1.19 Maximum Power Transfer Theorem

This theorem deals with transfer of maximum power from a source to load and may be stated as under:

In dc circuits, maximum power is transferred from a source to a load when the load resistance is made equal to the equivalent resistance of the network as viewed from the load terminals, with load removed and replacing all sources with their internal resistances.

Figure 1.371(a) shows a complex circuit supplying power to the load R_L. The circuit enclosed in a box can be replaced by Thevenin's equivalent circuit consisting of a single source of emf V_{TH} (called Thevenin voltage) in series with a single resistance R_{TH} (called Thevenin resistance), as shown in Fig. 1.371(b). Clearly, the resistance R_{TH} is the resistance measured between terminals A and B with R_L removed and replacing the sources with their internal resistances. According to maximum power transfer theorem, maximum power will be transferred from the circuit to the load when R_L is made equal to R_{TH}, the Thevenin's resistance at terminals A and B.

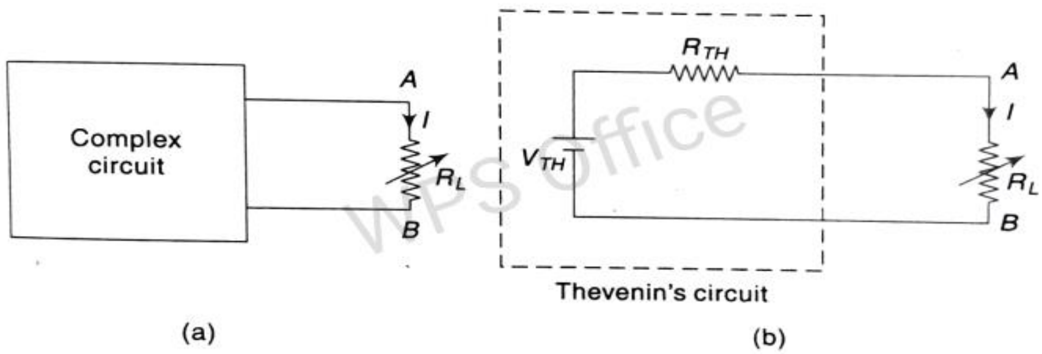

Fig. 1.371 Illustration of maximum power transfer theorem

Proof

Referring to Fig. 1.371(b), the current supplied to R_L is given by

$$I = \frac{V_{TH}}{R_L + R_{TH}}$$

Power delivered to R_L, $P = I^2 R_L = \left(\frac{V_{TH}}{R_L + R_{TH}}\right)^2 \times R_L = \frac{V_{TH}^2 R_L}{(R_L + R_{TH})^2} \qquad \text{(i)}$

For a given circuit, V_{TH} and R_{TH} are constants. Therefore, power delivered to the load depends upon R_L. In order to find the value of R_L for which the value of P is maximum, it is necessary to differentiate Eq. (i) w.r.t. R_L and set the result equal to zero,

i.e., for P_{max}, $\frac{dP}{dR_L} = 0.$

So, differentiating Eq. (i) w.r.t. R_L, we get

$$\frac{dP}{dR_L} = \frac{(R_{TH} + R_L)^2 V_{TH}^2 - V_{TH}^2 R_L (2R_{TH} + 2R_L)}{(R_{TH} + R_L)^4} = 0$$

or $\quad (R_{TH} + R_L)^2 V_{TH}^2 - V_{TH}^2 R_L (2R_{TH} + 2R_L) = 0$

or $\quad (R_{TH} + R_L)\ V_{TH}^2\ [(R_{TH} + R_L) - 2R_L] = 0$

or $\quad R_{TH} - R_L = 0$

or $\quad R_L = R_{TH}$

This proves the maximum power transfer theorem.

Power delivered to R_L is given by

$$P = \frac{V_{TH}^2 R_L}{(R_L + R_{TH})^2}$$

When $\quad R_L = R_{TH}$, $P = P_{max}$.

So, $\quad P_{max} = \dfrac{V_{TH}^2 R_{TH}}{(R_{TH} + R_{TH})^2}$

or $\quad P_{max} = \dfrac{V_{TH}^2}{4R_{TH}}$ W

Example 1.86 Calculate the value of R_L for it to absorb the maximum power and find out the maximum power in the circuit of Fig. 1.372.

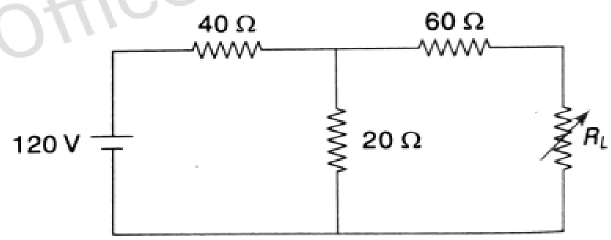

Fig. 1.372

Solution

According to maximum power transfer theorem, maximum power will be transferred from the circuit to the load when R_L is made equal to R_{TH}, the Thevenin's resistance at terminals A and B. Load terminals are marked as A and B.

Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage source by short circuit, we get the modified network as shown in Fig. 1.373.

In Fig. 1.373, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}.

By series-parallel circuit reduction techniques, we have

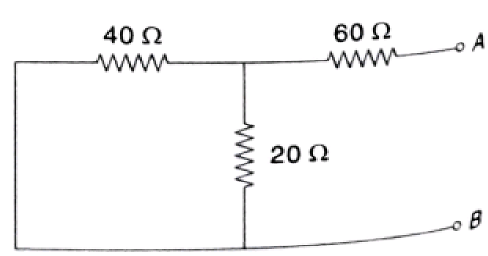

Fig. 1.373

$$R_{TH} = R_{AB} = (40 \parallel 20) + 60 = 13.33 + 60 = 73.33\ \Omega$$

Thus, when $R_L = 73.33\ \Omega$, it absorbs the maximum power.
The maximum power, P_{max} is given by

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}}\ \text{W}$$

Thus, for calculation of P_{max} (maximum power), V_{TH} is required.

Calculation of V_{TH}

Removing the load resistance from the network, we get the following network:

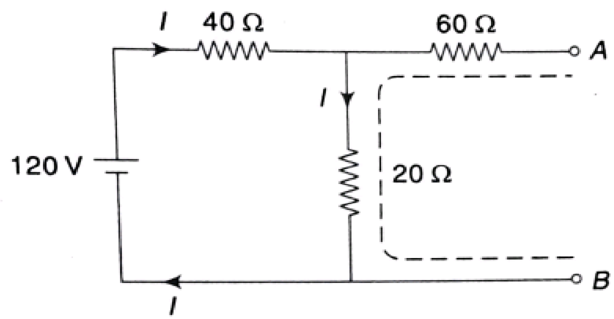

Fig. 1.374

In Fig. 1.374, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. 1.374. As this path contains the 20 Ω resistor, current through this resistance is required. The 120 V source produces the total current I A, which flows through 20 Ω resistor.

By Ohm's law,

$$\text{circuit current, } I = \frac{120}{40+20} = 2\ \text{A}$$

$$\begin{aligned}\text{Hence,}\quad V_{TH} &= V_{AB}\\ &= (20 \times 2) + (60 \times 0)\\ &= 40\ \text{V}\end{aligned}$$

Now, P_{max} can be calculated as

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{(40)^2}{4 \times 73.33} = 5.45\ \text{W}$$

Example 1.87 Find the magnitude of R_L for the maximum power transfer to the circuit shown in Fig. 1.375. Also find out the maximum power.

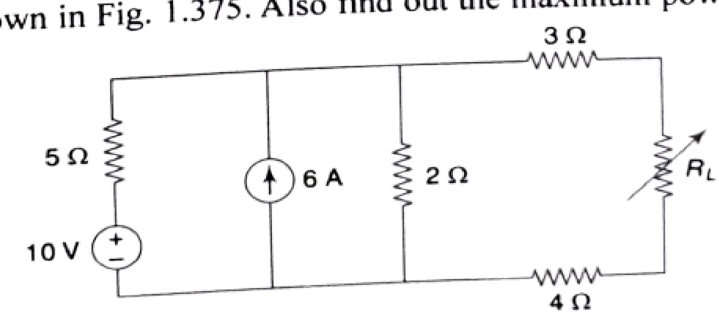

Fig. 1.375

Solution

According to maximum power transfer theorem, maximum power will be transferred from the circuit to the load when R_L is made equal to R_{TH}, the Thevenin's resistance at terminals A and B. Load terminals are marked as A and B.

Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage source by short circuit and current source by open circuit, we get the following network:

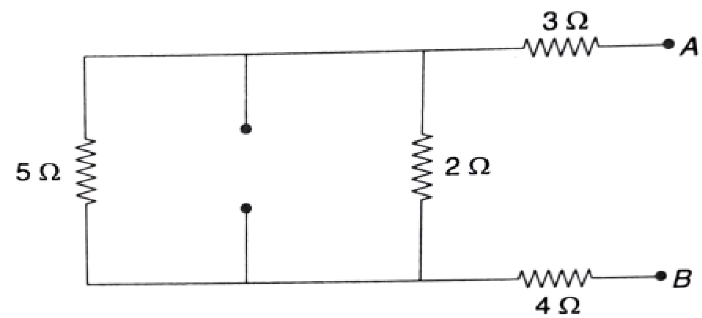

Fig. 1.376

In Fig. 1.376, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}.

By series-parallel circuit reduction techniques, we have

$$R_{TH} = R_{AB} = (5 \parallel 2) + 3 + 4 = 1.43 + 3 + 4 = 8.43\ \Omega$$

Thus, when $R_L = 8.43\ \Omega$, maximum power is transferred to the circuit.

Calculation of V_{TH}

Removing the load resistance from the network, we get the modified network as shown in Fig. 1.377.

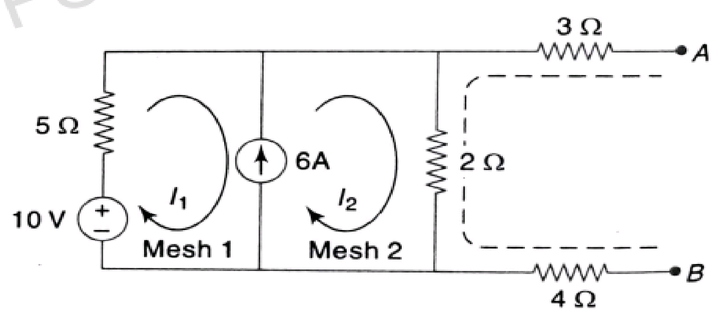

Fig. 1.377

In Fig. 1.377, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. 1.377. As this path contains the resistor 2 Ω, current through this resistance is required. The required current can be calculated by mesh analysis. Mesh 1 and mesh 2 form a supermesh.

By expressing the current in the common branch in terms of mesh currents, we get the current equation as

$$I_2 - I_1 = 6 \qquad \text{(i)}$$

By applying the KVL to the supermesh, we get the voltage equation as

$$10 - 5I_1 - 2I_2 = 0$$

or $$-5I_1 - 2I_2 = -10 \qquad \text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_2 = 5.71 \text{ A}$$

$$\begin{aligned} V_{TH} &= V_{AB} \\ &= (4 \times 0) + (2 \times 5.71) + (3 \times 0) \\ &= 11.42 \text{ V} \end{aligned}$$

Now, P_{max} can be calculated as

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{(11.42)^2}{4 \times 8.43} = 3.87 \text{ W}$$

Example 1.88 In the network of Fig. 1.378, determine the maximum power delivered to R_L.

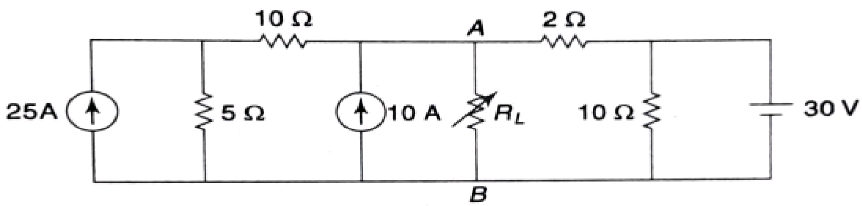

Fig. 1.378

Solution

According to maximum power transfer theorem, maximum power will be transferred from the circuit to the load when R_L is made equal to R_{TH}, the Thevenin's resistance at terminals A and B. Load terminals are marked as A and B.

Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage source by short circuit and current sources by open circuits, we get the following network:

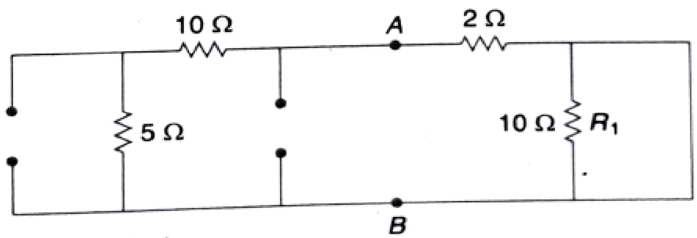

Fig. 1.379

In Fig. 1.379, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. Resistor R_1 gets short circuited, so removing it, we get the network as shown below:

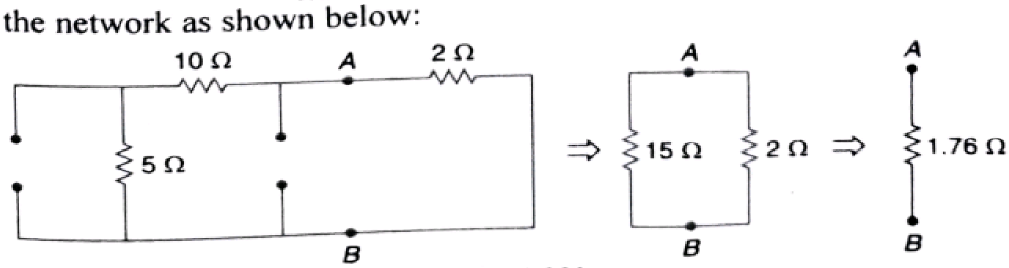

Fig. 1.380

We have $R_{TH} = R_{AB} = 1.76\ \Omega$

Thus, when $R_L = 1.76\ \Omega$, maximum power is transferred to the circuit.

Calculation of V_{TH}

Removing the load resistance from the network, we get the following network:

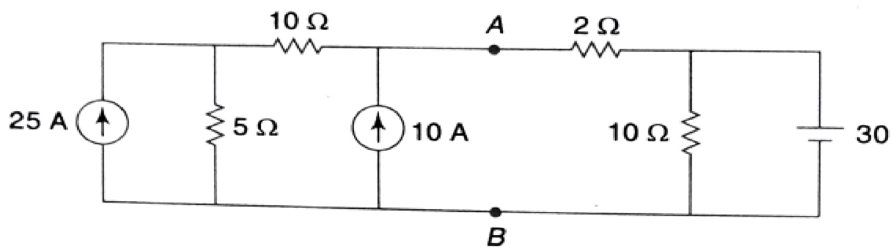

Fig. 1.381

By source transformation, i.e., converting parallel combination of current source of 25 A and resistor of 5 Ω into equivalent series combination of voltage source and resistor, we get the modified network as shown in Fig. 1.382.

In Fig. 1.382, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. 1.382. As this path contains the 2 Ω resistor, current through this resistance is required. The required current can be calculated by mesh analysis.

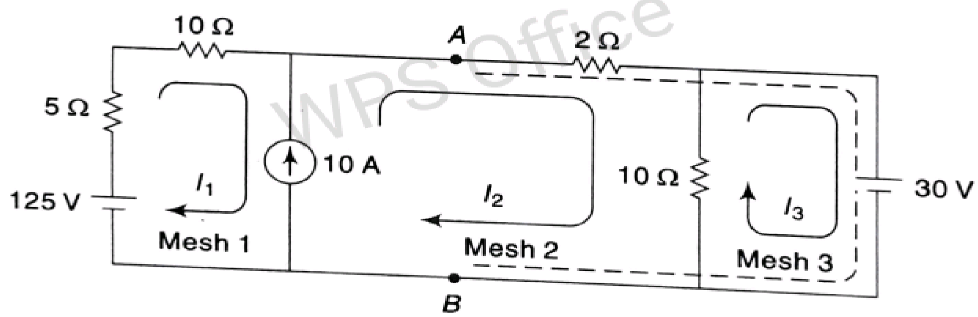

Fig. 1.382

Mesh 1 and mesh 2 form a supermesh.

Expressing the current in the common branch, we get the current equation as

$$I_2 - I_1 = 10 \quad \text{(i)}$$

Applying the KVL to the supermesh, we get the voltage equation as

$$-10I_1 - 2I_2 - 10(I_2 - I_3) + 125 - 5I_1 = 0$$

or $$-15I_1 - 12I_2 + 10I_3 = -125 \quad \text{(ii)}$$

Applying the KVL to mesh 3,

$$-10(I_3 - I_2) - 30 = 0$$

or $$10I_2 - 10I_3 = 30 \quad \text{(iii)}$$

Solving Eqs (i), (ii) and (iii),

$$I_2 = 14.41 \text{ A}$$

Hence, $I_{2\,\Omega} = 14.41\text{ A}(\rightarrow)$

Further, we get $V_{TH} = V_{AB}$

$$= 30 + (2 \times 14.41)$$
$$= 30 + 28.82$$
$$= 58.82\text{ V}$$

Now, P_{max} can be calculated as

$$P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{(58.82)^2}{4 \times 1.76} = 491.45\text{ W}$$

Example 1.89 Find the value of R_L for maximum power transfer and also calculate the maximum power transferred to R_L in the network of Fig. 1.383.

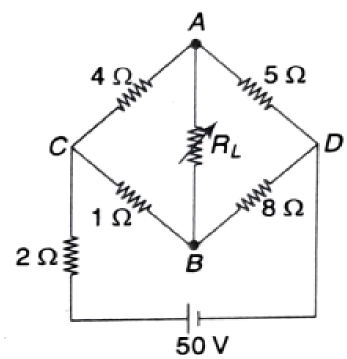

Fig. 1.383

Solution

According to maximum power transfer theorem, maximum power will be transferred from the circuit to the load when R_L is made equal to R_{TH}, the Thevenin's resistance at terminals A and B. Load terminals are marked as A and B.

Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage source by short circuit, we get the network as shown in Fig. 1.384.

In Fig. 1.384, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. For simplification, the circuit can be redrawn as shown in Fig. 1.385.

Converting the delta connections formed by 4 Ω, 5 Ω, and 2 Ω resistors ($\Delta\, CAD$) into equivalent star network, i.e., $\Delta\, CAD \Rightarrow Y\, CAD$, we obtain the modified network as shown in Fig. 1.386.

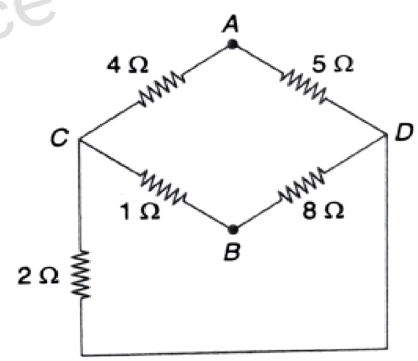

Fig. 1.384

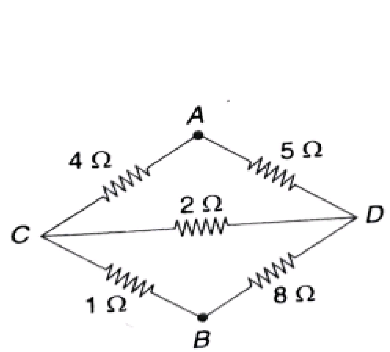

Fig. 1.385

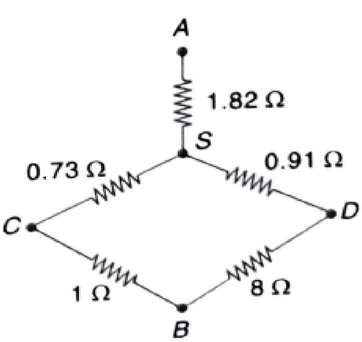

Fig. 1.386

By series-parallel circuit-reduction techniques, we get the following network:

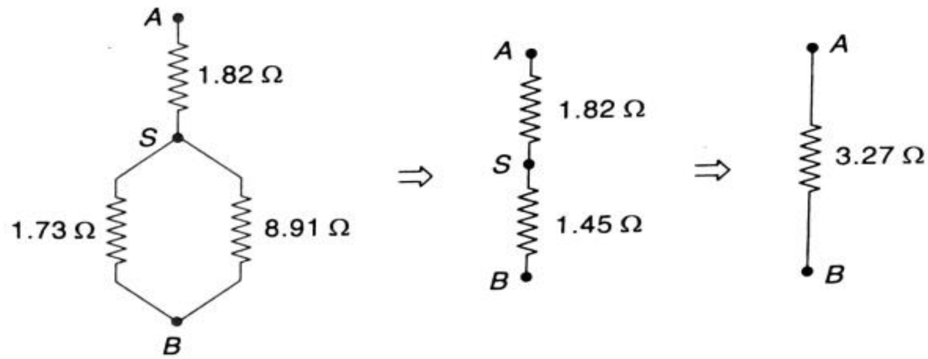

Fig. 1.387

Thus, $R_{TH} = R_{AB} = 3.27\ \Omega$. Hence, when $R_L = 3.27\ \Omega$, maximum power is transferred to the circuit.

Calculation of V_{TH}

Removing the load resistance from the network, we get the following circuit:

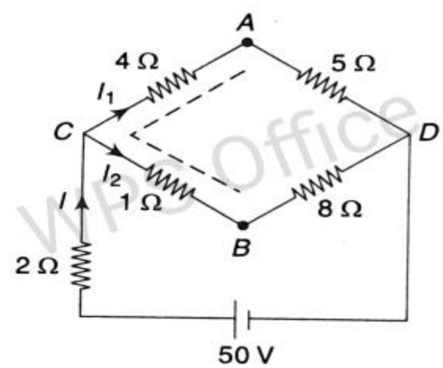

Fig. 1.388

In Fig. 1.388, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. 1.388. As this path contains the resistors 1 Ω and 4 Ω, currents through these resistances are required. In Fig. 1.388, the actual directions of currents are marked. The 50 V source produces the total current I A, which divides at node C. Let current through branch CAD is I_1 and current through branch CBD is I_2. By using series-parallel circuit-reduction techniques, we get the circuits as shown in Fig. 1.389.

From Fig. 1.389(b), the total circuit current, $I = \dfrac{50}{2 + 4.5} = 7.69$ A

From Fig. 1.389(a), the total circuit current divides at node C. By CDR,

$$I_1 = 7.69 \times \frac{9}{9+9} = 3.845 \text{ A}$$

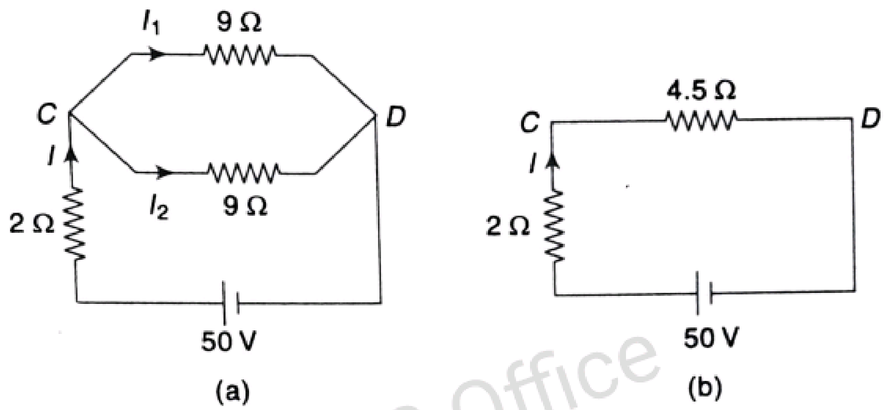

Fig. 1.389

$$I_2 = 7.69 \times \frac{9}{9+9} = 3.845 \text{ A}$$

We now have $V_{TH} = (1 \times I_2) - (4 \times I_1) = (1 \times 3.845) - (4 \times 3.845) = -11.54 \text{ V}$

Hence, $$P_{max} = \frac{V_{TH}^2}{4R_{TH}} = \frac{(11.54)^2}{4 \times 3.27} = 10.18 \text{ W}$$