PS 10

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4-9-15

(1)

$\dot{y} + y = x$ Step response

$y(t) = (1 - e^{-t})\mu(t)$

$y(t) = \mu(t) - e^{-t}\mu(t)$

when $x: \mu(t)$

Laplace transform of

$Y(s) = \frac{1}{s} - \frac{1}{s+1}$ $Re\{s\} > -1$

$Y(s) = \frac{1}{s} - \frac{1}{s+1} = \frac{s+1}{s(s+1)} - \frac{s}{s+1(s)}$

$Y(s) = \frac{1}{s(s+1)}$

Laplace transform

$sY(s) + Y(s) = X(s)$

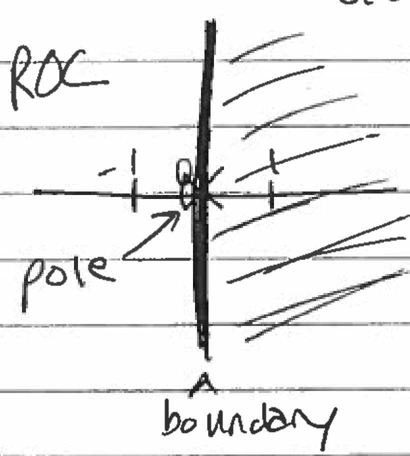
$Y(s)(s+1) = X(s)$

$Y(s) = X(s)\left(\frac{1}{s+1}\right)$

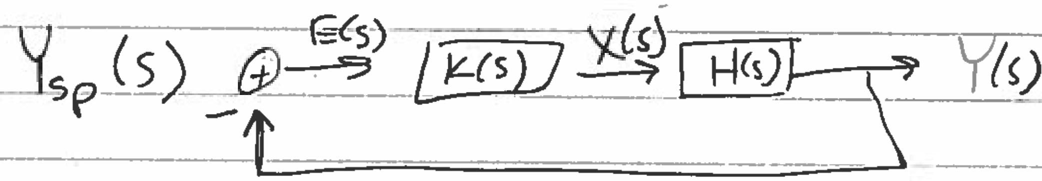
plug in above functions

$Y(s) = \frac{1}{s(s+1)} = X(s)\left(\frac{1}{s+1}\right) = \frac{1}{s}\left(\frac{1}{s+1}\right)$

equal!!



② Ⓐ find DC gain of system $\frac{Y(s)}{Y_{sp}(s)}$ (goal) if you use an integral controller $K(s) = K_I/s$ for any $H(s)$



$$E(s) = Y_{sp}(s) - Y(s)$$
$$X(s) = E(s)\,K(s)$$
$$Y(s) = H(s)\,X(s)$$
$$\frac{Y(s)}{H(s)} = X(s)$$

$$X(s) = \left[Y_{sp}(s) - Y(s)\right]K(s)$$
$$\cancel{X(s) = Y_{sp}(s)K(s) - Y(s)K(s)}$$
$$X(s) = \left[Y_{sp}(s) - H(s)X(s)\right]K(s)$$
$$X(s) = Y_{sp}(s)K(s) - H(s)X(s)K(s)$$
$$X(s) + H(s)X(s)K(s) = Y_{sp}(s)K(s)$$
$$X(s)\left[1 + H(s)\cancel{X(s)}K(s)\right] = Y_{sp}(s)K(s)$$
$$\frac{X(s)}{Y_{sp}(s)} = \frac{K}{1+K(s)H(s)} = \frac{Y(s)/H(s)}{Y_{sp}}$$

$$\star\ Y(s)/Y_{sp}(s) = H(s)K(s)/1+K(s)H(s)$$

~~Y(s)~~

DC Gain is $\lim_{s\to 0} H(s)$

Where $X(s) \to \boxed{H(s)} \to Y(s)$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{H(s)\ K_I/s}{1 + K_I/s\ H(s)}$$

~~as $s \to 0$ the numerator~~

~~$\to$ numerator $\to \infty$~~

~~denominator $\to$~~

as $s \to 0$

$$\left(\begin{array}{l} \underbrace{\frac{H(s)\ K_I}{\cancel{s}\to 0}}_{\text{to infinity! numerator}} \\ \\ \underbrace{1 + \frac{K_I\ H(s)}{\cancel{s}\to 0}}_{\text{1+ infinity! denominator}} \end{array} \right.$$

$\boxed{\text{close to 1!}}$

H(s) does not affect DC gain

K_I does not affect DC gain either

(B) assume $H(s) = \frac{1/\tau}{s + 1/\tau}$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{\frac{1/\tau}{s+1/\tau}\ K(s)}{1 + K(s)\ \frac{1/\tau}{s + 1/\tau}}$$

$$= \frac{1/\tau\ K(s)}{(s + \frac{1}{\tau}) + K(s)\frac{1}{\tau}} = \frac{K(s)}{\tau(s + \frac{1}{\tau}) + K(s)}$$

$$s = \frac{1}{\tau} - \frac{K_I}{s}$$

~~$$\boxed{= \frac{K(s)}{\tau s + 1 + K(s)}}$$~~ ~~$K \gg \frac{1}{\tau}$~~

~~$\tau s + 1 + K(s) = 0$~~

~~$\tau s + K(s) = -1$~~

~~$s + K(s) = -\frac{1}{\tau} - K(s)$~~

$$\tau s + 1 + K(s) = 0$$

$$\tau s + 1 + K_I/s = 0$$

$$\tau s^2 + s + K_I - 0$$

$$s = \frac{-1 \pm \sqrt{1^2 - 4\tau K_I}}{2\tau}$$

$$s = -\frac{1}{2\tau} \pm \frac{\sqrt{1^2 - 4\tau K_I}}{2\tau}$$

$$K_I \gg 1/\tau$$

$$4K_I \gg 4/\tau$$

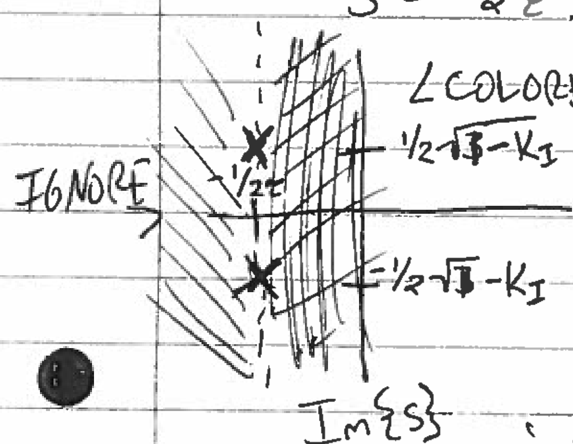
$$4K_I\tau \gg 4$$

$$s = -\frac{1}{2\tau} \pm \frac{\sqrt{1^2 - 4 \text{ (or greater)}}}{2\tau}$$

$$s = \frac{-1}{2\tau} \pm \frac{\sqrt{-3}}{2} \text{ (or greater)}$$

$$s = \frac{-1}{2\tau} \pm \frac{1}{2}\sqrt{-3} = -\frac{1}{2}\frac{1}{\tau} \pm \frac{1}{2}\sqrt{3}\, i$$

or $-4, -5 \ldots$ etc
as long as its less than -4



ROC

$\cancel{s = \frac{1}{2}\left(-\frac{1}{\tau} \pm \sqrt{-3}\right)}$

real component is the same for both poles
imaginary component is $\pm \frac{1}{2}\sqrt{3}$
depending on K_I, poles can move closer to or further from x axis

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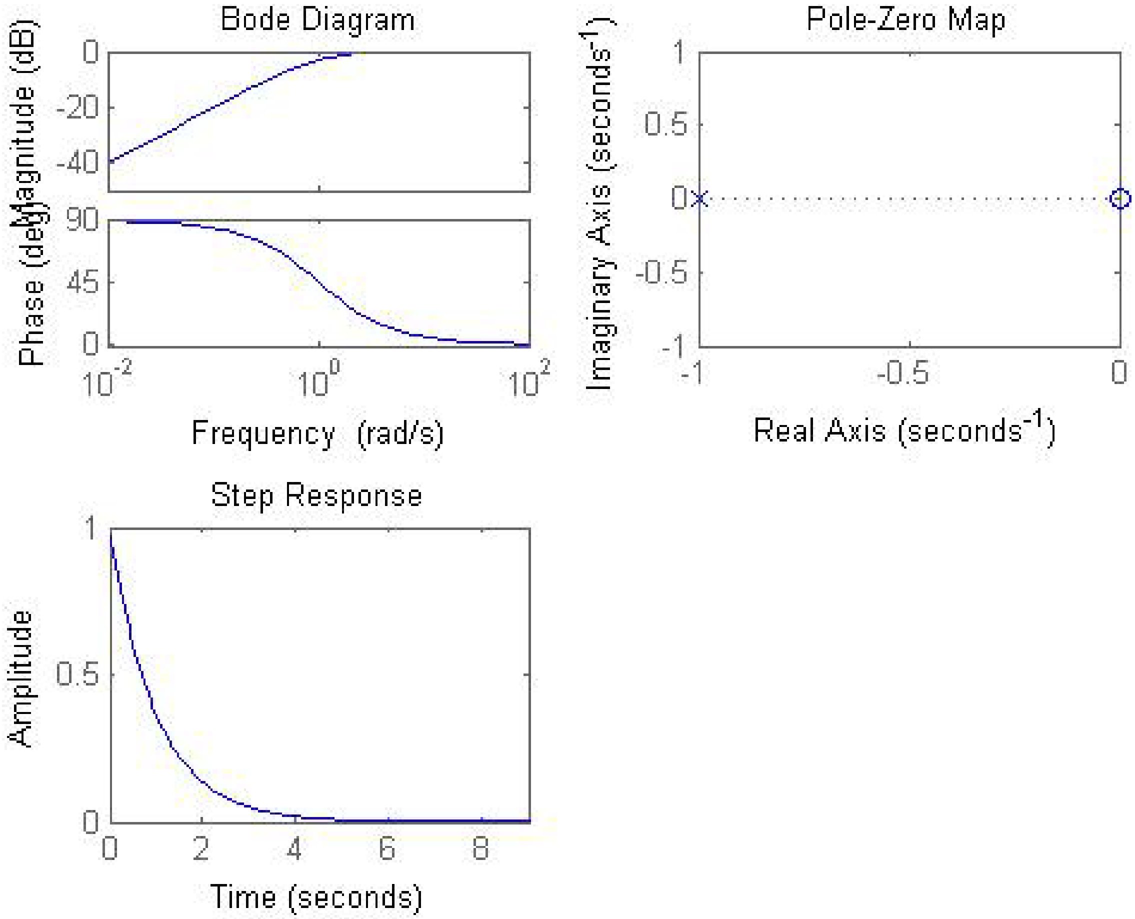
Problem Three

Analyze the behavior of the systems listed using a Bode plot, a pole-zero map, and the step response. Note the relationship between all three plots: order of the system, number of poles and zeros, real or complex poles, oscillations, and so forth.

A.

$$H(s) = \frac{s}{s+1}$$



The Bode magnitude diagram begins by increasing with around a 20 dB per decade slope indicating there is a zero around the origin and then evens out at around 1 indicating there is a pole at -1. Both of these are confirmed through the pole-zero map. The pole-zero map says that all poles and zeros lie along the real axis (have no imaginary part), which confirms the lack of oscillation in the step response plot. There is also only one pole and zero, so there is only one major impact on the step response, a decreasing force. This is a type of high-pass filter.

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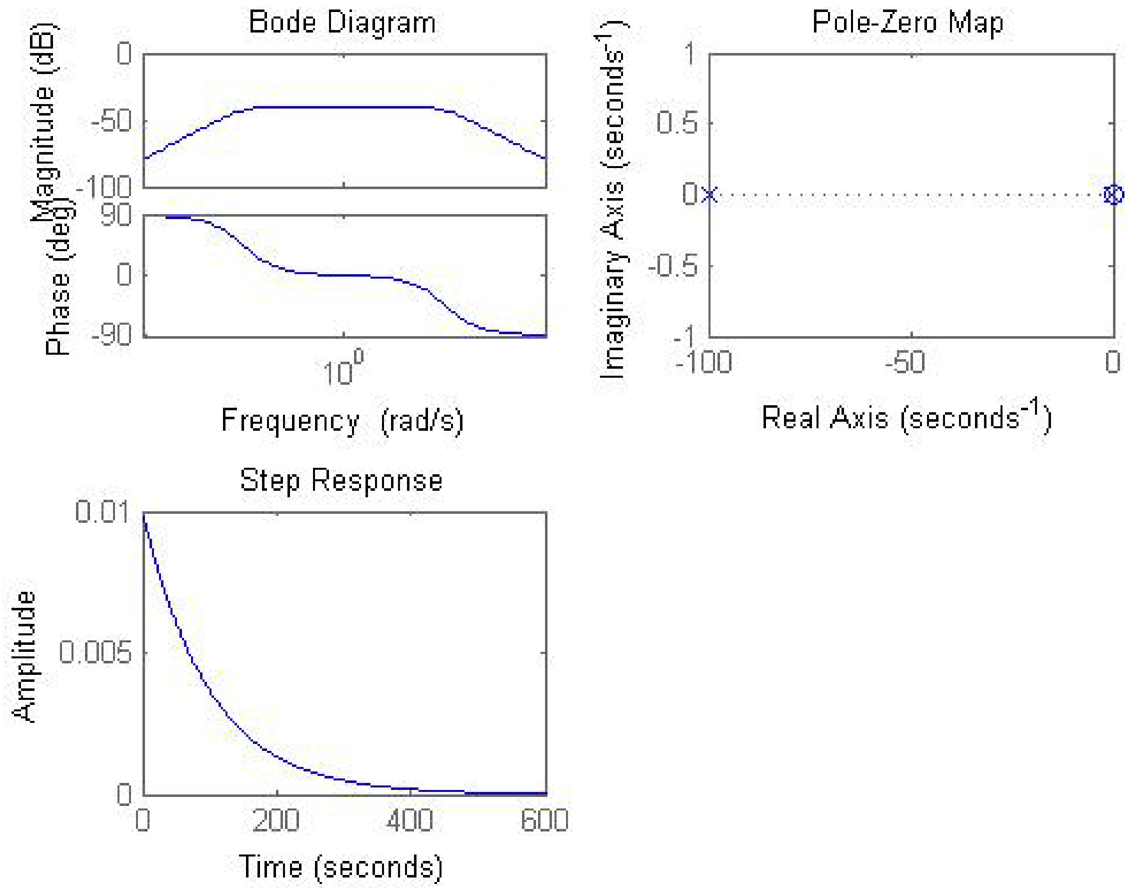
B.

$$H(s) = \frac{s}{s^2 + 100s + 1}$$



The Bode magnitude diagram begins by increasing with around a 20 dB per decade slope indicating there is a zero around the origin, evens out at around .01 indicating there is a pole at -.01, and then decreases at around 100 indicating there's a pole at -100. All of these are confirmed through the pole-zero map. The pole-zero map says that all poles and zeros lie along the real axis (have no imaginary part), which confirms the lack of oscillation in the step response plot. There is also two one poles, so there are two aspects you can see in the step response, something that causes the sharp quick increase at the origin up to 1 and then a decreasing force. The sharp increase is caused by the pole at -100 which is very far from the origin. That fact causes it to become minimally impactful after a short period of time. This is a type of band-pass filter.

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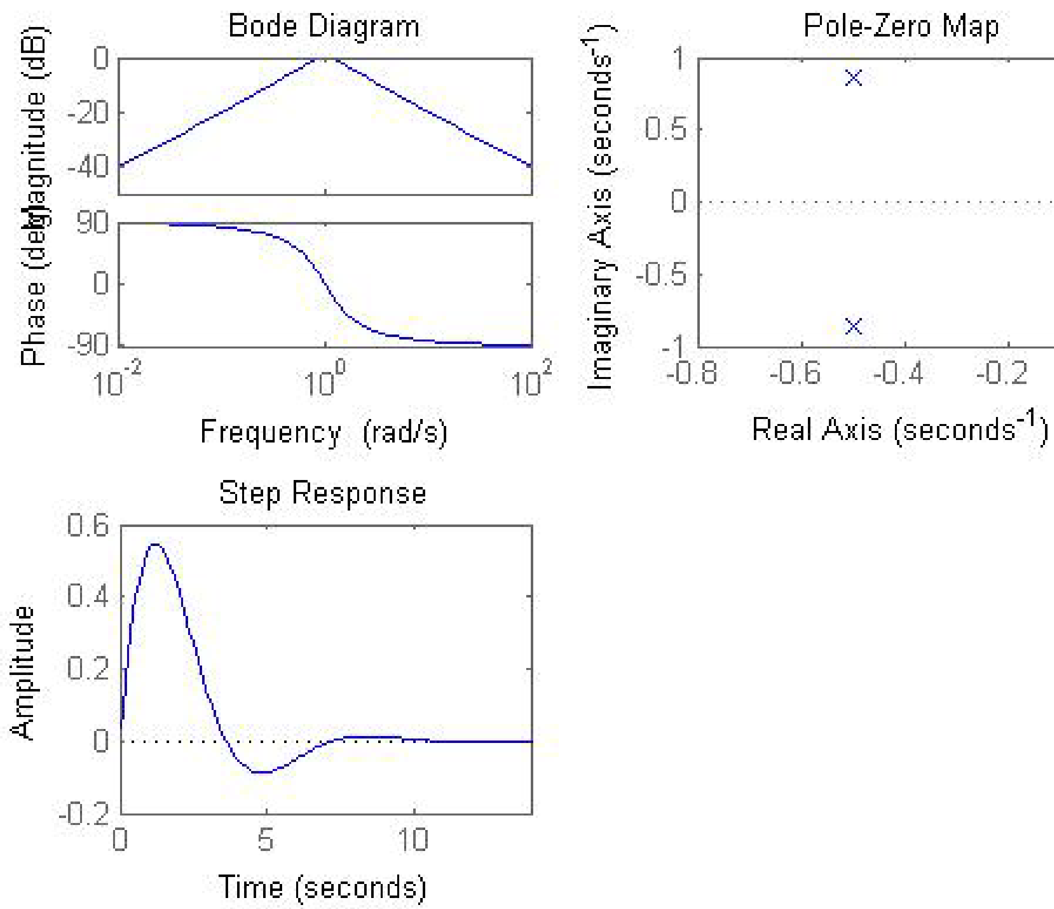
C.

$$H(s) = \frac{s}{s^2 + s + 1}$$



The Bode magnitude diagram begins by increasing with around a 20 dB per decade slope indicating there is a zero around the origin and then at 1 it begins a mirrored decrease indicating there isn't just one pole at -1, but two. All of these are confirmed through the pole-zero map—the poles have a -.5 real component which combined with the slightly-less-than-one imaginary component create a magnitude of 1. The pole-zero map says that the two poles have imaginary components, which creates an oscillation in the step response plot. There are two poles and a zero, so there are two major impacts on the step response, an oscillation force and a dampening force. This is a type of band-pass filter. The time constant is 1/(real component of the poles) = 1/.5 = 2. At 2 seconds, the envelope will thus have dropped by a factor of e, which attributes to the quick dampening of the oscillation.

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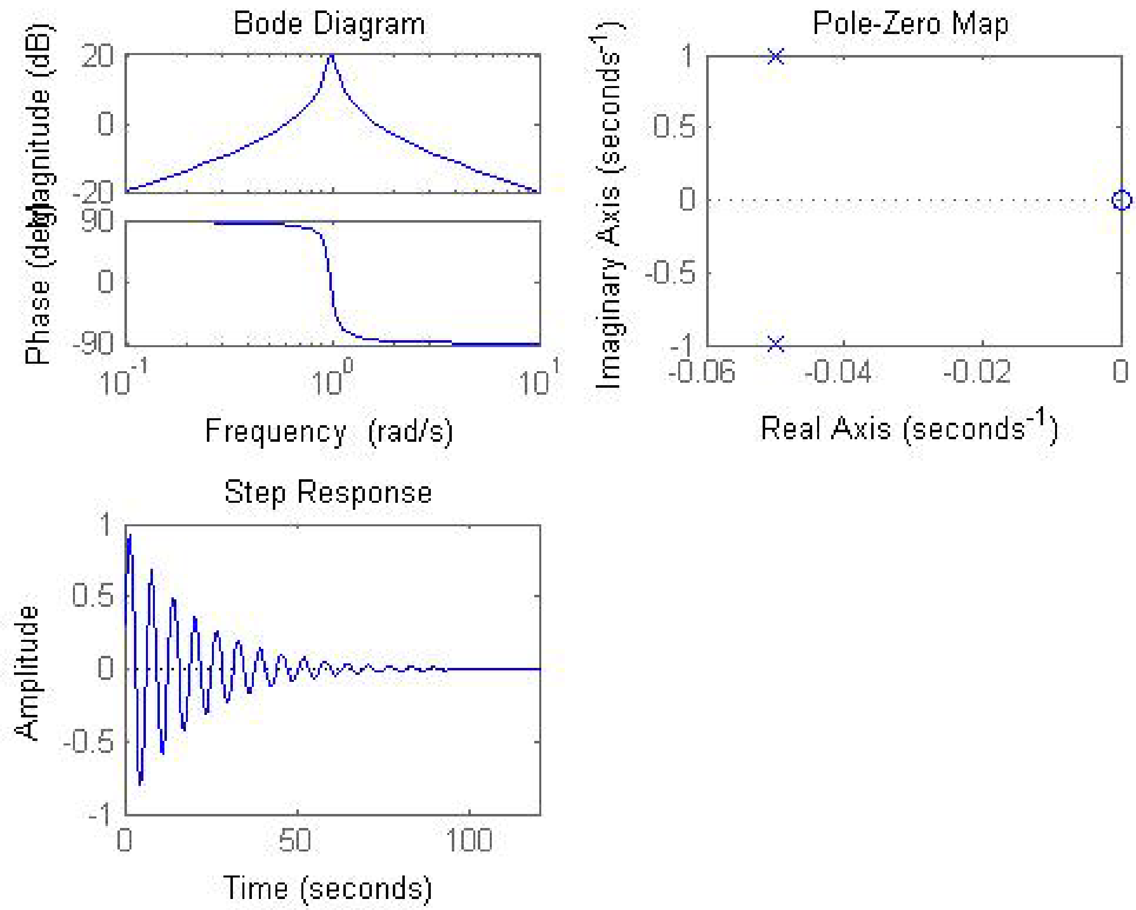
D.

$$H(s) = \frac{s}{s^2 + 0.1s + 1}$$



The behavior of this system is almost the same as C, but with higher time constant. It will oscillate faster and decay slower however, as the real component is smaller (thus causing a slower decay) and the imaginary component is larger (thus causing a shorter oscillation period).

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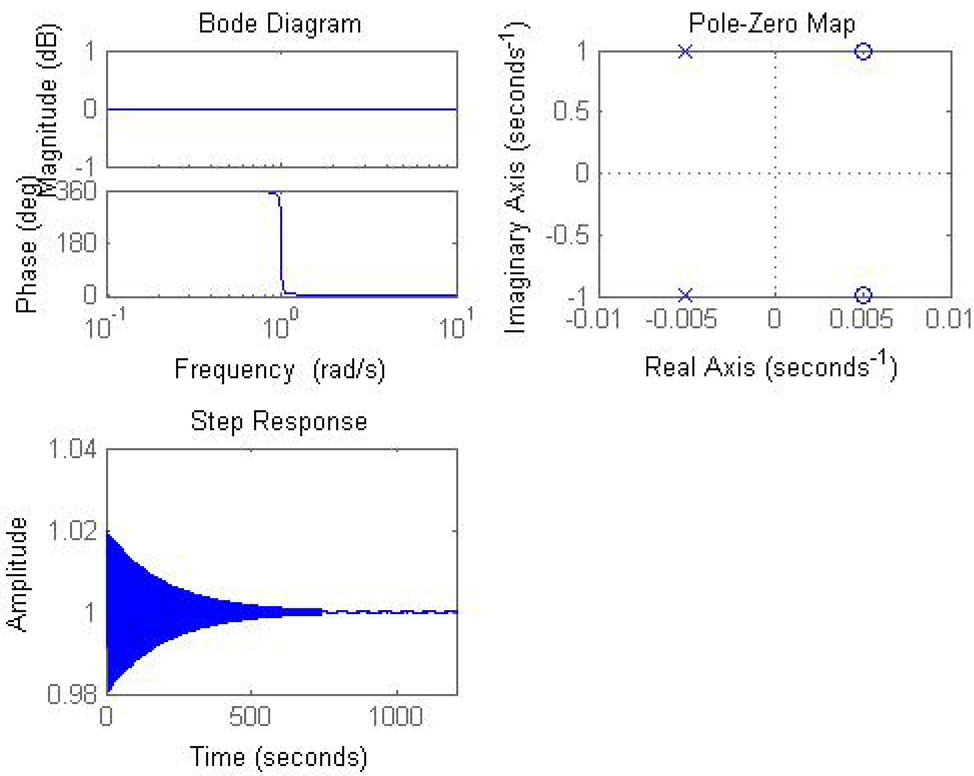
E.

$$H(s) = \frac{s^2 - 0.01s + 1}{s^2 + 100s + 1}$$



The Bode magnitude diagram is completely constant at 1. All of this is confirmed through the pole-zero map—the poles have an identical magnitude to the zeros opposite them. The pole-zero map says that the two poles have imaginary components, which creates an oscillation in the step response plot. There are two poles and two zeros, so there are two major impacts on the step response, an oscillation force and a dampening force. As the poles are very close to the origin, the oscillations are very quick and very slowly are dampened.

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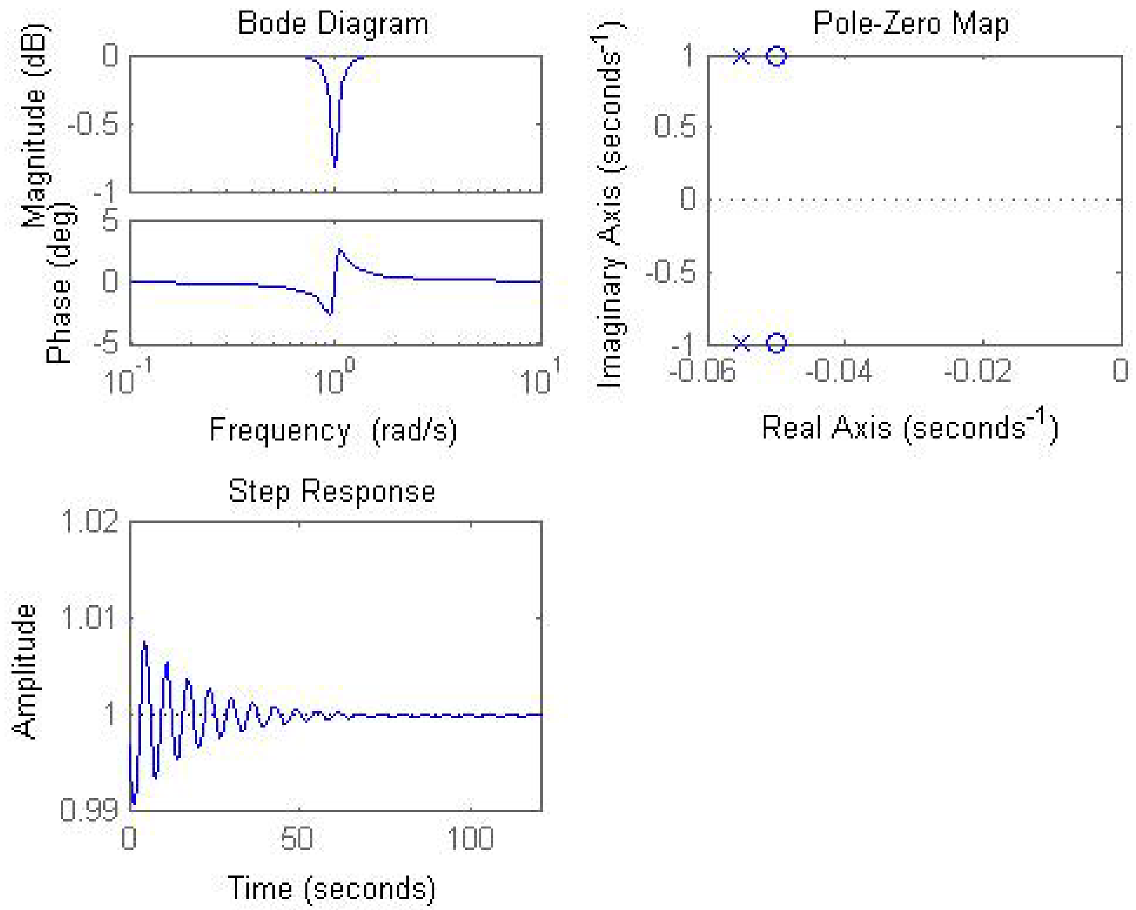
F.

$$H(s) = \frac{s^2 + 0.1s + 1}{s^2 + 100s + 1}$$



The Bode magnitude diagram begins by starting out at zero and then at around 1, there's a sharp drop to a magnitude of -1. All of these are confirmed through the pole-zero map—the poles have a -.05 real component which combined with the 1 imaginary component of 1 create a high magnitude higher than the last few parts of this question. The pole-zero map says that the two poles have imaginary components, which creates an oscillation in the step response plot. There are two poles and a zero, so there are two major impacts on the step response, an oscillation force and a dampening force. This is a type of notch filter.

(4) Stabilize System, $H(s) = \dfrac{1}{s^2 - 0.01s + 1}$

(A) plot step response & pole zero map

$Y(s) = H(s) X(s)$ for $u(t)$ as $x(t)$

$y(t) = ?$

$\ .01$
$\times .01$
$.0001$

if $x(t) = u(t)$

$X(s) = \dfrac{1}{s}$

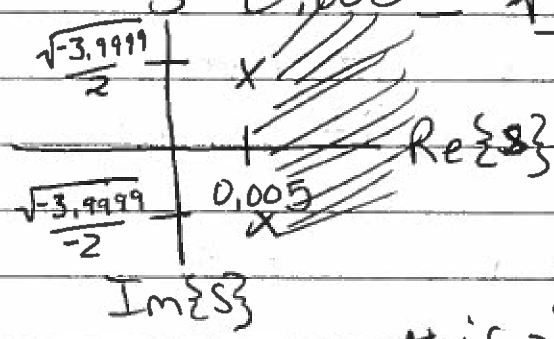
$Y(s) = 1/s \cdot \dfrac{1}{s^2 - 0.01s + 1}$

$Y(s) = \dfrac{1}{s^3 - 0.01s^2 + s}$

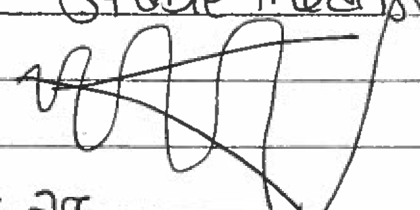
$s^2 - 0.01s + 1 = 0$ $\quad s = \dfrac{0.01 \pm \sqrt{0.01^2 - 4}}{2}$

$s = 0.005 \pm \dfrac{\sqrt{0.0001 - 4}}{2}$

$s = 0.005 \pm \dfrac{\sqrt{-3.9999}}{2}$



stable means

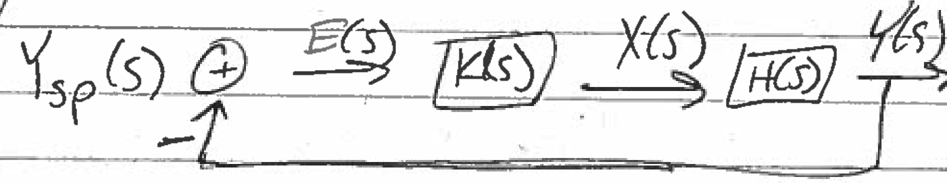


NOT this

~~if all poles are~~ on right side or on imaginary axis

put in coefficients [1, -0.01, 1, 0] as array in his ipython notebook

(4)



$$\frac{X(s)}{Y_{sp}(s)} = \frac{K(s)}{1 + K(s)H(s)} = \frac{Y(s)/H(s)}{Y_{sp}(s)}$$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{H(s)K(s)}{1 + K(s)H(s)}$$

(b) proportional control

$$K(s) = K_p$$

probably positive

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{\frac{1}{s^2 - .01s + 1} \cdot K_p}{1 + K_p \frac{1}{s^2 - .01s + 1}}$$

$$\frac{Y(s)}{Y_{sp}(s)} = \frac{K_p}{s^2 - .01s + 1 + K_p}$$

$$s^2 - .01s + 1 + K_p = 0$$

$$s = \frac{.01 \pm \sqrt{(.01)^2 - 4K_p}}{2}$$

$$s = .05 \pm \frac{\sqrt{.0001 - 4K_p}}{2}$$

if ~~$4K_p < .0001$~~ ~~$K_p <$~~ not possible for there to be all ~~positive~~ poles in left hand side of axis

(c) $K(s) = K_I/s$

~~int~~ $\frac{Y(s)}{Y_{sp}(s)} = \frac{\frac{1}{s^2-.01s+1} \cdot K_I/s}{1+ K_I/s\left(\frac{1}{s^2-.01s+1}\right)}$

$= \frac{K_I/s}{s^2-.01s+1+\frac{K_I}{s}}$

$\frac{Y(s)}{Y_{sp}(s)} = \frac{K_I}{s^3-.01s^2+s+K_I}$

$s^3-.01s^2+s+K_I=0$ wolfram alpha

[illegible]

no solutions!

(d) $K(s) = s\, K_s$

$\frac{Y(s)}{Y_{sp}(s)} = \frac{\frac{1}{s^2-.01s+1} \cdot K_s \cdot s}{1+K_s \cdot s\left(\frac{1}{s^2-.01s+1}\right)} = \frac{K_s \cdot s}{s^2-.01s+1+K_s \cdot s}$

$\cancel{s^2-.01s+1}\ \frac{Y(s)}{Y_{sp}(s)} = \frac{K_s \cdot s}{s^2+(K_s-.01)s+1}$

$s = \frac{-K_s+.01 \pm \sqrt{(K_s-.01)^2-4}}{2} = -\frac{K_s}{2}+.005 \pm$

need $-\frac{K_s}{2}+.005<0$

if it's going to be on the left hand side

$-K_s/2 < -.005$

$-K_s < -.010$

$K > 0.010$

totally possible

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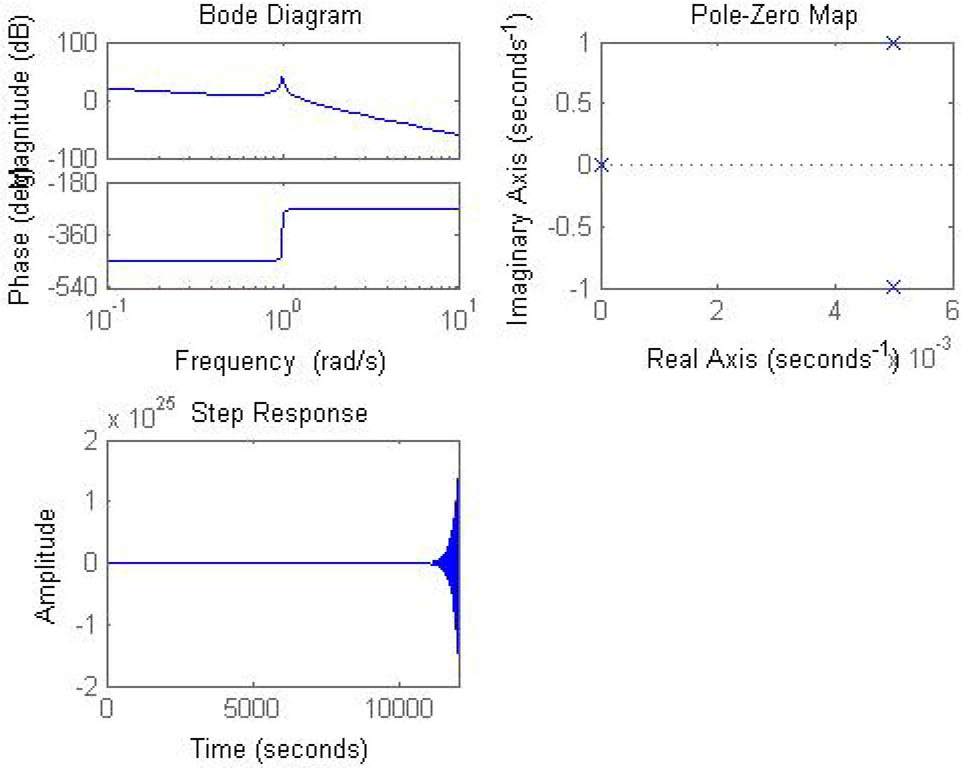
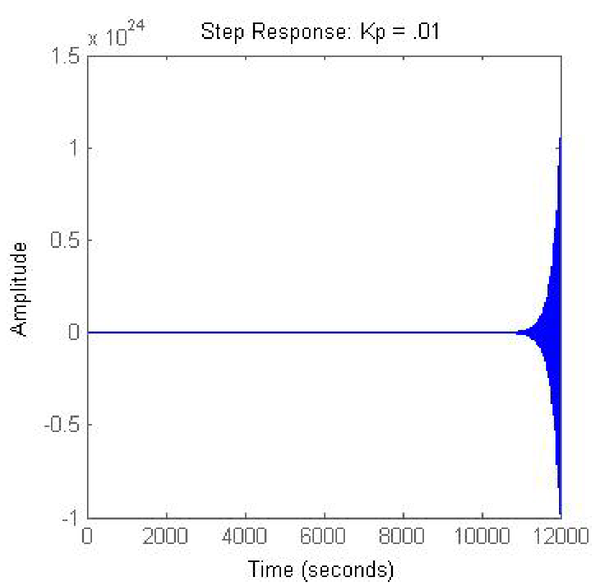
Problem Four

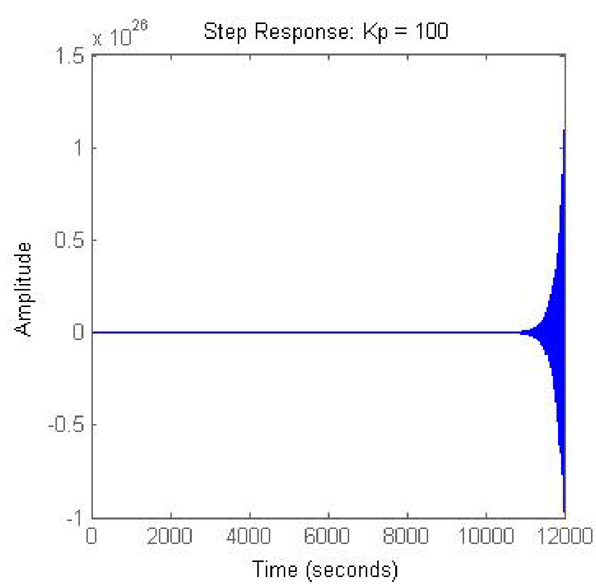
A.



B.

I set $K_p = .01$ in the first one and $K_p = 100$ in the second one.





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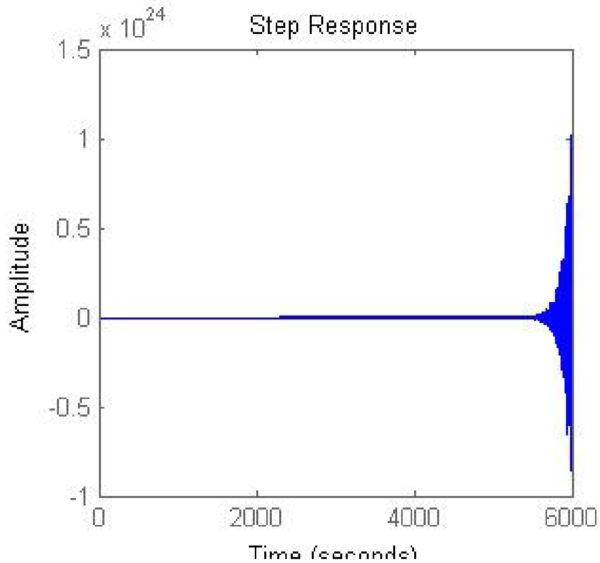
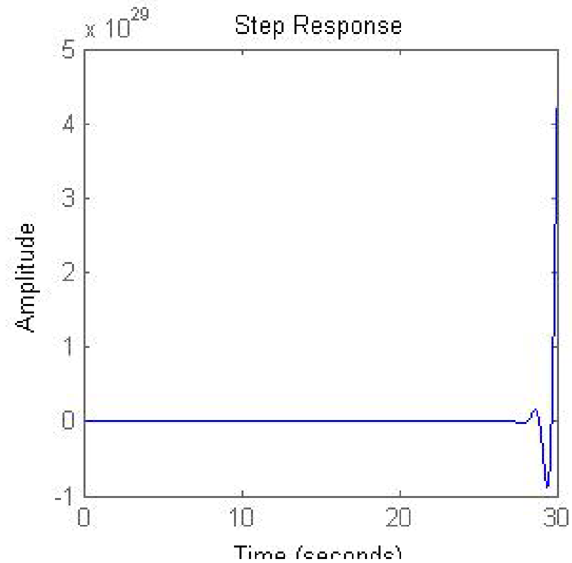
C.

I set K_I= .01 in the first one and K_I = 100 in the second one.





D.

I set K_s = .01 in the first one and K_s = 100 in the second one.