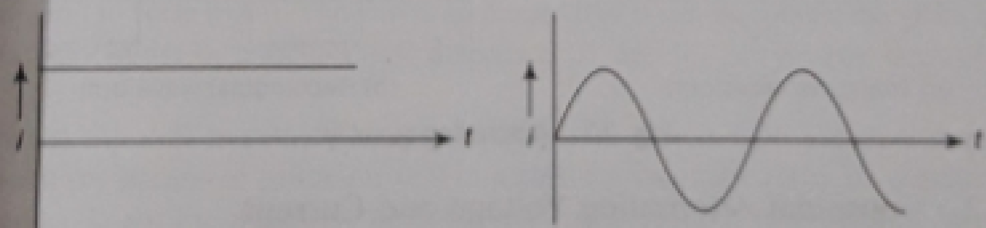
2

AC Circuits

2.1 AC Fundamentals

In the earlier chapter, we have dealt with direct currents, which flow continuosly in one direction only. If the applied voltage and the circuit resistance are kept constant, the magnitude of the current flowing through this circuit remains constant over time. When the current flowing in the circuit varies in magnitude as well as direction periodically, it is called **alternating current**. Thus, an alternating current or voltage is one that periodically passes through a definite cycle, consisting of two half cycles—during one of which the current or voltage varies in one direction and during the other half cycle varies in the opposite direction. The circuits in which alternating currents flow are called **ac circuits**.

The graphical representations of both dc and ac are shown in Figs 2.1(a) and (b). In order to produce an ac through an electric circuit, a source capable of reversing the emf periodically is required, e.g., an ac generator. On the other hand, to produce a dc, a source capable of developing a constant emf is necessary, e.g., a battery and a dc generator.



(a) Direct current (b) Alternating current

Fig. 2.1 Waveforms of dc and ac

Advantages of ac

1. The voltages in this system can be raised or lowered with the help of a transformer.
2. High-voltage ac transmission is possible and economical through the use of transformers.

3. AC motors are simple, cheap and require less attention from maintenance point of view.
4. An ac supply can be easily converted into a dc supply. This is required as dc is very much essential for the applications such as battery charging, printing process, cranes, and telephone systems.

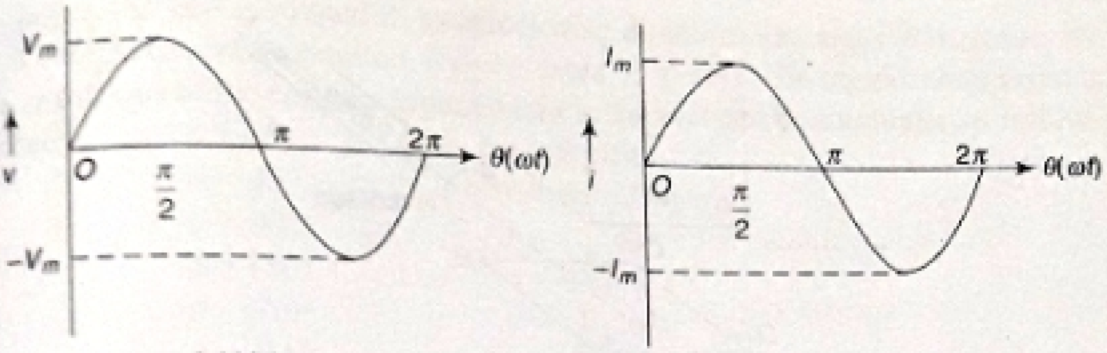
Due to these advantages, ac is used extensively in practice.

2.1.2 Sinusoidal Alternating Voltage and Current

A sinusoidal alternating quantity is one whose instantaneous value varies according to the sine function of time, i.e., it produces a sine wave. A sinusoidal alternating voltage can be produced by rotating a coil (winding) with a constant angular velocity (say ω rad/sec) in a uniform magnetic field. Figures 2.4(a) and (b) show the waveforms of sinusoidal alternating voltage and current respectively.

The sinusoidal alternating voltage shown in Fig. 2.4(a) can be expressed mathematically as:

$$v = V_m \sin\omega t$$



(a) Voltage (b) Current

Fig. 2.4 Sinusoidal alternating quantity

where v = Instantaneous value of alternating voltage
V_m = Maximum value of alternating voltage
ω = Angular velocity of coil (winding)

It may be noted that in the above equation of sinusoidal alternating voltage, V_m and ω are constant. Therefore, the instantaneous value v (i.e., value at any instant) changes with time according to the sine function. Similarly, an alternating current varying sinusoidally can be expressed as

$$i = I_m \sin \omega t$$

Out of all these types of alternating waveforms, a sinusoidal alternating waveform is preferred for ac system.

2.1.3 Generation of Alternating Voltage

The sinusoidal alternating voltage can be generated by rotating a coil (winding) in a magnetic field or by rotating a magnetic field in a stationary coil (winding). The machine which is used to generate a sinusoidal alternating voltage is called an **ac generator.**

Figure 2.5 shows the elementary simple ac generator. It consists of a permanent magnet of two poles. A single rectangular coil which is placed in the vicinity of the permanent magnet. The coil has two conductors AB and CD connected at one end to form a coil. The coil is so placed that it can be rotated about its own axis in clockwise or anticlockwise direction. 'P' and 'Q' are the two ends of the coil.

When the coil is rotated in anticlockwise direction with constant angular velocity (by means of prime mover) in a uniform magnetic field, its conductors AB and CD cut the magnetic lines of flux and according to Faraday's laws of electromagnetic induction, an emf gets induced in them (see Section C.5.1, tricai engineering

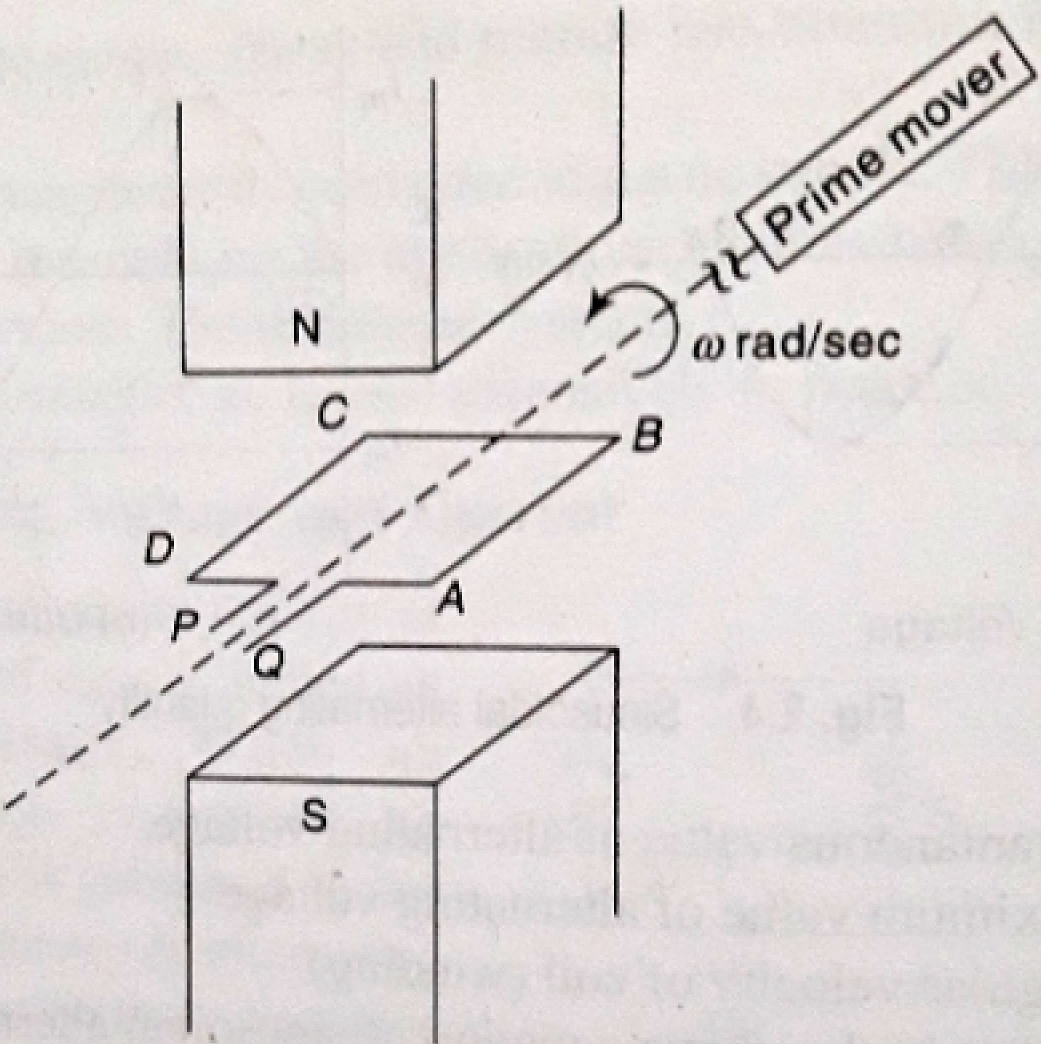


Fig. 2.5 Simple ac generator

Faraday's law of electromagnetic Induction :-

Whenever conductor cuts the magnetic flux, an emf is induced in the conductor.

Production of induced E.M.F. and Current.

The insulated coil whose terminals are connected to sensitive galvanometer G. It is placed close to stationary bar magnet initially at position AB so there is no deflection of the galvanometer.

Now, suppose that the magnet is suddenly brought closer to the coil in position CD. Then it is found that there is a jerk or a sudden but a momentary deflection in a galvanometer.

The deflection is reduced to zero when the magnet is stationary at its new position CD. so here in the phenomenon, flux linked with coil is increased.

Next, the magnet is suddenly withdrawn away from the coil, as in fig. 2. It is found that again there is a momentary deflection in a galvanometer. It is important to note that deflection is in a direction opposite to that of fig. 1. Due to withdrawal of magnet, flux linked with the coil is decreased.

The deflection of the galvanometer indicates the production of e.m.f in the coil. The only cause of production can be the sudden approach or withdrawal of magnet from the coil. It is found that the actual cause of e.m.f is the change of flux linking with the coil. This emf exist so long as the change in flux exists. Stationary flux, however strong, will never induce any e.m.f in a stationary conductor

ote- [In fact, same result can be obtained by keeping the bar magnet stationary and moving the coil suddenly away or towards the magnet.

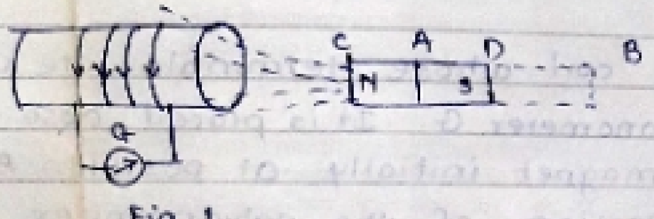


Fig. 1

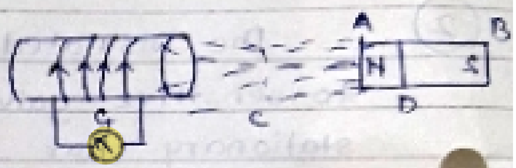
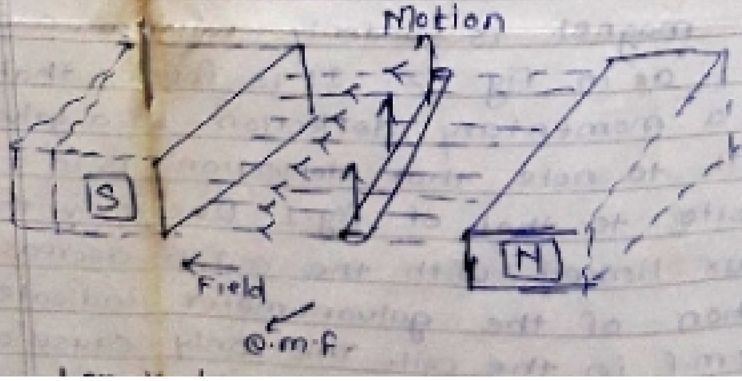


Fig. 2

<u>Direction of induced e.m.f and currents</u>

There exists a definite relation betn the direction of the induced current, the direction of flux and the direction of motion of the conductor.

(3) The direction of the induced current may be found easily by applying either Fleming's Right hand rule or Lenz's law.



& Produces emf in winding according to faraday's law of electromagnetic induction alternating voltage will produce.

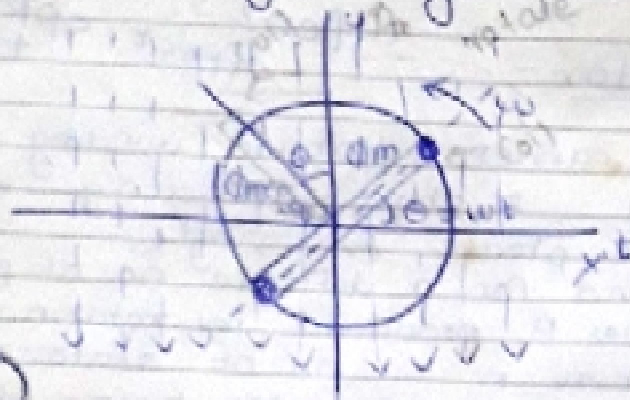
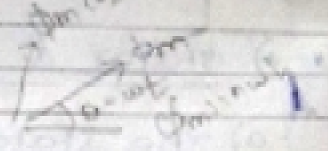



Fig. a
(a)

(5)

Magnetic field will produce from N to S.

Rectangular coil having N turns and rotating in uniform magnetic field with an angular velocity ω rad/sec.
let time be measured from the x axis

Maximum flux ϕ_m is linked with the coil when its plane coincides with x axis
In time t sec. the coil rotates through an angle $\theta = \omega t$. In this deflected position, the component of the flux which is perpendicular to the plane of coil $\phi = \phi_m \cos\omega t$
(N = No of flux linkages) Hence flux linkage of the coil at any time are $N\phi = N\phi_m \cos\omega t$

Important Notes
According to Faraday's law of electromagnetic induction, the induced e.m.f in the coil is given by the rate of change of flux linkages

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Mo	Tu	We	Th	Fr	Sa	Su
				1	2	3
4	5	6	7	8	9	10
11	12	13	14	15	16	17
18	19	20	21	22	23	24
25	26	27	28	29	30	

of the coil. Therefore instonteneous value of induced e.m.f is,

$$e = -\frac{d}{dt}(N\phi) \text{ volts} = -N\frac{d}{dt}\phi$$

$$= -N\frac{d}{dt}(\phi_m \cos\omega t) = \cancel{-N\phi\omega\sin\omega t}$$

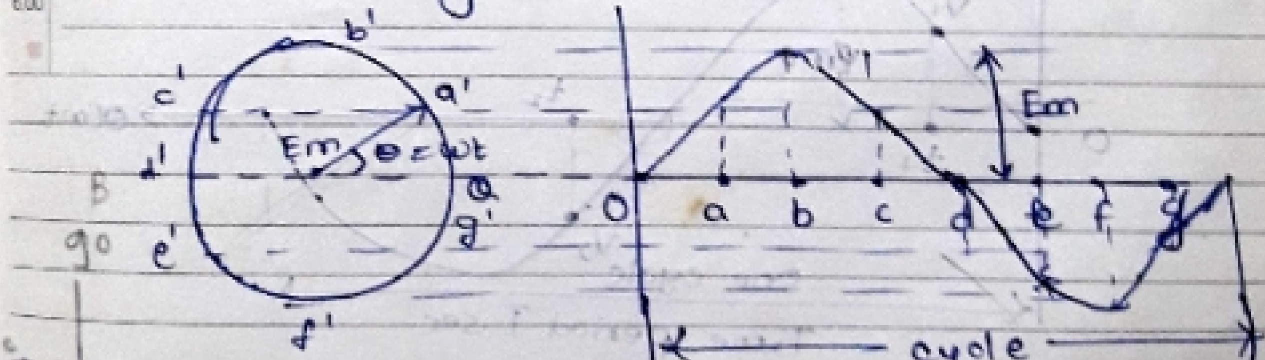
$$= -N\phi_m\omega(-\sin\omega t) \quad \boxed{e = N\phi_m\omega\sin\omega t}$$

Another eq^n we can write,

B = flux density
l = length of coil meter
v = Peripheral velocity. meter/sec

$$\boxed{e = N \times Blv \sin\theta}$$

* How alternating voltage will produce ?
From fig. (a)



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Mo	Tu	We	Th	Fr	Sa	Su
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

Important Notes

O = 0°
a = 45°
b = 90°
c = 135°
d = 180°
e = 225°
f = 270°
g = 315°
h = 360°

2.1.4 AC Terminology

Before further analysis of alternating quantity, it is necessary to be familiar with the different terms which are very frequently used related to the alternating quantities.

Instantaneous value (v) The value of the alternating quantity at a particular instant of time is known as its instantaneous value, e.g., v_1 and v_2 are the instantaneous values of alternating voltages at the instants t_1 and t_2 respectively shown in Fig. 2.7.

Cycle One complete set of positive and negative values of an alternating quantity is known as cycle. A cycle can be defined as each repetition of a set of positive and negative instantaneous values of an alternating quantity. One such cycle of an alternating quantity is shown in Fig. 2.7.

Time period (T) The time taken by an alternating quantity to complete its one cycle is known as its time period. It is denoted by T.

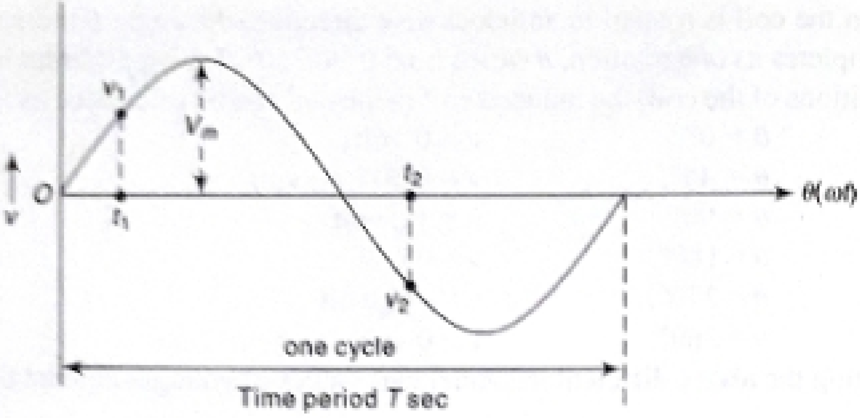


Fig. 2.7 One cycle of sinusoidal alternating voltage

Frequency (f) The number of cycles completed by an alternating quantity per second is known as its frequency. It is denoted by f and it is measured in cycles/second, also known as hertz (Hz).

As time period T is the time for one cycle, i.e., seconds/cycle and frequency is cycles/second, we can say that the frequency is reciprocal of the time period.

$$f = \frac{1}{T} \text{ Hz}$$

Amplitude (V_m) The maximum value attained by an alternating quantity during positive or negative half cycle is called its amplitude. It is denoted by V_m.

2.1.5 Standard Forms of Alternating Quantity

In an ac generator, if the coil is rotated with constant angular velocity ω rad/sec, the angle turned by the coil is given by

Angle turned, $\theta = \omega t$ rad

In one revolution of coil, the angle turned is 2π radian and the voltage wave completes one cycle. The time taken to complete one cycle is the time period T of the alternating voltage.

So, $$\text{Angular velocity } \omega = \frac{\text{Angle turned}}{\text{Time taken}} = \frac{2\pi}{T}$$

or $$\omega = 2\pi f \qquad \left(\because f = \frac{1}{T}\right)$$

The standard forms of a sinusoidal alternating voltage are given by

$$v = V_m \sin\theta$$

$$v = V_m \sin\omega t \qquad (\because \theta = \omega t)$$

$$v = V_m \sin 2\pi f t \qquad (\because \omega = 2\pi f)$$

$$v = V_m \sin \frac{2\pi}{T} t \qquad \left(\because f = \frac{1}{T}\right)$$

Similarly, the standard forms of a sinusoidal alternating current are given by

$$i = I_m \sin\theta \qquad (\because \theta = \omega t)$$

$$i = I_m \sin\omega t \qquad (\because \omega = 2\pi f)$$

$$i = I_m \sin 2\pi f t \qquad \left(\because f = \frac{1}{T}\right)$$

$$i = I_m \sin \frac{2\pi}{T} t$$

Example 2.1 An alternating current i is given by $i = 141.4 \sin 314t$. Find (i) the maximum value, (ii) the frequency, (iii) the time period, and (iv) the instantaneous value when time is 3 msec.

Solution

Comparing the given equation of alternating current with the standard form $i = I_m \sin \omega t$, we have

(i) Maximum value, $I_m = 141.4$ A

(ii) Frequency, $f = \dfrac{\omega}{2\pi} = \dfrac{314}{2\pi} = 50$ Hz

(iii) Time period, $T = \dfrac{1}{f} = \dfrac{1}{50} = 0.02$ sec

(iv) Given $i = 141.4 \sin 314t$

When $t = 3$ msec $= 3 \times 10^{-3}$ sec, we get

$$i = 141.4 \sin(314 \times 3 \times 10^{-3}) = 141.4 \sin\left[(314 \times 3 \times 10^{-3}) \times \frac{180}{\pi}\right]^{\circ}$$

So, $i = 114.36$ A

Example 2.2 An alternating current of frequency 60 Hz has a maximum value of 12A. (i) Write down the equation of the instantaneous value, (ii) find the value of current after $\dfrac{1}{360}$ sec, and (iii) find the time taken to reach 9.6 A for the first time.

Solution

(i) $i = I_m \sin \omega t = 12 \sin 2\pi \times 60t = 12 \sin 377t$

(ii) Value of current after $\dfrac{1}{360}$ sec,

$$i = 12 \sin 377t$$

or $$i = 12 \sin 377 \left(\frac{1}{360}\right)$$

or $$i = 12 \sin 1.047$$

or $$i = 12 \sin \left(1.047 \times \frac{180}{\pi}\right)^{\circ}$$

or $\quad i = 12 \sin 60°$

Hence, $\quad i = 10.39$ A

(iii) Time taken to reach 9.6 A for the first time is given by

$$i = I_m \sin 377t$$

or $$9.6 = 12 \sin 377t$$

or $$\sin 377t = 0.8$$

or $$\sin\left(377t \times \frac{180}{\pi}\right)^{\circ} = 0.8$$

or $$377t \times \frac{180}{\pi} = \sin^{-1} 0.8$$

or $$377t \times \frac{180}{\pi} = 53.13$$

Hence, $$t = 2.46 \times 10^{-3} \text{ sec}$$

Example 2.3 An alternating voltage is given by $v = 10 \sin 942t$. Determine the time taken from $t = 0$ for the voltage to reach a value of +6V for first and second time.

Solution

Figure 2.8 shows the waveform of the given alternating voltage. Let the voltage become +6V for the first time after t sec. Then

as $\quad v = V_m \sin \omega t,$

$\quad v = 10 \sin 942t$

So, $\quad 6 = 10 \sin 942t$

or $\quad \sin 942t = 0.6$

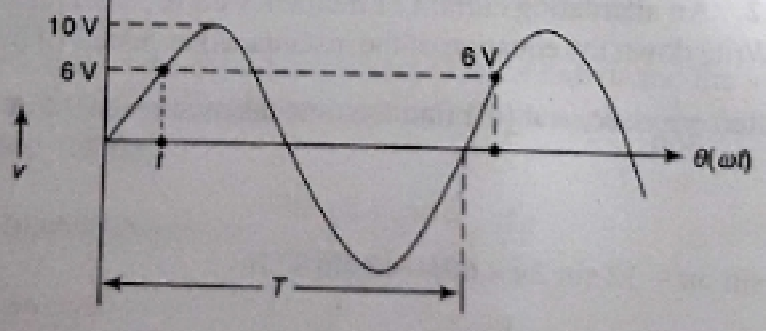


Fig. 2.8

In the above expression, the angle '942t' is in radians.

So, $$\sin\left(942t \times \frac{180}{\pi}\right)^{\circ} = 0.6$$

or $$942t \times \frac{180}{\pi} = \sin^{-1} 0.6$$

or $\qquad 942t \times \dfrac{180}{\pi} = 36.87$

or $\qquad t = 0.68 \times 10^{-3}$ sec

The frequency of the voltage is $f = \dfrac{942}{2\pi} = 150$ Hz and the time period of the wave is $T = \dfrac{1}{150} = 6.66 \times 10^{-3}$ sec.

So, the time taken to reach 6V for the second time $= 0.68 \times 10^{-3} +$ time period

$$= 0.68 \times 10^{-3} + 6.66 \times 10^{-3}$$

$$= 7.34 \text{ msec}$$

2.1.6 Values of Alternating Voltage and Current

In a dc system, the voltage and current are constant so that there is no problem of specifying their magnitudes. However, an alternating voltage or current varies from instant to instant. The magnitude of an alternating voltage or current is expressed by three ways, namely

1. peak value
2. average value
3. rms value or effective value

1. Peak value

It is the maximum value attained by an alternating quantity. The peak or maximum value of alternating voltage or current is represented by V_m or I_m.

2. Average value

The arithmetical average of all the values of an alternating quantity over one cycle is called its average value, i.e.,

$$\text{Average value} = \frac{\text{Area under the curve}}{\text{Base}}$$

The waveforms can be classified into two types: symmetrical waveform and unsymmetrical waveform. A symmetrical waveform has positive half cycle exactly equal to the negative half cycle. If positive half cycle is not equal to the negative half cycle, then the waveform is said to be unsymmetrical.

In case of symmetrical waveforms (e.g., sinusoidal voltage or current), the average value over one cycle is zero. It is because the positive half is exactly equal to the negative half so that the net area is zero. However, the average value of positive or negative half is not zero. Hence, in case of symmetrical waveforms, average value means the average value of half cycle or one alternation. In case of unsymmetrical waveform, average value is taken over full cycle.

Thus, for symmetrical waveforms (+ve half = –ve half),

$$\text{Average value} = \frac{\text{Area of half cycle}}{\text{Base length of half cycle}}$$

Similarly, for unsymmetrical waveforms (+ve half ≠ –ve half),

$$\text{Average value} = \frac{\text{Area of full cycle}}{\text{Base length of full cycle}}$$

Average value of sinusoidal ac

The equation of sinusoidal ac is given by

$$i = I_m \sin\theta$$

Figure 2.9 shows the waveform of sinusoidal ac.

As the given waveform is symmetrical,

$$\text{Average value} = \frac{\text{Area of half cycle}}{\text{Base length of half cycle}}$$

So, $$I_{average} = \frac{\int_0^{\pi} i d\theta}{\pi}$$

$$= \frac{1}{\pi}\int_0^{\pi} I_m \sin\theta \, d\theta$$

$$= \frac{I_m}{\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{I_m}{\pi}[-\cos\pi - (-\cos 0)]$$

$$= \frac{I_m}{\pi}[-(-1) - (-1)]$$

$$= \frac{2I_m}{\pi}$$

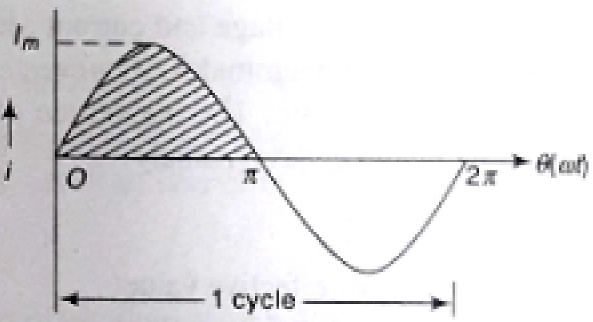


Fig. 2.9

or $$I_{average} = 0.637\, I_m$$

Similarly, it can be proved that for sinusoidal alternating voltage,

$$V_{average} = 0.637\, V_m$$

Example 2.4 Find the average value of the waveform shown in Fig. 2.10.

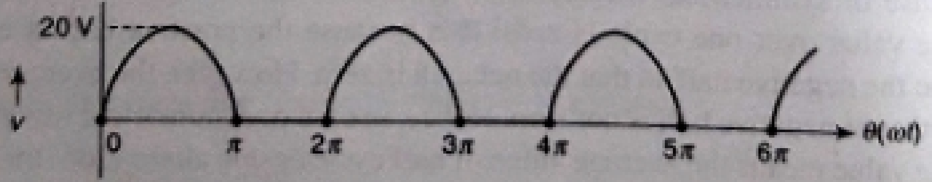
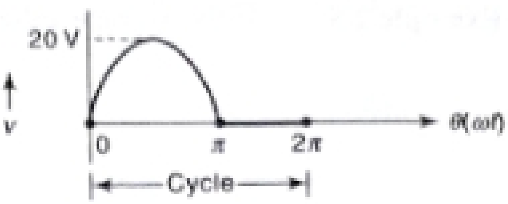


Fig. 2.10

Solution

Consider a full cycle of the given waveform. (Each repetition of a set of positive and negative values of a quantity is known as cycle.)

In a cycle, as positive half is not equal to the negative half, the waveform is unsymmetrical. So, we have to consider a full cycle for average value.



Now, mathematical equation of a cycle is required. It is not possible to write the single equation of a waveform. The given voltage waveform can be divided into two intervals.

First interval is $0 < \theta < \pi$, where the voltage is sinusoidal. Therefore, equation of the voltage is $v = 20 \sin\theta$.

Second interval is $\pi < \theta < 2\pi$, where the voltage remains zero. Therefore, equation of the voltage is $v = 0$.

Thus,

Interval	Equation
$0 < \theta < \pi$	$v = 20 \sin\theta$
$\pi < \theta < 2\pi$	$v = 0$

So, $$V_{\text{average}} = \frac{\text{Area of full cycle}}{\text{Base length of full cycle}}$$

or $$V_{\text{average}} = \frac{\int_0^{2\pi} v d\theta}{2\pi}$$

or $$V_{\text{average}} = \frac{1}{2\pi}\left\{\int_0^{\pi} v d\theta + \int_{\pi}^{2\pi} v d\theta\right\}$$

$$= \frac{1}{2\pi}\int_0^{\pi} 20 \sin\theta \, d\theta$$

$$= \frac{20}{2\pi}\int_0^{\pi} \sin\theta \, d\theta$$

$$= \frac{20}{2\pi}[-\cos\theta]_0^{\pi}$$

$$= \frac{20}{2\pi}[-\cos\pi - (-\cos 0)]$$

$$= \frac{20}{2\pi}[2]$$

$$= 6.366 \text{ V}$$

Example 2.5 Find the average value of the waveform shown in Fig. 2.11.

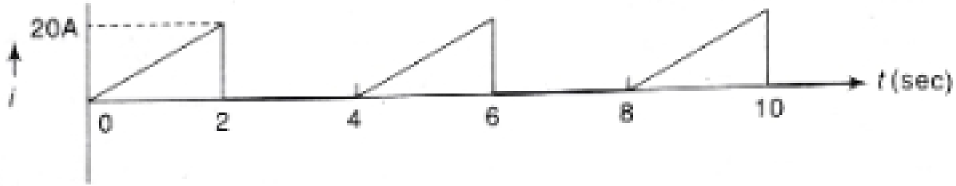
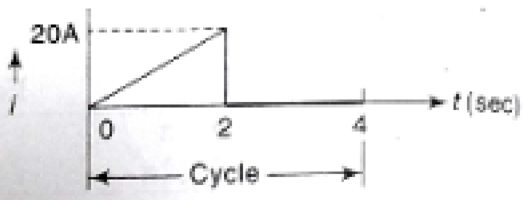


Fig. 2.11

Solution

Consider one cycle of a given waveform.

As positive half is not equal to negative half, given waveform is unsymmetrical. Therefore, for average value, we have to consider a full cycle.



In interval $0 < t < 2$, current increases with constant slope of 10 A/sec. Therefore, the equation of current is $i = 10t$.

In interval $2 < t < 4$, current remains zero. Therefore, the equation of current is $i = 0$.

Thus,

Interval	Equation
$0 < t < 2$	$i = 10t$
$2 < t < 4$	$i = 0$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20-0}{2-0} = 10$$

So, $$I_{\text{average}} = \frac{\int_0^4 i\,dt}{4}$$

$$= \frac{1}{4}\left\{\int_0^2 i\,dt + \int_2^4 i\,dt\right\}$$

$$= \frac{1}{4}\int_0^2 10t\,dt + 0$$

$$= \frac{10}{4}\left[\frac{t^2}{2}\right]_0^2$$

$$= \frac{10}{4}\left[\frac{(2)^2}{2} - \frac{(0)^2}{2}\right]$$

$$= 5\text{ A}$$

Example 2.6 Find the average value of the waveform shown in Fig. 2.12.

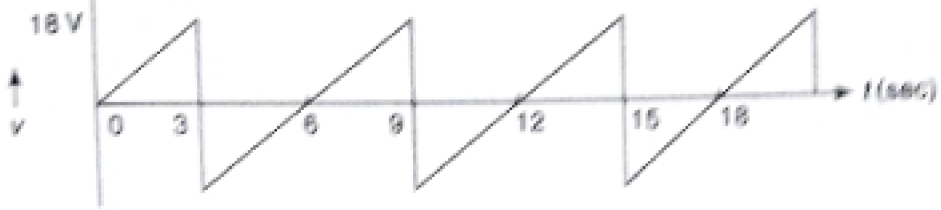
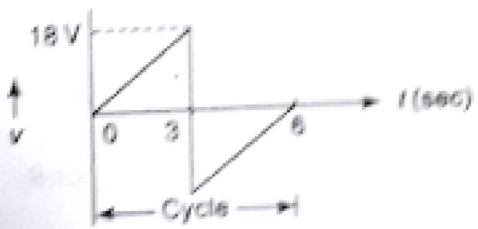


Fig. 2.12

Solution



Consider one cycle of the given waveform. As positive half is exactly equal to negative half, the given waveform is symmetrical. Therefore, for average value, we have to consider a half cycle, i.e., duration of 3 sec ($0 < t < 3$).

In interval $0 < t < 3$, voltage increases with constant slope of 6 V/sec. Therefore, the equation of voltage is $v = 6t$.

consider half cycle

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{18-0}{3-0} = 6$$

So, $$V_{average} = \frac{\int_0^3 v\, dt}{3}$$

$$= \frac{1}{3}\int_0^3 6t\, dt$$

$$= \frac{6}{3}\left[\frac{t^2}{2}\right]_0^3$$

$$= \frac{6}{3}\left[\frac{(3)^2}{2} - \frac{(0)^2}{2}\right]$$

$$= 9 \text{ V}$$

Example 2.7 Find the average value of the waveform shown in Fig. 2.13.

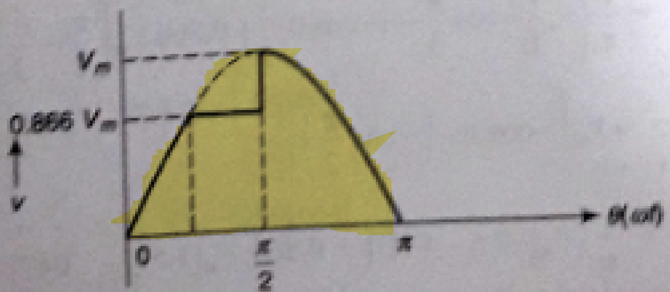


Fig. 2.13

Solution

The angle 'θ' at which the instantaneous value of the voltage becomes equal to 0.866 V_m is required. The angle 'θ' can be calculated as given below.

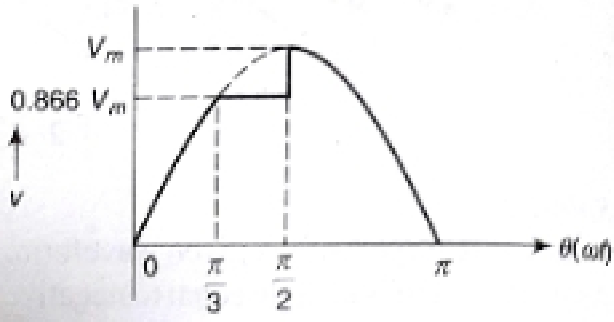
The equation of the waveform is given by

$$v = V_m \sin\theta$$

When $v = 0.866\, V_m$, $\theta = ?$

So, $\quad 0.866\, V_m = V_m \sin\theta$

Hence, $\theta = \dfrac{\pi}{3}$ rad



Now, the voltage waveform is given as follows:

Interval	Equation
$0 < \theta < \dfrac{\pi}{3}$	$v = V_m \sin\theta$
$\dfrac{\pi}{3} < \theta < \dfrac{\pi}{2}$	$v = 0.866\, V_m$
$\dfrac{\pi}{2} < \theta < \pi$	$v = V_m \sin\theta$

So, $$V_{\text{average}} = \frac{\int_0^{\pi} v\, d\theta}{\pi}$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi/3} v\, d\theta + \int_{\pi/3}^{\pi/2} v\, d\theta + \int_{\pi/2}^{\pi} v\, d\theta\right\}$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi/3} V_m \sin\theta\, d\theta + \int_{\pi/3}^{\pi/2} 0.866\, V_m\, d\theta + \int_{\pi/2}^{\pi} V_m \sin\theta\, d\theta\right\}$$

$$= \frac{1}{\pi}\left\{V_m[-\cos\theta]_0^{\pi/3} + 0.866\, V_m[\theta]_{\pi/3}^{\pi/2} + V_m[-\cos\theta]_{\pi/2}^{\pi}\right\}$$

$$= \frac{1}{\pi}\left\{V_m\left[-\cos\frac{\pi}{3} - (-\cos 0)\right] + 0.866\, V_m\left[\frac{\pi}{2} - \frac{\pi}{3}\right]\right.$$
$$\left. + V_m\left[-\cos\pi - \left(-\cos\frac{\pi}{2}\right)\right]\right\}$$

$$= \frac{1}{\pi}\{V_m[-0.5 - (-1)] + 0.866\, V_m[1.57 - 1.047]$$
$$+ V_m[-(-1) - (-0)]\}$$

$$= \frac{1}{\pi}\{0.5\,V_m + 0.45\,V_m + V_m\}$$

$$= 0.621\,V_m$$

Example 2.8 Find the average value of the waveform shown in Fig. 2.14.

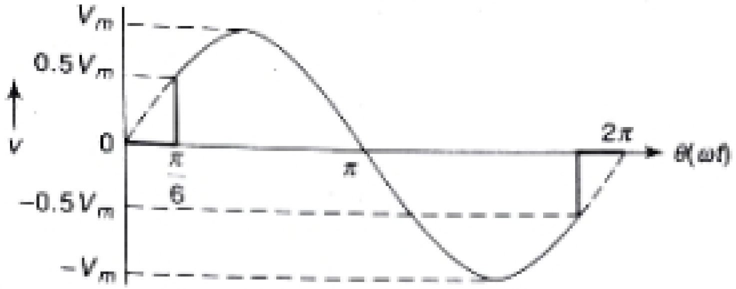


Fig. 2.14

Solution

As the given waveform is symmetrical, for average value we have to consider a half cycle.

So, $$V_{\text{average}} = \frac{\int_0^{\pi} v\,d\theta}{\pi}$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi/6} v\,d\theta + \int_{\pi/6}^{\pi} v\,d\theta\right\}$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi/6} 0\,d\theta + \int_{\pi/6}^{\pi} V_m \sin\theta\,d\theta\right\}$$

$$= \frac{V_m}{\pi}\int_{\pi/6}^{\pi} \sin\theta\,d\theta$$

$$= \frac{V_m}{\pi}[-\cos\theta]_{\pi/6}^{\pi}$$

$$= \frac{V_m}{\pi}\left[-\cos\pi - \left(-\cos\frac{\pi}{6}\right)\right]$$

$$= \frac{V_m}{\pi}[-(-1) - (-0.866)]$$

$$= 0.594\,V_m$$

Example 2.9 Find the average value of the waveform shown in Fig. 2.15.

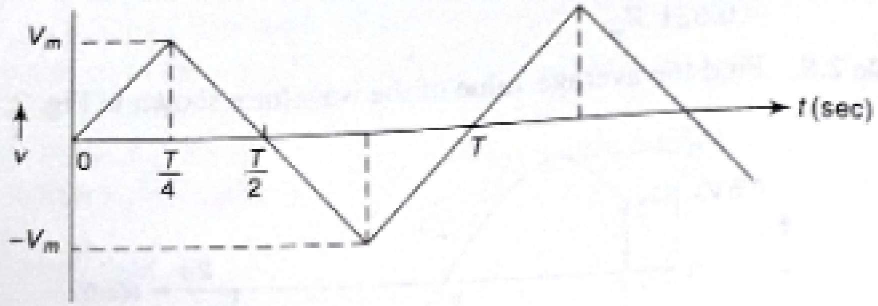


Fig. 2.15

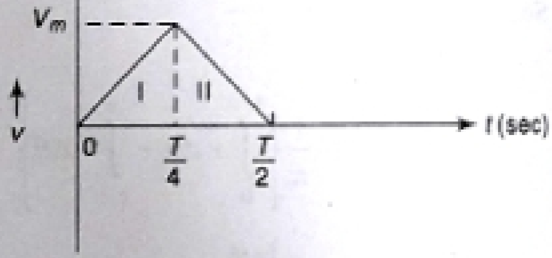
Solution

Time period of the given waveform is T sec. As positive half cycle is equal to negative half cycle, waveform is symmetrical. Therefore, for average value, we have to consider half cycle.

For interval $0 < t < \frac{T}{4}$, the equation of voltage is $v = \frac{4V_m}{T}t$.

As shown in the above figure, the area under the curve can be divided into two parts (Part I and Part II).

Area of Part I = Area of Part II



So, $$V_{\text{average}} = \frac{2\int_0^{T/4} v\,dt}{\frac{T}{2}}$$

$$= \frac{2}{T}\left[2\int_0^{T/4} \frac{4V_m}{T}t\,dt\right]$$

$$= \frac{2}{T}\times 2\times\frac{4V_m}{T}\left[\frac{t^2}{2}\right]_0^{T/4}$$

$$= \frac{2}{T}\times 2\times\frac{4V_m}{T}\left[\frac{(T)^2}{16}\cdot\frac{1}{2}-\frac{0}{2}\right]$$

$$= \frac{V_m}{2}$$

$$= 0.5\ V_m$$

3. RMS or effective value

The value of alternating current changes continuously with time, whereas the direct current remains constant with respect to time. The rms value is the criterion to measure the effectiveness of an alternating current (or voltage). The evident choice would be to measure it in terms of direct current that would do work (or produce heat) at the same average rate under similar conditions.

In Fig. 2.16(a), direct current of I A is passed through the resistance R for some time. In Fig. 2.16(b), an alternating current is passed through the same resistance for the same time. If the same amount of heat is produced in both cases, then the rms value of alternating current is said to be equal to direct current, i.e., I A. Thus, from the above example, the rms or effective value can be stated as follows:

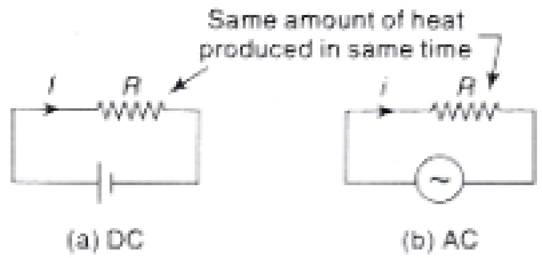


Fig. 2.16 Illustration of rms value

The effective or rms value of an alternating current is equal to that direct current which when flowing through a given resistance for a given time produces the same amount of heat as produced by the ac when flowing through the same resistance for the same time.

For symmetrical waveform, the rms or effective value can be found by considering half cycle or full cycle. However, for unsymmetrical waves, full cycle should be considered.

For symmetrical waveform (+ve half = –ve half),

$$\text{RMS value} = \sqrt{\frac{\text{Area of half/full cycle of squared wave}}{\text{Base length of half/full cycle}}}$$

For unsymmetrical waveform (+ve half ≠ –ve half),

$$\text{RMS value} = \sqrt{\frac{\text{Area of full cycle of squared wave}}{\text{Base length of full cycle}}}$$

RMS value of sinusoidal alternating current The equation of an alternating current varying sinusoidally is given by

$$i = I_m \sin\theta$$

Figure 2.17 shows the waveform of sinusoidal alternating current. The waveform of squared wave is shown by dotted line.

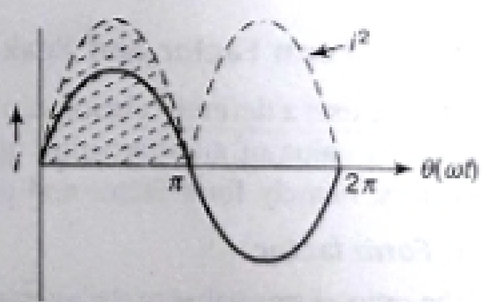


Fig. 2.17 Sinusoidal alternating current

As the given waveform is symmetrical, for rms value, we can consider half or full cycle.

So, $$\text{rms value} = \sqrt{\frac{\text{Area of half cycle of squared wave}}{\text{Base length of half cycle}}}$$

or $\quad I_{rms} = \sqrt{\dfrac{\int_0^{\pi} i^2\, d\theta}{\pi}}$

or $\quad I^2_{rms} = \dfrac{1}{\pi}\int_0^{\pi} I_m^2 \sin^2\theta\, d\theta$

$$= \frac{I_m^2}{\pi}\int_0^{\pi} \sin^2\theta\, d\theta$$

$$= \frac{I_m^2}{\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

$$= \frac{I_m^2}{2\pi}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{I_m^2}{2\pi}\left[\pi - \frac{\sin 2\pi}{2} - \left(0 - \frac{\sin(2\times 0)}{2}\right)\right]$$

$$= \frac{I_m^2}{2\pi}[\pi]$$

Hence, $\quad I_{rms} = \dfrac{I_m}{\sqrt{2}} = 0.707\, I_m$

Similarly, it can be proved that for sinusoidal alternating voltage,

$$V_{rms} = \frac{V_m}{\sqrt{2}} = 0.707\, V_m$$

2.1.7 Form Factor and Peak Factor

There exists a definite relationship among the peak value, the average value, and the rms value of an alternating quantity. The relationship is expressed by two factors, namely form factor and peak factor.

1. Form factor

The ratio of rms value to the average value of an alternating quantity is known as **form factor**.

Thus, $\quad$ Form factor $= \dfrac{\text{rms value}}{\text{Average value}}$

The value of form factor depends upon the waveform of the alternating quantity. The form factor for an alternating voltage or current varying sinusoidally is 1.11. i.e., for a sinusoidal voltage or current,

$$\text{Form factor} = \frac{\text{rms value}}{\text{Average value}} = \frac{0.707 \times \text{Maximum value}}{0.637 \times \text{Maximum value}} = 1.11$$

The form factor gives a measure of the 'peakiness' of the waveform. The peakier the wave, the greater is its form factor and vice versa. The form factor is useful in rectifier service.

2. *Peak factor*

The ratio of maximum value to the rms value of an alternating quantity is known as peak factor, i.e.,

$$\text{Peak factor} = \frac{\text{Maximum value}}{\text{rms value}}$$

The value of peak factor also depends upon the waveform of the alternating quantity. The peak factor for an alternating voltage or current varying sinusoidally is 1.414, i.e., for a sinusoidal voltage or current,

$$\text{Peak factor} = \frac{\text{Maximum value}}{\text{rms value}} = \frac{\text{Maximum value}}{0.707 \times \text{Maximum value}} = 1.414$$

Knowledge of this factor is important in dieletric insulation testing because the dielectric stress to which the insulation is subjected, is proportional to the maximum or peak value of the applied voltage.

Example 2.10 Find the rms value of the waveform shown in Fig. 2.18.

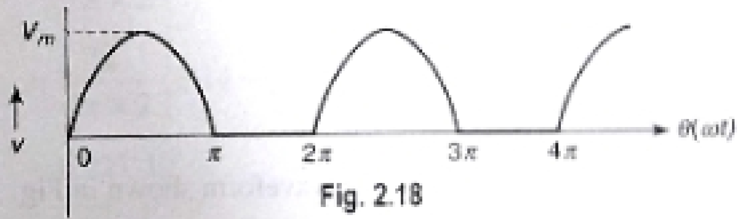
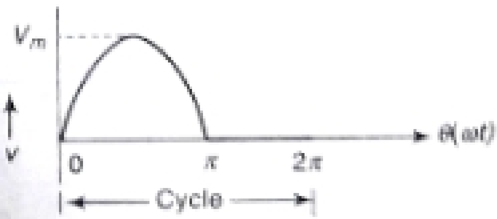


Fig. 2.18

Solution

As given waveform is unsymmetrical, for rms value, we have to consider full cycle. The equations of the voltage waveform are given by

$$v = V_m \sin\theta, \qquad 0 < \theta < \pi$$
$$v = 0, \qquad \pi < \theta < 2\pi$$



So, $$V_{rms} = \sqrt{\frac{\text{Area of full cycle of squared wave}}{\text{Base length of full cycle}}}$$

or $$V_{rms} = \sqrt{\frac{\int_0^{2\pi} v^2\, d\theta}{2\pi}}$$

or $$V_{\text{rms}}^2 = \frac{1}{2\pi}\int_0^{2\pi} v^2\, d\theta$$

$$= \frac{1}{2\pi}\left\{\int_0^{\pi} v^2\, d\theta + \int_{\pi}^{2\pi} v^2\, d\theta\right\}$$

$$= \frac{1}{2\pi}\left\{\int_0^{\pi} (V_m \sin\theta)^2\, d\theta + \int_{\pi}^{2\pi} (0)^2\, d\theta\right\}$$

$$= \frac{1}{2\pi}\int_0^{\pi} V_m^2 \sin^2\theta\, d\theta$$

$$= \frac{V_m^2}{2\pi}\int_0^{\pi}\left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{V_m^2}{2\pi\times 2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi}$$

$$= \frac{V_m^2}{2\pi\times 2}\left\{\pi - \frac{\sin 2\pi}{2} - \left[0 - \frac{\sin(2\times 0)}{2}\right]\right\}$$

$$= \frac{V_m^2}{2\pi\times 2}\{\pi\}$$

Hence, $V_{\text{rms}} = \dfrac{V_m}{2} = 0.5\, V_m$

Example 2.11 Find the rms value of the waveform shown in Fig. 2.19.

Solution

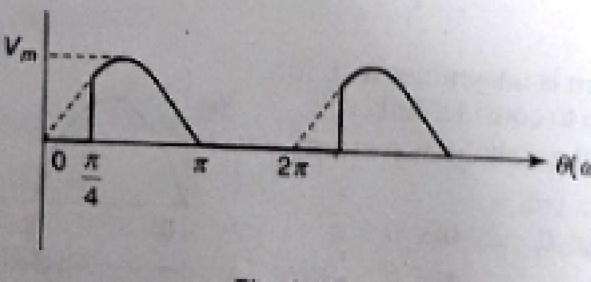
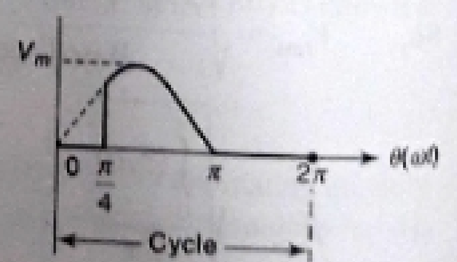


Fig. 2.19

As given waveform is unsymmetrical, for rms value we have to consider full cycle.

The equations of the voltage waveform are given as



$v = 0, \qquad 0 < \theta < \frac{\pi}{4}$

$v = V_m \sin\theta, \qquad \frac{\pi}{4} < \theta < \pi$

$v = 0, \qquad \pi < \theta < 2\pi$

So, $$V_{rms} = \sqrt{\frac{1}{2\pi}\int_0^{2\pi} v^2\,d\theta}$$

or $$V_{rms}^2 = \frac{1}{2\pi}\left\{\int_0^{\pi/4} v^2\,d\theta + \int_{\pi/4}^{\pi} v^2\,d\theta + \int_{\pi}^{2\pi} v^2\,d\theta\right\}$$

$$= \frac{1}{2\pi}\int_{\pi/4}^{\pi} (V_m \sin\theta)^2\,d\theta$$

$$= \frac{1}{2\pi}\int_{\pi/4}^{\pi} V_m^2 \sin^2\theta\,d\theta$$

$$= \frac{V_m^2}{2\pi}\int_{\pi/4}^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta$$

$$= \frac{V_m^2}{2\pi\times 2}\left\{\theta - \frac{\sin 2\theta}{2}\right\}_{\pi/4}^{\pi}$$

$$= \frac{V_m^2}{2\pi\times 2}\left\{\pi - \frac{\sin 2\pi}{2} - \left[\frac{\pi}{4} - \frac{\sin\left(2\times\frac{\pi}{4}\right)}{2}\right]\right\}$$

$$= \frac{V_m^2}{2\pi\times 2}\left\{3.14 - \frac{0}{2} - \left[0.785 - \frac{1}{2}\right]\right\}$$

$$= 0.227\ V_m^2$$

Hence, $V_{rms} = 0.476\ V_m$

Example 2.12 Find the rms value of the waveform shown in Fig. 2.20.

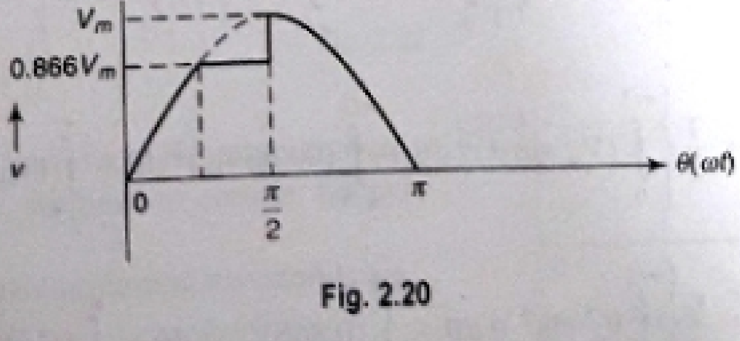


Fig. 2.20

Solution

The angle 'θ', at which the instantaneous value of the voltage becomes equal to 0.866 V_m, is required. This angle 'θ' can be calculated as given below.

The equation of the waveform is given by

$$v = V_m \sin\theta$$

When $v = 0.866\ V_m$, $\theta = ?$

we have $0.866\ V_m = V_m \sin\theta$

or $\theta = \sin^{-1} 0.866$

or $\theta = 60°$

Hence, $\theta = \dfrac{\pi}{3}$ rad

Now, the voltage waveform is shown in Fig. 2.21.

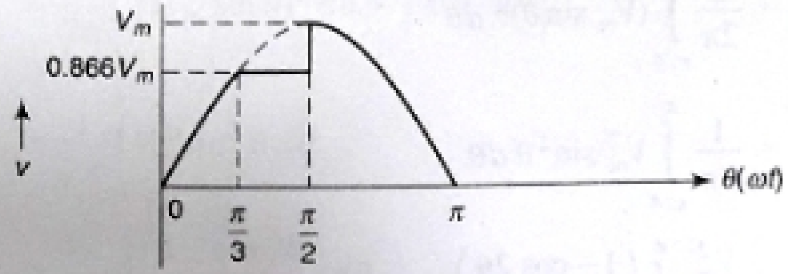


Fig. 2.21

The equations are as follows:

$$v = V_m \sin\theta, \qquad 0 < \theta < \frac{\pi}{3}$$

$$v = 0.866\ V_m, \qquad \frac{\pi}{3} < \theta < \frac{\pi}{2}$$

$$v = V_m \sin\theta, \qquad \frac{\pi}{2} < \theta < \pi$$

So, $$V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2\, d\theta}$$

or $$V_{rms}^2 = \frac{1}{\pi}\int_0^{\pi} v^2\, d\theta = \frac{1}{\pi}\left\{\int_0^{\pi/3} v^2\, d\theta + \int_{\pi/3}^{\pi/2} v^2\, d\theta + \int_{\pi/2}^{\pi} v^2\, d\theta\right\}$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi/3} (V_m \sin\theta)^2\, d\theta + \int_{\pi/3}^{\pi/2} (0.866V_m)^2\, d\theta + \int_{\pi/2}^{\pi} (V_m \sin\theta)^2\, d\theta\right\}$$

$$= \frac{1}{\pi}\left\{\int_0^{\pi/3} V_m^2 \sin^2\theta\, d\theta + \int_{\pi/3}^{\pi/2} (0.866V_m)^2\, d\theta + \int_{\pi/2}^{\pi} V_m^2 \sin^2\theta\, d\theta\right\}$$

$$= \frac{V_m^2}{\pi}\left\{\int_0^{\pi/3}\left(\frac{1-\cos 2\theta}{2}\right)d\theta + \int_{\pi/3}^{\pi/2} (0.866)^2\, d\theta + \int_{\pi/2}^{\pi}\left(\frac{1-\cos 2\theta}{2}\right)d\theta\right\}$$

$$= \frac{V_m^2}{\pi}\left\{\frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/3} + (0.866)^2[\theta]_{\pi/3}^{\pi/2} + \frac{1}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_{\pi/2}^{\pi}\right\}$$

$$= \frac{V_m^2}{\pi}\left\{\frac{1}{2}\left[\frac{\pi}{3} - \frac{\sin 2\left(\frac{\pi}{3}\right)}{2} - \left(0 - \frac{\sin 2\times 0}{2}\right)\right] + (0.866)^2\left[\frac{\pi}{2} - \frac{\pi}{3}\right]\right.$$

$$\left. + \frac{1}{2}\left[\pi - \frac{\sin 2\pi}{2} - \left(\frac{\pi}{2} - \frac{\sin 2\left(\frac{\pi}{2}\right)}{2}\right)\right]\right\}$$

$$= \frac{V_m^2}{\pi}\left\{\frac{1}{2}[1.047 - 0.433] + (0.866)^2[1.57 - 1.047]\right.$$

$$\left. + \frac{1}{2}[3.14 - 0 - (1.57 - 0)]\right\}$$

$$= \frac{V_m^2}{3.14}(0.307 + 0.392 + 0.785)$$

or $\quad V_{rms}^2 = 0.4726\ V_m^2$

Hence, $V_{rms} = 0.687\ V_m$

Example 2.13 Find the rms value of the waveform shown in Fig. 2.22.

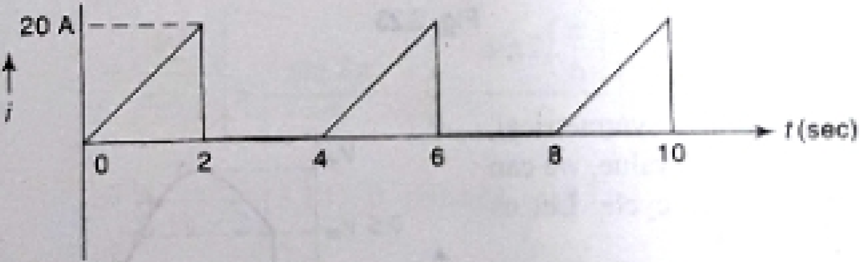
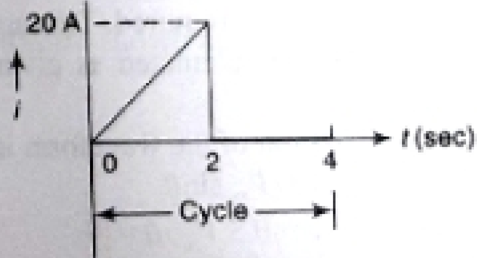


Fig. 2.22

Solution

As the given waveform is unsymmetrical, for rms value, we have to consider full cycle.



The equations of the current waveforms are:

$i = 10t \qquad 0 < t < 2$

$i = 0 \qquad 2 < t < 4$

So, $\quad I_{rms} = \sqrt{\frac{1}{4}\int_0^4 i^2\, dt}$

or $\quad I_{rms}^2 = \frac{1}{4}\int_0^4 i^2\, dt$

$$= \frac{1}{4}\left\{\int_0^2 i^2\, dt + \int_2^4 i^2\, dt\right\}$$

$$= \frac{1}{4}\int_0^2 (10t)^2\, dt$$

$$= \frac{(10)^2}{4}\left[\frac{t^3}{3}\right]_0^2$$

$$= \frac{(10)^2}{4}\left[\frac{(2)^3}{3} - \frac{(0)^3}{3}\right]$$

or $\qquad I_{rms}^2 = 66.66$

Hence, $I_{rms} = 8.16$ A

Example 2.14 Find the rms value of the waveform in Fig. 2.23.

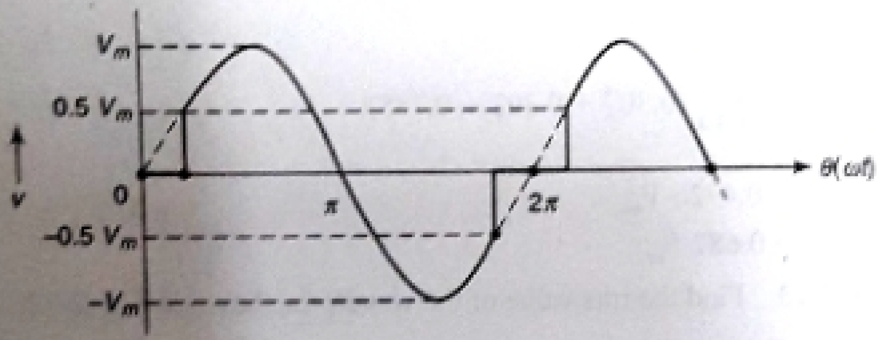
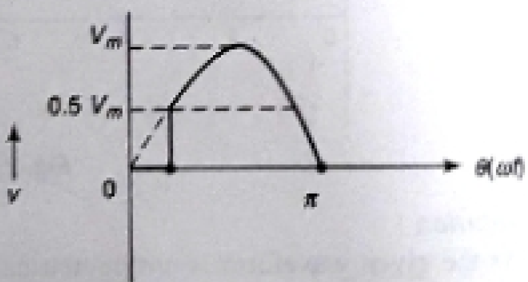


Fig. 2.23

Solution

As given waveform is symmetrical, for calculation of rms value, we can consider half or full cycle. Let us consider half cycle.

The angle 'θ' at which the instantaneous value of the voltage becomes equal to 0.5 V_m is required. This angle 'θ' can be calculated as given below.



The equation of the waveform is given as,

$$v = V_m \sin\theta$$

When $\qquad v = 0.5\ V_m,\ \theta = ?$

We have $\quad 0.5\ V_m = V_m \sin\theta$

or $\qquad \theta = \sin^{-1} 0.5$

or $\qquad \theta = 30°$

or $\qquad \theta = \dfrac{\pi}{6}$ rad

The equations are:

$$v = 0, \qquad 0 < \theta < \frac{\pi}{6}$$

$$v = V_m \sin\theta, \qquad \frac{\pi}{6} < \theta < \pi$$

We have $\quad V_{rms} = \sqrt{\frac{1}{\pi}\int_0^{\pi} v^2\, d\theta}$

or

$$V_{rms}^2 = \frac{1}{\pi}\left\{\int_0^{\pi/6} v^2\, d\theta + \int_{\pi/6}^{\pi} v^2\, d\theta\right\}$$

$$= \frac{1}{\pi}\int_{\pi/6}^{\pi} (V_m \sin\theta)^2\, d\theta$$

$$= \frac{V_m^2}{\pi}\int_{\pi/6}^{\pi} \sin^2\theta\, d\theta$$

$$= \frac{V_m^2}{\pi}\int_{\pi/6}^{\pi} \left(\frac{1-\cos 2\theta}{2}\right) d\theta$$

$$= \frac{V_m^2}{\pi \times 2}\left\{\theta - \frac{\sin 2\theta}{2}\right\}_{\pi/6}^{\pi}$$

$$= \frac{V_m^2}{\pi \times 2}\left\{\pi - \frac{\sin 2\pi}{2} - \left[\frac{\pi}{6} - \frac{\sin 2\left(\frac{\pi}{6}\right)}{2}\right]\right\}$$

$$= \frac{V_m^2}{3.14 \times 2}\{3.14 - 0 - (0.523 - 0.433)\}$$

$$= 0.4856\, V_m^2$$

Hence, $\quad V_{rms} = 0.697\, V_m$

6.00

From. B.L.T

(1) An alternating current varying sinusoidally with a freq. of 50 Hz has an RMS value of 20A. write down the eq^n for the instanteneous value and find this value (a) 0.0025 sec (b) 0.0125 sec after passing through a +ve maxi. value. At what time, measured from a +ve

Important Notes

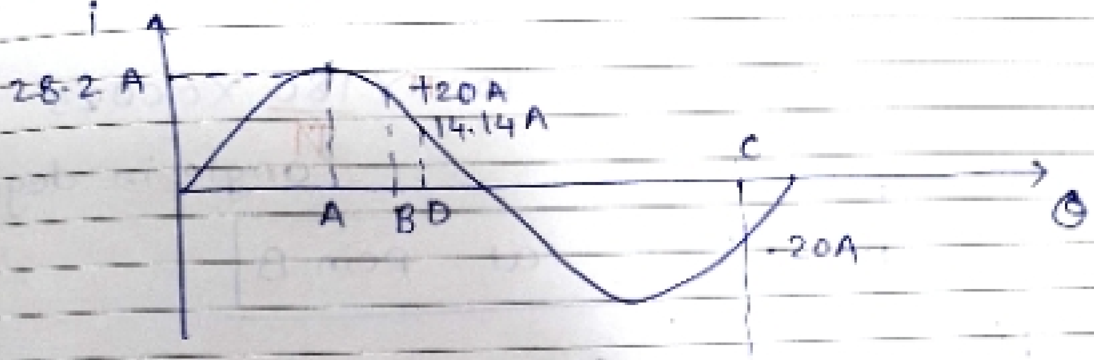
maxi value, will the instantaneous current be 14.14A ?

MAY '11

Mo	Tu	We	Th	Fr	Sa	Su
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

Sol^n - ~~$I = I_m$~~ $I = \frac{I_m}{\sqrt{2}}$

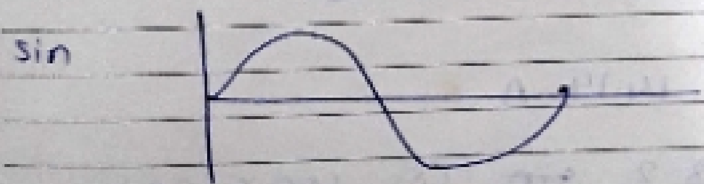
$I_m = 20\sqrt{2} = 28.2A$ $\omega = 2\pi \times 50 = 100\pi$ rad/sec



The eq^n of sinusoidal current wave with ref. to point O at zero time,

$$i = \boxed{28.2 \sin 100\pi t} \; A.$$

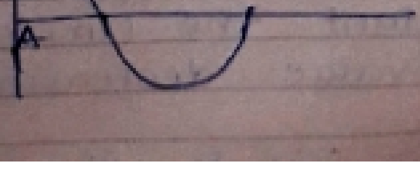
at A $T = 0.005$
total $T = 0.02$
half $T = 0.01$



Since time values are given from pt. A where voltage has +ve & maxi value, the eq^n may itself be referred to pt. A.

$$i = 28.2 \cos 100\pi t.$$

Cos wave



(i) when $t = 0.0025$ sec.

$i = 28.2 \cos 100\pi \times 0.0025$

angle in rad.

$= 28.2 \cos 100\cancel{\pi} \times \frac{180}{\cancel{\pi}} \times 0.0025$

angle in degree

$\boxed{i = 20\text{ A} \quad \text{at poin B}}$

(ii) when $t = 0.0125$ sec.

$i = 28.2 \cos 100\cancel{\pi} \times \frac{180}{\cancel{\pi}} \times 0.0125$

$\boxed{i = -20\text{ A} \quad \text{at pt C}}$

(iii) Here $i = 14.14$ A.

$14.14 = 28.2$ ~~sin~~ $\cos 100 \times 180 t$

8 Sunday

$\boxed{t = 1/300 \text{ sec}}$ 3.33 msec

(2) An alternating current of freq. 50 Hz has a maxi value of 100 A. Calculate (a) its value 1/600 sec. after the instant the current is zero and its value decreasing there after

(b) How many seconds after the

Important Notes

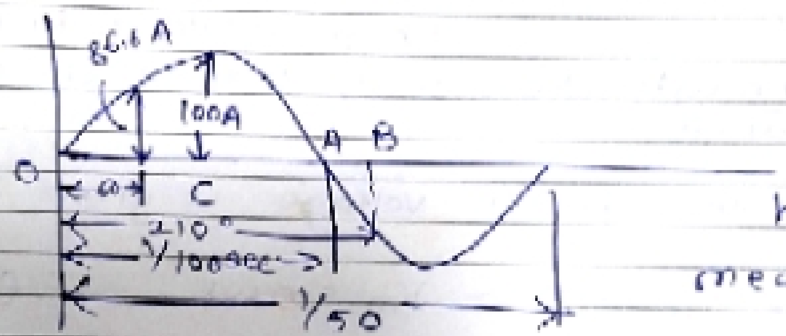
MAY '11

Mo	Tu	We	Th	Fr	Sa	Su
30	31					1
2	3	4	5	6	7	8
9	10	11	12	13	14	15
16	17	18	19	20	21	22
23	24	25	26	27	28	29

instant the current is zero (increasing thereafter wards) will the current attain the value of 86.6A?

→ $f = 50Hz$ $T_{total} = 1/50$

half time $1/100$



half time = $1/100$ means up to pt. A. half time

Eq^n at pt. O is,

$i = 100 \sin 2\pi \times 50t = 100 \sin 100\pi t$

(a) In this case, time is being measured from point A. (where current is zero and decreasing thereafter) & not from pt. O.

If above eq^n is to be utilized, then its time must be referred to point O. For this purpose, half time period i.e. $1/100$ sec has to be added to $1/600$ sec. The time as referred to pt. O is

$$= \frac{1}{100} + 1/600 = 7/600 \text{ sec.}$$

$$i = 100 \sin 100 \times 180 \times 7/600$$

$$\boxed{i = 100 \sin 210^\circ = -50A}$$

(b) In this case reference pt. is θ

$$86.6 = 100 \sin 100\cancel{\pi} \times \frac{180}{\cancel{\pi}} \times t$$

$$\boxed{t = 1/300 \text{ sec}}$$

$$\theta = 100 \times 180 \times 1/300 = 60^\circ$$

Example 2.17 The equation of an alternating current is $i = 62.35 \sin 323t$ A. Determine (i) maximum value, (ii) frequency, (iii) rms value, (iv) average value, and (v) form factor.

Solution

(i) $I_m = 62.35$ A

(ii) Frequency, $f = \dfrac{323}{2\pi} = 51.41$ Hz

(iii) rms value, $I_{rms} = \dfrac{I_m}{\sqrt{2}} = \dfrac{62.35}{\sqrt{2}} = 44.1$ A

(iv) Average value, $I_{average} = 0.637\, I_m = 39.7$ A

(v) Form factor$= \dfrac{I_{rms}}{I_{average}}$

$$= \frac{44.1}{39.7}$$
$$= 1.11$$

2.1.8 Phase Angle and Phasor

To understand the concept of phase angle and phasor, consider the following cases.

Case (i) The sinusoidal alternating current represented by the waveform shown in Fig. 2.25.

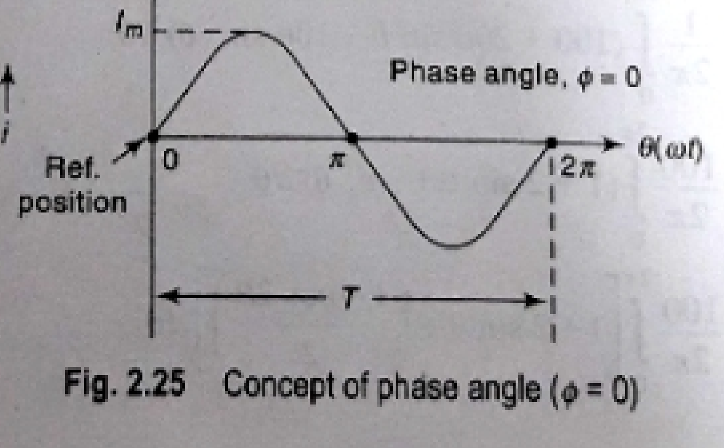


Fig. 2.25 Concept of phase angle ($\phi = 0$)

The X-axis is marked as 'ωt'. As 'ω' is constant, X-axis is the time axis. The instant from which time is measured (counted) is called reference position or zero (origin) position. In Fig. 2.25, the current attains its zero value exactly at reference position (or we can say that time is counted when the instant current is zero). Therefore, the phase angle of the current is said to be zero (i.e., $\phi = 0$). The given sinusoidal alternating current can be expressed by the equation

$$i = I_m \sin(\omega t \pm 0)$$

This equation is known as standard sinusoidal form. The alternating current can be represented by the phasor. Length of the phasor represents magnitude, i.e., rms value, and inclination w.r.t. reference axis such as X-axis is equal to the phase angle of that quantity. Thus, the given alternating current can be represented by phasor as shown in Fig. 2.26.

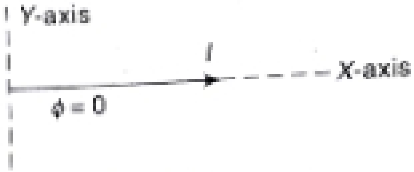


Fig. 2.26 Phasor

An alternating quantity is generally referred by its rms value. As a result, length of the phasor is drawn equal to the rms value (i.e., $I = \dfrac{I_m}{\sqrt{2}}$) instead of the maximum value. In practice, the rms values are denoted by capital letter such as 'I' or 'V', instead of I_{rms} or V_{rms}.

Case (ii) The sinusoidal alternating voltage represented by the waveform shown in Fig. 2.27.

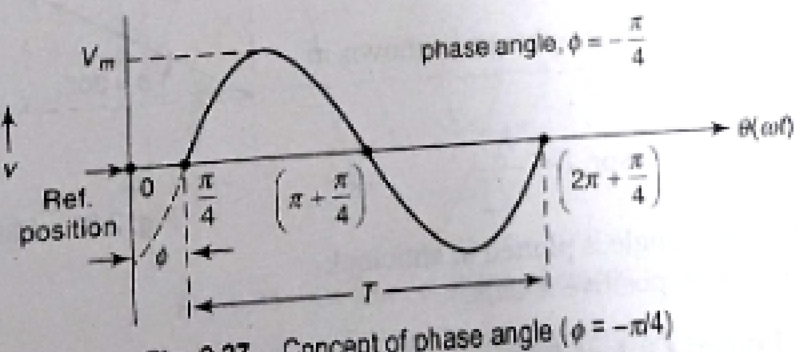


Fig. 2.27 Concept of phase angle ($\phi = -\pi/4$)

The voltage attains its zero value (first time) after a reference position by an angle $\pi/4$ rad or 45° (i.e., it lags behind reference). Therefore, phase angle of the voltage is $-\pi/4$ rad or $-45°$. Here the standard sinusoidal form of the given quantity is

$$v = V_m \sin\left(\omega t - \frac{\pi}{4}\right)$$

or $\quad v = V_m \sin(\omega t - 45°)$

The corresponding voltage phasor is shown in Fig. 2.28.

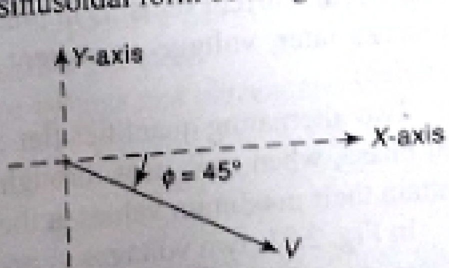


Fig. 2.28 Phasor

Length of phasor, $V = \frac{V_m}{\sqrt{2}}$

The negative phase angle is plotted in clockwise direction from positive X-axis.

Case (iii) The sinusoidal alternating current represented by the waveform shown in Fig. 2.29.

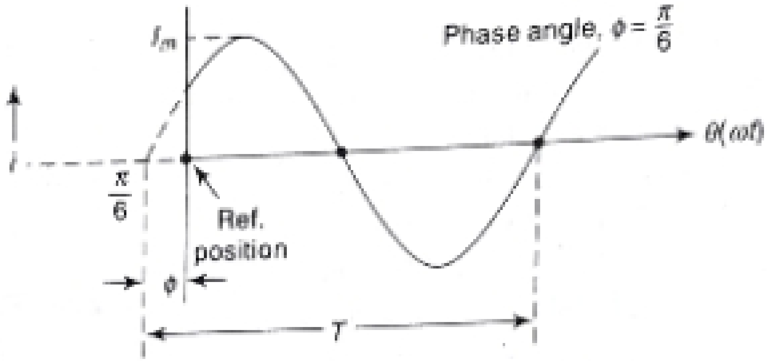


Fig. 2.29 Concept of phase angle ($\phi = \pi/6$)

The current attains its zero value (first time) before a reference position by an angle $\pi/6$ rad or 30° (i.e., it leads reference). Therefore, phase angle of the voltage is positive ($+\pi/6$ rad or 30°). The standard sinusoidal form of the given quantity is

$$i = I_m \sin\left(\omega t + \frac{\pi}{6}\right)$$

or $\quad i = I_m \sin(\omega t + 30)$

The corresponding current phasor is shown in Fig. 2.30.

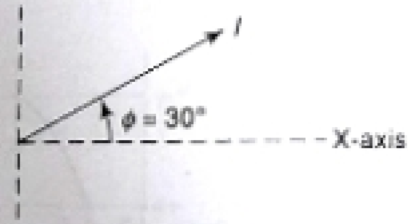


Fig. 2.30 Phasor

Length of phasor, $I = \frac{I_m}{\sqrt{2}}$

The positive phase angle is plotted in anticlockwise direction from positive X-axis.

2.1.9 In Phase, Out of Phase, and Phase Difference

When an alternating voltage is applied to the circuit, an alternating current of the same frequency flows through the circuit. In some circuits, the applied voltage and circuit current remains in phase. In most of the circuits, for reasons we will discuss later, voltage or current are out of phase (i.e., have different phase angles).

Two alternating quantities (let v_1 and v_2) of the same frequency are said to be in phase, when each pass through their zero value at the same instant and also attain their maximum values at the same instant in a given cycle (see Fig. 2.31).

In Fig. 2.31, two voltages v_1 and v_2 are in phase to each other. In other words, there is no phase difference between the two. The phase difference is zero ($\phi = 0$).

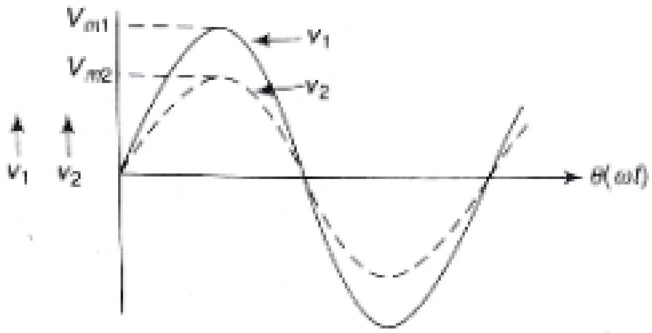


Fig. 2.31 Voltages in phase

The standard sinusoidal forms of the above two quantities are:

$$v_1 = V_{m1} \sin \omega t$$
$$v_2 = V_{m2} \sin \omega t$$

The above two voltages can be shown in the same phasor diagram (see Fig. 2.32).

Fig. 2.32 Phasor diagram

When two alternating quantities (let v and i) of the same frequency (same time period) reach their maximum or zero value at the different instants, then the alternating quantities are said to have a phase difference or out of phase (see Fig. 2.33).

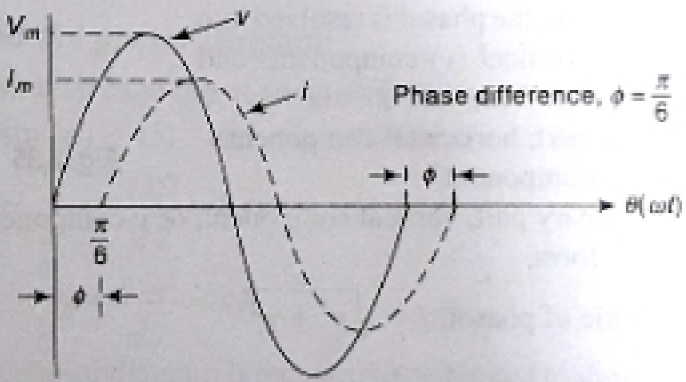


Fig. 2.33 Concept of phase difference

In Fig. 2.33, voltage passes through its zero value earlier than current by an angle ϕ (= $\pi/6$). Therefore, the voltage is said to be leading while the current is said to be lagging. It should be noted that those zero values of alternating quantities are to be considered where they pass in the same direction. Thus, if the voltage has passed through its zero value and is rising in the positive direction, then zero value considered for the current should have similar situation.

The equations (standard sinusoidal forms) of voltage and current are:

$$v = V_m \sin \omega t$$
$$i = I_m \sin \left(\omega t - \frac{\pi}{6}\right)$$

In the current equation, minus sign is used because the current lags behind the voltage.

Figure 2.34 shows the phasor diagram.

In this case, the current lags behind the voltage by an angle $\phi(= \pi/6$ rad or $30^\circ)$.

As phase angle of the voltage is zero, it acts as a reference quantity. Remember that the lagging and leading words are relative to the reference. In this case, if we take the current as reference, we have to say that the voltage leads current by angle ϕ.

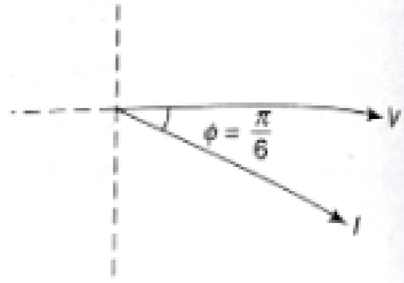


Fig. 2.34 Phasor diagram

2.1.10 Phasor Algebra

Mathematical representation of any phasor (current, voltage, or impedance) is known as **phasor algebra**.

The phasor can be represented mathematically in two ways: (a) rectangular form and (b) polar form.

(a) Rectangular form

It is a complex form in which operator 'j' is used. Operator 'j' is same as operator 'i' in complex form. Consider a current phasor (I) as shown in Fig. 2.35.

In rectangular form, the phasor is resolved into horizontal (x) and vertical (y) components and expressed in complex form, i.e., $\overline{I} = (x + jy)$ A

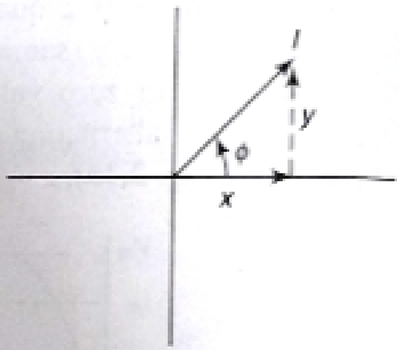


Fig. 2.35 Current phasor

where x = Real part, horizontal component, or x-component

y = Imaginary part, vertical component, or y-component

From rectangular form,

$$\text{Magnitude of phasor, } I = \sqrt{x^2 + y^2}$$

Its angle w.r.t. X-axis, $\phi = \tan^{-1}\frac{y}{x}$.

This form is used for addition and subtraction of alternating quantities.

(b) Polar form

In this form, current phasor is represented by $(I \angle \pm\phi)$ or voltage phasor is represented as $(V \angle \pm\phi)$.

Consider a voltage phasor as shown in Fig. 2.36.

The polar form of the voltage phasor, $\overline{V} = (V\angle\phi)$ V where V is the magnitude (rms value) and ϕ is the phase angle (i.e., angle made with +ve X-axis), which is taken as positive if measured anticlockwise direction while negative if measured clockwise direction w.r.t. positive X-axis. This form is used for multiplication and division of alternating quantities. This form is also used for drawing a phasor diagram of ac circuit.

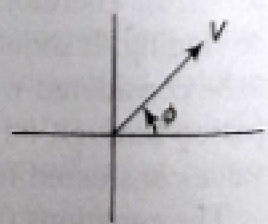


Fig. 2.36 Voltage phasor

Remember that the phase angle (ϕ), i.e., angle made by the phasor with +ve X-axis can be measured in two ways. For example, consider a current phasor of magnitude 10 A as shown in Fig. 2.37. The phase angle of current phasor is 190° if measured in clockwise direction, or 170° if measured in anticlockwise direction.

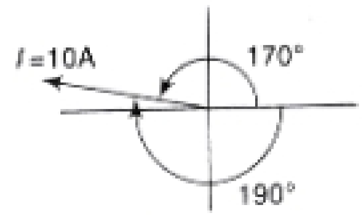


Fig. 2.37 Concept of +ve and –ve phase angles

The polar form of the current phasor is

$$\bar{I} = (10 \angle 170°)\ \text{A}$$

or

$$\bar{I} = (10 \angle -190°)\ \text{A}$$

2.1.11 Addition and Subtraction of Alternating Quantities

It is often required in ac analysis to add or subtract the two or more alternating quantities with same frequency but with different amplitudes and phases. The addition or subtraction using waveforms is much tedious and cumbersome. So, it is always preferred to add or subtract these quantities by using respective phasors.

Addtion and subtraction of alternating currents and voltages can be accomplished by one of the following methods:

(i) By phasor diagram (graphical method)
(ii) By phasor algebra

Graphical method (by phasor diagram)

Let us take an illustrative example of adding currents:

$$i_1 = 7.07 \sin(\omega t - 45) \quad \text{and} \quad i_2 = 4.24 \sin(\omega t + 30)$$

The rms values are: $$I_1 = \frac{7.07}{\sqrt{2}} = 5\ \text{A}$$

$$I_2 = \frac{4.24}{\sqrt{2}} = 3\ \text{A}$$

In this method, the phasor diagram is required to be plotted to the scale. Now, the polar forms of the two currents are:

$$\bar{I}_1 = (5 \angle -45)\ \text{A}$$

$$\bar{I}_2 = (3 \angle 30)\ \text{A}$$

Take scale 1 cm = 1 A. Draw I_1 = 5 cm at –45° and I_2 = 3 cm at 30°, with respect to positive X-axis (reference) as shown in Fig. 2.38. Complete the parallelogram. Then the diagonal (=6.4cm by measurement) represents the resultant current. Angle made with positive X-axis by the resultant is the phase angle, ϕ = –19° (by measurement).

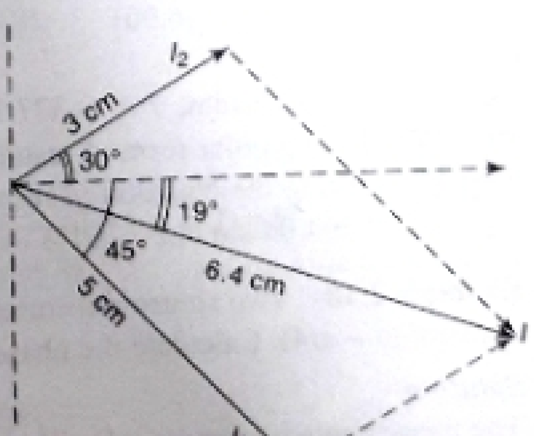


Fig. 2.38 Phasor diagram

From phasor diagram,

Resultant, I = 6.4 A

Phase angle, $\phi = -19°$

i.e., $\overline{I} = (6.4 \angle -19)$ A

So, $I_m = 6.4 \times \sqrt{2} = 9.05$ A

Hence, equation of the resultant current is given by

$$i = 9.05 \sin(\omega t - 19°)$$

By phasor algebra

Steps to be followed in phasor algebra method are as follows:

Step I Write all the given quantities in standard sinusoidal forms, i.e., for voltage, standard sinusoidal form is $v = V_m \sin(\omega t \pm \phi)$ and for current, it is $i = I_m \sin(\omega t \pm \phi)$.

The RHS of the equation must be positive and sin function only. Any non-standard sinusoidal form can be converted into standard sinusoidal form by using the following trigonometric formulae:

$$\cos \alpha = \sin(\alpha + 90°)$$
$$-\sin \alpha = \sin(\alpha + 180°)$$

Step II Convert the standard sinusoidal forms into polar forms.

Step III Convert the polar forms into rectangular forms. Add or subtract as required. The result will be in rectangular form.

Step IV Convert the rectangular form of result into polar form.

Step V Convert the result from polar form into standard sinusoidal form.

Let us take an illustrative example of adding the currents:

$$i_1 = -565.69 \sin(\omega t + 20) \quad \text{and} \quad i_2 = 10 \cos(\omega t + 10)$$

Step I Standard sinusoidal forms of the given quantities are:

$$i_1 = 565.69 \sin(\omega t + 200) \text{ A}$$
$$i_2 = 10 \sin(\omega t + 100) \text{ A}$$

Step II Converting the standard sinusoidal forms into polar forms,

$$\overline{I_1} = (400 \angle 200) \text{ A}$$
$$\overline{I_2} = (7.07 \angle 100) \text{ A}$$

Step III Converting the polar forms into rectangular forms,

$$\overline{I_1} = (-375.88 - j136.81)$$
$$\overline{I_2} = (-1.23 + j6.96)$$

Let $\overline{I} = \overline{I}_1 + \overline{I}_2$

Hence, resultant current, $\overline{I} = (-377.11 - j129.85)$ A

Step IV Taking polar form of resultant current, $\overline{I} = (398.84 \angle -161°)$ A

Step V Writing the equation of resultant current,

$$i = 564.04 \sin(\omega t - 161°)$$

Example 2.18 Two sinusoidal currents are given by $i_1 = 10\sin(\omega t + \pi/3)$ and $i_2 = 15\sin(\omega t - \pi/4)$. Calculate the phase difference between them in degrees.

Solution

The phase angle of current i_1 is $\pi/3$, i.e., 60°, while the phase angle of current $i_2 = -\pi/4$, i.e., −45°. This is shown in Fig. 2.39.

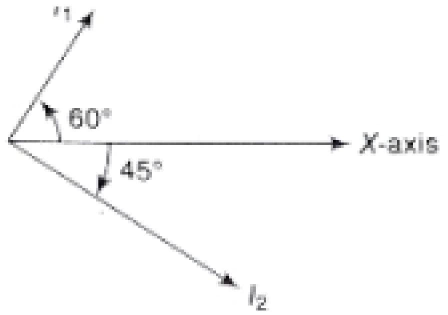


Fig. 2.39

Hence, the phase difference between the two currents is

$$\phi = \phi_1 - \phi_2 = 60 - (-45) = 105°$$

and I_2 lags behind I_1 by ϕ.

Example 2.19 The instantaneous values of two alternating voltages are represented respectively by $v_1 = 60 \sin\theta$ and $v_2 = 40 \sin(\theta + \pi/3)$. Derive the expression for the instantaneous values of (i) the sum and (ii) the difference of these voltages.

Draw neat phasor diagram with each and every value of the quantity marked.

Solution

The given voltages are, $v_1 = 60 \sin \theta$ and $v_2 = 40 \sin\left(\theta + \frac{\pi}{3}\right)$

The rms values of the two voltages are:

$$V_1 = \frac{60}{\sqrt{2}} = 42.43 \text{ V and } V_2 = \frac{40}{\sqrt{2}} = 28.28 \text{ V}$$

Writing the polar forms of the voltages,

$$\bar{V_1} = (42.43 \angle 0°) \text{ V and } \bar{V_2} = (28.28 \angle 60°) \text{ V}$$

Converting the polar forms into rectangular forms,

$$\bar{V_1} = (42.43 + j0) \text{ V and } \bar{V_2} = (14.14 + j24.49) \text{ V}$$

Case (i) Sum of the two voltages

Let the sum of the two voltages, i.e., resultant is V_A.

So, $\bar{V_A} = \bar{V_1} + \bar{V_2}$

or $\bar{V_A} = (42.43 + j0) + (14.14 + j24.49)$

or $\bar{V_A} = (56.57 + j24.49)$ volt

Converting into polar form,

$$\bar{V_A} = (61.64 \angle 23.41) \text{ volt}$$

Maximum value of the resultant voltage, $V_m = 61.64 \times \sqrt{2} = 87.17$ V

Expression for the instantaneous value of sum is

$$v_A = 87.17 \sin(\omega t + 23.41°)$$

Phasor diagram:

$$\bar{V_A} = \bar{V_1} + \bar{V_2}; \qquad \bar{V_1} = (42.43 \angle 0°) \text{ V}$$

$$\bar{V_2} = (28.28 \angle 60°) \text{ V}$$

Scale: 1 cm = 10 V

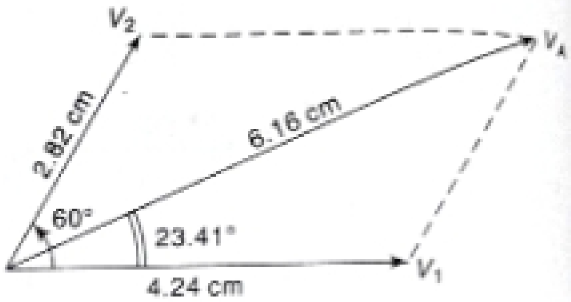


Fig. 2.40

Case (ii) Difference of the two voltages

Let the difference of the two voltages is V_B.

So, $\quad \bar{V}_B = \bar{V}_1 - \bar{V}_2$

or $\quad \bar{V}_B = (42.43 + j0) - (14.14 + j24.49)$

or $\quad \bar{V}_B = (28.29 - j24.49)$ V

Converting into polar form,

$\bar{V}_B = (37.42 \angle -40.88)$ V

Maximum value of the resultant voltage, $V_m = \sqrt{2} \times 37.42 = 52.92$ V.

Expression for the instantaneous value of difference is

$v_B = 52.92 \sin(\omega t - 40.88)$

Phasor diagram:

$\bar{V}_B = \bar{V}_1 - \bar{V}_2;\qquad \bar{V}_1 = (42.42 \angle 0)$

$\bar{V}_2 = (28.28 \angle 60)$

Scale: 1 cm = 10 V

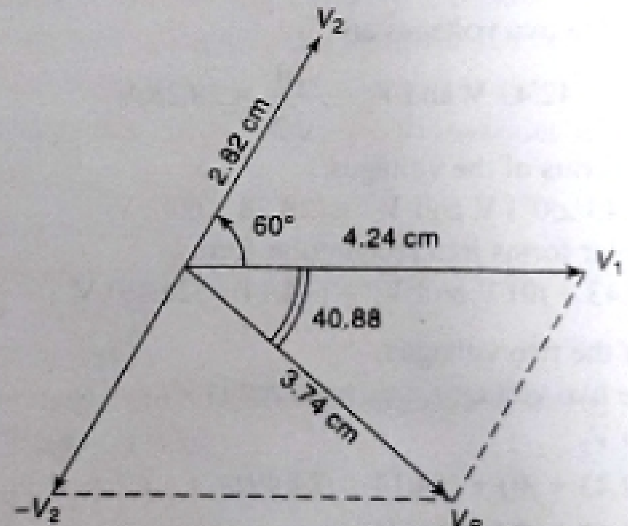


Fig. 2.41

Example 2.20 A voltage is defined as $-V_m \cos \omega t$. Express it in polar form.

Solution

To express a voltage in polar form, first we have to write the standard sinusoidal form as follows:

$v = -V_m \cos \omega t$

or $\quad v = -V_m \sin\left(\omega t + \frac{\pi}{2}\right) \qquad \because \cos\alpha = \sin\left(\alpha + \frac{\pi}{2}\right)$

or $$v = V_m \sin\left(\omega t + \frac{\pi}{2} + \pi\right) \qquad \because -\sin\alpha = \sin(\alpha + \pi)$$

or $$v = V_m \sin\left(\omega t + \frac{3}{2}\pi\right)$$

or $$v = V_m \sin(\omega t + 270)$$

Now, it can be expressed in polar form as

$$\bar{V} = \left(\frac{V_m}{\sqrt{2}} \angle 270^\circ\right) \text{ V}$$

or $$\bar{V} = \left(\frac{V_m}{\sqrt{2}} \angle -90^\circ\right) \text{ V} \qquad \text{(since +270° phase is nothing but −90°)}$$

Example 2.21 Find the resultant of the following voltages:

$$v_1 = 25 \sin \omega t$$

$$v_2 = 10 \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_3 = 30 \cos \omega t$$

$$v_4 = 20 \sin\left(\omega t - \frac{\pi}{4}\right)$$

Solution

Writing the standard sinusoidal forms of the given quantities,

$$v_1 = 25 \sin \omega t$$

$$v_2 = 10 \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_3 = 30 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$v_4 = 20 \sin\left(\omega t - \frac{\pi}{4}\right)$$

Converting the standard sinusoidal forms into polar forms,

$$\bar{V}_1 = (17.68 \angle 0) \text{ V}$$

$$\bar{V}_2 = (7.07 \angle 30) \text{ V}$$

$$\bar{V}_3 = (21.21 \angle 90) \text{ V}$$

$$\bar{V}_4 = (14.14 \angle -45) \text{ V}$$

Converting the polar forms into rectangular forms,

$$\bar{V}_1 = (17.68 + j0) \text{ V}$$

$$\bar{V}_2 = (6.12 + j3.54) \text{ V}$$

$$\bar{V}_3 = (0 + j21.21) \text{ V}$$

$$\bar{V}_4 = (10 - j10) \text{ V}$$

Let the resultant voltage is $\bar{V}$.

So, $\bar{V} = \bar{V}_1 + \bar{V}_2 + \bar{V}_3 + \bar{V}_4$

or $\bar{V} = (17.68 + j0) + (6.12 + j3.54) + (0 + j21.21) + (10 - j10)$

or $\bar{V} = (33.8 + j14.75)$ V

Converting into polar form,

$$\bar{V} = (36.88 \angle 23.58) \text{ V}$$

Converting into standard sinusoidal form,

$$v = 52.16 \sin (\omega t + 23.58)$$

Example 2.22 The voltage drops across the four series-connected impedances are:

$$v_1 = 60 \sin \left(\omega t + \frac{\pi}{6} \right)$$

$$v_2 = 75 \sin \left(\omega t - \frac{5}{6}\pi \right)$$

$$v_3 = 100 \cos \left(\omega t + \frac{\pi}{4} \right)$$

$$v_4 = V_{m4} \sin (\omega t + \phi_4)$$

Calculate the values of V_{m4} and ϕ_4 if the applied voltage across the series circuit is $140 \sin \left(\omega t + \frac{3\pi}{5} \right)$.

Solution

Let the four series-connected impedances are Z_1, Z_2, Z_3 and Z_4. The applied voltage is $v = 140 \sin \left(\omega t + \frac{3\pi}{5} \right)$. The voltage drops across the impedances are v_1, v_2, v_3, and v_4 (see Fig. 2.42).

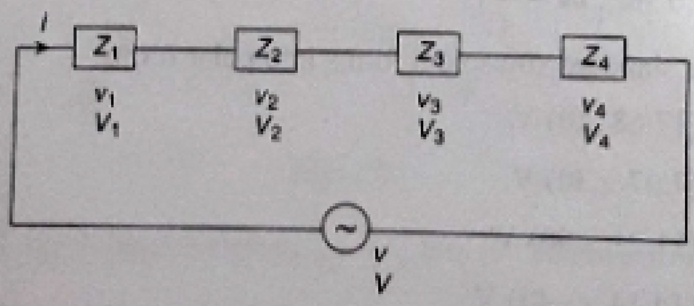


Fig. 2.42

The symbols v_1, v_2, v_3, v_4 represent the instantaneous values of the voltage drops and V_1, V_2, V_3, V_4 represent the rms values of the voltage drops.

Writing the standard sinusoidal forms of the given quantities, we get

$$v_1 = 60 \sin \left(\omega t + \frac{\pi}{6} \right)$$

$$v_2 = 75 \sin\left(\omega t - \frac{5}{6}\pi\right)$$

$$v_3 = 100 \sin\left(\omega t + \frac{3}{4}\pi\right)$$

applied voltage, $v = 140 \sin\left(\omega t + \frac{3\pi}{5}\right)$

Converting the standard sinusoidal forms into polar forms,

$$\bar{V_1} = (42.43 \angle 30) \text{ V}$$

$$\bar{V_2} = (53.03 \angle -150) \text{ V}$$

$$\bar{V_3} = (70.71 \angle 135) \text{ V}$$

$$\bar{V} = (99 \angle 108) \text{ V}$$

Converting the polar forms into rectangular forms,

$$\bar{V_1} = (36.75 + j21.22) \text{ V}$$

$$\bar{V_2} = (-45.93 - j26.52) \text{ V}$$

$$\bar{V_3} = (-50 + j50) \text{ V}$$

$$\bar{V} = (-30.59 + j94.15) \text{ V}$$

From the circuit diagram shown in Fig. 2.42,

$$\bar{V} = \bar{V_1} + \bar{V_2} + \bar{V_3} + \bar{V_4}$$

or $\quad \bar{V_4} = \bar{V} - (\bar{V_1} + \bar{V_2} + \bar{V_3})$

or $\quad \bar{V_4} = (-30.59 + j94.15) - [(36.75 + j21.22) + (-45.93 - j26.52) + (-50 + j50)]$

or $\quad \bar{V_4} = (-30.59 + j94.15) - (-59.18 + j44.7)$

or $\quad \bar{V_4} = (28.59 + j49.45) \text{ V}$

Converting into polar form,

$$\bar{V_4} = (57.12 \angle 59.97) \text{ V}$$

Converting into standard sinusoidal form,

$$v_4 = 80.78 \sin(\omega t + 59.97)$$

Hence, $V_{m4} = 80.78$ V and $\phi_4 = 59.97°$

Example 2.23 Four wires *p*, *q*, *r*, and *s* are connected to a common point. The currents in lines *p*, *q*, and *r* are $6 \sin\left(\omega t + \frac{\pi}{3}\right)$, $5 \cos\left(\omega t + \frac{\pi}{3}\right)$, and $3 \cos\left(\omega t + \frac{\pi}{3}\right)$.

Find the current in wire *s*.

Solution

Let the instantaneous values of the currents in lines *p*, *q*, *r*, and *s* are i_1, i_2, i_3, and i_4 respectively (see Fig. 2.43).

According to Kirchhoff's current law, sum of these currents must be zero. This means that current in the fourth wire (i.e., wire s) is the phasor sum of all the first three currents, i.e.,

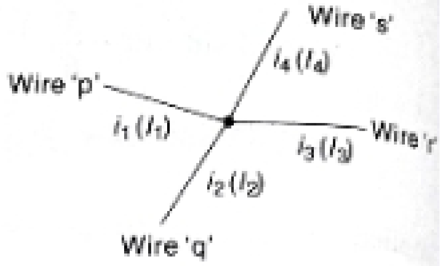


Fig. 2.43

$$\overline{I}_1 + \overline{I}_2 + \overline{I}_3 + \overline{I}_4 = 0$$

So, $$\overline{I}_4 = -(\overline{I}_1 + \overline{I}_2 + \overline{I}_3)$$

Thus,

$$i_1 = 6 \sin\left(\omega t + \frac{\pi}{3}\right)$$

$$i_2 = 5 \cos\left(\omega t + \frac{\pi}{3}\right)$$

$$i_3 = 3 \cos\left(\omega t + \frac{\pi}{3}\right)$$

The standard sinusoidal forms of the given currents are:

$$i_1 = 6 \sin\left(\omega t + \frac{\pi}{3}\right)$$

$$i_2 = 5 \sin\left(\omega t + \frac{5\pi}{6}\right)$$

$$i_3 = 3 \sin\left(\omega t + \frac{5\pi}{6}\right)$$

Converting the standard sinusoidal forms into polar forms,

$$\overline{I}_1 = (4.24 \angle 60)\text{ A}$$

$$\overline{I}_2 = (3.54 \angle 150)\text{ A}$$

$$\overline{I}_3 = (2.12 \angle 150)\text{ A}$$

Converting the polar forms into rectangular forms,

$$\overline{I}_1 = (2.12 + j3.67)\text{ A}$$

$$\overline{I}_2 = (-3.07 + j1.77)\text{ A}$$

$$\overline{I}_3 = (-1.84 + j1.06)\text{ A}$$

Current in wire s can be calculated as

$$\overline{I}_4 = -(\overline{I}_1 + \overline{I}_2 + \overline{I}_3)$$

or $$\overline{I}_4 = -[(2.12 + j3.67) + (-3.07 + j1.77) + (-1.84 + j1.06)]$$

or $$\overline{I}_4 = -(-2.79 + j6.5)\text{ A}$$

or $$\overline{I}_4 = (2.79 - j6.5)\text{ A}$$

Converting into polar form,

$$\overline{I}_4 = (7.07 \angle -66.77)\text{ A}$$

Converting into the standard sinusoidal form,

$$i_4 = 10 \sin(\omega t - 66.77)$$

Example 2.24 Three coils are connected in series. Each of them generates an emf of 230 V. The emf of the second coil leads that of the first coil by 120°, and the emf of the third coil lags behind that of the first by the same angle. What is the resultant emf across the series combination of the coils?

Solution

Let the emf generated in the first coil is E_1.

By taking this emf as reference, we can express in polar form as

$$\bar{E}_1 = (230 \angle 0)\ \text{V}$$

Let the emf generated in the second coil is E_2. As E_2 leads E_1 by 120°, we get the polar form as

$$\bar{E}_2 = (230 \angle 120)\ \text{V}$$

Let the emf generated in the third coil is E_3. As E_3 lags behind E_1 by 120°, we get the polar form as

$$\bar{E}_3 = (230 \angle -120°)\ \text{V}$$

Now, the resultant emf E_R across the series combination of the coils is

$$\begin{aligned}\bar{E}_R &= \bar{E}_1 + \bar{E}_2 + \bar{E}_3 \\ &= (230 \angle 0) + (230 \angle 120) + (230 \angle -120) \\ &= (230 + j0) + (-115 + j200) + (-115 - j200)\end{aligned}$$

So, $\bar{E}_R = 0\ \text{V}$

Example 2.25 The instantaneous voltages across each of four series-connected coils are given by

$$v_1 = 100 \sin \omega t$$

$$v_2 = 250 \cos \omega t$$

$$v_3 = 150 \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_4 = 200 \sin\left(\omega t - \frac{\pi}{4}\right)$$

Find the total potential difference (pd) and express the answer in similar form. What will be the resultant pd if the polarity of v_2 is reversed.

Solution

Writing the standard sinusoidal forms of the given instantaneous voltages,

$$v_1 = 100 \sin \omega t$$

$$v_2 = 250 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$v_3 = 150 \sin\left(\omega t + \frac{\pi}{6}\right)$$

$$v_4 = 200 \sin\left(\omega t - \frac{\pi}{4}\right)$$

Converting the standard sinusoidal forms into polar forms,

$$\bar{V}_1 = (70.71 \angle 0) \text{ V}$$

$$\bar{V}_2 = (176.78 \angle 90) \text{ V}$$

$$\bar{V}_3 = (106.07 \angle 30) \text{ V}$$

$$\bar{V}_4 = (141.42 \angle -45) \text{ V}$$

Let the total potential difference (pd) is V volt.

So, $\bar{V} = \bar{V}_1 + \bar{V}_2 + \bar{V}_3 + \bar{V}_4$

or $\bar{V} = (70.71 \angle 0) + (176.78 \angle 90) + (106.07 \angle 30) + (141.42 \angle -45)$

or $\bar{V} = (70.71 + j0) + (0 + j176.78) + (91.86 + j53.04) + (100 - j100)$

or $\bar{V} = (262.57 + j129.82)$ V

or $\bar{V} = (292.91 \angle 26.31)$ V

Converting into standard sinusoidal form,

$$v = 414.24 \sin(\omega t + 26.31)$$

It polarity of v_2 is reversed, the total pd (V) can be calculated as

$$\bar{V} = \bar{V}_1 - \bar{V}_2 + \bar{V}_3 + \bar{V}_4$$

or $\bar{V} = (70.71 \angle 0) - (176.78 \angle 90) + (106.07 \angle 30) + (141.42 \angle -45)$

or $\bar{V} = (70.71 + j0) - (0 + j176.78) + (91.86 + j53.04) + (100 - j100)$

or $\bar{V} = (262.57 - j223.74)$ V

or $\bar{V} = (344.97 \angle -40.43)$ V

Converting into standard sinusoidal form,

$$v = 487.86 \sin(\omega t - 40.43)$$