1.13 Star–Delta (Y–Δ) Transformation

We know that by using series/parallel circuit rules, we can reduce or simplify the circuit. But there are some networks in which the resistances are neither in series nor in parallel and are connected in Y- or Δ-connection. In such a situation, it is not possible to simplify the network by series/parallel circuit rules. However, converting Δ-connection into equivalent Y-connection and vice versa, a network can be simplified and application of series/parallel circuit rules is made possible.

Figure 1.146(a) shows three resistances R_{12}, R_{23}, and R_{31} connected in delta, while Fig. 1.146(b) shows three resistances R_1, R_2, and R_3 connected in star.

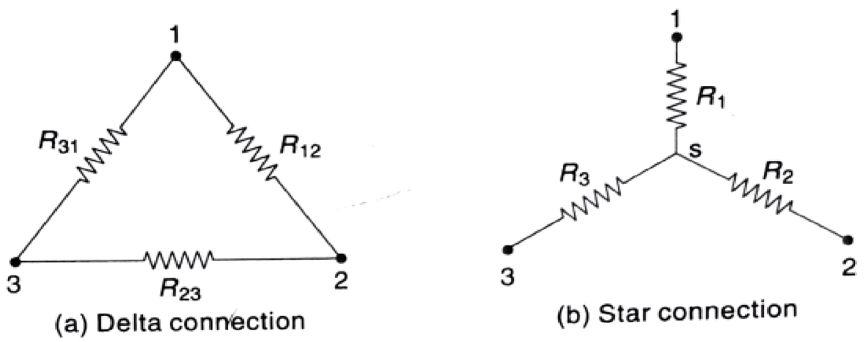

Fig. 1.146

The two connections will be electrically equivalent if the resistance as measured between any pair of terminals is the same in both arrangements.

1.13.1 Delta (Δ) to Star (Y) Transformation

Referring to delta network in Fig. 1.146(a),

$$\text{Resistance between terminals 1 and 2} = R_{12} \parallel (R_{23} + R_{31}) = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \tag{1.6}$$

Referring to star network in Fig. 1.146(b),

$$\text{Resistance between terminals 1 and 2} = R_1 + R_2 \tag{1.7}$$

Since two networks are electrically equivalent,

$$R_1 + R_2 = \frac{R_{12}(R_{23} + R_{31})}{R_{12} + R_{23} + R_{31}} \tag{1.8}$$

Similarly, it can be shown that between terminals 2 and 3 as well as 3 and 1,

$$R_2 + R_3 = \frac{R_{23}(R_{31} + R_{12})}{R_{12} + R_{23} + R_{31}} \tag{1.9}$$

and
$$R_3 + R_1 = \frac{R_{31}(R_{12} + R_{23})}{R_{12} + R_{23} + R_{31}} \tag{1.10}$$

Subtracting Eq. (1.9) from Eq. (1.8),

$$R_1 - R_3 = \frac{R_{12} R_{31} - R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \tag{1.11}$$

Adding Eqs (1.11) and (1.10),

$$R_1 = \frac{R_{12} R_{31}}{R_{12} + R_{23} + R_{31}} \tag{1.12}$$

Similarly, $$R_2 = \frac{R_{23} R_{12}}{R_{12} + R_{23} + R_{31}} \tag{1.13}$$

$$R_3 = \frac{R_{31} R_{23}}{R_{12} + R_{23} + R_{31}} \tag{1.14}$$

Thus, star resistance connected to terminal is equal to the product of the two delta resistances connected to same terminal divided by the sum of the delta resistances.

1.13.2 Star (Y) to Delta (Δ) Transformation

Multiplying Eqs (1.12) and (1.13),

$$R_1 R_2 = \frac{R_{12}^2 R_{23} R_{31}}{(R_{12} + R_{23} + R_{31})^2} \tag{1.15}$$

Multiplying Eqs (1.13) and (1.14),

$$R_2 R_3 = \frac{R_{23}^2 R_{31} R_{12}}{(R_{12} + R_{23} + R_{31})^2} \tag{1.16}$$

Multiplying Eqs (1.14) and (1.12),

$$R_3 R_1 = \frac{R_{31}^2 R_{12} R_{23}}{(R_{12} + R_{23} + R_{31})^2} \tag{1.17}$$

Adding Eqs (1.15), (1.16), and (1.17),

$$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12} R_{23} R_{31} (R_{12} + R_{23} + R_{31})}{(R_{12} + R_{23} + R_{31})^2}$$

or $$R_1 R_2 + R_2 R_3 + R_3 R_1 = \frac{R_{12} R_{23} R_{31}}{R_{12} + R_{23} + R_{31}}$$

or $$R_1 R_2 + R_2 R_3 + R_3 R_1 = R_{12} R_3 \qquad \left[\because R_3 = \frac{R_{23} R_{31}}{R_{12} + R_{23} + R_{31}} \right]$$

Hence, $$R_{12} = R_1 + R_2 + \frac{R_1 R_2}{R_3} \tag{1.18}$$

Similarly, $$R_{23} = R_2 + R_3 + \frac{R_2 R_3}{R_1} \tag{1.19}$$

and $$R_{31} = R_3 + R_1 + \frac{R_3 R_1}{R_2} \tag{1.20}$$

Thus, the delta resistance between the two terminals is the sum of the two star resistances connected to the same terminals plus the product of the two resistances divided by the remaining third star resistance.

Example 1.44 Find the equivalent resistance between the terminals X and Y in the network shown in Fig. 1.147.

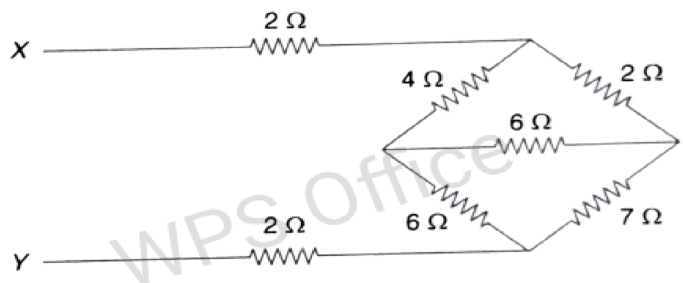

Fig. 1.147

Solution

Marking the different nodes, we get the following circuit:

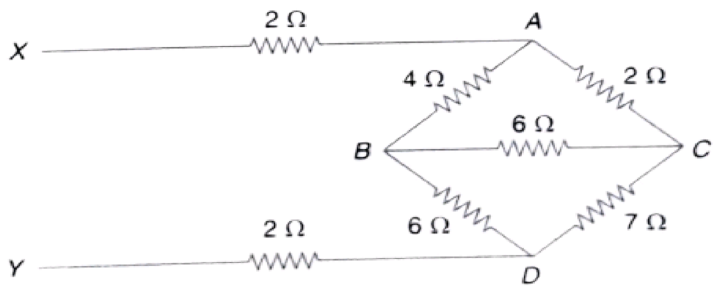

Fig. 1.148

Converting the delta connection formed by 4 Ω, 2 Ω, and 6 Ω resistors into equivalent star network, i.e., $\Delta ABC \Rightarrow Y\ ABC$, we get the following circuit:

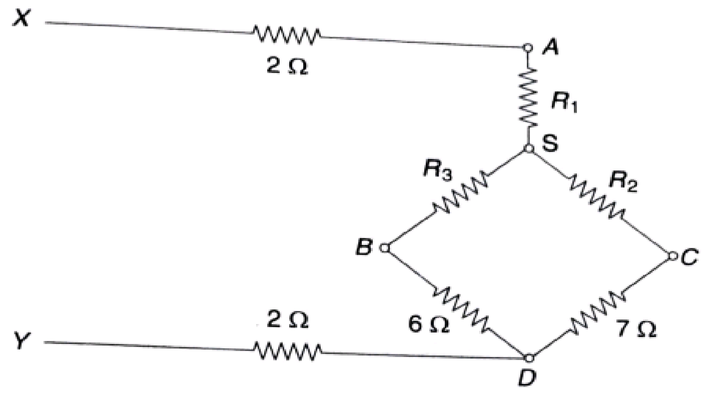

Fig. 1.149

$$R_1 = \frac{4\times 2}{4+2+6} = 0.67\ \Omega$$

$$R_2 = \frac{6\times 2}{4+2+6} = 1\ \Omega$$

$$R_3 = \frac{6\times 4}{4+2+6} = 2\ \Omega$$

The simplified network is shown in Fig. 1.150.

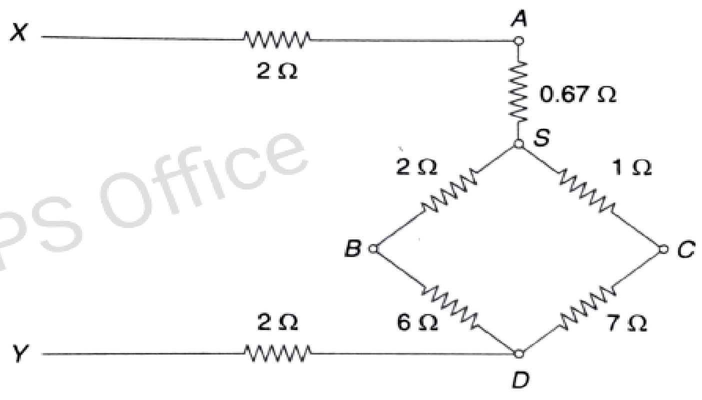

Fig. 1.150

In Fig. 1.150, resistors 6 Ω and 2 Ω are in series. Also resistors 1 Ω and 7 Ω are in series.

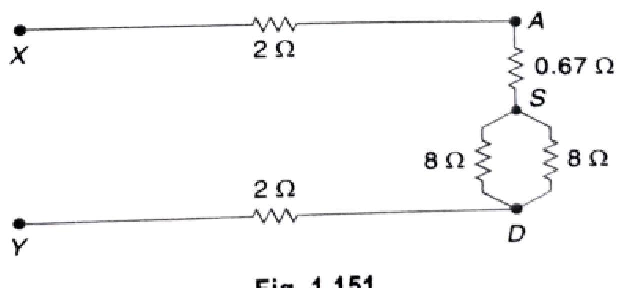

Fig. 1.151

In Fig. 1.151, two 8 Ω resistors are in parallel.

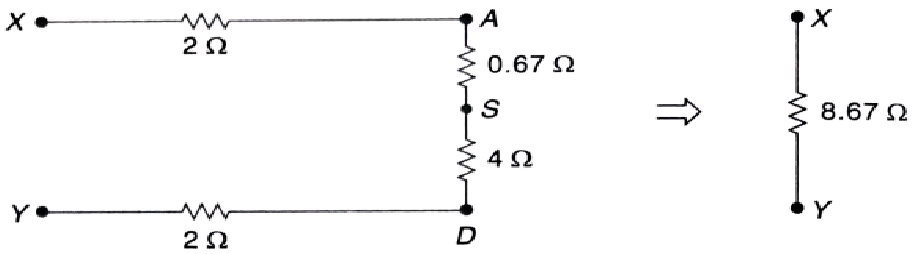

Fig. 1.152

Example 1.45 Find the equivalent resistance between the terminals A and B in the network shown in Fig. 1.153.

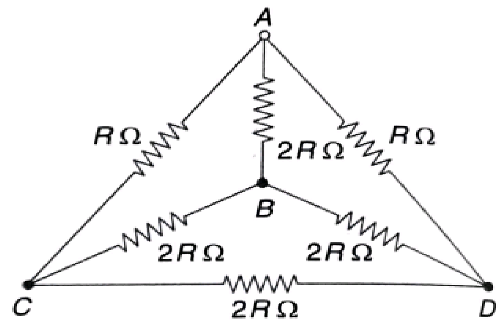

Fig. 1.153

Solution

Converting delta connection formed by three '$2R$' Ω resistors into equivalent star network, i.e., $\Delta\ CBD \Rightarrow Y\ CBD$, we get the network as shown in Fig. 1.154.

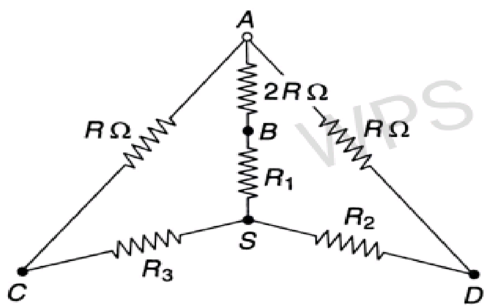

Fig. 1.154

We have $R_1 = R_2 = R_3 = \dfrac{2R \times 2R}{2R + 2R + 2R} = \dfrac{2}{3}R$

The simplified network is shown in Fig. 1.155.

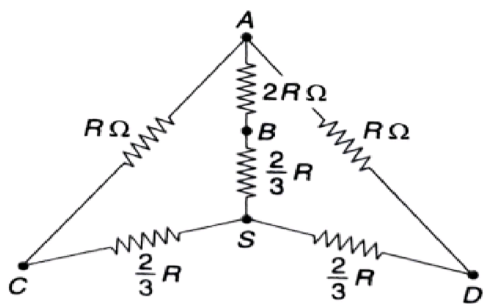

Fig. 1.155

In branch ACS, $R\ \Omega$ and $\frac{2}{3}R\ \Omega$ are in series. Also in branch ADS, $R\ \Omega$ and $\frac{2}{3}R\ \Omega$ resistors are in series.

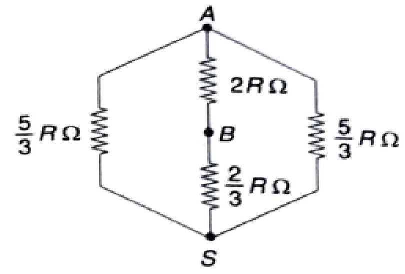

Fig. 1.156

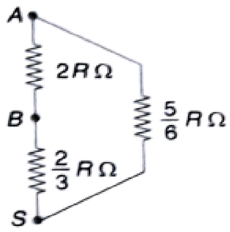

Fig. 1.157

In Fig. 1.156, two $\frac{5}{3}R\ \Omega$ resistors are in parallel.

In Fig. 1.157, resistors $\frac{2}{3}R\ \Omega$ and $\frac{5}{6}R\ \Omega$ are in series.

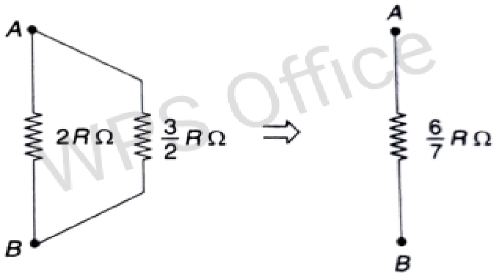

Fig. 1.158

Example 1.46 Find the current I in the network shown in Fig. 1.159.

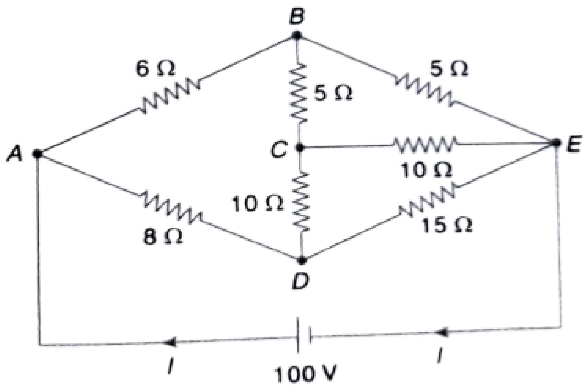

Fig. 1.159

Solution

Converting star connection formed by 5 Ω and two 10 Ω resistors into equivalent delta network, i.e., $Y\ BED \Rightarrow \Delta\ BED$, we get the network as shown in Fig. 1.160.

$$R_x = 5 + 10 + \frac{5 \times 10}{10} = 20\ \Omega,$$

$$R_y = 10 + 10 + \frac{10 \times 10}{5} = 40\ \Omega,$$

$$R_z = 5 + 10 + \frac{5 \times 10}{10} = 20\ \Omega$$

The simplified network is shown in Fig. 1.161.

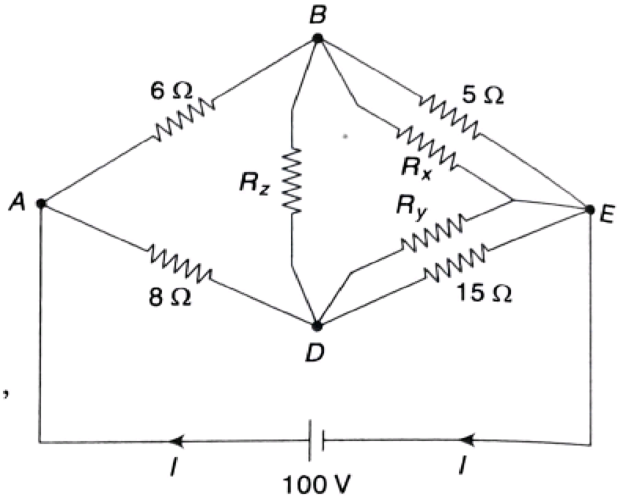

Fig. 1.160

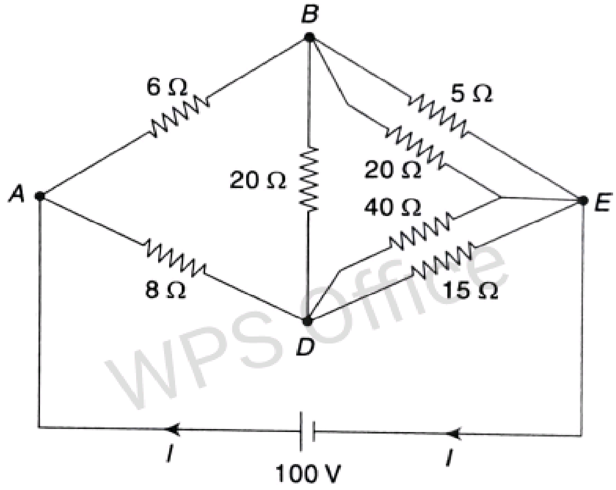

Fig. 1.161

In Fig. 1.161, resistors 20 Ω and 5 Ω are in parallel. Also resistors 40 Ω and 15 Ω are in parallel.

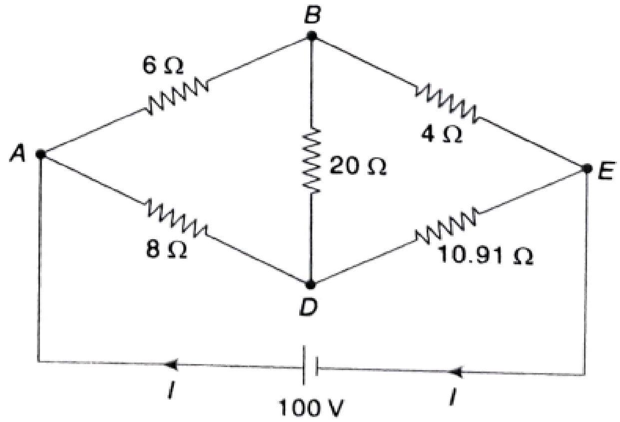

Fig. 1.162

Converting the delta connection formed by 6 Ω, 20 Ω, and 8 Ω resistors into equivalent star network, i.e., $\Delta\, ABD \Rightarrow Y\, ABD$, we get the following network:

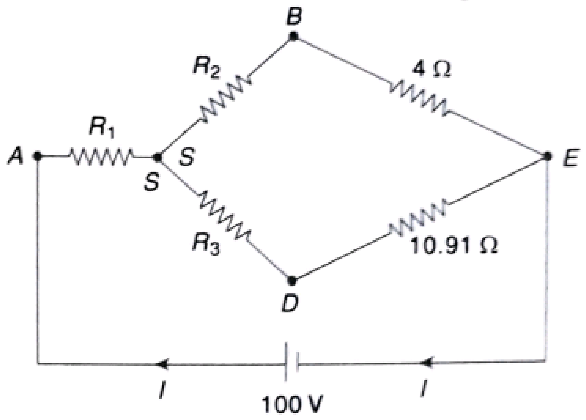

Fig. 1.163

We have

$$R_1 = \frac{6\times 8}{6+20+8} = 1.41\ \Omega,$$

$$R_2 = \frac{6\times 20}{6+20+8} = 3.53\ \Omega,$$

$$R_3 = \frac{20\times 8}{6+20+8} = 4.71\ \Omega$$

The simplified network is shown in Fig. 1.164.

In Fig. 1.164, resistors 3.53 Ω and 4 Ω are in series. Also resistors 4.71 Ω and 10.91 Ω are in series.

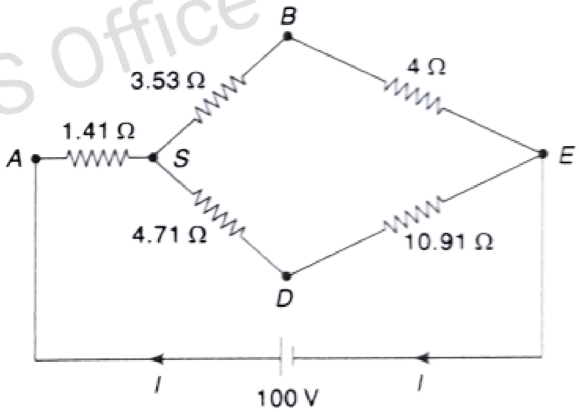

Fig. 1.164

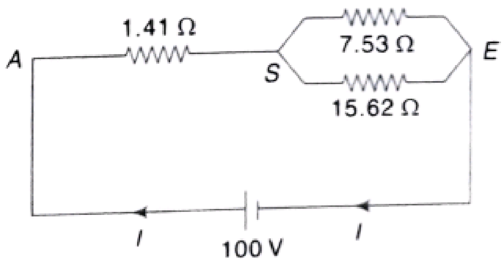

Fig. 1.165

In Fig. 1.165, resistors 7.53 Ω and 15.62 Ω are in parallel.

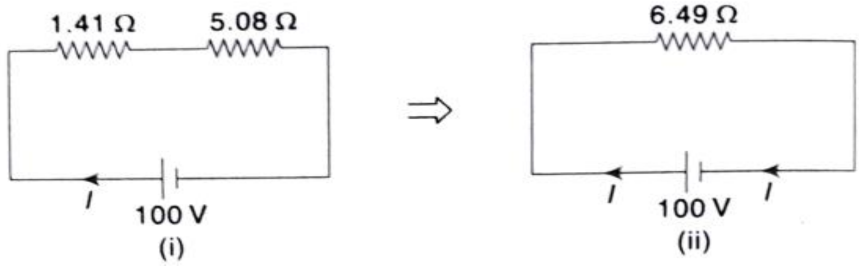

Fig. 1.166

By Ohm's law,

$$I = \frac{100}{6.49} = 15.41 \text{ A}$$

Example 1.47 Find the equivalent resistance between the terminals X and Y in the network shown in Fig. 1.167.

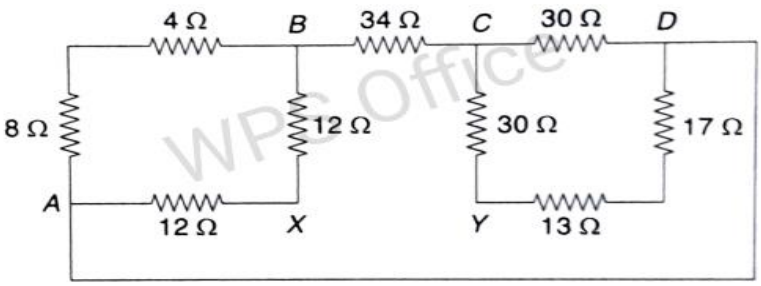

Fig. 1.167

Solution

In Fig. 1.167, resistors 8 Ω and 4 Ω are in series. Also resistors 17 Ω and 13 Ω are in series.

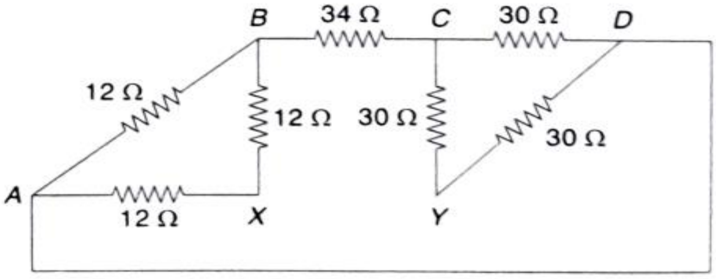

Fig. 1.168

Converting the delta connections formed by three 12 Ω resistors (ΔABX) and three 30 Ω resistors (ΔCDY) into equivalent star connections, i.e., $\Delta ABX \Rightarrow YABX$ and $\Delta CDY \Rightarrow YCDY$, we get the following network:

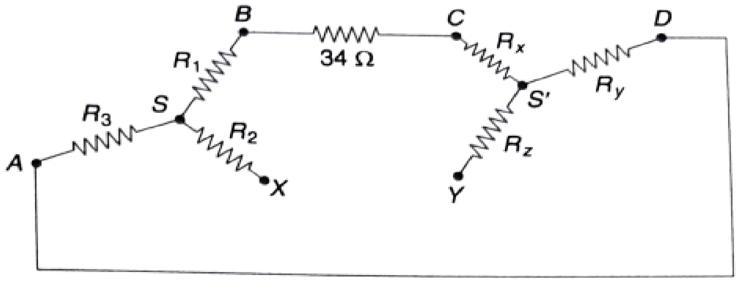

Fig. 1.169

$$R_1 = R_2 = R_3 = \frac{12 \times 12}{12 + 12 + 12} = 4\ \Omega$$

$$R_x = R_y = R_z = \frac{30 \times 30}{30 + 30 + 30} = 10\ \Omega$$

The simplified network is shown in Fig. 1.170.

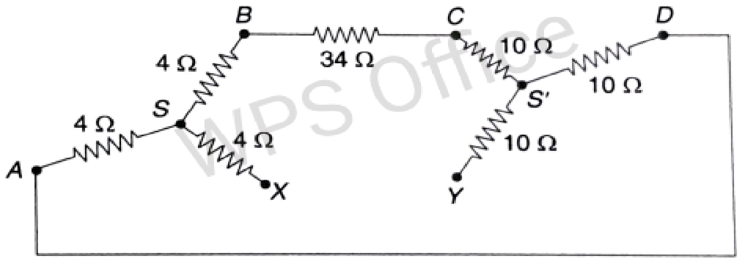

Fig. 1.170

In branch $SBCS'$, 4 Ω, 34 Ω, and 10 Ω are in series. Also in branch $SADS'$, 4 Ω and 10 Ω are in series.

In Fig. 1.171, 48 Ω and 14 Ω are in parallel.

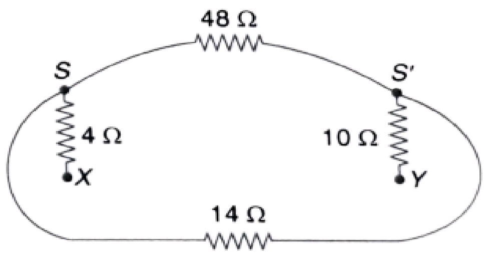

Fig. 1.171

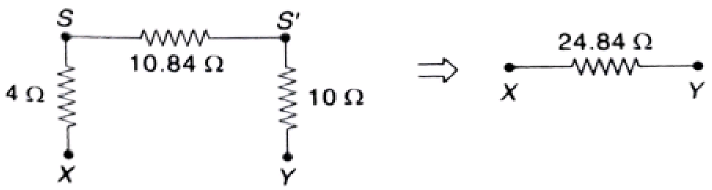

Fig. 1.172

Example 1.48 Find the equivalent resistance between the terminals A and B in the network shown in Fig. 1.173.

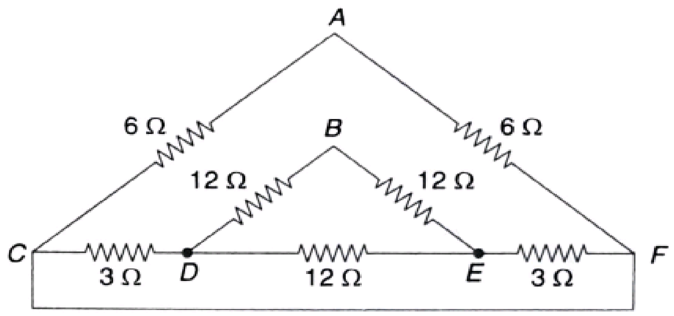

Fig. 1.173

Solution

Converting the delta connection formed by three 12 Ω resistors (Δ *BDE*) into equivalent star connection, i.e., $\Delta\, BDE \Rightarrow YBDE$, we get the network as shown in Fig. 1.174.

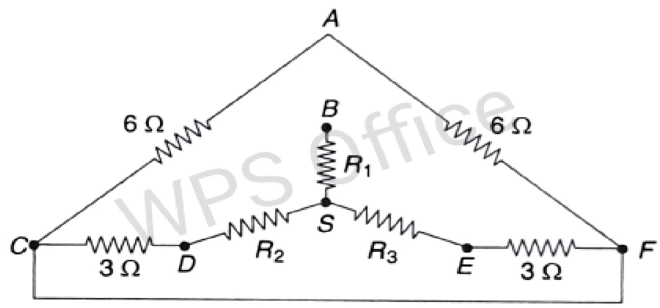

Fig. 1.174

$$R_1 = R_2 = R_3 = \frac{12 \times 12}{12 + 12 + 12} = 4\ \Omega$$

The simplified network is shown in Fig. 1.175.

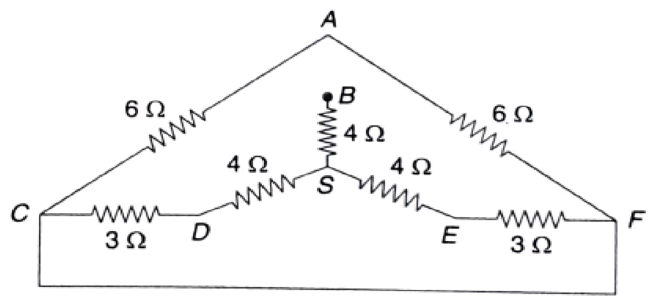

Fig. 1.175

In branch *CDS*, 3 Ω and 4 Ω are in series. Also in branch *SEF*, 4 Ω and 3 Ω are in series.

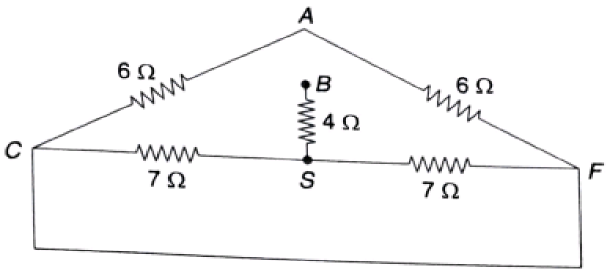

Fig. 1.176

In Fig. 1.176, the nodes C and F are same, and by joining them, the circuit simplifies as shown in Fig. 1.177.

In Fig. 1.177, two 7 Ω resistors are in parallel and two 6 Ω resistors are in parallel.

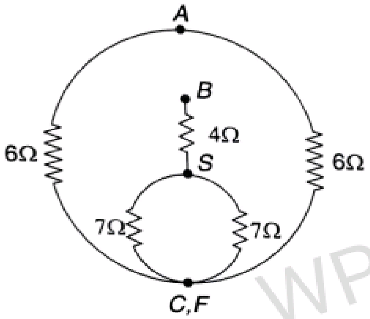

Fig. 1.177

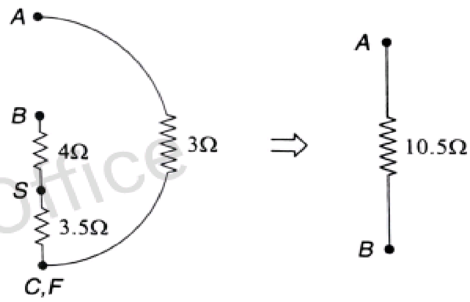

Fig. 1.178

Example 1.49 Find the equivalent resistance between the terminals A and B in the network shown in Fig. 1.179.

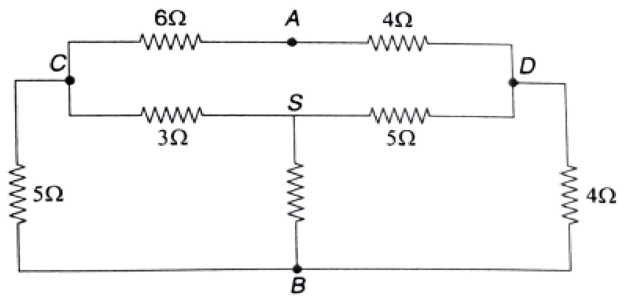

Fig. 1.179

Solution

Converting the star connection formed by 3 Ω, 5 Ω, and 2 Ω resistors (Y CDB) into equivalent delta connection, i.e., $Y\ CDB \Rightarrow \Delta\ CDB$, we get the circuit as shown in Fig. 1.180.

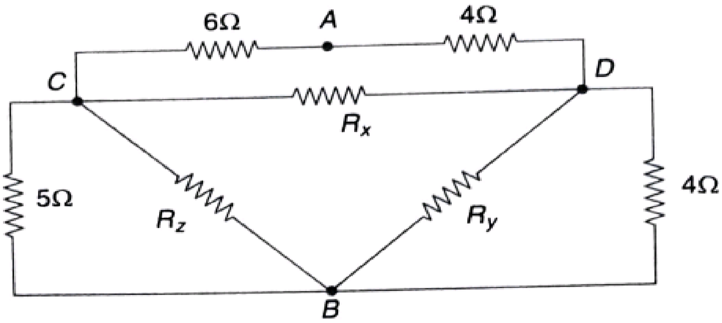

Fig. 1.180

$$R_x = 3 + 5 + \frac{3 \times 5}{2} = 15.5\ \Omega, \qquad R_y = 5 + 2 + \frac{5 \times 2}{3} = 10.33\ \Omega,$$

$$R_z = 2 + 3 + \frac{2 \times 3}{5} = 6.2\ \Omega$$

The simplified network is shown in Fig. 1.181.

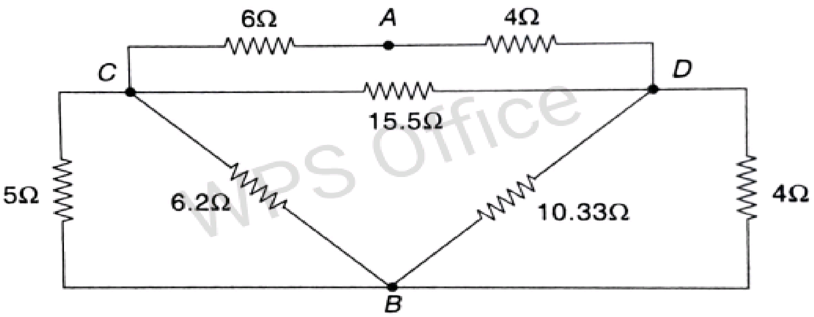

Fig. 1.181

In Fig. 1.181, 5 Ω and 6.2 Ω are in parallel. Also 10.33 Ω and 4 Ω are in parallel.

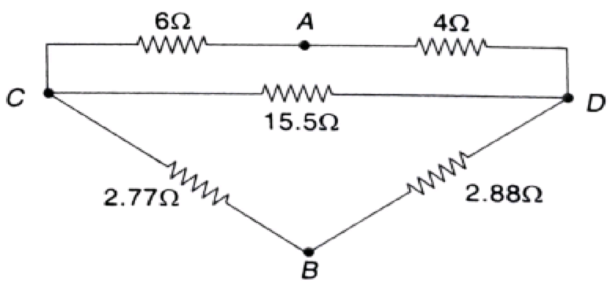

Fig. 1.182

Converting delta connection formed by 6 Ω, 4 Ω and 15.5 Ω resistors (Δ *ACD*) into equivalent star connection, i.e., Δ *ACD* ⇒ *Y ACD*, we get the network as shown in Fig. 1.183.

$$R_1 = \frac{6 \times 4}{6+4+15.5} = 0.94\ \Omega, \qquad R_2 = \frac{4 \times 15.5}{6+4+15.5} = 2.43\ \Omega,$$

$$R_3 = \frac{15.5 \times 6}{6+4+15.5} = 3.65\ \Omega$$

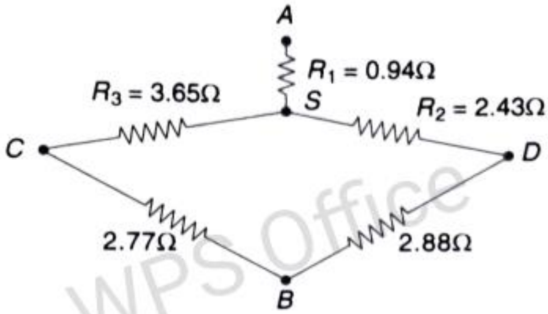

Fig. 1.183

In Fig. 1.183, 3.65 Ω and 2.77 Ω are in series. Also 2.43 Ω and 2.88 Ω are in series.

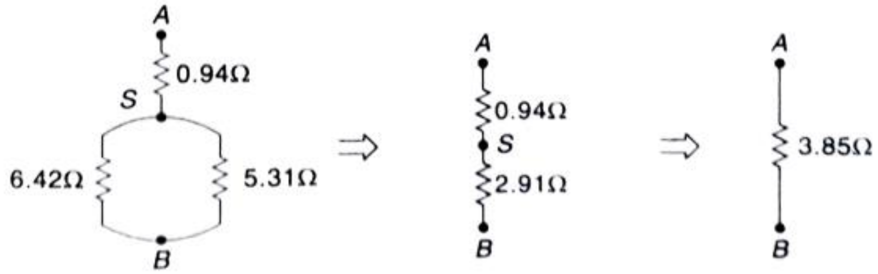

Fig. 1.184