Homework 3 - Mechanics of flexible structures and soft robots

Jessica Anz, December 10, 2024 (MAE 263F)

The following report will explore both linear and nonlinear fitting of a data set using gradient descent and backpropagation [1].

I. Linear Fit

In this section data will be fitted to the linear equation [2]:

$$y = mx + b$$

A. Data Comparison

Using the given hyperparameters of 10,000 epochs and a learning rate of 0.001, the data was fitted to the linear model. As seen in Figure 1, the linear model provides a reasonable but not perfect fit to the data. From visual inspection, some of the actual data points lie on the prediction line, while others are off by a small margin.

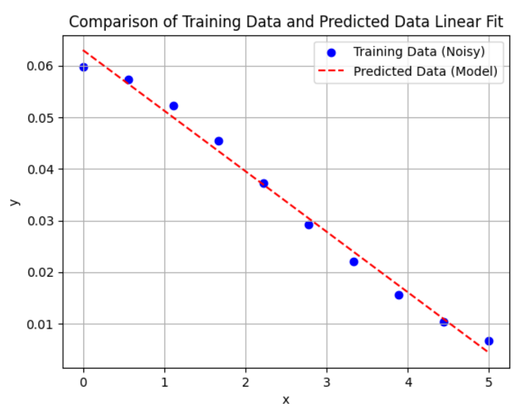

Fig. 1. Linear Fit of Data

B. Hyperparameter Experimentation

In order to find the best linear fit for the data, the hyperparameters of the epochs and learning rate can be modified to find the ideal values. To find a suitable epochs value, the learning rate will be kept at $\eta = 0.001$ while the epochs are varied. Plots of the results for varying epochs are given below. As seen in the figures, all of the tested epochs gave decent results for the model. The model with 5,000 epochs seems to have underfitted the data since the edges do not fit well. Based on the results, the higher the epochs the better the fit to the data is. From these tested values, 20,000 epochs is the best at fitting the data and minimizing loss.

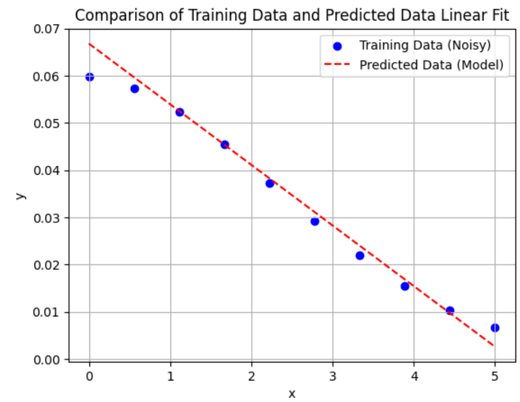

Fig. 2. Linear Fit with Epochs = 5,000 and η = 0.001

Fig. 3. Linear Fit with Epochs = 10,000 and η = 0.001

Fig. 4. Linear Fit with Epochs = 20,000 and η = 0.001

Similarly, the epochs value will be kept at 10,000 while the learning rate is varied to find an ideal η. Plots of the results are given below. As seen below, the learning rates of 0.01 and 0.001 have nearly identical results with 0.001 being a slightly better fit. However, with a learning rate of 0.0001 the model never converges and the predicted data does not match the actual data at all. Based on these results, a low learning rate can cause the fit to not converge. From the tested hyperparameters, 20,000 epochs with a learning rate of 0.001 was found to work best. Therefore, Figure 4 provides the best linear to the data.

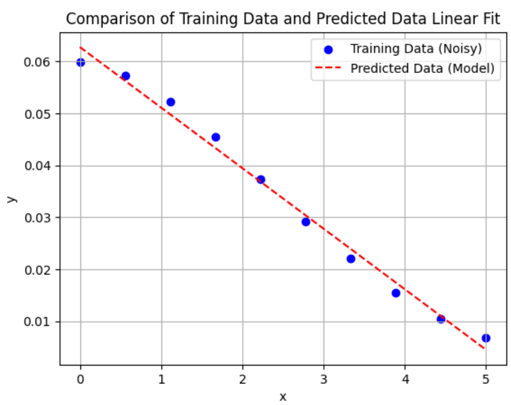

Fig. 5. Linear Fit with Epochs = 10,000 and η = 0.01

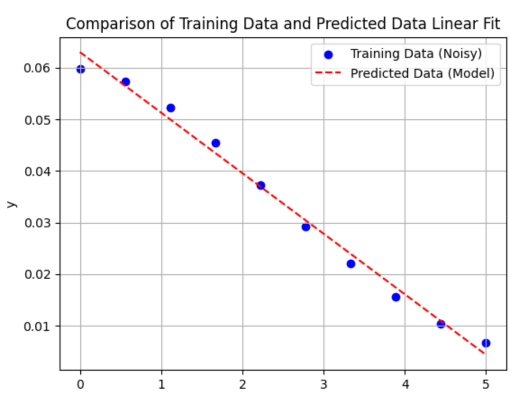

Fig. 6. Linear Fit with Epochs = 10,000 and η = 0.001

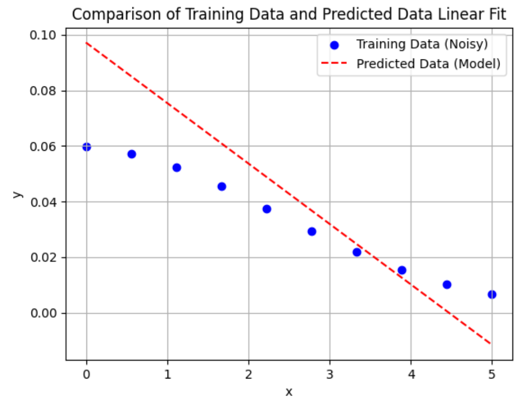

Fig. 7. Linear Fit with Epochs = 10,000 and η = 0.0001

II. Nonlinear Fit

In this section the same data set will be fitted to the nonlinear equation [3]:

$$y = n \cdot exp(-a \cdot y_{int}) \quad \text{where} \quad y_{int} = (mx + b)^2$$

A. Data Comparison

Using the given hyperparameters of 10,000 epochs and a learning rate of 0.001, the data was fitted to the linear model. As seen in Figure 8, the nonlinear shape fits the data better than the previous linear shapes. However, these parameters are also not perfect. The predicted data diverges from the actual data at some points, such as the beginning of the curve.

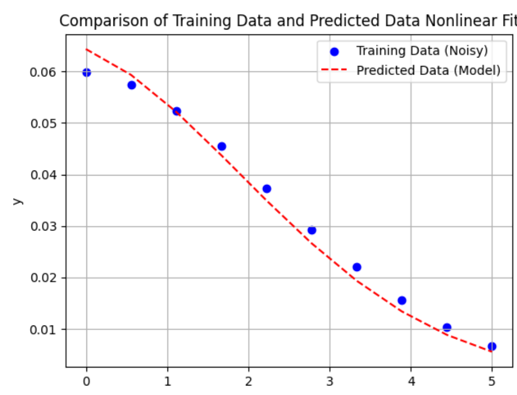

Fig. 8. Nonlinear Fit of Data

B. Hyperparameter Experimentation

In order to find the best nonlinear fit for the data, the hyperparameters of the epochs and learning rate can be modified to find the ideal values. Similarly to the previous problem, the learning rate will be kept at $\eta = 0.001$ while the epochs are varied. Plots of the results for varying epochs are given below. Similarly to the linear model, a too low number of epochs can result in an underfitted model. Figure 9 shows how with 5,000 epochs the nonlinear model is underfitted and does not match well. The best fitting curve is given in

Figure 10 with 10,000 epochs as it matches the actual data well capturing most of the curvature. The model with 20,000 epochs also works well, but is slightly less accurate than the model with 10,000 epochs.

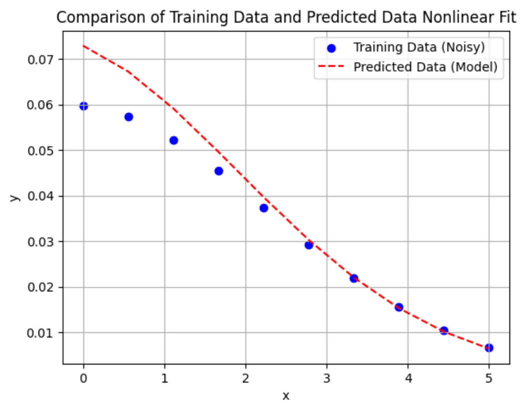

Fig. 9. Nonlinear Fit with Epochs = 5,000 and η = 0.001

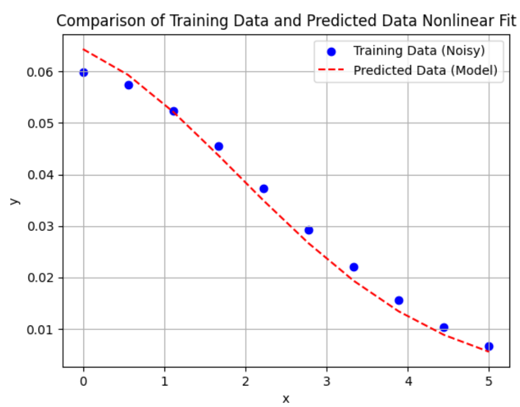

Fig. 10. Nonlinear Fit with Epochs = 10,000 and η = 0.001

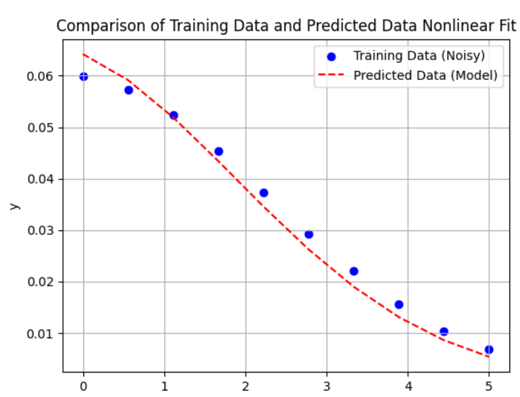

Fig. 11. Nonlinear Fit with Epochs = 20,000 and η = 0.001

The epochs will now be kept at 10,000 while the learning rate is varied. Results of this are given below. As seen in Figure 12, too high of a learning rate can cause issues with convergence, as the curve does not match the curves of the actual data. However, Figure 14 shows that too low of a learning rate can also cause issues with convergence as the model does not at all. The ideal learning rate for this data was found to be in the middle at $\eta = 0.001$. Figure 13 gives the best fit based on the chosen hyperparameters.

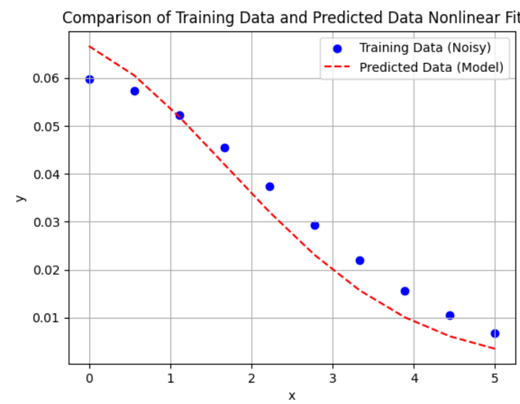

Fig. 12. Nonlinear Fit with Epochs = 10,000 and η = 0.01

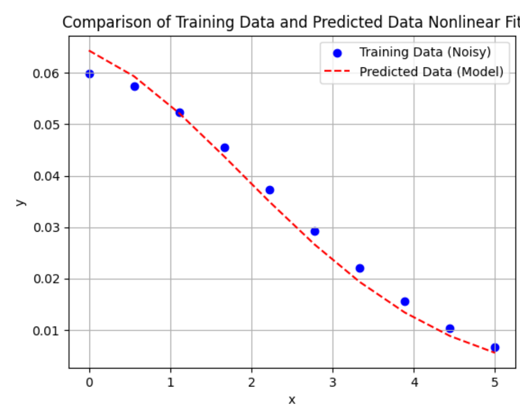

Fig. 13. Nonlinear Fit with Epochs = 10,000 and η = 0.001

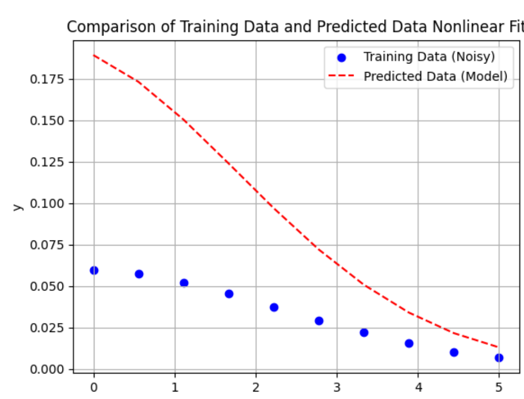

Fig. 14. Nonlinear Fit with Epochs = 10,000 and η = 0.0001

REFERENCES

[1] M. K. Jawed, "Homework 3 mae 263f," *BruinLearn*, Fall 2024.

[2] ——, "Lecture 16 and homework 3 - simple line fit - gradient descent and backpropagation.ipynb," *Google Colaboratory*, 2024.

[3] ——, "Lecture 16 and homework 3 - gradient descent and backpropagation - nonlinear fit.ipynb," *Google Colaboratory*, 2024.