Problem 1

A passenger in a train moving at 30 mps passes a man standing on a station platform at $t = t' = 0$. Twenty second after the train passes the station, the man on the platform determines that a bird flying along the tracks in the same directions as the train is 800 m away. Using Galilean transformation, find the coordinates of the bird as determined by the passenger.

Solution :

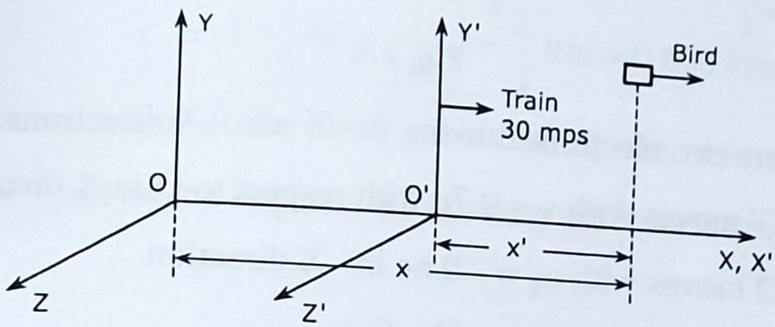

Fig. 4.5

By Galilean transformation $x' = x - vt$.

where, x = position of the bird on X-axis as seen by the stationary observer O on the station.

$= 800$ m

x' = position of the bird on X-axis as seen by the passenger O' in the train

v = 30 mps = velocity of the train

t = 20 sec.

Hence, $x' = 800 - (30 \times 20) = 200$ m

For the stationary observer the coordinates of the bird is

$$(x, y, z, t) = (800 \text{ m}, 0, 0, 20 \text{ s})$$

For the passenger the coordinates of the bird is

$$(x', y', z', t) = (200 \text{ m}, 0, 0, 20 \text{ s})$$

Problem 2

A sample of radioactive material, at rest in the laboratory, ejects two electrons in opposite directions. One of the electrons has a speed of 0.6 c and the other has a speed of

0.7c as measured by a laboratory observer. According to Galilean transformation, what will be the speed of one electron as measured from the other? Comment on your result.

Solution :

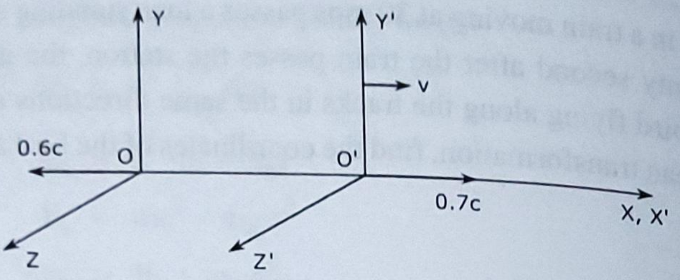

Fig. 4.6

Let O and O' are two electrons moving in –X and +X directions.

The electron O' moves with v = 0.7c with respect to O in X direction.

The electron O moves with $u_x = -0.6c$ in –X direction.

Hence, the velocity of O' measured by O is

$$u_x' = u_x - v = -0.6c - 0.7c = -1.3c$$

The result shows a velocity greater than c by Galilean transformation. This is inconsistent with the special theory of relativity.

Problem 3

A train moving with a velocity of 60 kmph passes through a rail station at 12.00 clock. Twenty seconds later a bolt of lightening strikes the rail track one km away from the station in the direction of the train. Using Galilean transformation, find the coordinates of the lightening flash as measured by an observer at the station and by the engineer of the train.

Solution :

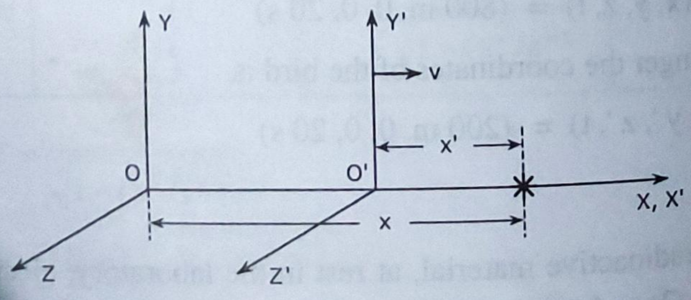

Fig. 4.7

The flash strikes the rail tract at

$$t = t' = \frac{20}{3600} = \frac{1}{180} \text{ hr}$$

Here O is the observer at the station and O' is the engineer in the train moving at $v = 60$ kmph with respect to O.

The observer at the station measures the x-coordinate of the flash as $x = 1$ km.

The engineer in the train measures the x-coordinates of the flash as

$$x' = x - vt$$

$$x' = 1 - \left(60 \times \frac{1}{180}\right) = \frac{2}{3} \text{ km} = 0.666 \text{ km}$$

Hence, the coordinates of the flash measured by the observer at the station is $\left(1 \text{ km}, 0, 0, \frac{1}{180} \text{ hr}\right)$ and by the engineering the train are $\left(0.666 \text{ km}, 0, 0, \frac{1}{180} \text{ hr}\right)$.

Problem 4

An event occurs at $x = 100$ m, $y = 10$ m, $z = 5$ m and $t = 1 \times 10^{-4}$ sec in a frame S. Find the coordinates of this event in a frame S' which is moving with a velocity 2.7×10^8 m/sec with respect to the frame S along the common XX' axes using (i) Galilean transformation and (ii) Lorentz transformation.

Solution :

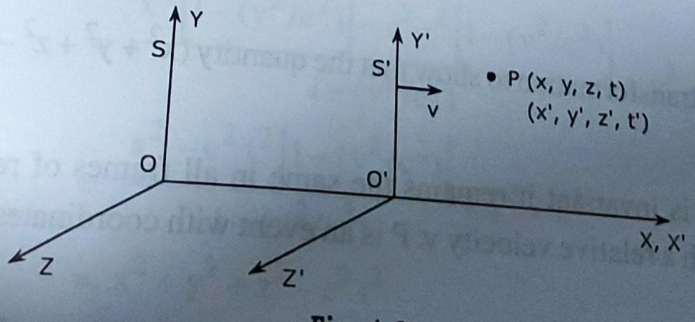

Fig. 4.8

P is the event with coordinates (x, y, z, t) in the stationary frame S and (x', y', z', t) in frame S' moving with $v = 2.7 \times 10^8$ m/sec. with respect to S.

$$x = 100 \text{ m}, \; y = 10 \text{ m}, \; z = 5 \text{ m}, \; t = 10^{-4} \text{ sec.}$$

According to Galilean transformation

$$x' = x - vt, \quad y' = y, \quad z' = z, \quad t' = t$$

$$x' = 100 - (2.7 \times 10^8)\, 10^{-4} = -26900 \text{ m}$$

$$y' = 10 \text{ m}, \; z' = 5 \text{ m}, \; t = 10^{-4} \text{ sec.}$$

So the coordinates of the event are (– 26900 m, 10 m, 5 m, 10^{-4} sec.)

(ii) According to Lorentz transformation

$$x' = \frac{x - vt}{\sqrt{1-(v^2/c^2)}}, \quad y' = y, \quad z' = z, \quad t' = \frac{t-(vx/c^2)}{\sqrt{1-(v^2/c^2)}}$$

Here, $$\sqrt{1-(v^2/c^2)} = \sqrt{1-\left(\frac{2.7\times10^8}{3\times10^8}\right)^2} = \sqrt{1-(0.9)^2} = 0.43588$$

$$\therefore \quad x' = \frac{100-(2.7\times10^8)(10^{-4})}{0.43588} = 6.712 \text{ m}$$

$$y' = 10 \text{ m}$$

$$z' = 5 \text{ m}$$

$$t' = \frac{t-(vx/c^2)}{0.43588} = \frac{10^{-4}-(2.7\times10^8)(100)/(3\times10^8)^2}{0.43588}$$

$$\therefore \quad t' = 2.2735\times10^{-4} \text{ sec}$$

Hence, the coordinates of the event in frame S' are

(61712 m, 10 m, 5 m, 2.2735×10^{-4} S).

Problem 5

Use Lorentz transformation to show that the quantity $(x^2+y^2+z^2-c^2t^2)$ is invariant.

Solution :

If a quantity is invariant it remains the same in all frames of reference. Consider frames S and S' with a relative velocity v. P is an event with coordinates (x, y, z, t) in S and (x', y', z', t') in S'.

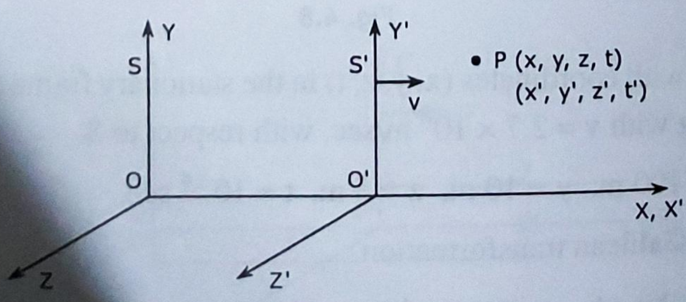

Fig. 4.9

According to Lorentz transformation, we have

$$x' = \frac{x - vt}{\sqrt{1-(v^2/c^2)}}, \quad y' = y, \quad z' = z, \quad t' = \frac{t-(vx/c^2)}{\sqrt{1-(v^2/c^2)}}$$

If $(x^2 + y^2 + z^2 - c^2 t^2)$ is invariant, we should have

$$(x^2 + y^2 + z^2 - c^2 t^2) = x'^2 + y'^2 + z'^2 - c^2 t'^2$$

$$\text{R.H.S.} = x'^2 + y'^2 + z'^2 - c^2 t'^2$$

$$= \left[\frac{x - vt}{\sqrt{1-(v^2/c^2)}}\right]^2 + y^2 + z^2 - c^2\left[\frac{t-(vx/c^2)}{\sqrt{1-(v^2/c^2)}}\right]^2$$

$$= \frac{(x - vt)^2 - c^2[t-(vx/c^2)]^2}{1-(v^2/c^2)} + y^2 + z^2$$

$$= \frac{x^2 - 2x\,vt + v^2t^2 - c^2\left(t^2 - 2\frac{vxt}{c^2} + \frac{v^2x^2}{c^4}\right)}{1-(v^2/c^2)} + y^2 + z^2$$

$$= \frac{x^2 - 2x\,vt + v^2t^2 - c^2t^2 + 2x\,vt - \frac{v^2x^2}{c^2}}{1-(v^2/c^2)} + y^2 + z^2$$

$$= \frac{x^2\left[1-(v^2/c^2)\right] - c^2t^2\left[1-(v^2/c^2)\right]}{1-(v^2/c^2)} + y^2 + z^2$$

$$= \frac{x^2 - c^2t^2\left[1-(v^2/c^2)\right]}{1-(v^2/c^2)} + y^2 + z^2$$

$$= x^2 + y^2 + z^2 - c^2t^2$$

$$\text{R.H.S.} = \text{L.H.S.}$$

Hence, proved.

Problem 6

Using Lorentz transformation, show that the circle $x^2 + y^2 = a^2$ in frame S appears to be an ellipse in frame S ' moving with a velocity v with respect to S.

Solution :

Data : Equation of a circle $x^2 + y^2 = a^2$ in the stationary frame S. 'a' is the radius of the circle, a constant.

Formula :
$$x' = \frac{x - vt}{\sqrt{1-(v^2/c^2)}}, \quad y' = y$$

Calculations : Assuming $t = 0$

$$x' = \frac{x}{\sqrt{1-(v^2/c^2)}}$$

$$x^2 + y^2 = a^2$$

$$\left(x'\sqrt{1-(v^2/c^2)}\right)^2 + y'^2 = a^2$$

$$\frac{x'^2}{a^2}\left(1-\frac{v^2}{c^2}\right) + \frac{y'^2}{a^2} = 1$$

Let $b = \dfrac{a}{\sqrt{1-(v^2/c^2)}}$, another constant.

$$\therefore \quad \frac{x'^2}{b^2} + \frac{y'^2}{a^2} = 1, \text{ the equation of an ellipse.}$$

Problem 7

What is the length of a meter stick moving parallel to its length when its mass is 3/2 times of its rest mass?

Solution :

Data : $L_o = 1$ m, $m = \frac{3}{2}\, m_o$.

Formula : $L = L_o\sqrt{1-(v^2/c^2)}$, $m = \dfrac{m_o}{\sqrt{1-(v^2/c^2)}}$

Calculations :
$$\sqrt{1-(v^2/c^2)} = \frac{m_o}{m}$$

$$L = L_o\frac{m_o}{m} = 1 \times \frac{m_o}{(3/2)\, m_o} = \frac{2}{3}\text{ m}$$

$$\therefore \quad L = 0.67 \text{ m}$$

.... Ans.

Problem 8

The length of a rod is found to be half of its length when at rest. What is the speed of the rod relative to the observer?

Solution :

Data : $L = \frac{L_0}{2}$

Formula : $L = L_0 \sqrt{1-(v^2/c^2)}$

$$\frac{L_0}{2} = L_0 \sqrt{1-(v^2/c^2)}$$

$$\sqrt{1-(v^2/c^2)} = \frac{1}{2} \quad \therefore \quad 1-\frac{v^2}{c^2} = \frac{1}{4}$$

$$\frac{v^2}{c^2} = \frac{3}{4} \quad \therefore \quad v = \frac{\sqrt{3}}{2} c = 0.866 \text{ c}$$

.... **Ans.**

Problem 9

A 1 m long rod is moving along its length with a velocity 0.6 c. Calculate its length as it appears to an observer on the earth.

Solution :

Data : $v = 0.6$ c, $L_0 = 1$ m.

Formula : $L = L_0 \sqrt{1-(v^2/c^2)}$

Calculations : $L = 1\sqrt{1-(0.6)^2} = 0.8$ m

.... **Ans.**

Problem 10

A rod has a length of 2 m. Find its length when it is carried in a rocket with a speed of 0.9 e.

Solution :

Data : $L_0 = 2$ m, $v = 0.9$ c

Formula : $L = L_0 \sqrt{1-(v^2/c^2)}$

Calculations : $L = 2\sqrt{1-(0.9)^2}$

$\therefore$ $L = 0.872$ m

.... **Ans.**

Problem 11

A rocket ship is 100 m long on the ground. When it is in flight, its length is 99 m to an observer on the ground. What is its speed?

Solution :

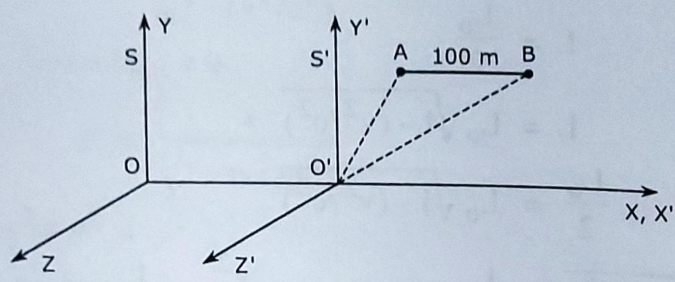

Fig. 4.10

Data : Observer O' in the rocket moves with a velocity $\dot{v}$ with respect to the observer O on the ground. Hence,

$l' = 100$ m as observed by O' and $l = 99$ m as observed by O.

Formula : $l = l' \sqrt{1-(v^2/c^2)}$

Calculations : $l^2 = l'^2 [1-(v^2/c^2)]$

$$\left(\frac{l}{l'}\right)^2 = 1 - \frac{v^2}{c^2}$$

$$v^2 = c^2\left(1 - \frac{l^2}{l'^2}\right)$$

$$v = c\sqrt{1-\left(\frac{l}{l'}\right)^2} = 3 \times 10^8 \sqrt{1-\left(\frac{99}{100}\right)^2}$$

$\therefore$ $v = 4.23 \times 10^7$ m/sec. .... **Ans.**

Problem 12

Calculate the percentage contraction in the length of a rod moving with a speed of 0.8 c in a direction at an angle of 60° with its own length.

Solution :

Data : $v = 0.8\,c,\ \theta = 60°$

Formula : $l = l_o \sqrt{1-(v^2/c^2)}$

Calculations : θ is the angle between frames S and S'. The rod AB lies in the XY plane in frame S. As it travels in S' at an angle of 60° with frame S.

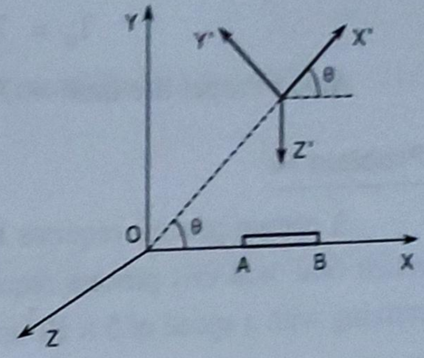

Fig. 4.11

$$l_x = l\cos 60°, \quad l_y = l\sin 60°.$$

$$l_x' = l_x\sqrt{1-(v^2/c^2)}$$

$$= l\cos 60°\sqrt{1-(0.8)^2}$$

$$\therefore \quad l_x' = 0.3\,l$$

$$l_y' = l_y = 1\sin 60° = \frac{\sqrt{3}\,l}{2}$$

as there is no motion perpendicular to the length of the rod.

Hence, $$l' = \sqrt{l_{x'}^2 + l_{y'}^2} = \sqrt{(0.3l)^2 + \left(\frac{\sqrt{3}\,l}{2}\right)^2}$$

$$\therefore \quad l' = 0.9165\,l$$

$$\text{Percentage contraction} = \frac{l-l'}{l}\times 100$$

$$= \frac{l-0.9165l}{l}\times 100 = 8.2\,\%$$

Problem 13

In the laboratory, the lifetime of a particle moving with speed 2.8×10^8 m/sec is found to be 2×10^{-7} sec. Calculate the proper life time of the particle.

Solution :

Data : $v = 2\times10^{-7}$ sec, $T = 2.8\times10^8$ m/sec.

Formula : $T = \dfrac{T_0}{\sqrt{1-(v^2/c^2)}}$; T = Life time measured, T_0 = Proper lifetime.

Calculations :

$$T_0 = T\sqrt{1-(v^2/c^2)}$$

$$= 2\times10^{-7}\sqrt{1-\left(\frac{2.8\times10^8}{3\times10^8}\right)^2}$$

$$\therefore \quad T_o = 7.18 \times 10^{-7} \text{ sec.}$$

Ans. : Proper life time $= 7.18 \times 10^{-7}$ sec.

Problem 14

A certain process requires 10^{-6} sec. to occur in an atom at rest in laboratory. How much time will this process require to an observer in the laboratory, when the atom is moving with a speed of 5×10^7 m/sec ?

Solution :

Data : $T_o = 10^{-6}$ sec., $v = 5 \times 10^7$ m/sec.

Formula :
$$T = \frac{T_o}{\sqrt{1-(v^2/c^2)}}$$

Calculations :
$$T = \frac{10^{-6}}{\sqrt{1-\left(\dfrac{5\times 10^7}{3\times 10^8}\right)^2}} = 1.014 \times 10^{-6} \text{ sec.}$$

Ans. : Time $= 1.014 \times 10^{-6}$ sec.

Problem 15

The mean life of a meson is 2×10^{-8} sec. Calculate the mean life of a meson moving with a velocity of 0.8 c.

Solution :

Data : $T_o = 2 \times 10^{-8}$ sec., $v = 0.8$ c.

Formula :
$$T = \frac{T_o}{\sqrt{1-(v^2/c^2)}}$$

Calculations :
$$T = \frac{2 \times 10^{-8}}{\sqrt{1-\left(\dfrac{0.8\,c}{c}\right)^2}} = \frac{2 \times 10^{-8}}{\sqrt{1-(0.8)^2}}$$

$$\therefore \quad T = 3.33 \times 10^{-8} \text{ sec.}$$

Ans. : Mean life of a meson $= 3.33 \times 10^{-8}$ sec.

Problem 16

What is the velocity of π mesons whose observed mean life is 2.5×10^{-7} sec. The proper mean life of these π mesons is 2.5×10^{-8} sec.

Solution :

Data : $T_o = 2.5 \times 10^{-8}$ sec., $T = 2.5 \times 10^{-7}$ sec.

Formula : $$T = \frac{T_o}{\sqrt{1-(v^2/c^2)}}$$

Calculations :

$$\sqrt{1-(v^2/c^2)} = \frac{T_o}{T}$$

$$v^2 = c^2\left[1-\left(\frac{T_o}{T}\right)^2\right]$$

$$v^2 = c^2\left[1-\left(\frac{2.5\times10^{-8}}{2.5\times10^{-7}}\right)^2\right]$$

$\therefore$ $$v = 0.995\ c$$ .... **Ans.**

Problem 17

A clock keeps correct time on the earth. It is put on the space ship moving uniformly with a speed of 10^8 m/sec. How many hours does it appear to lose per day?

Solution :

Data : $T = 24$ hrs as measured in the space ship

$T_o =$ the time observed by an observe on the earth.

$v = 10^8$ m/sec.

Formula : $$T = \frac{T_o}{\sqrt{1-(v^2/c^2)}}$$

Calculations : $$T_o = T\sqrt{1-\frac{v^2}{c^2}} = 24\sqrt{1-\left(\frac{10^{-8}}{3\times10^{-8}}\right)^2}$$

$\therefore$ $$T_o = 24 \times \frac{2\sqrt{2}}{3} = 22.63 \text{ sec.}$$ .... **Ans.**

Time lost per day $= 24 - 22.63 = 1.37$ hr

Problem 18

With what velocity should a rocket move, so that every year spent on it corresponds to 4 years on the earth?

Solution :

Data : T_o = correct time = 1 year on the rocket

$T = 4$ years, as appears from the earth

Formula :

$$T = \frac{T_o}{\sqrt{1-(v^2/c^2)}}$$

Calculations :

$$1-\frac{v^2}{c^2} = \frac{T_o}{T}$$

$$v^2 = c^2\left[1-\left(\frac{T_o}{T}\right)^2\right] = c^2\left[1-\left(\frac{1}{4}\right)^2\right]$$

$$\therefore \quad v = 0.97\ c$$

.... Ans.

Problem 19

With what velocity should a space ship fly so that every day spent on it may correspond to three days on the earth's surface.

Solution :

Data : $T = 3$ days as it appears on the earth's surface.

$T_o = 1$ day as measured in the spaceship.

Formula :

$$T = \frac{T_o}{\sqrt{1-(v^2/c^2)}}$$

Calculations :

$$1-\frac{v^2}{c^2} = \left(\frac{T_o}{T}\right)^2$$

$$\therefore \quad v^2 = c^2\left[1-\left(\frac{T_o}{T}\right)^2\right] = c^2\left[1-\frac{1}{9}\right]$$

$$\therefore \quad v = \frac{2\sqrt{2}}{3}\ c = 0.47\ c$$

.... Ans.

Problem 20

At what speed should a clock be moved so that it may appear to lose 1 minute in each hour?

Solution :

Data : The clock, loses 1 min in each hour. So it must record 59 min. for each hour. Hence, $T_o = 59$ min., $T = 1$ hour $= 60$ min.

Formulae :
$$T = \frac{T_o}{\sqrt{1-(v^2/c^2)}}$$

$$\therefore \quad \sqrt{1-\frac{v^2}{c^2}} = \frac{T_o}{T}$$

$$\therefore \quad v^2 = c^2\left[1-\left(\frac{T_o}{T}\right)^2\right] = c^2\left[1-\left(\frac{59}{60}\right)^2\right]$$

$$\therefore \quad v = 0.1818\ c$$
.... **Ans.**

Problem 21

At what velocity will the mass of a body is 2.25 times its rest mass?

Solution :

Data : $m = 2.25\ m_o$

Formula :
$$m = \frac{m_o}{\sqrt{1-(v^2/c^2)}}$$

Calculations :

$$1-\frac{v^2}{c^2} = \left(\frac{m_o}{m}\right)^2$$

$$v^2 = c^2\left[1-\left(\frac{m_o}{m}\right)^2\right] = c^2\left[1-\left(\frac{1}{2.25}\right)^2\right]$$

$$\therefore \quad v = 0.895\ c$$
.... **Ans.**

Problem 22

With what velocity a particle should move so that its mass appears to increase by 20 % of its rest mass?

Solution :

Data : $m = m_o + 20\ \%\ m_o = 1.2\ m_o$

Formula :
$$m = \frac{m_o}{\sqrt{1-(v^2/c^2)}}$$

Calculations :
$$\frac{m}{m_o} = \frac{1}{\sqrt{1-(v^2/c^2)}}$$

$$1-\frac{v^2}{c^2} = \left(\frac{m_o}{m}\right)^2$$

$$v^2 = c^2\left[1-\left(\frac{m_o}{m}\right)^2\right] = c^2\left[1-\left(\frac{1}{1.2}\right)^2\right]$$

$$\therefore \quad v^2 = c^2\ [0.30558]$$

$$\therefore \quad v = 0.553\ c$$

..............Ans.

Problem 23

If the kinetic energy of a body is double its rest mass energy calculate its velocity.

Solution :

Data : $E_k = 2\ m_o\ c^2$

Formulae : $E = E_k + m_o\ c^2,\ E = mc^2,\ m = \frac{m_o}{\sqrt{1-(v^2/c^2)}}$

Calculations :
$$mc^2 = 2\ m_o\ c^2 + m_o\ c^2$$

$$m = 3\ m_o$$

$$\therefore \quad \frac{m_o}{\sqrt{1-(v^2/c^2)}} = 3\ m_o$$

$$1-\frac{v^2}{c^2} = \frac{1}{9} \qquad \therefore \quad v^2 = c^2\ \frac{8}{9}$$

$$\therefore \quad v = \frac{2\sqrt{2}}{3}\ c = 0.94\ c$$

..............Ans.

Problem 24

The mass of a moving electron is 11 times its rest mass. Calculate its kinetic energy and momentum.

Solution :

Data : $m = 11\ m_o$

Formulae : $E = E_k + m_o c^2,\ E = mc^2,\ m = \dfrac{m_o}{\sqrt{1-(v^2/c^2)}}$

Calculations :

$$E_k = E_k + m_o c^2$$

$$= 11\ m_o c^2 - m_o c^2 = 10\ m_o c^2$$

$$= 10 \times 9.1 \times 10^{-31} \times (3 \times 10^8)^2 \text{ J}$$

$$= 8.2 \times 10^{-13} \text{ J} = \frac{8.2 \times 10^{-13}}{1.6 \times 10^{19}} \text{ eV}$$

$$\therefore \quad E_k = 5.1 \text{ MeV}$$

Now,
$$m = \frac{m_o}{\sqrt{1-(v^2/c^2)}} \quad \therefore \quad \frac{1}{\sqrt{1-(v^2/c^2)}} = \frac{m}{m_o} = 11$$

$$1 - \frac{v^2}{c^2} = \left(\frac{1}{11}\right)^2 \quad \therefore \quad \frac{v^2}{c^2} = 1 - \frac{1}{121}$$

$$\therefore \quad v = 2.98 \times 10^8 \text{ m/sec}$$

$$p = \frac{m_o\, v}{\sqrt{1-(v^2/c^2)}} = 11\ m_o\, v$$

$$= 11 \times 9.1 \times 10^{-31} \times 2.98 \times 10^8$$

$$\therefore \quad p = 2.98 \times 10^{-21} \text{ N-sec.}$$

.... **Ans.**

Problem 25

How fast must an electron move in order to have its mass equal to the rest mass of the proton (1.67×10^{-27} kg)?

Solution :

Data : $m = m_p$ = rest mass of a proton $= 1.67 \times 10^{-27}$ kg

$m_o = 9.1 \times 10^{-31}$ kg

Formula :
$$m = \frac{m_o}{\sqrt{1-(v^2/c^2)}}$$

Calculations :

$$1-\frac{v^2}{c^2} = \left(\frac{m_o}{m}\right)^2$$

$$v^2 = c^2\left[1-\left(\frac{m_o}{m}\right)^2\right] = c^2\left[1-\left(\frac{9.1\times10^{-31}}{1.67\times10^{-27}}\right)^2\right]$$

$$\therefore \quad v = 0.9985\ c$$

.... Ans.

Problem 26

Kinetic energy of a particle is (i) 3 times, (ii) equal to its rest mass energy. What is its velocity?

Solution :

Data : (i) $E_k = 3\ m_o\ c^2$, (ii) $E_k = m_o\ c^2$.

Formulae : $E = E_k + m_o\ c^2,\ E_k = mc^2 - m_o\ c^2,\ m = \dfrac{m_o}{\sqrt{1-(v^2/c^2)}}$

Calculations : *(i)* $E_k = mc^2 - m_o\ c^2$

$$3\ m_o\ c^2 = mc^2 - m_o\ c^2$$

$$m = 4\ m_o$$

$$\frac{m_o}{\sqrt{1-(v^2/c^2)}} = 4\ m_o \qquad \therefore\ 1-\frac{v^2}{c^2} = \frac{1}{16}$$

$$v^2 = c^2\left(1-\frac{1}{16}\right) \qquad \therefore\ v = 0.968\ c$$

.... Ans.

(ii)

$$E_k = mc^2 - m_o\ c^2$$

$$m_o\ c^2 = mc^2 - m_o\ c^2$$

$$mc^2 = 2\ m_o\ c^2$$

$$\therefore \quad m = 2\ m_o \qquad \therefore\ \frac{m_o}{\sqrt{1-(v^2/c^2)}} = 2\ m_o$$

$$1-\frac{v^2}{c^2} = \frac{1}{4}$$

$$v^2 = c^2\left(1-\frac{1}{4}\right) \qquad \therefore\ v = 0.866\ c$$

.... Ans.

Problem 27

Find the velocity of a 0.1 MeV electron according to classical and relativistic mechanics.

Solution :

Data : $E_k = 0.1 \text{ MeV} = 0.1 \times 10^6 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-14}$ Joules

$m = 9.1 \times 10^{-31}$ kg (classically),

$m_o = 9.1 \times 10^{-31}$ kg (relativistically)

Formulae : $E_k = \frac{1}{2} mv^2$ in classical mechanics and

$E_k = mc^2 - m_o c^2$ in relativistic mechanics.

$$m = \frac{m_o}{\sqrt{1-(v^2/c^2)}}$$

Calculations : In classical mechanics,

$$v = \sqrt{\frac{2}{m} E_k} = \sqrt{\frac{2}{9.1 \times 10^{-31}} \times 1.6 \times 10^{-14}}$$

$$\therefore \quad v = 1.87 \times 10^8 \text{ m/sec}$$

In relativistic mechanics,

$$E_k = \frac{m_o c^2}{\sqrt{1-(v^2/c^2)}} - m_o c^2$$

$$E_k = m_o c^2 \left(\frac{1}{\sqrt{1-(v^2/c^2)}} - 1 \right)$$

$$\frac{E_k}{m_o c^2} = \frac{1}{\sqrt{1-(v^2/c^2)}} - 1$$

$$\frac{1}{\sqrt{1-(v^2/c^2)}} = \frac{E_k}{m_o c^2} + 1$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{1}{1+\left(\frac{E_k}{m_o c^2}\right)} \qquad \therefore \quad 1-\frac{v^2}{c^2} = \frac{1}{\left(1+\frac{E_k}{m_o c^2}\right)^2}$$

$$\frac{v^2}{c^2} = 1 + \frac{1}{\left(1+\frac{E_k}{m_o c^2}\right)^2}$$

$$\therefore \quad v^2 = c^2\left[1+\frac{1}{\left(1+\frac{E_k}{m_o c^2}\right)^2}\right]$$

$$= c^2\left[1+\frac{1}{\left(1+\frac{1.6\times10^{-14}}{9.1\times10^{-31}\times3\times10^{8}}\right)^2}\right]$$

$$\therefore \quad v = 0.54\ c$$

.... **Ans.**