1.18 Norton's Theorem

Norton's theorem is converse of Thevenin's theorem. Norton's equivalent circuit uses a current source instead of voltage source and a resistance R_N (which is same as R_{TH}) in parallel with the source instead of being in series with it.

E.L. Norton, an engineer employed by the Bell Laboratory, USA, first developed this theorem. According to this theorem, any two-terminal network can be replaced by a single current source of magnitude I_N (called Norton current) in parallel with a single resistance R_N (called Norton resistance). Figure 1.328(a) shows a complex network enclosed in a box with two terminals A and B brought out. The network in the box may contain any number of resistors and sources connected in any fashion. But according to Norton, the entire circuit behind terminals AB can be replaced by a current source I_N in parallel with a resistance R_N as shown in Fig. 1.328(b). The current I_N is equal to the current that would flow when terminals A and B are short circuited, i.e., I_N, is equal to the current flowing through short-circuited terminals AB. The resistance R_N is the same as Thevenin's resistance R_{TH}, i.e., R_N is the resistance measured at AB with load removed and replacing all sources by their internal resistances.

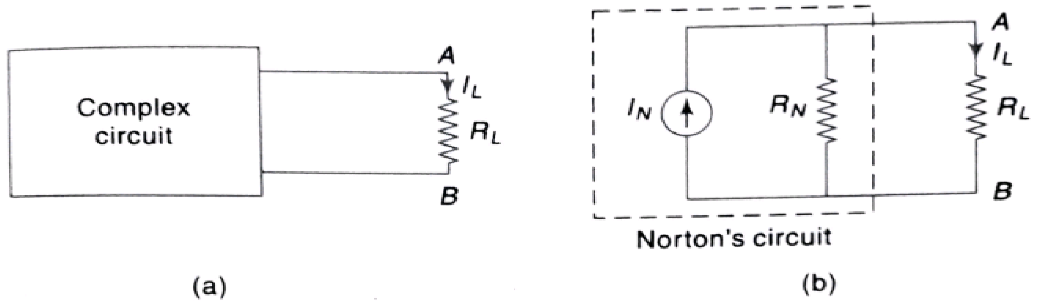

Fig. 1.328 Norton's equivalent circuit

Hence, Norton's theorem as applied to dc circuits may be stated as under:

Any complex network having two terminals A and B can be replaced by a current source of current output I_N in parallel with a resistance R_N.

(i) The output I_N of the current source is equal to the current that would flow through AB when A and B are short circuited.

(ii) The resistance R_N is the resistance of the network measured between A and B with load removed and replacing the source with their internal resistances.

Steps to apply Norton's theorem

Step 1: Short the branch resistance through which current is to be calculated.

Step 2: Obtain the current through this short-circuited branch, using any of the network-simplification techniques. This current is Norton's current I_N.

Step 3: Calculate R_N as viewed through the two terminals of the branch from which current is to be calculated by removing that branch resistance and replacing all sources by their internal resistances.

Step 4: Draw the Norton's equivalent circuit showing current source I_N, with the resistance R_N in parallel with it.

Step 5: Reconnect the branch resistance. Let it be R_L. The required current through the branch is given by

$$I_L = I_N \frac{R_N}{R_L + R_N}$$

Example 1.80 By Norton's theorem, find the current in 20 Ω in the network shown in Fig. 1.329.

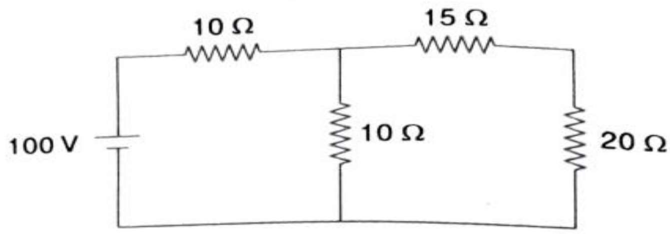

Fig. 1.329

Solution

Current through 20 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.

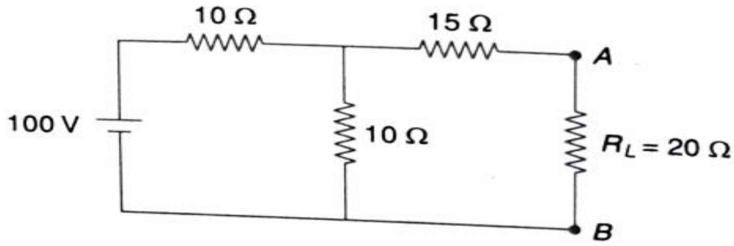

Fig. 1.330

Step I: Calculation of I_N

Removing the load resistance from the network and short circuiting the load terminals, we get the modified network as shown in Fig. 1.331.

In Fig. 1.331, current flowing through the short circuit placed across the load terminals A and B is called Norton current I_N. This current can be calculated by mesh analysis as shown below.

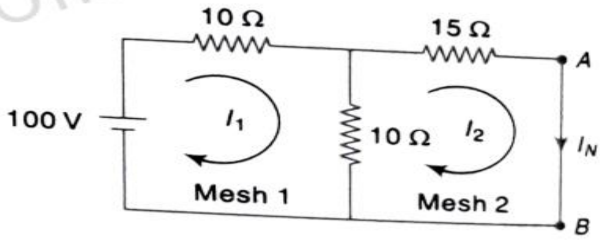

Fig. 1.331

Applying the KVL to mesh 1,

$$-10I_1 - 10(I_1 - I_2) + 100 = 0$$

or

$$-20I_1 + 10I_2 = -100 \quad \text{(i)}$$

Applying the KVL to mesh 2,

$$-15I_2 - 10(I_2 - I_1) = 0$$

or

$$10I_1 - 25I_2 = 0 \quad \text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_2 = 2.5 \text{ A}$$

Hence, $I_N = 2.5$ A, from A to B

Step II: Calculation of R_N

Removing the load resistance from the network and replacing the voltage source by short circuit, we get the network as shown in Fig. 1.332.

In Fig. 1.332, equivalent resistance across the load terminals A and B is called Norton's resistance R_N.

Thus, $R_N = R_{AB} = (10 \parallel 10) + 15 = 20\ \Omega$

Step III: Calculation of load current

Norton's equivalent circuit can be drawn as shown in Fig. 1.333.

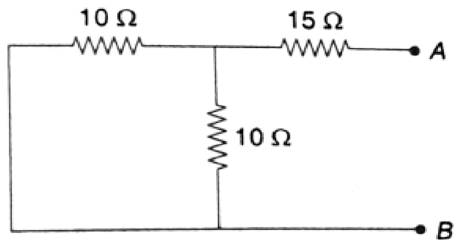

Fig. 1.332

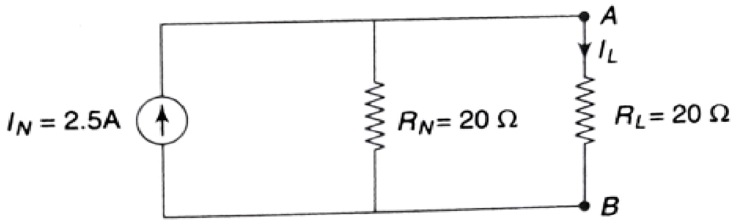

Fig. 1.333

By current division rule,

$$I_L = I_{20\Omega} = 2.5 \times \frac{20}{20+20} = 1.25\ \text{A}(\downarrow)$$

Example 1.81 By Norton's theorem, find the current in 4 Ω resistor in the network shown in Fig. 1.334.

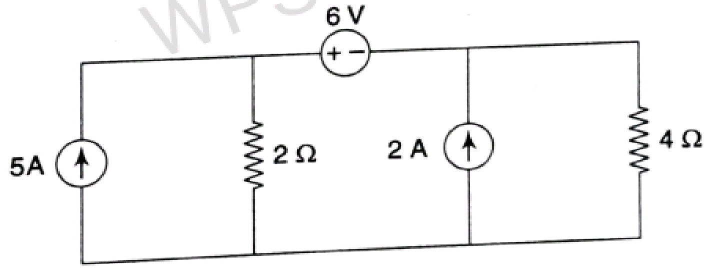

Fig. 1.334

Solution

Current through 4 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.

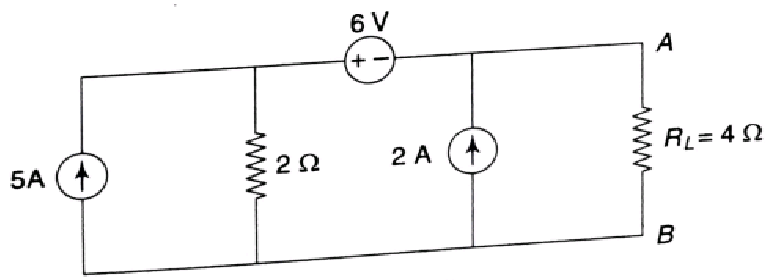

Fig. 1.335

Step I: Calculation of I_N

Removing the load resistance from the network and short circuiting the load terminals, we get the modified circuit as shown in Fig. 1.336.

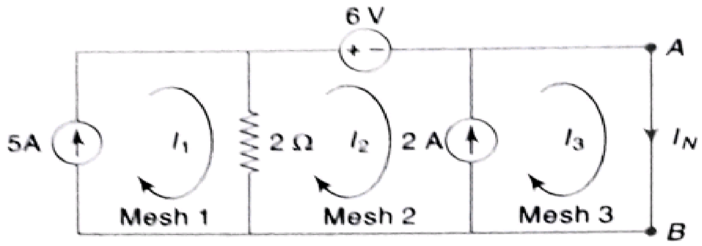

Fig. 1.336

In Fig. 1.336, current flowing through the short circuit placed across the load terminals A and B is called Norton's current I_N. This current can be calculated by mesh analysis as follows.

In mesh 1, current source of 5A is in the direction of mesh current I_1.

So, $\quad I_1 = 5 \quad$ (i)

Mesh 2 and mesh 3 form a supermesh.

By expressing the current in the common branch, we get the current equation as

$$(I_3 - I_2) = 2 \quad \text{(ii)}$$

By applying the KVL to the supermesh, we get the voltage equation as

$$-6 - 2(I_2 - I_1) = 0$$

or $\quad 2I_1 - 2I_2 = 6 \quad$ (iii)

Solving Eqs (i), (ii) and (iii),

$$I_3 = 4 \text{ A}$$

Hence, $I_N = 4$ A; from A to B

Step II: Calculation of R_N

Removing the load resistance from the network and replacing the voltage source by short circuit and current sources by open circuits, we get the network as shown in Fig. 1.337.

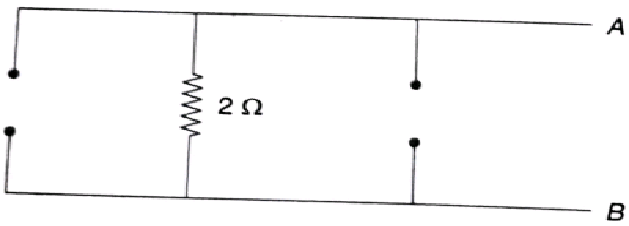

Fig. 1.337

Thus, $R_N = R_{AB} = 2\ \Omega$.

Step III: Calculation of load current

Norton's equivalent circuit can be drawn as shown in Fig. 1.338.

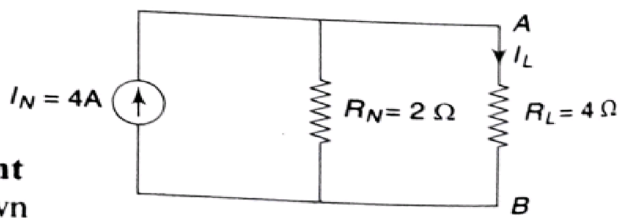

Fig. 1.338

By current division rule,

$$I_L = I_{4\Omega} = 4 \times \frac{2}{4+2} = 1.33 \text{ A}(\downarrow)$$

Example 1.82 By Norton's theorem, find the current in 4 Ω resistor in the network shown in Fig. 1.339.

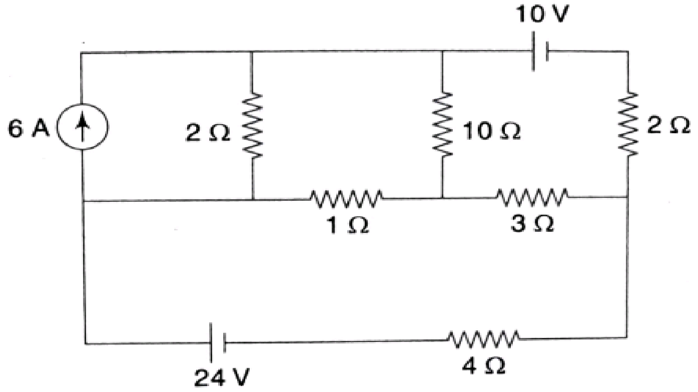

Fig. 1.339

Solution

Current through 4 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals (see Fig. 1.340).

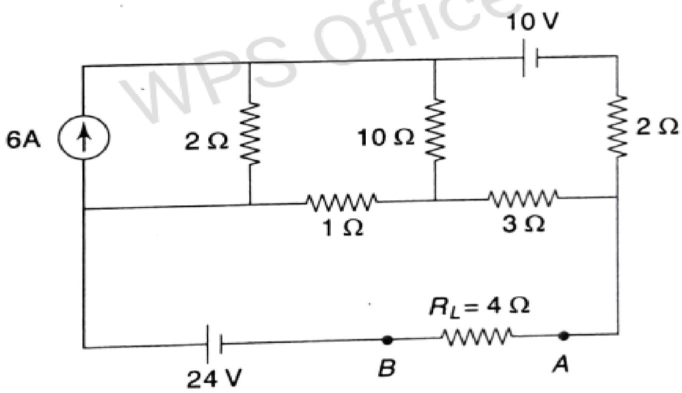

Fig. 1.340

Step I: Calculation of I_N

Removing the load resistance from the network and short circuiting the load terminals, we get the network as shown in Fig. 1.341.

In Fig. 1.341, current flowing through the short circuit placed across the load terminals A and B is called Norton's current I_N.

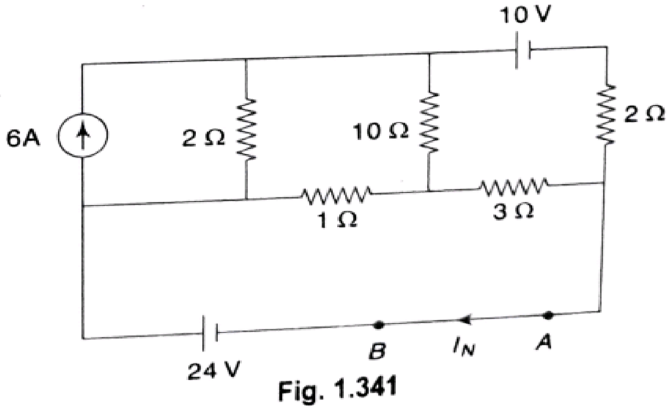

Fig. 1.341

By source transformation, i.e., converting parallel combination of current source of 6 A and resistor of 2 Ω into equivalent series combination of voltage source and resistor, we get the modified network as shown in Fig. 1.342.

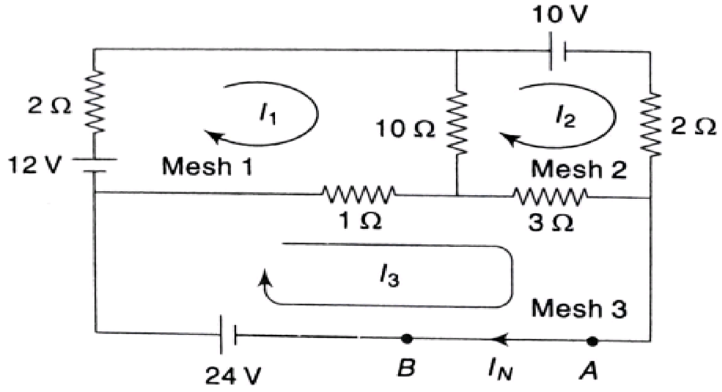

Fig. 1.342

Applying KVL to mesh 1,

$$-2I_1 - 10(I_1 - I_2) - (I_1 - I_3) + 12 = 0$$

or $$-13I_1 + 10I_2 + I_3 = -12 \qquad \text{(i)}$$

Applying the KVL to mesh 2,

$$-10(I_2 - I_1) - 10 - 2I_2 - 3(I_2 - I_3) = 0$$

or $$10I_1 - 15I_2 + 3I_3 = 10 \qquad \text{(ii)}$$

Applying the KVL to mesh 3,

$$-(I_3 - I_1) - 3(I_3 - I_2) + 24 = 0$$

or $$I_1 + 3I_2 - 4I_3 = -24 \qquad \text{(iii)}$$

The value of I_3 may be found by solving the above three simultaneous equations or by the method of determinants as given below:

Putting the above three equations in matrix form, we have

$$\begin{bmatrix} -13 & 10 & 1 \\ 10 & -15 & 3 \\ 1 & 3 & -4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} -12 \\ 10 \\ -24 \end{bmatrix}$$

So, $$\Delta = \begin{vmatrix} -13 & 10 & 1 \\ 10 & -15 & 3 \\ 1 & 3 & -4 \end{vmatrix} = -188, \qquad \Delta_3 = \begin{vmatrix} -13 & 10 & -12 \\ 10 & -15 & 10 \\ 1 & 3 & -24 \end{vmatrix} = -2330$$

By Cramer's rule,

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-2330}{-188} = 12.39 \text{ A}$$

Hence, $I_N = 12.39$ A, from A to B

Step II: Calculation of R_N

Removing the load resistance from the network and replacing the voltage source by short circuits and current source by open circuit, we get the following circuit:

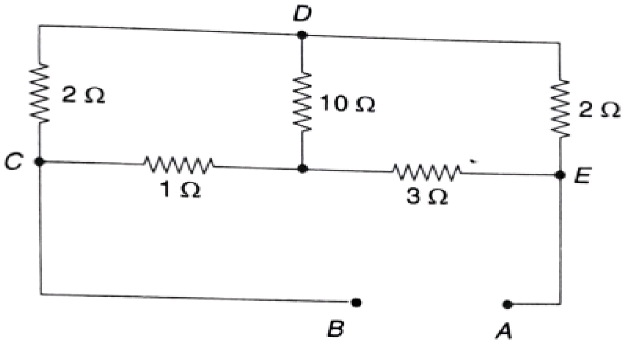

Fig. 1.343

Converting the star connection formed by 1 Ω, 10 Ω, and 3 Ω resistors ($Y\,CDE$) into equivalent delta connection, i.e., $Y\,CDE \Rightarrow \Delta\,CDE$, we get the circuit as shown in Fig. 1.344.

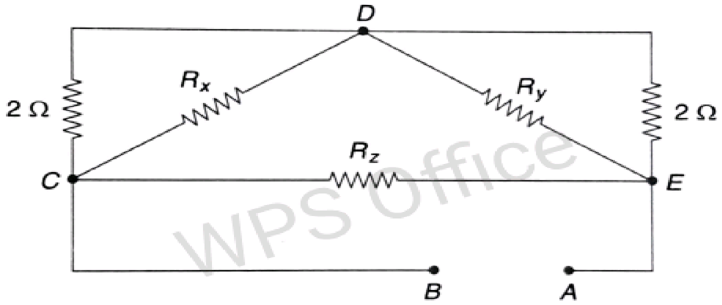

Fig. 1.344

We have $R_x = 1 + 10 + \dfrac{1 \times 10}{3} = 14.33\ \Omega,\qquad R_y = 10 + 3 + \dfrac{10 \times 3}{1} = 43\ \Omega,$

$$R_z = 1 + 3 + \frac{1 \times 3}{10} = 4.3\ \Omega$$

The simplified network is shown in Fig. 1.345.

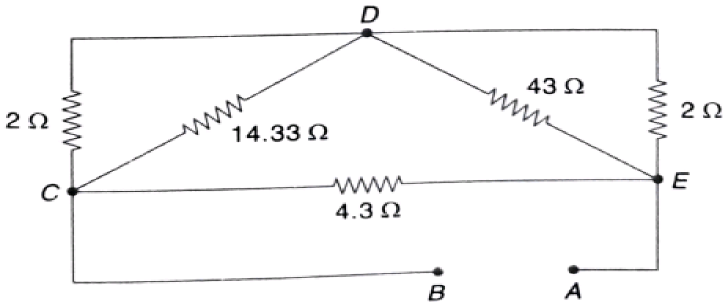

Fig. 1.345

By series-parallel circuit reduction techniques, we get the following network:
Thus, $R_N = R_{AB} = 1.98\ \Omega$.

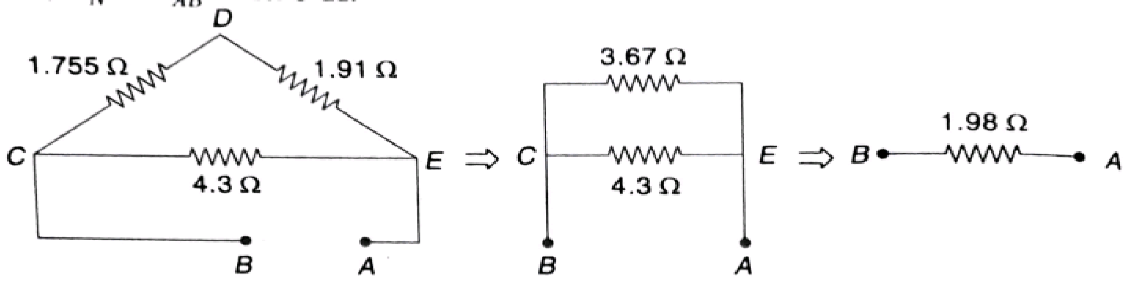

Fig. 1.346

Step III: Calculation of load current

Norton's equivalent circuit can be drawn as shown below:
By current division rule,

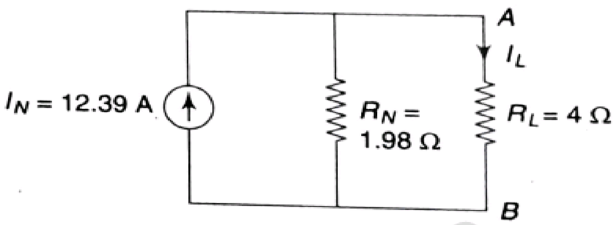

Fig. 1.347

$$I_L = I_{4\Omega} = 12.39 \times \frac{1.98}{1.98 + 4} = 4.1\ \text{A}(\leftarrow)$$

Example 1.83 By Norton's theorem, find the current in 5 Ω resistor in the network shown in Fig. 1.348.

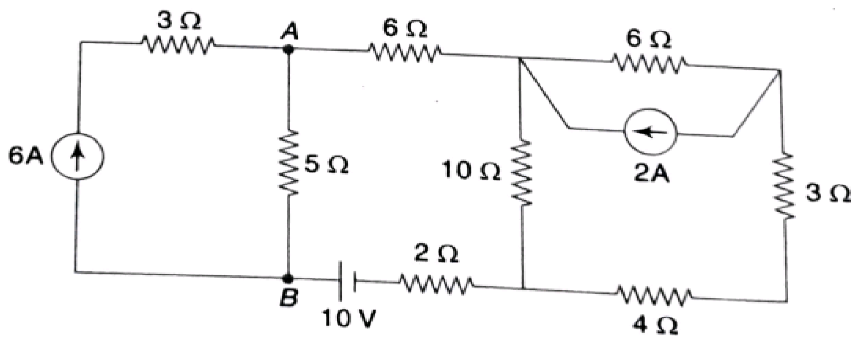

Fig. 1.348

Solution

Step I: Calculation of I_N

Current through 5 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals. Removing the load resistance from the network and short circuiting the load terminals, we get the following network:

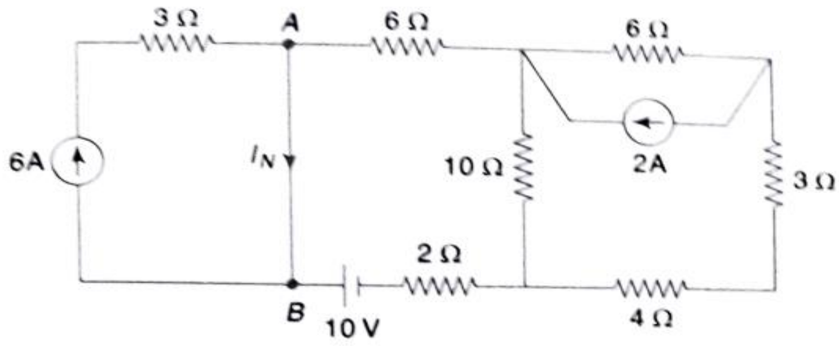

Fig. 1.349

In Fig. 1.349, current flowing through the short circuit placed across the load terminals A and B is called Norton's current I_N. By source transformation, i.e., converting parallel combination of current source of 2 A and resistor of 6 Ω into equivalent series combination of voltage source and resistor, we get the modified network as shown in Fig. 1.350.

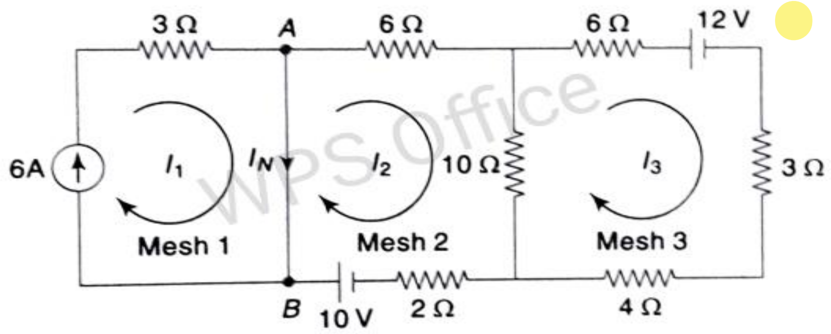

Fig. 1.350

The current I_N can be calculated by mesh analysis.

In mesh 1, current source of 6 A is in the direction of mesh current I_1.

So, $\quad I_1 = 6$ (i)

Applying the KVL to mesh 2,

$$6I_2 - 10(I_2 - I_3) - 2I_2 + 10 = 0$$

or $\quad -18I_2 + 10I_3 = -10$ (ii)

Applying the KVL to mesh 3,

$$-6I_3 - 12 - 3I_3 - 4I_3 - 10(I_3 - I_2) = 0$$

or $\quad 10I_2 - 23I_3 = 12$ (iii)

Solving Eqs (i), (ii) and (iii), we get

$$I_1 = 6\text{ A},\ I_2 = 0.35\text{ A}$$

or $\quad I_N = I_1 - I_2 = 5.65$ A, from A to B

Step II: Calculation of R_N

Removing the load resistance from the network and replacing the voltage source by short circuit and current sources by open circuits, we get the following network:

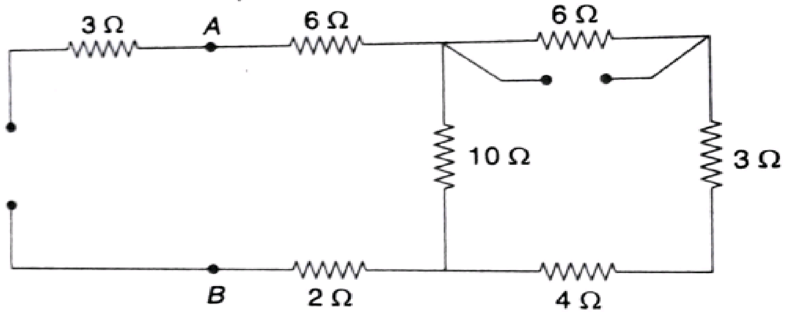

Fig. 1.351

By series-parallel circuit reduction techniques, we get the network as shown in Fig. 1.352.

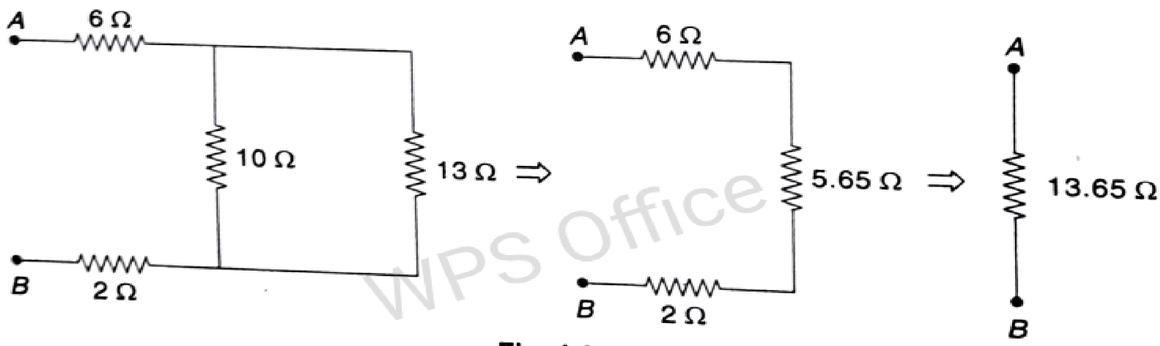

Fig. 1.352

Thus, $R_N = R_{AB} = 13.65\ \Omega$.

Step III: Calculation of load current

Norton's equivalent circuit can be drawn as shown in Fig. 1.353.

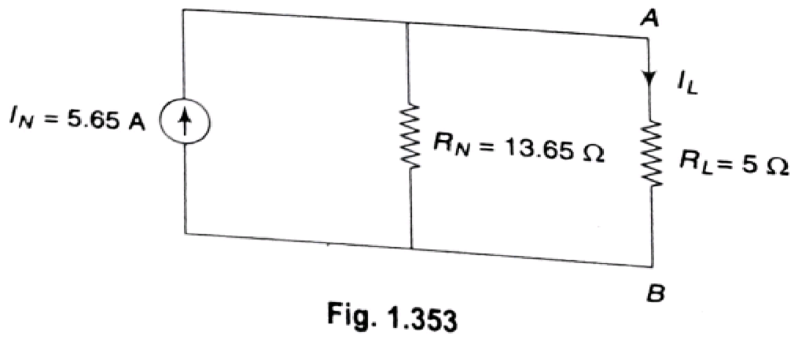

Fig. 1.353

By current division rule,

$$I_L = I_{5\Omega} = 5.65 \times \frac{13.65}{13.65 + 5} = 4.14\ \text{A}(\downarrow)$$

Example 1.84 Find the current I in the network of Fig. 1.354 by using Norton's theorem.

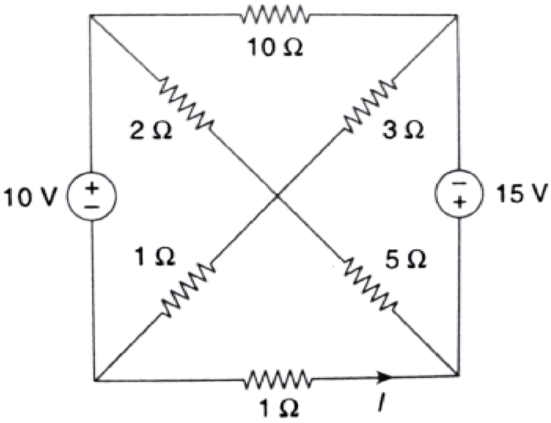

Fig. 1.354

Solution

Current through 1 Ω resistor is required. This resistance can be called as load resistance R_L. Its terminals A and B are called load terminals (see Fig. 1.355).

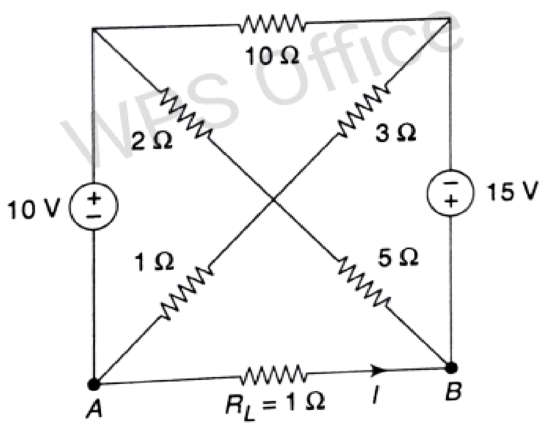

Fig. 1.355

Step I: Calculation of I_N

Removing the load resistance from the network and short circuiting the load terminals, we get the network as shown in Fig. 1.356.

In Fig. 1.356, current flowing through the short circuit placed across the load terminals A and B is called Norton's current I_N.

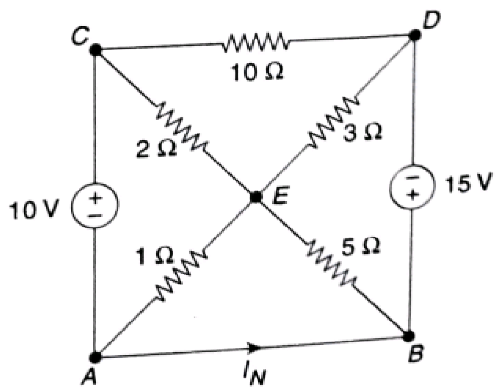

Fig. 1.356

Converting delta connection formed by three resistors, i.e., 2 Ω, 3 Ω, and 10 Ω into equivalent star network, i.e. ΔCDE ⇒ YCDE, we get the simplified network as shown in Fig. 1.357.

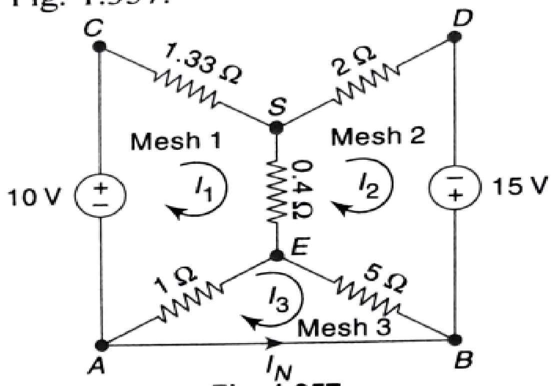

Fig. 1.357

The current I_N can be calculated by Mesh analysis.

Applying KVL to Mesh 1,

$$-1.33I_1 - 0.4(I_1 - I_2) - 1(I_1 - I_3) + 10 = 0$$

$$-2.73I_1 + 0.4I_2 + I_3 = -10 \quad \text{(i)}$$

Applying KVL to Mesh 2,

$$-2I_2 + 15 - 5(I_2 - I_3) - 0.4(I_2 - I_1) = 0$$

$$0.4I_1 - 7.4I_2 + 5I_3 = -15 \quad \text{(ii)}$$

Applying KVL to Mesh 3,

$$-5(I_3 - I_2) - 1(I_3 - I_1) = 0$$

$$I_1 + 5I_2 - 6I_3 = 0 \quad \text{(iii)}$$

Solving Eqs (i), (ii), and (iii), we get

$$I_3 = 7.506 \text{ A}$$

i.e.

$$I_N = -7.506 \text{ A}$$

Step II: Calculation or R_N

Removing the load resistance from the network and replacing the voltage sources by short circuit, we get the network as shown in Fig. 1.358. In Fig. 1.358, equivalent resistance across load terminals A and B is called Norton's resistance R_N.

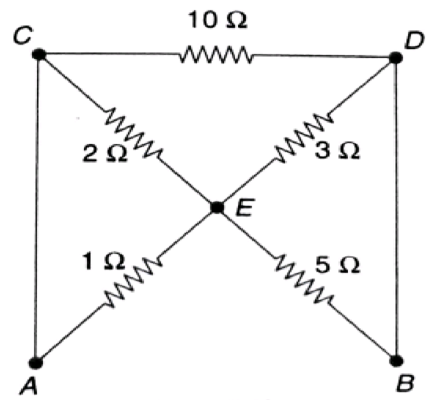

Fig. 1.358

In Fig.1.358, node C and node A are same, similarly node D and node B are same. By joining them, the circuit can be simplified as follows:

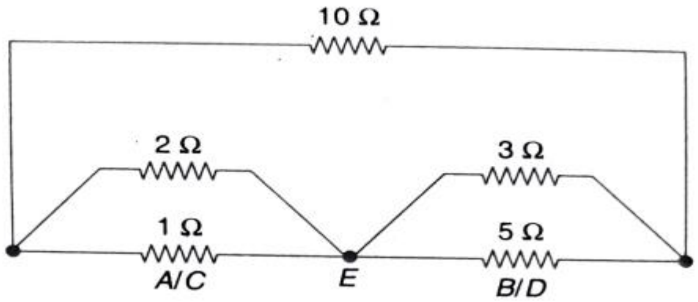

Fig. 1.359

By using series-parallel reduction technique, the circuit is reduced across A and B as follows:

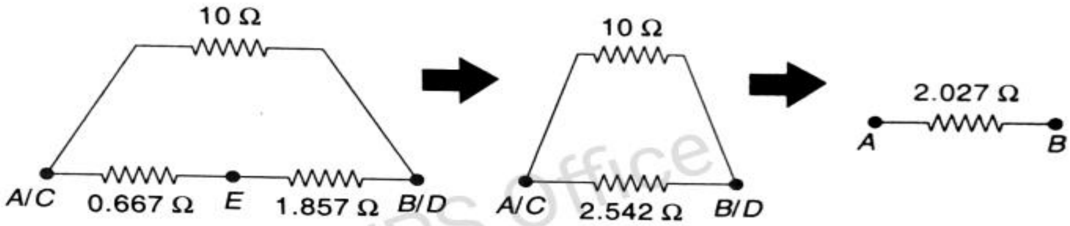

Fig. 1.360

Thus, $R_N = R_{AB} = 2.027\ \Omega$

Step III: Calculation of load current

Norton's equivalent circuit can be drawn as follows:

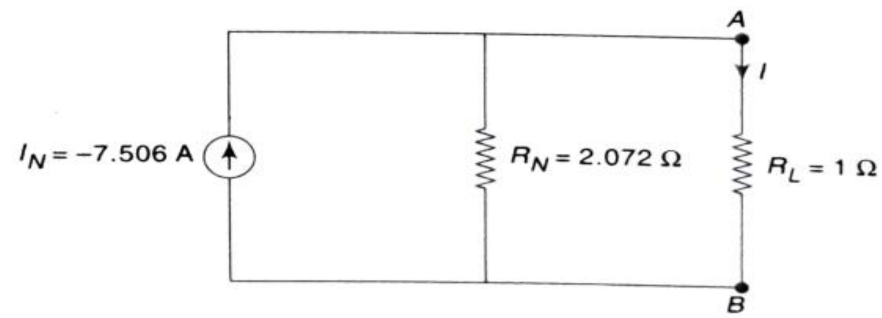

Fig. 1.361

By current division rule,

$$I = \frac{-7.506 \times 2.027}{2.027 + 1} = -5.026 \text{ A}$$

xample 1.85 Obtain Norton's equivalent circuit across A and B as shown in ig. 1.362.

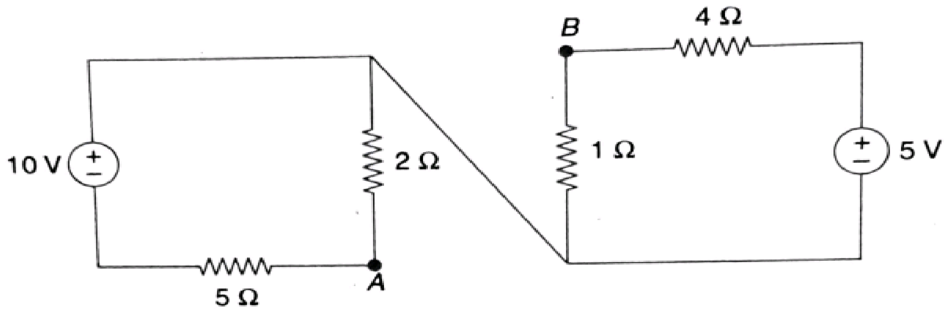

Fig. 1.362

Solution

For simplicity, the circuit shown in Fig. 1.362 can be redrawn as shown in Fig. 1.363.

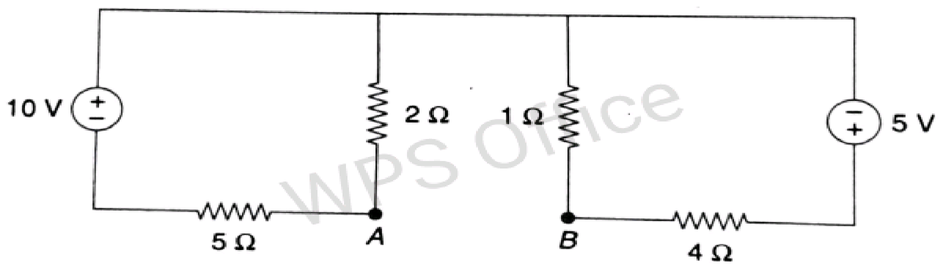

Fig. 1.363

Step I: Calculation of I_N

In Fig. 1.364, current flowing through the short circuit placed across the load terminals A and B is called Norton's current I_N.

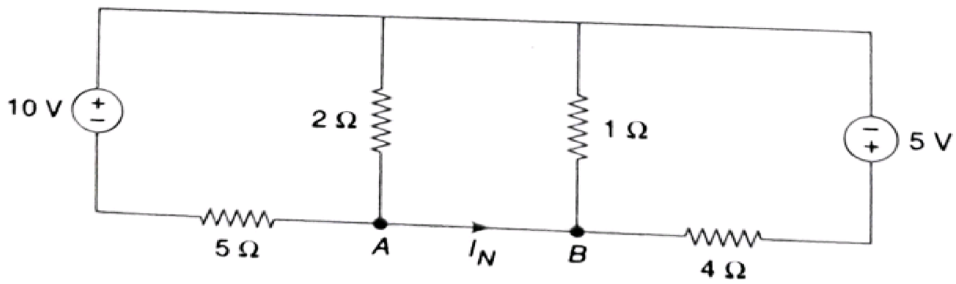

Fig. 1.364

In the circuit of Fig. 1.364, there are two combinations as follows:

(i) Series combination of voltage source of 10 V and resistor of 5 Ω

(ii) Series combination of voltage source of 5 V and resistor of 4 Ω

Converting the above combinations into equivalent combinations, we get the simplified circuit as follows:

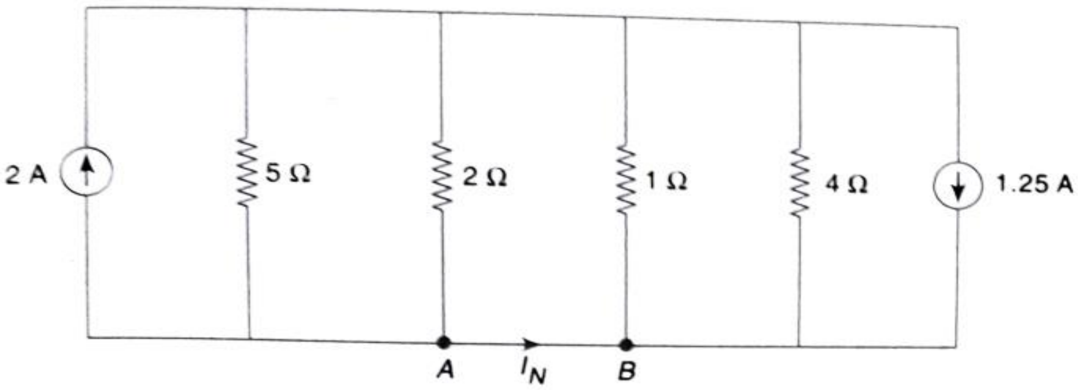

Fig. 1.365

In Fig. 1.365, all elements are in parallel. The resistors 5 Ω and 2 Ω are in parallel. Similarly resistors 1 Ω and 4 Ω are in parallel.

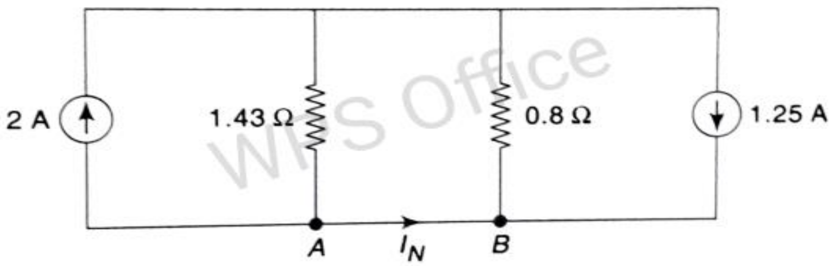

Fig. 1.366

In the circuit of Fig. 1.366, there are two combinations as follows:

(i) Parallel combination of current source of 2 A and resistor of 1.43 Ω

(ii) Parallel combination of current source of 1.25 A and resistor of 0.8 Ω

Converting the above combinations into equivalent combinations, we get the simplified circuit as follows:

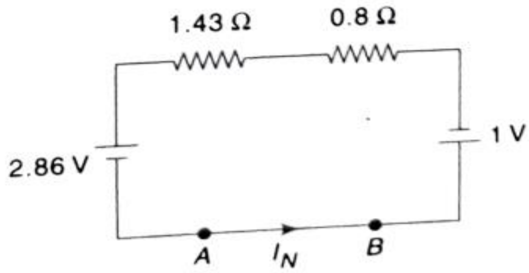

Fig. 1.367

By applying KVL to the loop, we get

$$2.86 + 1.43I_N + 0.8I_N + 1 = 0$$

$\therefore \qquad I_N = -1.73 \text{ A}$

Step II: Calculation of R_N

Replacing the voltage sources by short circuit, we get the network as shown in Fig. 1.368. In Fig. 1.368, equivalent resistance across load terminals A and B is called Norton's resistance R_N.

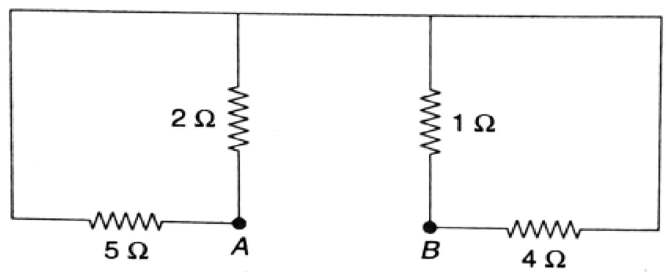

Fig. 1.368

In Fig. 1.368, resistors 5 Ω and 2 Ω are in parallel. Also resistors 1 Ω and 4 Ω are in parallel.

Fig. 1.369

Thus, $R_N = R_{AB} = 1.43 + 0.8 = 2.23 \ \Omega$

Step III: Norton's equivalent circuit

Norton's equivalent circuit can be drawn as follows:

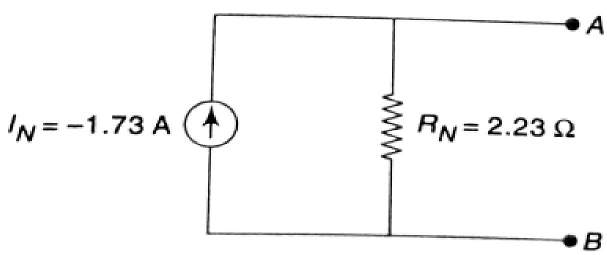

Fig. 1.370