Problems:-

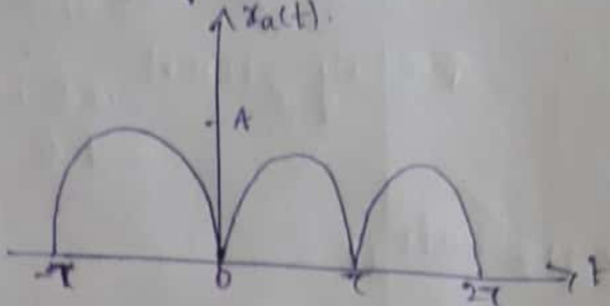
4.1) Consider the full wave rectifier.

(a) Determine its spectrum $X_a(F)$

(b) Compute the power of the signal

(c) Plot the power spectral density.

(d) Check the validity of parseval's relation for this signal.



A- (a) Since $x_a(t)$ is periodic, it can be represented by the fourier series.

$$x_a(t) = \sum_{k=-\infty}^{\infty} C_k e^{j\omega t} = \sum_{k=-\infty}^{\infty} C_k e^{j2\pi \frac{k}{\tau} t}.$$

Where $C_k = \frac{1}{T}\int_T A\sin(\pi t/\tau)\, e^{j2\pi kt/\tau}\, dt.$

$$= \frac{1}{T}\int_0^T A\sin\left(\frac{\pi t}{\tau}\right) e^{j2\pi kt/\tau}\, dt = \frac{A}{j2T}\int_0^T \left(e^{j\pi t/\tau} - e^{-j\pi t/\tau}\right) e^{-j2\pi kt/\tau}\, dt$$

$$= \frac{A}{j2T}\left[\int_0^T e^{j\pi(1-2k)\frac{t}{\tau}} - e^{-j\pi(1+2k)t/\tau}\, dt\right]$$

$$= \frac{A\tau}{j2\tau}\left[\frac{e^{j\pi(1-2k)\frac{t}{\tau}}}{j\pi(1-2k)} - \frac{e^{-j\pi(1+2k)\frac{t}{\tau}}}{-j\pi(1+2k)}\right]_0^\tau$$

$$= \frac{A}{j2}\left[\frac{1}{j\pi(1-2k)}\left(-1+e^{j\pi(1-2k)}\right) + \frac{1}{j\pi(1+2k)}\left(e^{-j\pi(1+2k)}-1\right)\right]$$

$$= \frac{-A}{2\pi}\left[\frac{e^{j\pi(1-2k)}-1}{(1-2k)} + \frac{e^{-j\pi(1+2k)}-1}{(1+2k)}\right].$$

$$= \frac{-A}{2\pi}\left[\frac{-1-1}{(1-2k)} + \frac{-1-1}{(1+2k)}\right] = \frac{-A}{2\pi}\times -2\left[\frac{1}{1-2k} + \frac{1}{1+2k}\right]$$

$$= \frac{A}{\pi}\left[\frac{1}{1-2k} + \frac{1}{1+2k}\right]$$

$$= \frac{-A}{\pi}\left[\frac{1+2k+1-2k}{(1-2k)(1+2k)}\right] \quad \left[\because e^{j\theta} = \cos\theta + j\sin\theta \quad \therefore e^{j\pi(1-2k)} = \cos(\pi(1-2k)) + j\sin\pi(1-2k)\right]$$

$$= \frac{2A}{\pi}\left[\frac{1}{1-4k^2}\right]$$

$\pi(1-2k)$ = odd multiples of π.

$$= \frac{2A}{\pi(1-4k^2)}$$

$\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

$\therefore e^{j\pi(1-2k)} = -1+0 = -1$

Similarly $e^{-j\pi(1+2k)} = \cos\theta - j\sin\theta$

$= -1-0 = -1$.

(b) $$X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi Ft}\, dt$$

$$\Rightarrow X_q(F) = \int_{-\infty}^{\infty} \sum_{k=-\infty}^{\infty} c_k e^{j2\pi kF_0 t} \cdot e^{-j2\pi Ft}\, dt$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} c_k\, e^{-j2\pi(F-kF_0)t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} c_k \int_{-\infty}^{\infty} e^{-j2\pi(F-kF_0)t}\, dt$$

$$= \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} e^{j2\pi kF_0 t} \cdot e^{-j2\pi Ft}$$

$$= \sum_{k=-\infty}^{\infty} c_k\, \delta(F-kF_0) \quad \left[e^{j\omega_0} x(t) \rightleftharpoons X(\omega-\omega_0)\right]$$

The power of the signal is.

$$P = \frac{1}{\tau}\int_0^{\tau} |x_q^2(t)|\, dt$$

$$= \frac{1}{\tau}\int_0^{\tau} \left(-A\sin\frac{\pi t}{\tau}\right)^2 dt$$

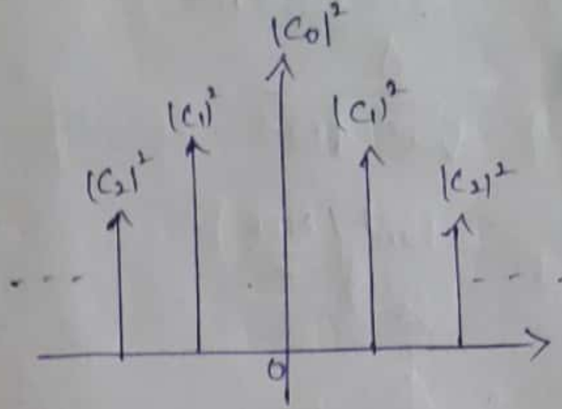
$$= \frac{A^2}{\tau}\int_0^{\tau} \sin^2\frac{\pi t}{\tau}\, dt.$$

$$= \frac{A^2}{\tau}\int_0^{\tau} \frac{1-\cos\frac{2\pi t}{\tau}}{2}\, dt = \frac{A^2}{\tau}\int_0^{\tau}\left(1-\cos\frac{2\pi t}{\tau}\right)dt.$$

$$= \frac{A^2}{2\tau}\left[[t]_0^{\tau} - \left[\frac{\sin\frac{2\pi t}{\tau}}{2\pi}\right]_0^{\tau}\right]$$

$$= \frac{A^2}{2\tau}\left[\tau - \frac{\tau}{2\pi}[0-0]\right] = \frac{A^2}{2\tau} \times \tau = \frac{A^2}{2}$$

(c)



(d)

$$\text{Power} = \sum_{k=-\infty}^{\infty} |C_k\, \delta(F - kF_0)|^2 = \sum_{k=-\infty}^{\infty} |C_k|^2.$$

$$= \sum_{k=-\infty}^{\infty} \left|\frac{2A}{\pi(1-4k^2)}\right|^2 = \sum_{k=-\infty}^{\infty} \frac{4A^2}{\pi^2(1-4k^2)^2}$$

$$= \frac{4A^2}{\pi^2} \sum_{k=-\infty}^{\infty} \frac{1}{\pi^2(1-4k^2)^2} = \frac{4A^2}{\pi^2} \sum_{k=-\infty}^{\infty} \frac{1}{(4k^2-1)^2}$$

$$= \frac{4A^2}{\pi^2}\left[1 + 2\sum_{k=1}^{\infty} \frac{1}{(4k^2-1)^2}\right]$$

$$= \frac{4A^2}{\pi^2}\left[1 + \frac{2}{3^2} + \frac{2}{15^2} + \dots\right]$$

$$= \frac{4A^2}{\pi^2}(1.2337) \;\; \hookrightarrow \text{Infinite series sum to } \pi^2/8$$

$$\therefore \sum_{k=-\infty}^{\infty} |C_k|^2 = \frac{4A^2}{\pi^2}(1.2337) = \frac{A^2}{2}$$

(4.2) Compute and sketch the magnitude and phase spectra for the following signals.

(a) $x_a(t) = \begin{cases} Ae^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases}$

A:-

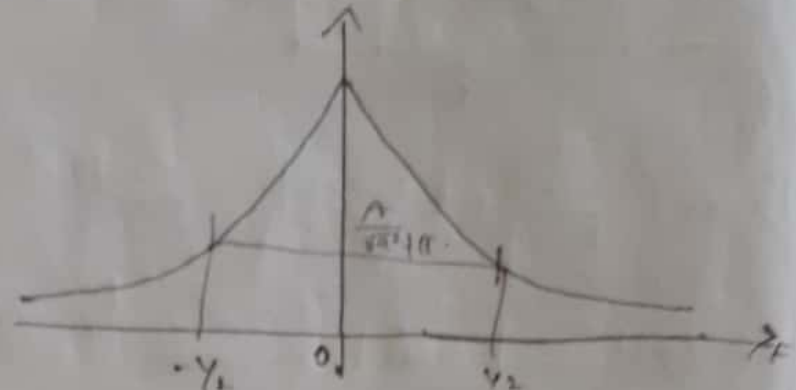
$$X_a(F) = \int_{-\infty}^{\infty} x_a(t)\, e^{-j2\pi Ft} = \int_0^{\infty} A e^{-at} e^{-j2\pi Ft}\, dt.$$

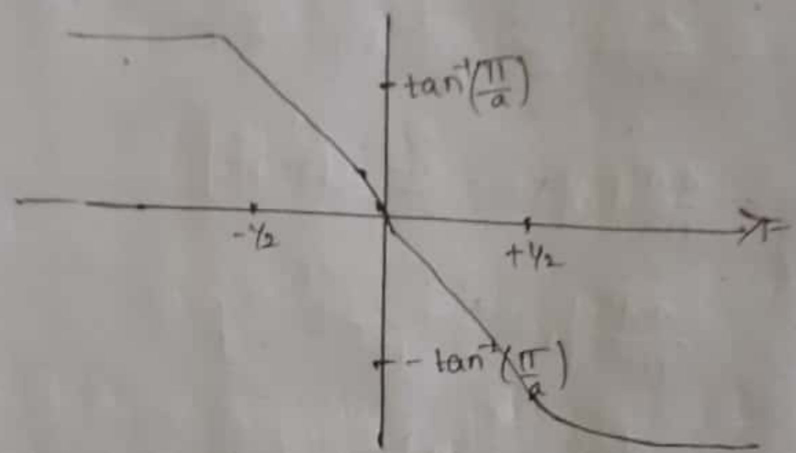
$$= A\int_0^{\infty} e^{-(a+j2\pi F)t}\, dt = A \left.\frac{e^{-(a+j2\pi F)t}}{-(a+j2\pi F)}\right]_0^{\infty}$$

$$= \frac{A}{(a+j2\pi F)}[0-1] = \frac{A}{a+j2\pi F}$$

$$|X_a(F)| = \frac{A}{\sqrt{a^2+4\pi F^2}}$$

$$\angle X_a(F) = -\tan^{-1}\left(\frac{2\pi F}{a}\right)$$





(b) $x_a(t) = Ae^{-a|t|}$

$$X_a(F) = \int_{-\infty}^{\infty} x_a(t)e^{-j2\pi Ft}\,dt$$

$$\int_{-\infty}^{\infty} x_a(t)e^{-j2\pi Ft}\,dt = \int_{-\infty}^{\infty} Ae^{-a|t|}\cdot e^{-j2\pi Ft}\,dt$$

$$= \int_{-\infty}^{0} Ae^{at}e^{-j2\pi Ft}\,dt + \int_{0}^{\infty} Ae^{-at}\cdot e^{-j2\pi Ft}\,dt$$

$$= A\int_{-\infty}^{0} e^{(a-j2\pi F)t}\,dt + A\int_{0}^{\infty} e^{-(a+j2\pi F)t}\,dt$$

$$= A\left.\frac{e^{(a-j2\pi F)t}}{a-j2\pi F}\right]_{-\infty}^{0} + A\left.\frac{e^{-(a+j2\pi F)t}}{-(a+j2\pi F)}\right]_{0}^{\infty}$$

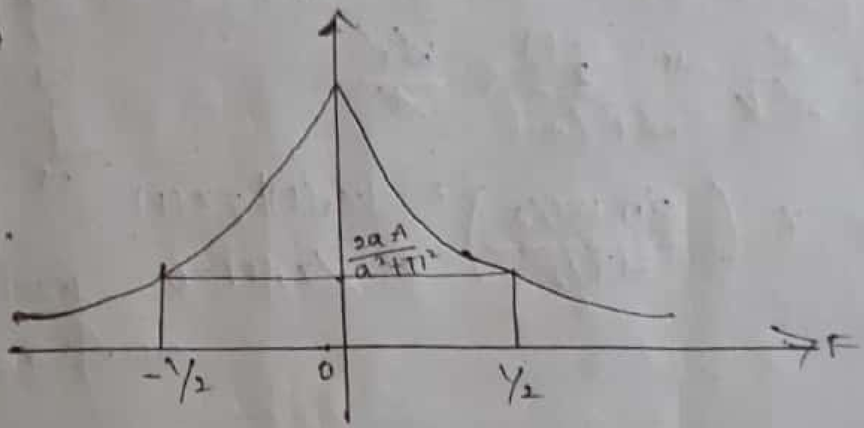
$$= \frac{-A}{a-j2\pi F}(1-0) + \frac{-A}{-(a+j2\pi F)}[0-1].$$

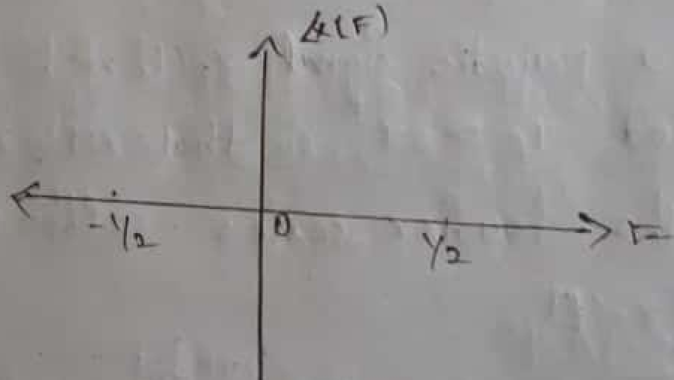
$$= \frac{A}{a-j2\pi F} + \frac{A}{a+j2\pi F} = \frac{A(a+j2\pi F) + A(a-j2\pi F)}{a^2+4\pi F^2}$$

$$= \frac{2aA}{a^2+4\pi F^2}.$$

$$\therefore |X_a(F)| = \frac{2aA}{a^2+4\pi^2F^2}$$

$$\therefore \angle X_a(F) = \tan^{-1}\left(\frac{0}{\frac{2aA}{a^2+4\pi F^2}}\right) = 0$$





(4.3) Consider the signal

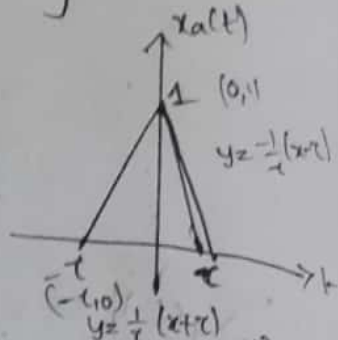
$$x(t) = \begin{cases} 1 - \frac{|t|}{\tau} & |t| \le \tau \\ 0 & \text{elsewhere.} \end{cases}$$

(a) Determine and sketch its magnitude and phase spectra $|X_a(F)|$ and $\angle X_a(F)$ respectively.

$$x(t) = \begin{cases} 1 - \frac{|t|}{\tau} & -\tau < t < \tau \\ 0 & \text{otherwise} \end{cases}, \quad X(F) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi F t}\, dt$$

$$\frac{d x^2(t)}{dt} = \frac{1}{\tau}\left[\delta(t+\tau) - 2\delta(t) + \delta(t-\tau)\right]$$

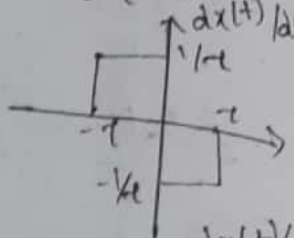
$\downarrow$ FT



$$(j\omega)^2 X(\omega) = \frac{1}{\tau}\left(e^{j\omega\tau} - 2 + e^{-j\omega\tau}\right)$$

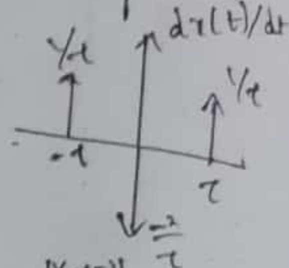
$$\Rightarrow X(\omega) = \frac{-1}{\omega^2 \tau}\left[e^{j\omega\tau} + e^{-j\omega\tau} - 2\right]$$

$$= +\frac{2}{\omega^2 \tau}\left(1 - \cos\omega\tau\right)$$



$$= \frac{2}{\omega^2\tau}\left(2 \cdot \sin^2\frac{\omega\tau}{2}\right) = \frac{4}{\omega^2\tau}\sin^2\frac{\omega\tau}{2}$$

$$= \frac{\cancel{4}}{\cancel{\omega^2}\cancel{\tau}} \frac{\sin^2\left(\frac{\omega\tau}{2}\right)}{\omega^2\tau^2/4} \times \frac{\cancel{\omega^2}\cancel{\tau^2}}{\cancel{4}}$$



$$= \tau\left(\frac{\sin \omega\tau/2}{\omega\tau/2}\right)^2 \qquad |X_a(F)| = X(F)$$

$$\angle X_a(F) = 0$$

(b) Consider a periodic signal $x_p(t)$ with fundamental time period $T_p \geq 2\tau$, so that $x(t) = x_p(t)$ for $|t| < T_p/2$. What are the fourier series coefficients c_k for the signal $x_p(t)$.

$$c_k = \frac{1}{T_p}\int_{T_p} x_p(t)\, e^{-j2\pi k \frac{1}{T_p} t}\, dt$$

$$= \frac{1}{T_p}\int_{-T_p/2}\left(1 - \frac{|t|}{\tau}\right) e^{-j2\pi k t/T_p}\, dt$$

$$= \frac{1}{T_p}\left[\int_{-\tau}^{0}\left(1 + \frac{t}{\tau}\right) e^{-j2\pi k t/T_p}\, dt + \int_{0}^{\tau}\left(1 - \frac{t}{\tau}\right) e^{-j2\pi k t/T_p}\, dt\right]$$

$$= \frac{\tau}{T_p}\left[\frac{\sin \pi k \tau/T_p}{\pi k \tau/T_p}\right]^2$$

$$c_k = \frac{1}{T_p}\int_{T_p} x_p(t)\, e^{-j2\pi k t/T_p}\, dt.$$

(c) using the results in part (a) and (b) show that

$C_k = \frac{1}{T_p} x_a(k/T_p)$.

$$\frac{1}{T_p} x_a\left(\frac{k}{T_p}\right) = \frac{1}{T_p} \tau \left[\frac{\sin \frac{\pi k \tau}{T_p}}{\pi k \tau / T_p}\right]^2$$

$$= \frac{\tau}{T_p}\left(\frac{\sin \frac{\pi k \tau}{T_p}}{\pi k \tau / T_p}\right)^2 = C_k$$

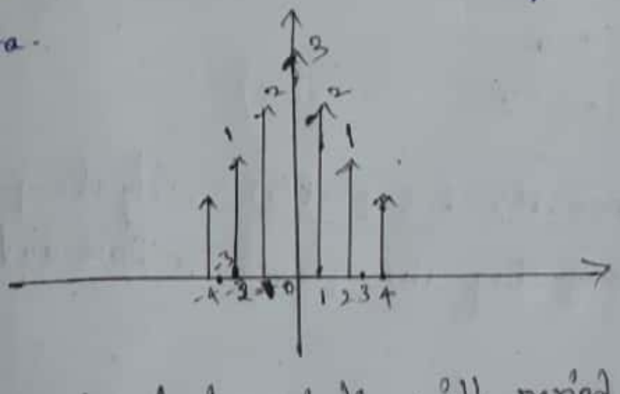
$$\therefore C_k = \frac{1}{T_p} x_a\left(\frac{k}{T_p}\right)$$

Hence proved.

(4.4) Consider the following periodic signal.

$$x[n] = \{\ldots \ldots 1, 0, 1, 2, 3, 2, 1, 0, 1 \ldots\ldots\}.$$

(a) Sketch the $x[n]$ and its magnitude and phase spectra.



The given signal is periodic with period $N=6$

$$\therefore C_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n]\, e^{-j2\pi kn/N}.$$

$$\therefore C_k = \frac{1}{6} \sum_{n=0}^{5} x[n]\, e^{-j2\pi kn/6}$$

(i) for $n=0$, $x(0) \sum_{n=0} e^{-\frac{j2\pi k(0)}{6}} = 3\times 1 = 3.$

$n=1$, $x(1)\, e^{-\frac{j2\pi k}{6}} = 2e^{-\frac{j2\pi k}{6}} = 2e^{j\pi k/3}$

$n=2$, $x(2)\, e^{-\frac{j4\pi k}{6}} = e^{-j2\pi k/3}$.

$n=3$, $x(3)\, e^{-j6\pi k/6} = 0$.

$n=4 \rightarrow x(4)\, e^{-j8\pi k/6} = e^{-j4\pi k/3}$

$n=5 \rightarrow x(5)\, e^{-j10\pi/6} = 2e^{-j5\pi k/3}$

— for $k=0$

$= \frac{1}{6}[3+2+1+0+1+2] = \frac{1}{6}\times 9 = 3/2$.

— for $k=1 = \frac{1}{6}\left[3+2e^{j\pi/3}+e^{-j2\pi/3}+0+e^{-j4\pi/3}+2e^{-j5\pi/3}\right]$

$= \frac{1}{6}\Big[3+2\cos\frac{\pi}{3} - 2j\sin\pi/3 + \left(\cos\left(\frac{2\pi}{3}\right) - j\sin\left(\frac{2\pi}{3}\right)\right) +$

$\cos\left(\frac{4\pi}{3}\right) - j\sin\left(\frac{4\pi}{3}\right) + 2\cos\left(\frac{5\pi}{3}\right) - 2j\sin\left(\frac{5\pi}{3}\right)\Big]$

$= \frac{1}{6}\left[3+1-\cancel{j\sqrt{3}}+\left(\frac{-1}{2}\right)-\cancel{\frac{j\sqrt{3}}{2}}+\left(\frac{-1}{2}\right)+\cancel{\frac{j\sqrt{3}}{2}}+1+\cancel{j\sqrt{3}}\right]$

$= \frac{4}{6}$.

$k=2 \Rightarrow C_2 = 0$

$k=3 \Rightarrow C_3 = 1/6$

$k=4 \Rightarrow C_4 = 0$

$k=5 \Rightarrow C_5 = 4/6$.

(b) using the results in part (a) verify the parseval's relation by computing the power in time and frequency domain.

$P_t = 1/6 \sum_{k=0} |x(n)|^2$

$= \frac{1}{6}\left[1^2+0^2+1^2+2^2+3^2+2^2\right] = \frac{1}{6}[1+1+4+9+4]$

$= 19/6$

$P_f = \sum_{n=0} |c(n)|^2 = \left(\frac{9}{6}\right)^2 + \left(\frac{4}{6}\right)^2 + 0 + \left(\frac{1}{6}\right)^2 + \left(\frac{4}{6}\right)^2$

$= \frac{114}{36} = \frac{19}{6}$

— Hence proved.

4.5) Consider the signal.

$$x(n) = 2 + 2\cos\left(\frac{\pi n}{4}\right) + \cos\left(\frac{\pi}{2}n\right) + \frac{1}{2}\cos\left(\frac{3\pi n}{4}\right)$$

(a) Determine and sketch its power density spectrum.

A:- $x(n) = 2 + 2\cos\left(\frac{\pi}{4}n\right) + \cos\left(\frac{\pi}{2}n\right) + \frac{1}{2}\cos\left(\frac{3\pi}{4}n\right)$.

$$= 2 + 2\left(\frac{e^{j\pi/4 n} + e^{-j\pi/4 n}}{2}\right) + \left(\frac{e^{-j\pi/2 n} + e^{j\pi/2 n}}{2}\right) + \frac{e^{-j\frac{3\pi}{4}n} + e^{j\frac{3\pi}{4}n}}{4}.$$

$$x(n) = 2 + e^{\frac{j\pi}{4}n} + e^{-\frac{j\pi}{4}n} + \frac{1}{2}e^{j\pi/2 n} + \frac{1}{2}e^{-j\pi/2 n} + \frac{1}{4}e^{j3\pi/4 n} + \frac{1}{4}e^{-j3\pi/4 n}$$

$\therefore N = 8$

$$C_k = \frac{1}{8}\sum_{n=0}^{7} x(n)\, e^{-\frac{j\pi k n}{4}}$$

$$\therefore x(n) = \left\{\frac{11}{2}, 2 + \frac{(\sqrt{3})^2}{4}\sqrt{2}, 1, 2 - \frac{3\sqrt{2}}{4}, \frac{1}{2}, 2 - \frac{3\sqrt{2}}{4}, 1, 2 + \frac{3\sqrt{2}}{4}\right\}$$

$$x(0) = 2 + 2 + 1 + \tfrac{1}{2} = 5 + \tfrac{1}{2} = \tfrac{11}{2}.$$

$$C_0 = \frac{1}{8}\sum_{n=0}^{7} x(n) = \frac{1}{8}\left[\frac{11}{2} + \cancel{2} + \cancel{\frac{3}{4}\sqrt{2}} + 1 + \cancel{2} - \cancel{\frac{3\sqrt{2}}{4}} + \frac{1}{2} + 2 - \cancel{\frac{3}{4}\sqrt{2}} + 1 + 2 + \cancel{\frac{3}{4}\sqrt{2}}\right]$$

$$= \frac{1}{8}\left[\frac{11}{2} + 10 + \frac{1}{2}\right] = \frac{1}{8}\left[\frac{12}{2} + 10\right] = \frac{1}{8}\left[6 + 10\right] = \frac{16}{8} = \underline{\underline{2}}.$$

Similarly,

$C_1 = C_7 = 1$, $C_2 = C_6 = \frac{1}{2}$, $C_3 = C_5 = \frac{1}{4}$, $C_4 = 0$.

(b) Determine and sketch the magnitude and phase spectrum of following periodic signal.

Evaluate the power of the signal.

A:- $\text{Power} = \sum_{k=0}^{7} |c(k)|^2 = 1 + 1 + \frac{1}{4} + \frac{1}{4} + \frac{1}{16} + \frac{1}{16} + 4$

$$= 6 + \frac{2}{4} + \frac{2}{16} = 6 + \frac{1}{2} + \frac{1}{8} = 6 + \frac{5}{8} = \underline{\underline{\frac{53}{8}}}.$$

4.6) Determine and sketch the magnitude and phase spectrum of following periodic signals.

(a) $x(n) = 4\sin\left(\frac{\pi(n-2)}{3}\right)$.

A:- $x(n) = 4\sin\left(\frac{\pi(n-2)}{3}\right) = 4\left(\frac{e^{\frac{j\pi(n-2)}{3}} - e^{\frac{-j\pi(n-2)}{3}}}{2j}\right)$

$$= \frac{2}{j}\left(e^{\frac{j2\pi(n-2)}{6}} - e^{\frac{-j2\pi(n-2)}{6}}\right)$$

$\therefore N = 6$.

$$\therefore C_k = \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{\frac{-j2\pi kn}{6}}.$$

$$= \frac{j}{3}\sum_{n=0}^{5}\left[e^{\frac{j\pi(n-2)}{3}} - e^{\frac{-j\pi(n-2)}{3}}\right] e^{\frac{j2\pi kn}{6}}.$$

$$= \frac{j}{3}\sum_{n=0}^{5}\left[e^{\frac{j\pi(n-2)}{3}} - e^{\frac{-j\pi(n-2)}{3}}\right] e^{\frac{j2\pi kn}{6}}.$$

$x[n] = \{-2\sqrt{3}, -2\sqrt{3}, 0, 2\sqrt{3}, 2\sqrt{3}, 0\}$

$C_0 = \frac{1}{6}\sum_{n=0}^{5} x[n] = 0$

$C_1 = \frac{1}{6}\sum_{n=0}^{5} x(n)\, e^{-\frac{j\pi}{3}n}$.

(b) $x(n) = \cos\left(\frac{2\pi}{3}n\right) + \sin\left(\frac{2\pi}{5}n\right)$ .. $c_{1k} + c_{2k}$

$N = LCM(3,5) = 15$.

$$\cos\left(\frac{2\pi n}{3}\right) = \frac{1}{2}\left(e^{\frac{j2\pi}{3}n} + e^{-\frac{j2\pi}{3}n}\right)$$

$$C_{1k} = \frac{1}{15}\sum_{n=0}^{14} x(n)\, e^{-\frac{j2\pi kn}{15}}.$$

$$C_{1k} = \begin{cases} \frac{1}{2} & k = 5, 10. \\ 0 & \text{otherwise} \end{cases}$$

$$\sin\left(\frac{2\pi}{5}n\right) = \frac{1}{2j}\left(e^{\frac{j2\pi n}{5}} - e^{-\frac{j2\pi n}{5}}\right)$$

$$C_{2k} = \begin{cases} \frac{1}{2j} & k = 3 \\ -\frac{1}{2j} & k = 12 \\ 0 & \text{otherwise.} \end{cases}$$

$$\therefore C_k = C_{1k} + C_{2k} = \begin{cases} \frac{1}{2j} & k = 3 \\ \frac{1}{2} & k = 5 \\ \frac{1}{2} & k = 10 \\ -\frac{1}{2j} & k = 12 \\ 0 & \text{otherwise.} \end{cases}$$

(c) $x(n) = \cos\left(\frac{2\pi}{3}n\right)\cdot \sin\left(\frac{2\pi}{5}n\right)$

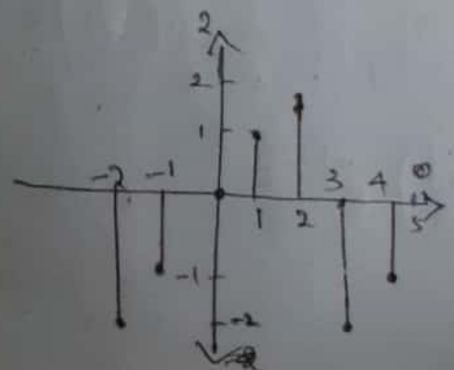
$$x(n) = \cos\left(\frac{2\pi}{3}n\right)\sin\left(\frac{2\pi}{5}n\right).$$

$$= \frac{1}{2}\left(\sin\left(\frac{16\pi}{15}n\right) - \sin\left(\frac{4\pi}{15}n\right)\right)$$

Hence $N = 5$

$$\therefore C_k = \begin{cases} -\frac{1}{4j} & k = 2, 7 \\ \frac{1}{4j} & k = 8, 13 \\ 0 & \text{otherwise} \end{cases}$$

(d) $x(n) = \{\ldots\ldots -2, -1, 0, 1, 2, -2, -1, 0, 1, 2, \ldots\}$



$\Rightarrow N = 5$

$\therefore C_K = \frac{1}{5}\sum_{K=0}^{4} x(n)\, e^{-\frac{j2\pi kn}{5}}$

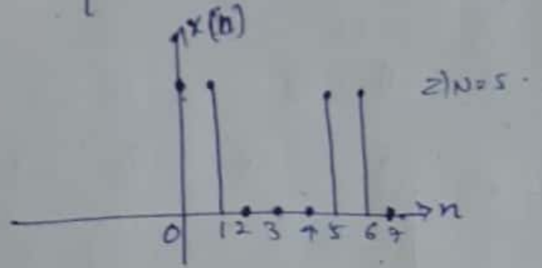
$= \frac{1}{5}\left[e^{-\frac{j2\pi k}{5}} + 2e^{-\frac{j4\pi k}{5}} + -2e^{\frac{j4\pi k}{5}} - e^{\frac{j2\pi k}{5}}\right]$

$= \frac{2j}{5}\left[-\sin\left(\frac{2\pi k}{5}\right) - 2\sin\left(\frac{4\pi k}{5}\right)\right]$

$\therefore C_0 = 0,\ C_1 = \frac{2j}{5}\times(\ ) = j0.6155.$

$C_2 = j0.145,\ C_3 = j(-0.145),\ C_4 = -j0.6155$

c) $x(n) = \{\cdots 0, 0, 1, 1, 0, 0, 0, 1, 1, 0, 0, \cdots\}$



$\Rightarrow C_K = \frac{1}{5}\sum_{n=0}^{4} x(n)\, e^{-\frac{j2\pi nk}{5}}$

$\Rightarrow C_K = \frac{1}{5}\left[1 + e^{-\frac{j\pi 2k}{5}} + 0 + 0 + 0\right] = \frac{1}{5}\left[1 + e^{-j2\pi k/5}\right]$

$\therefore C_0 = \frac{2}{5},\ C_1 = \frac{1}{5}\left[1 + e^{-j2\pi/5}\right]$

$C_K = \frac{1}{5}\left[1 + e^{-j2\pi k/5}\right]\frac{e^{j\pi k/5}}{e^{j\pi k/5}} = \frac{1}{5}\left[e^{\frac{j\pi k}{5}} + e^{-\frac{j\pi k}{5}}\right] e^{-\frac{j\pi k}{5}}$

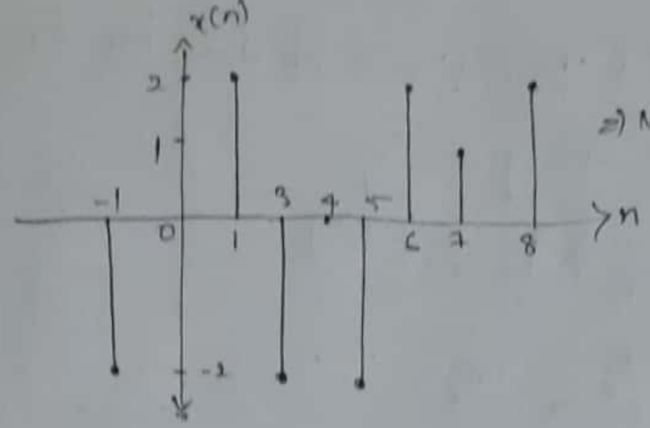
$\therefore C_K = \frac{2}{5}\left[\frac{e^{\frac{j\pi k}{5}} + e^{-\frac{j\pi k}{5}}}{2}\right] e^{-\frac{j\pi k}{5}} = \frac{2}{5}\cos\left(\frac{\pi k}{5}\right) e^{-\frac{j\pi k}{5}}$

$\therefore C_0 = \frac{2}{5},\ C_1 = \frac{2}{5}\cos\left(\frac{\pi}{5}\right) e^{-\frac{j\pi}{5}}$

$C_2 = \frac{2}{5}\cos\left(\frac{2\pi}{5}\right) e^{-\frac{j2\pi}{5}},\ C_3 = \frac{2}{5}\cos\left(\frac{3\pi}{5}\right) e^{-j3\pi/5}$

$C_4 = \frac{2}{5}\cos\left(\frac{4\pi}{5}\right) e^{-j4\pi/5}$

(f) $x(n) = \{\ldots -1, 2, \underset{\uparrow}{1}, 2, -1, 0, -1, 2, 1, 2 \ldots\ldots\}$.



$\Rightarrow N=6$.

$$C_K = \frac{1}{6}\sum_{K=0}^{5} x(n)\, e^{\frac{-j2\pi kn}{6}}.$$

$$= \frac{1}{6}\left[1 + 2e^{\frac{-j\pi k}{3}} + (-1)e^{\frac{-j2\pi k}{3}} + 0 + (-1)e^{\frac{-j4\pi k}{3}} + 2e^{\frac{-j5\pi k}{3}}\right]$$

$$= \frac{1}{6}\left[1 + 2e^{\frac{-j\pi k}{3}} - e^{\frac{-j2\pi k}{3}} - e^{\frac{-j4\pi k}{3}} + e^{\frac{-j5\pi k}{3}}\right].$$

$$C_0 = \frac{1}{6}\left[1 + 2 - \cancel{1} - \cancel{1} + \cancel{2}\right] = \frac{3}{6} = 1/2.$$

$$C_K = \frac{1}{6}\left[1 + 4\cos\left(\frac{\pi k}{3}\right) - 2\cos\left(\frac{2\pi k}{3}\right)\right].$$

$$C_1 = \frac{1}{6}\left(1 + 4\left(\frac{1}{2}\right) + 1\right) = \frac{1}{6}(1+2+1) = \frac{4}{6} = 2/3.$$

$$C_2 = \frac{1}{6}\left(1 + 4\left(\frac{-1}{2}\right) + 1\right) = 0.$$

$$C_3 = \frac{1}{6}\left[1 + 4(-1) - 2\right] = \frac{1}{6}[1-6] = \frac{-5}{6}.$$

$$C_4 = \frac{1}{6}\left[1 + 4\left(\frac{-1}{2}\right) - 2\left(\frac{-1}{2}\right)\right] = \frac{1}{6}[1-2+1] = 0.$$

$$C_5 = \frac{1}{6}\left[1 + 4\left(\frac{1}{2}\right) - 2\left(\frac{-1}{2}\right)\right] = \frac{1}{6}[1+2+1] = \frac{4}{6} = \frac{2}{3}.$$

(g) $x(n) = 1, -\infty < n < \infty$.

$N = 1$, $C_K = x(0) = 1$ (or) $C_0 = 1$.

(h) $x(n) = (-1)^n, -\infty < n < \infty$ $\quad N=2$

$$C_K = 1/2 \sum_{n=0}^{1} x(n)\, e^{-j\pi nk}$$

$$C_K = \frac{1}{2}\left[1 - e^{-j\pi k}\right], \quad C_0 = 0, \ C_1 = 1.$$

4.7) Determine the periodic signals $x(n)$, with fundamental period $N=8$, if their fourier co-efficients are given by.

(a) $c_k = \cos\left(\frac{k\pi}{4}\right) + \sin\left(\frac{3k\pi}{4}\right)$.

$$x(n) = \sum_{k=0}^{7} c_k e^{\frac{j2\pi nk}{8}}.$$

if $c_k = e^{\frac{j2\pi pk}{8}}$ then,

$$x(n) = \sum_{k=0}^{7} e^{\frac{j2\pi pk}{8}} e^{\frac{j2\pi nk}{8}} = \sum_{k=0}^{7} e^{\frac{j2\pi(p+n)k}{8}}.$$

$= 8$ when $p = -n$

0 when $p \neq -n$.

Since $c_k = \frac{1}{2}\left(e^{\frac{j\pi k}{4}} + e^{\frac{-j\pi k}{4}}\right) + \frac{1}{2j}\left(e^{\frac{j3\pi k}{4}} - e^{\frac{-j3\pi k}{4}}\right)$.

$$\therefore x(n) = \left\{4\delta(n+1) + 4\delta(n-1) - 4j\,\delta(n+3) + 4j\,\delta(n-3)\right\}$$
$-3 \leq n \leq 5$.

(b). $c_k = \begin{cases} \sin\frac{k\pi}{3} & 0 \leq k \leq 6 \\ 0 & k = 7. \end{cases}$

$c_0 = 0$, $c_1 = \frac{\sqrt{3}}{2}$, $c_2 = \frac{\sqrt{3}}{2}$, $c_3 = 0$, $c_4 = -\frac{\sqrt{3}}{2}$, $c_5 = -\frac{\sqrt{3}}{2}$.

$c_6 = c_7 = 0$.

$$x(n) = \sum_{k=0}^{7} c_k e^{\frac{j2\pi nk}{8}}.$$

$$= \frac{\sqrt{3}}{2}\left[e^{\frac{j\pi}{4}n} + e^{\frac{j\pi n2}{4}} - e^{\frac{j\pi n4}{4}} - e^{\frac{j5\pi n}{4}}\right]$$

$$= \frac{\sqrt{3}}{2}\left[\sin\frac{\pi}{2}n + \sin\frac{\pi}{4}n\right] e^{\frac{j\pi(3n-2)}{4}}.$$

(c) $\{a\} = \{, \ldots 0, \frac{1}{4}, \frac{1}{2}, 1, 2, \frac{1}{2}, \frac{1}{4}, 0, \ldots\}$

$$\Rightarrow x(n) = \sum_{k=-3}^{4} c_k e^{\frac{j2\pi nk}{8}}$$

$$= 2 + e^{\frac{j\pi n}{4}} + \frac{1}{2} e^{\frac{j\pi n}{2}} + \frac{1}{4} e^{\frac{j3\pi n}{4}} + e^{\frac{-j\pi n}{4}} + \frac{1}{2} e^{\frac{-j\pi n}{2}} + \frac{1}{4} e^{\frac{-j3\pi n}{4}}.$$

$$= 2 + 2\cos\frac{\pi}{4}n + \cos\frac{\pi}{2}n + \frac{1}{2}\cos\left(\frac{3\pi n}{4}\right).$$

4.9) Compute the fourier transform of following signals

(a) $x(n) = u(n) - u(n-6)$.

$$x(n) = u(n) - u(n-6)$$

$$x(\omega) = \sum_{m=-\infty}^{\infty} x(n) e^{-j\omega n} = \sum_{m=0}^{5} e^{-j\omega n}$$

$$= \frac{1-e^{-j6\omega}}{1-e^{-j\omega}} \quad \left[\because \sum_{m=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}\right]$$

(b) $x(n) = 2^n u(-n)$.

$$x(\omega) = \sum_{m=-\infty}^{0} 2^n e^{-j\omega n}, \text{ let } m = -n.$$

$\Rightarrow n = -\infty \Rightarrow m = \infty$

$n = 0 \Rightarrow m = 0$

$$\therefore x(\omega) = \sum_{m=0}^{\infty} 2^{-m} e^{j\omega n} = \sum_{m=0}^{\infty} \left(\frac{e^{j\omega}}{2}\right)^m$$

$$= \frac{1}{1-\frac{1}{2}e^{j\omega}} = \frac{2}{2-2e^{j\omega}}$$

(c) $x(n) = \left(\frac{1}{4}\right)^n u(n+4)$.

$$x(\omega) = \sum_{m=-4}^{\infty} \left(\frac{1}{4}\right)^n e^{-j\omega n}$$

Let $n+4 = m \Rightarrow$ when $n = -4 \Rightarrow m = 0$

$n = \infty \Rightarrow m = \infty$

$$\therefore x(\omega) = \sum_{m=0}^{\infty} \left(\frac{1}{4}\right)^{m-4} e^{-j\omega(m-4)}$$

$$= \sum_{m=0}^{\infty} (4^4) e^{j4\omega} \left(\frac{1}{4} e^{-j\omega}\right)^m = \frac{4^4 e^{j4\omega}}{1-\frac{1}{4}e^{-j\omega}}.$$

(d) $x(n) = (\alpha^n \sin w_0 n)\, u(n)$, $|\alpha| < 1$

$$X(\omega) = \sum_{n=0}^{\infty} \alpha^n \sin w_0 n \, e^{-j\omega n}.$$

$$= \sum_{n=0}^{\infty} \alpha^n \left[\frac{e^{jw_0 n} - e^{-jw_0 n}}{2j} \right] e^{-j\omega n}.$$

$$= \frac{1}{2j} \sum_{n=0}^{\infty} \alpha^n e^{-j(\omega - w_0)n} - \frac{1}{2j} \sum_{n=0}^{\infty} \alpha^n e^{-j(\omega + w_0)n}.$$

$$= \frac{1}{2j} \left[\frac{1}{1 - \alpha e^{-j(\omega - w_0)}} - \frac{1}{1 - \alpha e^{-j(\omega + w_0)}} \right].$$

$$= \frac{1}{2j} \left[\frac{1 - \alpha e^{-j(\omega + w_0)} - 1 + \alpha e^{-j(\omega - w_0)}}{1 + \alpha^2 e^{-j(\omega - w_0)} e^{-j(\omega + w_0)} - \alpha e^{-j(\omega + w_0)} - \alpha e^{-j(\omega - w_0)}} \right]$$

$$= \frac{1}{2j} \left[\frac{-\alpha e^{-j\omega} + \alpha e^{-j\omega} \, \alpha e^{jw_0} - \alpha e^{-jw_0}}{1 + \alpha^2 e^{-j2\omega} - 2\alpha \cos w_0 e^{-j\omega}} \right]$$

$$X(\omega) = \frac{\alpha \sin w_0 \, e^{-j\omega}}{1 - 2\alpha \cos(w_0) e^{-j\omega} + \alpha^2 e^{-j2\omega}}$$

(e) $x(n) = |\alpha|^n \sin(w_0 n)$, $|\alpha| < 1$.

$x(n) = |\alpha|^n \sin(w_0 n)$, $|\alpha| < 1$.

$$\sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=-\infty}^{\infty} |x(n)| = \sum_{n=-\infty}^{\infty} |\alpha^n| \, |\sin w_0 n|.$$

when $w_0 = \frac{\pi}{2}$, then $|\sin w_0 n| = 1$

$$\sum_{n=-\infty}^{\infty} |\alpha|^n = \sum_{n=-\infty}^{\infty} |x(n)| \rightarrow \infty$$

Therefore it is not absolute summable signal

∴ DTFT does not exist.

(f) $x(n) = \begin{cases} 2-(\frac{1}{2})^n & |n| \le 4 \\ 0 & \text{elsewhere.} \end{cases}$

$$x(\omega) = \sum_{n=-4}^{4} x(n)\, e^{-j\omega n}$$

$$= \sum_{n=-4}^{4} \left(2-\frac{1}{2}n\right) e^{-j\omega n}$$

$$= \sum_{n=-4}^{4} 2e^{-j\omega n} - \sum_{n=-4}^{4} \left(\frac{1}{2}\right)^n e^{-j\omega n}$$

$$= \sum_{n=-4}^{4} 2e^{-j\omega n} - \sum_{n=-4}^{4} \left(\frac{1}{2} e^{-j\omega}\right)^n$$

Let $m = n+4 \Rightarrow m = 0$ when $n = -4$

$m = 8$ when $n = 4$

$$x(\omega) = \sum_{m=0}^{8} 2e^{-j\omega(m+4)} - \sum_{m=0}^{8} \left(\frac{1}{2} e^{-j\omega}\right)^{m+4}.$$

$$= 2e^{-j4\omega} \sum_{m=0}^{8} e^{-j\omega m} - \left(\frac{1}{2} e^{-j\omega}\right)^4 \sum_{m=0}^{8} \left(\frac{1}{2} e^{-j\omega}\right)^m$$

$$= \frac{2e^{-j4\omega}}{1-e^{-j\omega}} - \left(\frac{1}{2} e^{-j\omega}\right)^4 \frac{1-\left(\frac{1}{2}\right)^9}{1-\frac{1}{2}e^{-j\omega}}$$

$$= \frac{2e^{-j4\omega}}{1-e^{-j\omega}} - \frac{1}{2}\Big[-4e^{j4\omega} + 4e^{-j4\omega} - 3e^{j3\omega} + e^{-j3\omega} - 2e^{j2\omega} + 2e^{-j2\omega} + 2e^{-j2\omega} + e^{-j\omega} - e^{j\omega}$$

$$= \frac{2e^{j4\omega}}{1-e^{-j\omega}} + j\left[4\sin(4\omega) + 3\sin(3\omega) + 2\sin(2\omega) + \sin(\omega)\right]$$

(g) $x(n) = \{-2, -1, 0, 1, 2\}$

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} = \sum_{n=-2}^{2} x(n)\, e^{-j\omega n}$$

$$= -2e^{j2\omega} - e^{j\omega} + e^{-j\omega} + 2e^{-j2\omega}$$

$$= -2j\left[2\sin(2\omega) + \sin(\omega)\right]$$

(h) $x(n) = \begin{cases} A(2M+1-|n|); & |n| \le M \\ 0 & |n| > M \end{cases}$

$$X(\omega) = \sum_{n=-M}^{M} x(n) e^{-j\omega n} = A \sum_{n=-M}^{M} (2M+1-|n|) e^{-j\omega n}$$

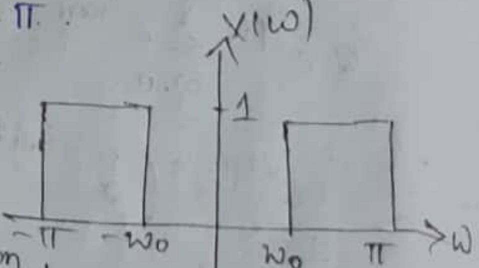
$$= (2M+1)A + A \sum_{k=1}^{M} (2M+1-k) \left(e^{-j\omega k} + e^{j\omega k}\right)$$

$$= (2M+1)A + 2A \sum_{k=1}^{M} (2M+1-k) \left(\frac{e^{-j\omega k} + e^{j\omega k}}{2}\right)$$

$$\therefore X(\omega) = (2M+1)A + 2A \sum_{k=1}^{M} (2M+1-k) \cos(\omega k).$$

4.10) Determine the signal having the following fourier transform.

(a) $X(\omega) = \begin{cases} 0 & 0 < |\omega| < \omega_0 \\ 1 & \omega_0 < |\omega| \le \pi \end{cases}$



A:- $x(n) = \frac{1}{2\pi} \int_{2\pi} X(\omega) e^{j\omega n} \, d\omega$

$$= \frac{1}{2\pi} \int_{-\pi}^{-\omega_0} e^{j\omega n} d\omega + \frac{1}{2} \int_{\omega_0}^{\pi} e^{j\omega n} d\omega.$$

$$= \frac{1}{2\pi} \left[\left[\frac{e^{j\omega n}}{jn}\right]_{-\pi}^{-\omega_0} + \frac{1}{2} \left[\frac{e^{j\omega n}}{jn}\right]_{\omega_0}^{\pi} \right]$$

$$= \frac{1}{2\pi} \left[\frac{e^{-j\omega_0 n} - e^{-j\pi n}}{jn}\right] + \frac{1}{2\pi} \left[\frac{e^{j\pi n} - e^{j\omega_0 n}}{jn}\right]$$

$$= \frac{1}{\pi n} \left[\frac{e^{-j\omega_0 n} - e^{-j\pi n}}{2j} + \frac{e^{j\pi n} - e^{j\omega_0 n}}{2j}\right]$$

$$= \frac{1}{\pi n} \left[-\left[\frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j}\right] + \frac{e^{j\pi n} - e^{-j\pi n}}{2j} \right]$$

$$= -\frac{1}{\pi n}\left[\sin(n\omega_0) + \sin^{0}(n\pi)\right]$$

$$= -\sin(n\omega_0)/\pi n$$

(b) $X(\omega) = \cos^2(\omega)$.

$$X(\omega) = \cos^2\omega = \left(\frac{1}{2}e^{j\omega} + \frac{1}{2}e^{-j\omega}\right)^2$$

$$= \frac{1}{4}e^{j2\omega} + \frac{1}{4}e^{-j2\omega} + 2\times\frac{1}{4} = \frac{1}{4}\left(e^{j2\omega} + 2 + e^{-j2\omega}\right).$$

$$x(n) = \frac{1}{2\pi}\int_{2\pi} X(\omega)\, e^{j\omega n}\, d\omega.$$

$$= \frac{1}{2\pi}\int_{2\pi}\frac{1}{4}\left(e^{j(\omega+2)n} + 2 + e^{j(\omega-2)n}\right) d\omega.$$

$$x(n) = \frac{1}{4}\left(\delta(n+2) + 2\delta(n) + \delta(n-2)\right).$$

(c) $$X(\omega) = \begin{cases} 1 & \omega_0 - \frac{\delta\omega}{2} \le |\omega| \le \omega_0 + \frac{\delta\omega}{2}. \\ 0 & \text{elsewhere.} \end{cases}$$

$$\Rightarrow x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega.$$

$$= \frac{1}{2\pi}\int_{\omega_0 - \frac{\delta\omega}{2}}^{\omega_0 + \frac{\delta\omega}{2}} e^{j\omega n}\, d\omega.$$

$$= \frac{1}{2\pi}\left[\frac{e^{j\omega n}}{jn}\right]_{\omega_0 - \frac{\delta\omega}{2}}^{\omega_0 + \frac{\delta\omega}{2}}.$$

$$= \frac{1}{2\pi}\times\frac{1}{jn}\left[e^{j(\omega_0 + \frac{\delta\omega}{2})n} - e^{j(\omega_0 - \frac{\delta\omega}{2})n}\right]$$

$$= \frac{1}{(2j)\pi n}\left[e^{j\omega_0 n} \cdot e^{+j\omega_0 n} + e^{\frac{j\delta\omega n}{2}} - e^{-\frac{j\delta\omega n}{2}}\right].$$

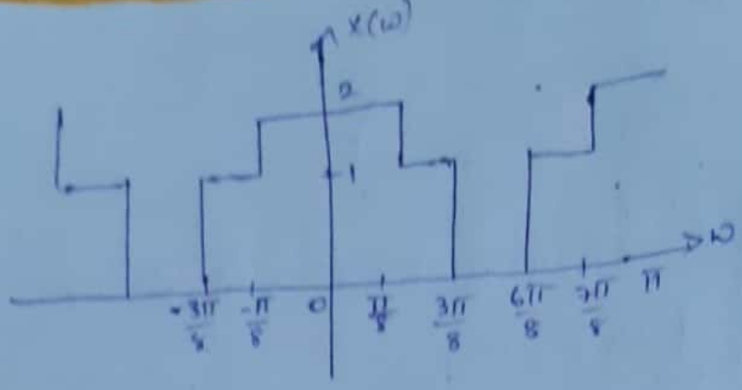
$$= \frac{1}{(2j)\pi n}\left[e^{\frac{j\delta\omega n}{2}} - e^{-\frac{j\delta\omega n}{2}}\right] e^{j\omega_0 n}.$$

$$= \frac{1}{\pi n}\left[\sin\left(\frac{\delta\omega n}{2}\right)\right] e^{j\omega_0 n}$$

$$= \frac{2}{2\pi}\delta\omega\left(\frac{\sin(n\delta\omega/2)}{n\delta\omega/2}\right) e^{jn\omega_0}.$$

$$= \frac{1}{\pi}\delta\omega\; Sa\left(n\delta\omega/2\right) e^{jn\omega_0}.$$

(d)



$$x(n) = \frac{1}{2\pi}\int_{2\pi} X(\omega) e^{j\omega n} d\omega$$

$$= \frac{1}{2\pi} Re\left\{ \int_0^{\pi/8} 2e^{j\omega n} + \int_{\pi/8}^{3\pi/8} e^{j\omega n} d\omega + \int_{6\pi/8}^{7\pi/8} e^{j\omega n} + \int_{7\pi/8}^{\pi} 2e^{j\omega n} \right\}$$

$$= 2 \times \frac{1}{2\pi}\left[\int_0^{\pi/8} 2\cos(\omega n) + \int_{\pi/8}^{3\pi/8} \cos(\omega n) d\omega + \int_{6\pi/8}^{7\pi/8} \cos(\omega n) d\omega + \int_{7\pi/8}^{\pi} 2\cos(\omega n) d\omega \right]$$

$$= \frac{1}{\pi n}\left[-2\sin\omega n \Big]_0^{\pi/8} - \sin\omega n \Big]_{\pi/8}^{3\pi/8} + -\sin\omega n \Big]_{6\pi/8}^{7\pi/8} + -2\sin\omega n \Big]_{7\pi/8}^{\pi} \right.$$

$$= \frac{1}{n\pi}\left[-2\sin\left(\frac{\pi}{8}n\right) - \sin\left(\frac{3\pi}{8}n\right) + \sin\left(\frac{\pi}{8}n\right) - \sin\left(\frac{7\pi}{8}n\right) + \sin\left(\frac{6\pi}{8}n\right) - \overset{0}{2\sin(\pi n)} + 2\sin\left(\frac{7\pi}{8}n\right) \right]$$

$$= \frac{1}{n\pi}\left[-\sin\left(\frac{\pi}{8}n\right) - \sin\left(\frac{3\pi n}{8}\right) + \sin\left(\frac{7\pi}{8}n\right) + \sin\left(\frac{6\pi n}{8}\right) \right]$$

4.11) consider the signal

$$x(n) = \{1, 0, 1, -1, 2, 3\}$$

with fourier transform $X(\omega) = X_R(\omega) + jX_I(\omega)$.
Determine and sketch the signal $y(n)$ with fourier transform.

$$Y(\omega) = X_I(\omega) + X_R(\omega) e^{j2\omega}.$$

A:- $x(n) = \{1,0,-1,2,3\}$

$x_e(n) = \frac{x(n)+x(-n)}{2} = \{\frac{1}{2}, 0, 1, \underset{\uparrow}{2}, 1, 0, \frac{1}{2}\}$

when $n=0$, $x_e[0] = \frac{2+2}{2} = \frac{4}{2} = 2.$

$n=1$, $x_e[1] = \frac{3-1}{2} = \frac{2}{2} = 1.$

$n=2$, $x_e[2] = \frac{0+0}{2} = 0$

$n=3$, $x_e[3] = \frac{0+1}{2} = \frac{1}{2}$

$n=-1$, $x_e[-1] = \frac{3-1}{2} = \frac{2}{2} = 1$

$n=-2$, $x_e[-2] = 0$

$n=-3$, $x_e[-3] = \frac{1}{2}$

Similarly we get $x_o(n) = \frac{x(n)-x(-n)}{2}$.

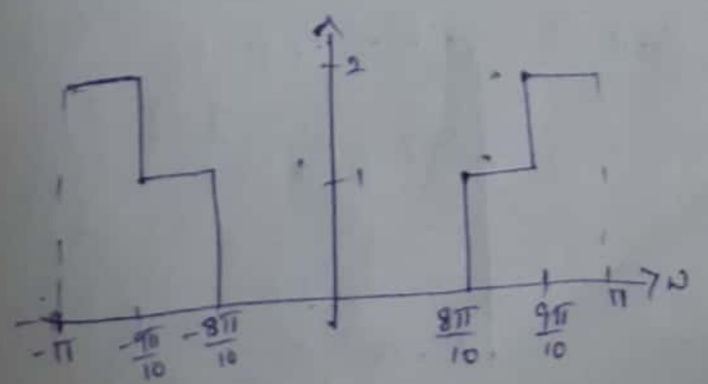
$x_o(n) = \{\frac{1}{2}, 0, 2, 0, \frac{1}{2}\}$

$$X_R(\omega) = \sum_{n=-3}^{3} x_e(n) e^{-j\omega n}$$

$$j X_I(\omega) = \sum_{n=-3}^{3} x_o(n) e^{-j\omega n}.$$

$X_R(\omega) = \{\frac{1}{2} e^{j3\omega}, 0, e^{j\omega}, 2, e^{-j\omega}, 0, \frac{1}{2} e^{-j\omega 3}\}.$

4.12) Determine the signal $x(n)$ if the fourier transfo[rm] is given in



1:- $x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{-9\pi/8} 2e^{j\omega n}\,d\omega + \int_{-9\pi/10}^{-8\pi/10} e^{j\omega n}\,d\omega + \int_{8\pi/10}^{9\pi/10} e^{j\omega n}\,d\omega + \int_{9\pi/10}^{\pi} 2e^{j\omega n}\right]$$

$$= \frac{1}{2\pi}\left[\left[2\frac{e^{jn\omega}}{jn}\right]_{-\pi}^{+\frac{9\pi}{10}} + \left[\frac{e^{jn\omega}}{jn}\right]_{-\frac{9\pi}{10}}^{-\frac{8\pi}{10}} + \left[\frac{e^{jn\omega}}{jn}\right]_{\frac{8\pi}{10}}^{9\pi/10} + \left[2\frac{e^{jn\omega}}{jn}\right]_{\frac{9\pi}{10}}^{\pi}\right]$$

$$= \frac{1}{j2\pi n}\left[2\left(e^{-j\frac{9\pi n}{10}} - e^{-j\pi n}\right) + \left(e^{-j\frac{8\pi n}{10}} - e^{-j\frac{n9\pi}{10}}\right) + e^{+j\frac{9\pi n}{10}} - e^{j\frac{8\pi n}{10}} + 2\left(e^{jn\pi} - e^{j\frac{9\pi n}{10}}\right)\right]$$

$$\cancel{= \frac{1}{j2\pi n}\left[2\left(e^{-j\frac{9\pi n}{10}} - 2e^{-j\pi n}\left(+e^{-j\frac{8\pi n}{10}}\right)\right)\right]}$$

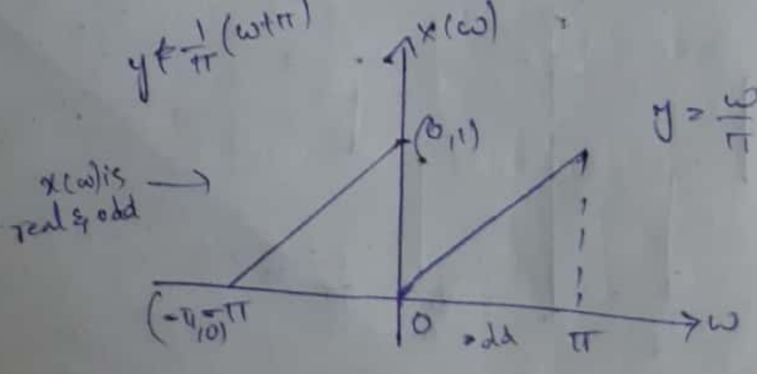
$$= \frac{1}{j2\pi n}\left[2e^{-j\frac{9\pi n}{10}} - 2e^{-j\pi n} + e^{-j\frac{8\pi n}{10}} - e^{-j\frac{n9\pi}{10}} + e^{j\frac{9\pi n}{10}} - e^{j\frac{8\pi n}{10}} + 2e^{jn\pi} - 2e^{j9\pi/10}\right]$$

$$= \frac{1}{j2\pi n}\left[e^{-j\frac{n9\pi}{10}} + 2\left(e^{jn\pi} - e^{-j\pi n}\right) + e^{-j\frac{\pi 8n}{10}} - e^{j\frac{\pi 8n}{10}} - e^{j\frac{n9\pi}{10}}\right]$$

$$= \frac{1}{n\pi}\left[-\sin\left(\frac{9\pi n}{10}\right) + 2\overset{0}{\sin}(n\pi) - \sin\left(\frac{8\pi n}{10}\right)\right]$$

$$= -\frac{1}{n\pi}\left[\sin\left(\frac{9\pi n}{10}\right) + \sin\left(\frac{4\pi n}{10}\right)\right].$$

(ii)



A:- $x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\omega)\, e^{j\omega n}\, d\omega$

$$\therefore x(n) = \frac{1}{2\pi}\left[\int_{-\pi}^{0}\left(\frac{\omega}{\pi}+1\right) e^{j\omega n}\, d\omega + \int_{0}^{\pi}\frac{\omega}{\pi} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\int_{-\pi}^{0}\frac{\omega}{\pi} e^{j\omega n}\, d\omega + \int_{-\pi}^{0} e^{j\omega n}\, d\omega + \int_{0}^{\pi}\frac{\omega}{\pi} e^{j\omega n}\, d\omega\right]$$

$$= \frac{1}{2\pi}\left[\frac{1}{\pi}\int_{-\pi}^{\pi}\omega e^{j\omega n}\, d\omega + \int_{-\pi}^{0} e^{j\omega n}\right]$$

$$\frac{1}{\pi}\int_{-\pi}^{\pi}\omega e^{j\omega n} = \frac{1}{\pi}\left(\left[\frac{\omega e^{j\omega n}}{jn}\right]_{-\pi}^{\pi} - \int_{-\pi}^{\pi}\frac{e^{j\omega n}}{jn}\, d\omega\right)$$

$$= \frac{1}{\pi}\left[\frac{1}{jn}\left(\pi e^{jn\pi} + \pi e^{-jn\pi}\right) - \frac{1}{jn}\left.\frac{e^{j\omega n}}{jn}\right]_{-\pi}^{\pi}\right]$$

$$= \frac{1}{\pi}\left[\frac{\pi\left(e^{jn\pi}+e^{-jn\pi}\right)}{jn} + \frac{1}{n^2}\left(e^{jn\pi}-e^{-jn\pi}\right)\right]$$

$$= \frac{2}{n}\left(\frac{e^{jn\pi}+e^{-jn\pi}}{2j}\right) + \frac{2j}{n^2}\left(\frac{e^{jn\pi}-e^{-jn\pi}}{2j}\right)$$

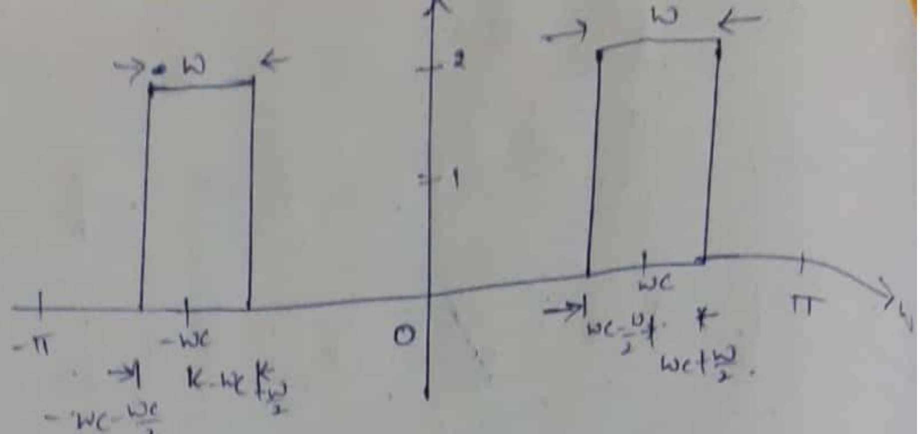
$$= \frac{2}{jn}\cos(n\pi) + \underbrace{\frac{2j}{n^2}\sin n\pi}_{=0}$$

$$\int_{-\pi}^{0} e^{j\omega n} = \left.\frac{e^{j\omega n}}{jn}\right]_{-\pi}^{0} = \frac{1}{jn}\left[1-e^{-jn\pi}\right] \times \frac{e^{j\pi/2\, n}}{e^{j\pi/2\, n}}$$

$$= \frac{1}{2jn}\left[e^{j\pi/2\, n} - e^{-jn\pi/2}\right] e^{-j\pi/2\, n} = \frac{2}{n}\sin\left(\frac{\pi}{2}n\right) e^{-j\pi/2\, n}$$

$$\Rightarrow = \frac{1}{2\pi}\int_{-\pi}^{0} e^{j\omega n} = \frac{1}{\pi n}\sin\left(\frac{\pi}{2}n\right) e^{-j\pi/2\, n}$$

(ii)



$$x(n) = \frac{1}{2\pi}\int_{-\pi}^{\pi} x(n)\, e^{jwn}.dw$$

$$= \frac{1}{2\pi}\left[\int_{-w_c-\frac{w}{2}}^{-w_c+\frac{w}{2}} 2e^{jwn}.dw + \int_{w_c-\frac{w}{2}}^{w_c+\frac{w}{2}} 2e^{jwn}\,dw\right]$$

$$= \frac{1}{\pi}\left[\left[\frac{e^{jnw}}{jn}\right]_{-w_c-\frac{w}{2}}^{-w_c+\frac{w}{2}} + \left[\frac{e^{jwn}}{jn}\right]_{w_c-\frac{w}{2}}^{w_c+\frac{w}{2}}\right]$$

$$= \frac{1}{j\pi n}\left[e^{jn(-w_c+\frac{w}{2})} - e^{jn(-w_c-\frac{w}{2})} + e^{jn(w_c+\frac{w}{2})} - e^{jn(w_c}\right.$$

$$= \frac{1}{j\pi n}\left[e^{-jnw_c}\left[e^{jnw/2} - e^{-jnw/2}\right] + e^{jnw_c}\left[e^{jnw/2} - e^{-jnw}\right.\right.$$

$$= \frac{1}{j\pi n}\left[\left[e^{-jnw_c} + e^{jnw_c}\right]\left[e^{jnw/2} - e^{-jnw/2}\right]\right]$$

$$\therefore x(w) = \frac{4}{\pi n}\left[\cos w_c \;\; \sin\left(\frac{wn}{2}\right)\right]$$

4.13) Given the fourier transform of the signal.

$$x(n) = \begin{cases} 1 & ; -M \le n \le M \\ 0 & ; \text{otherwise} \end{cases}$$ was shown to be.

$x(w) = 1 + 2\sum_{n=1}^{M}\cos(wn)$ then show that fourier transform ~~fouri~~ of

$$a_1(n) = \begin{cases} 1 & ; 0 \le n \le M \\ 0 & ; \text{otherwise} \end{cases}$$ is $x_1(w) = \dfrac{1-e^{-jw(n+1)}}{1-e^{-jw}}$

$$a_2(n) = \begin{cases} 1 & ; -M \le n \le -1 \\ 0 & ; \text{otherwise} \end{cases}$$ is $x_2(w) = \dfrac{e^{jw} - e^{jw(n+1)}}{1-e^{jw}}$.

A:- $X_1(\omega) = \sum_{n=-\infty}^{\infty} x_1(n) e^{-j\omega n}$

$$= \sum_{n=0}^{M} e^{-j\omega n} = \sum_{n=0}^{M} (e^{-j\omega})^n = \frac{1-(e^{-j\omega})^{M+1}}{1-e^{-j\omega}}$$

$$\because \left[\sum_{n=0}^{N} a^n = \frac{1-a^{N+1}}{1-a}\right]$$

$$X_2(\omega) = \sum_{n=-M}^{-1} x_2[n] e^{-j\omega n}$$

$$= e^{j\omega} + e^{2j\omega} + \dots + e^{j\omega M}$$

$$= e^{j\omega}\left[1 + e^{j\omega} + \dots e^{j\omega M-1}\right].$$

$$= e^{j\omega} \sum_{n=0}^{M-1} e^{j\omega n} = e^{j\omega}\left[\frac{1-(e^{j\omega})^M}{1-e^{j\omega}}\right]$$

$$= e^{j\omega}\left[\frac{1-e^{j\omega M}}{1-e^{j\omega}}\right]$$

$\therefore X_1(\omega) + X_2(\omega) = X(\omega)$

$$= \frac{1-e^{-j\omega(M+1)}}{1-e^{-j\omega}} + \frac{1-e^{j\omega M}}{1-e^{j\omega}} e^{j\omega}$$

$$= \frac{(1-e^{-j\omega(M+1)})(1-e^{j\omega}) + e^{j\omega}(1-e^{j\omega M})(1-e^{-j\omega})}{1-e^{j\omega}-e^{-j\omega}+1 = 2-e^{j\omega}-e^{-j\omega}}$$

$$= 1-e^{j\omega}-e^{-j\omega(M+1)}+e^{-j\omega M} + e^{j\omega}\left[1-e^{-j\omega}-e^{j\omega M}+e^{j\omega(M-1)}\right]$$

$$= \cancel{1} - \cancel{e^{j\omega}} - e^{-j\omega(M+1)} + e^{-j\omega M} + \cancel{e^{j\omega}} - \cancel{1} - e^{j\omega(M+1)} + e^{j\omega M}$$

$$= \frac{-\left[e^{-j\omega(M+1)} + e^{j\omega(M+1)}\right] + (e^{j\omega M} + e^{-j\omega M})}{2-e^{j\omega}-e^{-j\omega}}$$

$$= \frac{-\left[e^{-j\omega(M+1)} + e^{j\omega(M+1)}\right] + (e^{j\omega M} + e^{-j\omega M})}{2(1-\cos\omega)}$$

$$= \frac{2\cos(\omega M) - 2\cos(\omega(M+1))}{2(1-\cos\omega)}$$

$$= \frac{\cancel{2} \sin(\omega M + \frac{\omega}{2}) (\cos\frac{\omega}{2})}{\cancel{2} \sin^2 \frac{\omega}{2}}$$

$$= \frac{\sin(M+\frac{1}{2})\omega}{\sin(\frac{\omega}{2})}$$

4.14) Consider the signal.

$x(n) = \{-1, 2, -3, 2, -1\}$ with fourier transform

$X(\omega)$. Compute the following quantities, without explicitly computing $X(\omega)$.

(a) $X(0)$.

$$X(0) = \sum_{n=-\infty}^{\infty} x(n) = -1+2-3+2-1 = 1$$

(b) $\angle X(\omega)$

(c) $\int_{-\pi}^{\pi} X(\omega)\, d\omega$.

$$\int_{-\pi}^{\pi} X(\omega) \cdot d\omega = 2\pi X(0) = -6\pi.$$

(d) $X(\pi)$.

$$X(\pi) = \sum_{n=-\infty}^{\infty} x(n) e^{-jn\pi}$$

$$= \sum_{n=-2}^{2} x(n) e^{-jn\pi}$$

$$= -e^{j\pi 2} + 2e^{j\pi} - 3 + 2e^{-j\pi} - e^{-j2\pi}$$

$$= -(e^{j2\pi} + e^{-j2\pi}) + 2(e^{j\pi} + e^{-j\pi}) - 3$$

$$= -2\cos(2\pi) + 2\cos(\pi) - 3 = -2-2-3 = -7.$$

(c) $\int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$

$$\int_{-\pi}^{\pi} |X(\omega)|^2 \, d\omega = \sum_{n=-\infty}^{\infty} |x(n)|^2.$$

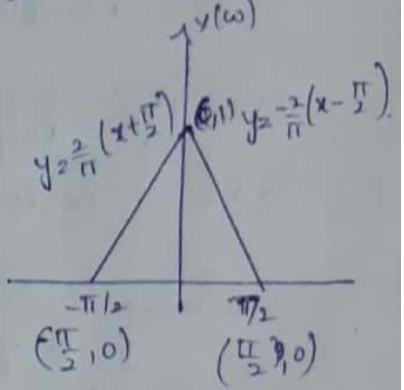
↳ according parsvel's energy theorem.

$= 1+4+9+4+1 = \underline{\underline{19}}.$

4.15) The center of gravity of a signal $x(n)$ is defined as $c = \dfrac{\sum_{n=-\infty}^{\infty} n\,x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$ and provides a measure of the "time delay" of the signal.

(a) Express 'c' in terms of $X(\omega)$

(b) compute the 'c' for the signal $x(n)$ whose fourier transform is shown in figure.



A:- Given $c = \dfrac{\sum_{n=-\infty}^{\infty} n\,x(n)}{\sum_{n=-\infty}^{\infty} x(n)}$.

We know that $X(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega n}$.

$$\therefore X(0) = \sum_{n=-\infty}^{\infty} x(n) \rightarrow (1)$$

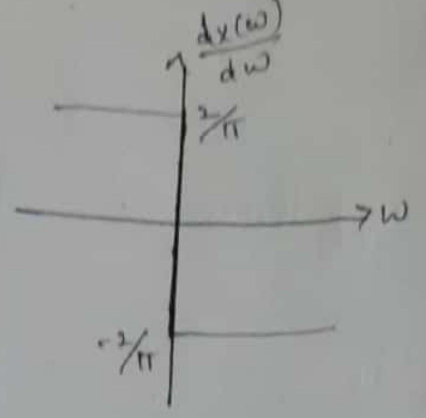
$$\frac{j\,dX(\omega)}{d\omega} = \sum_{n=-\infty}^{\infty} n\,x(n)\, e^{-j\omega n}.$$

$$\Rightarrow \left.\frac{dX(\omega)}{d\omega}\right|_{\omega=0} = -j \sum_{n=-\infty}^{\infty} n\,x(n).$$

$$\therefore c = \frac{\left.\frac{j\,dX(\omega)}{d\omega}\right|_{\omega=0}}{X(0)}$$

(b) $x(0) = 1$.

$$\therefore \left.\frac{dx(\omega)}{d\omega}\right|_{\omega=0} = \frac{2}{\pi} - \frac{2}{\pi}$$



$$\therefore c = \frac{\left.\frac{j\,dx(\omega)}{d\omega}\right|_{\omega=0}}{x(0)} = \frac{0}{1} = 0.$$

4.16) Consider the fourier transform pair

$$a^n u(n) \xrightarrow{F.T} \frac{1}{1-ae^{-j\omega}}, \quad |a| < 1.$$

use the differentiation in frequency theorem and induction to show that,

$$x(n) = \frac{(n+l-1)!}{n!(l-1)!} a^n u(n) \xleftrightarrow{F.T} x(\omega) = \frac{1}{(1-ae^{-j\omega})}.$$

A:- $x_1(n) = a^n u(n) \xrightarrow{F.T} \frac{1}{1-ae^{-j\omega}}$

$$x_{1e}(n) = \frac{(n+l-1)!}{n!(l-1)!} a^n u(n) \xleftrightarrow{F.T} \frac{1}{(1-ae^{-j\omega})}$$

let $l = k+1$

$$\Rightarrow x(n) = \frac{(n+k+1-1)!}{n!(k+1-1)!} a^n u(n) = \frac{(n+k)!}{n!\,k!} a^n . u(n).$$

$$= \frac{(n+k)(n+k-1)!}{n!\,k(k-1)!} a^n u(n) = \frac{n+k}{k} \frac{(n+k-1)!}{n!(k-1)!} a^n u(n)$$

$$= \frac{n+k}{k} x_k(n).$$

$$\therefore X_{k+1}(\omega) = \frac{1}{k}\sum_{n=-\infty}^{\infty} n x_k[n] e^{-j\omega n} + \sum_{n=-\infty}^{\infty} x_k[n] e^{-j\omega n}.$$

$$= \frac{1}{k}\, j\frac{d X_k(\omega)}{d\omega} + X_k(\omega)$$

$$= \frac{j}{k}\frac{d}{d\omega}\left(\frac{1}{(1-ae^{-j\omega})^k}\right) + \frac{1}{(1-ae^{-j\omega})^k}$$

$$= \frac{j}{k}\, k\,(1-ae^{-j\omega})^{-k-1}\left(-ae^{-j\omega}(-1)\right) + \frac{1}{(1-ae^{-j\omega})^k}$$

$$= \frac{jae^{-j\omega}}{(1-ae^{-j\omega})^{k+1}} + \frac{1}{(1-ae^{-j\omega})^k}.$$

4.17) Let $x(n)$ be an arbitrary signal, not necessarily real valued, with fourier transform $X(\omega)$. Express the fourier transform of following in terms of $X(\omega)$.

(a) $x^*[n]$.

We know that $X(\omega) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$

$$\therefore \sum_{n=-\infty}^{\infty} x^*[n] e^{-j\omega n} = \sum_{n=-\infty}^{\infty} [x[n] e^{j\omega n}]^*$$

$$= \left[\sum_{n=-\infty}^{\infty} x[n] e^{-j(-\omega) n}\right]^* = [X(-\omega)]^* = X^*(-\omega)$$

(b) $x^*[-n]$

$$\sum_{n=-\infty}^{\infty} x^*[-n] e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x^*[n] e^{j\omega n}$$

$$= \left[\sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}\right]^* = [X(\omega)]^* = X^*(\omega).$$

(c) $y[n] = x[n] - x[n-1]$

$$x[n] \longrightarrow x(\omega)$$

$$x[n-1] \longrightarrow e^{-j\omega} x(\omega).$$

$\therefore y[n] = x[n] - x[n-1] \longrightarrow x(\omega) - e^{-j\omega} x(\omega).$

$\therefore y(\omega) = (1 - e^{-j\omega})\, x(\omega).$

(d) $y(n) = \sum_{k=-\infty}^{\infty} x(k).$

$y(n) = \sum_{k=-\infty}^{\infty} x(k) = y(n) - y(n-1)$

$y(n) = x(-\infty) + \dots + x(n)$

$y(n-1) = x(-\infty) + \dots\dots + x(n-1)$

$\therefore y(n) - y(n-1) = x[n]$

$\therefore x(\omega) = (1 - e^{-j\omega})\cancel{x}(\omega)$

$\therefore Y(\omega) = \frac{X(\omega)}{1 - e^{-j\omega}}.$

(e) $y(n) = x(2n)$

$y(n) = x(2n)$

$\therefore y(\omega) = \sum_{n=-\infty}^{\infty} x[2n] e^{-j\omega n}$

Let $m = n/2$ when $n = -\infty, m = -\infty$

$n = \infty \Rightarrow m = \infty.$

$\therefore Y(\omega) = \sum_{m=-\infty}^{\infty} x(m) e^{-j\frac{\omega}{2}m} = \sum_{n=-\infty}^{\infty} x(n) e^{-j\omega\left(\frac{n}{2}\right)}$

$= \sum_{n=-\infty}^{\infty} x(n) e^{-j\left(\frac{\omega}{2}\right)n} = X\left(\frac{\omega}{2}\right).$

(f) $y(n) = \begin{cases} x(\frac{n}{2}) & n\text{-even} \\ 0 & n\text{-odd} \end{cases}$

$$y(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x(\tfrac{n}{2})\, e^{-j\omega n}$$

replacing n by 2n we get.

$$y(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega(2n)} = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j(2\omega)n} = x(2\omega).$$

4.18) Determine and sketch the fourier transform $X_1(\omega), X_2(\omega)$ & $X_3(\omega)$ of following signals.

(a) $x_1(n) = \{1,1,1,1,1\}$

$x_1(n) = \delta(n+2) + \delta(n+1) + \delta(n) + \delta(n-1) + \delta(n-2)$.

$\delta(n) \longrightarrow 1$

$\delta(n-n_0) \longrightarrow e^{-j\omega n_0}(1)$.

$\therefore X_1(\omega) = e^{j2\omega} + e^{j\omega} + 1 + e^{-j\omega} + e^{-j2\omega} = 1 + 2\cos(\omega) + 2\cos(2\omega)$.

(b) $x_2(n) = \{1,0,1,0,1,0,1,0,1\}$.

$x_2(n) = x_1(\frac{n}{2}) = \delta(n+4) + \delta(n+2) + \delta(n) + \delta(n-2) + \delta(n-4)$.

$\downarrow FT$

$X_1(2\omega)$

$\therefore X_2(\omega) = e^{j4\omega} + e^{j2\omega} + 1 + e^{-j4\omega} + e^{-j2\omega}$

$= 1 + 2\cos(2\omega) + 2\cos(4\omega)$.

(c) $x_3(n) = \{1,0,0,1,0,0,\underset{\uparrow}{1},0,0,1,0,0,1\}$

$x_3(n) = x_1(\frac{n}{3}) \rightleftharpoons X_1(3\omega)$

$\therefore X_3(\omega) = X_1(3\omega)$

$\therefore X_3(\omega) = e^{j6\omega} + e^{j3\omega} + 1 + e^{-j3\omega} + e^{-j6\omega}$

$= 1 + 2\cos(3\omega) + 2\cos(6\omega)$.

(d) Is there any relationship between $X_1(\omega)$, $X_2(\omega)$, $X_3(\omega)$? what is it's physical meaning?

Yes,

$X_2(\omega) = X_1(2\omega)$

$X_3(\omega) = X_1(3\omega)$

$\Rightarrow$ Expansion in one domain corresponds to compression in another domain and vice versa.

(e) Show that if

$$x_k(n) = \begin{cases} x\left(\frac{n}{k}\right) & \text{if } n/k \text{ integer.} \\ 0 & \text{otherwise.} \end{cases}$$

then $X_k(\omega) = X(\omega)$.

$$X_k(\omega) = \sum_{n=-\infty}^{\infty} x_k(n) e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{k}\right) e^{-j\omega n}$$

replacing n by m we get.

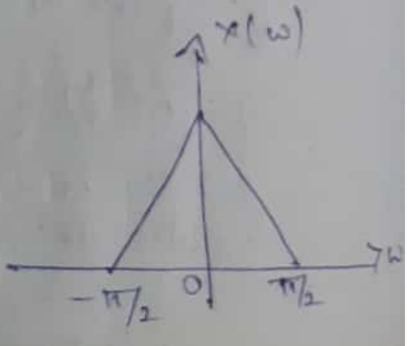
$$X_k(\omega) = \sum_{m=-\infty}^{\infty} x(n) e^{-j(k\omega)m} = \underline{\underline{X(k\omega)}}.$$

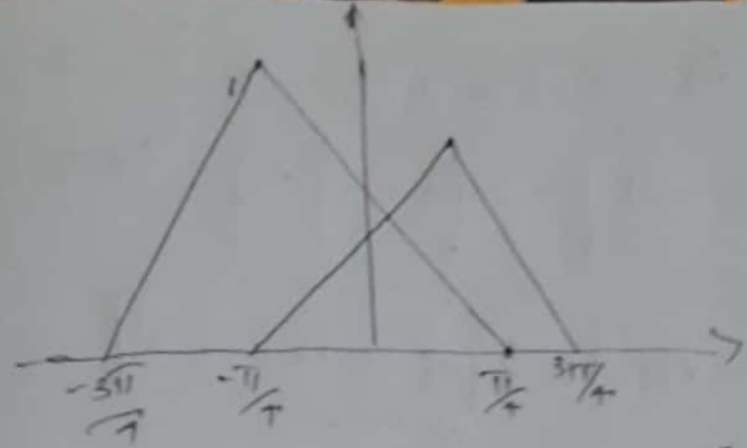
4.19) Let $x[n]$ be a signal with Fourier transform as shown in figure. Determine and sketch the fourier transform of following signals.

a) $x_1(n) = x(n)\cos\left(\frac{\pi n}{4}\right)$.

$$x_1(n) = x(n)\left[\frac{e^{j\frac{\pi}{4}n} + e^{-j\frac{\pi}{4}n}}{2}\right]$$

$$= \frac{1}{2} e^{\frac{j\pi n}{4}} x(n) + \frac{1}{2} e^{-j\pi/4\, n} x(n)$$





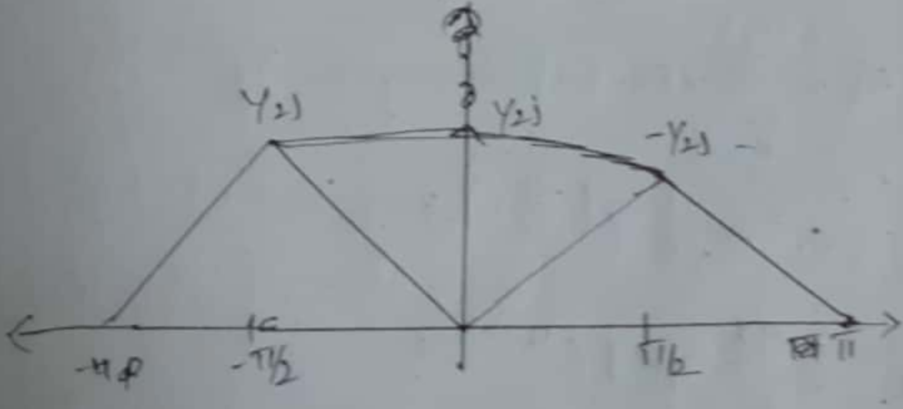
$$\therefore X_1(\omega) = \frac{1}{2}\left[X(\omega+\pi/4) + X(\omega-\pi/4)\right]$$

(b) $x_2(n) = x(n)\sin(\pi n/2)$.

$$x_2(n) = x(n)\left[\frac{e^{j\frac{\pi}{2}n} - e^{-j\pi/2\, n}}{2j}\right]$$

$$x_2(n) = \frac{1}{2j}e^{j\pi/2\, n}x(n) - \frac{1}{2j}e^{-j\pi/2\, n}x(n)$$

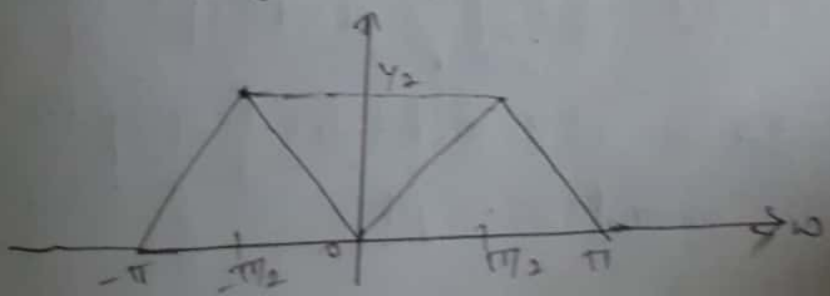
$$\therefore X_2(\omega) = \frac{1}{2j}\left[X(\omega+\pi/2) - X(\omega-\pi/2)\right]$$



(c) $x_3(n) = x(n)\cos(\pi n/2)$

$$x_3(n) = x(n)\left[\frac{e^{j\frac{\pi}{2}n} + e^{-j\frac{\pi}{2}n}}{2}\right] = \frac{1}{2}e^{j\frac{\pi}{2}n}x(n) + \frac{1}{2}e^{-j\frac{\pi}{2}n}x(n).$$

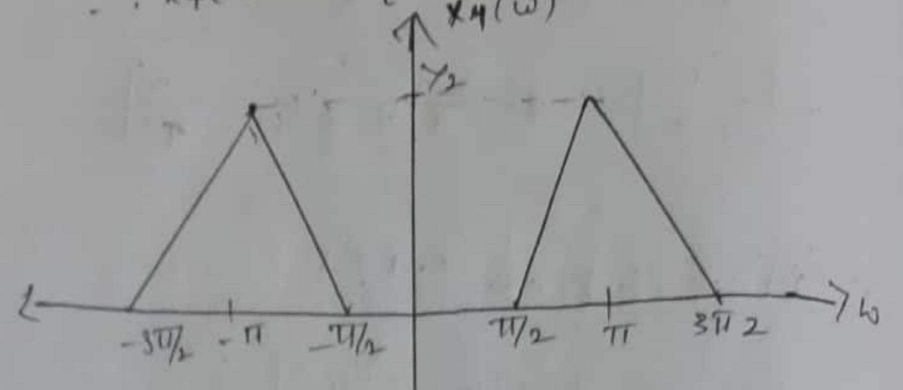
$$\therefore X_3(\omega) = \frac{1}{2}\left(X(\omega+\pi/2) + X(\omega-\pi/2)\right).$$



(d) $x_4(n) = x(n)\cos(\pi n)$.

$x_4(n) = x(n)\left[\frac{e^{j\pi n}+e^{-j\pi n}}{2}\right] = \frac{1}{2}e^{j\pi n}x(n)+\frac{1}{2}e^{-j\pi n}$

$\therefore X_4(\omega) = \frac{1}{2}\left[X(\omega+\pi)+X(\omega-\pi)\right]$.



4.20) Consider an aperiodic signal $x(n)$ with ~~frequency~~ fourier transform $X(\omega)$. Show that the fourier series coefficients C_k^y of periodic signal

$y(n] = \sum_{l=-\infty}^{\infty} x(n-lN]$ are given by

$C_k^y = \frac{1}{N}X\left(\frac{2\pi}{N}k\right), k=0,1,\ldots N-1.$

A:- $C_k^y = \frac{1}{N}\sum_{n=0}^{N-1} y(n]\, e^{-j2\pi kn/N}$.

↳ for a discrete time periodic signal, the fourier series co-efficients are also periodic with period 'N'.

$\therefore C_k^y = \frac{1}{N}\sum_{n=0}^{N-1}\left[\sum_{l=-\infty}^{\infty} x(n-lN]\right] e^{-j2\pi kn/N}$.

let $m = n-lN$ and interchanging the summations we get,

$$C_k y = \frac{1}{N} \sum_{l=-\infty}^{\infty} \sum_{m=-lN}^{N-1-lN} x(m) e^{-j2\pi k(m+lN)/N}.$$

we can consider.

$$\sum_{l=-\infty}^{\infty} \sum_{m=-lN}^{N-1-lN} x(m) e^{-j\omega(m+lN)} = X(\omega).$$

$$\therefore C_{yk} = \frac{1}{N} X(\omega) = \frac{1}{N} X\left(\frac{2\pi k}{N}\right).$$

4.21) Prove that.

$$X_N(\omega) = \sum_{n=-N}^{N} \frac{\sin(\omega_c n)}{\pi n} e^{-j\omega n}$$ May be expressed as

$$X_N(\omega) = \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} \frac{\sin\left[(2N+1)(\omega-\theta/2)\right]}{\sin((\omega-\theta)/2)} d\theta.$$

Let $x_N(n) = \frac{\sin \omega_c n}{\pi n} = x[n]\,\omega[n], \; -N \leq n \leq N.$

where $x[n] = \frac{\sin \omega_c n}{\pi n}, \; -\infty \leq n \leq \infty.$

$$\omega[n] = \begin{cases} 1 & -N \leq n \leq N \\ 0 & \text{otherwise.} \end{cases}$$

$$\therefore X_N(\omega) = X(\omega) * W(\omega).$$

4.22) A signal $x(n)$ has the following fourier transform. $X(\omega) = \frac{1}{1-ae^{-j\omega}}$ determine the F.T of signals.

(a) $x[2n+1]$.

$$X(\omega) = \sum_{n=-\infty}^{\infty} x(2n+1)\, e^{-j\omega n}.$$

Let $m = 2n+1$ when $n = -\infty$ then $m = -\infty$
$n = \infty$ then $m = -\infty$

$$\therefore X(\omega) = \sum_{m=-\infty}^{\infty} x[m]\, e^{-j\omega\left(\frac{m-1}{2}\right)}.$$

$$= \sum_{m=-\infty}^{\infty} x(m)\, e^{-j\left(\frac{\omega}{2}\right)m} e^{j\omega/2} = e^{j\omega/2} \sum_{m=-\infty}^{\infty} x(m) e^{-j\left(\frac{\omega}{2}\right)m}$$

$$\therefore X_1(\omega) = e^{j\omega/2} X\left(\frac{\omega}{2}\right) = \frac{e^{j\omega/2}}{1-\alpha e^{-j\omega/2}}.$$

(b) $e^{j\pi/2 n} x(n+2)$.

$$X_2(\omega) = \sum_{n=-\infty}^{\infty} e^{j\pi/2 n} x(n+2) e^{-jn\omega}.$$

$$= \sum_{n=-\infty}^{\infty} x(n+2)\, e^{-j(\omega-\pi/2)n}$$

Let $m = n+2$.

$$\Rightarrow X_2(\omega) = \sum_{m=-\infty}^{\infty} x(m)\, e^{-j(\omega-\pi/2)(m-2)}.$$

$$= \sum_{m=-\infty}^{\infty} x(m)\, e^{-j(\omega-\pi/2)(m-2)}.$$

$$= e^{j2\omega} e^{j\pi} \sum_{m=-\infty}^{\infty} x(m)\, e^{-j(\omega-\pi/2)m} = e^{j2\omega} e^{j\pi} X(\omega-\pi/2)$$

$$= e^{j\pi} = \cos\pi + j\sin\pi = -1$$

$$\therefore X_2(\omega) = \frac{e^{j2\omega}}{1-ae^{-j(\omega-\pi/2)}} = \frac{e^{j2\omega}}{1-ae^{-j\omega}e^{j\pi/2}}$$

$$= \frac{e^{j2\omega}}{1-jae^{-j\omega}}$$

c) $x[-2n]$

$$x_3(\omega) = \sum_{n=-\infty}^{\infty} x[-2n]\, e^{-j\omega n}$$

Let $m = \frac{-n}{2}$ replacing we get.

$$x_3(\omega) = \sum_{n=-\infty}^{\infty} x(n)\, e^{-j\omega(\frac{-n}{2})} = \sum_{n=-\infty}^{\infty} x(n) e^{-j(\frac{\omega}{2})n}$$

$$X_3(\omega) = X\left(\frac{-\omega}{2}\right) = \frac{1}{1-ae^{j\omega/2}}$$

(d) $x(n) \cdot \cos(0.3\pi n)$

$$x_4(n) = x(n)\left[\frac{e^{j0.3\pi n} + e^{-j0.3\pi n}}{2}\right]$$

$$= \frac{1}{2} e^{j0.3\pi n} x(n) + \frac{1}{2} e^{-j0.3\pi n} x(n) = \frac{1}{2}\left(x(\omega+0.3\pi n) + x(\omega-0.3\pi n)\right)$$

$$\therefore X_4(\omega) = \frac{1}{2}\left[\frac{1}{1-ae^{-j(\omega+0.3\pi)}} + \frac{1}{1-ae^{-j(\omega-0.3\pi)}}\right]$$

(e) $x[n] * x(n-1)$

$x[n] \longrightarrow x(\omega)$, $x(n-1] \rightarrow e^{-j\omega} x(\omega)$

convolution in one domain corresponds to multiplication in other domain.

$$x_5(\omega) = e^{-j\omega} X^2(\omega) = \frac{e^{-j\omega}}{(1-ae^{-j\omega})^2}$$

(f) $x[n] * x(-n]$

$$x[n] * x(-n] \longrightarrow x(\omega)\, x(-\omega)$$

$$= \frac{1}{(1-ae^{-j\omega})(1-ae^{j\omega})} = \frac{1}{1+a^2-2a\cos(\omega)}$$

4.23) From a discrete time signal $x(n)$ with $X(\omega)$ is shown in figure. Determine and sketch the fourier transform of the following signals.

(a) $y_1(n) = \begin{cases} x(n) & n \text{ even} \\ 0 & n \text{ odd.} \end{cases}$

A:- $y_1[n] = \begin{cases} y_2(n/2) & , n \text{ even} \\ 0 & \text{odd} \end{cases}$ (or).

$= \begin{cases} y_3(2n) & , n \text{ even} \\ 0 & , n \text{ odd.} \end{cases}$

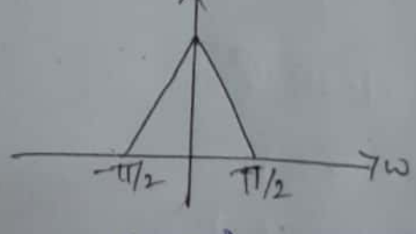
$\therefore Y_1(\omega) = Y_2(2\omega)$ (or) $Y_3(\omega/2)$

(b) $y_2[n] = x[2n]$

$Y_2(\omega) = \sum_{n=-\infty}^{\infty} x(2n) e^{-j\omega n}$

$n = \frac{n}{2}$ ← replace n by $n/2$.

$\therefore Y_2(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j\left(\frac{\omega}{2}\right)n} = X\left(\frac{\omega}{2}\right)$



(c) $y_3[n] = \begin{cases} x(n/2) & , n \text{ even} \\ 0 & , 'n' \text{ odd.} \end{cases}$

$Y_3(\omega) = \sum_{n=-\infty}^{\infty} x\left(\frac{n}{2}\right) e^{-j\omega n}$

Let replace n by $2n$ we get.

$Y_3(\omega) = \sum_{n=-\infty}^{\infty} x(n) e^{-j(2\omega)n} = X(2\omega)$.