Homework 2 - Mechanics of flexible structures and soft robots

Jessica Anz, November 6, 2024 (MAE 263F)

I. ASSIGNMENT 1

This assignment shall simulate the deformation of a discrete elastic rod undergoing bending, stretching, and twisting [1]. To clarify the implementation of this simulation, high-level pseudocode of the main Discrete Elastic Rod algorithm is given below [2].

Algorithm 1 Discrete Elastic Rod

Physical Parameters:
  $nv,\ ne,\ ndof$ ▷ Nodes, edges, DOFs
  $rodLength,\ natR,\ r_0$ ▷ Rod's physical parameters
  $nodes$ ▷ Matrix of nodes at t=0
  $Y,\ nu,\ G,\ EA,\ EI,\ GJ$ ▷ Stiffness parameters

Time Parameters:
  $totaltime,\ dt$ ▷ Time stepping parameters
  tol ▷ Tolerance

Masses and Forces:
  $massvector,\ mMat$ ▷ Mass vector and matrix
  Fg ▷ Gravity
  q_0 ▷ Initial displacement vector
  u ▷ Velocity

Reference Lengths and Frames:
  $a_1,\ a_2,\ m_1,\ m_2$ ▷ Reference and material frames
  $refLen$ ▷ Undeformed length of edges
  $voronoiRefLen$ ▷ Undeformed length of nodes
  $refTwist$ ▷ Reference twist
  $kappaBar$ ▷ Natural curvature
  $twistBar$ ▷ Natural twist

Initialize Loop Values:
  $Nsteps$ ▷ Number of time steps
  $(fixedIndex,\ freeIndex)$ ▷ Boundary conditions

Time-Stepping Loop:
for $timeStep = 1$ to $Nsteps$ **do**
    $qGuess \leftarrow q_0$ ▷ Guess current displacement
    $q, u, a_1, a_2 \leftarrow$ objfun$(\dots)$ ▷ Objective function
    $ctime \leftarrow ctime + dt$ ▷ Update time
    $q_0 \leftarrow q$ ▷ Update displacement
    $endZ[timeStep] \leftarrow q[-1]$ ▷ Store z-coordinate
end for

Plotting:
  $Plot(time,\ endZ)$ ▷ Plot z-coordinate of last node

A. Algorithm Explanation

As seen in the pseudocode, the algorithm begins by defining key variables, such as the physical parameters of the rod and nodes for discretization. The total time for the simulation to run and time step are then defined. Next, initial values for masses, forces, displacements, and velocities are defined. Then the reference and material frames are calculated using pre-defined helper functions. The undeformed length of each edge and node are calculated by looping through each of these values and geometrically solving for the length. The natural curvature, natural twist, and reference twist are all calculated using helper functions. Values for the time-stepping loop are then initialized, such as the number of time steps and the boundary conditions of the nodes.

The algorithm now begins the time-stepping loop which loops through all the time steps. The current displacement vector is used as a guess for the new displacement. The objective function which runs an iterative Newton-Raphson method is then used to calculate the current displacement. This uses elastic forces and the Jacobian of those forces to solve for the current displacement, and iteratively improves the solution until it is within an acceptable error margin. Time and displacement variables are then updated, and the z-coordinate of the last node is stored for plotting later. Following the end of the time-stepping loop, the z-coordinate of the last node is plotted against time to finish the algorithm.

B. Algorithm Outputs

The elastic rod was discretized into $N = 20$ nodes with a time step of $\Delta t = 0.01s$ [3]. The simulation of this discrete elastic rod under gravity was run from 0 to 5 seconds. The following plots give the output of the simulation at $t = 0s$ and $t = 5s$.

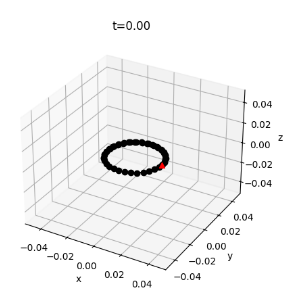

Fig. 1. Discrete Elastic Rob Structure at t=0 s

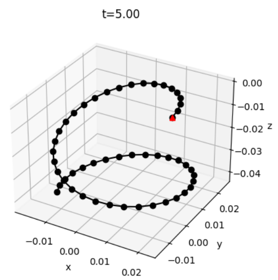

Fig. 2. Discrete Elastic Rob Structure at t=5 s

By comparing the initial and final structures, it is clear that the rod has undergone bending, twisting, and stretching based on the final deformed shape.

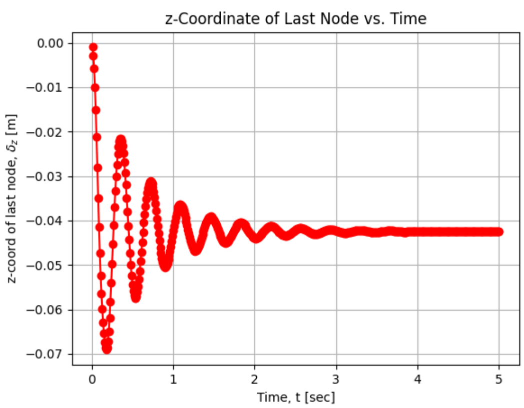

Fig. 3. z - Coordinate of Last Node vs. Time

Based on the plotted z-coordinate of the last node against time, the node initially oscillates up and down, but eventually reaches a steady-state of $\delta_z = -0.04m$. The shape of the plot resembles that of a damped oscillator.

REFERENCES

[1] M. K. Jawed and S. Lim, "Discrete simulation of slender structures," *BruinLearn*, 2024.

[2] Overleaf, "Algorithms in latex," *Overleaf.com*, 2024. [Online]. Available: https://www.overleaf.com/learn/latex/Algorithms

[3] M. K. Jawed, "Discrete_elastic_rods.ipynb," *Google Colaboratory*, 2024.