JESSY

Module 1
DIFFRACTION

Syllabus:

Fraunhofer diffraction at single slit, Diffraction grating, Resolving power of grating, Application of diffraction grating - Determination of wavelength of light using plane transmission grating

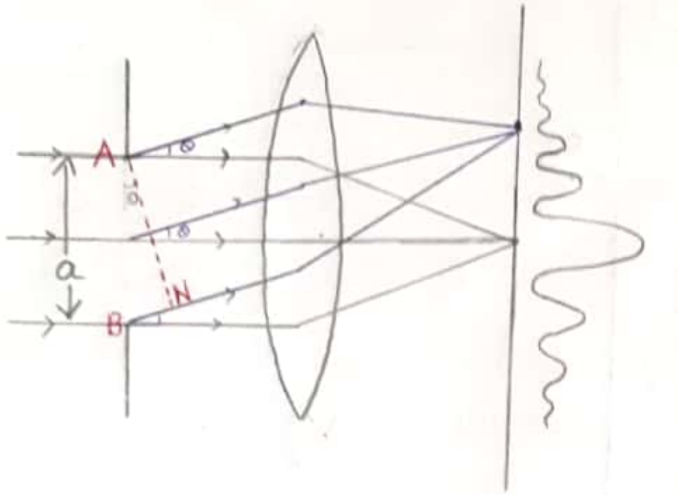
Introduction

The bending of waves around an obstacle (or small aperture) and deviation from a rectilinear path is called diffraction. The condition for diffraction is the size of the obstacles must be comparable to wavelength of waves. The sound waves are bend around the obslactes, but not light waves.
The bending of light occurs if the size of obstacle is of the order of wavelength of light. That is very small obstacles are needed to create diffraction of light. Hence it is not evident in daily life easily.

Diffraction

	Fresnel Diffraction	Fraunhofer Diffraction
1	The source of light and screen are at finite distance from obstacle	Source of light and screen are at infinite distance from obstacle
2	Lenses are not used to make the rays parallel or convergent.	Two convex lenses are used to make rays parallel and for focussing diffracted rays on screen
3	The wavefront incident is either spherical or cylindrical	The wavefront incident is plane.
4	The phase of secondary wavelets is not same at all points in the plane of obstacle	The phase of secondary wavelets is same at all points in the plane of obstacle

FRAUNHOFFER DIFFRACTION AT A SINGLE SLIT



we are finding

① Path difference
② Phase difference
③ Resultant Amp.
④ Resultant Intensity
⑤ Cond^n for minimum
⑥ Cond^n for centre Max.
⑦ Cond^n for secondary Max.
⑧ Width of centre Max.

Let a planewavefront of light of wavelength λ be incident normally on a narrow slit AB. Let AB = d be the width of slit. The diffracted light be focussed on screen using a lens. This diffraction pattern consists of a central bright band with a number of alternate ~~b~~ dark and bright bands of decreasing intensity arranged symmetrically on either side of central bright.

According to Huygen's principle, each point of plane wavefront incident on AB is a source of secondary wavelets. These wavelets along the direction of incident ray get focussed at P and produces a central bright image. (At P optical path difference is zero). The rays diffracted through an angle θ with normal to AB are focussed at Q on screen. The point Q is of maximum or minimum intensity depending on pathdifference between secondary wavelets from corresponding points of wavefront

The path difference of secondary wavelets in reaching Q from edges of A and B = BN

Δ ABN

$\sin\theta = \frac{BN}{AB}$

$$BN = AB \sin\theta$$

$\therefore$ path difference $= d \sin\theta$ ——— ①

Corresponding phase difference is

$$\text{Phase difference } \phi = \frac{2\pi}{\lambda} \times \text{Path difference}$$

$$= \frac{2\pi}{\lambda} d\sin\theta$$

Let the slit AB is devided into n equal parts where n is very large. Then

$$\boxed{\text{Phase difference } \phi = \frac{1}{n}\left(\frac{2\pi}{\lambda} d\sin\theta\right)} \quad \text{——} \ (2)$$

To find resultant amplitude, we can use the theory of n harmonic vibrations of same amplitude 'a' and having common difference ϕ, That is

$$R = \frac{a\sin\left(\frac{n\phi}{2}\right)}{\sin\frac{\phi}{2}}$$

$$R = \frac{a\sin\left(\frac{n}{2}\frac{1}{n}\frac{2\pi}{\lambda}d\sin\theta\right)}{\sin\left(\frac{1}{2}\frac{1}{n}\frac{2\pi}{\lambda}d\sin\theta\right)}$$

$$R = \frac{a\sin\left(\frac{\pi}{\lambda}d\sin\theta\right)}{\sin\left(\frac{\pi}{n\lambda}d\sin\theta\right)}$$

Let $\frac{\pi}{\lambda}d\sin\theta = \alpha$

$$R = \frac{a\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}$$

since n is very large $\frac{\alpha}{n}$ = very small

$\therefore \sin\left(\frac{\alpha}{n}\right) = \frac{\alpha}{n}$

$$R = \frac{na\sin\alpha}{\alpha}$$

$$\boxed{R = \frac{A\sin\alpha}{\alpha}}$$

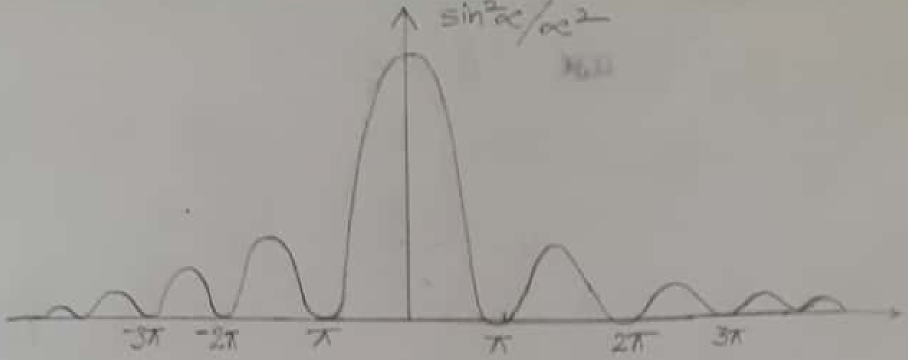
where $A = na$

Resultant intensity $I = R^2$

$$\boxed{I = \frac{A^2\sin^2\alpha}{\alpha^2}}$$

The value of α depends on angle of diffraction θ $\left[\because \alpha = \frac{\pi}{\lambda}d\sin\theta\right]$. A graph of $\frac{\sin^2\alpha}{\alpha^2}$ as a function α or $\sin\theta$ is given.

④



Most of the light is concentrated on central principal maximum. The intensity of secondary maxima decreases very rapidly. The intensity of first secondary maximum is about $\frac{1}{22}$ and that of second is $\frac{1}{61}$ of the centre maximum.

Condition for Minimum

$I = \frac{A^2 \sin^2\alpha}{\alpha^2}$ should be minimum $(I=0)$

$\therefore \quad \sin\alpha = 0 \quad$ but $\alpha \neq 0 \quad$ ($\alpha = 0 \Rightarrow$ maximum Intensity)

i.e $\quad \alpha = \pm m\pi \quad$ where $m = 1, 2, 3 \ldots$

$\frac{\pi}{\lambda} d\sin\theta = \pm m\pi \qquad$ but $\alpha = \frac{\pi}{\lambda} d\sin\theta$

$$\therefore \quad \boxed{d\sin\theta = \pm m\lambda}$$

Condition for Centre Maximum

Intensity $I = \frac{A^2 \sin^2\alpha}{\alpha^2}$ should be maximum $\Rightarrow$

$\alpha = 0 \Longrightarrow I_{max} = A^2 \lim_{\alpha \to 0} \frac{\sin^2\alpha}{\alpha^2}$

$\frac{\pi}{\lambda} d\sin\theta = 0$

$\sin\theta = 0 \qquad I_{max} = A^2 = I_0 \qquad \Big| \lim_{\alpha \to 0} \frac{\sin\alpha}{\alpha} = 1$

$\theta = 0$

The condition $\theta = 0$ means that the centre maximum of is formed by parts of secondary wavelets which travel normal to the slit and produced at point P.

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Condition for Secondary maximum

For this $\frac{dI}{d\alpha} = 0$

$$\frac{d}{d\alpha}\left(\frac{A^2 \sin^2\alpha}{\alpha^2}\right) = 0$$

$$A^2\left\{\frac{[\alpha^2(2\sin\alpha\cos\alpha)] - [(\sin^2\alpha)\,2\alpha]}{\alpha^4}\right\} = 0$$

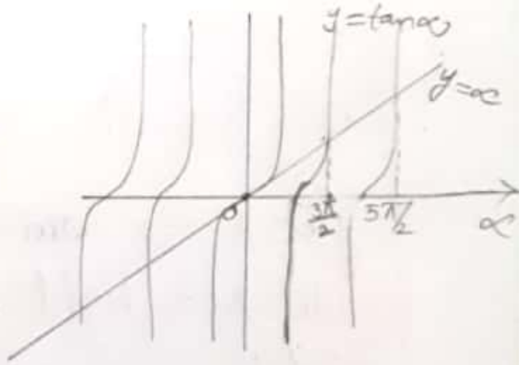
$$A^2\,\frac{2\alpha\sin\alpha}{\alpha^4}\,(\alpha\cos\alpha - \sin\alpha) = 0$$

This is possibly only if $\alpha\cos\alpha - \sin\alpha = 0$

$$\alpha\cos\alpha = \sin\alpha$$

$$\alpha = \tan\alpha$$

This eqn can be solved graphically. Curves $y=\alpha$ and $y=\tan\alpha$ are plotted, then the point of intersection of two curves gives the value of α corresponding to maximum intensity.



Those points corresponding to

$\alpha = 0, \frac{3\pi}{2}, \frac{5\pi}{2} \text{-----} (2m+1)\frac{\pi}{2}$

$\alpha = 0$ corresponds to the position of centre maximum.

Condition for secondary maximum is

$$\alpha = (2m+1)\frac{\pi}{2}$$

$$\frac{\pi}{\lambda}\, d\sin\theta = (2m+1)\frac{\pi}{2}$$

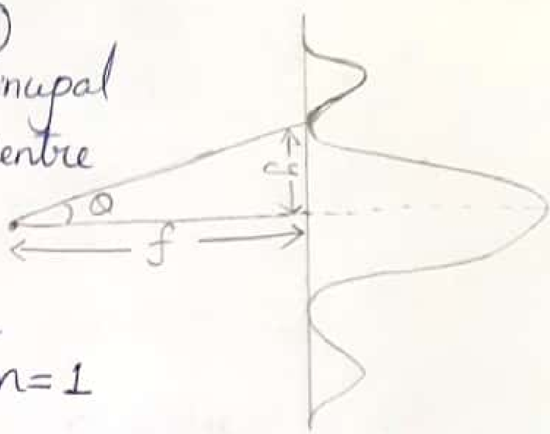
$$\boxed{d\sin\theta = (2m+1)\frac{\lambda}{2}}$$

where $m = 1, 2, 3$ -----

Cond^n for secondary Max.

Linear width of centre Maximum

Let y be the distance of first minimum from centre principal maximum. Then width of centre maximum is $2y$



Condition for first minimum

$$d \sin\theta = m\lambda \quad \text{with } m=1$$

$$\sin\theta = \frac{\lambda}{d}$$

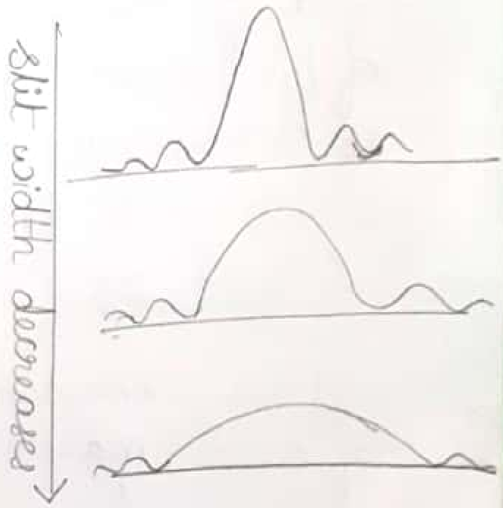
Also $\tan\theta = \dfrac{y}{f}$

θ is very small ($\sin\theta \approx \theta$ & $\tan\theta \approx \theta$) $\therefore$ $\theta = \dfrac{\lambda}{d}$ and also $\theta = \dfrac{y}{f}$

$$\frac{\lambda}{d} = \frac{y}{f}$$

$$\therefore \boxed{y = \frac{f\lambda}{d}}$$ $\Rightarrow$ Distance of 1st minimum from centre

$$\boxed{\text{Linear width } 2y = \frac{2f\lambda}{d}}$$



$y \propto \lambda \Rightarrow$ width of centre max. with red light is greater than violet.

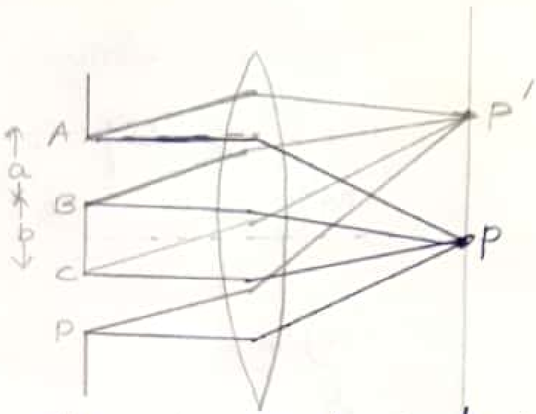
$y \propto \dfrac{1}{d} \Rightarrow$ Centre maximum increases as slit width decreases.

DIFFRACTION DUE TO DOUBLE SLIT

Consider two narrow slits AB and CD on which a light of wavelength λ is incident normally. Let

$a \rightarrow$ width of each slit

$b \rightarrow$ width of opaque portion



In this case diffraction pattern has to be in two parts

1) Interference due to secondary waves from corresponding points of two slits
2) Diffraction pattern due to secondary waves from each individual slits

Interference Maxima & Minima	Diffraction Maxima & Minima
path difference $= CN = (a+b)\sin\theta$	path difference $= a\sin\theta$
Maxima: $(a+b)\sin\theta = m\lambda$	Secondary Max: $a\sin\theta = (2m+1)\frac{\lambda}{2}$
Minima: $(a+b)\sin\theta = (2m+1)\frac{\lambda}{2}$	Minima: $a\sin\theta = m\lambda$

The Resultant Intensity $I = R^2$

$$I = 4A^2 \frac{\sin^2\alpha}{\alpha^2} \cos^2\beta$$

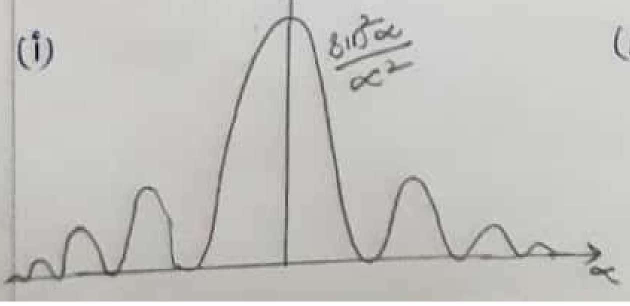
where $\alpha = \dfrac{\pi a \sin\theta}{\lambda}$ and $\beta = \dfrac{\pi (a+b)\sin\theta}{\lambda}$

Thus the resultant intensity depends on

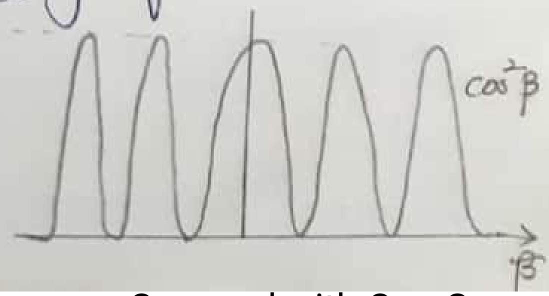
(i) The factor $\dfrac{A^2\sin^2\alpha}{\alpha^2} \Rightarrow$ Diffraction due to each individual single slits

(ii) The factor $\cos^2\beta \Rightarrow$ Interference pattern due to waves overlapping from two slits.

(i)



(ii)



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Dependence of Intensity $\left(I = 4A^2\frac{\sin^2\alpha}{\alpha^2}\cos^2\beta\right)$ on $\cos^2\beta$

Intensity Maximum

$I = max \Rightarrow \cos^2\beta = 1$

$\beta = \pm m\pi$

where $m = 0, 1, 2, 3 \ldots$

$\frac{\pi(a+b)\sin\theta}{\lambda} = \pm m\pi$

$(a+b)\sin\theta = \pm m\lambda$

when $m=0$, path diff $= 0$

$\therefore \theta = 0 \Rightarrow$ Centre maximum of interference pattern known as zeroth order principal maximum.

Intensity Minimum

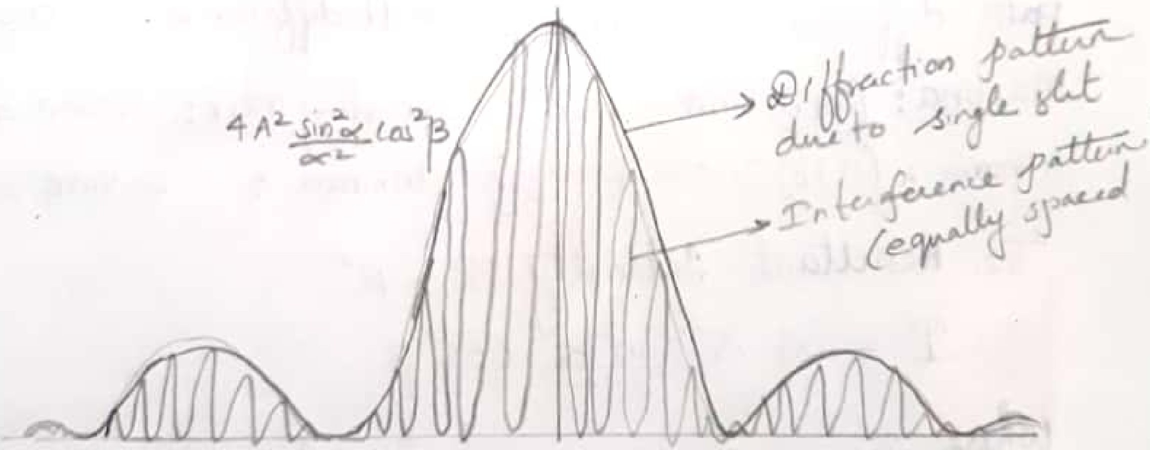
$I = 0 \Rightarrow \cos^2\beta = 0$

$\beta = \pm(2m+1)\frac{\pi}{2}$

$\frac{\pi(a+b)\sin\theta}{\lambda} = \pm(2m+1)\frac{\pi}{2}$

$(a+b)\sin\theta = \pm(2m+1)\frac{\lambda}{2}$

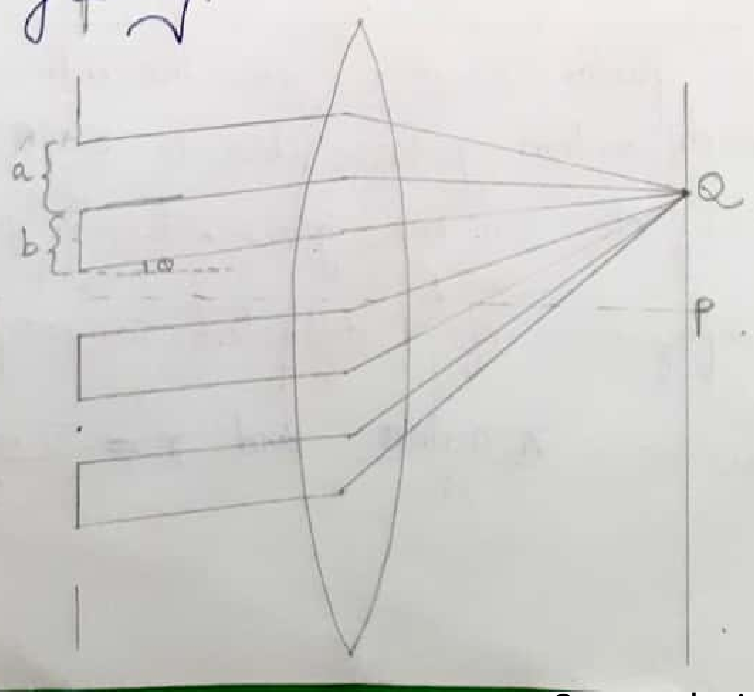
where $m = 0, 1, 2 \ldots$



DIFFRACTION GRATING — DIFFRACTION DUE TO N SLITS

An arrangement consisting of a large number of equally spaced parallel slits of equal width, separated from each other by opaque spaces which are also of equal width is called diffraction grating. It may be made by ruling a large number of fine, equidistant and parallel lines on an optically plane glass plate with a diamond point. The rulings are opaque to light while space between rulings is transparent. Such a grating is called plane transmission grating. There will be about 15000 rulings per inch.

Due to technical difficulties replicas of these are made for practical purposes. This is done by pouring a thin layer of collodion solution over the ruled grating and the solution is allowed to harden. It is stripped off from the surface & this film has the impression of original grating.



Let us consider the diffraction produced by N slits each of width 'a'. The separation between consecutive slits is

$$\boxed{d = a+b}$$

where $a \rightarrow$ width of each slit
$b \rightarrow$ width of opaque portion.

$d = a+b$ is called grating element or grating period

Reciprocal of grating element $\left(\frac{1}{a+b}\right)$ is the number of ruled lines per unit width of grating.

Let a plane wavefront λ be incident normally on the slit. Then each point in the slits give rise to secondary wavelets in all directions. The resultant amplitude of light from a single slit of width 'a' in a direction making an angle θ with incident ray direction is at Q

$$R = A \frac{\sin\alpha}{\alpha} \frac{\sin N\gamma}{\sin\gamma}$$

The resultant intensity is $\qquad I_0 = A^2$

$$\boxed{I = I_0 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\gamma}{\sin^2\gamma}}$$

→ The factor $I_0 \frac{\sin^2\alpha}{\alpha^2}$ gives the intensity distribution in diffraction pattern due to single slits.

→ The factor $\frac{\sin^2 N\gamma}{\sin^2\gamma}$ gives intensity pattern due to inferference of N diffracted waves from N slits.

where $\alpha = \frac{\pi a \sin\theta}{\lambda}$ and $\gamma = \frac{\pi d \sin\theta}{\lambda}$ (where $d = a+b$)

Principal Maxima

$$I \propto \frac{\sin^2 N\gamma}{\sin^2\gamma}$$

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\gamma}{\sin^2\gamma}$$

I = Maximum when $\sin\gamma = 0$

$$\gamma = \pm m\pi$$

$$\Rightarrow \quad \frac{\pi d \sin\theta}{\lambda} = \pm m\pi$$

$$\boxed{d\sin\theta = \pm m\lambda} \rightarrow \text{grating Equation}$$

$(a+b)\sin\theta = \pm m\lambda$ where $m = 0, 1, 2, 3 \dots$

If $m=0$, $\theta = 0$. In this direction waves from all slits arrive in phase and the central bright image is got. This is called zero order principal maximum. If $m = 1, 2, 3 \dots$ first order, second order etc. principal maxima are obtained.

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2} \lim_{\gamma \to \pm m\pi} \frac{\sin^2 N\gamma}{\sin^2\gamma}$$

$$\lim_{\gamma \to \pm m\pi} \frac{\sin N\gamma}{\sin\gamma} = N \quad \text{(L' Hospital's rule)}$$

$$\therefore \boxed{I = I_0 \left(\frac{\sin^2\alpha}{\alpha^2}\right) N^2} \rightarrow \text{Resultant Intensity of maxima}$$

The intensity of principal maximum increases with increase in N.

MINIMA

I = minimum = zero, when $\sin N\gamma = 0$ or

$$N\gamma = \pm m\pi$$

$$N \frac{\pi d \sin\theta}{\lambda} = \pm m\pi$$

$$N d\sin\theta = \pm m\lambda$$

$$\boxed{d\sin\theta = \pm \frac{m\lambda}{N}}$$

$\pm$ sign only mean that minima of any order lie symmetrically on both sides of central principal maximum

if $m = N$

$d\sin\theta = \pm\lambda$

$\Rightarrow$ Ist order principal max

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Here m can take all integral values except 0, N, 2N, 3N etc ⇒ (These give position of principal maxima)

$m = 0 \Rightarrow$ principal maximum of zeroth order

$m = N \Rightarrow$ principal max. of first order.

$m = 1, 2, 3 \ldots (N-1) \Rightarrow$ Minima

Thus between zero order and first order principal maxima, we have $(N-1)$ minima. Between two such consecutive minima, the intensity has to be maximum and these maxima are known as secondary maxima.

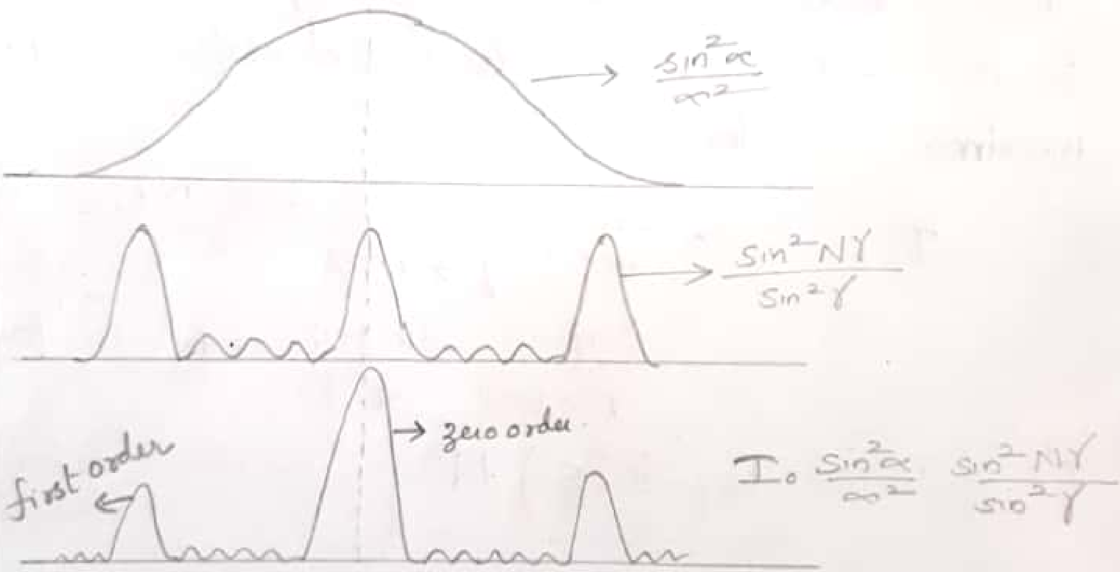


Fig: Intensity pattern

Maximum number of orders possible

Grating eqn → $d\sin\theta = m\lambda$

$$m = \frac{d\sin\theta}{\lambda} \qquad \sin\theta_{max} = 1$$

$$m_{max} = \frac{d}{\lambda} \qquad d = a+b$$

$$= \frac{a+b}{\lambda} = \frac{1}{\lambda \times (\text{No: of lines/cm})}$$

Case 1: if $(a+b) < \lambda \Rightarrow m_{max} < 1 \Rightarrow$ only central maximum

Case 2: if $(a+b) < 2\lambda \Rightarrow m_{max} < 2 \Rightarrow$ central max & first order

Missing of orders

If in any direction $\frac{\sin^2 N\gamma}{\sin^2\gamma}$ is maximum and $\frac{\sin^2\alpha}{\alpha^2}$ is zero, the principal maximum will not be present in that direction. Thus if $(a+b)\sin\theta = m\lambda$ (condn for maximum in N slits) and $a\sin\theta = n\lambda$ (condn for minimum in single slit) are simultaneously satisfied, the principal maxima of order 'm' will be absent in the grating spectrum

$$(a+b)\sin\theta = m\lambda \quad \text{——} ①$$

$$a\sin\theta = n\lambda \quad \text{——} ②$$

$\frac{①}{②}$ $$\frac{a+b}{a} = \frac{m}{n}$$

$$\boxed{m = \frac{(a+b)}{a}\cdot n}$$

This is the condition for mth order principal maximum to be absent in grating spectrum.

For example:

If slit width and ruling width are equal:

i.e $a = b$

Then $m = \left(\frac{a+b}{a}\right) n$

$m = 2n$ Here $n = 1, 2, 3 \ldots$

$m = 2, 4, 6 \ldots$

$\therefore$ 2^{nd}, 4^{th}, 6^{th} order spectra will be absent if $a = b$

RESOLVING POWER

The ability of an optical instrument to produce distinctly separate images of two objects located very close to each other is called its resolving power. It is also defined as the reciprocal of the smallest angle subtended at the objective of optical instrument by two point objects, which can just be distinguished as separate.

Rayleigh's Criterion

According to Rayleigh's criterion, two closely spaced point sources of light are said to be just resolved if the principal maximum of diffraction pattern due to one falls on the first minimum of the other.

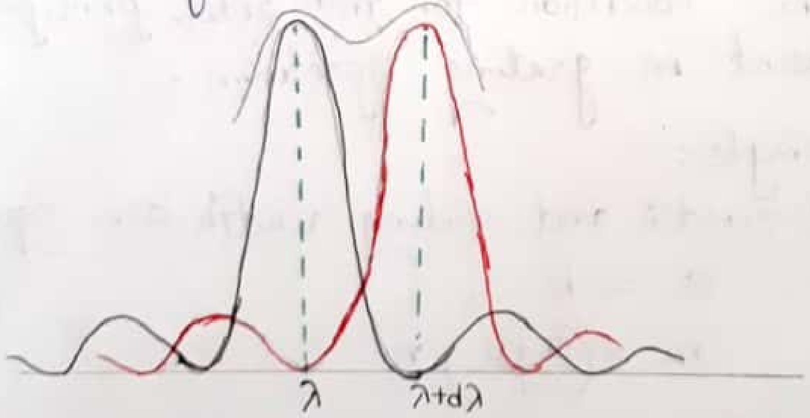


Fig: just resolved - Rayleigh's criteria

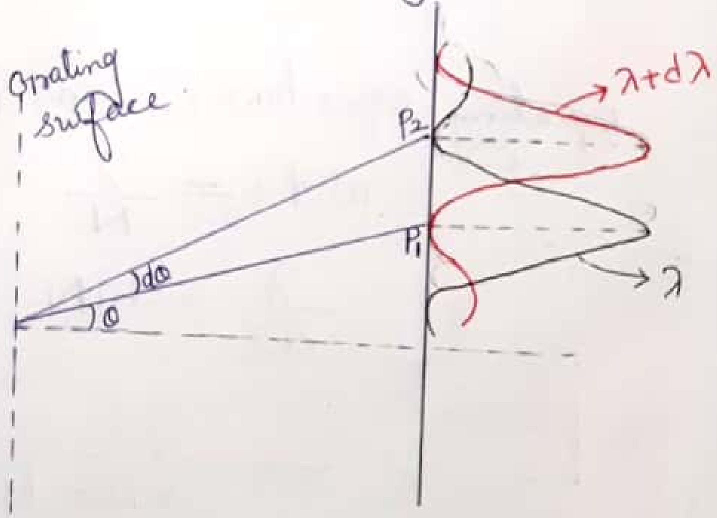
Resolving power of a grating

Resolving power of a grating is its ability to show two spectral lines of very close wavelengths clearly separated. To determine resolving power of a grating, consider a beam of light containing two wavelengths λ and $\lambda+d\lambda$ very close to each other, incident on grating surface with grating element $(a+b)$

Resolving power of grating $= \dfrac{\lambda}{d\lambda}$

where $\lambda \rightarrow$ wavelength of any of two spectral line (or can take average of two wavelength)

$d\lambda \rightarrow$ Difference in their wavelength



Let at $P_1 \rightarrow m^{th}$ order principal maximum for spectral line with λ wavelength,
Angle of diffraction is θ

at $P_2 \rightarrow m^{th}$ order principal maximum for spectral line with $(\lambda+d\lambda)$ wavelength,
Angle of diffraction at P_2 is $\theta+d\theta$

According to Rayleigh criterion, the first minimum due to λ should coincide with principal maximum of $\lambda+d\lambda$

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Condition for m^{th} order principal maximum for wavelength λ and $\lambda + d\lambda$ at P_1 and P_2 respectively are

$$P_1 : \quad (a+b)\sin\theta = m\lambda \quad \text{---} (1)$$

$$P_2 : \quad (a+b)\sin(\theta + d\theta) = m(\lambda + d\lambda) \quad \text{---} (2)$$

The extra path difference introduced at P_2 is

$$P_1 P_2 = \underbrace{(a+b)\sin(\theta+d\theta)} - (a+b)\sin\theta$$

$$= m(\lambda + d\lambda) - m\lambda$$

$$= m\,d\lambda \quad \text{---} (3)$$

This distance is also same as the distance between first minimum and principal maximum of wavelength λ

Cond^n for I^{st} minimum is $\Rightarrow (a+b)\sin\theta = \frac{m\lambda}{N}$ with $m=1$

$$= \frac{\lambda}{N} \quad \text{---} (4)$$

Equating equation (3) and (4)

$$m\,d\lambda = \frac{\lambda}{N}$$

$$\frac{\lambda}{d\lambda} = mN$$

$\therefore$ $\boxed{\text{Resolving power of grating is } \frac{\lambda}{d\lambda} = mN}$

where $N \rightarrow$ Minimum No: of lines on grating required to just resolve wavelengths λ & $\lambda + d\lambda$

→ The advantage of increasing the number of lines in grating?

When the number of lines in a grating increases, the principal maxima become intense and sharp. The dispersive power and resolving power will be large as N increases.

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