1.17 Thevenin's Theorem

Thevenin's theorem is a powerful tool in the hands of engineers to simplify a complex problem and obtain the circuit solution quickly. It reduces the complex circuit to a simple circuit. This theorem is particularly useful to find the current in a particular branch of a network as the resistance of that branch is varied while all other resistances and sources remains constant.

This theorem was first stated by French engineer M.L. Thevenin in 1883. According to this theorem, any two terminal networks, however complex, can be replaced by a single source of emf V_{TH} (called Thevenin voltage) in series with a single resistance R_{TH} (called Thevenin resistance). Figure 1.256(a) shows a complex network enclosed in a box with two terminals A and B brought out. The network in the box may consist of any number of resistors and emf sources connected in any manner. But according to Thevenin, the entire circuit behind terminals A and B can be replaced by a single source of emf V_{TH} in series with a single resistance R_{TH} as shown in Fig. 1.256(b). The voltage V_{TH} is the voltage that appears across terminals A and B with load removed. The resistance R_{TH} is the resistance obtained with load removed and looking back into the terminals A and B when all the sources in the circuit are replaced by their internal resistances. Once Thevenin's equivalent circuit is obtained, current through any load R_L connected across AB can be readily obtained.

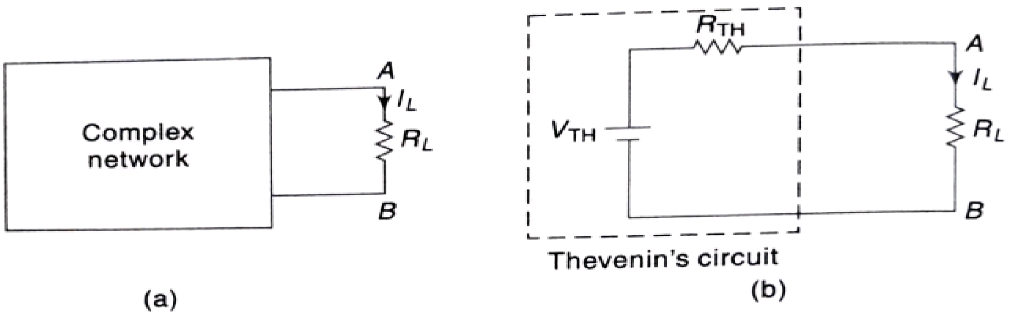

Fig. 1.256 Thevenin's equivalent circuit

Hence, Thevenin's theorem as applied to dc circuits may be stated as under: Any network having terminals A and B can be replaced by a single source of emf V_{TH} (called Thevenin voltage) in series with a single resistance R_{TH} (called Thevenin resistance).

(i) The emf V_{TH} is the voltage obtained across terminals A and B with load, if any, removed, i.e., it is open-circuited voltage between A and B.

(ii) The resistance R_{TH} is the resistance of the network measured between A and B with load removed and replacing all the voltage/current sources by their internal resistances[4].

Illustration

The concept of Thevenin's equivalent circuit across the load terminals can be explained by considering the circuit shown in Fig. 1.257(a). The Thevenin's equivalent circuit is shown in Fig. 1.257(b).

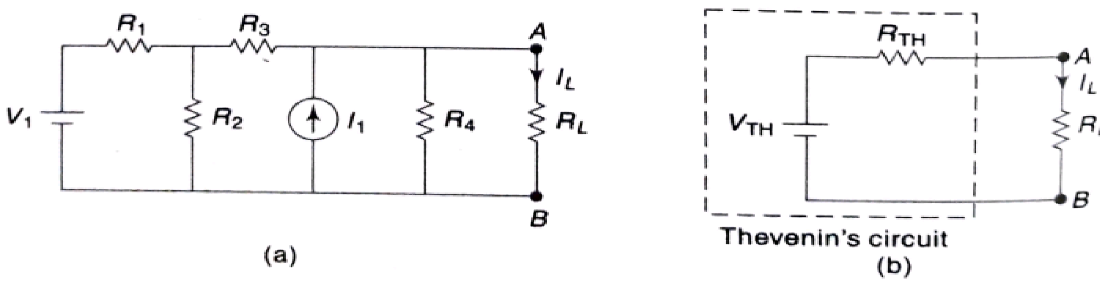

Fig. 1.257 Illustration of Thevenin's theorem

[4]The internal resistance of an ideal voltage source is zero and that of an ideal current source is infinite. Hence, while finding out R_{TH}, voltage sources are replaced by short circuits and current sources by open circuits.

The voltage V_{TH} is obtained across the terminals A and B with R_L removed. Hence, V_{TH} is also called open circuit Thevenin's voltage. The circuit to be used to calculate V_{TH} is shown in Fig. 1.258(a). While R_{TH} is the equivalent resistance obtained as viewed through the terminals A and B with R_L removed and replacing voltage source with short circuit and current source by open circuit as shown in Fig. 1.258(b).

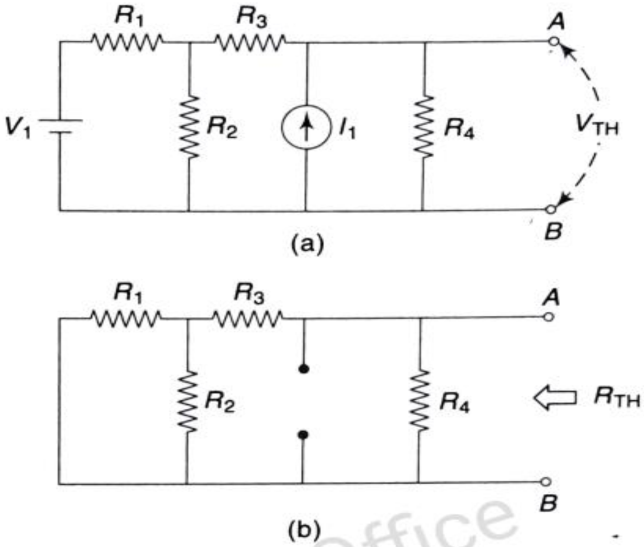

Fig. 1.258 Thevenin's voltage and Thevenin's resistance

While obtaining V_{TH}, any of the network simplification techniques can be used. When the circuit is replaced by Thevenin's equivalent across the load resistance, the load current can be obtained as

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

Steps to apply Thevenin's Theorem

Step 1: Remove the branch resistance through which current is to be calculated.

Step 2: Calculate the voltage across these open circuited terminals, by using any one of the network simplification techniques. This is V_{TH}.

Step 3: Calculate R_{TH} as viewed through the two terminals of the branch from which current is to be calculated by removing that branch resistance and replacing all sources by their internal resistances.

Step 4: Draw the Thevenin's equivalent circuit showing source V_{TH} with the resistance R_{TH} in series with it.

Step 5: Reconnect the branch resistance. Let it be R_L. The required current through the branch is given by

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

Example 1.70 Determine the current through 5 Ω resistor in the network shown in Fig. 1.259 by Thevenin's theorem.

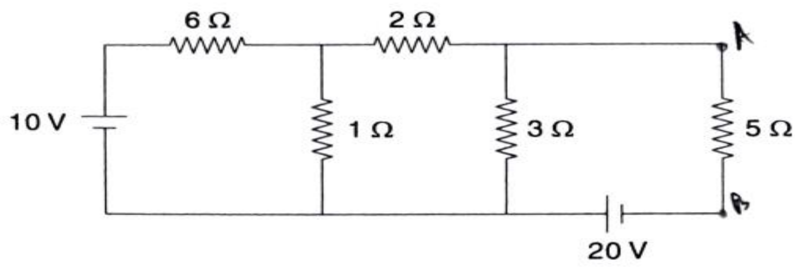

Fig. 1.259

Solution

Current through 5 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.

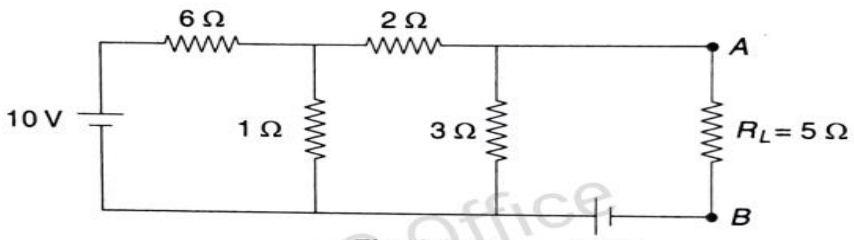

Fig. 1.260

Step I: Calculation of V_{TH}

Removing the load resistance from the network, we get the following network:

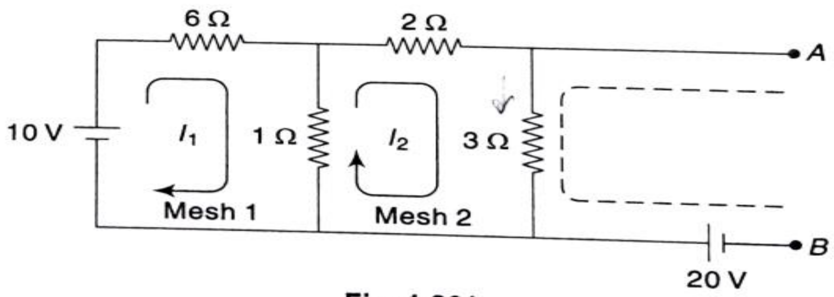

Fig. 1.261

In Fig. 1.261, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. The voltage V_{TH}, i.e., V_{AB}, can be calculated as follows:

Select any path from A to B and marked the selected path by dotted line as shown in Fig. 1.261. By using any circuit simplification techniques, calculate the current through all the resistances present in the selected path. Then travel through the selected path from B to A and add all voltage drops and emf's algebraically.

In Fig. 1.261, 3 Ω resistor is present in the selected path, and for calculation of V_{TH}, current through the 3 Ω resistor is required. By using mesh analysis, $I_{3\Omega}$ can be calculated.

Applying KVL to mesh 1,

$$-6I_1 - (I_1 - I_2) + 10 = 0$$

or $\quad 7I_1 - I_2 = 10 \qquad$ (i)

Applying KVL to mesh 2,

$$-2I_2 - 3I_2 - (I_2 - I_1) = 0$$

or $\quad I_1 - 6I_2 = 0 \qquad$ (ii)

Solving (i) and (ii),

$$I_2 = 0.244 \text{ A}$$

Hence, $I_{3\Omega} = 0.244 \text{ A} (\downarrow)$

Thus, $V_{TH} = V_{AB}$

$$= 20 + 3I_2$$
$$= 20 + 3(0.244)$$
$$= 20.732 \text{ V}$$

Step II: Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage sources by short circuit, we get the following network:

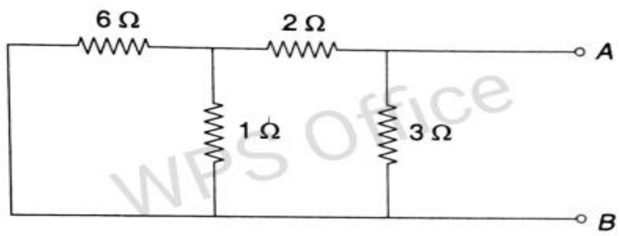

Fig. 1.262

In Fig. 1.262, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. By series-parallel circuit reduction techniques, we obtain the circuit as shown below:

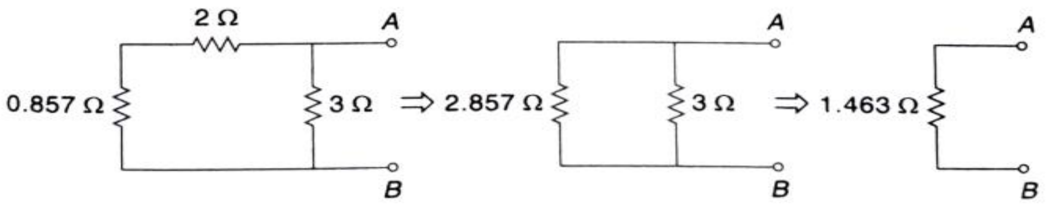

Fig. 1.263

Thus, $R_{TH} = R_{AB} = 1.463\ \Omega$.

Step III: Calculation of load current

Thevenin's equivalent circuit can be drawn as follows:

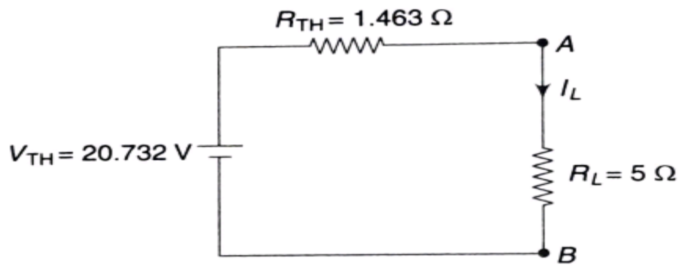

Fig. 1.264

By Ohm's law,

$$I_L = \frac{V_{\text{TH}}}{R_L + R_{\text{TH}}}$$

Hence, $I_L = I_{5\Omega} = \dfrac{20.732}{5+1.463} = 3.21\ \text{A}(\downarrow)$

Example 1.71 Determine the current through 1.5 Ω resistor in the network shown in Fig. 1.265 by Thevenin's theorem.

Solution

Current through 1.5 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.

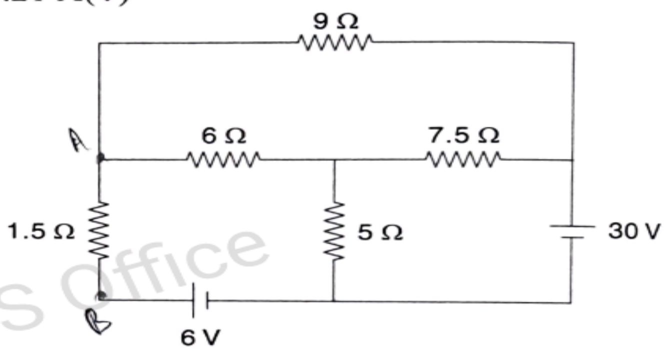

Fig. 1.265

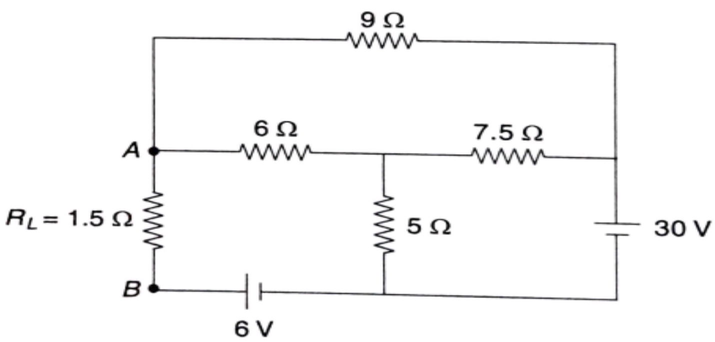

Fig. 1.266

Step I: Calculation of V_{TH}

Removing the load resistance from the network, we get the circuit as shown in Fig. 1.267.

In Fig. 1.267, voltage appears across the load terminals A and B, which is called

Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. 1.267. As this path contains the resistors 5 Ω and 6 Ω, currents through these resistances are required. By using mesh analysis, these required currents can be calculated.

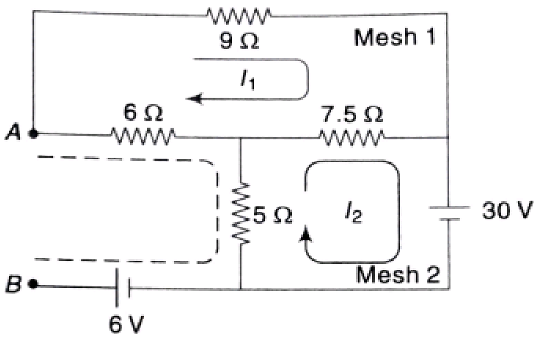

Fig. 1.267

Applying KVL to mesh 1,

$$-9I_1 - 7.5(I_1 - I_2) - 6I_1 = 0$$

or $$-22.5I_1 + 7.5I_2 = 0 \quad \text{(i)}$$

6(-1) + 5(-3) +6

-6 -15 + 6

-15

Applying KVL to mesh 2,

$$-7.5(I_2 - I_1) - 30 - 5I_2 = 0$$

or $$7.5I_1 - 12.5I_2 = 30 \quad \text{(ii)}$$

Solving (i) and (ii),

$$I_1 = -1 \text{ A}, \qquad I_2 = -3 \text{ A}$$

Hence, $V_{TH} = V_{AB}$

$$= -6 - 5I_2 - 6I_1$$
$$= -6 - 5(-3) - 6(-1)$$
$$= 15 \text{ V}$$

Step II: Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage sources by short circuits, we get the circuit as shown in Fig. 1.268.

In Fig. 1.268, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. The circuit can be redrawn as shown in Fig. 1.269.

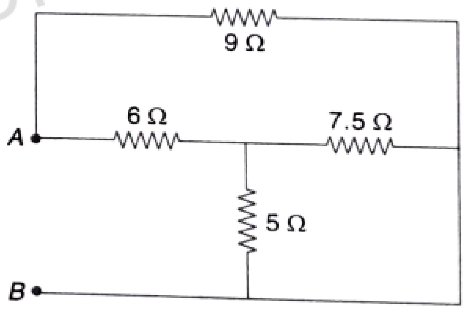

Fig. 1.268

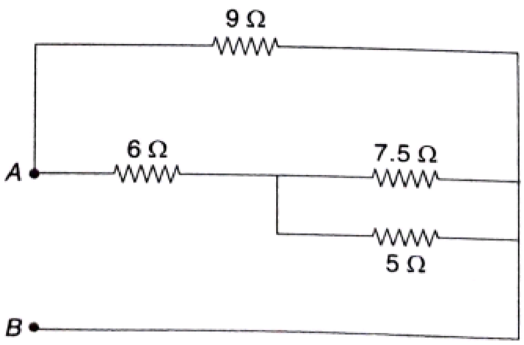

Fig. 1.269

By series-parallel circuit reduction techniques, we get the following circuit:

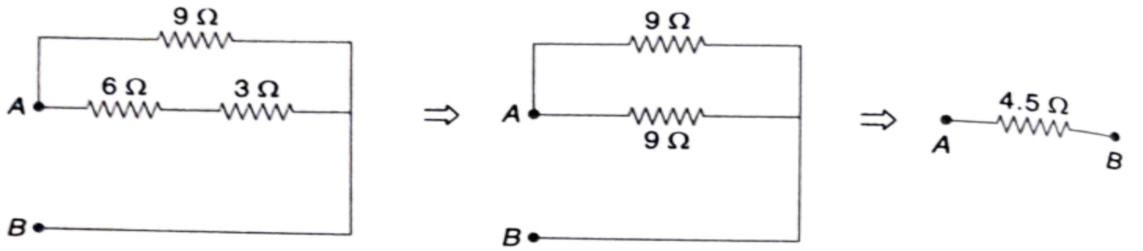

Fig. 1.270

Thus, $R_{TH} = R_{AB} = 4.5\ \Omega$

Step III: Calculation of load current

Thevenin's equivalent circuit can be drawn as shown in Fig. 1.271.

By Ohm's law,

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

Hence, $I_L = I_{1.5\Omega} = \dfrac{15}{1.5 + 4.5} = 2.5\ \text{A}(\downarrow)$

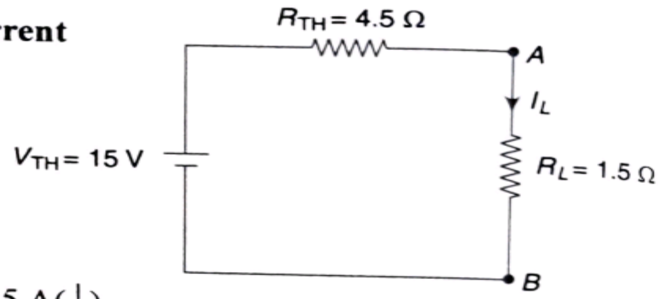

Fig. 1.271

Example 1.72 Determine the current through 8 Ω resistor in the network shown in Fig. 1.272 by Thevenin's theorem.

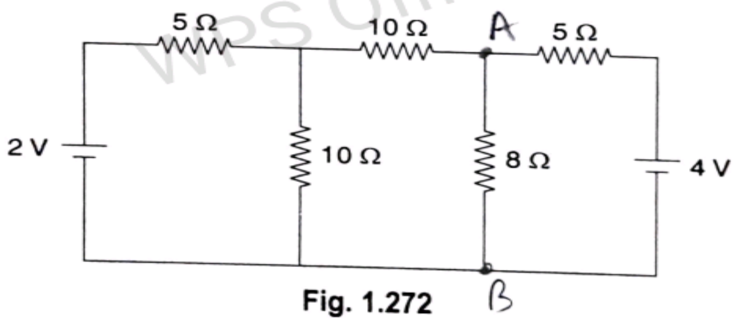

Fig. 1.272

Solution

Current through 8 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.

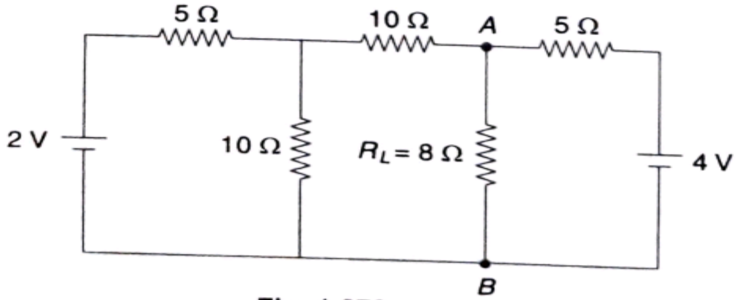

Fig. 1.273

Step I: Calculation of V_{TH}

Removing the load resistance from the network, we get the following network:

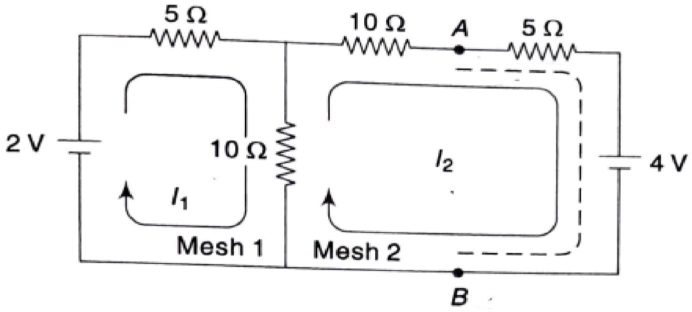

Fig. 1.274

In Fig. 1.274, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. 1.274. As this path contains the resistor 5 Ω, current through this resistance is required. By using mesh analysis, this required current can be calculated as follows.

Applying KVL to mesh 1,

$$-5I_1 - 10(I_1 - I_2) + 2 = 0$$

or $$-15I_1 + 10I_2 = -2 \quad \text{(i)}$$

Applying KVL to mesh 2,

$$-10I_2 - 5I_2 - 4 - 10(I_2 - I_1) = 0$$

or $$10I_1 - 25I_2 = 4 \quad \text{(ii)}$$

Solving (i) and (ii),

$$I_2 = -0.145 \text{ A}$$

Hence, $V_{TH} = V_{AB}$

$$= +4 + 5I_2$$
$$= +4 + 5(-0.145)$$
$$= 3.275 \text{ V}$$

Step II: Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage sources by short circuit, we get the following circuit:

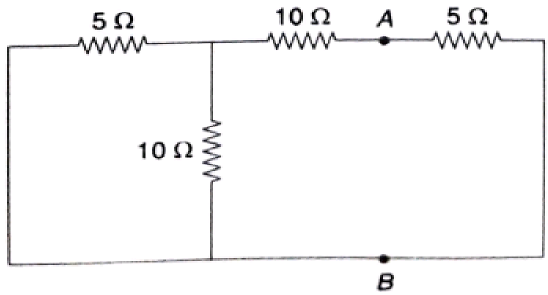

Fig. 1.275

In Fig. 1.275, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. By series-parallel circuit reduction techniques, we modify the circuit as shown below:

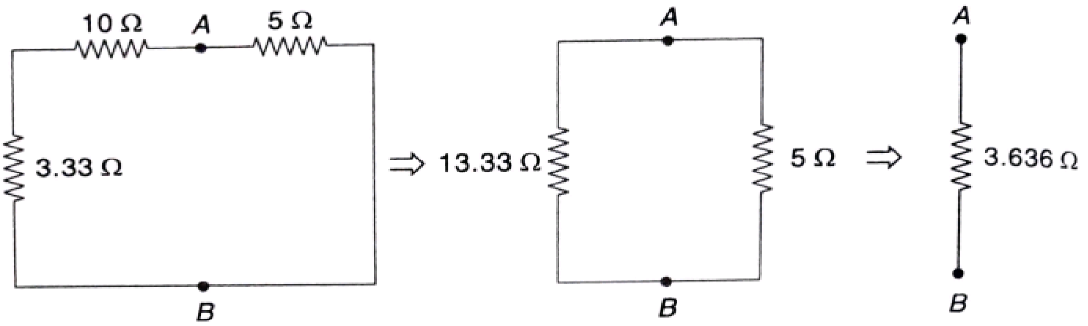

Fig. 1.276

Thus, $R_{TH} = R_{AB} = 3.636\ \Omega$

Step III: Calculation of load current

Thevenin's equivalent circuit can be drawn as shown in Fig. 1.277.

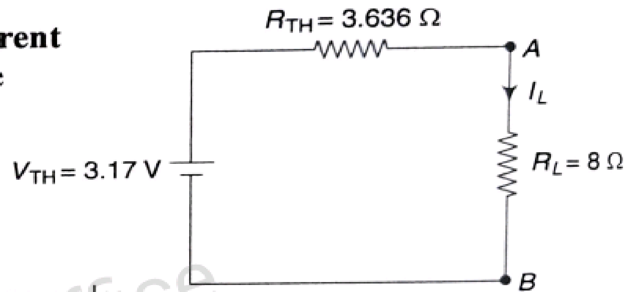

Fig. 1.277

By Ohm's law,

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

Hence, $I_L = I_{8\Omega} = \dfrac{3.275}{8 + 3.636} = 0.281\ \text{A}(\downarrow)$

Example 1.73 Determine the current through 10 Ω resistor in the network shown in Fig. 1.278 by Thevenin's theorem.

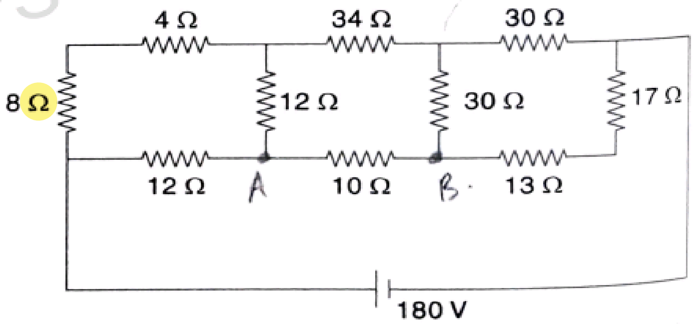

Fig. 1.278

Solution

Current through 10 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.

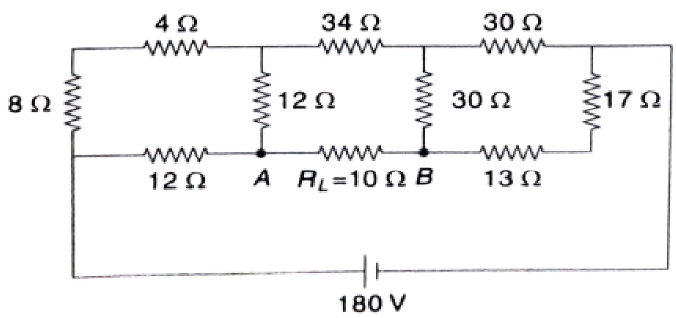

Fig. 1.279

Step I: Calculation of V_{TH}

Removing the load resistance from the network, we get the following modified network:

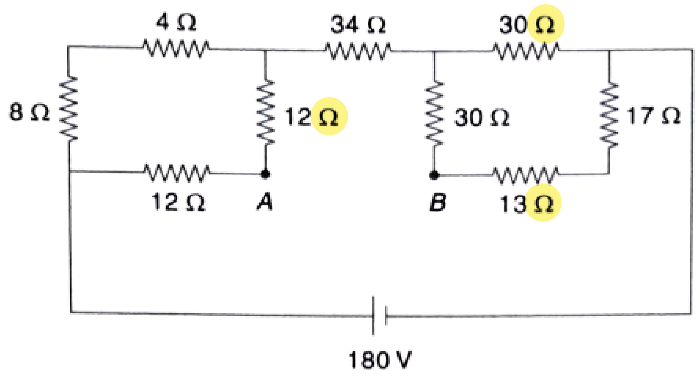

Fig. 1.280

In Fig. 1.280, resistors 8 Ω and 4 Ω are in series. Also resistors 17 Ω and 13 Ω are in series.

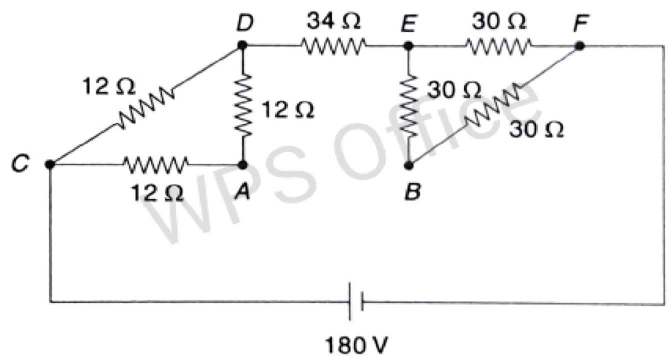

Fig. 1.281

Converting the delta connections formed by three 12 Ω resistors (Δ *CDA*) and three 30 Ω resistors (Δ *EFB*) into equivalent star connections, i.e., $\Delta\, CDA \Rightarrow Y\, CDA$ and $\Delta\, EFB \Rightarrow Y\, EFB$, we get the circuit as shown in Fig. 1.282.

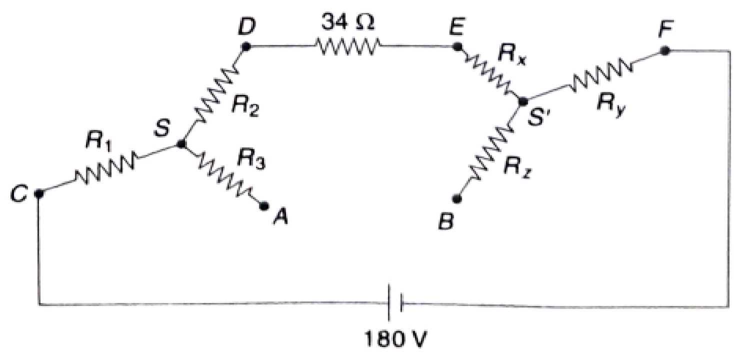

Fig. 1.282

$$R_1 = R_2 = R_3 = \frac{12 \times 12}{12 + 12 + 12} = 4\ \Omega$$

$$R_x = R_y = R_z = \frac{30 \times 30}{30 + 30 + 30} = 10\ \Omega$$

The simplified network is shown in Fig. 1.283.

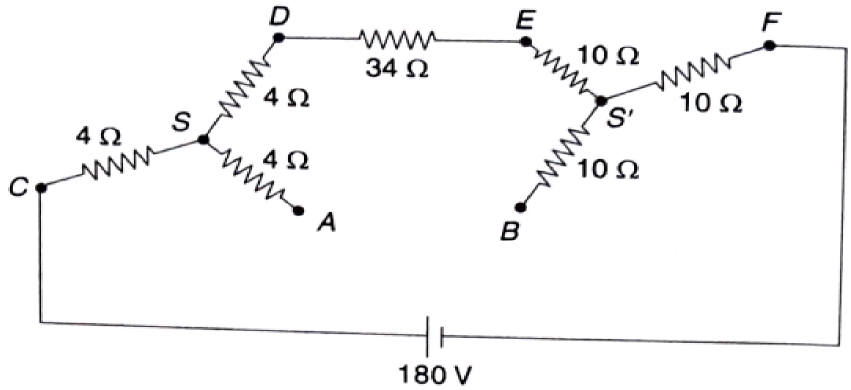

Fig. 1.283

In branch $SDES'$, 4 Ω, 34 Ω, and 10 Ω resistors are in series.
Let the current delivered by the battery is I A.

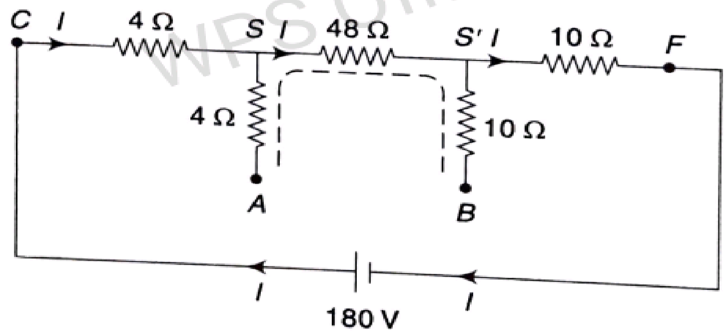

Fig. 1.284

For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. 1.284. As branch SA and branch $S'B$ are open circuited, $I_{SA} = 0$ and $I_{S'B} = 0$. By using Ohm's law, total circuit current I can be calculated as

$$I = \frac{180}{4 + 48 + 10} = \frac{180}{62} = 2.903\ \text{A}$$

Hence, $V_{TH} = V_{AB}$

$$= (10 \times 0) + (48 \times I) + (4 \times 0)$$
$$= (10 \times 0) + (48 \times 2.903) + (4 \times 0)$$
$$= 139.34\ \text{V}$$

Step II: Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage source by short circuit, we get the following network:

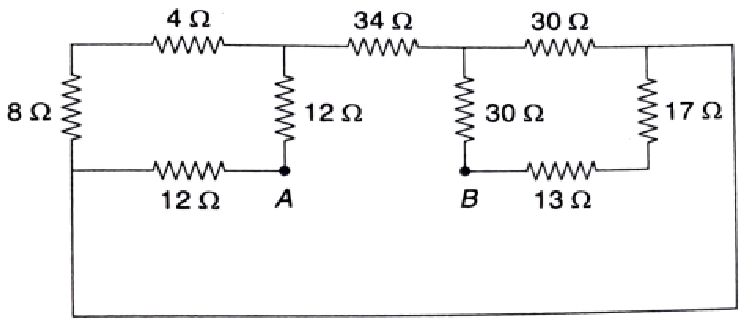

Fig. 1.285

In Fig. 1.285, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}.

In Fig. 1.285, 8 Ω and 4 Ω resistors are in series. Also 17 Ω and 13 Ω resistors are in series.

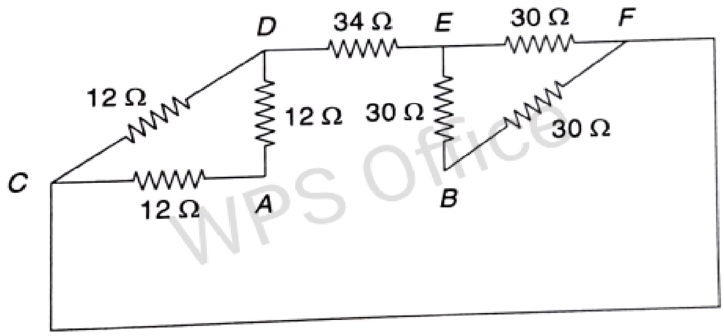

Fig. 1.286

Converting the delta connections formed by three 12 Ω resistors (Δ CDA) and three 30 Ω resistors (Δ EFB) into equivalent star network, i.e. Δ $CDA \Rightarrow Y\ CDA$ and Δ $EFB \Rightarrow Y\ EFB$, we get the network as shown in Fig. 1.287.

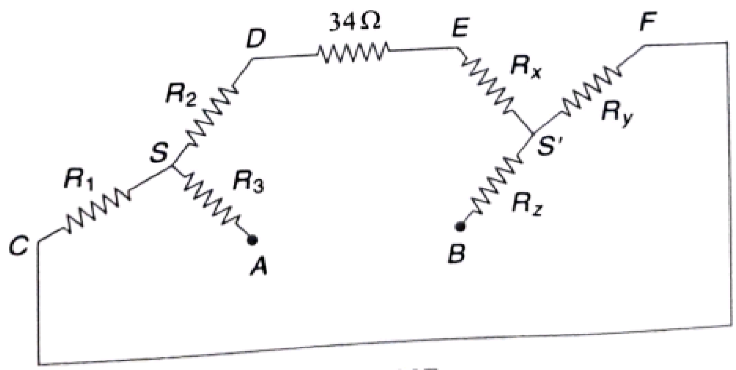

Fig. 1.287

$$R_1 = R_2 = R_3 = \frac{12 \times 12}{12 + 12 + 12} = 4\ \Omega$$

$$R_x = R_y = R_z = \frac{30 \times 30}{30 + 30 + 30} = 10\ \Omega$$

The simplified network is shown in Fig. 1.288.

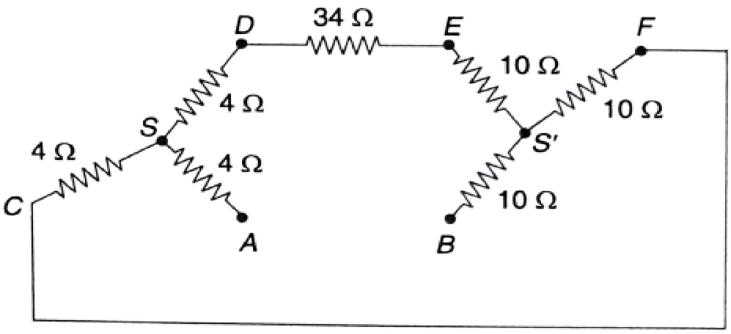

Fig. 1.288

In branch $SDES'$, 4 Ω, 34 Ω, and 10 Ω are in series. Also in branch $SCFS'$, 4 Ω and 10 Ω resistors are in series.

In Fig. 1.289, 48 Ω and 14 Ω resistors are in parallel.

Thus, $R_{TH} = R_{AB} = 24.84\ \Omega$.

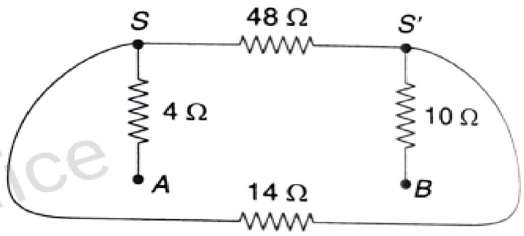

Fig. 1.289

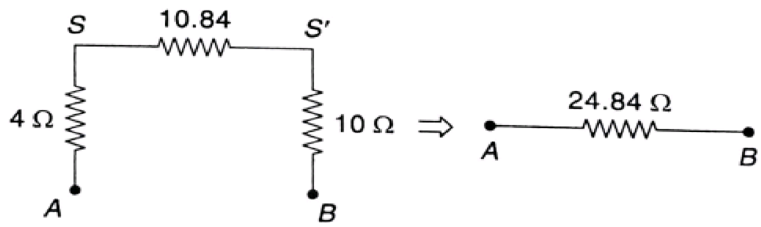

Fig. 1.290

Step III: Calculation of load current

Thevenin's equivalent circuit can be drawn as shown in Fig. 1.291.

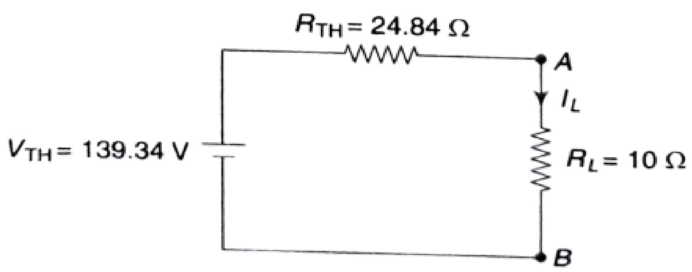

Fig. 1.291

By Ohm's law,

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

Hence, $I_L = I_{10\Omega} = \dfrac{139.34}{10 + 24.84} = 4\text{ A}(\rightarrow)$

Example 1.74 Using Thevenin's theorem, obtain the power drawn by 20 Ω resistor in the network shown in Fig. 1.292.

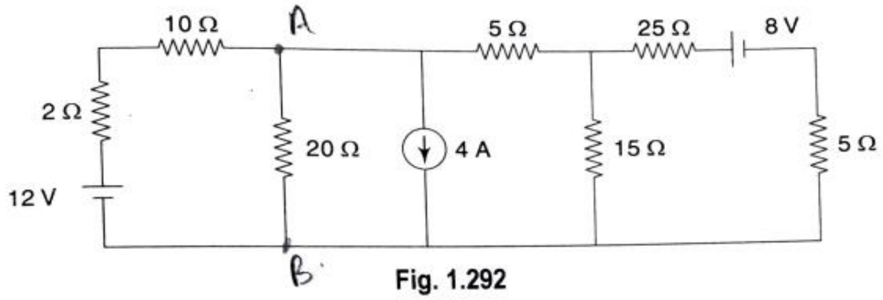

Fig. 1.292

Solution

Power drawn by 20 Ω resistor can be calculated as $P_{20\Omega} = I_{20\Omega}^2 \times 20$. Thus, current through 20 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.

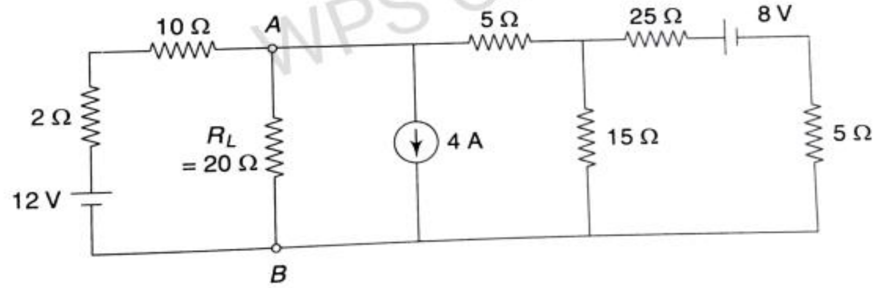

Fig. 1.293

Step I: Calculation of V_{TH}

Removing the load resistance from the network, we get the following network:

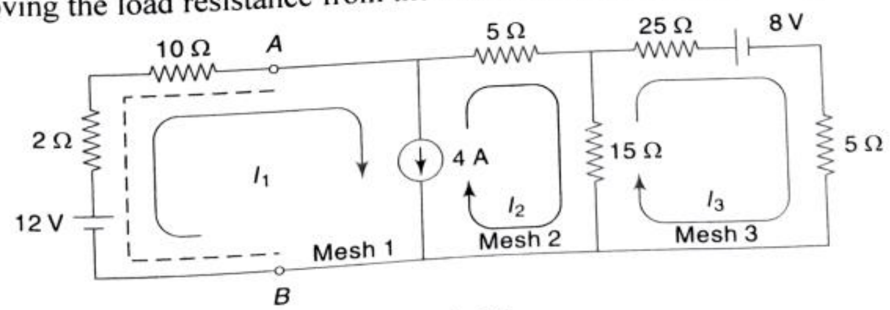

Fig. 1.294

In Fig. 1.294, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. 1.294. As this path contains the resistors 2 Ω and 10 Ω, currents through these resistances are required. By using mesh analysis, these required currents can be calculated.

Mesh 1 and mesh 2 form a supermesh.

By expressing the current in the common branch, we get the current equation as

$$(I_1 - I_2) = 4 \qquad \text{(i)}$$

By applying the KVL to the supermesh, we get the voltage equation as

$$-10I_1 - 5I_2 - 15(I_2 - I_3) + 12 - 2I_1 = 0$$

or $$-12I_1 - 20I_2 + 15I_3 = -12 \qquad \text{(ii)}$$

Now, by applying the KVL to mesh 3 (which does not contain any current source), we have

$$-25I_3 - 8 - 5I_3 - 15(I_3 - I_2) = 0$$

or $$15I_2 - 45I_3 = 8 \qquad \text{(iii)}$$

The value of I_1 may be found by solving the above three simultaneous equations or by the method of determinants as given below.

Putting the above three equations in matrix form, we have

$$\begin{bmatrix} 1 & -1 & 0 \\ -12 & -20 & 15 \\ 0 & 15 & -45 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 4 \\ -12 \\ 8 \end{bmatrix}$$

So, $$\Delta = \begin{vmatrix} 1 & -1 & 0 \\ -12 & -20 & 15 \\ 0 & 15 & -45 \end{vmatrix} = 1215, \qquad \Delta_1 = \begin{vmatrix} 4 & -1 & 0 \\ -12 & -20 & 15 \\ 8 & 15 & -45 \end{vmatrix} = 3120$$

By Cramer's rule,

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{3120}{1215} = 2.57 \text{ A}$$

Hence, $$V_{TH} = V_{AB}$$
$$= 12 - 2I_1 - 10I_1$$
$$= 12 - 2(2.57) - 10(2.57)$$
$$= -18.84 \text{ V}$$

Step II: Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage sources by short circuit and current source by open circuit, we get the following circuit:

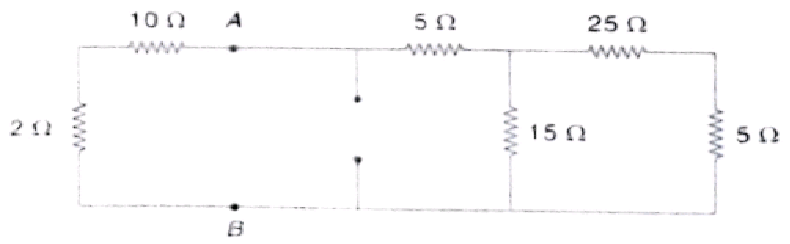

Fig. 1.295

In Fig. 1.295, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. In Fig. 1.296, 10 Ω and 2 Ω resistors are in series. Also 25 Ω and 5 Ω resistors are in series.

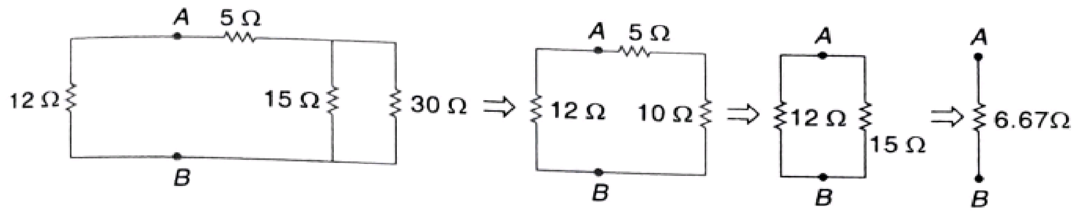

Fig. 1.296

Thus, $R_{TH} = R_{AB} = 6.67\ \Omega$.

Step III: Calculation of load current

Thevenin's equivalent circuit can be drawn as shown below:

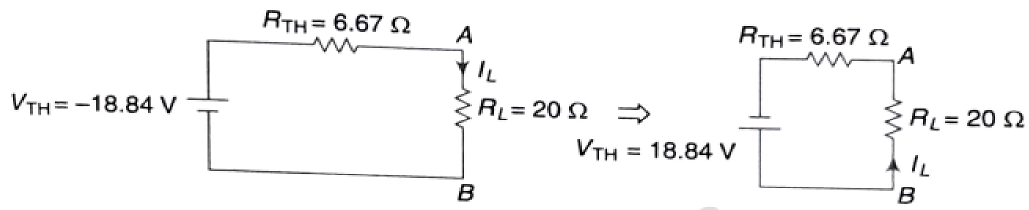

Fig. 1.297

By Ohm's law,

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

Hence, $I_L = I_{20\Omega} = \dfrac{18.84}{20 + 6.67} = 0.706\ \text{A}(\uparrow)$

Power drawn by 20 Ω resistor,

$$P_{20\Omega} = I_{20\Omega}^2 \times 20 = (0.706)^2 \times 20 = 9.97\ \text{W}$$

Example 1.75 Determine Thevenin's equivalent circuit and hence, find the current through 30 Ω resistor in the network shown in Fig. 1.298.

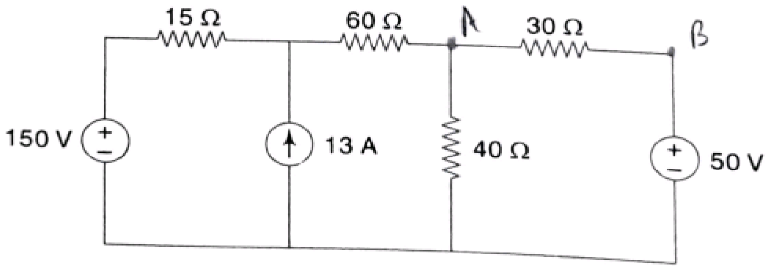

Fig. 1.298

Solution

Current through 30 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals.

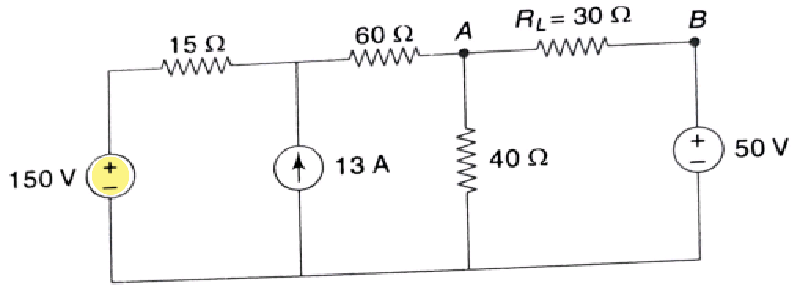

Fig. 1.299

Step I: Calculation of V_{TH}

Removing the load resistance from the network, we get the following modified network:

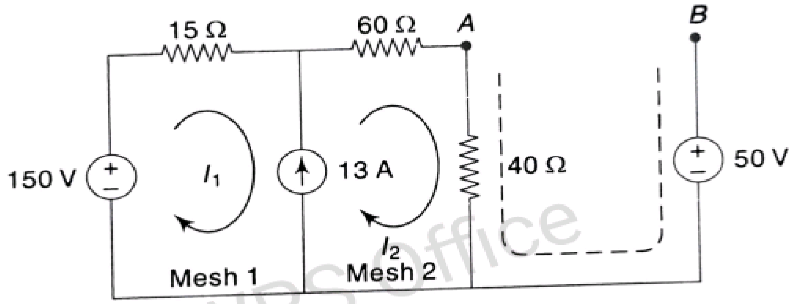

Fig. 1.300

In Fig. 1.300, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. 1.300. As this path contains the 40 Ω resistor, current through this resistance is required. By using mesh analysis, this required current can be calculated.

Mesh 1 and mesh 2 form a supermesh.

By expressing the current in the common branch, we get the current equation as

$$(I_2 - I_1) = 13 \quad \text{(i)}$$

By applying the KVL to the supermesh, we get the voltage equation as

$$-15I_1 - 60I_2 - 40I_2 + 150 = 0$$

or $$-15I_1 - 100I_2 = -150 \quad \text{(ii)}$$

Solving Eqs (i) and (ii),

$$I_2 = 3 \text{ A}$$

Hence, $$V_{TH} = V_{AB}$$
$$= -50 + 40I_2$$
$$= -50 + 40(3)$$
$$= 70 \text{ V}$$

Step II: Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage sources by short circuit and current source by open circuit, we get the circuit as shown in Fig. 1.301.

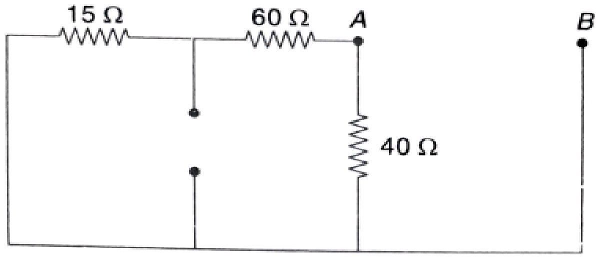

Fig. 1.301

In Fig. 1.301, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. In Fig. 1.301, resistors 60 Ω and 15 Ω are in series.

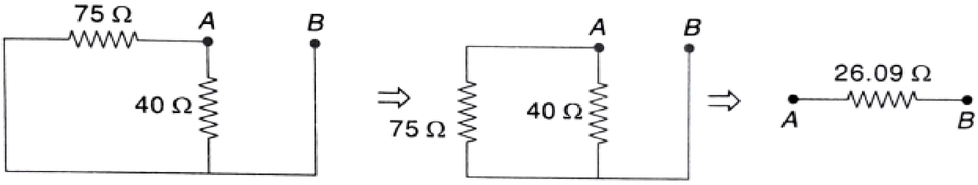

Fig. 1.302

Thus, $R_{TH} = R_{AB} = 26.09\ \Omega$.

Step III: Calculation of load current

Thevenin's equivalent circuit can be drawn as shown in Fig. 1.303.

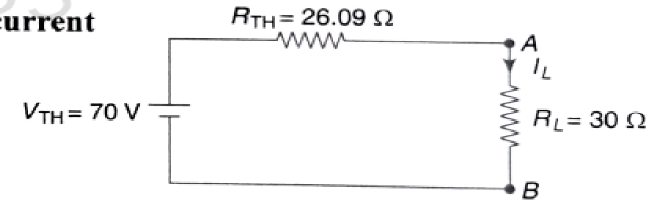

Fig. 1.303

By Ohm's law,

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

Hence, $I_L = I_{30\Omega} = \dfrac{70}{30 + 26.09} = 1.248\ \text{A}(\rightarrow)$

Example 1.76 Find the current in R_L in the network shown in Fig. 1.304, when R_L takes up values 5 Ω, 10 Ω, and 20 Ω.

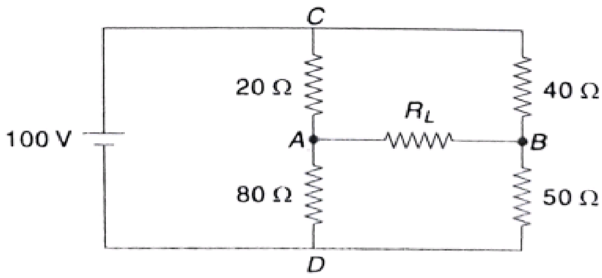

Fig. 1.304

Solution

Step I: Calculation of V_{TH}

Current through R_L is required. Its terminals A and B are called load terminals. Removing the load resistance from the network, we get the following network:

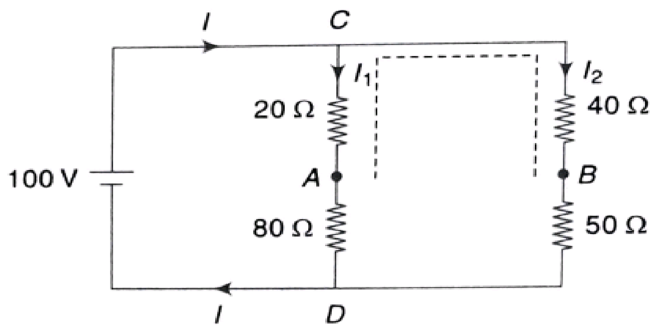

Fig. 1.305

In Fig. 1.305, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. 1.305. As this path contains the resistors 20 Ω and 40 Ω, currents through these resistances are required. In Fig. 1.305, the actual directions of currents are marked. The 100 V source produces the total current I A, which divides at node C. Let current through branch CAD is I_1 and current through branch CBD is I_2.

By Ohm's law,

$$I_1 = \frac{100}{20+80} = 1 \text{ A}$$

$$I_2 = \frac{100}{40+50} = 1.111 \text{ A}$$

Hence,
$$\begin{aligned} V_{TH} &= V_{AB} \\ &= 40I_2 - 20I_1 \\ &= 40(1.111) - 20(1) \\ &= 24.44 \text{ V} \end{aligned}$$

Step II: Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage source by short circuit, we get the following modified network:

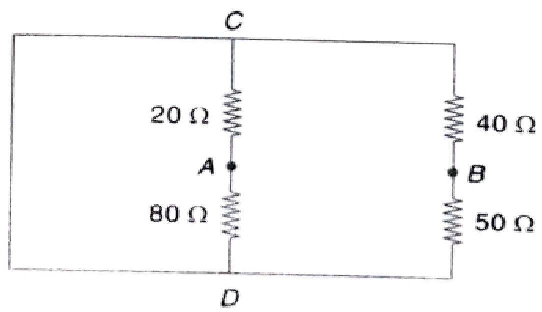

Fig. 1.306

In Fig. 1.306, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. Nodes C and D are same and by shorting them, the circuit can be redrawn as follows:

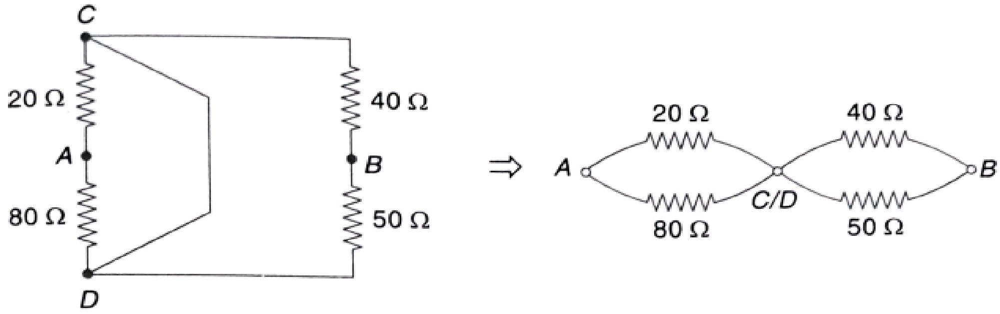

Fig. 1.307

By series-parallel circuit reduction techniques, we get the following circuit:
Thus, $R_{TH} = R_{AB} = 38.22\ \Omega$.

Fig. 1.308

Step III: Calculation of load current

Thevenin's equivalent circuit can be drawn as shown below:

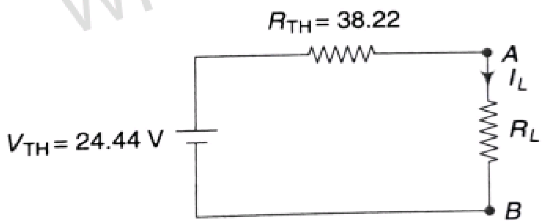

Fig. 1.309

By Ohm's law,

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

Hence, when $R_L = 5\ \Omega$, $I_L = I_{5\Omega} = \dfrac{24.44}{5 + 38.22} = 0.565\ \text{A}(\rightarrow)$

when $R_L = 10\ \Omega$, $I_L = I_{10\Omega} = \dfrac{24.44}{10 + 38.22} = 0.507\ \text{A}(\rightarrow)$

when $R_L = 20\ \Omega$, $I_L = I_{20\Omega} = \dfrac{24.44}{20 + 38.22} = 0.4198\ \text{A}(\rightarrow)$

Example 1.77 By Thevenin's theorem, find the current in 40 Ω resistor in the network shown in Fig. 1.310.

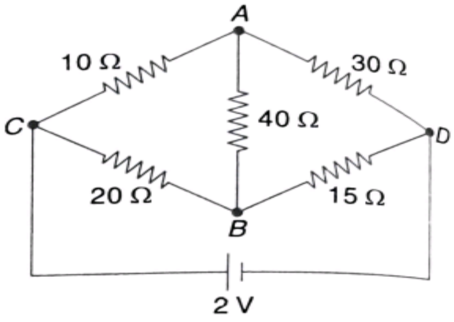

Fig. 1.310

Solution

Step I: Calculation of V_{TH}

Current through 40 Ω resistor is required. This resistance can be called load resistance R_L. Its terminals A and B are called load terminals. Removing the load resistance from the network, we get the modified network as shown in Fig. 1.311.

In Fig. 1.311, voltage appears across the load terminals A and B, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from A to B is marked by dotted line in Fig. 1.311. As this path contains the resistors 20 Ω and 10 Ω, currents through these resistances are required. In Fig. 1.311, the actual directions of currents are marked. The 2V source produces the total current I A, which divides at node C. Let current through branch CAD is I_1 and current through branch CBD is I_2.

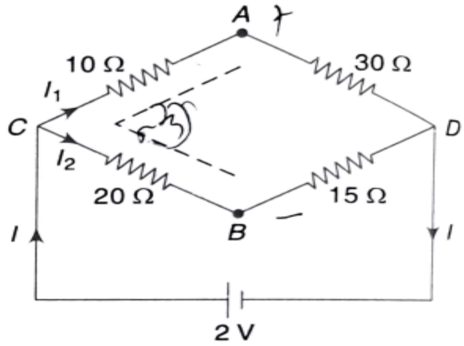

Fig. 1.311

By Ohm's law,

$$I_1 = \frac{2}{10+30} = 0.05 \text{ A}$$

$$I_2 = \frac{2}{20+15} = 0.0571 \text{ A}$$

Hence, $V_{TH} = V_{AB}$
$= 20I_2 - 10I_1$
$= 20(0.0571) - 10(0.05)$
$= 0.642$ V

Step II: Calculation of R_{TH}

Removing the load resistance from the network and replacing the voltage source by short circuit, we get the modified network as shown in Fig. 1.312.

In Fig. 1.312, equivalent resistance across the load terminals A and B is called Thevenin's resistance R_{TH}. Nodes C and D are same and by shorting them, the circuit can be redrawn as shown in Fig. 1.313.

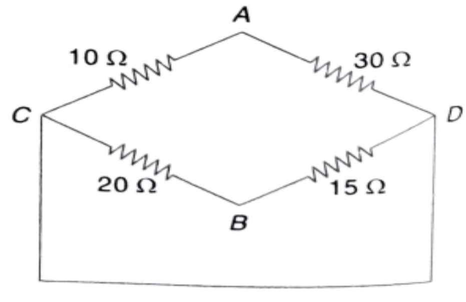

Fig. 1.312

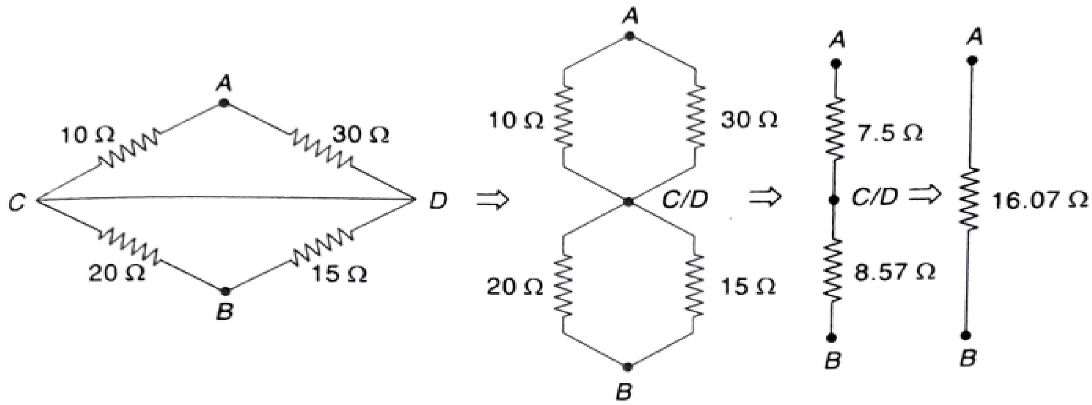

Fig. 1.313

Thus, $R_{TH} = R_{AB} = 16.07\ \Omega$.

Step III: Calculation of load current

Thevenin's equivalent circuit can be drawn as shown below:

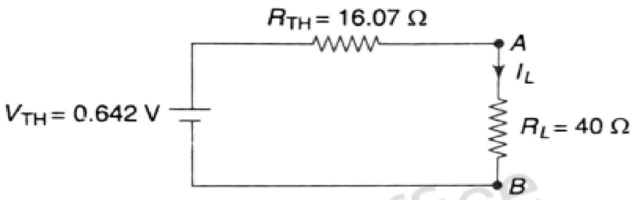

Fig. 1.314

By Ohm's law,

$$I_L = \frac{V_{TH}}{R_L + R_{TH}}$$

Hence, $I_L = I_{40\Omega} = \dfrac{0.642}{40 + 16.07} = 11.45\ \text{mA}(\downarrow)$

Example 1.78 For circuit shown in Fig. 1.315, find the Thevenin's equivalent circuit across a-b and hence find the current through the load of 10 Ω.

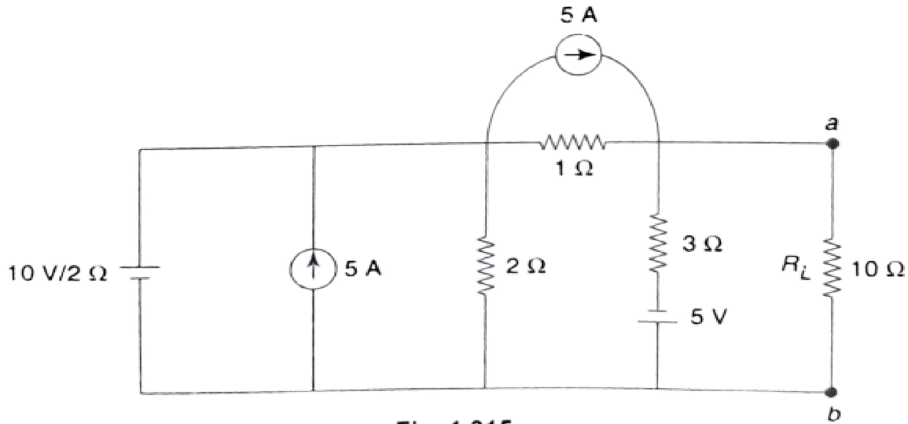

Fig. 1.315

Solution

Step1 : Calculation for V_{TH}

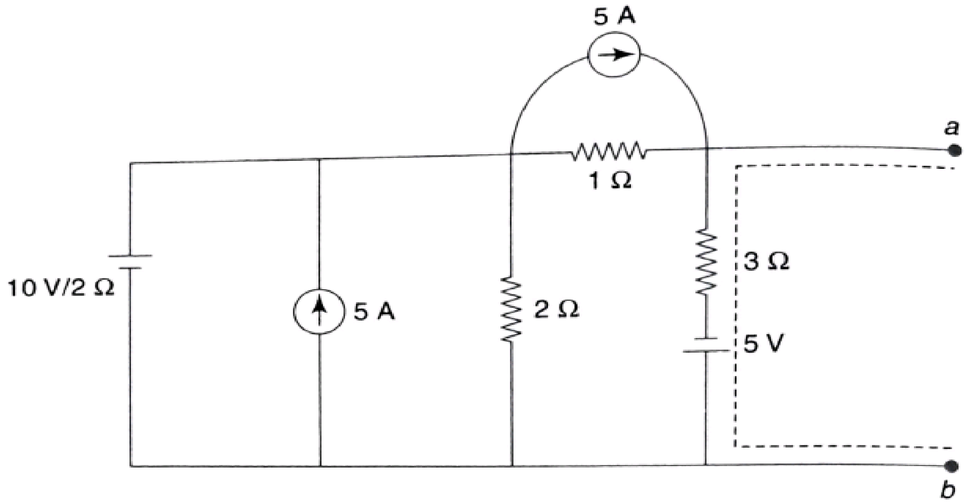

Fig. 1.316

In Fig. 1.316, voltage appears across the load terminal a and b, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{ab}, the selected path from a to b is marked by dotted line in Fig 1.316. As this path contains the resistor 3 Ω, current through this resistor is required. By using source transformation technique, this current can be calculated as follows.

By converting the 10V/2Ω source into equivalent parallel combinations (of current source and resistor), and by converting the parallel combinations (of 5 A current source and 1 Ω resistor) into equivalent series combination, we get the circuit as shown in Fig. 1.317.

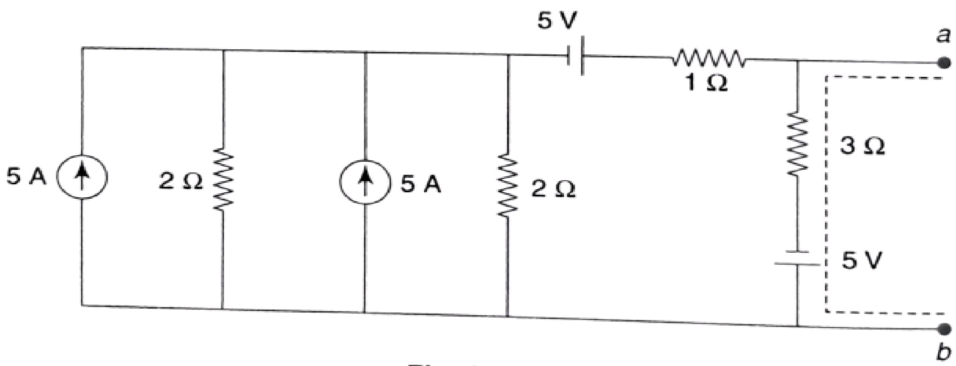

Fig. 1.317

By adding the parallel current sources, we get the circuit as shown in Fig. 1.318.

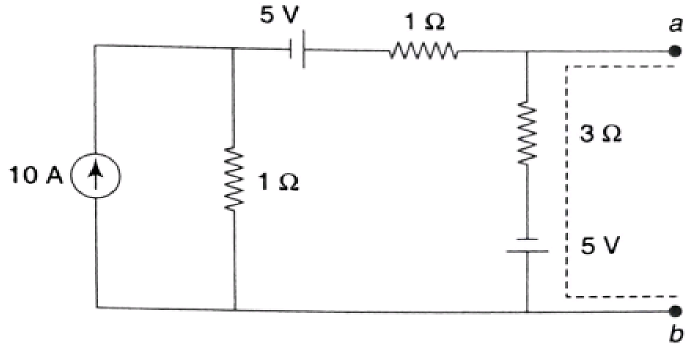

Fig. 1.318

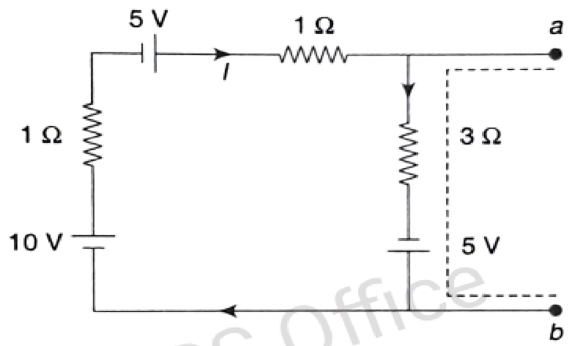

Fig. 1.319

By converting the parallel combinations into equivalent series combinations (of voltage source and resistor), we get the circuit as shown in Fig. 1.319. Let the loop current is I A, which can be calculated as follows:

Applying KVL the loop we get,

$$5 - I - 3I + 5 + 10 - I = 0$$

$$I = 4\text{ A}$$

Hence,

$$V_{TH} = V_{ab}$$

$$= -5 + 3I$$

$$= -5 + 3(4)$$

$$= 7\text{ V}$$

Step II: Calculation for R_{TH}

Removing the load resistance from the network and replacing the sources with their internal resistances, we get the circuit as shown in Fig. 1.320. In Fig. 1.320, equivalent resistance across the load terminal a to b is called Thevenin's resistance R_{TH}.

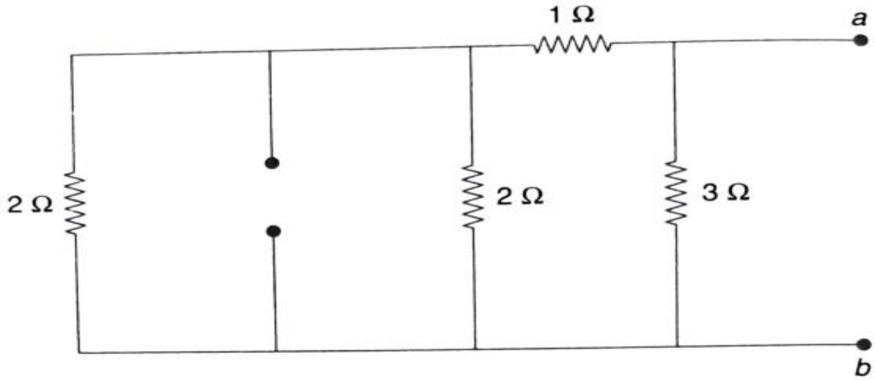

Fig. 1.320

By series–parallel circuit reduction technique, we get the following circuit:

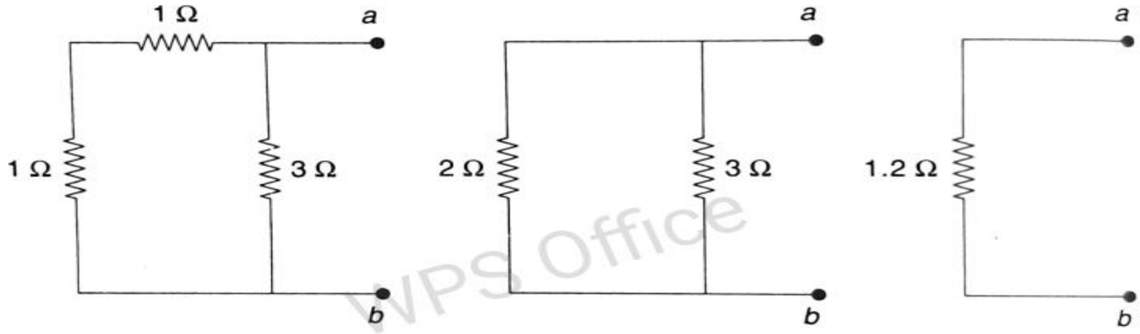

Fig. 1.321

Thus $R_{TH} = R_{ab} = 1.2\ \Omega$

Step III: Calculation for load current

Thevenin's equivalent circuit can be drawn as shown in Fig. 1.322.

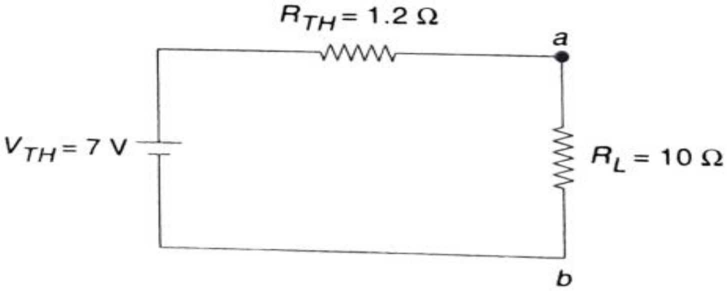

Fig. 1.322

By Ohm's law,

$$I_{10\Omega} = \frac{V_{TH}}{R_L + R_{TH}}$$

$$I_{10\Omega} = \frac{7}{10 + 1.2} = 0.625 \text{ A } (\downarrow)$$

Example 1.79 For the circuit shown in Fig. 1.323, find the Thevenin's equivalent circuit across *A-B*.

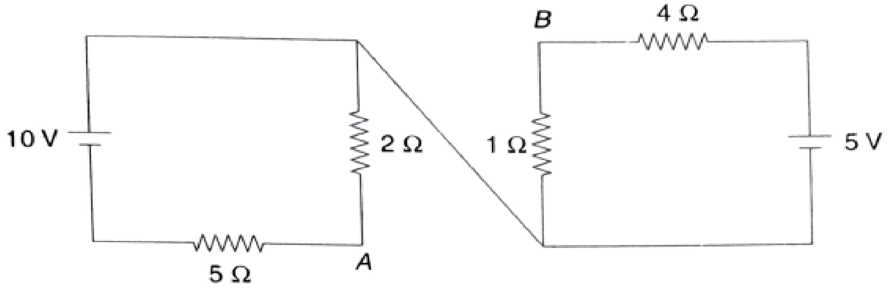

Fig. 1.323

Solution

For simplicity, the circuit shown in Fig. 1.323 can be redrawn as shown in Fig. 1.324.

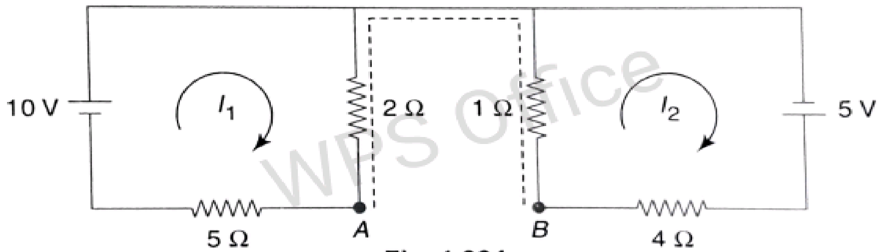

Fig. 1.324

Step I: Calculation for V_{TH}

In Fig. 1.324, voltage appears across the load terminal *A* and *B*, which is called Thevenin's voltage V_{TH}. For calculation of V_{TH}, i.e., V_{AB}, the selected path from *A* to *B* is marked by dotted line in Fig 1.324. As this path contains the resistors 1 Ω and 2 Ω, current through these resistors are required.

Let the loop currents are marked as I_1 and I_2.

From Fig. 1.324,

$$I_1 = \frac{10}{2 + 5} = 1.43 \text{ A}$$

$$I_2 = \frac{5}{4 + 1} = 1 \text{ A}$$

Hence, $V_{TH} = V_{AB}$

$= -1I_2 - 2I_1$

$= -1 \times 1 - 2 \times 1.43$

$= -3.86 \text{ V}$

Step II: Calculation for R_{TH}

In Fig. 1.324, replacing the sources with their internal resistances, we get the circuit as shown in Fig. 1.325. In Fig. 1.325, equivalent resistance across the load terminal A to B is called Thevenin's resistance R_{TH}.

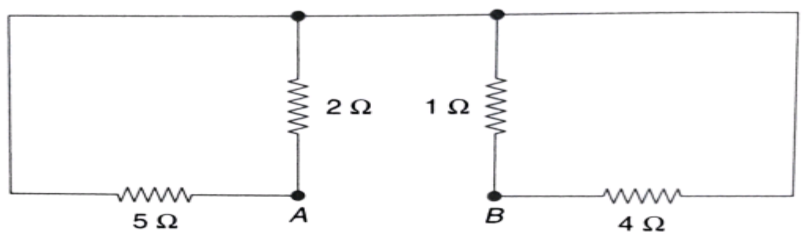

Fig. 1.325

In Fig. 1.325, resistors 5 Ω and 2 Ω are in parallel. Also, resistors 1 Ω and 4 Ω are in parallel.

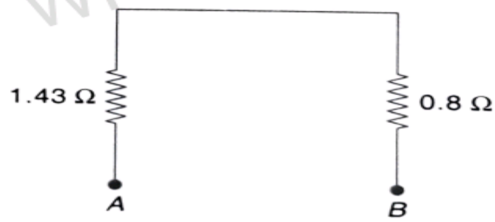

Fig. 1.326

From Fig. 1.326,

$R_{TH} = R_{AB} = 1.43 + 0.8 = 2.23\ \Omega$

Step III: Thevenin's equivalent circuit

Thevenin's equivalent circuit can be drawn as shown in Fig. 1.327.