Homework 1 - Mechanics of flexible structures and soft robots

Jessica Anz, October 23, 2024 (MAE 263F)

I. ASSIGNMENT 1

This assignment shall simulate the motion of three rigid spheres on an elastic beam falling inside a viscous fluid.

1) The shape of the structure at select times are given below. Plots of both the implicit and explicit simulation are provided. As seen in the plots, the implicit and explicit simulations yielded the same results. The structure undergoes minimal deformation and downward displacement in the first second of the simulation. By $t = 1.0\ s$, the middle node has a slight angle and is barely displaced below the x-axis. However, by $t = 10.0\ s$ the structure has fallen below $y = -0.04\ m$ and has a deeper angle in the middle node. Overall, the structure falls downward with time and curves to create an angle in the middle node. This behavior makes sense given that the simulation aims to model a beam falling inside viscous fluid.

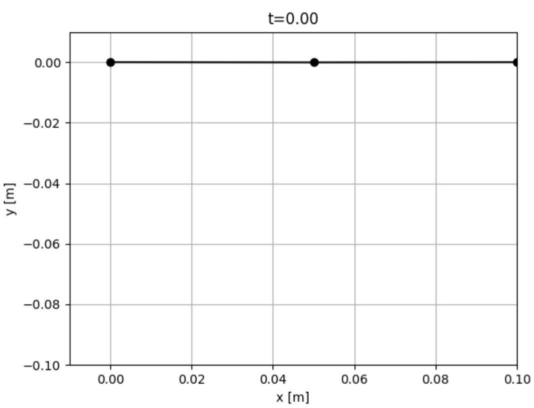

Fig. 1. Implicit Simulation of 3 Rigid Spheres at $t = 0.0\ s$

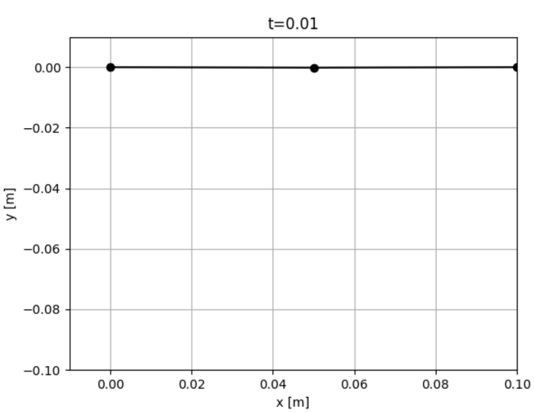

Fig. 2. Implicit Simulation of 3 Rigid Spheres at $t = 0.01\ s$

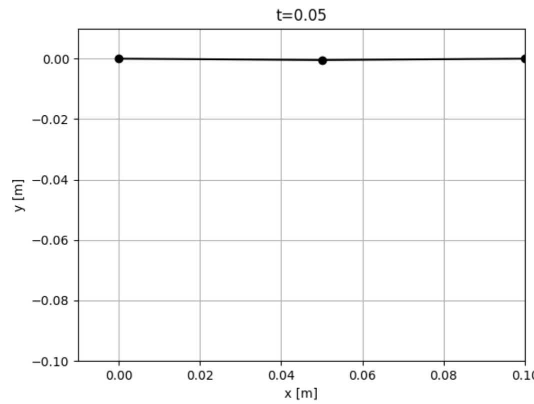

Fig. 3. Implicit Simulation of 3 Rigid Spheres at $t = 0.05\ s$

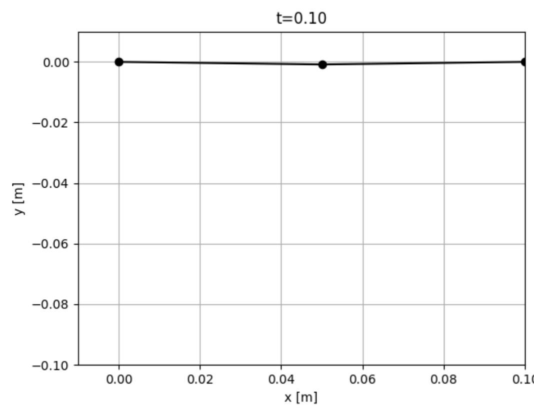

Fig. 4. Implicit Simulation of 3 Rigid Spheres at $t = 0.10\ s$

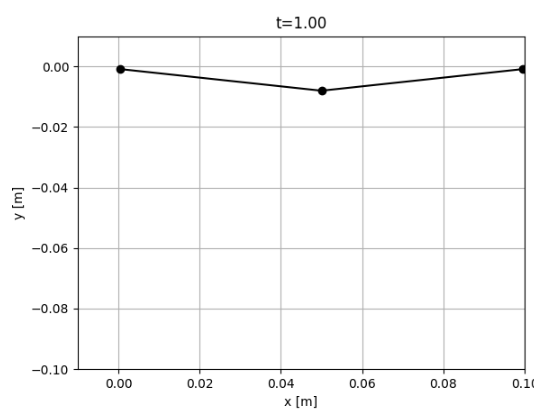

Fig. 5. Implicit Simulation of 3 Rigid Spheres at $t = 1.0\ s$

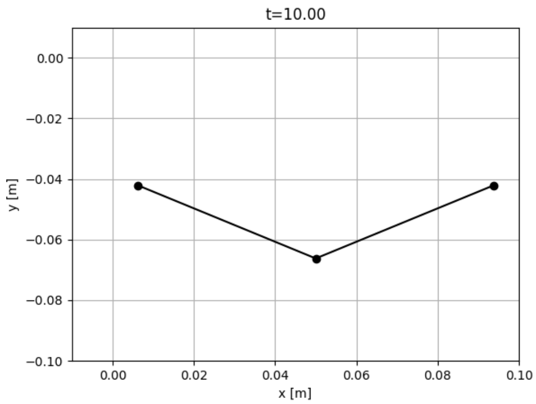

Fig. 6. Implicit Simulation of 3 Rigid Spheres at $t = 10.0$ s

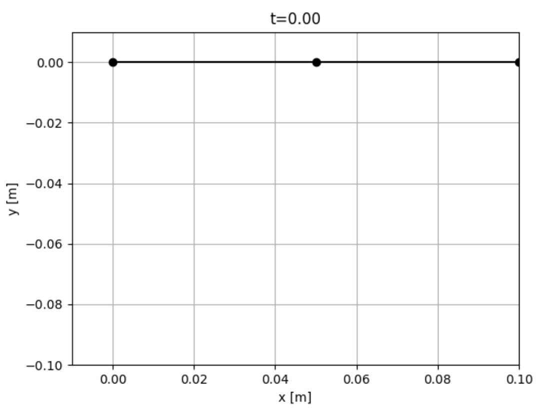

Fig. 7. Explicit Simulation of 3 Rigid Spheres at $t = 0.0$ s

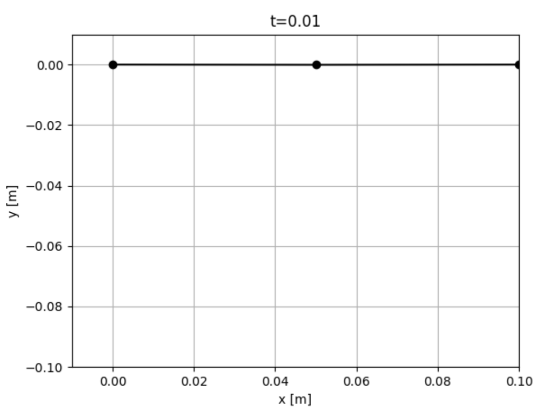

Fig. 8. Explicit Simulation of 3 Rigid Spheres at $t = 0.01$ s

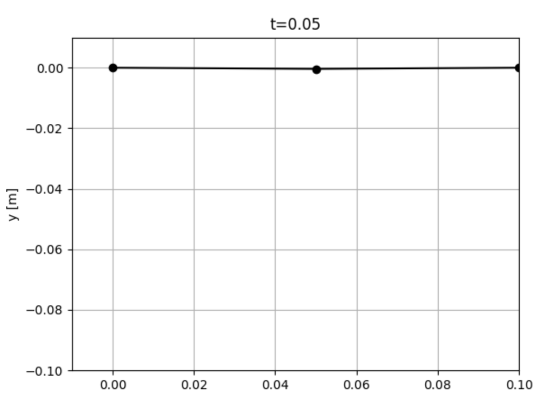

Fig. 9. Explicit Simulation of 3 Rigid Spheres at $t = 0.05$ s

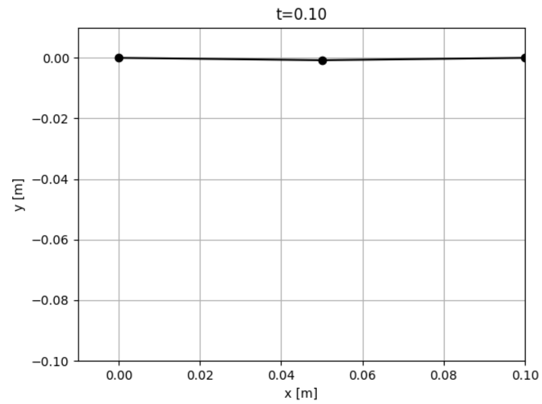

Fig. 10. Explicit Simulation of 3 Rigid Spheres at $t = 0.10$ s

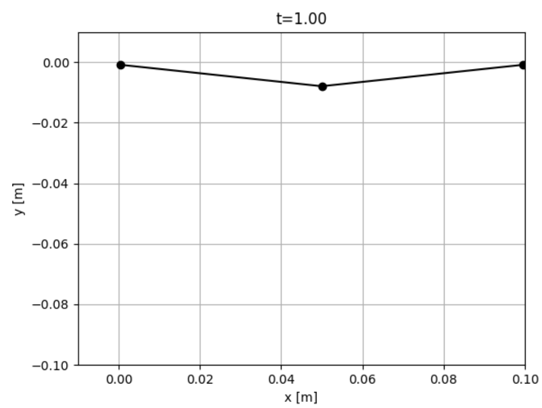

Fig. 11. Explicit Simulation of 3 Rigid Spheres at $t = 1.0$ s

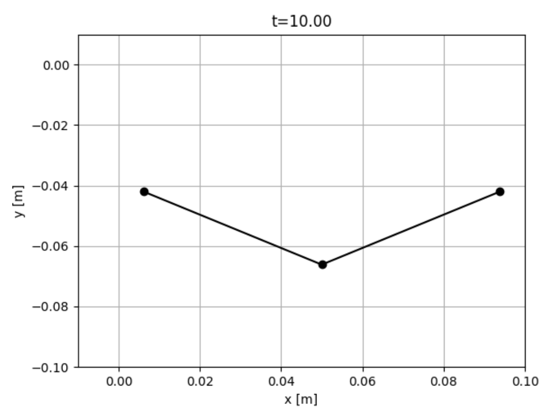

Fig. 12. Explicit Simulation of 3 Rigid Spheres at $t = 10.0$ s

Plots of the position and velocity of R_2 as a function of time are given below. The implicit and explicit results of these plots are the same. Based on the plots, the position of the middle node decreases over time with an approximately linear relationship. The velocity of the middle node starts at a negative value and slowly increases to a steady negative velocity. This steady velocity is known as the terminal velocity.

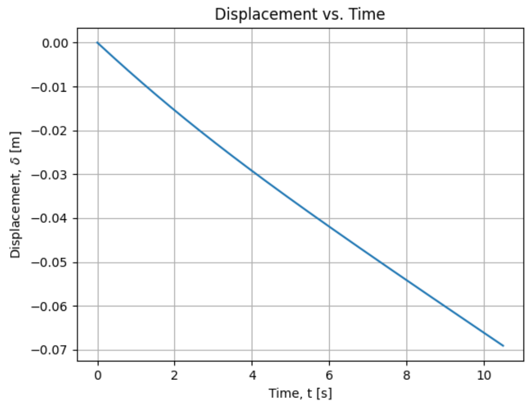

Fig. 13. Implicit Position of R_2 as a Function of Time

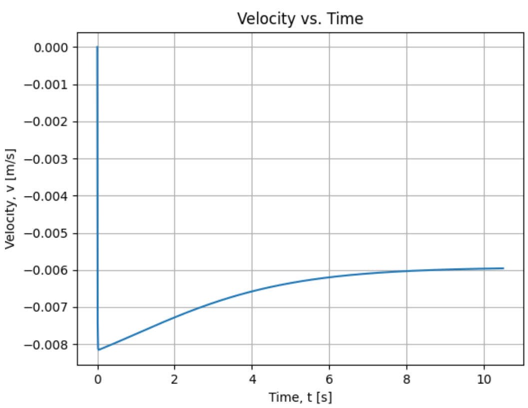

Fig. 14. Implicit Velocity of R_2 as a Function of Time

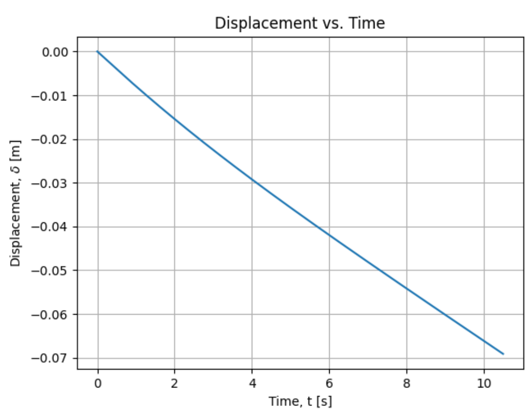

Fig. 15. Explicit Position of R_2 as a Function of Time

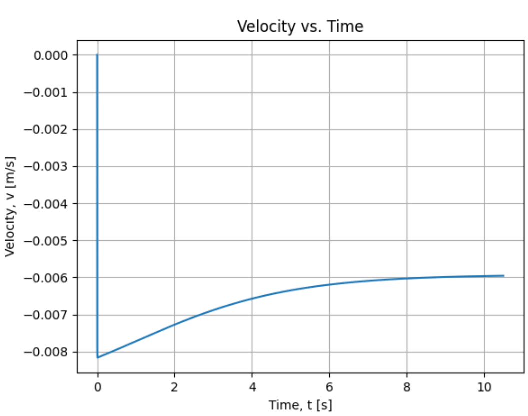

Fig. 16. Explicit Velocity of R_2 as a Function of Time

2) As seen in Figures 14 and 16, the terminal velocity of the middle node R_2 is $v_{terminal} = -0.00596\frac{m}{s}$.
3) If the radii of all nodes are the same, then the turning angle $= 0$ throughout the simulation. To test this, all nodes were set to have a radius of 0.005 m and the simulation was run to model the behavior. In the simulation, the beam remains flat as it slowly falls through the viscous fluid. This result aligns with intuition since if all the nodes are equally spaced with the same radii, then the mass is evenly distributed along the beam. With the mass balanced, there is no deformation at the center which was previously caused by the additional mass in R_2.
4) Testing with both the implicit and explicit versions of the simulation reveals benefits and drawbacks of both. Changing the step size in the explicit simulation to anything larger than $\Delta t = 1 \times 10^{-5}$ s prevented the simulation from running properly. With a $\Delta t = 1 \times 10^{-5}$ s, the explicit simulation took a long time to run, completing within 12 minutes. The implicit simulation was able to run much faster, completing within 10 seconds. The implicit simulation also remained fairly accurate at larger step sizes, which could allow it to run even faster. The explicit approach required a much smaller step size in order to run accurately, making it an inefficient option. However from a programming perspective, the explicit simulation was more simple to implement and understand. Overall, both simulation methods produced accurate results and each have their own benefits.

II. ASSIGNMENT 2

This assignment shall simulate the motion of N-connected spheres falling inside viscous fluid.

1) Plots of the vertical position and velocity of the middle node with respect to time are given below. With 21 nodes and a step size of $\Delta t = 1 \times 10^{-2}$ s, the terminal velocity is $-0.002211\frac{m}{s}$. Similarly to the 3 node simulation, the position of the middle node decreases over time almost linearly. The velocity also begins at a negative value, and then increases to a steady terminal velocity. However, in the 21 node simulation the terminal velocity is reached must faster than in the 3 node simulation.

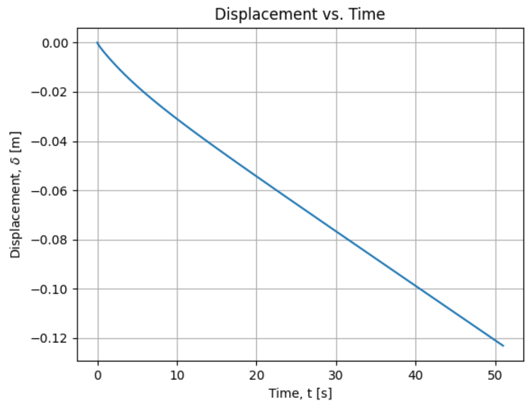

Fig. 17. Position of Middle Node as a Function of Time

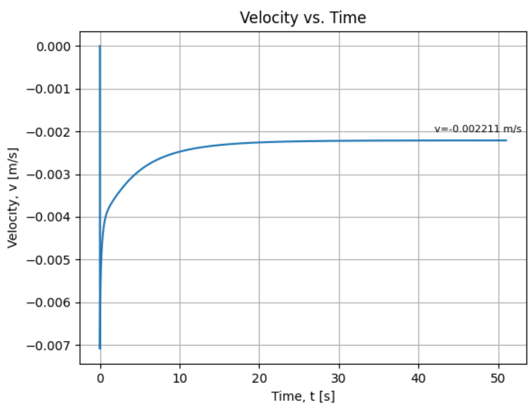

Fig. 18. Velocity of Middle Node as a Function of Time

2) The final deformed shape of the beam is given below. As seen in Figure 19, the deformed shape is a concave up parabola that has fallen below in the y-direction. These results are similar to the deformation in the 3-node simulation, but have more nodes allowing for a better discretized curve.

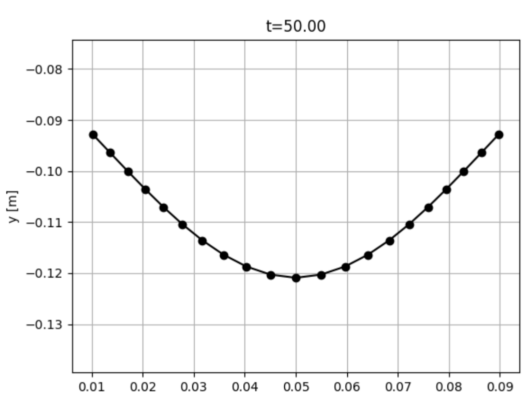

Fig. 19. Deformed Shape at $t = 50.0\ s$

3) For any discrete simulation to be accurate, the discrete steps should be small enough that the quantifiable metrics do not vary much when the step size is decreased. For this simulation, both time and the number of nodes are discretized. In order to evaluate if the spatial discretization (number of nodes) and temporal discretization (time step size) are accurate enough, plots of the terminal velocity vs. discrete steps can be analyzed to see when the terminal velocities converge. Plots of the terminal velocity vs. the number of nodes and terminal velocity vs. the time step size are given below.

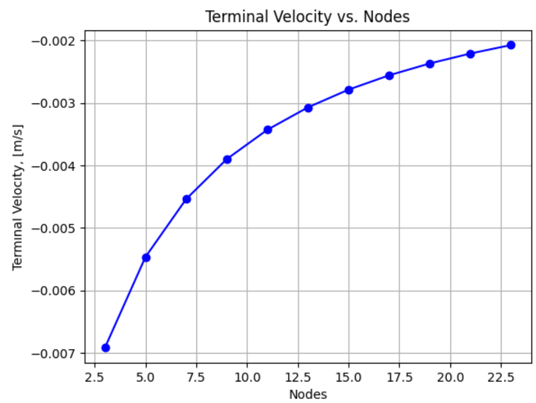

Fig. 20. Terminal Velocity vs. Nodes

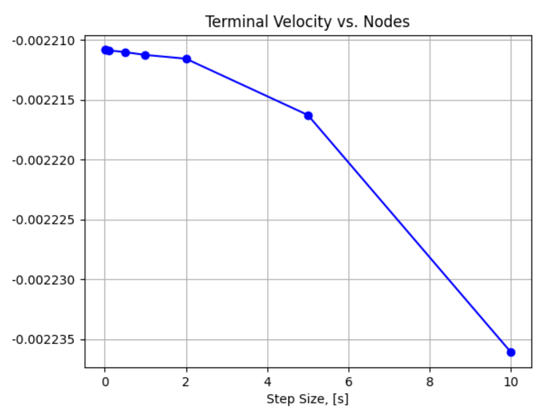

Fig. 21. Terminal Velocity vs. Step Size

As seen in Figure 20, at a small number of nodes the terminal velocity increases rapidly, and then slowly levels out as the number of nodes increases. The curve begins to reach a steady value between 21 and 23 nodes, meaning these values or larger are a good choice for the spatial discretization. In Figure 21, the terminal velocity is relatively constant for time steps less than $\Delta t = 0.1\ s$. After $\Delta t = 0.1\ s$, the terminal velocity drops in a downward curve. Therefore, any Δt less than $0.1\ s$ would be a good choice for the temporal discretization.

III. ASSIGNMENT 3

This assignment shall simulate the deformation of elastic beams and compare the results with Euler-Bernoulli beam theory.

1) The deformed shape of the beam with a load of 2,000 N is shown below. The maximum vertical beam displacement as a function of time is also plotted. As seen in Figure 23, the beam rapidly deforms within the first 0.1 seconds that the load is applied, and then has a very slight return to a steady level of deformation. This steady displacement of $y_{max} = -0.0371\ m$ remains constant through the rest of the simulation.

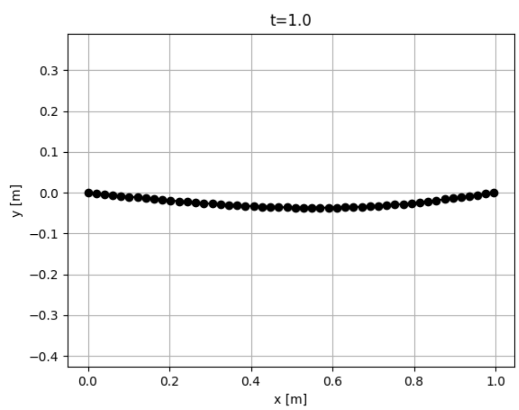

Fig. 22. Deformed Shape of Beam

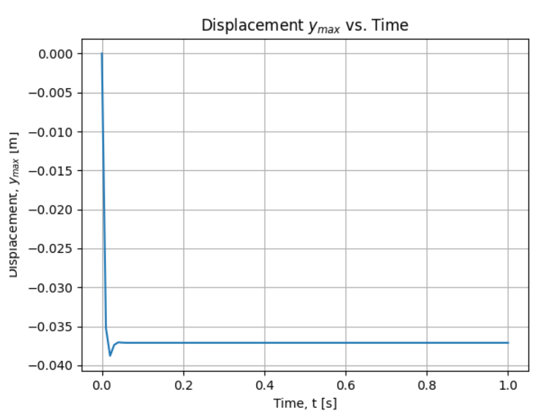

Fig. 23. Maximum Displacement vs. Time

According to Euler-Bernoulli beam theory, a simply supported beam with a single point load has a maximum displacement of

$$y_{max} = \frac{Pc(L^2 - c^2)^{1.5}}{9\sqrt{3}EIl} \quad \text{where } c = min(d, l-d)$$

For the simply-supported aluminum beam with a point load of 2,000 N applied at 0.75m from the left edge, beam theory predicts a maximum displacement of $y_{max} = -0.0380\ m$. For this small displacement, beam theory matches the results from the simulation with a percent error of only 2.37 %.

2) The results from Euler-Bernoulli beam theory are only accurate for small displacements, while the simulated model is accurate for large displacements. For example, when a point load of 20,000 N is applied instead of the 2,000 N load the results differ drastically. The simulated deformed shape of the beam and maximum vertical displacement vs. time are plotted below.

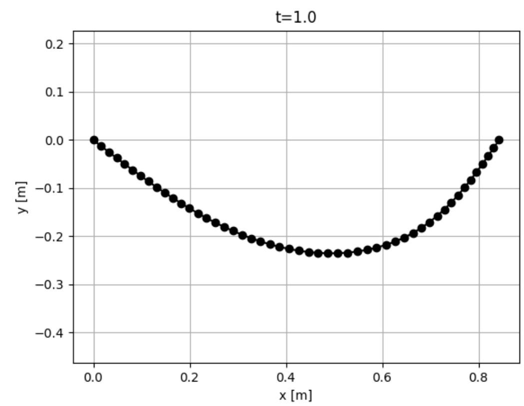

Fig. 24. Deformed Shape of Beam with 20,000 N Load

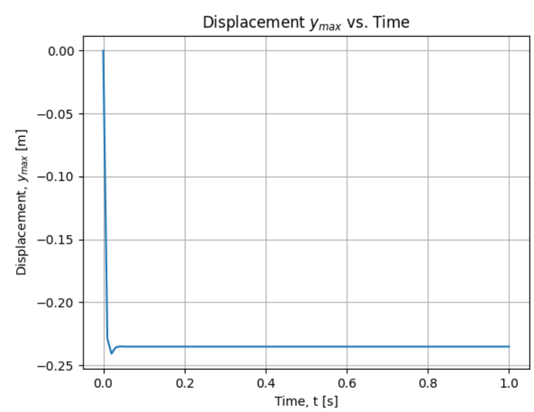

Fig. 25. Maximum Displacement vs. Time with 20,000 N Load

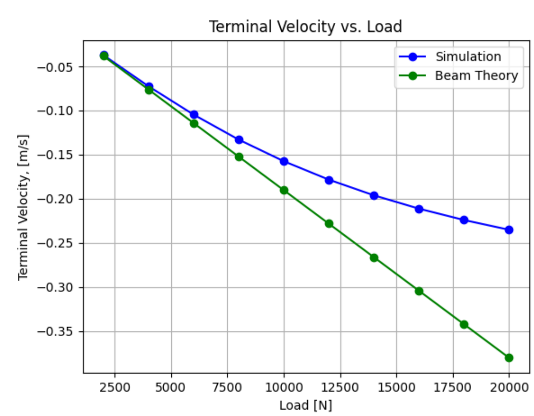

Fig. 26. Terminal Velocity vs. Load

As seen in Figure 25, the maximum vertical displacement from the simulation is $y_{max} = -0.235\ m$. By plugging values into the same theoretical equation from the 2,000 N load, a theoretical displacement of $y_{max} = -0.380\ m$ is obtained. These values are much father apart than the previous results, and yield a percent error of 518.4%. This error shows why beam theory is only valid for very small deformations, as the theoretical equation was far off from the simulated result. To find where the simulated and theoretical solutions diverge, the terminal velocity vs. load was plotted for various loads. As seen in Figure 26, at a load of 4,000 N, the simulated and theoretical solutions begin to diverge. Past 4,000 N it can be clearly seen that the simulation and beam theory produce different results. The accuracy of the simulated model for larger loads is an advantage over the inaccuracy of theoretical results.

REFERENCES

[1] Khalid Jawed, M, and Sangmin Lim. ”Discrete Simulation of Slender Structures”. *BruinLearn*, 2022.

[2] Khalid Jawed, M. ”Main_Falling_Beam_3_Nodes.ipynb”. *Google Colaboratory*, 2024.

[3] Khalid Jawed, M. ”Main_Falling_Beam_N_Nodes.ipynb”. *Google Colaboratory*, 2024.

[4] Khalid Jawed, M. ”Main_Falling_Beam_N_Nodes_Demo_Fixed_DOFs.ipynb”. *Google Colaboratory*, 2024.