MODULE 3
Equilibrium of Forces

PRINCIPLES OF EQUILIBRIUM

Though there are many principles of equilibrium, yet the following three are important from the subject point of view :

1. Two force principle. As per this principle, if a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.

2. Three force principle. As per this principle, if a body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third force.(LAMI'S THEOREM)

3.

Equilibrium of Rigid Bodies

- **Equilibrium of non-concurrent forces system:**
- A non-concurrent forces system will be in equilibrium if the resultant of all forces and moment is zero.

$$\sum Fx = 0 \qquad \sum Fy = 0 \qquad \sum M = 0$$

- **Equilibrium of concurrent forces system:**
- For the concurrent forces, the line of actions of all forces meet at a point, and hence the moment of those forces about that point will be zero automatically.

$$\sum Fx = 0 \qquad \sum Fy = 0$$

2 - 50

4. For general force system including couples,

condition of equilibrium(COE) will be

$$\sum Fx = 0 \quad \sum Fy = 0 \quad \sum M = 0$$

FREE BODY DIAGRAM(FBD)

IT SHOWS ALL EXTERNAL THE FORCES ACTING ON THE RIGID BODY IN EQUILIBRIUM INCLUDING IT'S SELF WEIGHT

STEPS TO DRAW FBD

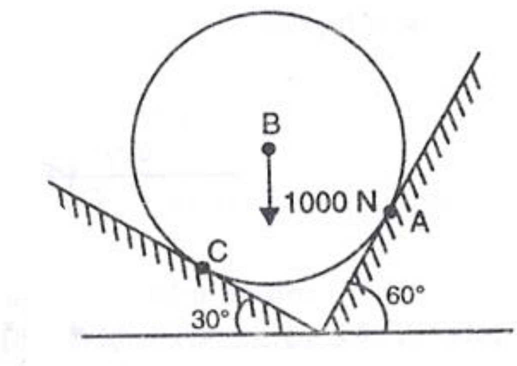
a. Draw diagram of selected body from the system in EQUILIBRIUM,

b. show self weight at the center of the body,

c. find out all contact points of this body with others bodies in the given system,

d. draw normals at all contact points

e. show normal support reactions along this normal towards the body.(number of support reactions will be equal to number of contact points)

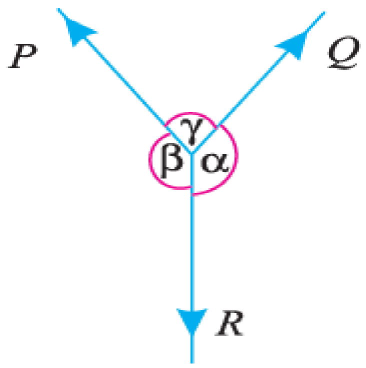


LAMI'S THEOREM

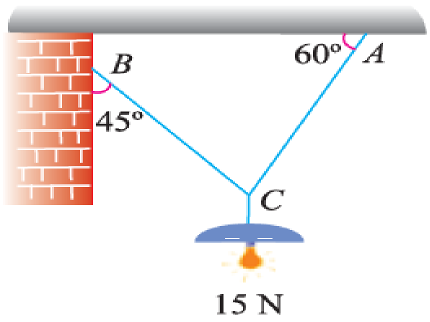
It states, "If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two." Mathematically,

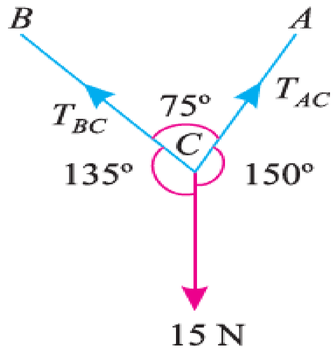
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

where, P, Q, and R are three forces and α, β, γ are the angles as shown in Fig.



Example E1. An electric light fixture weighting 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown in Fig.





Given : Weight at C = 15 N
Let TAC = Force in the string AC, and TBC = Force in the string BC.
The system of forces is shown in Fig. From the geometry of the figure, we find that angle between TAC and 15 N is 150° and angle between TBC and 15 N is 135°.
∴ ∠ ACB = 180° – (45° + 60°) = 75°
Applying Lami's equation at C,

$$\frac{15}{\sin 75^\circ} = \frac{T_{AC}}{\sin 135^\circ} = \frac{T_{BC}}{\sin 150^\circ}$$

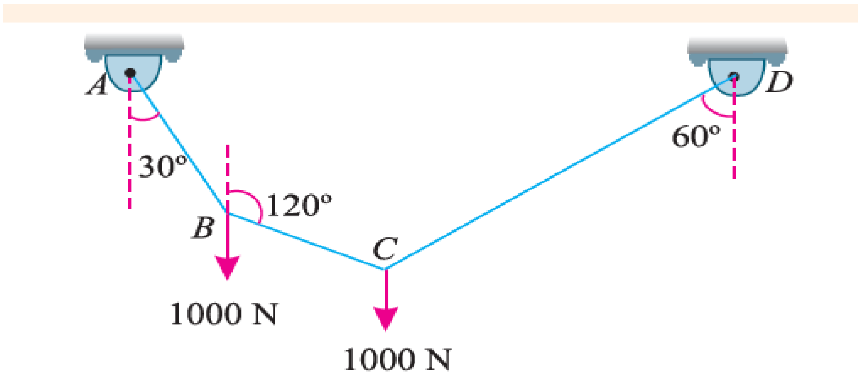
$$\frac{15}{\sin 75^\circ} = \frac{T_{AC}}{\sin 45^\circ} = \frac{T_{BC}}{\sin 30^\circ}$$

$$T_{AC} = \frac{15 \sin 45^\circ}{\sin 75^\circ} = \frac{15 \times 0.707}{0.9659} = 10.98\,\text{N} \quad \textbf{Ans.}$$

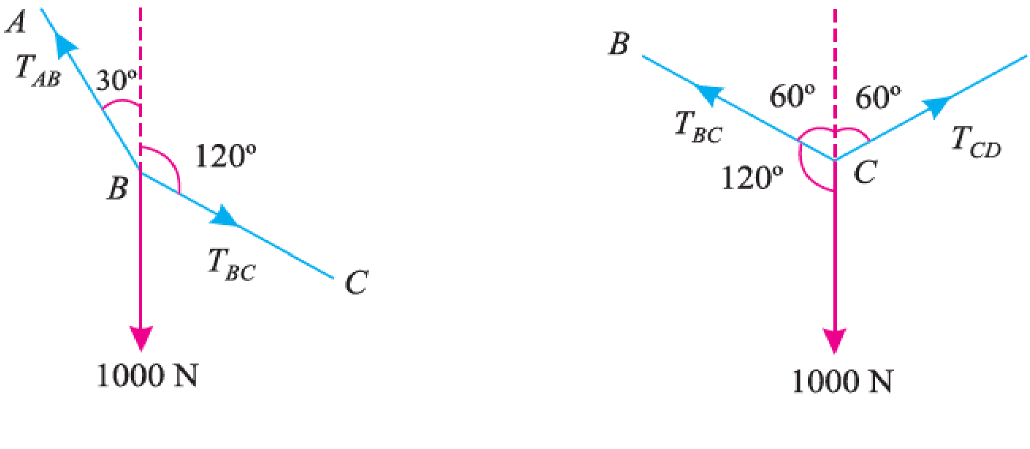
$$T_{BC} = \frac{15 \sin 30^\circ}{\sin 75^\circ} = \frac{15 \times 0.5}{0.9659} = 7.76\,\text{N} \quad \textbf{Ans.}$$

Example E2. A string ABCD, attached to fixed points A and D has two equal weights of 1000 N attached to it at B and C. The weights rest with the portions AB and CD inclined at angles as shown in Fig.

Find the tensions in the portions AB, BC and CD of the string, if the inclination of the portion BC with the vertical is 120°.



Solution. Given : Load at B = Load at C = 1000 N For the sake of convenience, let us split up the string ABCD into two parts. The system of forces at joints B and is shown in Fig. (a) and (b).



(*a*) Joint *B* (*b*) Joint *C*

Let TAB = Tension in the portion AB of the string,
TBC = Tension in the portion BC of the string, and
TCD = Tension in the portion CD of the string.
Applying Lami's equation at joint B,

$$\frac{T_{AB}}{\sin 60°} = \frac{T_{BC}}{\sin 150°} = \frac{1000}{\sin 150°}$$

$$\frac{T_{AB}}{\sin 60°} = \frac{T_{BC}}{\sin 30°} = \frac{1000}{\sin 30°} \qquad \text{...[} \because \sin (180° - \theta) = \sin \theta \text{]}$$

$$T_{AB} = \frac{1000 \sin 60°}{\sin 30°} = \frac{1000 \times 0.866}{0.5} = 1732 \text{ N} \quad \textbf{Ans.}$$

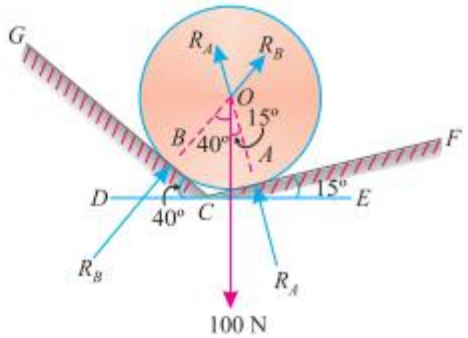
$$T_{BC} = \frac{1000 \sin 30°}{\sin 30°} = 1000 \text{ N} \quad \textbf{Ans.}$$

Again applying Lami's equation at joint C,

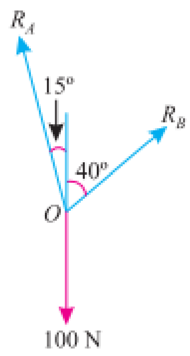
$$\frac{T_{BC}}{\sin 120°} = \frac{T_{CD}}{\sin 120°} = \frac{1000}{\sin 120°}$$

$$\therefore \qquad T_{CD} = \frac{1000 \sin 120°}{\sin 120°} = 1000 \text{ N} \quad \textbf{Ans.}$$

Example E3. A smooth circular cylinder of radius 1.5 meter is lying in a triangular groove, one side of which makes 15° angle and the other 40° angle with the horizontal. Find the reactions at the surfaces of contact, if there is no friction and the cylinder weighs 100 N.



Solution. Given : Weight of cylinder = 100 N



FBD

Let RA = Reaction at A, and RB = Reaction at B. The smooth cylinder lying in the groove is shown in Fig. 5.12 (a). In order to keep the system in equilibrium, three forces i.e. RA, RB and weight of cylinder (100 N) must pass through the centre of the cylinder. Moreover, as there is no *friction, the reactions RA and RB must be normal to the surfaces as shown in Fig. 5.12 (a). The system of forces is shown in Fig

Applying Lami's equation, at O,

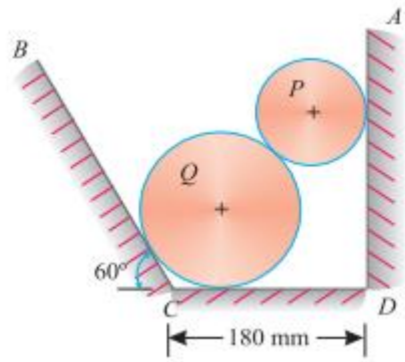
$$\frac{R_A}{\sin(180° - 40°)} = \frac{R_B}{\sin(180° - 15°)} = \frac{100}{\sin(15° + 40°)}$$

$$\frac{R_A}{\sin 40°} = \frac{R_B}{\sin 15°} = \frac{100}{\sin 55°}$$

$$R_A = \frac{100 \times \sin 40°}{\sin 55°} = \frac{100 \times 0.6428}{0.8192} = 78.5\text{ N} \qquad \textbf{Ans.}$$

$$R_B = \frac{100 \times \sin 15°}{\sin 55°} = \frac{100 \times 0.2588}{0.8192} = 31.6\text{ N} \qquad \textbf{Ans.}$$

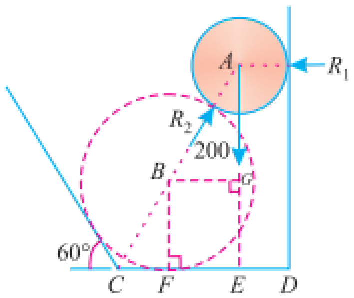
Example E4. Two cylinders P and Q rest in a channel as shown in Fig.



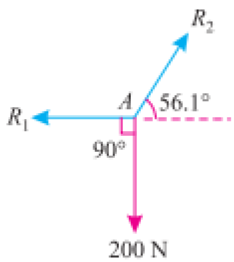
The cylinder P has diameter of 100 mm and weighs 200 N, whereas the cylinder Q has diameter of 180 mm and weighs 500 N.If the bottom width of the box is 180 mm, with one side vertical and the

other inclined at 60°, determine the pressures at all the four points of contact

Solution. Given : Diameter of cylinder P = 100 mm ; Weight of cylinder P = 200 N ; Diameter of cylinder Q = 180 mm ; Weight of cylinder Q = 500 N and width of channel = 180 mm. First of all, consider the equilibrium of the cylinder P. It is in equilibrium under the action of the following three forces which must pass through A i.e., the centre of the cylinder P as shown in Fig. 5.14 (a). 1. Weight of the cylinder (200 N) acting downwards. 2. Reaction (R1) of the cylinder P at the vertical side. 3. Reaction (R2) of the cylinder P at the point of contact with the cylinder Q. From the geometry of the figure, we find that



FBD of the cylinder A



$$ED = \text{Radius of cylinder } P = \frac{100}{2} = 50 \text{ mm}$$

Similarly $$BF = \text{Radius of cylinder } Q = \frac{180}{2} = 90 \text{ mm}$$

and $$\angle BCF = 60°$$

$\therefore$ $$CF = BF \cot 60° = 90 \times 0.577 = 52 \text{ mm}$$

$\therefore$ $$FE = BG = 180 - (52 + 50) = 78 \text{ mm}$$

and $$AB = 50 + 90 = 140 \text{ mm}$$

$\therefore$ $$\cos \angle ABG = \frac{BG}{AB} = \frac{78}{140} = 0.5571$$

or $$\angle ABG = 56.1°$$

Applying Lami's equation at A,

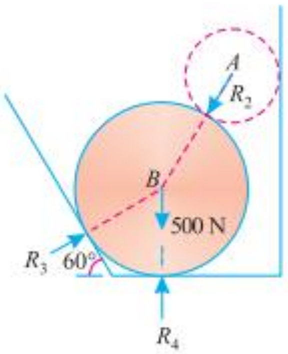
$$\frac{R_1}{\sin(90° + 56.1°)} = \frac{R_2}{\sin 90°} = \frac{200}{\sin(180° - 56.1°)}$$

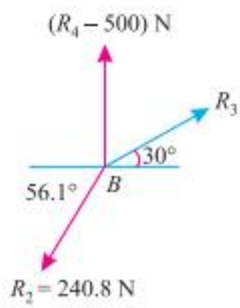
$$\frac{R_1}{\cos 56.1°} = \frac{R_2}{1} = \frac{200}{\sin 56.1°}$$

$\therefore$ $$R_1 = \frac{200 \cos 56.1°}{\sin 56.1°} = \frac{200 \times 0.5571}{0.830} = 134.2 \text{ N} \quad \textbf{Ans.}$$

and $$R_2 = \frac{200}{\sin 56.1°} = \frac{200}{0.8300} = 240.8 \text{ N} \quad \textbf{Ans.}$$

Now consider the equilibrium of the cylinder Q. It is in equilibrium under the action of the following four forces, which must pass through the centre of the cylinder as shown in Fig. . 1. Weight of the cylinder Q (500 N) acting downwards. 2. Reaction R2 equal to 240.8 N of the cylinder P on cylinder Q. 3. Reaction R3 of the cylinder Q on the inclined surface. 4. Reaction R4 of the cylinder Q on the base of the channel.





FBD of cylinder B

A little consideration will show that the weight of the cylinder Q is acting downwards and the reaction R4 is acting upwards. Moreover, their lines of action also coincide with each other. ∴ Net downward force = (R4 – 500) N The system of forces is shown in Fig. Applying Lami's equation at B,

$$\frac{R_3}{\sin(90° + 56.1°)} = \frac{240.8}{\sin 60°} = \frac{R_4 - 500}{\sin(180° + 30° - 56.1°)}$$

$$\frac{R_3}{\cos 56.1°} = \frac{240.8}{\sin 60°} = \frac{R_4 - 500}{\sin 26.1°}$$

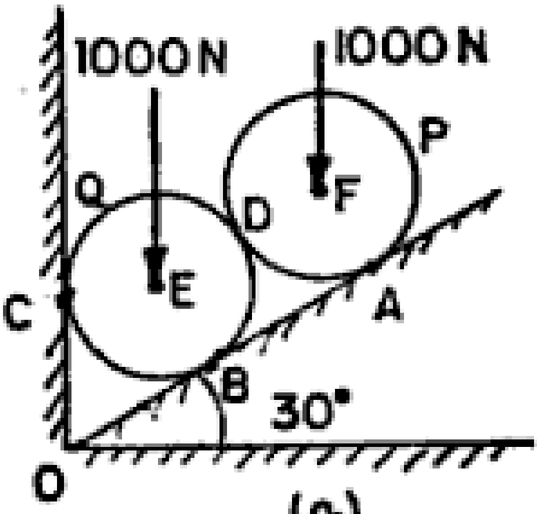
$$\therefore \quad R_3 = \frac{240.8 \times \cos 56.1°}{\sin 60°} = \frac{240.8 \times 0.5577}{0.866} = 155 \text{ N} \quad \textbf{Ans.}$$

$$R_4 - 500 = \frac{240.8 \times \sin 26.1°}{\sin 60°} = \frac{240.8 \times 399}{0.866} = 122.3 \text{ N}$$

$$\therefore \quad R_4 = 122.3 + 500 = 622.3 \text{ N} \quad \textbf{Ans.}$$

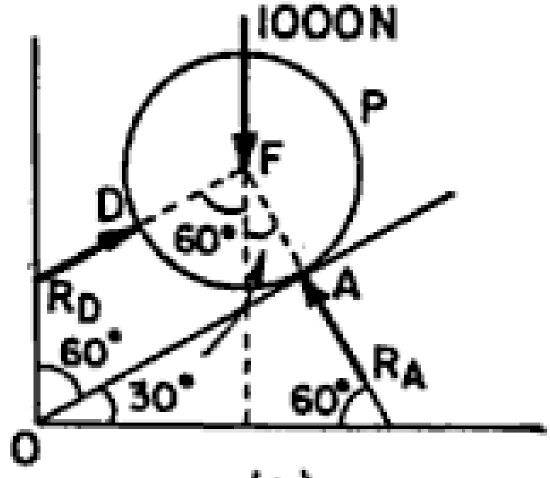
Example E5. Two identical rollers P and Q, each of weight W, are supported by an inclined plane and a vertical wall as shown in Fig.
Assume all the surfaces to be smooth. Draw the free body diagrams of:
(i) roller Q, (ii) roller P and (iii) rollers P and Q taken together. (iv) If W = 1000 N find reactions at all contact surfaces.

(Ans : Ra = 866 , Rc = 1154.7, Rd = 500 , Rb = 1443.3)



(a)

Solution



(c)

Weight of each roller = 1000 N

Radius of each roller is same. Hence line *EF* will be parallel to *AB*.

Equilibrium of Roller P

First draw the free-body diagram of roller P as shown in Fig. 4.17 (c). The roller P has points of contact at A and D. Hence the forces acting on the roller P are :

(i) Weight 1000 N acting vertically downward.

(ii) Reaction R_A at point A. This is normal to OA.

(iii) Reaction R_D at point D. This is parallel to line OA.

The resultant force in x and y directions on roller P should be zero.

For $\Sigma F_x = 0$, we have

$$R_D \sin 60° - R_A \sin 30° = 0 \quad \text{or} \quad R_D \sin 60° = R_A \sin 30°$$

$$\therefore \quad R_D = R_A \frac{\sin 30°}{\sin 60°} = 0.577\, R_A$$

For $\Sigma F_y = 0$, we have

$$R_D \cos 60° + R_A \cos 30° - 1000 = 0$$

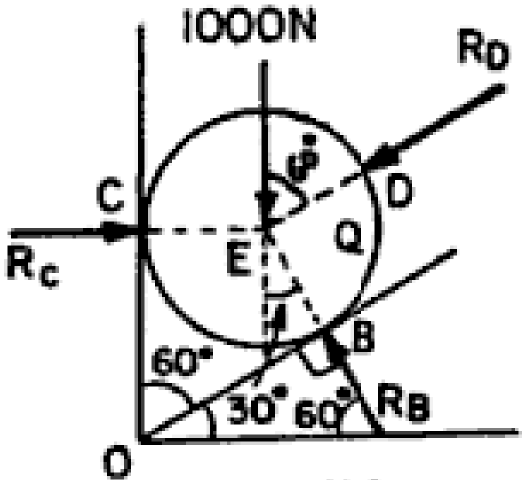
$$(0.577\, R_A) \cos 60° + R_A \cos 30° = 1000 \qquad (\because \; R_D = 0.577\, R_A)$$

$$0.577 \times 0.5\, R_A + R_A \times 0.866 = 1000$$

$$1.1545\, R_A = 1000 \quad \text{or} \quad R_A = \frac{1000}{1.1545} = 866.17 \text{ N.} \quad \textbf{Ans.}$$

Substituting this value in equation (i), we get

$$R_D = 0.577 \times 866.17 = 499.78$$



(b)

Equilibrium of Roller Q

The free-body diagram of roller Q is shown in Fig. 4.17 (b). The roller Q has points of contact at B, C and D.

The forces acting on the roller Q are :

(i) Weight $W = 1000$ N ;

(ii) Reaction R_B at point B and normal to BO ;

(iii) Reaction R_C at point C and normal to CO ; and

(iv) Reaction R_D at point D and parallel to BO.

For $\Sigma F_x = 0$, we have

$$R_B \sin 30° + R_D \sin 60° - R_C = 0$$

or $$R_B \times 0.5 + 449.78 \times 0.866 - R_C = 0$$

or $$R_C = 0.5\, R_B + 432.8 \qquad ...(ii)$$

For $\Sigma F_y = 0$, we have

$$R_B \times \cos 30° - 1000 - R_D \times \cos 60° = 0$$

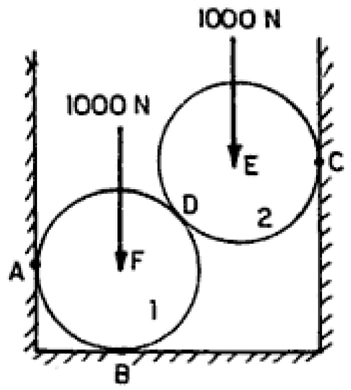
or $$R_B \times 0.866 - 1000 - 499.78 \times 0.5 = 0 \qquad (\because\ R_D = 499.78)$$

or $$0.866\, R_B - 1249.89 = 0 \quad \text{or} \quad R_B = \frac{1249.89}{0.866} = \mathbf{1443.3\ N.}\quad \textbf{Ans.}$$

Substituting this value in equation (ii), we get

$$R_C = 0.5 \times 1443.3 + 432.8 = \mathbf{1154.45\ N.}\quad \textbf{Ans.}$$

Problem 4.15. *Two spheres, each of weight 1000 N and of radius 25 cm rest in a horizontal channel of width 90 cm as shown in Fig. 4.18. Find the reactions on the points of contact A, B and C.*



Solution

Weight of each sphere, $W = 1000$ N

Radius of each sphere, $R = 25$ cm

$\therefore$ $AF = BF = FD = DE = CE = 25$ cm

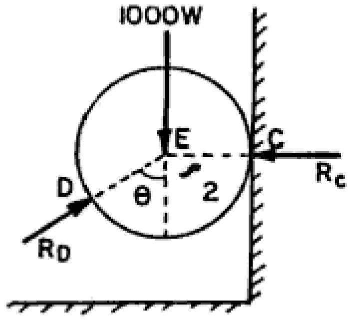
Width of horizontal channel = 90 cm

Join the centre E to centre F as shown in Fig. 4.18 (*b*).

Now $EF = 25 + 25 = 50$ cm, $FG = 40$ cm

In $\Delta EFG, EG = \sqrt{EF^2 - FG^2} = \sqrt{50^2 - 40^2} = \sqrt{2500 - 1600} = 30$

$$\cos\theta = \frac{EG}{EF} = \frac{30}{50} = \frac{3}{5} \quad \text{and} \quad \sin\theta = \frac{FG}{EF} = \frac{40}{50} = \frac{4}{5}.$$



Equilibrium of Sphere No. 2

For $\Sigma F_x = 0$, we have $R_D \sin\theta = R_C$

For $\Sigma F_y = 0$, we have $R_D \cos\theta = 1000$

$$R_D = \frac{1000}{\cos\theta} = \frac{1000}{\left(\frac{3}{5}\right)}$$

$$= 1000 \times \frac{5}{3} = \frac{5000}{3}\ \text{N}$$

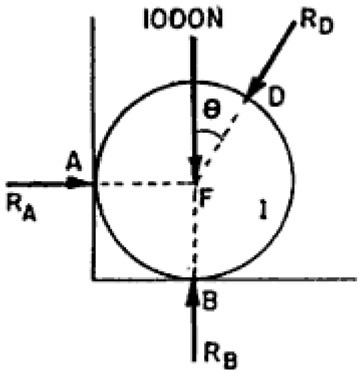
Substituting the value of R_D in equation (*i*),

$$\frac{5000}{3} \times \sin\theta = R_C \quad \text{or} \quad \frac{5000}{3} \times \frac{4}{5} = R_C$$

$$1333.33 = R_C$$

$$\therefore \qquad R_C = 1333.33\ \text{N.} \quad \textbf{Ans.}$$

Equilibrium of sphere No. 1.



For $\Sigma F_x = 0$, we have

$$R_A - R_D \sin\theta = 0$$

$$R_A = R_D \sin\theta$$

$$= \frac{5000}{3} \times \frac{4}{5}$$

$$\left(\because \quad R_D = \frac{5000}{3} \text{ and } \sin\theta = \frac{4}{5}\right)$$

$= 1333.33$ **N. Ans.**

For $\Sigma F_y = 0$, we have

$$R_B - 1000 - R_D \cos\theta = 0$$

$$\therefore \quad R_B = 1000 + R_D \cos\theta$$

$$= 1000 + \frac{5000}{3} \times \frac{3}{5}$$

$= 2000$ **N. Ans.**

Support Reactions/Beams

TYPES OF LOADING

Though there are many types of loading, yet the following are important from the subject point of view :

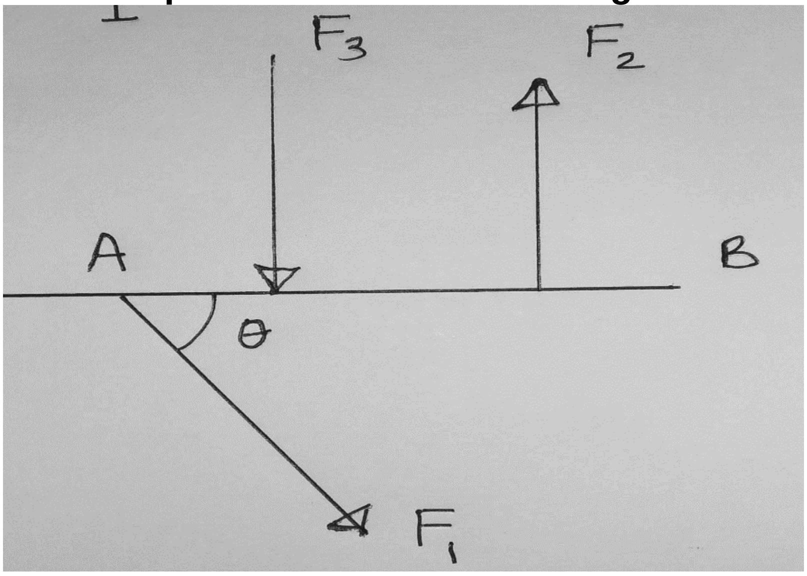
1. Concentrated or point load,

2. Uniformly distributed load,

3. Uniformly varying load

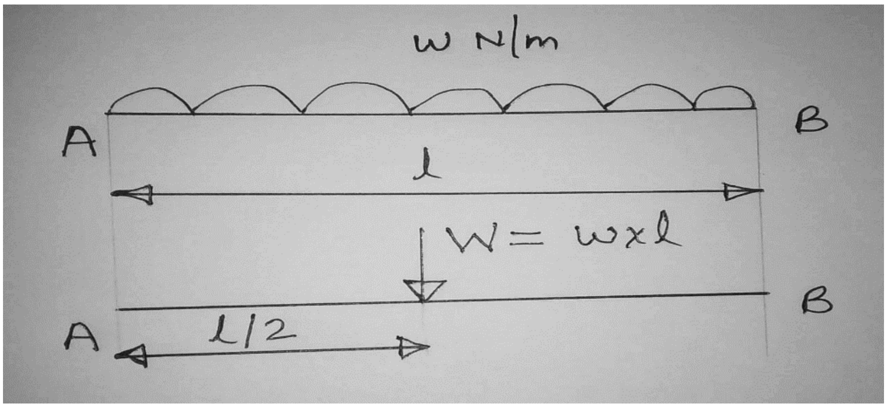
CONCENTRATED OR POINT LOAD

A load, acting at a point on a beam is known as a concentrated or a point load as shown in Fig.



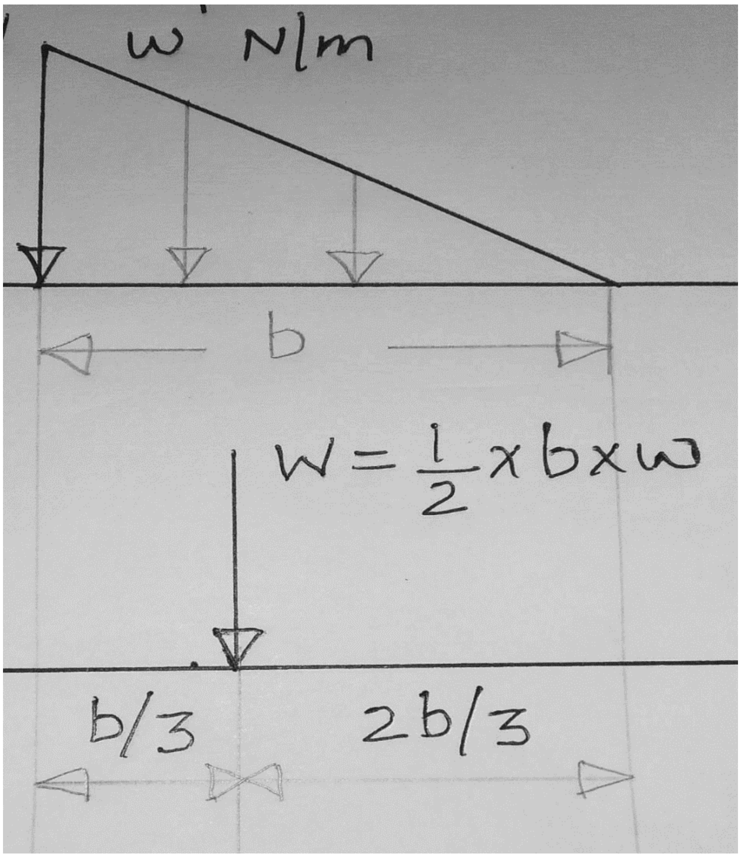
UNIFORMLY DISTRIBUTED LOAD

A load, which is spread over a beam, in such a manner that each unit length is loaded to the
same extent, is known as uniformly distributed load (briefly written as U.D.L.) as shown in Fig



UNIFORMLY VARYING LOAD

A load, which is spread over a beam, in such a manner that its extent varies uniformly on each unit length (say from w1 per unit length at one support to w2 per unit length at the other support) is known as uniformly varying load as shown in Fig.

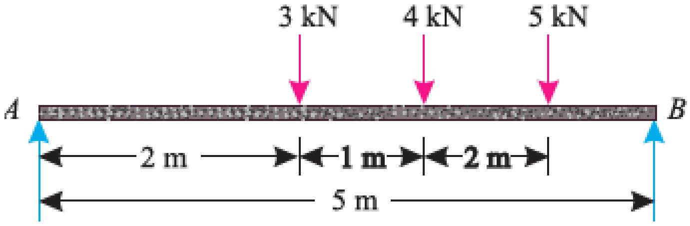


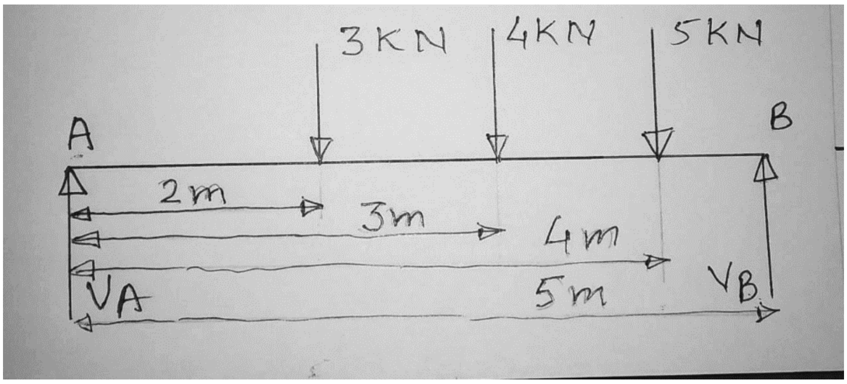
TYPES OF BEAMS

SIMPLY SUPPORTED BEAMS

It is a theoretical case, in which the end of a beam is simply supported over one of its support.
In such a case the reaction is always vertical to the line of contact.

Example 12.1. A simply supported beam AB of span 5 m is loaded as shown in Fig. 12.7.
Find the reactions at A and B.



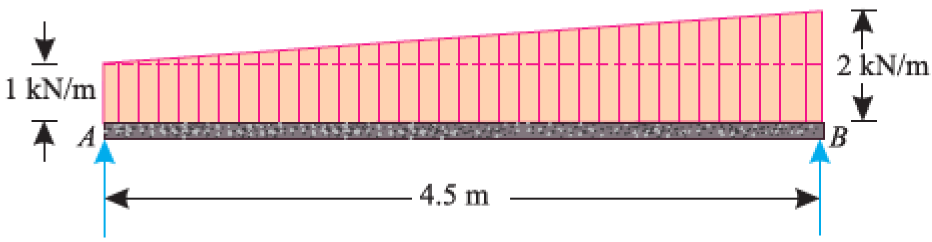


FBD

Solution. **Given: Span (l) = 5 m**
Let VA = Reaction at A, and VB = Reaction at B.
$\sum M_A = 0$

$= -(3 \times 2) - (4 \times 3) - (5 \times 4) + 5VB = 0$
VB = 7.6 kN
$\sum FY = 0 = -3-4-5+7.6$
and VA = 4.4 kN Ans.

Example 12.3. **A simply supported beam AB of span 4.5 m is loaded as shown in Fig.convert it into single point wt. acting on the beam**



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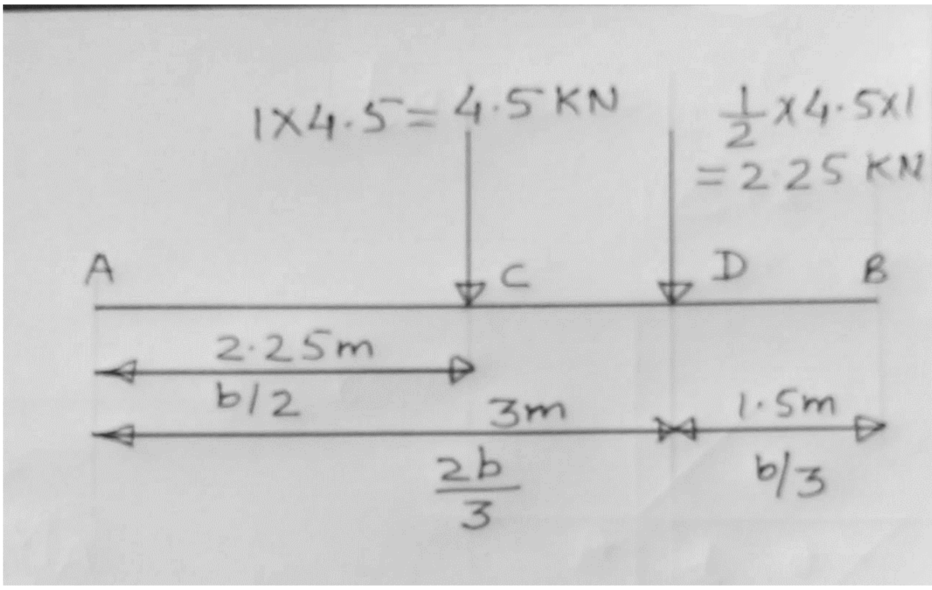
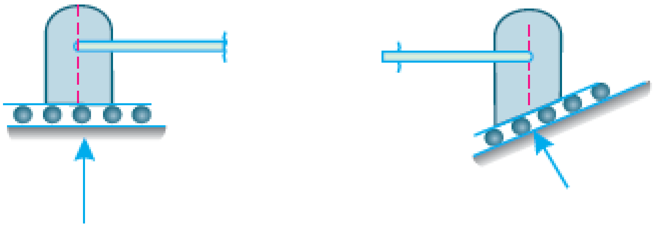


FBD

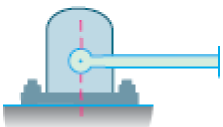
ROLLER SUPPORTED BEAMS



In such a case, the end of a beam is supported on rollers, and the reaction on such an end is always normal to the support, as shown in Fig.) and The main advantage, of such a support, is that the beam can move easily towards left or right,

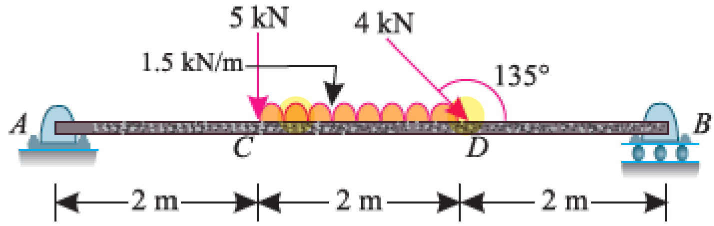
HINGED BEAMS

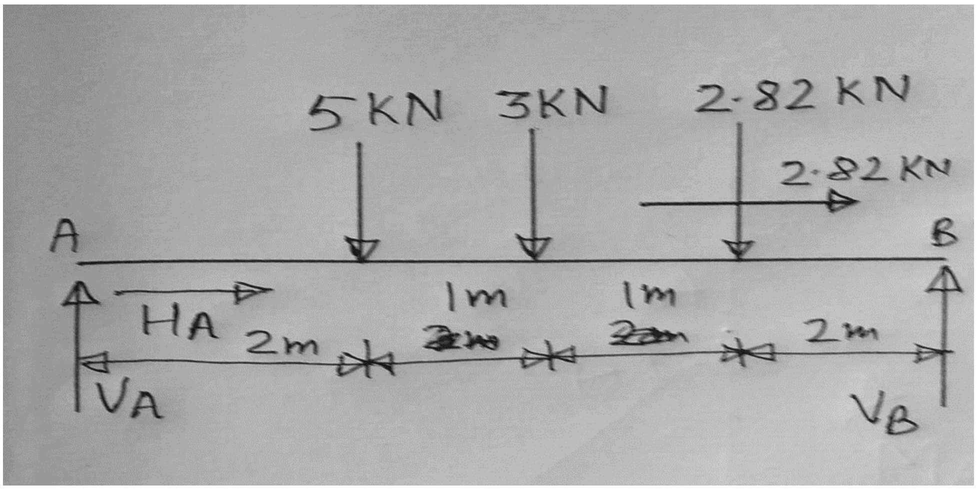
In such a case, the end of a beam is hinged to the support as shown in Fig.



The reaction on such an end may be horizontal, vertical or inclined, depending upon the type of loading. All the steel trusses of the
bridges have one of their end roller supported, and the other hinged.

Example **A beam AB of 6 m span is loaded as shown in Fig.. Determine the reactions at A and B.**





FBD

Solution. **Given: L = 6 m**

Let HA & VA = Reaction at A, and VB = Reaction at B.

$\sum M_A = 0$

= -5X2-3X3-2.82X4+VBX6 = 0

VB = 5.05 kN

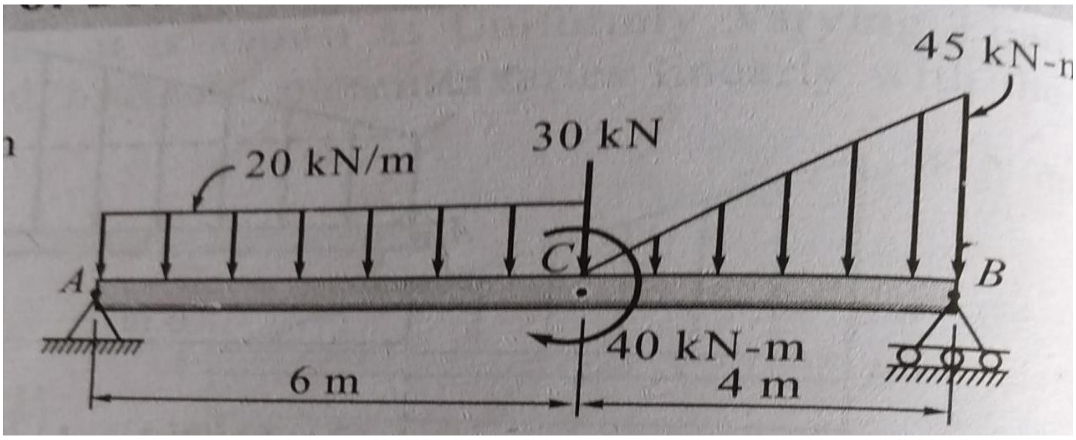
∑FY = 0 = -5-3-2.82+5.05+VA

VA=6.44 KN

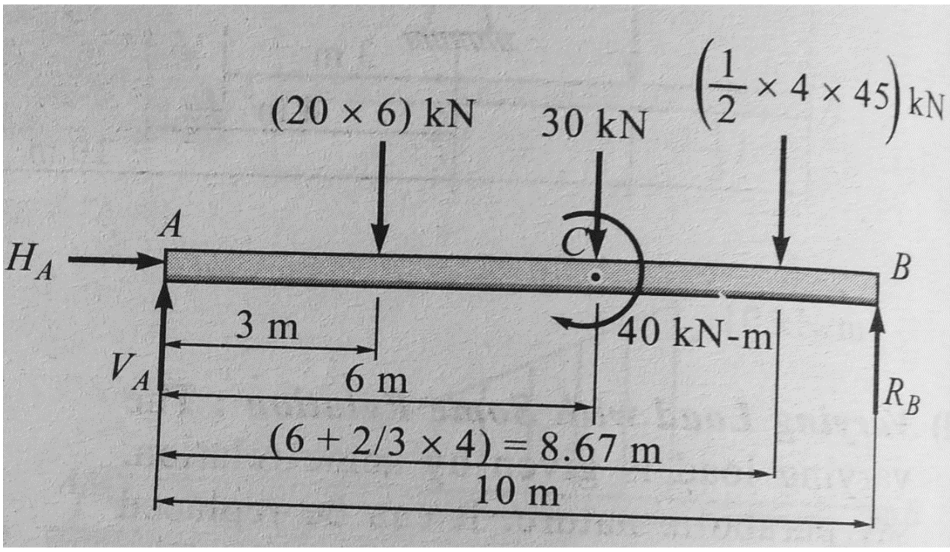
∑FX = 0 = HA+2.82

HA = -2.82 KN

EXAMPLE : FIND OUT SUPPORT REACTIONS AT A & B



SOL:



FBD

Let HA & VA = Reaction at A, and RB = Reaction at B.

$\sum M_A = 0$

= -120X3-30X6-40-90X8.67+RBX10 = 0

RB = 136.03 kN

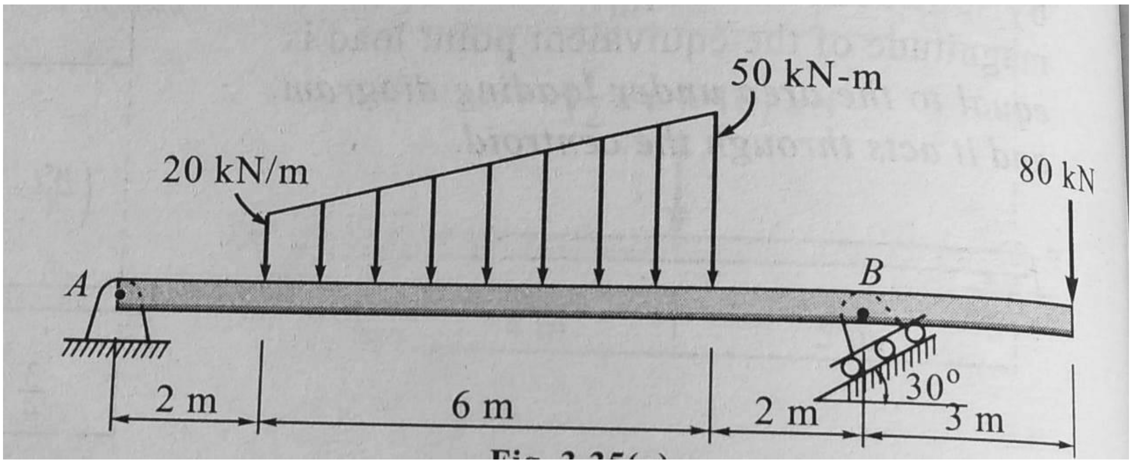
∑FY = 0 = VA-120-30-90+136.03

VA= 103.97 KN

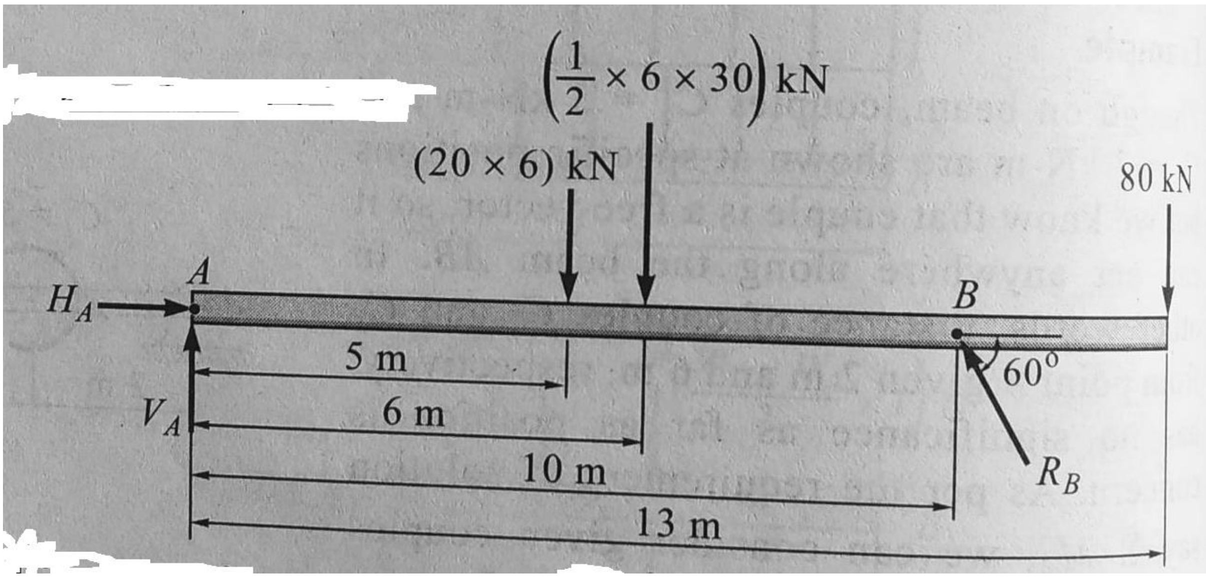
∑FX = 0 = HA

HA = 0 KN

EX:FIND OUT SUPPORT REACTIONS AT A & B



SOL:



FBD

Let HA & VA = Reaction at A, and RB = Reaction at B.
$\sum M_A = 0$
=RBSin60X10-120X5-90X6-80X13= 0
RB =251.7 kN↖

∑FY = 0 = VA-120-90+251.73Sin60-80
VA=72 KN↑
∑FX = 0 = HA-251.73Cos60=0
HA = 125.87 0 KN→
B1

, Determine the reactions at all the supports of the beam/ structure shown in Fig. 3.E47.

[Ans. V_A = 3.6 kN (↑), V_D = 10.4 kN (↑), and H_A = 0]

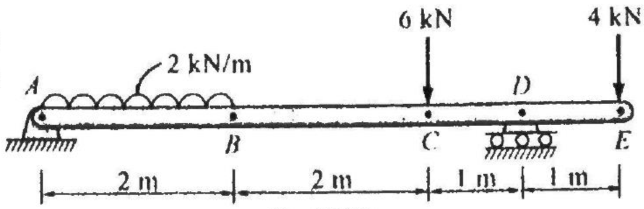


Fig. 3.E47

B2

, Determine the reactions at all the supports of the beam/ structure shown in Fig. 3.E48.

[Ans. R_B = 44.17 kN (60°), V_A = 36.75 kN (↑), and H_A = 22.1 kN (→)]

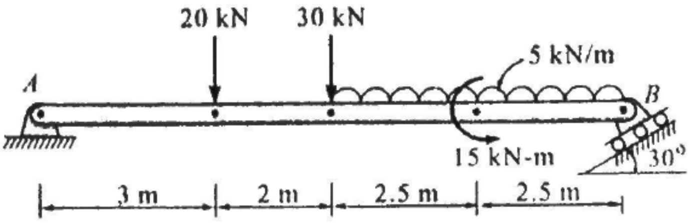


Fig. 3.E48

6 kN/m

B3

H_A = 22.1 kN (→)

Fig. 3.E48

, Determine the reactions at all the supports of the beam shown in Fig. 3.E49.

[Ans. H_A = 0, V_A = 10.56 kN (↑), and R_B = 15.44 kN (↑)]

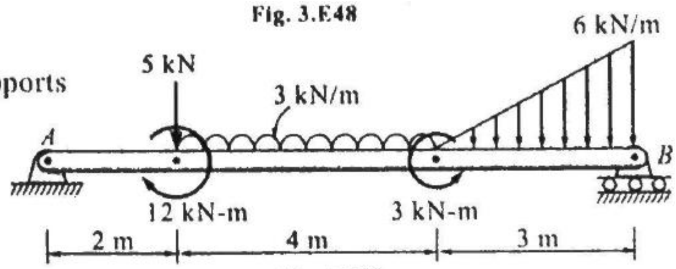


Fig. 3.E49

B4

, Determine the reactions at all the supports of the beam shown in Fig. 3.E50.

[Ans. H_A = 8.66 kN (→), V_A = 8.79 kN (↑), and V_B = 9.21 kN (↑)]

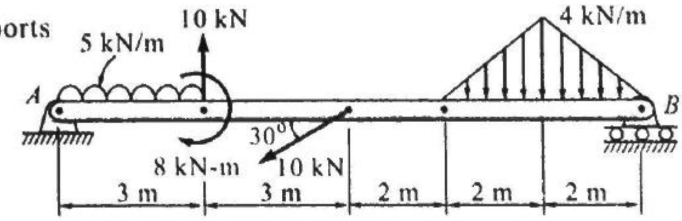


Fig. 3.E50

B5

. Calculate the reactions at A and B for the beam subjected to two linearly distributed loads as shown in Fig. 3.E52.

[Ans. $H_A = 0$,
$V_A = 21.1$ kN (↑), and
$V_B = 20.9$ kN (↑)]

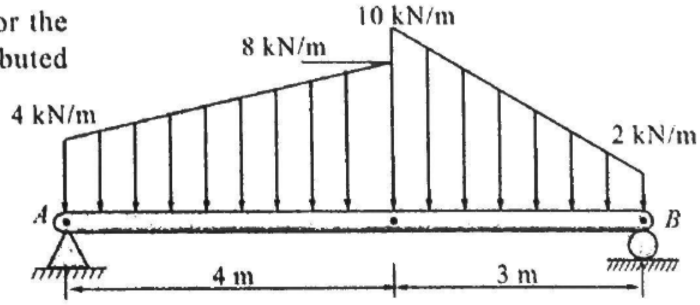


Fig. 3.E52

B6

. Determine the reactions at all the supports of the beam shown in Fig. 3.E51.

[Ans. $H_A = 10$ kN (←),
$V_A = 127.32$ kN (↑), and
$M_A = 694.6$ kNm (↺)]

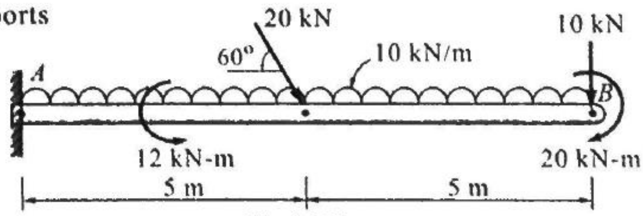
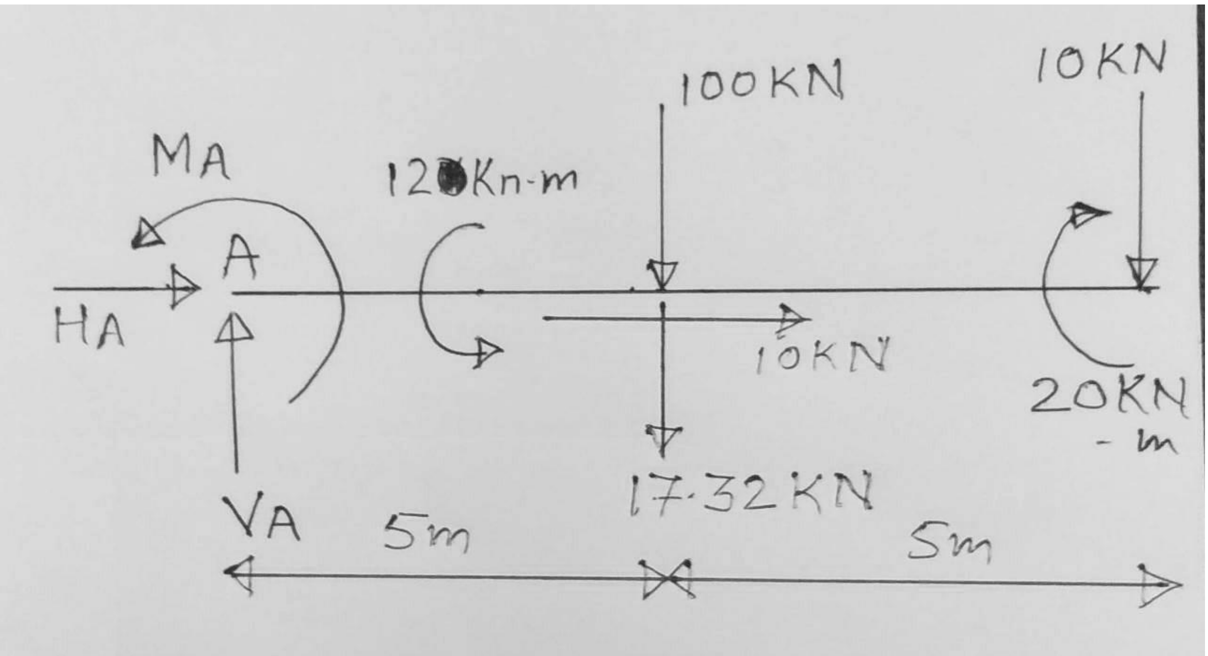
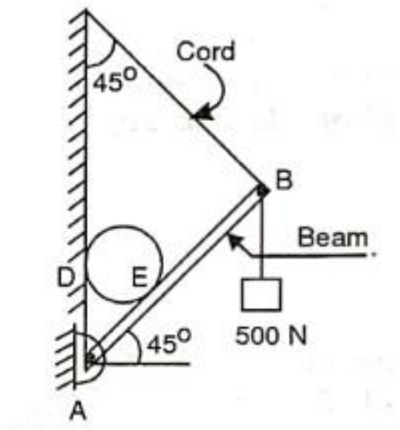


Fig. 3.E51

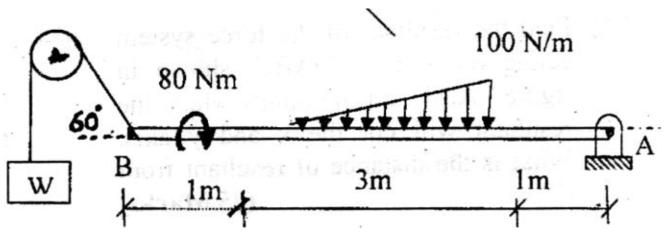
HINT:FBD GIVEN



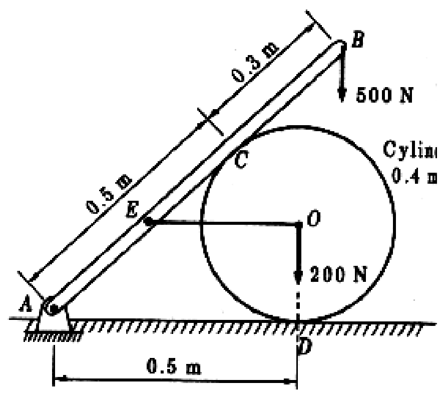
1)A cylinder of diameter 1m and weighing 1000N and another block weighing 500N are supported by beam of length 7m weighing 250N with the help of a cord as shown. If the surfaces of contact are frictionless determine tension in cord and reaction at points of contact.



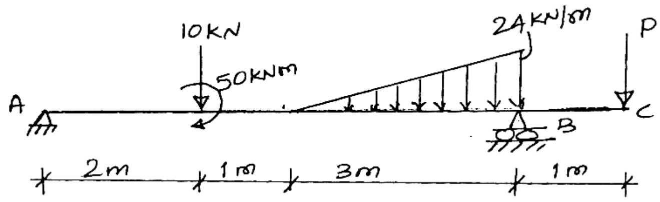
2) Determine minimum weight of the block required to keep beam in horizontal equilibrium. Assume smooth pulley.(Dec 10)



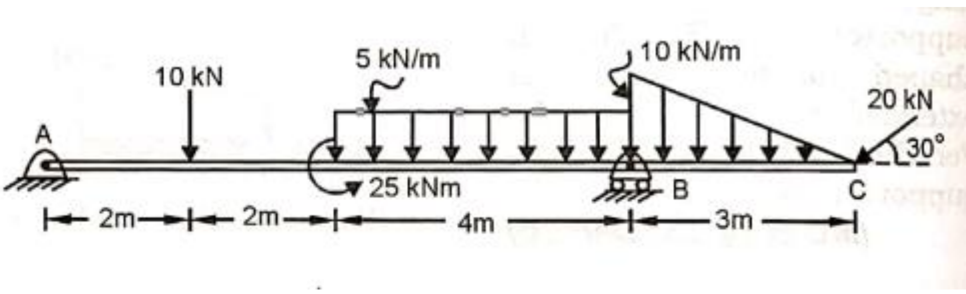
3)A bar is hinged at A and rests on cylinder at C. String connects bar at E and cylinder centre at O. If weight 500N is suspended at free end of the bar at B, Determine (a) The reaction of hinge at A.



4) Find analytically the support reaction at B and load P for the beam shown in figure if the reaction at support A is zero.



5)



6)