

## MODULE 5

### Kinematics of Rigid Bodies (ICR)

#### 1. 5.4 Prerequisite:

Knowledge of fundamentals of relationship between linear velocity and angular velocity and mathematical formulation learnt at higher secondary level of education (trigonometry & geometry).

#### 2. 5.5 Objective:

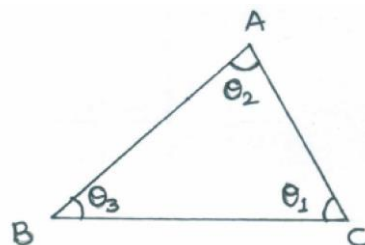
The main goal of this chapter is to elaborate the basic concepts of the Instantaneous Centre of Rotation theory, through a number of problems. The problems selected in this chapter will also be covered in the lectures.

#### 3. 5.6 Formulae:

Velocity:  $v = r.\omega$

Sine rule:  $\frac{AB}{\sin \theta_1} = \frac{BC}{\sin \theta_2} = \frac{CA}{\sin \theta_3}$

Cosine rule:  $AB^2 = BC^2 + CA^2 - 2BC \times CA \cos \theta_1$

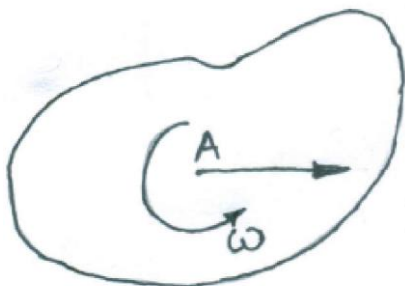


#### 4. 5.7 Key Definition:

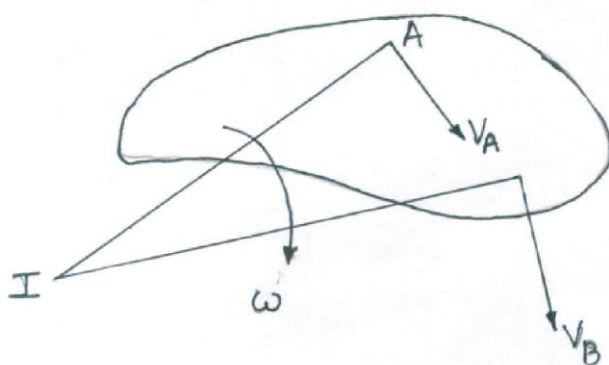
Instantaneous centre of rotation is the point around which the body is supposed to be virtually rotated. I.C.R. is not moving and exists for just an instant. When the body continues to move, for that particular instant, a new instantaneous centre of rotation may be located and can be determined.

#### 5. Instantaneous center of rotation (ICR)

The general plane motion of a rigid body can be considered as the sum of a plane translation motion and rotational motion about an axis perpendicular to the plane of motion. The velocity of a rigid body can therefore be completely specified by specifying the translation velocity of a point A i.e.  $v_A$  and the rotational velocity ( $\omega$ ) about an axis through the point as shown in fig 1.



The rotational velocity ( $\omega$ ) is the same for every point of the body and the displacement or translation is different for different points which suggest that a point can be considered to exist on the body with respect to which the body can be considered to rotate about an axis passing through this point at that instant. Such a point is known as *instantaneous centre of rotation*. The point of instantaneous centre of rotation has zero velocity at that instant. The point of instantaneous centre can be located on the body or outside the body. The velocity of other points in the body can be found out by comparing the perpendicular distances of the points from the instantaneous centre of rotation as shown in the fig.2.



The velocities at two points A and B of the body are

$$v_A = (IA) \times \omega$$

$$v_B = (IB) \times \omega$$

$$v_B / v_A = (IB) / (IA)$$

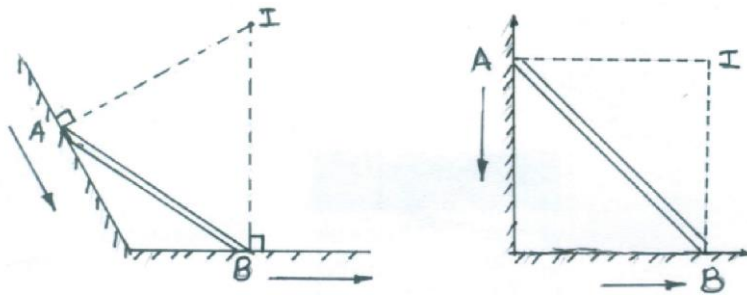
$$v_B = (IB) / (IA) \times v_A$$

Hence, it is possible to find  $v_B$  in case velocity  $v_A$  is known and distance IB and IA are measured. This fact provides another method of locating the instantaneous centre of rotation (point I) if the directions of the velocities at any points on the rigid body are known. The perpendiculars to the directions of the velocities at two points will intersect at the instantaneous centre of rotation as shown in fig 3. AI and BI are drawn perpendicular to the velocities  $v_A$  at point A and  $v_B$  at point B of the body. The perpendiculars intersect at point I which is now the point of instantaneous centre of rotation

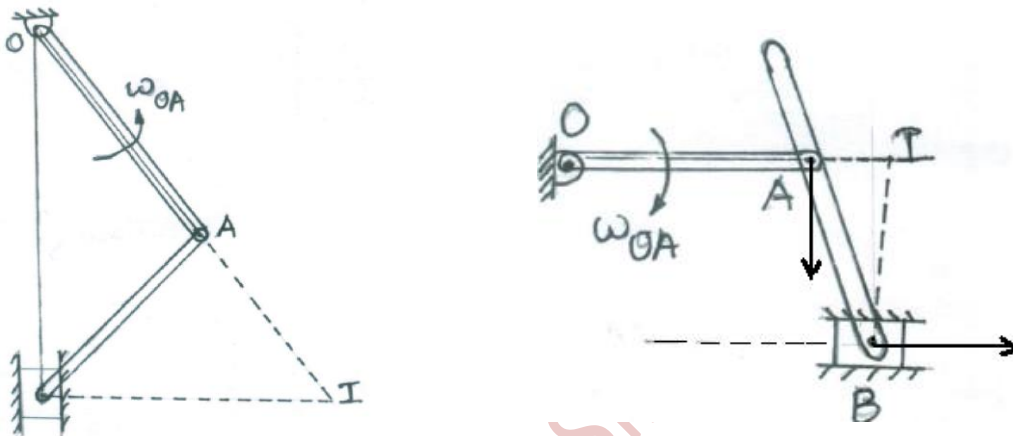
## 6. Methods used to find instantaneous centre of rotation

**Method 1:** When body slides on two surfaces,

$$\omega = v_A / IA = v_B / IB$$



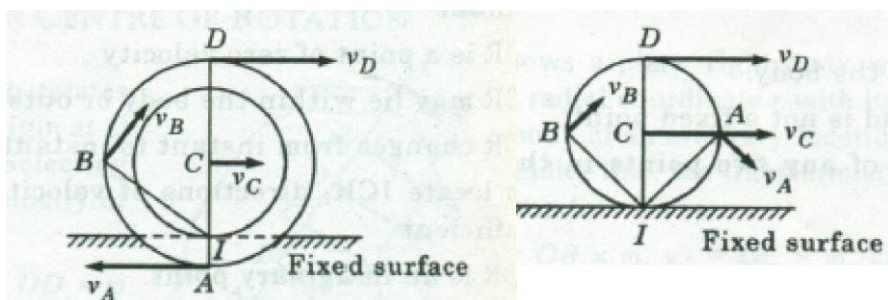
**Method 2:** When one part of the body slides & another part rotates about a hinge point.



To locate ICR draw a line perpendicular to the sliding surface from point B and extend rotating link about point O

$$\omega = v_A / IA = v_B / IB \text{ and } v_A = (OA) \times \omega_{OA}$$

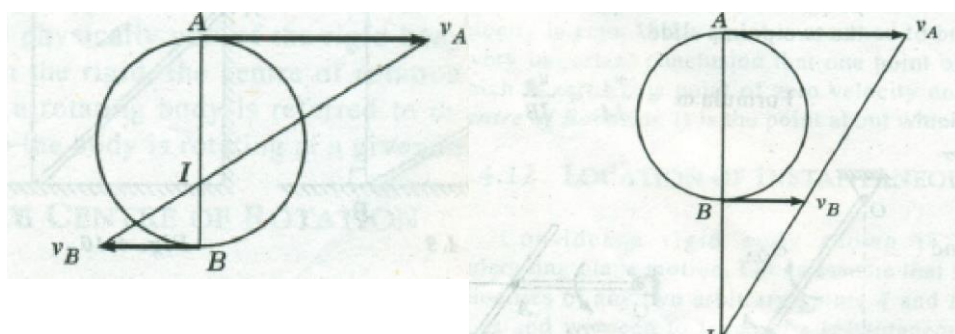
**Method :** When body rolls on fixed surface.



The point which is in contact with fixed surface becomes Instantaneous Centre of Rotation.

$$\omega = v_A / IA = v_B / IB = v_C / IC = v_D / ID$$

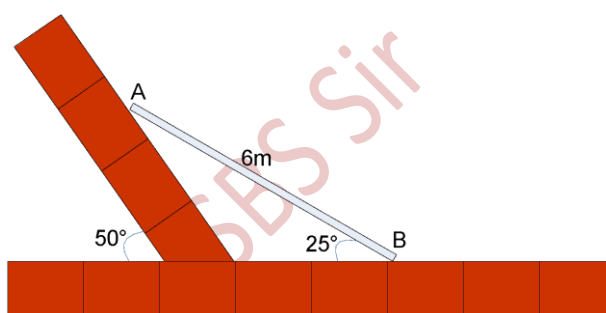
**Method 5:** When body lies between two moving surfaces.



$$\omega = v_A / IA = v_B / IB$$

### Problem 1:

The rod is in contact with two smooth stationary surfaces. At the instant shown in figure its end B has velocity 2 m/s rightward. Find velocity of end A and angular velocity of the rod. Also find velocity of a point on the rod, which is two meters from end B, at the same instant.

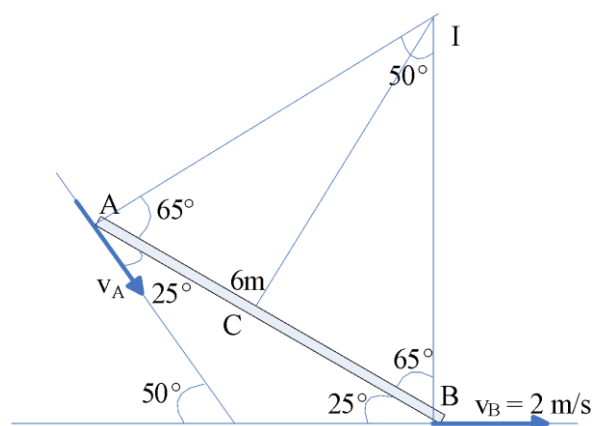


**Solution:** We have,

$$\omega = v_A / IA = v_B / IB \text{ ----- (I)}$$

$$\text{And } v_B = 2 \text{ m/s;}$$

$$\text{So, } \omega = 2 / IB$$



Applying Sine Rule for  $\Delta IAB$

$$6 / \sin 50 = IA / \sin 65 = IB / \sin 65$$

$$IA = IB = 7.0986 \text{ m}$$

Put these in equation (I)

$$\Omega = 2 / 7.0986 = 0.2817 \text{ r/s}$$

$$V_A = 2 \text{ m/s}$$

To find velocity of centre 'C' which is 2m away from point 'B'

Applying Cosine Rule for triangle IBC, We have;

$$IC^2 = IB^2 + BC^2 - 2 \times IB \times BC \times \cos 65$$

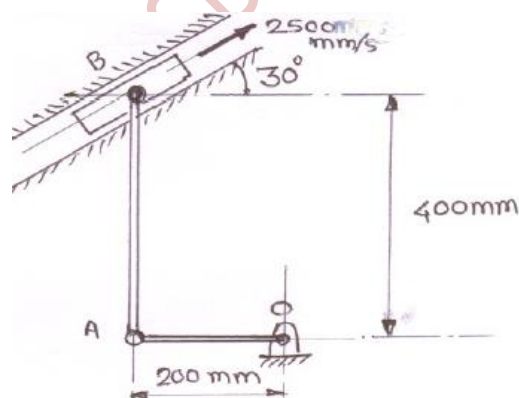
$$IC^2 = 7.0986^2 + 2^2 - 2 \times 2 \times 7.0986 \times \cos 65$$

$$IC = 6.5108 \text{ m}$$

$$\text{Now, } v_c = IC \times \omega = 6.5108 \times 0.2817 = \mathbf{1.8344 \text{ m/s Ans.}}$$

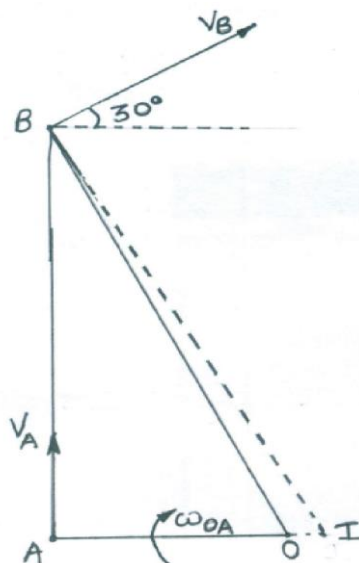
## 7. Problem 2:

Locate the instantaneous centre of rotation of link AB. Find also the angular velocity of link OA. Take velocity of slider at B = 2500 mm/s. The link and slider mechanism is as shown in the figure.



**Solution:**

Rod OA rotates about point 'O' with angular velocity  $\omega_{OA}$ . Velocity of point A is perpendicular to OA. Slider at B moves along the incline with velocity  $v_B$ . Drawing lines perpendicular to  $v_A$  and  $v_B$ , Intersect at ICR 'I'



$$v_B = 2500 \text{ mm/s} = IB \times \omega \text{ -----(I)}$$

$$v_A = OA \times \omega_{OA} = IA \times \omega$$

$$v_A = 200 \times \omega_{OA} = IA \times \omega \text{ -----(II)}$$

To find length IA and IB, we use geometry from  $\triangle BAI$

$$\cos 30 = AB / IB = 400 / IB$$

$$\text{i.e. } IB = 461.88 \text{ mm}$$

$$\tan 30 = IA / AB = IA / 400$$

$$\text{i.e. } IA = 230.94 \text{ mm}$$

Substituting these values in equations (I) & (II)

$$2500 = 461.88 \times \omega$$

Angular Velocity of the rod AB is  $\omega = 5.4127 \text{ r/s}$

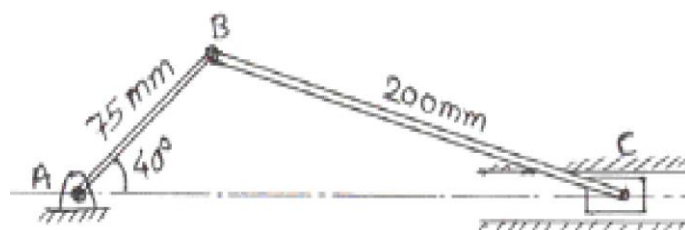
From eq. (II),

$$200 \omega_{OA} = IA \times \omega = 230.94 \times 5.4127$$

Angular velocity of the link OA =  $6.25 \text{ r/s}$

### 8. Problem 3:

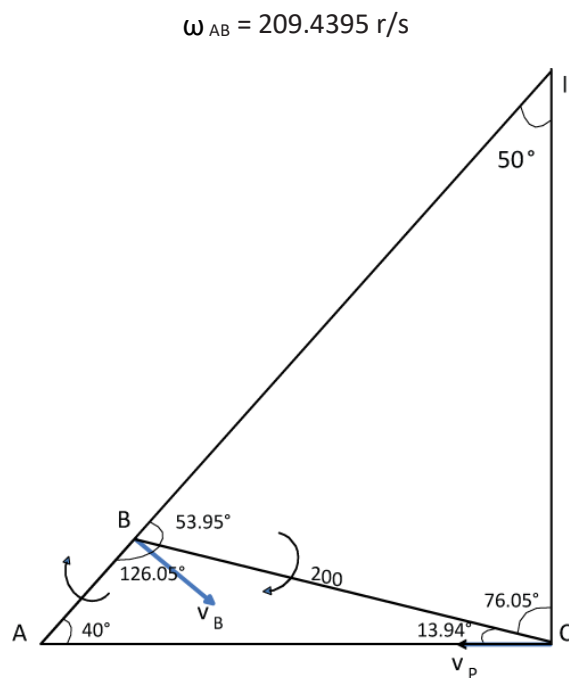
In the engine system shown, the crank AB has a constant clockwise angular velocity of 2000 rpm. For the crank position indicated, determine the angular velocity of the connecting rod BD and the velocity of the piston P.



**Solution:**

Now AB has a constant clockwise angular velocity of 2000 r.p.m. Then,

$$\omega_{AB} = 2\pi N / 60 = 2\pi \times 2000 / 60$$



The angular velocity of the connecting rod BC

For angle ACB by Sine Rule,

$$200 / \sin 40 = 75 / \sin \theta$$

$$\sin \theta = 75 \times \sin 40 / 200$$

$$\theta = 13.94^\circ$$

Now

$$v_B = \omega_{AB} \times AB$$

$$v_B = 15.7073 \text{ m/s}$$

Now by sine rule in  $\triangle IBC$

$$IB / \sin 76.0518 = 200 / \sin 50 = IC / \sin 53.94$$

$$IB = 0.253 \text{ m}$$

$$IC = 211.1135 \text{ mm} = 0.211 \text{ m}$$

So,

$$v_B = \omega_{BC} \times IB$$

$$15.707 = \omega_{BC} \times 0.253$$

$$\omega_{BC} = 62.083 \text{ r/s}$$

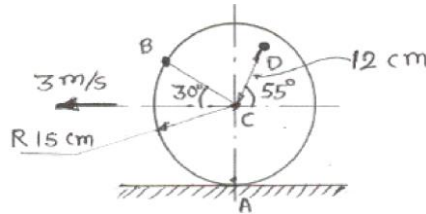
The velocity of piston P

$$v_P = \omega_{BC} \times IC$$

$$v_P = 62.083 \times 0.211 = 13.099 \text{ m/s}$$

## 9. Problem 4:

The roller is rolling without slipping on a stationary surface with velocity 3 m/s leftward as shown in the figure. Find velocities of points A, B, C, and D at the instant.



### Solution:

Point I which is in Contact with flat surface becomes Instantaneous Centre of Rotation. Hence Velocity of Point I i.e.  $v_A = 0$ . Let the Wheel rotates with angular velocity  $\omega$ .

$$\omega = v_C / IC = v_B / IB = v_D / ID \text{-----(I)}$$

hence,

$$\omega = v_C / IC = 3 / 0.15 = 20 \text{ rad/s}$$

Now velocities of point B and D,

$$v_B = \omega \times IB$$

$$v_C = \omega \times IC$$

To get IB in  $\triangle ICB$

,

$$\begin{aligned} IB^2 &= BC^2 + IC^2 - 2 \times BC \times IC \times \cos 120 \\ &= 0.15^2 + 0.15^2 - 2 \times 0.15 \times 0.15 \times \cos 120 \end{aligned}$$

$$\therefore IB = 0.259 \text{ m}$$

In  $\triangle ICD$

$$\begin{aligned} ID^2 &= IC^2 + CD^2 - 2 \times CD \times IC \times \cos 145 \\ &= 0.15^2 + 0.12^2 - 2 \times 0.12 \times 0.15 \times \cos 145 \end{aligned}$$

$$\therefore ID = 0.257 \text{ m}$$

Substituting these values in equation (I) we get,

$$v_B = 5.1962 \text{ m/s}, v_D = 5.14 \text{ m/s}$$

$$v_C = 3 \text{ m/s}, v_A = 0$$

## 10. Problem 5:

In a given mechanism roller D is constrained to slide in horizontal slot. Find velocity of D and angular velocity of rods for given instant if  $v_A = 5 \text{ m/s}$  in vertically upward direction.

**Solution:**

To find length IB & IC;

From  $\triangle ABC$  using cosine rule

$$IB^2 = BC^2 + AC^2 - 2 \times BC \times AC \times \cos 120^\circ$$

$$IB^2 = 152 + 152 - 2 \times 15 \times 15 \times \cos 120^\circ$$

$$IB = 25.98 \text{ cm} = 0.2598 \text{ m}$$

In  $\triangle IDC$  using cosine rule

$$ID^2 = DC^2 + AC^2 - 2 \times DC \times AC \times \cos 145^\circ$$

$$ID^2 = 122 + 152 - 2 \times 12 \times 15 \times \cos 145^\circ$$

$$ID = 25.7662 \text{ cm} = 0.25766 \text{ m}$$

a) For rod AB (ICR is point  $I_1$ )

$$v_A = AI_1 \times \omega_{AB}$$

$$\omega_{AB} = v_A / AI_1 = 5/0.4$$

$$\omega_{AB} = 12.5 \text{ rad/sec}$$

$$v_C = CI_1 \times \omega_{AB}$$

$$= 0.360 \times 12.5$$

$$v_C = 4.506 \text{ m/sec}$$

b) For rod CD (ICR is point  $I_2$ )

$$v_A = CI_2 \times \omega_{CD}$$

$$\omega_{CD} = v_C / CI_2 = 4.506/0.7209$$

$$\omega_{CD} = 6.25 \text{ rad/sec}$$

$$v_D = DI_2 \times \omega_{CD}$$

$$v_D = 0.2 \times 6.25$$

$$v_D = 1.25 \text{ m/sec}$$

In triangle  $CMI_1$

$$CI_1 = \sqrt{CM^2 + MI_1^2}$$

$$= \sqrt{(0.2^2 + 0.3^2)}$$

$$CI_1 = 0.360 \text{ m}$$

$$\text{Also, } \tan \theta = MI_1/CM = 0.3/0.2$$

$$\theta = 56.30^\circ$$

In triangle  $CNI_2$

$$\cos \theta = CN/CI_2$$

$$CI_2 = CN/\cos \theta = 0.4/\cos 56.30^\circ$$

$$\text{i.e. } CI_2 = 0.7209 \text{ m}$$

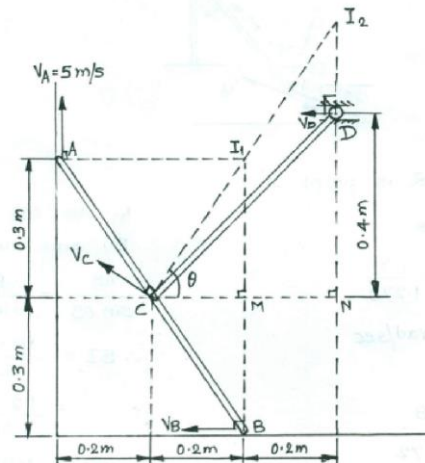
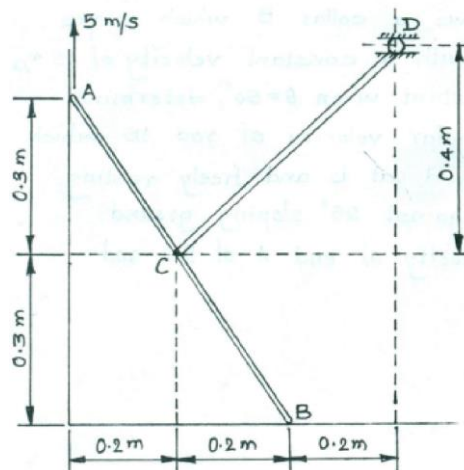
$$\text{Also, } \sin \theta = NI_2/CI_2$$

$$NI_2 = CI_2 \times \sin \theta = 0.7209 \times \sin 56.30^\circ$$

$$NI_2 = 0.5999 \text{ m} \approx 0.6 \text{ m}$$

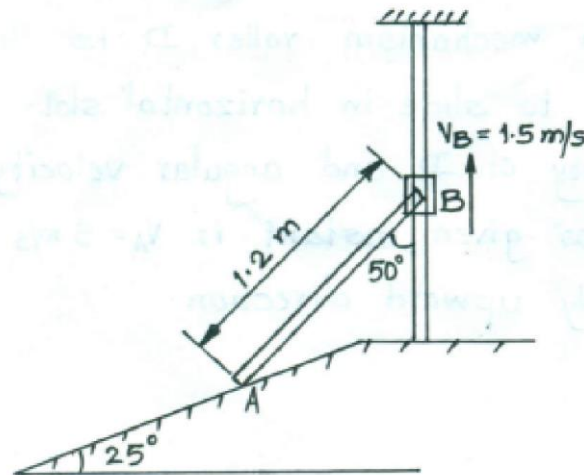
$$DI_2 = NI_2 - DN = 0.6 - 0.4$$

$$DI_2 = 0.2 \text{ m}$$



### 11. Problem 6:

Figure shows a collar B which moves upwards with a constant velocity of 1.5 m/s. At the instant when  $\theta = 50^\circ$ , determine (i) the angular velocity of rod AB which is pinned at B and freely resting at A against  $25^\circ$  sloping ground. (ii) the velocity of end A of the rod.



Solution:-

For rod AB : (ICR is point I)

i)

$$v_B = B I \times \omega_{AB}$$

$$\omega_{AB} = v_B / B I = 1.5 / 1.279$$

$$\omega_{AB} = 1.172 \text{ rad/sec}$$

$$v_A = A I_1 \times \omega_{AB}$$

$$= 0.850 \times 1.172 \approx 1 \text{ m/s}$$

$$v_A = 1 \text{ m/s}, \theta = 25^\circ$$

In triangle ABI

By Sine rule,

$$\frac{AB}{\sin 65^\circ} = \frac{BI}{\sin 75^\circ} = \frac{AI}{\sin 40^\circ}$$

$$BI = AB \times \sin 75^\circ / \sin 65^\circ$$

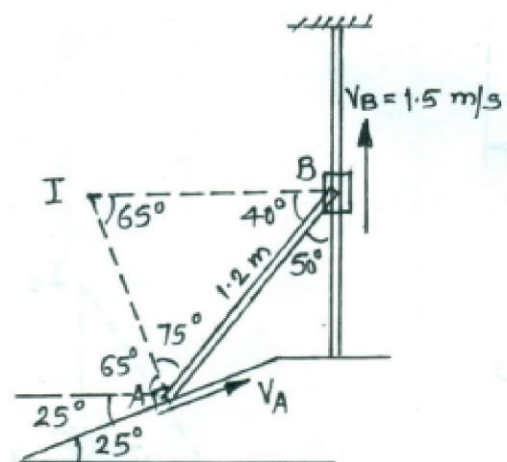
$$= 1.2 \times 0.966 / 0.906$$

$$BI = 1.279 \text{ m}$$

$$AI = AB \times \sin 40^\circ / \sin 65^\circ$$

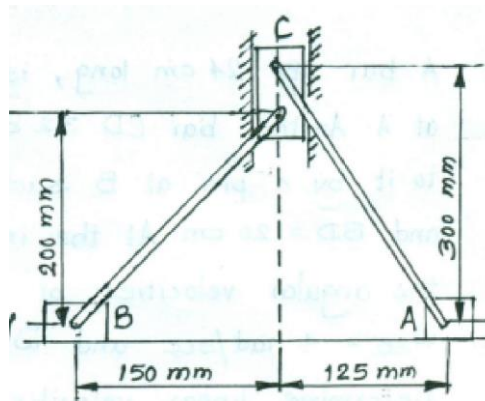
$$= 1.2 \times 0.642 / 0.906$$

$$AI = 0.850 \text{ m}$$



### 12. Problem 7:

In a mechanism shown in figure, piston C is constrained to move in a vertical slot. A and B moves on horizontal surface. Rods CA and CB are connected with smooth hinges. If  $v_A = 0.45 \text{ m/s}$  to the right, find velocity of C and B. Also find angular velocity of two rods.



**Solution:**

**a) For rod AC: (ICR is point  $I_1$ )**

$$v_A = AI_1 \times \omega_{AC}$$

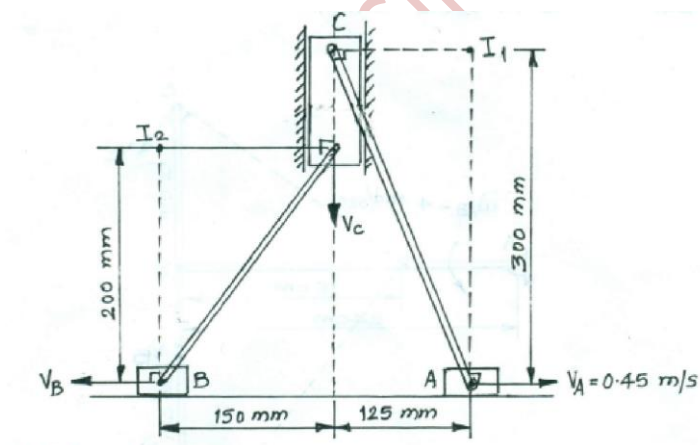
$$\omega_{AC} = v_A / AI_1 = 0.45 / 0.3$$

$$\omega_{AC} = 1.5 \text{ rad/sec}$$

$$v_C = CI_1 \times \omega_{AC}$$

$$= 0.125 \times 1.5$$

$$v_C = 0.1875 \text{ m/sec (}\downarrow\text{)}$$



**b) For rod BC: (ICR is point  $I_2$ )**

$$v_C = CI_2 \times \omega_{BC}$$

$$\omega_{BC} = v_C / CI_2 = 0.1875 / 0.150$$

$$\omega_{BC} = 1.25 \text{ rad/sec}$$

$$v_B = BI_2 \times \omega_{BC}$$

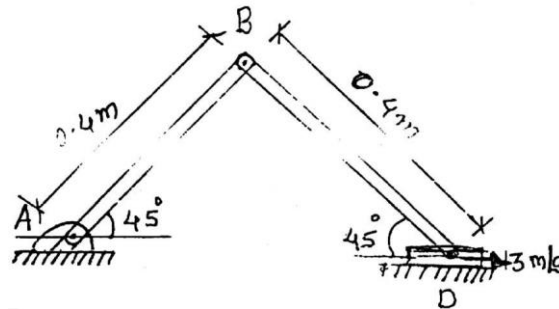
$$= 0.2 \times 1.25$$

$$v_C = 0.25 \text{ m/sec (}\leftarrow\text{)}$$

### 13. 5.10 University Problems:

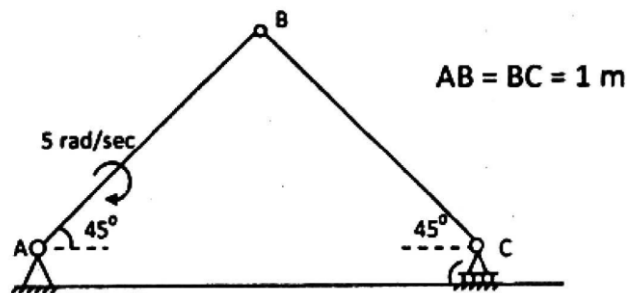
#### Problem

Block 'D' shown in the fig. moves with the speed of 3m/s. Find the angular velocity of links BD and AB and the velocity of point B at the instant shown. Use method of instantaneous centre of zero velocity. (Dec'09)



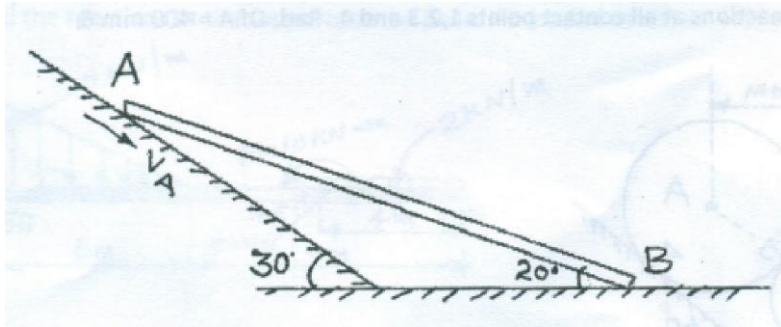
#### Problem

In the mechanism shown find the velocity of point C and angular velocity of link BC if angular velocity of link AB is 5rad/s. solve the problem when the link AB and link BC make an angle of 45 degrees with the horizontal as shown in the figure. (May'10)

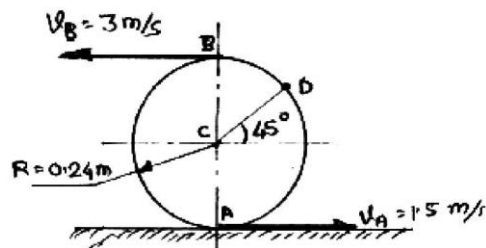


#### Problem

Rod AB of length 3m is kept on smooth planes as shown in the figure. The velocity of the end A is 5 m/sec. along the inclined plane. Locate the ICR and find the velocity of the end B [May 11](8M)

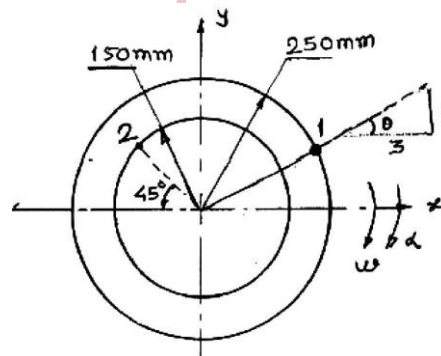


**Problem 5:** Due to slipping points A and B on the rim of the disk have the velocities shown. Determine the velocities of the center point C and point D at this instant.



[Ans.:  $v_c = 0.76 \text{ m/s}$ ,  $v_d = 2.87 \text{ m/s}$ ]

**Problem 6:** A circular lamina rotates in XY plane about an axis perpendicular to the XY plane and passing through a centre point O as shown. Angular velocity of the lamina is  $2 \text{ rad/sec}$  clockwise and it has an angular acceleration  $1 \text{ rad/sec}^2$ . Find the velocity and acceleration of the points 1 and 2 for the position shown.



[Ans :  $300 \text{ mm/s}$ ,  $618.46 \text{ mm/sec}^2$ ]