, it is assumed that a quantity  $\psi$  represents a de Broglie wave. This quantity  $\psi$  describes matter waves as a function of position and time, is called a wave function. In general,  $\psi$  is a complex valued function. It has no direct physical significance.  $\psi$  gives the probability amplitude.

Group Velocity equals the velocity of particle.

A particle moving with a velocity  $v$  is associated with a wavepacket. The group velocity of wave packet is

$$V_g = \frac{d\omega}{dk}$$

$$\left| \begin{array}{l} \omega = 2\pi f \\ k = \frac{2\pi}{\lambda} \end{array} \right.$$

$$\text{The energy } E = h\nu = \hbar\omega \quad \Rightarrow dE = \hbar d\omega$$

$$\text{Momentum } p = \frac{h}{\lambda} = \hbar k \quad \Rightarrow dp = \hbar dk$$

$$\frac{dE}{dp} = \frac{d\omega}{dk}$$

Then group velocity of wave packet becomes

$$V_g = \frac{d\omega}{dk} = \frac{dE}{dp} \quad \text{But } E = \frac{p^2}{2m}$$

$$V_g = \frac{d}{dp} \left( \frac{p^2}{2m} \right) = \frac{2p}{2m} = \frac{p}{m} \quad (\because p = mv)$$

$$\therefore V_g = v$$

Group velocity = Particle velocity

Relation between Group velocity, Phase velocity & velocity of light

Consider a particle of mass 'm' moving with velocity  $v$  is associated with wavegroup having group velocity  $V_g$  and phase velocity  $V_p$

$$\text{Phase velocity } V_p = \frac{\omega}{k}$$

$$V_p = \frac{E/\hbar}{P/\hbar} = \frac{E}{P}$$
$$= \frac{mc^2}{mv}$$

$$\begin{aligned} E &= \hbar\omega \\ P &= \hbar k \end{aligned}$$

$$\begin{aligned} E &= mc^2 \\ P &= mv \end{aligned}$$

$$V_p = \frac{c^2}{v} = \frac{c^2}{V_g} \quad (\text{because groupvelocity } V_g = \text{particle velocity } v)$$

$$V_g V_p = c^2$$

## Physical Interpretation of wavefunction.

given by Max Born in 1926.

The square of the magnitude of wavefunction  $|\psi|^2$  evaluated in a particular region represents the probability of finding the particle in that region.

$|\psi|^2$  is called the probability density

$\psi$  is probability amplitude

The probability of finding the particle in a small volume  $dv$  is

$$P \propto |\psi|^2 dv$$

Since the particle is certainly somewhere in space, the probability  $P=1$  and the integral of  $|\psi|^2 dv$  over the entire space must be equal to unity.

That is  $\int_{-\infty}^{\infty} \{|\psi|^2 dv\} = 1$

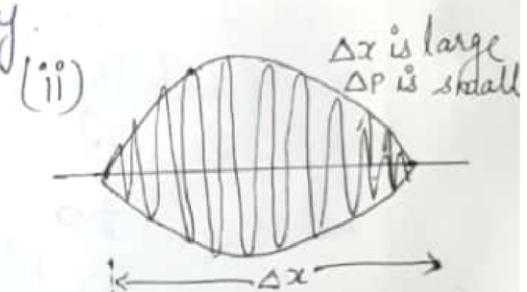
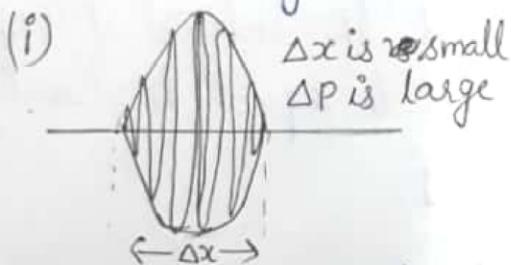
or  $\int_{-\infty}^{\infty} \psi \psi^* dv = 1$  where  $\psi^*$   $\rightarrow$  complex conjugate of  $\psi$

The condition given by above equation is called normalized condition and then the wavefunction is called normalized wave function.

Thus  $\psi$  has no physical significance, but  $|\psi|^2$  gives the probability of finding the particle in a particular region.

## Heisenberg's Uncertainty Principle

A moving particle is equivalent to a wavepacket. Although the particle is somewhere within the wavepacket, it is difficult to locate the exact position of microparticle. There is an uncertainty  $\Delta x$  (linear spread of wavepacket) in the position of particle. As a result, the momentum of the particle at that instant cannot be determined precisely. There is an uncertainty in the determination of momentum ( $\Delta p$ ) of particle.  $\therefore$  The position and momentum of a microparticle cannot be determined simultaneously with accuracy.



Heisenberg's uncertainty principle states that it is not possible to determine the position and momentum of the particle simultaneously and with exactness. The uncertainty  $\Delta x$  in position and uncertainty  $\Delta p$  in momentum are related as

$$\Delta x \Delta p \geq \frac{\hbar}{2}, \text{ where } \hbar = \frac{h}{2\pi}, h \rightarrow \text{Planck's const.}$$

In Quantum mechanics, there are pairs of measurable quantities called conjugate quantities which follows uncertainty principle. They are

(i) position ( $x$ ) — momentum ( $p$ )

(ii) Energy ( $E$ ) — Time ( $t$ )

(iii) Angular momentum — Angular displacement ( $\theta$ )

$$\text{That is } \Delta E \Delta t \geq \frac{\hbar}{2}, \quad \Delta J \Delta \theta \geq \frac{\hbar}{2}$$

In general if  $q$  &  $p$  are two canonically conjugate variables, then  $\Delta q \Delta p \geq \frac{\hbar}{2}$

(9)

## Non-existence of electron in nucleus

We apply uncertainty principle to find electrons are present in the nucleus or not.

If electrons were to be inside the nucleus, the maximum uncertainty  $\Delta x$  in its position is equal to the diameter of the nucleus.

The radius of the nucleus is of the order of  $5 \times 10^{-15} \text{ m}$

$$\therefore \Delta x = 2 \times 5 \times 10^{-15} \text{ m} = 10^{-14} \text{ m}$$

Then minimum uncertainty in its momentum is  $(\Delta p = \frac{\hbar}{2 \Delta x})$

$$\begin{aligned}\Delta p &= \frac{\hbar}{2 \Delta x} = \frac{\hbar}{4\pi \Delta x} \\ &= \frac{6.626 \times 10^{-34}}{4 \times 3.14 \times 2 \times 10^{-14}} \\ &= 5.275 \times 10^{-21} \text{ kg m/sec}\end{aligned}$$

$$\begin{aligned}\text{Also } \Delta p &= m \Delta v \\ \Delta v &= \frac{\Delta p}{m} \\ &= \frac{5.275 \times 10^{-21}}{9.1 \times 10^{-31}} \\ &= 5.8 \times 10^9 \text{ m/s} \\ &>c, \text{ not possible}\end{aligned}$$

If this is the uncertainty in momentum of the electron, the momentum of the electron must be at least comparable with its magnitude, i.e,

$$P \approx 5.275 \times 10^{-21} \text{ Kg. m/sec}$$

Then the minimum energy of the electron in the nucleus is

$$\begin{aligned}E_{\min} &= \frac{P^2}{2m} \\ &= \frac{(5.275 \times 10^{-21})^2}{2 \times 9 \times 10^{-31}} \text{ J} = \frac{\text{eV}}{1.6 \times 10^{-19}} \\ &= 9.55 \times 10^7 \text{ eV} \\ &\approx 96 \text{ MeV}\end{aligned}$$

This means that if the electrons exist inside the nucleus, their kinetic energy is of the order of 96 MeV. But experimental observations show that no electron in an atom possess energy greater than 4 MeV. Clearly the inference is that the electrons do not exist in the nucleus. (10)

## Schrodinger Time Dependent Wave Equation

Consider a particle of mass 'm' is moving along X direction. Let ' $\psi$ ' be the wavefunction of associated de Broglie wave, which is a function of co-ordinates x and t. The wavefunction can be written as

$$\begin{aligned}\psi &= A e^{i(kx - \omega t)} \\ &= A e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)}\end{aligned}\quad \left| \begin{array}{l} \text{but momentum } p = \hbar k \Rightarrow k = \frac{p}{\hbar} \\ \text{Energy } E = \hbar \omega \Rightarrow \omega = \frac{E}{\hbar} \end{array} \right.$$

$$\psi = A e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)} \quad \text{O}$$

Differentiating  $\psi$  w.r.t x

$$\frac{\partial \psi}{\partial x} = A e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)} \cdot \frac{i}{\hbar} p$$

$$\frac{\partial^2 \psi}{\partial x^2} = A e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)} \cdot \frac{i^2}{\hbar^2} (4 - \frac{p^2}{\hbar^2})$$

$$\frac{\partial^2 \psi}{\partial x^2} = -\frac{p^2}{\hbar^2} \psi$$

$$p^2 \psi = -\hbar^2 \frac{\partial^2 \psi}{\partial x^2} \quad \text{--- (2)}$$

Differentiating  $\psi$  w.r.t t

$$\frac{\partial \psi}{\partial t} = A e^{i\left(\frac{p}{\hbar}x - \frac{E}{\hbar}t\right)} \cdot -\frac{i}{\hbar} E = -\frac{i}{\hbar} E \psi$$

$$\frac{\partial \psi}{\partial t} = -\frac{i}{\hbar} E \psi$$

$$E \psi = -\frac{\hbar}{i} \frac{\partial \psi}{\partial t}$$

$$E \psi = i \hbar \frac{\partial \psi}{\partial t} \quad \text{--- (3)}$$

Total energy of particle  $E = \text{kinetic energy} + \text{potential energy}$

$$E = KE + V \quad \text{where } V \rightarrow \text{Potential energy}$$

$$KE = \frac{p^2}{2m}$$

$$E = \frac{p^2}{2m} + V$$

Multiplying above eqn with  $\psi$

(11)

$$E\psi = \frac{p^2}{2m}\psi - iV\psi$$

using relations (2) and (3) in above equation, we get

$$i\hbar \frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V\psi$$

$$\left[ -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V \right] \psi = i\hbar \frac{\partial \psi}{\partial t} \quad (4)$$

The above equation is known as time dependent Schrödinger eqn. (One dimensional)

Three dimensional case

$$\left[ -\frac{\hbar^2}{2m} \nabla^2 \psi + V \right] \psi = i\hbar \frac{\partial \psi}{\partial t}$$

$$\text{where } \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

$$H\psi = E\psi$$

where  $H$  is called Hamiltonian.

$$H = -\frac{\hbar^2}{2m} \nabla^2 + V \text{ and}$$

$$E\psi = i\hbar \frac{\partial \psi}{\partial t}$$

## TIME INDEPENDENT SCHRODINGER EQUATION

If the potential energy  $V$  of a particle does not depend on time, and varies only with position of the particle only, then the field is said to be stationary. In stationary problems Schrödinger equation can be solved by separating out position and time dependent parts, i.e

$$\Psi(x, t) = \Psi(x) \phi(t)$$

Using this the time dependent Schrödinger eqn (4) becomes

$$-\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} (\Psi \phi) + V(\Psi \phi) = i\hbar \frac{\partial}{\partial t} (\Psi \phi)$$

$$\begin{aligned} \Psi &= A e^{\frac{i}{\hbar} (Px - Et)} \\ \Psi_{(x,t)} &= \underbrace{A e^{\frac{i}{\hbar} Px}}_{= \Psi(x)} \cdot \underbrace{e^{\frac{i}{\hbar} Et}}_{= \phi(t)} \end{aligned}$$

(12)

$$-\frac{\hbar^2}{2m} \phi \frac{\partial^2 \psi}{\partial x^2} + V \psi t = i\hbar \phi \frac{\partial \phi}{\partial t}$$

Dividing above eqn with  $\psi \phi$

$$-\frac{\hbar^2}{2m} \frac{1}{\psi} \frac{d^2 \psi}{dx^2} + V = i\hbar \frac{\phi}{\psi} \frac{d\phi}{dt}$$

LHS of above equation is a function of  $x$  only and RHS is a function of  $t$  only. Then each side of above equation is equal to a constant called separation constant. Here the separation constant should be the total energy, thus LHS is

$$-\frac{\hbar^2}{2m} \frac{1}{\psi} \frac{d^2 \psi}{dx^2} + V = E$$

$$\boxed{-\frac{\hbar^2}{2m} \frac{d^2 \psi}{dx^2} + V \psi = E \psi}$$

The above equation is called time independent Schrödinger equation. Also it can be written as.

$$\frac{d^2 \psi}{dx^2} + \frac{2m}{\hbar^2} (E - V) \psi = 0$$

$$\frac{d^2 \psi}{dx^2} + \frac{8\pi^2 m}{\hbar^2} (E - V) \psi = 0$$

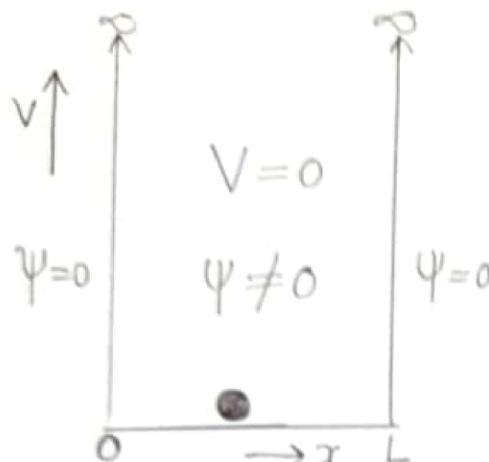
### Application Schrödinger eqn.

In Quantum mechanics, the Schrödinger equation is the basic equation which is helpful for solving the Energy values, Wavefunction and probability density of various quantum mechanical problems.

We now apply the Schrödinger wave equation to a particle confined to move in a box.

# Particle Trapped in One Dimensional Infinite Potential Well

(note: our goal find Energy, wavefunction  $\psi$ )  
of Probability density



A potential well is a potential energy function  $V(x)$  given by  
 $V(x) = 0$  at  $0 < x < L$   
 $= \infty$  at  $x \leq 0$  &  $x \geq L$

When a particle is left in the well, we say that the particle is trapped inside the infinite potential well and such a trapped state is called bound state. Inside the well,  $V=0$  and the particle can propagate along  $x$  axis and is described a wavefunction  $\psi$ .

The Schrodinger eqn inside the well is ( $V=0$ )

$$\frac{d^2\psi}{dx^2} + \frac{8\pi^2 m E}{h^2} \psi = 0$$

$$\text{Put } K^2 = \frac{8\pi^2 m E}{h^2} \quad \text{---} \quad ①$$

$$\frac{d^2\psi}{dx^2} + K^2 \psi = 0$$

$$\left| \begin{array}{l} p^2 = h^2 k^2 \\ k^2 = \frac{p^2}{h^2} \\ K^2 = \frac{2mE}{h^2} \\ K = \frac{\sqrt{2mE}}{h} \end{array} \right.$$

The particle can move back and forth freely between  $x=0$  and  $x=L$ , the general soln of above eqn becomes

$$\psi_{(x)} = A \sin Kx + B \cos Kx$$

Where the constants A and B can be find by applying boundary conditions  $\rightarrow$   $\psi = 0$  at  $x = 0$  --- (i)  
 $\psi = 0$  at  $x = L$  --- (ii)

$$0 = B \quad (\text{by applying (i)})$$

$$\psi = A \sin Kx \quad \text{---} \quad ②$$

$$0 = A \sin KL \quad (\text{by applying (ii)})$$

(14)

$$\therefore \sin kL = 0$$

$$kL = n\pi \Rightarrow k = \frac{n\pi}{L} \quad \text{--- (3)}$$

Sub in eqn

$$k^2 = \frac{8\pi^2 m E}{h^2} \Rightarrow E = \frac{n^2 \pi^2}{L^2} \frac{h^2}{8\pi^2 m}$$

$$\boxed{\therefore E = \frac{n^2 h^2}{8m L^2}} \quad \text{--- (4)}$$

Now the wavefunction  $\psi$  is (using eqn 2 & 3)

$$\psi = A \sin \frac{n\pi}{L} x \quad \text{--- (5)}$$

Applying normalisation condition, we can find  $A$

$$\int_0^L \psi \psi^* dx = 1$$

$$A^2 \int_0^L \sin^2 \frac{n\pi x}{L} dx = 1$$

$$\frac{A^2}{2} \int_0^L \left[ 1 - \cos \frac{2n\pi x}{L} \right] dx = 1$$

$$\frac{A^2}{2} \left[ x - \frac{\sin \frac{2n\pi x}{L}}{\frac{2n\pi}{L}} \right]_0^L = 1$$

$$\frac{A^2}{2} [L] = 1 \Rightarrow A^2 = \frac{2}{L} \Rightarrow A = \sqrt{\frac{2}{L}}$$

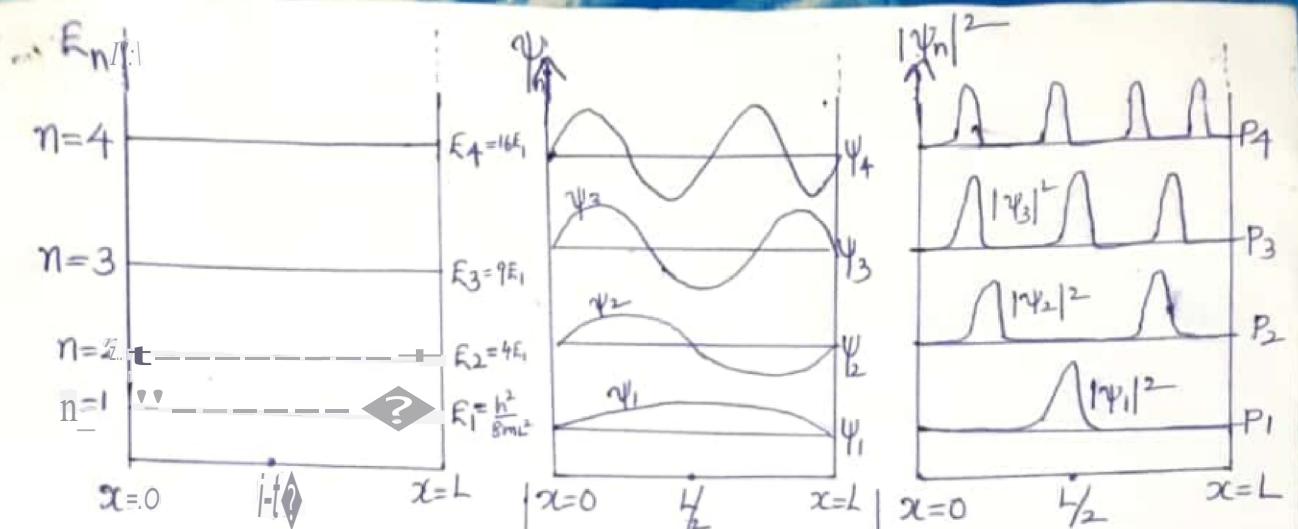
$\therefore$  Eqn (5) becomes

$$\boxed{\psi = \sqrt{\frac{2}{L}} \sin \frac{n\pi}{L} x} \quad \text{--- (6)}$$

Now Probability density is

$$\boxed{P = |\psi|^2 = \frac{2}{L} \sin^2 \left( \frac{n\pi x}{L} \right)} \quad \text{--- (7)}$$

$\therefore$  Eqn 4, 6 and 7 gives the energy, wavefunction and probability density of a particle trapped in an infinite potential well.



(i) Energy levels ( $E_n$ )

$$E_n = \frac{n^2 h^2}{8mL^2}$$

where  $n = \text{quantum no.}$   
= 1, 2, 3, ...

$$E_1 = \frac{h^2}{8mL^2}$$

$$E_2 = 4 \frac{h^2}{8mL^2}$$

$$E_3 = 9 \frac{h^2}{8mL^2} \text{ and so on.}$$

i.e Particle can possess discrete energy values. The particle cannot have zero energy.

(ii) Wavefunction ( $\Psi_n$ )

$$\Psi_n = \sqrt{\frac{2}{L}} \sin \frac{n\pi x}{L}$$

Probability density is maximum

$$\text{when } \frac{n\pi x}{L} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$$

$$\text{or } x = \frac{L}{2n}, \frac{3L}{2n}, \frac{5L}{2n}, \dots$$

For the state  $\Psi_1$ , probability is largest at  $x = \frac{L}{2}$

For the state  $\Psi_2$ , probability is largest at  $x = \frac{L}{4}$  and  $\frac{3L}{4}$

The number of peaks in the wavefunction  $\Psi$  at any state is equal to the order<sup>(n)</sup> of that state.

At boundary walls, wavefunction have nodes.

At the nodes of wavefunction, the probability of finding the particle is zero. The ~~number~~ of peaks in probability density is equal to quantum number 'n'