EXPERIMENT : No.

1\. Show that every skew Hermitian matrix can be expressed in the form of $P+iQ$ where P is real skew-symmetric and Q is real symmetric matrix.

Solun.- Let R be any skew Hermitian Matrix

$$\therefore R^* = -\bar{R}^T = -\bar{R}^T = -R$$

To Prove :- $R = P + iQ$

where P = real skew-symmetric matrix

Q = symmetric matrix

Let $\bar{A}$ be the conjugate matrix of given skew Hermitian matrix R,

Then

$$R = \frac{1}{2}\left[R - \bar{R} + R + \bar{R}\right]$$

$$= \frac{1}{2}\left[R - \bar{R}\right] + i\cdot\frac{1}{2i}\left[\bar{R} + \bar{R}\right]$$

$$R = P + iQ$$

To Prove :-

$P = \frac{1}{2}\left[R - \bar{R}\right]$ is real skew-symmetric matrix

and $Q = \frac{1}{2i}\left[R + \bar{R}\right]$ is real symmetric matrix

From Complex nos., we know that if $z = x+iy$, $\bar{z} = x - iy$ are complex conjugates of each other then

$$\frac{1}{2}(z - \bar{z}) = x$$ is real and

$$\frac{1}{2i}(z+\bar{z}) = y \quad \text{is real}$$

Hence

$$P = \frac{1}{2}\left[R - \bar{R}\right] \quad \text{and}$$

$$Q = \frac{1}{2i}\left[R + \bar{R}\right]$$

are real matrices

Now,

$$P^T = \left[\frac{1}{2}(R - \bar{R})\right]^T$$

$$= \frac{1}{2}\left(R^T - (\bar{R})^T\right)$$

$$= \frac{1}{2}\left[\left((\bar{R})^T\right)^T - (\bar{R})^T\right]$$

$$= \frac{1}{2}\left[\bar{R} - R\right] = -P$$

$$\therefore \underline{P^T = -P}$$

i.e $P = \frac{1}{2}(R - \bar{R})$ is skew-symmetric

Also $$Q^T = \left[\frac{1}{2i}(R + \bar{R})\right]^T$$

$$= \frac{1}{2i}\left(R^T + (\bar{R})^T\right)$$

$$= \frac{1}{2i}\left[\left((\bar{R})^T\right)^T + (\bar{R})^T\right]$$

$$= \frac{1}{2i}\left[\bar{R} + R\right] = Q$$

$$\therefore \underline{Q^T = Q}$$

i.e. $Q = \frac{1}{2i}(R + \bar{R})$ is symmetric

→ Hence, every Skew Hermitial matrix R can be expressed as $R = P + iQ$ where P is real skew-symmetric and Q is real symmetric matrix

2. Find the rank of matrix by reducing in Row Echelon form

$$\begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 4 & 3 & 3 \\ 6 & 18 & 7 & 15 \\ 6 & 12 & 6 & 10 \end{bmatrix}$$

Solnⁿ :- Let $A = \begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 4 & 3 & 3 \\ 6 & 18 & 7 & 15 \\ 6 & 12 & 6 & 10 \end{bmatrix}$

$R_3 \sim R_3 - 6R_1$; $R_4 \sim R_4 - 6R_1$

$$\sim \begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 4 & 3 & 3 \\ 0 & -18 & -11 & -21 \\ 0 & -24 & -12 & -26 \end{bmatrix}$$

$R_2 \sim R_2 (1/4)$; $R_4 \sim R_4 (-1/2)$

$$\sim \begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 1 & 3/4 & 3/4 \\ 0 & -18 & -11 & -21 \\ 0 & 12 & 6 & 13 \end{bmatrix}$$

$R_3 \sim R_3 + 18R_2$; $R_4 \sim R_4 - 12R_2$

$$\sim \begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 1 & 3/4 & 3/4 \\ 0 & 0 & 5/2 & -15/2 \\ 0 & 0 & -3 & 4 \end{bmatrix}$$

$R_3 \sim R_3 (2/5)$

$$\sim \begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 1 & 3/4 & 3/4 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & -3 & 4 \end{bmatrix}$$

$R_4 \sim R_4 + 3R_3$

$$\sim \begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 1 & 3/4 & 3/4 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & -5 \end{bmatrix}$$

$R_4 \sim R_4(-1/5)$

$$\sim \begin{bmatrix} 1 & 6 & 3 & 6 \\ 0 & 1 & 3/4 & 3/4 \\ 0 & 0 & 1 & -3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Since, there are 4 non-zero rows in the reduced matrix

$\therefore$ Rank of matrix $r(A) = \underline{4}$

3. Find the rank of matrix by reducing it to Normal form

$$\begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 1 & 3 & -2 & 3 & 0 \\ 2 & 4 & -3 & 6 & 4 \\ 1 & 1 & -1 & 4 & 6 \end{bmatrix}$$

Solⁿ:- Let $A = \begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 1 & 3 & -2 & 3 & 0 \\ 2 & 4 & -3 & 6 & 4 \\ 1 & 1 & -1 & 4 & 6 \end{bmatrix}$

$R_2 \sim R_2 - R_1$; $R_3 \sim R_3 - 2R_1$; $R_4 \sim R_4 - R_1$

$$\sim \begin{bmatrix} 1 & 2 & -2 & 3 & 1 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & -1 & 1 & 1 & 5 \end{bmatrix}$$

$C_2 \sim C_2 - 2C_1$; $C_3 \sim C_3 + 2C_1$; $C_4 \sim C_4 - 3C_1$; $C_5 \sim C_5 - C_1$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & -1 & 1 & 1 & 5 \end{bmatrix}$$

$R_4 \sim R_4 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 & 4 \end{bmatrix}$$

$C_5 \sim C_5 + C_2$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 & 4 \end{bmatrix}$$

$R_4 \sim R_4 - R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$C_5 \sim C_5 - 2C_3$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 \end{bmatrix}$$

$C_5 \sim C_5 - 2C_4$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

$$\sim \begin{bmatrix} I_4 & O \end{bmatrix}$$

rank of a matrix $r(A) = O(I_4) = \underline{4}$

4: Test the consistency and solve those which are solvable.

$$x_1 - x_2 + x_3 - x_4 + x_5 = 1$$
$$2x_1 - x_2 + 3x_3 + 4x_5 = 2$$
$$3x_1 - 2x_2 + 2x_3 + x_4 + x_5 = 1$$
$$x_1 + x_3 + 2x_4 + x_5 = 0$$

Soln:- Rewriting a system in a matrix form

$$AX = B$$

$$\begin{bmatrix} 1 & -1 & 1 & -1 & 1 \\ 2 & -1 & 3 & 0 & 4 \\ 3 & -2 & 2 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \\ 0 \end{bmatrix}$$

$\therefore$ Augmented Matrix $[A:B]$

$$[A:B] = \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 2 & -1 & 3 & 0 & 4 & 2 \\ 3 & -2 & 2 & 1 & 1 & 1 \\ 1 & 0 & 1 & 2 & 1 & 0 \end{array}\right]$$

$R_2 \sim R_2 - 2R_1$; $R_3 \sim R_3 - 3R_1$; $R_4 \sim R_4 - R_1$

$$\sim \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 0 & 1 & -1 & 4 & -2 & -2 \\ 0 & 1 & 0 & 3 & 0 & -1 \end{array}\right]$$

$R_3 \sim R_3 - R_2$; $R_4 \sim R_4 - R_2$

$$\sim \left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 0 & 0 & -2 & 2 & -4 & -2 \\ 0 & 0 & -1 & 1 & -2 & -1 \end{array}\right]$$

$R_4 \sim R_4 - R_3(1/2)$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & -1 & 1 & 1 \\ 0 & 1 & 1 & 2 & 2 & 0 \\ 0 & 0 & -2 & 2 & -4 & -2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Here $r(A) = r(A|B) = 3$ total no. of unknowns $= 5$

$\therefore r(A) = r(A|B) <$ total no. of unknowns

$\therefore$ given system is consistent and has infinite sol^n

Hence, no of parameters is $\underline{\underline{2}}$

$$x_1 + x_2 + x_3 - x_4 + x_5 = 1$$
$$x_2 + x_3 + 2x_4 + 2x_5 = 0$$
$$-2x_3 + 2x_4 - 4x_5 = -2$$

Let $x_4 = t_1 \in R$, $x_5 = t_2 \in R$

$x_3 = t_1 - 2t_2 + 1$

$x_2 + (t_1 - 2t_2 + 1) + 2t_1 + 2t_2 = 0$

$\therefore x_2 = -1(1 + 3t_1)$

$x_1 + 1 + 3t_1 + t_1 - 2t_2 + 1 - t_1 + t_2 = 0$

$\therefore x_1 = -(1 + 3t_1 + t_2)$

$\therefore$
$$x_1 = -(1 + 3t_1 + t_2)$$
$$x_2 = -(1 + 3t_1)$$
$$x_3 = 1 + t_1 - 2t_2$$
$$x_4 = t_1$$
$$x_5 = t_2$$

5. For what value of λ and μ, the system has no solution, a unique solution, infinite numbers of solution

$$3x - 2y + z - \mu = 0$$
$$5x - 8y + 9z - 3 = 0$$
$$2x + y + \lambda z + 1 = 0$$

Solⁿ:-

When the system has no solⁿ,
Rewriting in matrix form $AX = B$

~~Let AX~~ $= \begin{bmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & \lambda \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \mu \\ 3 \\ -1 \end{bmatrix}$

Augmented form $[A:B]$

$$[A:B] = \left[\begin{array}{ccc|c} 3 & -2 & 1 & \mu \\ 5 & 8 & 9 & 3 \\ 2 & 1 & \lambda & -1 \end{array}\right]$$

$R_1 \leftrightarrow R_3$

$$\sim \left[\begin{array}{ccc|c} 2 & 1 & \lambda & -1 \\ 5 & -8 & 9 & 3 \\ 3 & -2 & 1 & \mu \end{array}\right]$$

$R_1 \leftrightarrow R_1 (1/2)$

$$\sim \left[\begin{array}{ccc|c} 1 & 1/2 & \lambda/2 & -1/2 \\ 5 & -8 & 9 & 3 \\ 3 & -2 & 1 & \mu \end{array}\right]$$

$R_2 \sim R_2 - 5R_1$; $R_3 \sim R_3 - 3R_1$

$$\sim \left[\begin{array}{ccc|c} 1 & 1/2 & \lambda/2 & -1/2 \\ 0 & -21/2 & (18-5\lambda)/2 & 11/2 \\ 0 & -7/2 & \frac{(2-3\lambda)}{2} & (2\mu+3)/2 \end{array}\right]$$

$R_3 \sim R_3 - R_2(1/3)$

$$\sim \left[\begin{array}{ccc|c} 1 & 1/2 & \lambda/2 & -1/2 \\ 0 & -21/2 & (18-5\lambda)/2 & 11/2 \\ 0 & 0 & -2(3+\lambda)/3 & \mu - 1/3 \end{array}\right]$$

(1) To get no solution for the given equations

$$r(A|B) = r(A)$$

It is possible to get this situation when $r(A) = 2$ and $r(A|B) = 3$

Hence, to obtain this

$$-\frac{2}{3}(3+\lambda) = 0 \quad \text{and} \quad \mu - \frac{1}{3} \neq 0$$

$\therefore \lambda + 3 = 0$ & $\mu \neq \frac{1}{3}$

$\therefore \lambda = -3$

→ $\therefore$ For $\lambda = 3$ and all value of $\mu \in R - \{1/3\}$ there exists no solution.

(2) To get a unique solution, the below condition should be satisfied.

$$r(A|B) = r(A) = \cancel{3} = \text{total no. of unknowns}$$

Here total no. of unknowns $= 3$

$$\therefore r(A|B) = r(A) = 3$$

$$\therefore -\frac{2}{3}(3+\lambda) \neq 0 \quad \text{and} \quad \mu - \frac{1}{3} \neq 0$$

$3 + \lambda \neq 0$ $\therefore \mu \neq \frac{1}{3}$

$\therefore \lambda \neq -3$

→ Hence, for all values of $\lambda \in R - \{-3\}$ and $\mu \in R - \{\frac{1}{3}\}$, there exists a unique solution.

(3) To get an infinite solution, the below condition is to satisfied

$$r(A|B) = r(A) < \text{total no. of unknowns}$$

Here Total no. of unknowns = 3

Condition is satisfied only when $r(A|B) = r(A) = 2$

To get this,

$$\frac{-2}{3}(3+\lambda) = 0 \quad \text{and} \quad \mu - \frac{1}{3} = 0$$

$$\therefore \lambda = -3 \quad \text{and} \quad \mu = \frac{1}{3}$$

→ Hence, for $\lambda = -3$ and $\mu = \frac{1}{3}$, there exists infinite solution for the given system of equations

6. Does the following system have trivial or non-trivial solution ? Obtain the non-trivial solution if it exists.

$$3x_1 + 4x_2 - x_3 - 9x_4 = 0$$
$$2x_1 + 3x_2 + 2x_3 - 3x_4 = 0$$
$$2x_1 + x_2 - 14x_3 - 12x_4 = 0$$
$$x_1 + 3x_2 + 13x_3 + 3x_4 = 0$$

Ans:- Rewriting the system in matrix form $AX = B$

$$\begin{bmatrix} 3 & 4 & -1 & -9 \\ 2 & 3 & 2 & -3 \\ 2 & 1 & -14 & -12 \\ 1 & 3 & 13 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Augmented form $[A : B]$

$$[A : B] = \left[\begin{array}{cccc|c} 3 & 4 & -1 & -9 & 0 \\ 2 & 3 & 2 & -3 & 0 \\ 2 & 1 & -14 & -12 & 0 \\ 1 & 3 & 13 & 3 & 0 \end{array}\right]$$

$R_1 \leftrightarrow R_4$

$$\left[\begin{array}{cccc|c} 1 & 3 & 13 & 3 & 0 \\ 2 & 3 & 2 & -3 & 0 \\ 2 & 1 & -14 & -12 & 0 \\ 3 & 4 & -1 & -9 & 0 \end{array}\right]$$

$R_2 \sim R_2 - 2R_1$; $R_3 \sim R_3 - 2R_1$; $R_4 \sim R_4 - 3R_1$

$$\left[\begin{array}{cccc|c} 1 & 3 & 13 & 3 & 0 \\ 0 & -3 & -24 & -9 & 0 \\ 0 & -5 & -40 & -18 & 0 \\ 0 & -5 & -40 & -18 & 0 \end{array}\right]$$

$R_2 \sim R_2(1/(-3))$; $R_4 \sim R_4 - R_3$

$$\sim \left[\begin{array}{cccc|c} 1 & 3 & 13 & 3 & 0 \\ 0 & 1 & 8 & 3 & 0 \\ 0 & -5 & -40 & -18 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$R_3 \sim R_3 + 5R_2$

$$\left[\begin{array}{cccc|c} 1 & 3 & 13 & 3 & 0 \\ 0 & 1 & 8 & 3 & 0 \\ 0 & 0 & 0 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

Here $r(A|B) = 3$ $r(A) = 3$

total no. of unknowns $= 4$

$\therefore r(A|B) = r(A) <$ Total no. of unknowns

Hence the system of equation is consistent and has non-trivial solution.

Since pivot element is absent for x_3 and x_4, they will be considered as free variables

$x_3 = t_1$ where $t_1 \in R$ & $x_4 = t_2$ where $t_2 \in R$

From the matrix

$$x_1 + 3x_2 + 13x_3 + 3x_4 = 0 \quad -(1)$$

$$x_2 + 8x_3 + 3x_4 = 0 \quad -(2)$$

Substituting the values of x_3 and x_4 in (2)

$$x_2 + 8t_1 + 3t_2 = 0$$

$$\therefore x_2 = -3t_2 - 8t_1$$

Substituting the values of x_2, x_3, x_4 in ①

$$x_1 + 3(-3t_1 - 8t_2) + 13t_1 + 3t_2 = 0$$
$$x_1 + (-9t_1 - 24t_2) + 13t_1 + 3t_2 = 0$$
$$x_1 = 11t_1 + 6t_2$$

→ The given system has non-trivial solution where
$$x_1 = 11t_1 + 6t_2$$
$$x_2 = -8t_1 - 3t_2$$
$$x_3 = t_1$$
$$x_4 = t_2$$
where $t_1, t_2 \in R$

EXPERIMENT : No.

7. For what value of λ the following system of equations possesses a non-trivial solution? Obtain the solution for real value of λ.

$$x + 2y + 3z = \lambda x$$
$$3x + y + z = \lambda y$$
$$2x + 3y + z = \lambda z$$

Ans:-

$$(1-\lambda)x + 2y + 3z = 0$$
$$3x + y(1-\lambda) + 2z = 0$$
$$2x + 3y + (1-\lambda)z = 0$$

As the given system has a non-trivial solution,

$$\begin{vmatrix} (1-\lambda) & 2 & 3 \\ 3 & (1-\lambda) & 2 \\ 2 & 3 & (1-\lambda) \end{vmatrix} = 0$$

$$(1-\lambda)\left[(1-\lambda)^2 - 2(3)\right] - 2\left[3(1-\lambda) - 2(2)\right] + 3\left[3(3) - 2(1-\lambda)\right] = 0$$

$$(1-\lambda)^3 - 18(1-\lambda) + 35 = 0$$
$$-3\lambda + 3\lambda^2 - \lambda^3 - 18 + 18\lambda + 35 = 0$$
$$-\lambda^3 + 3\lambda^2 + 15\lambda + 18 = 0$$
$$\lambda^3 - 3\lambda^2 - 15\lambda - 18 = 0$$
$$(\lambda - 6)(\lambda^2 + 3\lambda + 3) = 0$$

Hence, the only real value of λ which satisfies the given condition is 6.

Sundaram

Teacher's Sign.: