1.14 Maxwell's Mesh Current Method

In this method, Kirchhoff's voltage law is applied to each mesh in terms of mesh currents instead of branch currents. Each mesh is assigned a separate mesh current. This mesh current is assumed to flow in clockwise direction around the perimeter of the mesh without splitting at a junction into branch currents. Kirchhoff's voltage law is applied to write equation in terms of unknown mesh currents. Once the mesh currents are known, the branch currents can be easily determined.

Maxwell's mesh current method consists of the following steps:

(i) Each mesh is assigned a separate mesh current. For convenience, all mesh currents are assumed to flow in clockwise direction.[1] For example, in Fig. 1.185, meshes *ABDA* and *BCDB* have been assigned mesh currents I_1 and I_2 respectively.

(ii) If two mesh currents are flowing through a circuit element, the actual current in the circuit element is the algebraic sum of two. Thus, in Fig. 1.185,

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[1] It is convenient to consider all mesh currents in one direction (clockwise or anticlockwise). The same result will be obtained if mesh currents are given arbitrary directions.

there are two mesh currents I_1 and I_2 flowing in R_2. If we go from B to D, current is $I_1 - I_2$ and if we go in the other direction (i.e. from D to B), current is $I_2 - I_1$.

(iii) Kirchhoff's voltage law is applied to write equation for each mesh in terms of unknown mesh currents.

(iv) If the value of any mesh current comes out to be negative in the solution, it means that true direction of that mesh current is anticlockwise, i.e., opposite to the assumed clockwise direction.

Consider a circuit as shown in Fig. 1.185.

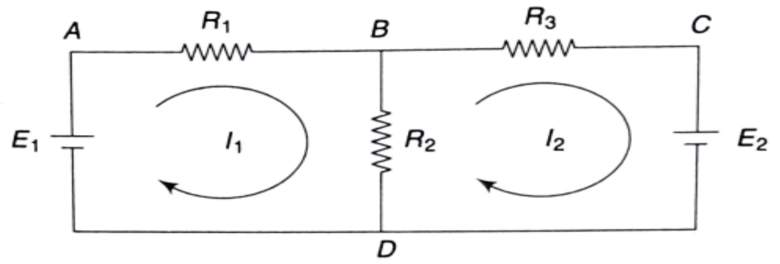

Fig. 1.185 Mesh current method

Applying Kirchhoff's voltage law to mesh $ABDA$,

$$-I_1R_1 - (I_1 - I_2)R_2 + E_1 = 0$$

or $$I_1(R_1 + R_2) - I_2R_2 = E_1 \tag{1.21}$$

Applying Kirchhoff's voltage law to mesh $BCDB$,

$$-I_2R_3 - E_2 - (I_2 - I_1)R_2 = 0$$

or $$-I_1R_2 + (R_2 + R_3)I_2 = -E_2 \tag{1.22}$$

Solving Eqs (1.21) and (1.22) simultaneously, mesh currents I_1 and I_2 can be calculated. Once the mesh currents are known, the branch current can be readily obtained.

Note: Branch currents are the real currents because they actually flow in the branches and can be measured. However, mesh currents are fictitious quantities and can not be measured directly. Hence, mesh current is concept rather than a reality.

Example 1.50 With the help of mesh current method, find the magnitude and direction of the current flowing through the 1 Ω resistor in the network of Fig. 1.186.

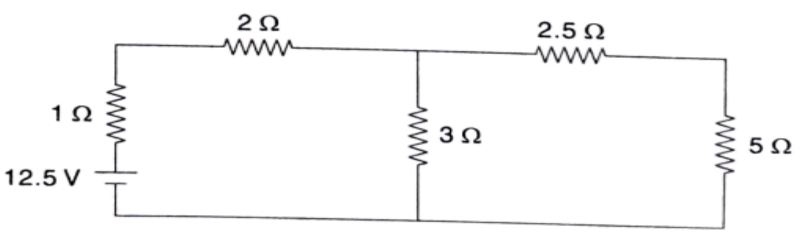

Fig. 1.186

Solution

Marking the different nodes and assigning the separate mesh current for each mesh, we have

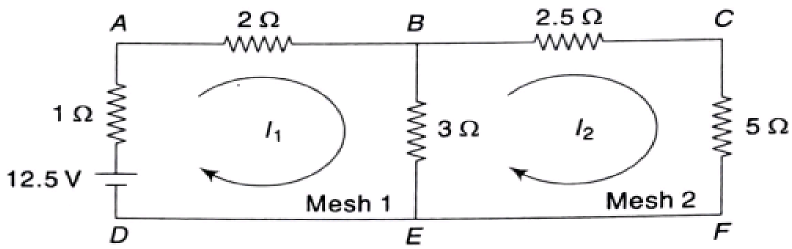

Fig. 1.187

Applying KVL to mesh 1,

$$-2I_1 - 3(I_1 - I_2) + 12.5 - I_1 = 0$$

or $\quad -6I_1 + 3I_2 = -12.5 \qquad$ (i)

Applying KVL to mesh 2,

$$-2.5I_2 - 5I_2 - 3(I_2 - I_1) = 0$$

or $\quad 3I_1 - 10.5I_2 = 0 \qquad$ (ii)

From Eqs (i) and (ii), we get

$$I_1 = 2.43 \text{ A}$$

Hence, $\; I_{1\Omega} = 2.43 \text{ A}(\uparrow)$

Example 1.51 By mesh analysis, find mesh currents I_1, I_2, and I_3 in the network of Fig. 1.188.

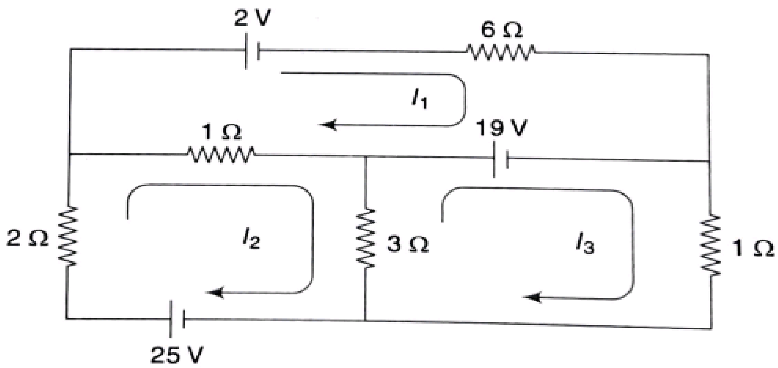

Fig. 1.188

Solution

Marking the different nodes, we get the circuit as shown in Fig. 1.189.

Applying KVL to mesh *ABEDCA*,

$$-2 - 6I_1 + 19 - 1(I_1 - I_2) = 0$$

or $\quad 7I_1 - I_2 = 17 \qquad$ (i)

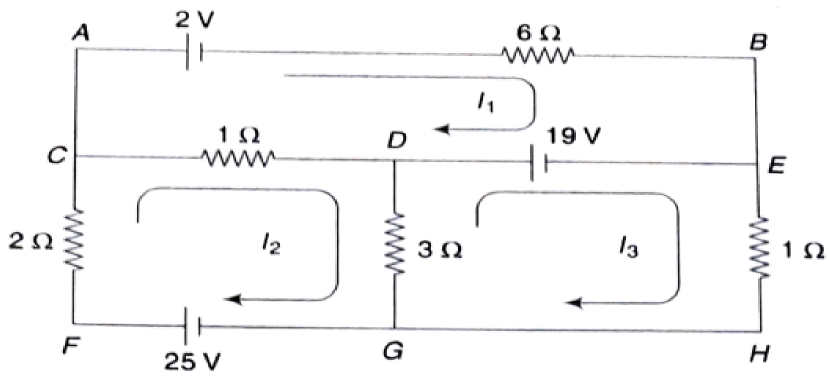

Fig. 1.189

Applying KVL to mesh $CDGFC$,

$$-(I_2 - I_1) - 3(I_2 - I_3) + 25 - 2I_2 = 0$$

or $$I_1 - 6I_2 + 3I_3 = -25 \qquad \text{(ii)}$$

Applying KVL to mesh $DEHGD$,

$$-19 - I_3 - 3(I_3 - I_2) = 0$$

or $$3I_2 - 4I_3 = 19 \qquad \text{(iii)}$$

The values of I_1, I_2, and I_3 may be found by solving the above three simultaneous equations or by the method of determinants as given below:

Putting the above three equations in matrix form, we have

$$\begin{bmatrix} 7 & -1 & 0 \\ 1 & -6 & 3 \\ 0 & 3 & -4 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 17 \\ -25 \\ 19 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 7 & -1 & 0 \\ 1 & -6 & 3 \\ 0 & 3 & -4 \end{vmatrix} = 101, \qquad \Delta_1 = \begin{vmatrix} 17 & -1 & 0 \\ -25 & -6 & 3 \\ 19 & 3 & -4 \end{vmatrix} = 298$$

$$\Delta_2 = \begin{vmatrix} 7 & 17 & 0 \\ 1 & -25 & 3 \\ 0 & 19 & -4 \end{vmatrix} = 369, \qquad \Delta_3 = \begin{vmatrix} 7 & -1 & 17 \\ 1 & -6 & -25 \\ 0 & 3 & 19 \end{vmatrix} = -203$$

By Cramer's rule,

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{298}{101} = 2.95 \text{ A}, \qquad I_2 = \frac{\Delta_2}{\Delta} = \frac{369}{101} = 3.65 \text{ A},$$

$$I_3 = \frac{\Delta_3}{\Delta} = \frac{-203}{101} = -2.01 \text{ A}$$

Example 1.52 By mesh analysis, find the unknown emf V in the network of Fig. 1.190, which causes the mesh current I_1 to be zero.

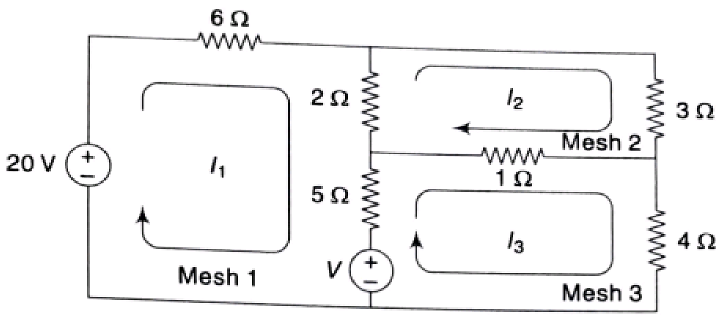

Fig. 1.190

Solution

Applying KVL to mesh 1,

$$-6I_1 - 2(I_1 - I_2) - 5(I_1 - I_3) - V + 20 = 0$$

or $$13I_1 - 2I_2 - 5I_3 = 20 - V \quad \text{(i)}$$

Applying KVL to mesh 2,

$$-3I_2 - (I_2 - I_3) - 2(I_2 - I_1) = 0$$

or $$2I_1 - 6I_2 + I_3 = 0 \quad \text{(ii)}$$

Applying KVL to mesh 3,

$$-(I_3 - I_2) - 4I_3 + V - 5(I_3 - I_1) = 0$$

or $$5I_1 + I_2 - 10I_3 = -V \quad \text{(iii)}$$

As mesh current I_1 is to be zero, i.e., by Cramer's rule, we have

$$\frac{\Delta_1}{\Delta} = 0 \quad \text{or} \quad \Delta_1 = 0$$

Hence $$\begin{vmatrix} 20-V & -2 & -5 \\ 0 & -6 & 1 \\ -V & 1 & -10 \end{vmatrix} = 0$$

or $$(20 - V)\{[(-6)(-10)] - [(1)(1)]\} - (-2)\{0 - (-V)\} + (-5)\{0 - [(-V)(-6)]\} = 0$$

or $$V = 43.7 \text{ V}$$

Example 1.53 By mesh analysis, find the current through 2 Ω resistor in the circuit of Fig. 1.191.

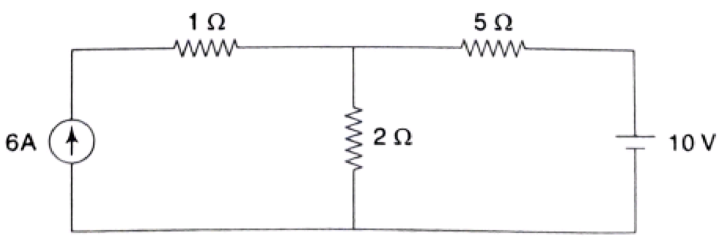

Fig. 1.191

Solution
Marking the different nodes and assigning the separate mesh current for each mesh, we get the following circuit:

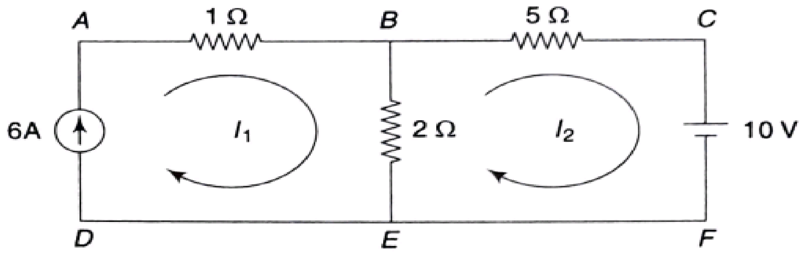

Fig. 1.192

In the circuit, current source is present in a branch not common to any other mesh. If their exists a current source in any mesh of a circuit, then Kirchhoff's voltage law can not be applied to such mesh as voltage drop across the current source is unknown. In Fig. 1.192, current source is present in a mesh *ABEDA*, so KVL can not be applied to this mesh.

In such case, to get the required equations, the current source is expressed in terms of mesh current, i.e., current source of 6 A in the direction of mesh current I_1. We can write the equation as

$$I_1 = 6 \tag{i}$$

Then apply the KVL to the remaining meshes existing without involving the branches consisting of current source. In Fig. 1.192, mesh *BCFEB* does not contain any current source. So, applying the KVL to mesh *BCFEB*, we can write the equation as

$$-5I_2 - 10 - 2(I_2 - I_1) = 0$$

or
$$2I_1 - 7I_2 = 10 \tag{ii}$$

From Eqs (i) and (ii), we get

$$I_2 = 0.2857 \text{ A}$$

Hence, $I_{2\Omega} = (I_1 - I_2)(\downarrow) = 5.71 \text{ A}(\downarrow)$

Example 1.54 By mesh analysis, find the current through 100 Ω resistor in the circuit of Fig. 1.193.

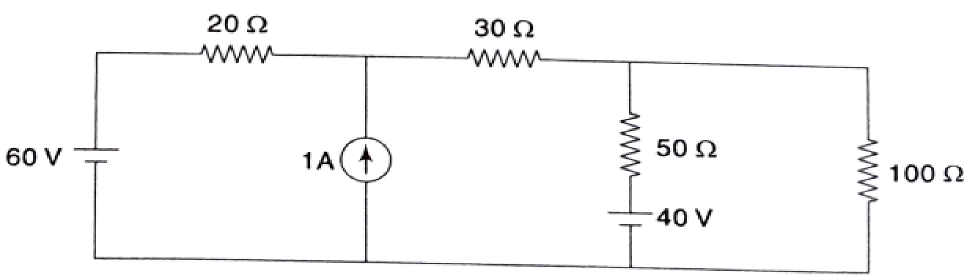

Fig. 1.193

Solution

Marking the different nodes and assigning the separate mesh current for each mesh, we get the following circuit:

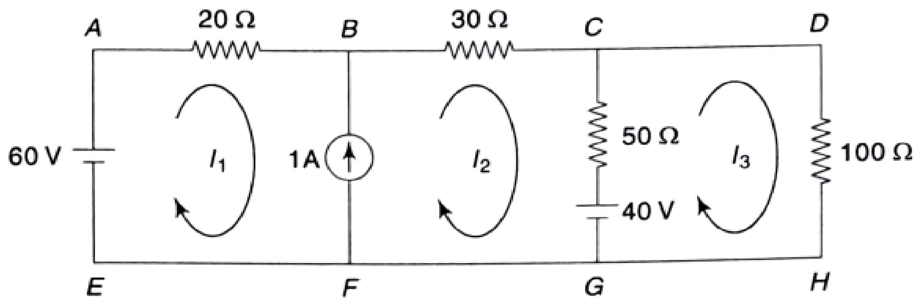

Fig. 1.194

In this example, current source of 1 A is present in a branch common to two meshes, i.e., mesh *ABFEA* and mesh *BCGFB*. When current source is present in a branch common to any two meshes, we need some manipulation and concept of supermesh can be used to solve such problems.

Supermesh is made out of two meshes. By opening the common branch in which current source is present (i.e., branch *BF*), we get the supermesh, i.e., supermesh *ABCGFEA*.

By marking the supermesh, we obtain Fig. 1.195.

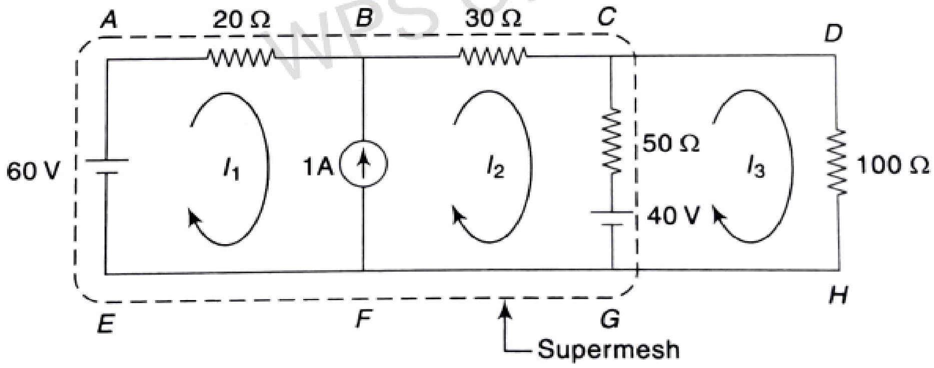

Fig. 1.195

In such a case, to get the required equations for the supermesh we can write the two equations, namely current equation and voltage equation. By expressing the current in the common branch in terms of mesh current, we get the current equation as

$$I_2 - I_1 = 1 \qquad \text{(i)}$$

By applying the KVL to supermesh *ABCGFEA*,we get the voltage equation as

$$-20I_1 - 30I_2 - 50(I_2 - I_3) - 40 + 60 = 0$$

or

$$-20I_1 - 80I_2 + 50I_3 = -20 \qquad \text{(ii)}$$

Now, by applying the KVL to the remaining mesh *CDHGC* (which does not contain any current source), we have

$$-50(I_3 - I_2) - 100I_3 + 40 = 0$$

or $\quad 50I_2 - 150I_3 = -40 \qquad$ (iii)

From Eqs (i), (ii), and (iii), we get

$$I_3 = 0.48 \text{ A}$$

Hence, $I_{100\Omega} = 0.48 \text{ A} (\downarrow)$

Example 1.55 By mesh analysis, find the current through 5 Ω resistor in the circuit of Fig. 1.196.

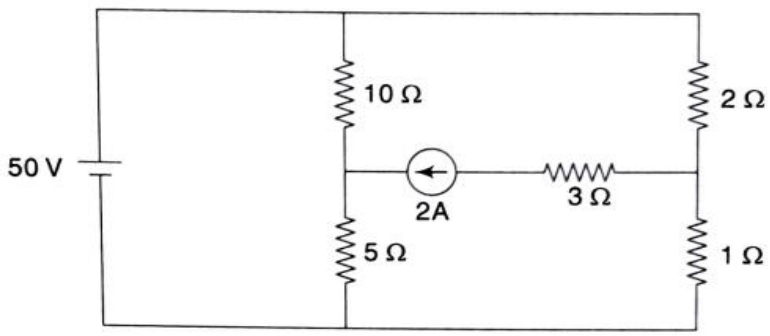

Fig. 1.196

Solution

Assigning the separate mesh current for each mesh, we obtain Fig. 1.197.

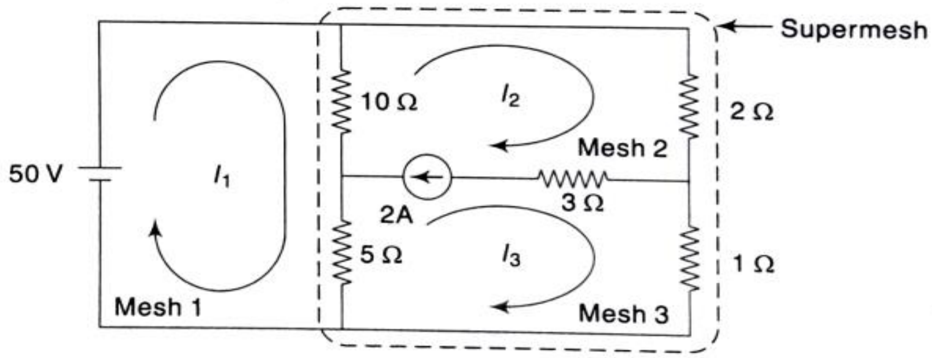

Fig. 1.197

Mesh 2 and mesh 3 form a supermesh.

By expressing the current in the common branch in terms of mesh currents, we get the current equation as

$$(I_2 - I_3) = 2 \qquad \text{(i)}$$

By applying the KVL to the supermesh, we get the voltage equation as

$$-2I_2 - I_3 - 5(I_3 - I_1) - 10(I_2 - I_1) = 0$$

or $\quad 15I_1 - 12I_2 - 6I_3 = 0 \qquad$ (ii)

Now, by applying the KVL to mesh 1 (which does not contain any current source), we have

$$-10(I_1 - I_2) - 5(I_1 - I_3) + 50 = 0$$

or $$-15I_1 + 10I_2 + 5I_3 = -50 \qquad \text{(iii)}$$

The values of I_1 and I_3 may be found by solving the above three simultaneous equations or by the method of determinants as given below:

Putting the above three equations in matrix form, we have

$$\begin{bmatrix} 0 & 1 & -1 \\ 15 & -12 & -6 \\ -15 & 10 & 5 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -50 \end{bmatrix}$$

$$\Delta = \begin{vmatrix} 0 & 1 & -1 \\ 15 & -12 & -6 \\ -15 & 10 & 5 \end{vmatrix} = 45, \qquad \Delta_1 = \begin{vmatrix} 2 & 1 & -1 \\ 0 & -12 & -6 \\ -50 & 10 & 5 \end{vmatrix} = 900,$$

$$\Delta_3 = \begin{vmatrix} 0 & 1 & 2 \\ 15 & -12 & 0 \\ -15 & 10 & -50 \end{vmatrix} = 690$$

By Cramer's rule,

$$I_1 = \frac{\Delta_1}{\Delta} = \frac{900}{45} = 20 \text{ A}, \qquad I_3 = \frac{\Delta_3}{\Delta} = \frac{690}{45} = 5.33 \text{ A}$$

Hence, $I_{5\Omega} = (I_1 - I_3)\,(\downarrow) = 14.67 \text{ A}\,(\downarrow)$