JESSY

RELATIVITY

Syllabus:- Inertial and non-inertial frames of reference, Galilean transformations, Lorentz transformations (space-time coordinates, Time Dilation, Length Contraction, and Mass-Energy relations. (Numericals)

FRAME OF REFERENCE

1) Inertial Frame of Reference

A frame of reference in which Newton's law of inertia hold good is called an inertial frame of reference. In this frame, an object is not subjected by an external force. It is at rest or moves with a constant velocity. Inertial frames are unaccelerated frame of reference. e.g: class room

2) Non inertial frame of Reference

Non inertial frame of reference is the frame of reference in which Newton's law of inertia is not valid. This is an accelerated frame of reference. In this type of frame of reference velocity is NOT constant. e.g: starting bus.

GALILEAN TRANSFORMATION

The transformation from one inertial frame of reference to another is called Galilean transformation.

Let s & s' be two inertial frames. Let s be at rest and s' is moving with a constant velocity V relative to S. For convenience if we assume that
(i) the origins of the two frames coincide at t=0
(ii) the coordinate axes of the second frame are parallel to first.

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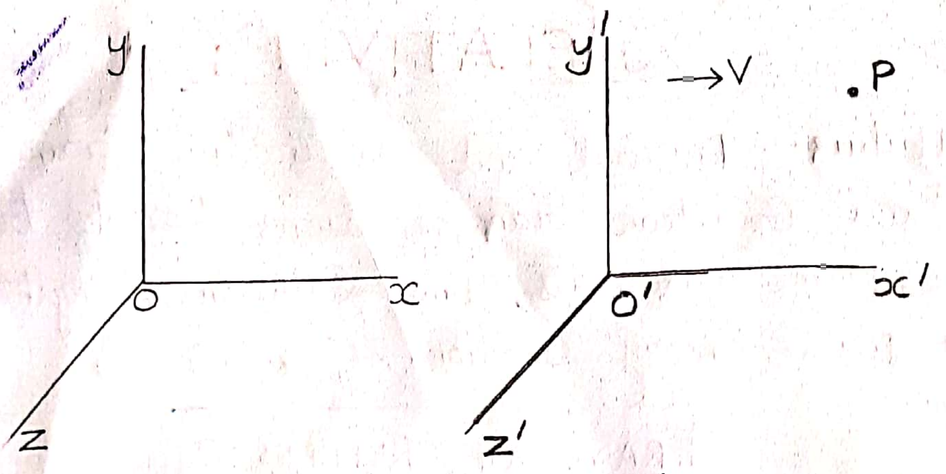


Suppose some event occurs at the point P.

1. Galilean Coordinate Transformation

The observer 'O' in frame S determines the position of the event by coordinates x, y, z, t. The observer O' in frame S' determines the position of the event by x', y', z', t'. There is no relative motion between S and S' along the axes y and z.
Hence $y = y'$ and $z = z'$. The distance measured by S' in positive x direction in time t is vt. So x coordinate of two frames differ by vt.
Hence $x' = x - vt$
Then transformation eqn from S to S' are given by

$$\boxed{\begin{aligned} x' &= x - vt \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}}$$

These four equations are called Galilean coordinate transformation equations.

2. Galilean velocity Transformation

The velocity coordinates of the object in event E observed by O in (S frame) as (u_x, u_y, u_z) and by O' in S' frame as (u_x', u_y', u_z') respectively. Then

$$u_x' = \frac{dx'}{dt'} = \frac{d}{dt}(x - vt)\frac{dt}{dt'} = \frac{dx}{dt} - V \qquad \text{as } \frac{dt}{dt'} = 1$$

Altogether, the Galilean velocity transformations are

$$\boxed{\begin{aligned} u_x' &= u_x - V \\ u_y' &= u_y \\ u_z' &= u_z \end{aligned}}$$

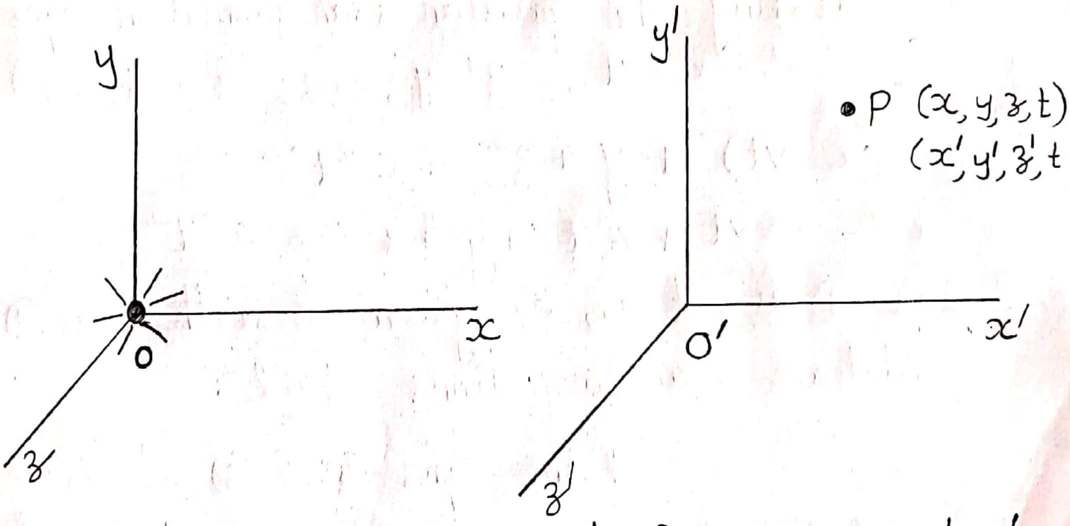
3- Galilean Acceleration Transformation

Let a_x', a_y', a_z' be the acceleration coordinate in S' frame and a_x, a_y, a_z be in S frame respectively then

$$\boxed{\begin{aligned} a_x' &= a_x \\ a_y' &= a_y \\ a_z' &= a_z \end{aligned}}$$

Lorentz Transformations

Galilean transformations are not valid, when the speed of ~~light~~ particle approaches velocity of light c. The transformation equations that apply for all speeds up to 'c' and incorporate the invariance of speed of light were developed by Lorentz in 1890. These are known as Lorentz transformations.



Consider two inertial frames S and S'. S' is moving with a velocity V along the ~~+ve~~ positive x direction. Let t and t' be the time recorded in two frames. For our convenience we will assume that the origins O & O' of the two coordinate system coincide at $t = t' = 0$

Now suppose a source of light is situated at origin O in frame S, from which a wavefront of light is emitted at $t=0$. When the light reaches at point P, the position & time measured by observers O and O' be (x, y, z, t) and (x', y', z', t') respectively.

$c \rightarrow$ velocity of light.

The time measured by light signal in travelling the distance OP in frame S is

$$t = \frac{OP}{c} = \frac{(x^2+y^2+z^2)^{1/2}}{c} \quad \text{or} \quad (x^2+y^2+z^2)^{1/2} = ct$$

In S' frame, the time required by light signal in travelling the distance O'P' is

$$t' = \frac{O'P'}{c} = \frac{(x'^2+y'^2+z'^2)^{1/2}}{c} \quad \text{or} \quad (x'^2+y'^2+z'^2)^{1/2} = ct'$$

$$\therefore \quad x^2+y^2+z^2 = c^2t^2 \quad \text{——— (1)}$$

$$x'^2+y'^2+z'^2 = c^2t'^2 \quad \text{——— (2)}$$

The velocity of light wave is same in both system.

Substituting the Galilean transformation eqns in eqn (2) (i.e $x' = x - vt$, $y' = y$, $z' = z$, $t' = t$) it becomes

$$(x-vt)^2 + y^2 + z^2 = c^2t^2$$

$$x^2 - 2xvt + v^2t^2 + y^2 + z^2 = c^2t^2 \quad \text{——— (3)}$$

Eqn (3) is not in agreement with eqn (1), therefore Galilean transformations fail.

From eqns (1) & (3), it is clear that the terms y and z are in agreement. Hence it may be concluded that $y = y'$ and $z = z'$. The additional term in eqn (3) namely $(-2xvt + v^2t^2)$ indicates that the transformation in x and t require modification.

The transformation for x & t can be taken in the form

$$x' = K(x - vt)$$

$$t' = At + Bx \quad \text{where } K, A, B \text{ \& constants.}$$

∴ New set of equations

$$\left.\begin{aligned} x' &= K(x - vt) \\ y' &= y \\ z' &= z \\ t' &= At + Bx. \end{aligned}\right\} \text{——— (4)}$$

Next try to find K, A, B

Substituting eqns (4) in eqn (2) $\quad x'^2 + y'^2 + z'^2 = c^2t'^2$

$$K^2(x - vt)^2 + y^2 + z^2 = c^2(At + Bx)^2$$

$$K^2(x^2 - 2xvt + v^2t^2) + y^2 + z^2 = c^2(A^2t^2 + B^2x^2 + 2ABxt)$$

$$(K^2 - B^2c^2)x^2 + y^2 + z^2 - (A^2c^2 - K^2v^2)t^2 - 2(ABc^2 + K^2v)xt = 0 \quad \text{——— (5)}$$

Comparing eqn (5) with eqn (1) $\left(x^2 + y^2 + z^2 - c^2t^2 = 0\right)$

$$K^2 - B^2c^2 = 1 \longrightarrow (6.a)$$

$$A^2c^2 - K^2v^2 = c^2 \longrightarrow (6.b)$$

$$ABc^2 + K^2v = 0 \longrightarrow (6.c)$$

$(6.a) \Rightarrow \quad B^2c^2 = K^2 - 1 \quad \therefore \quad B = \sqrt{\dfrac{K^2-1}{c^2}}$ ——— (7)

$(6.b) \Rightarrow \quad A^2c^2 = c^2 + K^2v^2 \quad \quad A = \sqrt{\dfrac{c^2+K^2v^2}{c^2}}$ ——— (8)

substitute A & B in eqn (6.c)

$$\sqrt{\frac{c^2+K^2v^2}{c^2}}\sqrt{\frac{K^2-1}{c^2}}\,c^2 + K^2v = 0$$

$$\sqrt{c^2+K^2v^2}\,\sqrt{K^2-1} = -K^2v$$

squaring $\quad (c^2 + K^2v^2)(K^2 - 1) = K^4v^2$

$$c^2K^2 - c^2 + \cancel{K^4v^2} - K^2v^2 = \cancel{K^4v^2}$$

$$\cancel{K^2}\ \cancel{=}\ K^2(c^2 - v^2) = c^2 \Rightarrow K^2 = \frac{c^2}{c^2 - v^2}$$

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$$K^2 = \frac{1}{\frac{c^2 - v^2}{c^2}}$$

$$\therefore \quad K = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{——— (9)}$$

sub. (9) in eqn (8)

$$A = \sqrt{\frac{c^2 + K^2 v^2}{c^2}} = \sqrt{1 + \left(\frac{\cancel{c^2}}{c^2 - v^2}\right)\frac{v^2}{\cancel{c^2}}} = \sqrt{\frac{c^2 - \cancel{v^2} + \cancel{v^2}}{c^2 - v^2}}$$

$$A = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{——— (10)} \quad \therefore \; A = K$$

eqn (6.c) $\Rightarrow$ $KBc^2 + K^2 V = 0 \quad \Rightarrow Bc^2 = -KV$

$$\therefore \; B = \frac{-KV}{c^2} \quad \text{——— (11)}$$

Substitute the value of constants A, B, K in eqn (4)

$$\boxed{\begin{aligned} x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \end{aligned}} \Rightarrow \text{Lorentz Transformation Equations.}$$

$t' = At + Bx$

$t' = Kt - \frac{KV}{c^2}x$

$= K\left(t - \frac{vx}{c^2}\right)$

$$\boxed{\begin{aligned} x' &= \gamma(x - vt) \\ y' &= y \\ z' &= z \\ t' &= \gamma\left(t - \frac{vx}{c^2}\right) \end{aligned}}$$

where $\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$

The Inverse transformations from reference frame s' to s is

$$\left.\boxed{\begin{aligned} x &= \gamma(x' + vt') \\ y &= y' \\ z &= z' \\ t &= \gamma\left(t' + \frac{vx'}{c^2}\right) \end{aligned}}\right\} \Rightarrow \text{Inverse Lorentz Transformation Equations.}$$

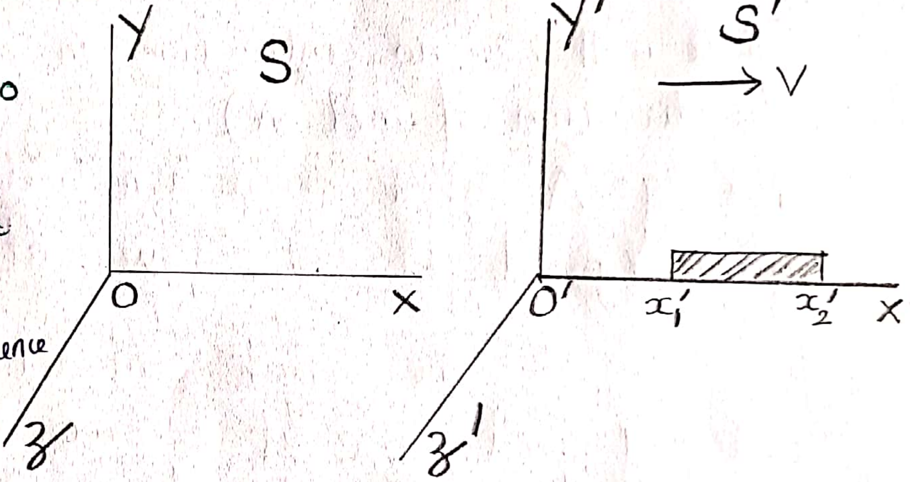
When $v \ll c$, i.e for values of speed v negligibly small compared to c; $\frac{v}{c} \to 0$ and the Lorentz transformation reduces to Galilean transformation.

Consequences of Lorentz Transformation

1 LENGTH CONTRACTION

Consider two identical frame of reference S & S' with their axes parallel and S' is moving along positive x direction. Imagine a rigid rod with its length L_0 parallel to x direction in system S', in which it is at rest.

The length L_0 is called Proper length (The length of body measured in reference frame in which body is at rest)



Proper length $L_0 = x_2' - x_1'$

Let x_1 and x_2 be the x coordinate of the ends of the rod as measured by an observer in stationary frame S; Then in S frame observed length of rod is

Observed length $L = x_2 - x_1$

By Lorentz transformation

$$x_1' = \gamma (x_1 - vt) \quad \text{—— ①} \qquad \text{where } \gamma = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$x_2' = \gamma (x_2 - vt) \quad \text{—— ②}$$

② — ① $\quad x_2' - x_1' = \gamma (x_2 - x_1)$

$$\boxed{L_0 = \gamma L}$$

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$L_0 = \gamma L \quad \therefore L = \frac{L_0}{\gamma}$ where $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$

$$\boxed{L = L_0\sqrt{1-\frac{v^2}{c^2}}}$$

As the factor $\sqrt{1-\frac{v^2}{c^2}}$ is always less than 1, [or $\gamma > 1$]

$\therefore \boxed{L < L_0}$

Thus the length of the rod appears to be smaller than its rest length. This phenomenon of shortening or contraction in the length of an object along direction of motion relative to an observer is called length contraction.

(There will be no contraction in directions perpendicular to direction of motion)

For eg: A sphere in motion will appear to be an ellipsoid to an observer at rest

The length contraction is negligible at ordinary speed. But it becomes important at relativistic speeds, i.e at speeds comparable to that of light.

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2. TIME DILATION

Consider a frame s' is moving along the positive x direction with a velocity v relative to another frame s. Let a clock be fixed at a point x' in system s'. Hence the clock is in motion with velocity v relative to an observer s. Let this clock measure times of two events in s' as t_1' and t_2'. Then time interval between this two events is

$$\Delta t' = t_2' - t_1'$$

For an observer in s, the time interval between this events is

$$\Delta t = t_2 - t_1$$

using Inverse Lorentz transformation,

$$t_1 = \gamma\left(t_1' + \frac{vx'}{c^2}\right)$$

$$t_2 = \gamma\left(t_2' + \frac{vx'}{c^2}\right)$$

$$t_2 - t_1 = \gamma\,(t_2' - t_1')$$

$$\boxed{\Delta t = \gamma\, \Delta t'}$$

$\Delta t'$ is proper time

$$\Delta t = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}}$$

As $\gamma = \frac{1}{\sqrt{1-v^2/c^2}} > 1$ $\therefore$ $\boxed{\Delta t > \Delta t'}$

Thus the time interval measured in frame S is greater than the time in frame s'. That means to a stationary observer, the moving clock will appear to go slow. This effect is called Time dilation or lengthening of time interval. That is moving clock run slower compared to observer's own clock.

If v is very small, $\frac{v^2}{c^2} \to 0$ and $\Delta t = \Delta t'$
(Then time interval between two events in both frames are same)

MASS ENERGY RELATION

Einstein mass energy relation is

$$E = mc^2$$

where $m = \dfrac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$ or $m = \gamma m_0$

where $m \rightarrow$ Relativistic mass (moving mass with $v \approx c$)
$m_0 \rightarrow$ rest mass of body
$v \rightarrow$ velocity of motion
$c \rightarrow$ velocity of light.

Proof

Let a force 'F' be acting on a ~~par~~ body so that its kinetic energy increases. The gain in KE will be work done on body.

The gain in KE; $dE_K = dW = F\,dx$ $\rightarrow$ where $F = \frac{dP}{dt} = \frac{d}{dt}(mv)$

$$dE_K = \frac{d}{dt}(mv) \cdot dx$$

$$= \left(m\frac{dv}{dt} + v\frac{dm}{dt}\right)\frac{dx}{dt} \cdot dt \quad \rightarrow \text{where } \frac{dx}{dt} = v$$

$$dE_K = mv\,dv + v^2 dm \quad \text{——— (1)}$$

The Relativistic mass $m = \gamma m_0$

$$m = \frac{m_0}{\sqrt{1-\frac{v^2}{c^2}}}$$

(differentiate this eqn; for easy steps; first square this)

$$m^2\left(1-\frac{v^2}{c^2}\right) = m_0^2$$

$$m^2c^2 - m^2v^2 = m_0^2c^2$$

Differentiating above eqn (with m_0 & $c \rightarrow$ constants)

$$2m\,dm\,c^2 - 2m\,dm\,v^2 - 2v\,dv\,m^2 = 0 \quad \text{(cancel 2m)}$$

$$c^2 dm = v^2 dm + mv\,dv \quad \text{——— (2)}$$

Comparing eqn (1) & (2); $c^2 dm = dE_K$ ——— (3)

For a rest object $v = 0$; $m = m_0$

For a particle move with velocity v, mass $= m$

Integrating equation (3)

$$\int dE_K = c^2 \int_{m_0}^{m} dm$$

$$E_K = c^2(m - m_0)$$

$$E_K = mc^2 - m_0c^2$$

$$mc^2 = E_K + m_0c^2$$

$$\boxed{E = mc^2} \longrightarrow (4)$$

$\therefore$ Total energy $E = E_K + m_0c^2$ where Rest energy

$E = E_K + E_0$ $\qquad E_0 = m_0c^2$

Eqn (4) is known as Einstein mass energy relation. This imply that energy manifests as mass.

Extra:

Postulates of Special Theory of Relativity

(1) Principle of relativity :– All the laws of physics are the same in all inertial frames of reference.

(2) The principle of independence of velocity of light:
The speed of light in vacuum is constant and is independent of the motion of light source or receiver

PROBLEMS

Qn: A 1 m long rod is moving along its length with a velocity 0.6c. Calculate its length as it appears to an observer on the earth?

Solu^n:

$$\boxed{L_o = \gamma L} \quad \text{where } \gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$$

Data
$L_o = 1m$
$v = 0.6c$
$L = ?$

note: $L_o \rightarrow$ proper length

$$L_o = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} L$$

$$L = L_o\sqrt{1-\frac{v^2}{c^2}}$$

$$= 1m \times \sqrt{1-\left(\frac{0.6c}{c}\right)^2}$$

$$= \sqrt{1-0.36} = \sqrt{0.64}$$

$$= \underline{0.8m}$$

Qn: The length of a rocket ship is 100m long on ground. During its flight, the apparent length is found to be 99m when measured from the ground. What is its speed?

Solu^n:

$$\boxed{\begin{aligned} L_o &= \gamma L \\ L &= L_o\sqrt{1-\frac{v^2}{c^2}} \end{aligned}}$$

Data
$L_o = 100m$
$L = 99m$
note: $\boxed{L_o > L}$
$v = ?$

$$1-\frac{v^2}{c^2} = \left(\frac{L}{L_o}\right)^2$$

$$\frac{v^2}{c^2} = 1-\left(\frac{L}{L_o}\right)^2$$

$$= 1-\left(\frac{99}{100}\right)^2$$

$$v^2 = 0.0199\, c^2$$

$$v = \underline{0.141\, c\ m/s} = \underline{0.423 \times 10^8\ m/s}$$

Qn: What is the length of a meter stick moving parallel to its length when its mass is $\frac{3}{2}$ times of its rest mass?

Solun:

$$L_o = \gamma L$$

$$L = L_o\sqrt{1-\frac{v^2}{c^2}}$$

$$m = \gamma m_0$$

$$m = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} m_0$$

$$\sqrt{1-\frac{v^2}{c^2}} = \frac{m_0}{m} = \frac{m_0}{\frac{3}{2}m_0} = \frac{2}{3}$$

$$\therefore \quad L = L_o\sqrt{1-\frac{v^2}{c^2}} = 1\times\frac{2}{3} = \underline{0.67m}$$

Data

$L_o = 1m$

$m = \frac{3}{2}m_0$

$L = ?$

Qn: In the laboratory, the life time of a particle moving with speed 2.8×10^8 m/sec is found to be 2×10^{-7} sec. Calculate the proper life time of the particle?

Soln:

$$\Delta t = \gamma \Delta t'$$

$$\Delta t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\Delta t'$$

$$\Delta t' = \Delta t\sqrt{1-\frac{v^2}{c^2}}$$

$$= 2\times10^{-7}\sqrt{1-\left(\frac{2.8\times10^8}{3\times10^8}\right)^2}$$

$$= 2\times10^{-7}\times .3590$$

$$= \underline{0.718\times10^{-7}} \text{ sec}$$

Data

$\Delta t > \Delta t'$

$\Delta t' \Rightarrow$ proper life time

$V = 2.8\times10^8 m/s$

$\Delta t = 2\times10^{-7} sec$

$\Delta t' = ?$

Qn: What is the velocity of π mesons whose observed mean life is 2.5×10^{-7} sec. The proper mean life of these π mesons is 2.5×10^{-8} sec.

Solun:

$$\Delta t = \gamma \Delta t'$$

$$\Delta t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\Delta t'$$

$$1-\frac{v^2}{c^2} = \left(\frac{\Delta t'}{\Delta t}\right)^2$$

$$\frac{v^2}{c^2} = 1-\left(\frac{\Delta t'}{\Delta t}\right)^2$$

$$\frac{v^2}{c^2} = 1-\left(\frac{2.5\times10^{-8}}{2.5\times10^{-7}}\right)^2 = 0.99$$

$$V = \underline{0.995c \ m/s}$$

Data

$\Delta t = 2.5\times10^{-7}$ sec

$\Delta t' = 2.5\times10^{-8}$ sec

$V = ?$

Remember $\boxed{\Delta t > \Delta t'}$

Qn: (14) At what speed should a clock to be moved so that it may appear to lose 1 minute in each hour?

Soln:

$$\Delta t = \gamma \Delta t'$$

$$\Delta t = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \Delta t'$$

$$\left(1-\frac{v^2}{c^2}\right) = \left(\frac{\Delta t'}{\Delta t}\right)^2$$

$$\frac{v^2}{c^2} = 1 - \left(\frac{\Delta t'}{\Delta t}\right)^2$$

$$= 1 - \left(\frac{59}{60}\right)^2 = 0.0331$$

$$V = \underline{0.1819 c}$$

Data

$\Delta t = 1 hr = 60 minute$

$\Delta t' = 59 minute$

$\Delta t > \Delta t'$

$V = ?$

Qn: At what velocity will the mass of a body is 2.25 times its rest mass?

Soln:

$$m = \gamma m_0$$

$$m = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}} m_0$$

$$1 - \frac{V^2}{c^2} = \left(\frac{m_0}{m}\right)^2 = \left(\frac{\cancel{2.25} m_0}{2.25 m_0}\right)^2$$

$$\frac{V^2}{c^2} = 1 - 0.1975 = 0.8025$$

$$V = \underline{0.9083 c}$$

Data

$V = ?$

$m = 2.25 m_0$

$c = 3 \times 10^8 m/s$

Qn: The sun radiates away energy at the rate of 4×10^{26} J/s. Calculate the rate at which its mass is decreasing?

Soln:

$$E = mc^2$$

$$\Delta E = \Delta m \, c^2$$

Change in mass $\Delta m = \frac{\Delta E}{c^2}$

$$\Delta m = \frac{4 \times 10^{26} J/S}{9 \times 10^{16} m^2/s^2} = \underline{0.444 \times 10^{10}} \text{ kg}$$

$kg \frac{m^2}{s^2} \cdot \frac{s}{m^2}$

Data

$\Delta E = 4 \times 10^{26} J/s$

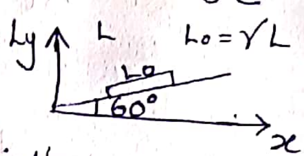
$\Delta m = ?$

$c = 3 \times 10^8 m/s$

Qn: Calculate the percentage contraction in the length of a rod moving with a speed of 0.8c in a direction at an angle 60° with its own length

Soln: Let the rod of length L_o at rest in frame S. Let S′ be a reference frame moves with a speed of 0.8c in a direction making an angle 60° with x axis.

Data
$V = 0.8c$
$L_o = \gamma L$



The component of L_o along x axis $L_x = L_o \cos 60$

∴ observed length in x direction : L_x'

$\therefore L_x = \gamma L_x'$

$$L_x' = L_x \sqrt{1 - \frac{v^2}{c^2}}$$

$$= L_o \cos 60 \sqrt{1 - \frac{v^2}{c^2}}$$

$$= \frac{L_o}{2} \sqrt{1 - \left(\frac{0.8c}{c}\right)^2} = 0.3\, L_o$$

$1.00 - .64 = .36$

The component of L_o along the y axis in S & S′ frame

$$L_y' = L_y = L_o \sin 60 = \frac{\sqrt{3}\, L_o}{2}$$

∴ Resultant length $L' = \sqrt{L_x'^2 + L_y'^2}$

$$L' = \sqrt{(0.3L_o)^2 + \left(\frac{\sqrt{3}L_o}{2}\right)^2} = 0.87\, L_o$$

$$\%\ \text{contraction} = \frac{\text{original length} - \text{observed length}}{\text{original length}} \times 100$$

$$= \frac{L_o - L'}{L_o} \times 100 = \frac{L_o - 0.87 L_o}{L_o} \times 100$$

$$= \underline{13}$$

Qn: The average life time of a free neutron at rest is 15 minutes. It disintegrates spontaneously into an electron, proton and neutrino. What is the average velocity with which a neutron must leave the Sun in order to reach the earth before breaking up? Given the distance of earth from sun is 11×10^{10} m.

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solⁿ: The average life time Δt of a moving neutron, as measured by an observer on earth is

$$\boxed{\Delta t = \gamma \Delta t'}$$

Data

proper life time $\Delta t' = 15$ minute

$\Delta t' = 15 \times 60$ seconds

$d = 11 \times 10^{10}$

$V = ?$

Also Δt is the time for neutron to reach earth after leaving the sun

$$\Delta t = \frac{d}{V} = \frac{11\times10^{10}}{V} \quad \text{---(1)}$$

$$\Delta t = \gamma \Delta t' = \frac{15\times60\ s}{\sqrt{1-\frac{V^2}{c^2}}} \quad \text{---(2)}$$

$$\frac{11\times10^{10}}{V} = \frac{15\times60}{\sqrt{1-\frac{V^2}{c^2}}}$$

$$(15\times60)^2 V^2 = \left(1-\frac{V^2}{c^2}\right)(11\times10^{10})^2$$

$$V^2\left[900^2+\left(\frac{11\times10^{10}}{c}\right)^2\right] = (11\times10^{10})^2$$

$$V^2(9.444\times10^5) = (11\times10^{10})^2$$

$$V = \frac{11\times10^{10}}{971.825}$$

$$= \underline{\underline{1.13\times10^8\ m/s}}$$

Qn: A meter stick is projected into space at such a velocity that its length appears to become half the original length. Calculate velocity of metre stick

solⁿ:

$$L_0 = \gamma L$$

Data

proper length $L_0 = 1m$

$L = \frac{L_0}{2}$

$V = ?$

$$L_0 = \frac{1}{\sqrt{1-\frac{V^2}{c^2}}}\frac{L_0}{2}$$

$$\sqrt{1-\frac{V^2}{c^2}} = \frac{1}{2}$$

$$1-\frac{V^2}{c^2} = \frac{1}{4}$$

$$V^2 = \frac{3}{4}c^2$$

$$V = \underline{\underline{\frac{\sqrt{3}}{2}c\ m/s}}$$

Study well

Regards

JESSY