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PS06

Jessica Diller

① explain how the recorded audio signal of a gun being fired in a shooting range can be convolved with a violin recording to approximate how the violin would sound if played there using what you know about impulse & impulse response

~~Fas~~ A gunshot ~~sound~~ is similar to an impulse - it is very brief time length and loud (high amplitude)

the recorded sound of this gunshot in the shooting range acts an impulse response as it records the output of the room (the system) to this gunshot

to determine the effect of a system on a potential input signal you convolve this signal w/ the "impulse response"

thus, when you convolve the violin signal (sound) with the gunshot recorded sound (impulse response) you get the hypothetical output signal → the ideal sound of a violin being played in the shooting range

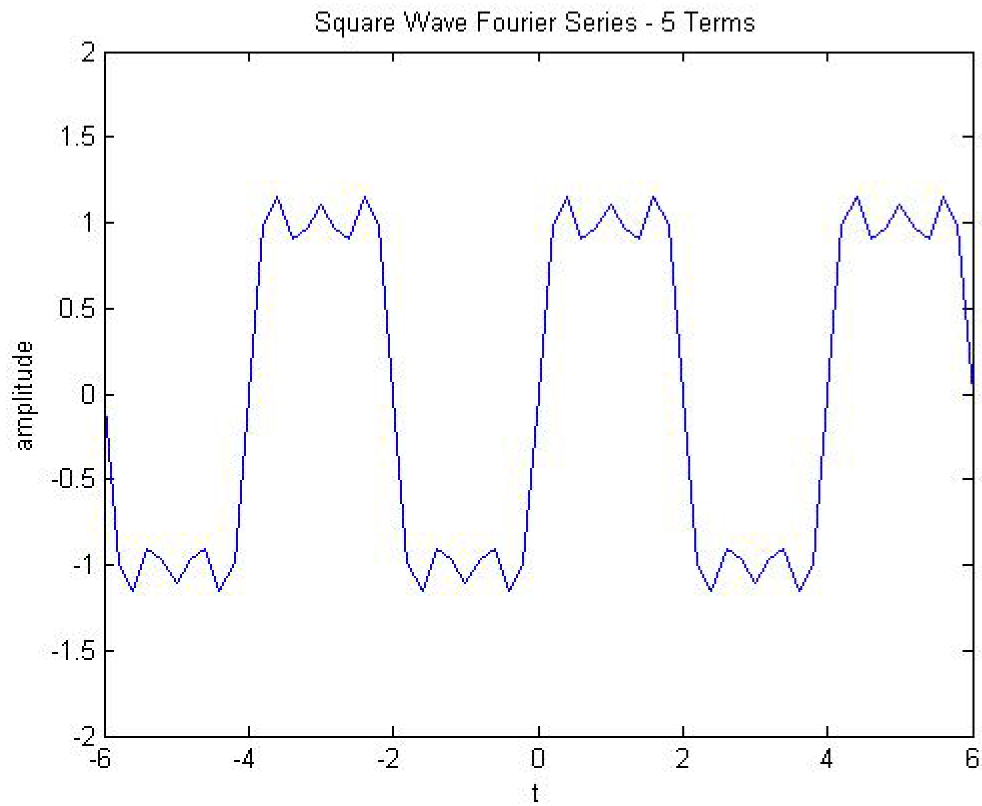
(2) Simple model of an echo channel

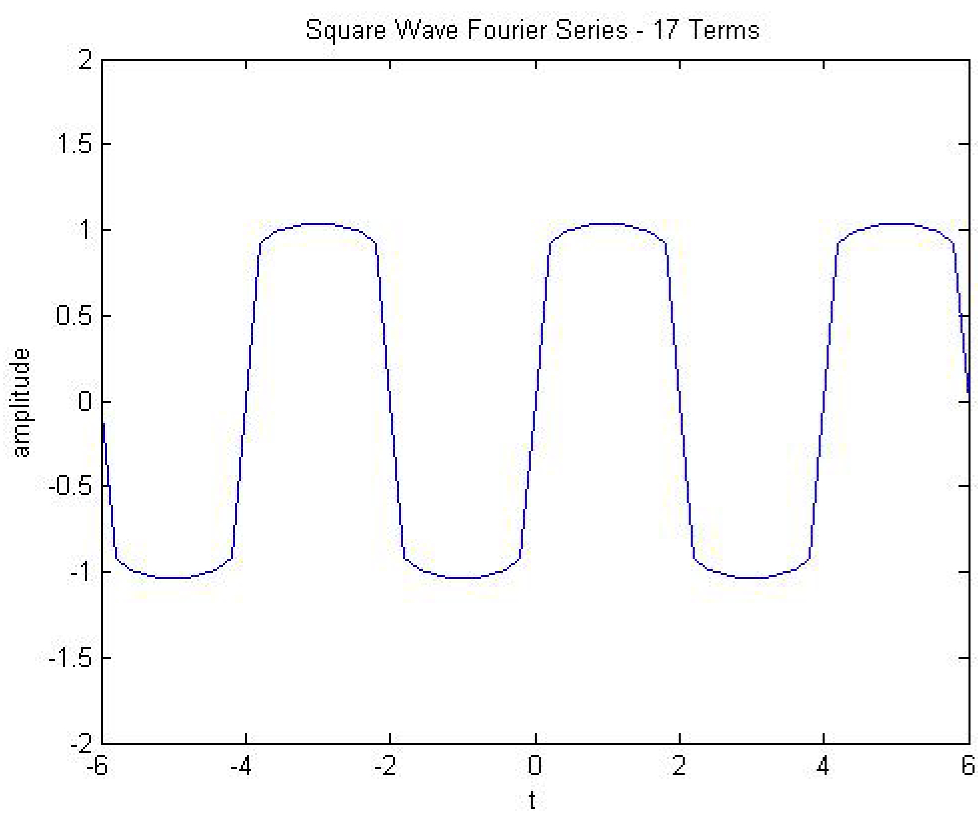
output = $y(t)$ input = $x(t)$

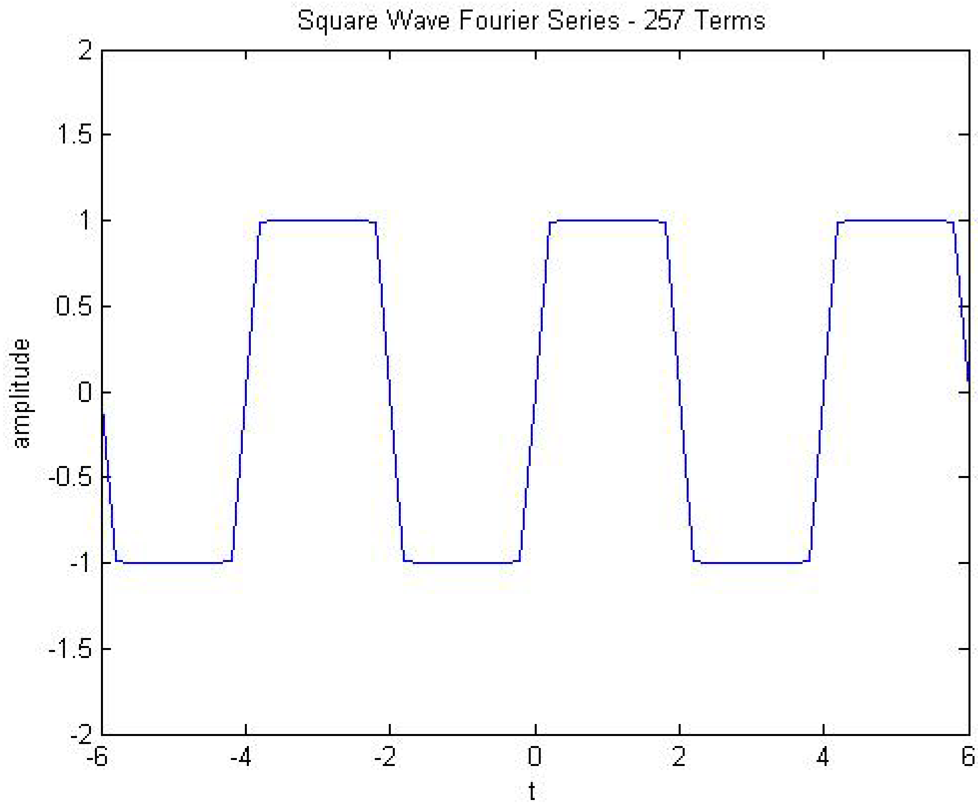
$y(t) = \frac{1}{2} x(t-1) + \frac{1}{4} x(t-10)$

explain why its reasonable to call this an echo channel & find expressions for the impulse response of this system & sketch

An echo channel has signal "echos", ~~that~~ ~~responses~~ system responses, that are delayed from the signal and scaled down. If a signal was input it would respond in decreasing amplitude


Square Wave Fourier Series - 5 Terms
amplitude
t
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-6
-4
-2
0
2
4
6


Square Wave Fourier Series - 17 Terms
amplitude
t
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-6
-4
-2
0
2
4
6


Square Wave Fourier Series - 257 Terms
amplitude
t
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-6
-4
-2
0
2
4
6

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Poly

③ a) find fourier series of sq. wave example

$$\sin(\theta) = \frac{1}{2j} e^{j\theta} - \frac{1}{2j} e^{-j\theta}$$

$$x(t) = \begin{cases} \text{if } \cancel{\;} -T < x < -\frac{3T}{4} & 1 \\ \text{if } -\frac{3T}{4} < x < \frac{T}{-4} & 0 \\ \text{if } \frac{T}{-4} < x \frac{T}{4} & 1 \\ \text{if } \frac{T}{4} < x \frac{3T}{4} & 0 \\ \text{if } \frac{3T}{4} < x < T & 1 \end{cases}$$

$$X(t) = \begin{cases} \text{if } -\frac{T}{2} < x < -\frac{T}{4} & 0 \\ \text{if } -\frac{T}{4} < x < \frac{T}{4} & 1 \\ \text{if } \frac{T}{4} < x < \frac{T}{2} & 0 \end{cases}$$

$$\tilde{X}_K(t) = \sum_{k=-K}^{K} C_k e^{j2\frac{\pi}{T}kt}$$

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

$$C_k = \frac{1}{T}\left[\int_{-T/2}^{-T/4} 0 \cdot e^{-j\frac{2\pi}{T}kt}\, dt + \int_{-T/4}^{T/4} 1 \cdot e^{-j\frac{2\pi}{T}kt}\, dt + \int_{T/4}^{T/2} 0 \cdot e^{-j\frac{2\pi}{T}kt}\, dt\right]$$

$$C_k = \frac{1}{T}\left[\cancel{\int_{-T/2}^{-T/4} 0\, dt} + \int_{-T/4}^{T/4} e^{-j\frac{2\pi}{T}kt}\, dt + \cancel{\int_{T/4}^{T/2} 0\, dt}\right]$$

$$C_k = \frac{1}{T}\left[\frac{1}{j\frac{2\pi}{T}kt} e^{-j\frac{2\pi}{T}kt}\Big|_{-T/4}^{T/4}\right]$$

$$\cancel{C_k = \frac{1}{T}\left[\frac{1}{2j} \cdot \frac{1}{\frac{2\pi}{T}k\frac{T}{4}} e^{-j\frac{2\pi}{T}k\frac{T}{4}}\right]}$$

$$C_k = \frac{1}{T} \frac{1}{j\frac{2\pi}{T}k}\left[\frac{1}{T/4} e^{-j\frac{2\pi}{T}k\,T/4} + \frac{1}{T/4} e^{+j\frac{2\pi}{T}k\,\frac{T}{4}}\right]$$

$$C_k = \frac{1}{2\pi j k}\left[\frac{4}{T} e^{-j\frac{\pi}{2}k} + \frac{4}{T} e^{j\frac{\pi}{2}k}\right]$$

$$C_k = \frac{4}{T\pi k}\left[\frac{1}{2j} e^{-j\frac{\pi}{2}k} + \frac{1}{2j} e^{j\frac{\pi}{2}k}\right]$$

I keep rechecking my math and signs, but can't seem to figure out the way to put it into $\sin(\theta)$ form from here to get further along

$$x(t) = \sum_{k=-\infty}^{\infty} \frac{4}{T\pi k}\left[\frac{1}{2j} e^{-j\frac{\pi}{2}k} + \frac{1}{2j} e^{j\frac{\pi}{2}k}\right] e^{j\frac{2\pi}{T}kt}$$

b) using a computer, plot the Fourier Series representation of a square ~~wave~~ w/ $T=4$ & 5, 17, & 257 terms in Fourier series

I know I didn't finish the problem above & didn't get a correct equation useable in this part so I went online to find the actual Fourier series to plot

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c) describe what you see at the discontinuous points of the square wave

well (10) of the notes says $\int_{-T/2}^{T/2} |x(t) - \tilde{x}_K(t)|^2 dt \to 0$

as the $K \to \infty$

it isn't a straight line (doesn't jump) even at large N values, there is some limit. it's because Fourier series ~~can't~~ don't approximate discrete jumps very well. in (10) for discrete jumps, ~~in one~~ the value of $x(t)$ will change a lot, but $\tilde{x}_K(t)$ will not change as much

(4) a) $x(t)$ periodic w/ fundamental period T

Fourier Series w/ coefficients C_K

consider new signal $y(t) = x(t - T_1)$

where $|T_1| < T$ $\quad$ $y(t)$ = delayed version of $x(t)$

find Fourier Series coefficients for $y(t)$ in terms of C_K

$$C_K = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

$$= \frac{1}{T}\int_{-T/2-w}^{T/2-w} x(t)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

~~$C_{K_y} = \frac{1}{T}\int_{-T/2-w}^{T/2-w} x(t)\, e^{-j\frac{2\pi}{T}kt}\, dt$~~

well given you could do it over any interval, they ($x(t)$ & $y(t)$) have the same C_K. to solve, I would just pick the interval of $y(t)$ that matches $x(t)$'s $-T/2$ to $T/2$

b) So using the same logic, the C_k would be the same for a shifted triangle wave

I don't have the code from class and I was having trouble finding resources online - I got lost in the amount of data out there & I couldn't ~~find the~~ find the exact information I needed ☹