



Semi-device-independent certification of indefinite causal order in a photonic quantum switch

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Quantum processes are transformations that act on quantum operations. Their study led to the discovery of the phenomenon of indefinite causal order: some quantum processes, such as the quantum switch, act on independent quantum operations in such a way that the order in which the operations are acted upon not only cannot be determined but is simply undefined. This is the property that we experimentally certify in this work. We report an experimental certification of indefinite causal order that relies only on the characterization of the operations of a single party. We do so in the semi-device-independent scenario with the fewest possible assumptions of characterization of the parties' local operations in which indefinite causal order can be demonstrated with the quantum switch. To achieve this result, we introduce the concept of semi-device-independent causal inequalities and show that the correlations generated in a photonic quantum switch, in which all parties are able to collect local outcome statistics, achieve a violation of this inequality of 224 standard deviations. This result consists of the experimental demonstration of indefinite causal order with the fewest device-characterization assumptions to date.

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1. INTRODUCTION

Quantum mechanics challenges the viewpoint that physical quantities are locally pre-defined independently of measurement [1]. In recent years, pioneering work in quantum information has shown that, by assuming local operations respect quantum mechanics but dropping the assumption that the events must occur in a definite order, we may also challenge the viewpoint of well-defined causality [2,3]. While extending the quantum circuit formalism [4,5] and the notion of quantum combs [6–9], these works predicted processes with an indefinite causal order (ICO) that nevertheless do not lead to contradictions or paradoxes in the collected statistical data [3].

The study of such causal structures shows that ICO brings advantages to the performance of several quantum tasks [10–19]. From a fundamental perspective, the investigation of quantum

causal structures not only renews our understanding of causality in nature but also helps to address the long-standing problem of reconciling quantum theory and general relativity in a theory of quantum gravity [20–22]. A well-studied process with ICO is the quantum switch [2], upon which all experimental investigations of ICO to date have been based [23–31]. There has been some discussion regarding what is to be considered a valid implementation of the quantum switch as proposed in [2], with some of the opinion that current experiments are simulations [32–34], while others conclude that the experiments have an ICO [35] or at least have a quantifiable resource advantage [36]. To avoid ambiguity, here we call these experimental implementations "photonic" quantum switches.

An experimental certification of ICO that depends exclusively on the collected statistical data, and critically does not rely on any assumptions about the description of the local operations or

the process, is called device-independent certification. It can be achieved via the violation of a causal inequality [3,37], similar to how entanglement can be device-independently certified through the violation of a Bell inequality [38]. However, not all processes with ICO are able to generate noncausal correlations that can be observed in a device-independent way [12,39]; one such example is the quantum switch [39,40]. Although there exist theoretical processes that are able to violate causal inequalities [3,37], currently, no experimental implementations of them are known. ICO has, on the other hand, been certified in a device-dependent scenario in a photonic quantum switch, where the operations of all parties must be fully characterized. In this scenario, certification can be achieved through a causal witness [39], analogous to an entanglement witness [41]. All but one experimental demonstration of ICO to date [23–25,27–31] have critically relied on fully device-dependent assumptions, essentially assuming a perfect implementation of all local operations. One recent experiment was reported [26] in which the measurements performed by the final party of the photonic quantum switch were treated device-independently, but still assuming a full characterization of the operations of the other two parties inside the switch. It also employed device-dependent assumptions in the analysis of the initial target system, leaving as an open question whether a certification of ICO that relies on fewer assumptions would be possible. Recently, new theoretical proposals have positively answered this question [42,43].

Here, we experimentally confirm this stronger form of certification by only making assumptions about the characterization of the operations of *a single party*—in a semi-device-independent scenario. Our certification relies on strictly fewer device-characterization assumptions than previous implementations. Moreover, the assumptions upon which we rely are the minimal set of complete device-characterization assumptions in which the quantum switch demonstrates noncausal properties [42], without the need for any further assumptions. We would like to point out that, in an alternative certification scenario with the aid of ancillary entangled pairs and an assumption of locality on the space-like separation of observers, a recent work has demonstrated that all device-characterization assumptions can be dropped [44]. By extending the framework of [42], we introduce the concept of tailored semi-device-independent causal inequalities, whose violation certifies ICO, parallel to how the violation of a steering inequality [45,46] certifies entanglement in a semi-device-independent way. We experimentally test our inequality by implementing a photonic quantum switch. We develop a compact interferometer array, which incorporates multiple-outcome instruments for all parties acting on the quantum switch. This enables each party to generate local outcomes. This novel design allows us to experimentally test our inequality, yielding a violation by more than 224 standard deviations.

2. RESULTS

A. Semi-Device-Independent Causal Inequalities

The quantum switch [2] is a process that describes the following experimental situation. Consider an experiment in which two local parties, Alice and Bob, act on a qubit target system in an order determined by the state of a qubit control system (see Fig. 1). If the control system is in the state $|0\rangle$ ($|1\rangle$), Alice will act on the target system before (after) Bob. However, if the control system is in the coherent superposition $|+\rangle := (|0\rangle + |1\rangle)/\sqrt{2}$, then

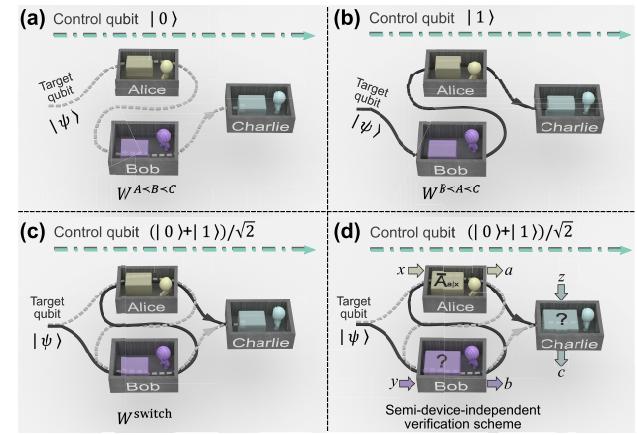


Fig. 1. Schematic diagram of the quantum switch. (a) and (b) Alice and Bob act on the target qubit in an order determined by the state of the control qubit, either $|0\rangle$ or $|1\rangle$. (c) When the control qubit is in the superposition state $\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, Alice and Bob act on the target qubit in an ICO. (d) We certify the indefinite causal properties of the quantum switch in a semi-device-independent scenario, where only the operations of Alice are characterized while no assumptions are made about the operations of Bob and Charlie.

the target state will be acted upon by Alice and Bob in an ICO. Finally, a third party, Charlie, that is in the well-defined future of Alice and Bob, performs a measurement in both target and control systems, regardless of the causal order between Alice and Charlie. Such a process, depicted in Fig. 1(d), allows for the events marked by the local operations of Alice and Bob to occur in what can be interpreted as a superposition of causal orders.

In the process matrix formalism [3], the quantum switch can be expressed as an operator [39] $W^{\text{switch}} \in \mathcal{L}(\mathcal{H}^{A_I} \otimes \mathcal{H}^{A_O} \otimes \mathcal{H}^{B_I} \otimes \mathcal{H}^{B_O} \otimes \mathcal{H}^{C_t} \otimes \mathcal{H}^{C_c})$ that acts on the linear spaces of Alice and Bob's input (A_I, B_I) and output (A_O, B_O) systems, and on Charlie's input, where he receives the future states of the target (C_t) and control (C_c) systems. The fact that the quantum switch exhibits an ICO is formalized by the statement that this process cannot be expressed as a classical mixture of a process $W^{A < B < C}$ (Alice acting before Bob, and Bob before Charlie), with a process $W^{B < A < C}$ (Bob acting before Alice, and Alice before Charlie) [39]. That is,

$$W^{\text{switch}} \neq q W^{A < B < C} + (1 - q) W^{B < A < C}, \quad (1)$$

for any $0 \leq q \leq 1$. This property is called *causal nonseparability* [39] and is also referred to simply as indefinite causal order.

Causal nonseparability can be certified through the correlations that arise when the independent parties involved in the experiment collect local statistics by making different choices of operations and recording their outcomes. Such operations are modeled by quantum instruments, which are the most general operations that can measure and transform a quantum system. Then, the joint probability distributions over the outcomes of Alice, Bob, and Charlie are given by a function of their instrument elements and the quantum switch process, according to a generalized Born rule [3],

$$p(abc|xyz) = \text{Tr} \left[(A_{a|x} \otimes B_{b|y} \otimes M_{c|z}) W^{\text{switch}} \right], \quad (2)$$

where $\{x, y, z\}$ label the inputs, $\{a, b, c\}$ label the outputs, and $A_{a|x} \in \mathcal{L}(\mathcal{H}^{A_I} \otimes \mathcal{H}^{A_O})$, $B_{b|y} \in \mathcal{L}(\mathcal{H}^{B_I} \otimes \mathcal{H}^{B_O})$, and

$M_{c|z} \in \mathcal{L}(\mathcal{H}^{C_t} \otimes \mathcal{H}^{C_c})$ are the instrument elements of Alice and Bob, and the measurements of Charlie, respectively. Here, instruments are represented in the Choi picture [47–49].

Following the framework developed in [42], consider an experiment that generates the correlations $\{p(abc|xyz)\}$ in a semi-device-independent scenario by having Alice, Bob, and Charlie act on an uncharacterized process with a set of known operations $\{\bar{A}_{a|x}\}$ for Alice, and unknown operations for Bob and Charlie. This experiment certifies ICO if and only if, for some a, b, c, x, y, z ,

$$p(abc|xyz) \neq \text{Tr}[(\bar{A}_{a|x} \otimes B_{b|z} \otimes M_{c|z}) W^{\text{SEP}}], \quad (3)$$

for all sets of quantum instruments $\{B_{b|z}\}$ and measurements $\{M_{c|z}\}$, and for all tripartite causally separable process matrices W^{SEP} that have a well-defined last party [i.e., of the form of the right-hand side of Eq. (1)]. That is, ICO is certified when the experimentally measured correlations cannot be explained by a causally separable process regardless of the operations of Bob and Charlie, assuming only knowledge of Alice's operations. This statement is equivalent to showing that the experiment described by $\{p(abc|xyz)\}$ and $\{\bar{A}_{a|x}\}$ cannot be explained by a semi-device-independent causal model or causal assemblage [42]. A causal assemblage is a mathematical object that carries the information of all possible correlations that the uncharacterized parties (Bob and Charlie) could generate in a definite causal manner, without even assuming that their operations are restricted by quantum mechanics. See [42] or [Supplement 1](#) for the precise definition of semi-device-independent causal models.

We now define semi-device-independent causal inequalities, whose violation witnesses the fact that the experimental data cannot be explained by a semi-device-independent causal model. Given a set of experimentally measured correlations $\{p(abc|xyz)\}$ and a description for the operations of Alice $\{\bar{A}_{a|x}\}$, the existence of a causal model that describes this experiment is a membership problem that can be solved via semidefinite programming (SDP) [42,50]. Should the experimental data *not be able* to certify an ICO, this SDP provides us with the exact causal model that describes the data. Alternatively, should the experimental data *be able* to certify ICO, it is guaranteed that there exists a hyperplane that separates the experimental data from the set of correlations that can be described by semi-device-independent causal models. This hyperplane can be obtained by the solution of the dual problem associated with the membership (also called the primal) SDP and used to construct an inequality of the following form:

$$S := \sum_{\substack{abc \\ xyz}} \alpha_{xyz}^{abc} p(abc|xyz) \geq 0, \quad (4)$$

where $\{\alpha_{xyz}^{abc}\}$ is a set of real coefficients obtained as the solution of the dual problem. We show that this inequality is satisfied if and only if the data comes from an experiment that (1) implements a process that is causally separable and (2) implements the specific instruments $\{\bar{A}_{a|x}\}$, regardless of what operations are performed by Bob and Charlie. Therefore, the violation of this inequality implies that whichever process being analyzed demonstrates ICO as long as one can guarantee the hypothesis that Alice holds the exact instruments $\{\bar{A}_{a|x}\}$ that were implemented. The derivation of Eq. (4), as well as the primal and dual problems, can be found in the [Supplement 1](#).

Any set of coefficients $\{\alpha_{xyz}^{abc}\}$ that satisfies the constraints of the dual problem defines a valid inequality of the form of Eq. (4). However, when the information of the theoretical prediction of the correlations generated in the experiment is available, our method allows one to derive a specific inequality that is tailored to a particular experiment and able to better unveil the noncausal properties of the process being implemented. In order to derive an inequality tailored to our experiment, we first calculate the theoretically expected set of probability distributions $\{p_{\text{theory}}(abc|xyz)\}$, using the instruments and quantum switch process provided in the [Supplement 1](#). Then, by computationally evaluating the dual SDP problem for this set of theoretical probability distributions and the proposed operations of Alice, we obtain the coefficients $\{\alpha_{xyz}^{abc}\}$, available at the repository in [51]. From these coefficients, we calculate an expected theoretical value of $S_{\text{theory}} = -0.0794$ for a perfect quantum switch.

B. Experimental Results with a Photonic Quantum Switch

To experimentally test this semi-device-independent causal inequality, we devised and carried out a photonic quantum switch experiment in which all three involved parties are able to implement multiple-outcome instruments. The experiment starts with the preparation of a heralded single photon. As shown in Fig. 2, twin photons are generated by spontaneous parametric downconversion (SPDC). While one is directly detected as a trigger, the heralded signal photon goes through a half-wave plate (HWP) for the initial state preparation and is then fed into a photonic quantum switch. The target qubit is encoded in the polarization of the signal photon, and the control qubit in its path degree of freedom. The path (control) qubit is introduced by the first beam splitter (BS), and the superposition of causal orders is completed when the paths are coherently combined as in a Mach–Zehnder interferometer (MZI) at the second BS, projecting the path qubit into the diagonal basis $\{|\pm\rangle\}$. Note that, in all photonic quantum switches proposed or implemented to date, the operations of Alice and Bob must act identically on two orthogonal optical modes. In most cases, as in our experiment, these modes are two spatial modes that traverse the same optical element, but polarization modes [25] have been demonstrated, and temporal modes have been proposed [52].

The core of our experimental implementation is the incorporation of multiple-outcome instruments acting on the quantum switch. Only one experiment so far generated local outcomes for two of the three parties of the quantum switch by coherently adding the outputs of measure-and-reprepare instruments of a single party with two interferometer loops [24]. Here, our experiment is based on a novel design of a compact setup that allows the incorporation of multiple-outcome instruments for all three parties. Specifically, Alice and Bob perform two different two-outcome measure-and-reprepare instruments on the target system, and Charlie performs two different four-outcome projective measurements on target and control systems. Overall, this constitutes 8 joint input settings and 16 possible joint outcome sets.

Multiple-outcome instruments lead to a stronger certification of ICO, as compared with certification schemes involving only single-outcome instruments (typically unitaries), by allowing a certification that does not rely on assumptions about the instruments of Bob and Charlie. However, they also impose the experimental

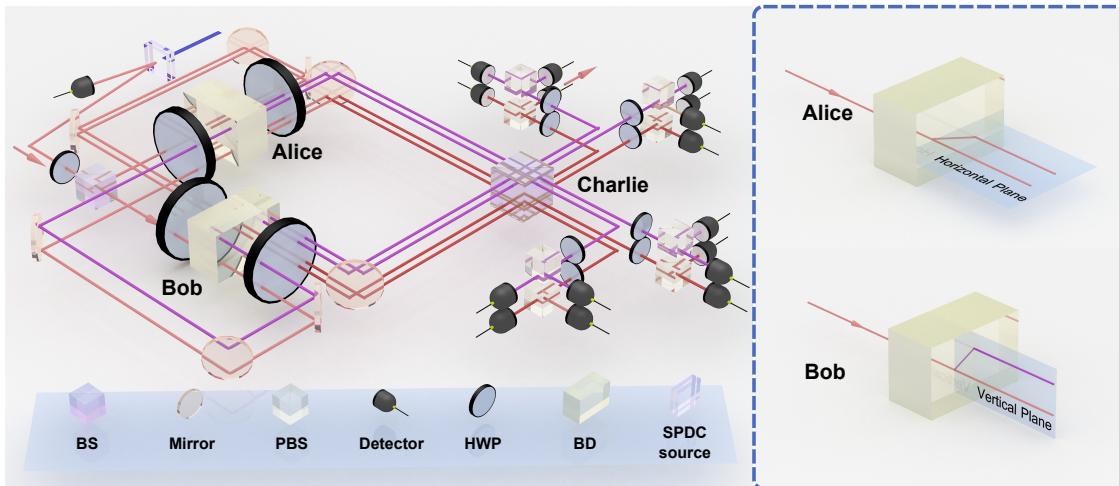


Fig. 2. Experimental setup. A 390 nm violet laser pumps β -barium borate (BBO) crystals cut for beam-like emission to generate a 780 nm heralded single photon. Notice that the four interferometer loops form a stereoscopic 2×2 optical path array, corresponding to the various outcomes of Alice and Bob's instruments. Bob's outcomes are differentiated by the optical paths in the lower or upper layers (denoted by red and purple beams), while Alice's outcomes are differentiated by the optical paths in the right or left layers.

challenge of collecting the outcome statistics generated by Alice and Bob without destroying the coherent superposition of causal orders. We overcome this challenge in the following way: the measure-and-reprepare instruments of Alice and Bob are realized by coupling the polarization mode to additional spatial modes. The measurement step is realized by a HWP followed by a beam displacer (BD) and a subsequent HWP that applies the same unitary rotation to the polarization state in both spatial modes. This reprepares the target qubit into different *orthogonal* states given by the measurement outcome. The deflecting direction of the BDs of the two parties is set to be orthogonal. Alice's instruments deflect her outcomes horizontally, while Bob's instruments deflect his outcomes vertically (Fig. 2, insets). Consequently, the interferometer loops introduced by the outcome sets (a, b) of both parties constitute a 2×2 interferometer array, with the beams in the lower and upper layers represented in Fig. 2 by red and purple beams, respectively. These interferometric loops are introduced to coherently recombine the different spatial modes in order to erase the information about the path through which they propagated. In this way, Alice and Bob operate on the target system locally but do not read out their classical outcomes until the information about the order of their operations is erased, preserving a coherent superposition of causal orders.

The compact interferometer loops are spatially close, in such a way that they undergo essentially the same environmental disturbance, which is inherently nearly phase-synchronous. Hence, they can be simultaneously stabilized with a single phase-locking system. An additional reference beam (not shown in Fig. 2) is fed into the quantum switch, and an active locking system is applied to simultaneously lock the phase of all four interferometers (see Section 4). Due to the compact setup and the active locking, an average Mach-Zehnder (MZ) interference visibility of $>99.1\%$ is achieved for all four interferometer loops over more than 1800 s (see Section 4).

To ensure that the device-dependent assumption of Alice's operations holds, we performed quantum process tomography of her local instruments. Fidelities of $>99.8\%$ for all instrument elements of Alice's instruments are achieved, confirming that Alice

performs the assumed operations (see Supplement 1). We recall that the operations of Bob and Charlie are experimentally constructed with the aim of implementing a specific set of operations that is known to allow for the certification of an ICO. However, this information is not taken into account in the analysis of the data. Therefore, our certification does not depend on the implementation of Bob and Charlie's operations being accurate or even in any way close to what is theoretically proposed.

Figure 3 displays the joint probability distributions over the outcomes of all combinations of instruments performed by each party, both the theoretical prediction and the experimentally collected data. From this experimental data, we achieve a value of $S_{\text{exp}} = -0.0673 \pm 0.0003$. The uncertainty of S_{exp} represents a single standard deviation via 50 samples of Poisson-distributed photon counts generated by a Monte Carlo simulation. This constitutes a violation by more than 224 standard deviations.

3. DISCUSSION

While a fully device-independent certification is the ultimate goal in the demonstration of ICO, currently it is unknown whether the required processes can be physically implemented. Therefore, in the state-of-the-art of quantum process implementation [53], the strongest possible method to experimentally certify ICO is the one that relies on the lowest possible number of device-characterization assumptions. In that sense, we have improved upon previous experimental demonstrations by requiring that only the quantum operations of a single party are characterized. Our proof was based on the conclusive violation of what we introduce as a semi-device-independent causal inequality. Since the quantum switch is known not to produce correlations that can be fully device-independently observed [39,40], our technique constitutes the minimal set of complete device-characterization assumptions necessary for the certification of the non-causal properties of the quantum switch [42]. We ensured these minimal assumptions to hold by performing process tomography of the characterized party. Our setup was based on a novel design of a photonic quantum switch with compact interferometer arrays, which enabled

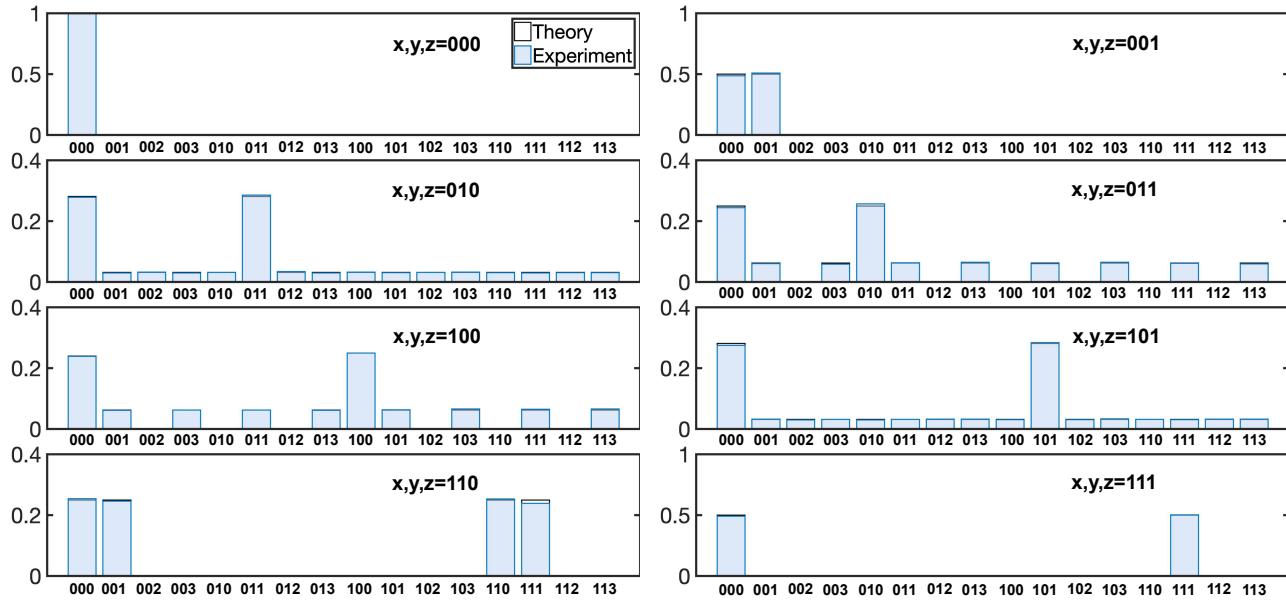


Fig. 3. Theoretically predicted and experimentally measured frequencies. Each subplot corresponds to the distribution over one set of joint outcomes, denoted in the horizontal axis by the labels $\{a, b, c\}$, of one input setting, denoted in the title of the subplots by the labels $\{x, y, z\}$. The experimental data are plotted with blue-colored pillars, while the theoretical predictions are plotted with transparent pillars. For each subplot, the probability distribution is normalized. The error bars have been omitted as they are negligible due to the high number of counts collected in our experiment.

the additional experimental improvement of locally generating multiple outcomes by all parties, instead of the more usual implementation of local (single-outcome) unitary operations. Our active phase-locking system created a very stable and high-performance quantum switch. This demonstration, therefore, contributes with stronger experimental evidence of the occurrence of ICO.

Although our experiment treats all but one party device independently, our demonstration could still be susceptible to loopholes. This is intimately related to the debate about whether photonic quantum switches are valid implementations or simulations of the quantum switch [2], which often revolves around the number of uses of each local operation in the implementations of the quantum switch. This “many-copy loophole” comes from the fact that it is known that the effect of a quantum switch that acts *only on unitary operations* can be also reproduced by a causally ordered circuit that has access to extra copies of these unitary operations [2]. For a switch that acts on non-unitary operations, such as the one reported here, it is unknown whether the action of the quantum switch can be simulated in an ordered fashion even with access to an arbitrary number of extra copies of these operations. To conclusively discard the many-copy loophole, in principle, the number of uses of each operation should be certified. It has been proposed that this could be achieved with a counter device [35] or by quantifying how much energy has been expended in a single run of the experiment [36]. In the case of our experiment, additional copies could potentially correspond to the different spatial modes that pass through the same optical element. If taken to the extreme, one could use an unfolded MZI, with instruments A1 and B1 in one arm and instruments B2 and A2 on the other, and this would behave the same as our experiment. However, since our multiple-outcome instruments are not unitary operations, it remains an open question whether access to these potential extra copies would be sufficient to explain our experimental results in a causally ordered way. Hence, it is unclear whether our experiment in particular also suffers from the many-copy loophole. This

current lack of understanding illustrates that the certification of ICO is a field in its infancy, and more work is required to identify all potential loopholes and devise methods to close them. We hope that our work will further motivate the study of potential loopholes in general quantum switches and that our novel method of implementing operations beyond unitaries will motivate the use of more complex operations in future protocols concerning indefinite causality, stimulating further investigation of this phenomenon based on even less assumptions.

4. MATERIALS AND METHODS

A. Phase-Locking System Setup

The violation of our semi-device-independent causal inequality relies on integrating measure-and-reprepare operations in both interrogating agents inside the quantum switch. This particular demand can be well addressed in our optical quantum switch with compact interferometer loops. Although the interferometer loops of 2×2 interferometer arrays are inherently phase-synchronous, they still undergo the identical environmental noise. An actively phase-locking system is applied to stabilize the path difference of the two spatial paths introduced by the first BS.

The phase-locking system consists of a reference light with 780 nm, a photon detector (PD) and a proportional-integral-derivative controller (PID) module. As shown in Fig. 4, the vertical polarized reference light (the yellow line) is first fed into the quantum switch from its outcome path and counterpropagated through the spatial path. Notice that the reference light is set slightly higher than, but close to, the single photon’s optical path, so that it can be conveniently fed into, and separated from, the experimental setup while undergoing the same environmental noise. The reference light is split into two branches by the BS of Charlies’ instrument. Each branch travels along its corresponding arm of the Mach–Zehnder interferometer (MZI). The reference light is kept

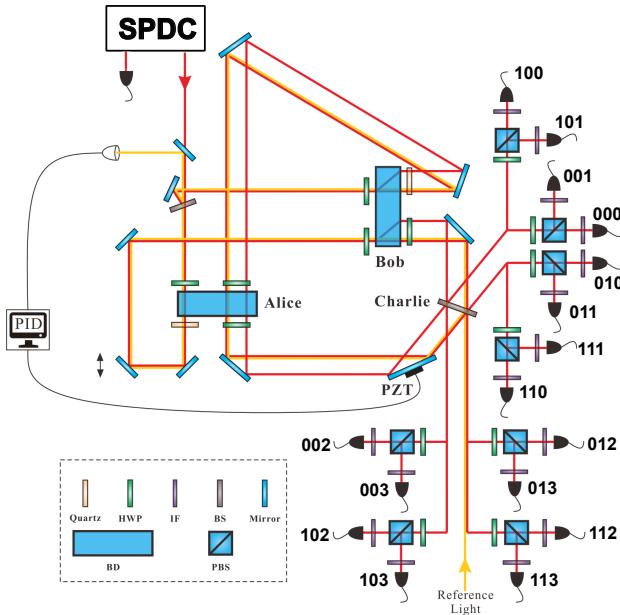


Fig. 4. Sketch of the experimental setup with locking systems. PID, proportional–integral–derivative controller; PZT, piezoelectric ceramic; HWP, half-wave plate; IF, interference filter; BS, beam splitter; BD, beam displacer; PBS, polarized beam splitter. Here, the lower-layer and upper-layer interferometers are implicitly shown in a single path. An explicit structure of the interferometer loops is shown in the main text. The single-photon target system is denoted by the red line, and the reference light for the locking system is represented in yellow. Two mirrors in the left-bottom part of the figure, mounted in translation stages, work as a trombone-arm delay line to tune the path difference of the two branches of the Mach–Zehnder interferometer (MZI). With the observed interference, the quartz plates are used to independently finely tune the phase of each interferometer loop. Notice that Alice and Bob perform multiple-outcome instruments inside the switch, but they do not record the result to avoid destroying the indistinguishability of causal orders until Charlie has performed his measurements. The label in the final single-photon detectors {000, 001, 002, 003 ··· , 110, 111, 112, 113} denotes the joint measurement outcomes $\{abc\}_{a,b,c}$ of Alice, Bob, and Charlie, respectively. Note that Charlie’s instruments include both the measurement on the control qubit (by the BS) and the subsequent measurement on the target qubit (by the polarization analysis system).

overhead of all the wave plates of Alice and Bob’s instrument and only travels through the BDs, such that the polarization is always kept in vertical direction and not deflected by the BD when Alice and Bob run over all settings. The two branches of the MZI are recombined again in first BS. One of the outcomes is reflected by a mirror overhead and continuously monitored by a PD. The power recorded by the PD reveals the phase relation of the MZI and is sent into the PID. The feedback voltage generated by PID drives the piezoelectric ceramic (PZT) attached to the mirror to actively stabilize the phase of the MZI.

B. Performance of Interferometer Array

When the locking system is applied, the phases of the four interferometer loops can be efficiently stabilized, while still not sharing precisely the same phase relations because they do not strictly overlap in space. To further finely tune the phases and make them strictly synchronous, additional quartz plates are inserted in each branch of the interferometer. As shown in Fig. 4, for each spatial branch of the interferometer, one quartz is inserted behind the

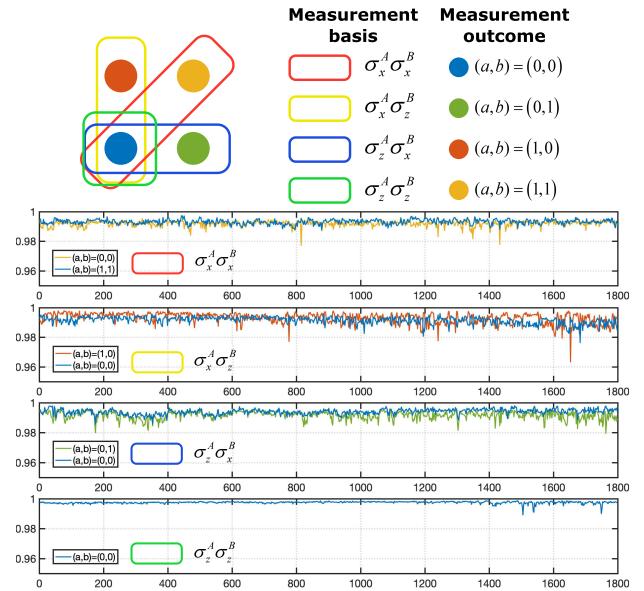


Fig. 5. Visibilities of the interferometer array. For each specific setting, only the parts of the spatial mode containing interference are shown. In the upper inset, the colored boxes include the interfering spatial modes. Different colored boxes correspond to different settings, while different colored spatial modes correspond to different measurement outcomes. The interference fringes of these interfering spatial modes are provided in the lower plots, where each setting is plotted individually.

BD. The optical axes of the quartz are both set along horizontally, such that the quartz will introduce birefringence between horizontal and vertical polarization. By tuning the yaw of the quartz, its effective thickness can be changed to introduce a tunable relative phase between outcome spatial modes of BD. The quartz in Alice’s side is used to synchronize the phase between the lower (vertical polarization) and upper (horizontal polarization) interferometers (in Fig. 4 they are represented by a single optical line as overlapped in a bird’s-eye view); the quartz in Bob’s side is used to synchronize the phase between the left and right interferometers.

The 2×2 interferometer loops will share stabilized and identical phases after the above-mentioned procedures are completed. To characterize the performance of our quantum switch, we measure the interfering visibilities of the 2×2 arrays with a single photon in a real experimental scenario. In this real experimental scenario, Alice and Bob may have two possible measurement settings along the direction of σ_x and σ_z . There are four possible setting combinations in the experiment $\{\sigma_x^A \sigma_x^B, \sigma_x^A \sigma_z^B, \sigma_z^A \sigma_x^B, \sigma_z^A \sigma_z^B\}$; in each setting, there are at most two interferometer loops containing interference, while the others have no interference because in such setting those outcomes may have vanishing probability. In the upper inset of Fig. 5, we provide the cross section of the interferometer array and interconnect the spatial modes with corresponding measurement outcomes by different colors. The interferometer loops that contain interference are also included by different color boxes for different settings. The lower plots of Fig. 5 depict the measured visibilities of these interferometer loops. Visibilities for each setting are plotted in an individual subplot, and each interfering outcome is plotted with a corresponding colored fringe in the upper inset. All fringes show averaged visibility of more than 99.1% in more than 1800s, which suffices to collect the data of the whole experiment. The high visibilities of the interferometer arrays suggest a strong proof that we faithfully implemented a

high-performance quantum switch, which incorporates distinct outcomes in both Alice's and Bob's side.

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Disclosures. The authors declare no conflicts of interest.

Data availability. The code developed for this work is made available by J. B. at [51]. All relevant data are presented in the paper and/or [Supplement 1](#). Additional data related to this paper may be requested from the authors.

Supplemental document. See [Supplement 1](#) for supporting content.

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