Martingale Roulette Strategy Analysis

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1 BACKGROUND

On an American roulette wheel, the odds of a player winning when betting on black favor the house at $\frac{9}{19}$; if players did not change their bet amounts over time, they should expect to lose money. In attempt to overcome the player's slanted odds, the proposed betting algorithm uses a variation of the Martingale roulette strategy introduced by French mathematician Paul Pierre Levy in which players increase their expected net winnings by doubling bets at each instance of a loss. However, in following this strategy, players must weather wide positive and negative swings, have the cash to fund any losing streaks that occur, and the time to bet on enough rounds needed to achieve a net positive cashflow.

A 80$maximumwinningsvariationoftheMartingalestrategyisusedinthefollowingex$

```
episode_winnings = $0
n_bet = 1

while episode_winnings < $80 and n_bet <= 1000:
        won = False
        bet_amount = $1
        n_bet += 1

        while not won and n_bet <= 1000:
                wager bet_amount on black
                won = result of roulette wheel spin
                if won == True:
                        episode_winnings += bet_amount
                else:
                        episode_winnings -= bet_amount
                        bet_amount *= 2

                n_bet += 1
```

2 EXPERIMENT 1

2.1 Question 1: Based on the experiment results calculate and provide the estimated probability of winning $80 within 1000 sequential bets.

For a given net winnings amount, we can backwards-solve for the number of betting rounds expected using the fact that each bet is expected to increase the player's total net winnings by $1 *if* the bet is won.[1] For instance, a player may lose n bets in a row resulting in a total loss of $\Sigma_{x=0}^{n} 2^{x-1} = 2^n - 1$. To offset the current loss streak, the next bet will be 2^n and, if the bet is won, the player's total winnings will increase by $1 over the value at the start of the losing streak.

Therefore, if the goal is to earn $80, there will need to be at least 80 completed losing streaks of length $\geqslant 0$ – in other words, 80 or more wins. As such, the likelihood of winning $80 within 1000 sequential bets could be modeled as a binomial random variable where $p = \frac{9}{19}$, $n = 1000$, $x = 80$, and P(80+ wins across 1000 bets) is nearly 100%:

$$\begin{aligned} \text{P(80+ wins across 1000 bets)} &= \Sigma_{x=80}^{1000} \binom{1000}{x} (0.47)^{x} (0.53)^{1000-x} && (1) \\ &> 0.999999 && (2) \end{aligned}$$

Further, a player could reasonably expect to win at least $80 in far fewer than 1000 bets as evidenced by the 10 randomized episodes converging before bet #200 in Figure 1.

1 If the bet is lost, the bet amount logic extends to the next round.

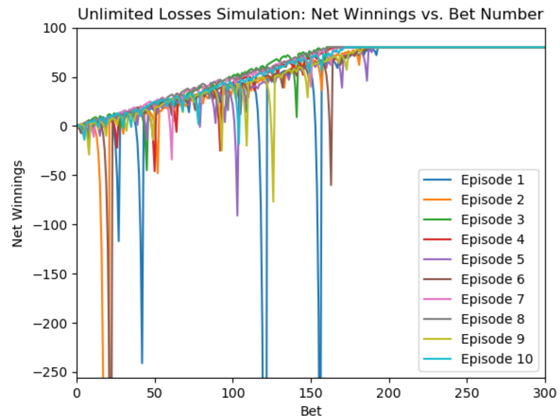

Figure 1—10 randomized episodes (sets of up to 1000 sequential bets) all converged to the maximum net winnings threshold of \$80 within 200 bets apiece.

2.2 Question 2: What is the estimated expected value of winnings after 1000 sequential bets?

Given the professor's betting strategy, the amount won from 1000 sequential bets is capped at \$80. Using the fact that a player has a near-100% likelihood of winning \$80 as demonstrated in Section 2.1, the expected value of a player's winnings across 1000 bets equates to $E[\text{expected winnings/episode}] = \min(\$80, (1.0 * (\$80)) + (0.0 * < 80)) = \min(\$80, \$80) = \$80$.

2.3 Question 3: Do the upper standard deviation line (mean + stdev) and lower standard deviation line (mean – stdev) reach a maximum (or minimum) value and then stabilize? Do the standard deviation lines converge as the number of sequential bets increases?

As shown below in Figures 2 and 3, the standard deviation of the mean and median values of multiple episodes' winnings fluctuated the most across the earlier bets since the gap between the bet amount and net winnings was much larger earlier on. By definition, the bands' magnitude could only be as big as $2^n - 1$ at any point, but they typically didn't reach those extremes due to the multiple samples generated for the experiment.

That said, both the bands of the mean and median of the sample instances leveled out to 0 as the players' net winnings approached the threshold of \$80. This is because a player can expect to eventually win \$80 each episode with near-100% certainty, hence little room for variation.

Additionally, while Figure 2 shows that the mean has potential to oscillate whereas the median increases at a constant rate (Figure 3), the mean follows the same overall trend of a \$1/bet increase. The average winnings amount converged to the \$1/bet slope line as betting continued since the relative magnitude of bet outcomes had less variation among the most likely outcomes.

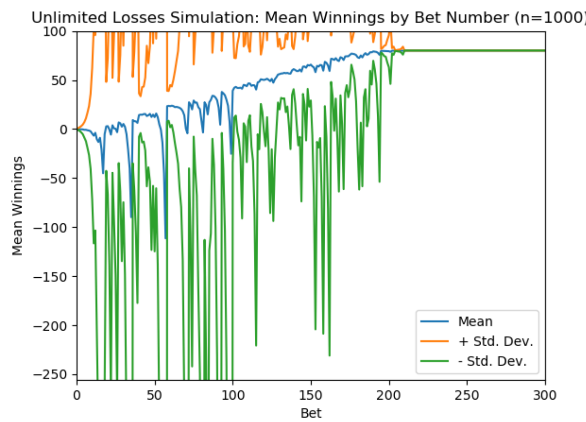

***Figure* 2**—Across 1,000 episodes containing up to 1,000 bets each with unlimited losses, average net winnings met the upper threshold of \$80 before bet #200.

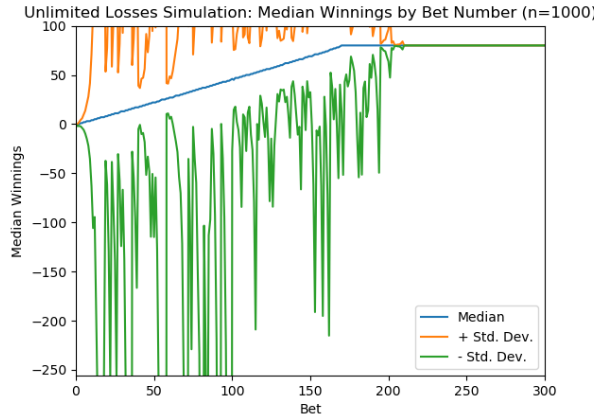

Figure 3—Across 1,000 episodes containing up to 1,000 bets each with unlimited losses, standard deviation of the median outcomes at each bet oscillated somewhat sporadically as a result of the varying loss potentials at each bet.

3 EXPERIMENT 2

3.1 Question 4: Based on the experiment results calculate and provide the estimated probability of winning \$80 within 1000 sequential bets.

In this experiment, losses were capped at a maximum of \$256 which acted as a limiting factor to the number of losses a player could experience.

Since the betting schematic remained the same, a player could still expect to come out of each loss streak with \$1 more than before it but the episode would end if the player ever lost all of the original money. Since the bet doubles each time, the original money would only cover $256 = \Sigma_{x=0}^{n} 2^n = 2^{n+1} - 1$->$n = 8$ full bets + 1 partial bet each ending in a loss before the episode ended unsuccessfully. The probability of reaching \$80 in net winnings is the probability of obtaining 80 wins (ceil$(80 * 1/\frac{19}{9}) = 169$ bets needed on average) before a period of 9 sequential losses (ceil$(1/(\frac{19}{10}^9) = 322$ bets needed on average) or 1000 total bets, which is approximately 62.1%.[2]

2 This average was obtained by averaging the outcomes of 10 instances of Experiment 2.

3.2 Question 5: What is the estimated expected value of winnings after 1000 sequential bets?

Since we can expect a player to either win the full \$80 or lose all the initial \$256 in a series of 1000 bets with approximately 62.1% odds that a win will occur before a loss, the expected value of winnings for a random episode is $E[\text{winnings}] = 0.621 * \$80 + 0.379 * (-\$256) = -\$47.04$.

3.3 Question 6: Do the upper standard deviation line (mean + stdev) and lower standard deviation line (mean – stdev) reach a maximum (or minimum) value and then stabilize? Do the standard deviation lines converge as the number of sequential bets increases?

Given the limits to how much a player could lose, the standard deviation bands are generally tighter and smoother (Figure 3 below) than that of Experiment 1. The eventual standard deviation lines of the expected value of winnings stabilized to their largest values of roughly $-47.04 - 154.81 = -201.85$ and $-47.04 + 154.81 = 107.77$ as betting continued since there were now two possible outcomes with unequal winnings amounts and likelihoods.

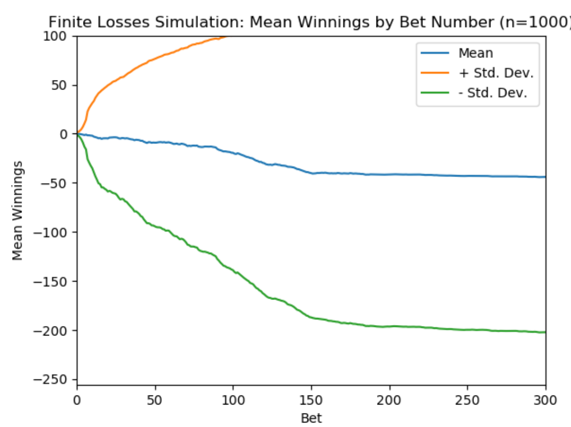

Figure 4—Across 1,000 episodes containing up to 1,000 bets each, the average net winnings converged to a negative amount since players were forced to quit after losses of the initial \$256 bankroll.

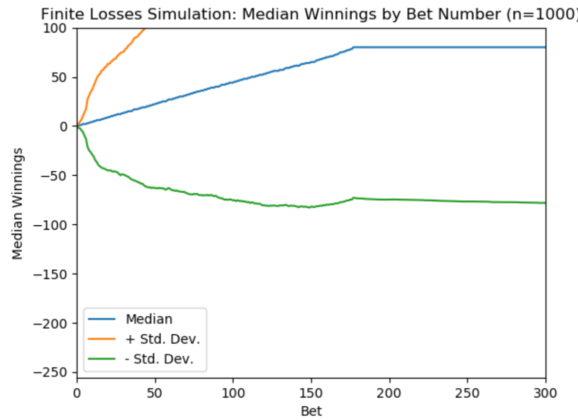

Figure 5—When losses were capped at \$256, the median winnings across 1,000 episodes still grew by \$1/bet, but the standard deviation of the median was much smaller than that of the mean shown in Figure 3 since relative magnitudes did not carry as much influence.

4 CONCLUSION

4.1 Question 7: What are some of the benefits of using expected values when conducting experiments instead of simply using the result of one specific random episode?

Using means/expected values from many observations meant that the above findings were less influenced by random chance and instead represented a more balanced picture of what someone could reasonably *expect* given the underlying probability distribution of outcomes as opposed to what *could* happen. As shown by the wide error bars in Figures 2-5, there were a wide range of potential values that were not always particularly reflective of an outcome.

Expected values (modeled as the mean outcomes of 1000 episodes) were more indicative of the set of potential outcomes than a single isolated sample due to wide differences in outcomes and their relative likelihoods. In fact, in the case of Figure 4 and 5, the median winnings of Experiment 2 was positive despite a negative mean since there was more money to be lost than won if a player stopped at -\$256 as compared to \$80.