1.16 Superposition Theorem

This theorem is applicable for linear[2] and bilateral[3] networks. According to this theorem, if there are number of sources acting simultaneously in any linear bilateral networks, then each source acts independently of the others, i.e., as if other source did not exit. Hence, this theorem may be stated as in a linear network containing more than one sources, the resulted current in any branch is the algebraic sum of the currents that would be produced by each source, acting alone, all other sources of emf being replaced by their respective internal resistances.

Illustration

Consider a network shown in Fig. 1.218, having two voltage sources V_1 and V_2. Let us calculate the current in branch AB of the network by using superposition theorem.

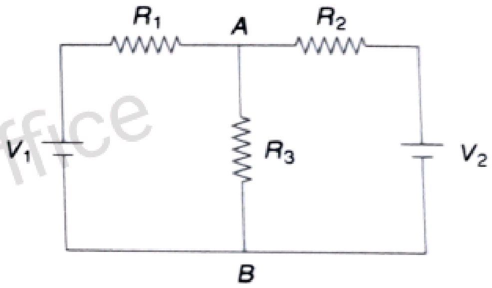

Fig. 1.218 DC circuit

Case (i) According to superposition theorem, each source acts independently. Consider source V_1 acting independently. At this time, other sources must be replaced by internal resistances. But as internal resistance of V_2 is not given, i.e., it is assumed to be zero, V_2 must be replaced by short circuit. Thus, the circuit becomes as shown in Fig. 1.219(a). Using any of the techniques, obtain the current through branch AB, i.e., I_{AB} due to source V_1 alone.

Case (ii) Now, consider source V_2 alone, with V_1 replaced by a short circuit, to obtain the current through branch AB. The corresponding circuit is shown in Fig. 1.219(b). Obtain I_{AB} due to V_2 alone by using any of the techniques such as mesh analysis, nodal analysis, and source transformation.

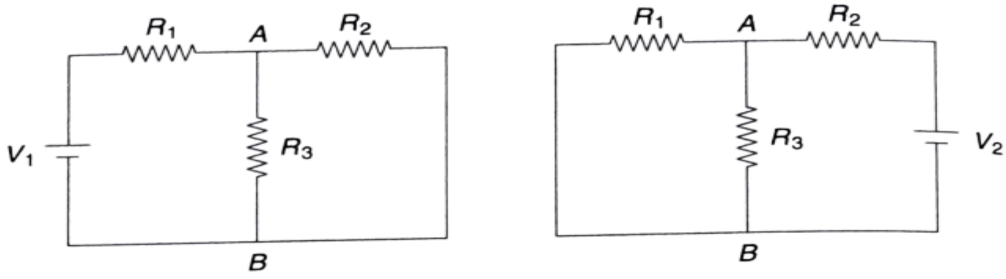

Fig. 1.219 Illustration of superposition theorem

Case (iii) According to superposition theorem, the resultant current through branch AB is the sum of the currents through branch AB produced by each source acting independently.

Hence, $I_{AB} = I_{AB}$ due to $V_1 + I_{AB}$ due to V_2

Example 1.64 In the circuit shown in Fig. 1.220, find the different branch currents by using superposition theorem.

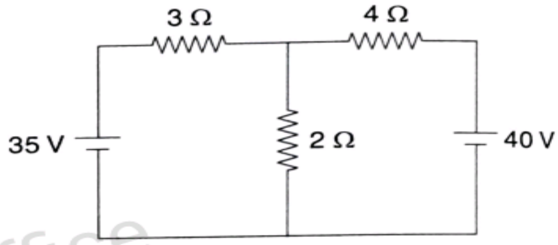

Fig. 1.220

Solution

Step I: Considering 35 V source acting alone, replacing 40 V source by short circuit, we get the circuit as shown in Fig. 1.221.

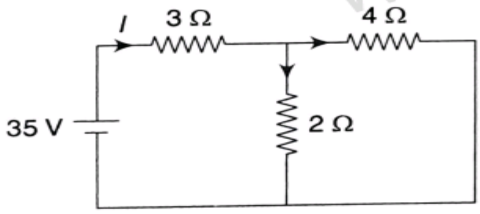

Fig. 1.221

In circuit of Fig. 1.221, as single source is acting, the actual directions of currents are marked. These branch currents can be calculated as follows:

The equivalent resistance across the source,

$$R_{eq} = 3 + (2 \parallel 4) = 3 + \frac{2 \times 4}{2 + 4} = 4.33\ \Omega$$

By Ohm's law, total circuit current, $I = \dfrac{V}{R_{eq}} = \dfrac{35}{4.33} = 8.08$ A

Hence, current in 3 Ω resistor, $I_{3\Omega} = 8.08$ A $(\rightarrow)$

By current division rule, current in 4 Ω resistor, $I_{4\Omega} = 8.08 \times \dfrac{2}{2+4} = 2.69$ A$(\rightarrow)$

Current in 2 Ω resistor, $I_{2\Omega} = 8.08 - 2.69 = 5.39$ A$(\downarrow)$

Step II: Considering 40 V source acting alone, replacing 35 V source by short circuit, we get the circuit as shown in Fig. 1.222.

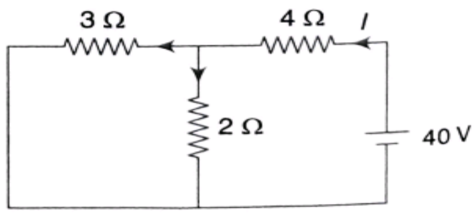

Fig. 1.222

In circuit of Fig. 1.222, as single source is acting, the actual directions of currents are marked. These branch currents can be calculated as follows:

The equivalent resistance across the source, $R_{eq} = 4 + (3 \parallel 2) = 4 + \frac{3 \times 2}{3 + 2} = 5.2$

Ω

By Ohm's law, total circuit current, $I = \frac{V}{R_{eq}} = \frac{40}{5.2} = 7.69$ A

$\therefore$ Current in 4 Ω resistor, $I_{4\Omega} = 7.69$ A($\leftarrow$)

By current division rule, current in 3 Ω resistor, $I_{3\Omega} = 7.69 \times \frac{2}{3+2} = 3.08$ A($\leftarrow$)

Current in 2 Ω resistor, $I_{2\Omega} = 7.69 - 3.08 = 4.61$ A($\downarrow$)

Step III: By principle of superposition, the resultant branch currents can be calculated as follows:

Current in 3 Ω resistor, $I_{3\Omega} = 8.08$ A($\rightarrow$) + 3.08 A($\leftarrow$) = 5 A($\rightarrow$)

Current in 2 Ω resistor, $I_{2\Omega} = 5.39$ A($\downarrow$) + 4.61 A ($\downarrow$) = 10 A ($\downarrow$)

Current in 4 Ω resistor, $I_{4\Omega} = 2.69$ A($\rightarrow$) + 7.69 A($\leftarrow$) = 5 A($\leftarrow$)

Example 1.65 Determine the current in 1 Ω resistor between *A* and *B* from the network shown in Fig. 1.223 by superposition theorem.

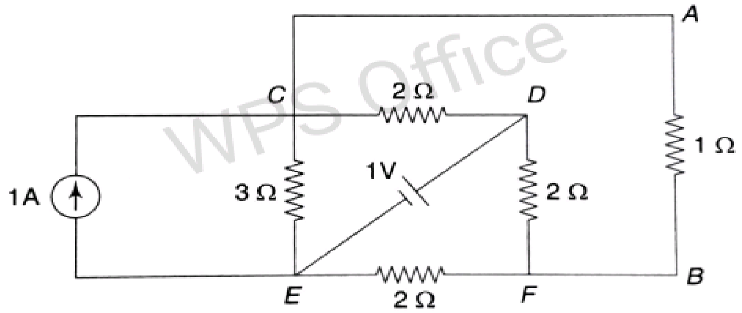

Fig. 1.223

Solution

Step I: Considering 1 A source acting alone, replacing 1 V source by short circuit, we get Fig. 1.224. Nodes *D* and *E* are same, and by joining them, we get the circuit

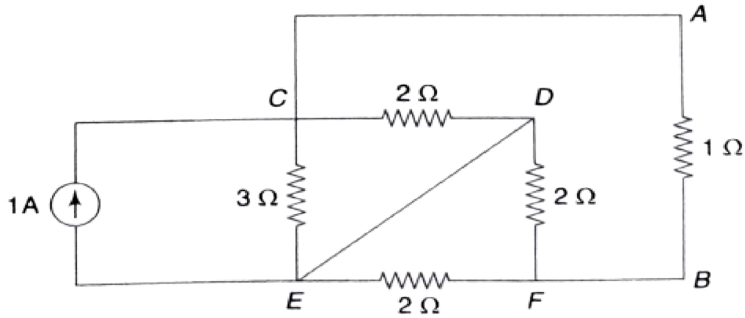

Fig. 1.224

as shown in Fig. 1.225(a). Replacing the parallel combinations, we get the circuit as shown in Fig. 1.225(b), where branch CE is parallel to branch $CABE$. The total current in the circuit is 1 A, which divides at node C.

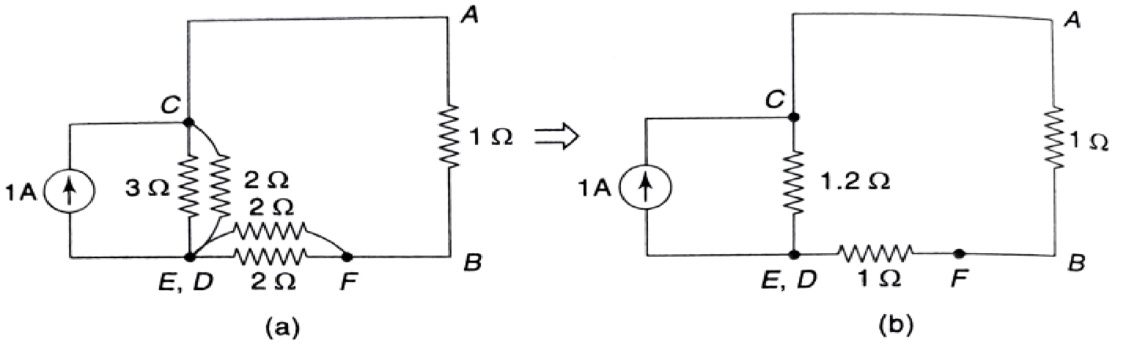

Fig. 1.225

Hence, by applying current division rule, current in 1 Ω resistor between A and B is given as

$$I_{AB} = I_{1\Omega} = 1 \times \frac{1.2}{1.2 + (1+1)} = 0.375\ \text{A}(\downarrow)$$

Step II: Considering 1 V source acting alone, replacing 1 A source by open circuit, we obtain the following circuit:
Assuming the separate mesh current for each mesh, we get the circuit as shown in Fig. 1.227.

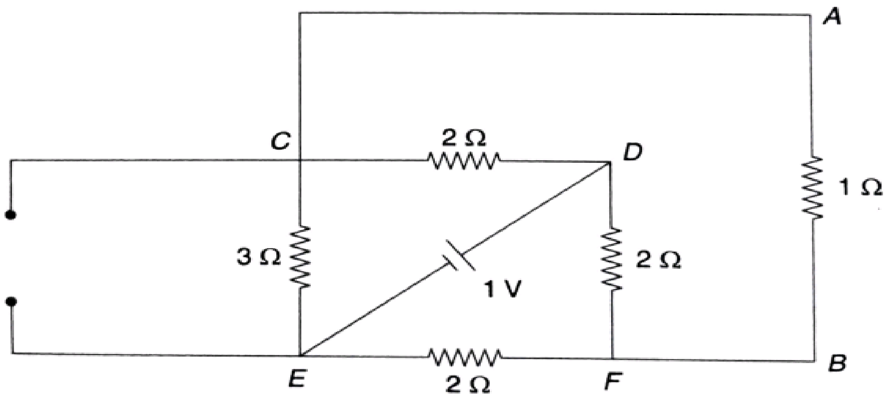

Fig. 1.226

Applying KVL to mesh 1,

$$-3I_1 - 2(I_1 - I_3) - 1 = 0$$

or $\quad 5I_1 - 2I_3 = -1 \qquad$ (i)

Applying KVL to mesh 2,

$$1 - 2(I_2 - I_3) - 2I_3 = 0$$

or $\quad 4I_2 - 2I_3 = 1 \qquad$ (ii)

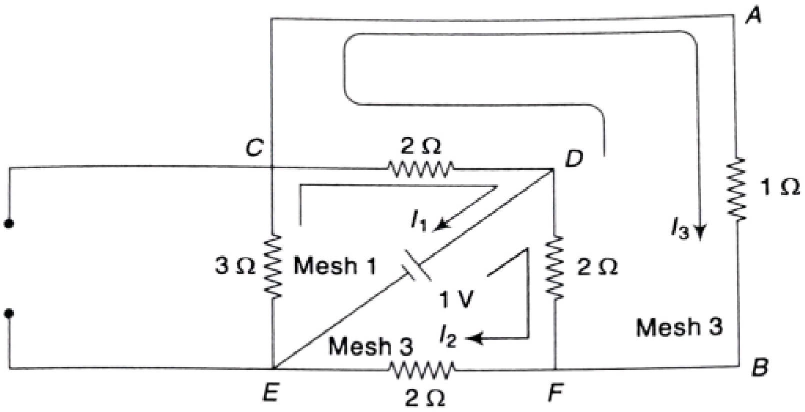

Fig. 1.227

Applying KVL to mesh 3,

$$-2(I_3 - I_1) - I_3 - 2(I_3 - I_2) = 0$$

or $\quad 2I_1 + 2I_2 - 5I_3 = 0 \qquad$ (iii)

The values of I_3 may be found by solving the above three simultaneous equations or by the method of determinants as given below. Putting the above three equations in the matrix form, we have

$$\begin{bmatrix} 5 & 0 & -2 \\ 0 & 4 & -2 \\ 2 & 2 & -5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 5 & 0 & -2 \\ 0 & 4 & -2 \\ 2 & 2 & -5 \end{vmatrix} = -64, \qquad \Delta_3 = \begin{vmatrix} 5 & 0 & -1 \\ 0 & 4 & 1 \\ 2 & 2 & 0 \end{vmatrix} = -2$$

By Cramer's rule,

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-2}{-64} = 0.031 \text{ A}$$

Hence, $I_{AB} = I_{1\Omega} = 0.031 \text{ A}(\downarrow)$

Step III: According to superposition theorem, resultant current through 1 Ω resistor, i.e., through branch AB, is algebraic sum of currents due to individual sources acting alone.

Hence, $I_{AB} = I_{1\Omega} = 0.375 \text{ A}(\downarrow) + 0.031 \text{ A}(\downarrow) = 0.406 \text{ A}(\downarrow)$

Example 1.66 Determine the current in 20 Ω resistor in the network shown in Fig. 1.228 by superposition theorem.

Solution

Step I: Considering 10 V source acting alone, replacing 8 V source and 12 V source by short circuit, we obtain Fig. 1.229.

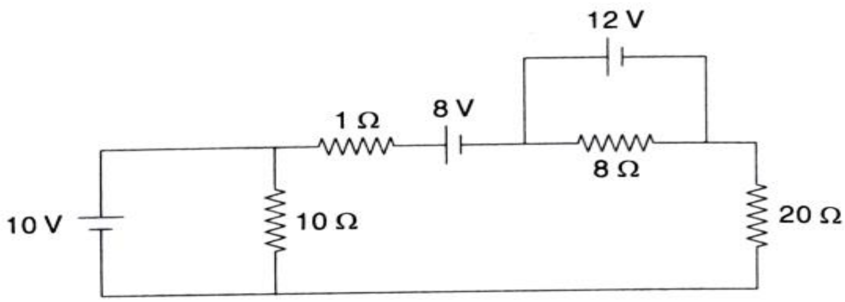

Fig. 1.228

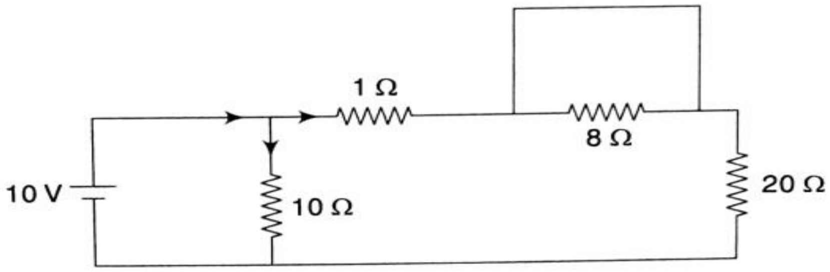

Fig. 1.229

In the circuit of Fig. 1.229, the 8 Ω resistor gets short circuited, i.e., $I_{8\Omega} = 0$ A. For circuit simplification, the 8 Ω resistor can be removed. By removing 8 Ω resistor, we get the following circuit:

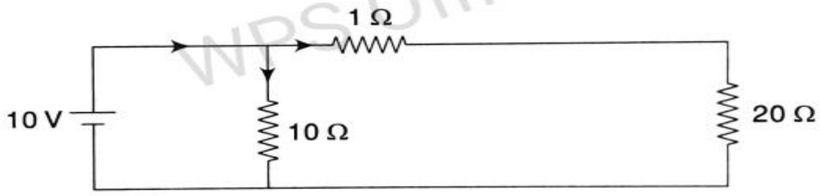

Fig. 1.230

By Ohm's law, $I_{20\Omega} = \dfrac{10}{1+20} = 0.476 \text{ A}(\downarrow)$

Step II: Considering 12 V source acting alone, replacing 8 V source and 10 V source by short circuit, we get Fig. 1.231.

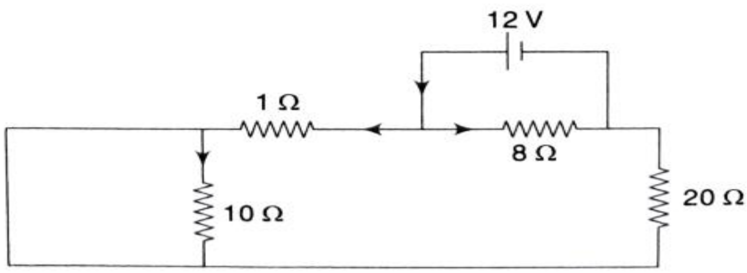

Fig. 1.231

In the circuit of Fig. 1.231, the 10 Ω resistor gets short circuited, i.e., $I_{10\Omega} = 0$ A. For circuit simplification, the resistor 10 Ω can be removed. By removing 10 Ω resistor, we get the circuit as shown in Fig. 1.232(a). Now, replacing series combination (1 Ω + 20 Ω), we get the circuit as shown in Fig. 1.232(b).

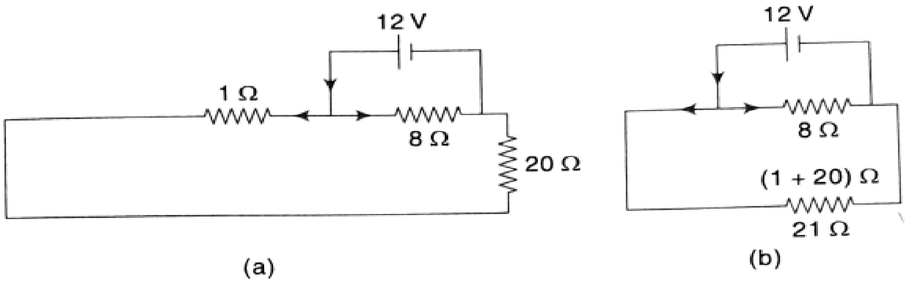

Fig. 1.232

By Ohm's law, $I_{20\Omega} = \dfrac{12}{1+20} = 0.571\ \text{A}(\uparrow)$

Step III: Considering 8 V source acting alone, replacing 10 V source and 12 V source by short circuit, we get the following circuit:

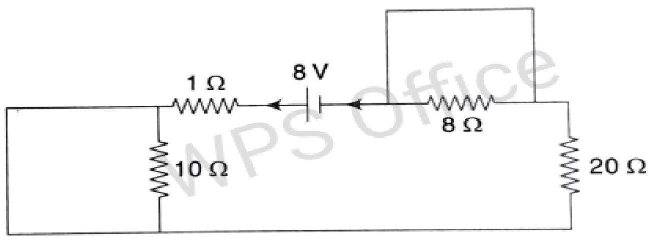

Fig. 1.233

In the circuit of Fig. 1.233, the resistors 10 Ω and 8 Ω get short circuited. For circuit simplification, the resistors 10 Ω and 8 Ω can be removed. By removing 10 Ω and 8 Ω resistors, we get the circuit as shown in Fig. 1.234.

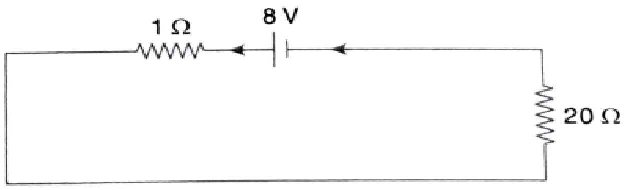

Fig. 1.234

By Ohm's law, $I_{20\Omega} = \dfrac{8}{1+20} = 0.381\ \text{A}(\uparrow)$

Step IV: According to superposition theorem, resultant current through 20 Ω resistor is algebraic sum of currents due to individual sources acting alone.
Hence, $I_{20\Omega} = 0.476\ \text{A}(\downarrow) + 0.571\ \text{A}(\uparrow) + 0.381\ \text{A}(\uparrow) = 0.476\ \text{A}(\uparrow)$

Example 1.67 Determine the current in 1 Ω resistor in the network shown in Fig. 1.235 by superposition theorem.

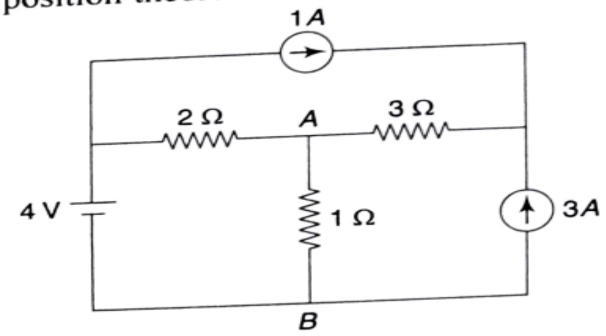

Fig. 1.235

Solution

Step I: Considering 4 V source acting alone, replacing 1 A source and 3 A source by open circuit, we get the following circuit:

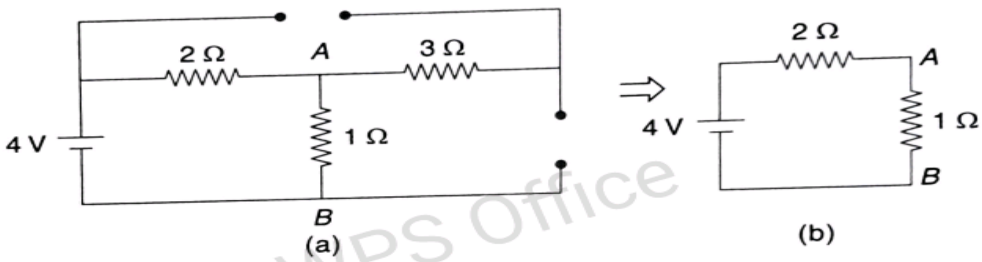

Fig. 1.236

By Ohm's law, $I_{AB} = I_{1\Omega} = \dfrac{4}{2+1} = 1.33 \text{ A}(\downarrow)$

Step II: Consider 3 A source acting alone, replacing 4 V source by short circuit and 1 A source by open circuit, we obtain the circuit as shown below:

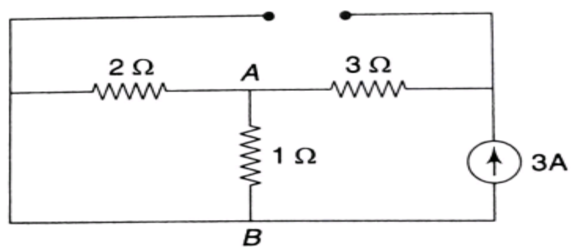

Fig. 1.237

In Fig. 1.237, current flowing through 3 Ω resistor, $I_{3\Omega} = 3 \text{ A}(\leftarrow)$. This current divides at node A. Resistors 2 Ω and 1 Ω are in parallel. By current division rule,

$$I_{AB} = I_{1\Omega} = 3 \times \frac{2}{2+1} = 2 \text{ A}(\downarrow)$$

Step III: Consider 1A source acting alone, replacing 4 V source by short circuit and 3 A source by open circuit, we obtain the following circuit:

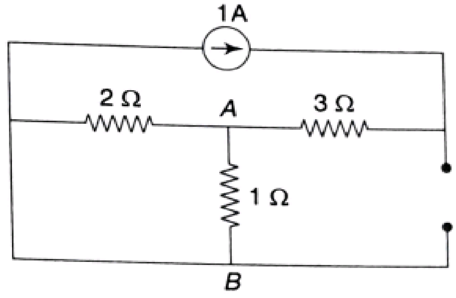

Fig. 1.238

In Fig. 1.238, current flowing through 3 Ω resistor, $I_{3\Omega} = 1\ A(\leftarrow)$.This current divides at node A. Resistors 2 Ω and 1 Ω are in parallel. By current division rule,

$$I_{AB} = I_{1\Omega} = 1 \times \frac{2}{2+1} = 0.67\ A(\downarrow)$$

Step IV: According to superposition theorem, resultant current through 1 Ω resistor is algebraic sum of currents due to individual sources acting alone. Hence,

$$I_{1\Omega} = 1.33\ A(\downarrow) + 2\ A(\downarrow) + 0.67\ A(\downarrow) = 4\ A(\downarrow)$$

Example 1.68 Determine the current in 5 Ω resistor in the network shown in Fig. 1.239 by superposition theorem.

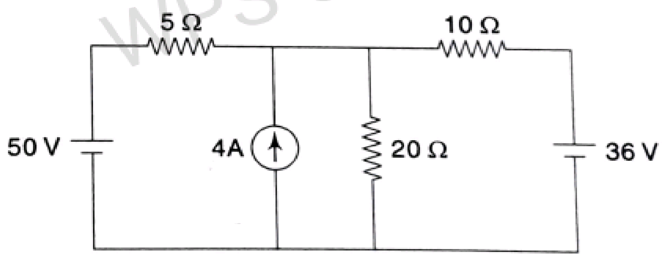

Fig. 1.239

Solution

Step I: Consider 50 V source acting alone, replacing 4 A source by open circuit and 36 V source by short circuit, we get the following circuit:

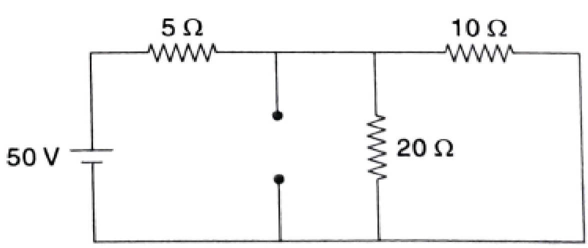

Fig. 1.240

In Fig. 1.240, resistors 20 Ω and 10 Ω are in parallel. So, by circuit reduction technique, we get the simplified circuit as shown in Fig. 1.241.

By Ohm's law, $I_{5\Omega} = \dfrac{50}{5+6.67} = 4.28\ \text{A}(\rightarrow)$

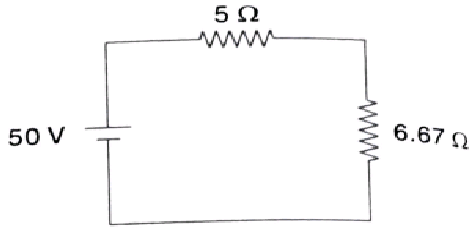

Fig. 1.241

Step II: Considering 4 A source acting alone, replacing 50 V source and 36 V source by short circuit, we obtain the circuit as shown in Fig. 1.242.

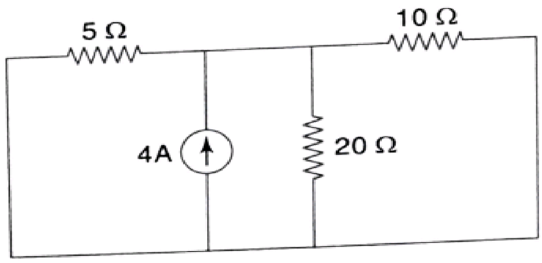

Fig. 1.242

In Fig. 1.242, resistors 20 Ω and 10 Ω are in parallel. By circuit reduction technique, we get the circuit as shown in Fig. 1.216.

In Fig. 1.243, resistors 5 Ω and 6.67 Ω are in parallel. By current division rule,

$$I_{5\Omega} = 4 \times \frac{6.67}{5+6.67} = 2.286\ \text{A}(\leftarrow)$$

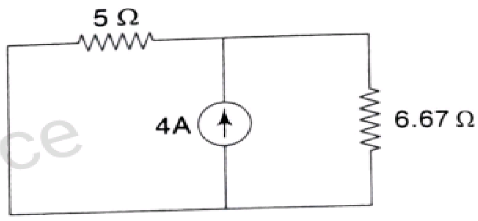

Fig. 1.243

Step III: Consider 36 V source acting alone, replacing 50 V source by short circuit and 4 A source by open circuit. We obtain the circuit as shown in Fig. 1.244.

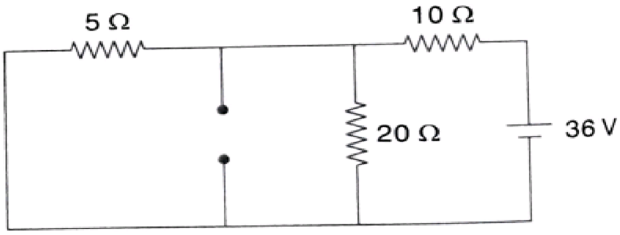

Fig. 1.244

By source transformation, i.e., converting series combination of voltage source of 36 V and resistor of 10 Ω into equivalent parallel combination of current source and resistor, we get the following circuit:

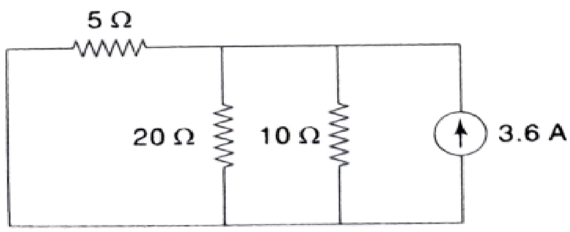

Fig. 1.245

By circuit reduction technique, we get the circuit as shown in Fig. 1.246.

In Fig. 1.246, resistors 6.67 Ω and 5 Ω are in parallel. By current division rule,

$$I_{5\Omega} = 3.6 \times \frac{6.67}{6.67 + 5} = 2.057 \text{ A}(\leftarrow)$$

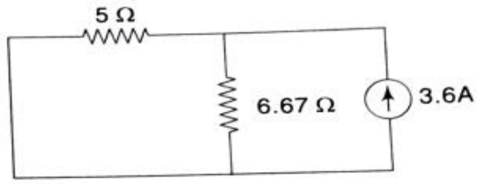

Fig. 1.246

Step IV: According to superposition theorem, resultant current through 5 Ω resistor is algebraic sum of currents due to individual sources acting alone. Hence,

$$I_{5\Omega} = 4.28 \text{ A}(\rightarrow) + 2.286 \text{ A}(\leftarrow) + 2.057 \text{ A}(\leftarrow) = 0.063 \text{ A}(\leftarrow)$$

Example 1.69 Determine the current through 10 Ω resistor in the network shown in Fig. 1.247 by superposition theorem.

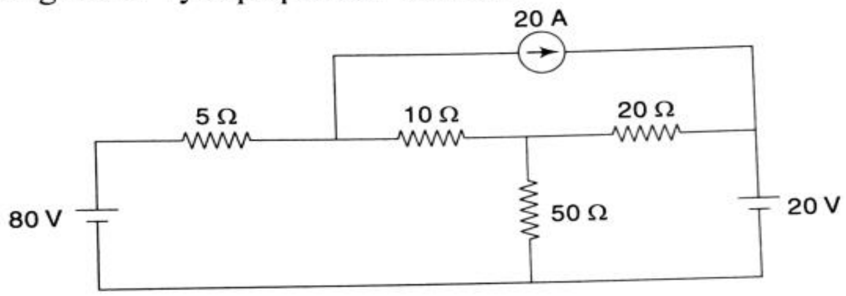

Fig. 1.247

Solution

Step I: Considering 80 V source acting alone, replacing 20 A source by open circuit and 20 V source by short circuit, we get the following circuit (Fig. 1.248):

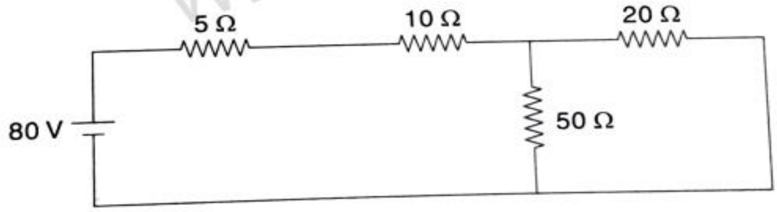

Fig. 1.248

In Fig. 1.248, resistors 20 Ω and 50 Ω are in parallel. By circuit reduction technique, we obtain the simplified circuit as shown below:

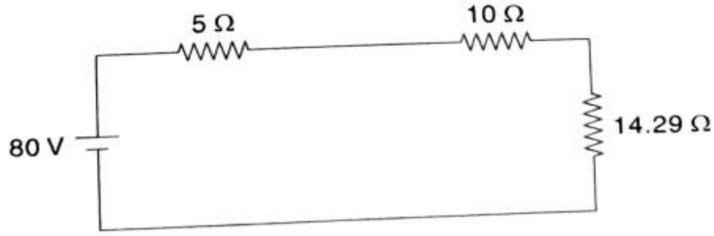

Fig. 1.249

By Ohm's law, $I_{10\Omega} = \dfrac{80}{5 + 10 + 14.29} = 2.73 \text{ A}(\rightarrow)$

Step II: Consider 20 V source acting alone, replacing 20 A source by open circuit and 80 V source by short circuit, we get the following circuit:

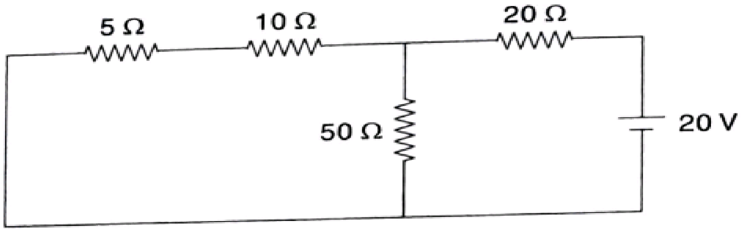

Fig. 1.250

By source transformation, i.e., converting series combination of voltage source of 20 V and resistor of 20 Ω into equivalent parallel combination of current source and resistor, we get the circuit as shown in Fig. 1.251.

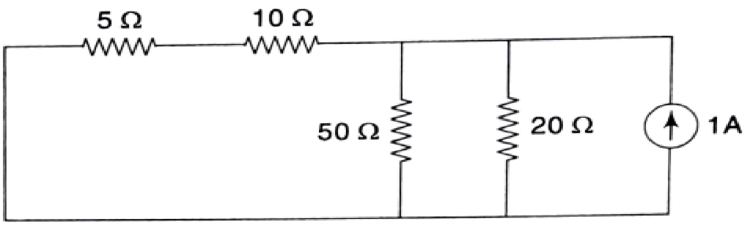

Fig. 1.251

By circuit reduction technique, we get the following circuit:

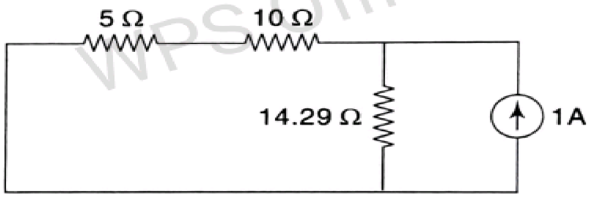

Fig. 1.252

By current division rule, $I_{10\Omega} = 1 \times \dfrac{14.29}{5+10+14.29} = 0.488\ \text{A}(\leftarrow)$

Step III: Consider 20 A source acting alone, replacing 80 V source and 20 V source by short circuit, we get Fig. 1.253.

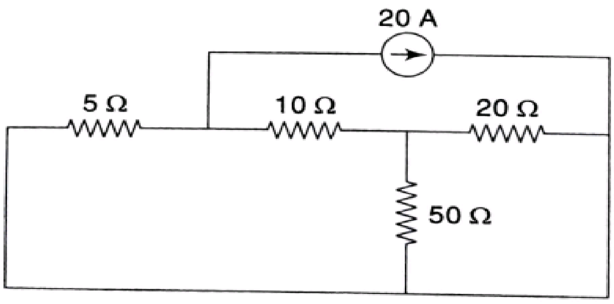

Fig. 1.253

The circuit can be redrawn as follows:

42760

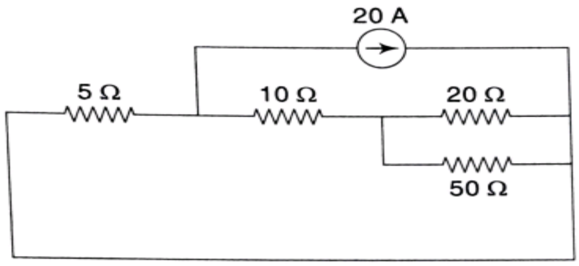

Fig. 1.254

By circuit reduction technique, we get the following circuit:

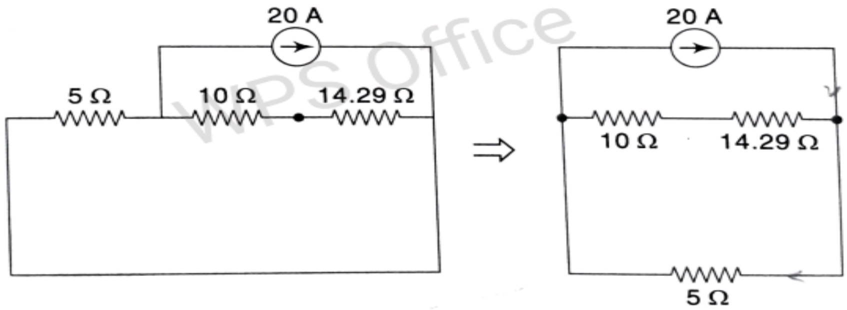

Fig. 1.255

By current division rule, $I_{10\Omega} = 20 \times \dfrac{5}{5+10+14.29} = 3.41\ \text{A}(\leftarrow)$

Step IV: According to superposition theorem, resultant current through 5 Ω resistor is algebraic sum of currents due to individual sources acting alone. Hence,

$$I_{10\Omega} = 2.73\ \text{A}(\rightarrow) + 0.488\ \text{A}(\leftarrow) + 3.41\ \text{A}(\leftarrow) = \underline{1.17\ \text{A}(\leftarrow)}$$