Jessica Diller

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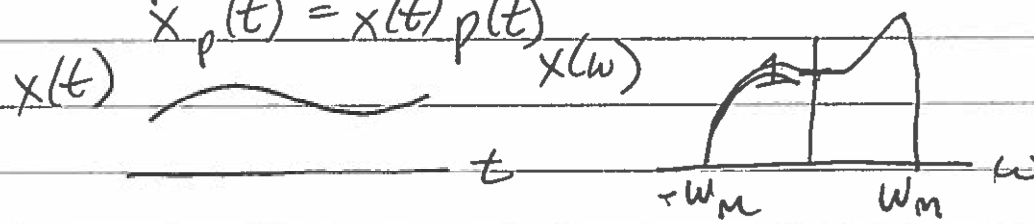
PS 08

(1) Consider signal $x(t)$ band limited to ω_M

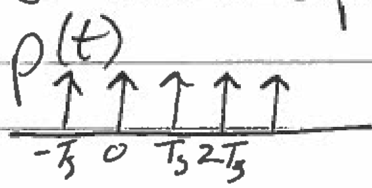
$p(t)$ is an impulse train w/impulses by T_s

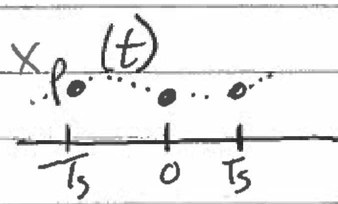
$$p(t) = \sum \delta(t - kT_s)$$

$$x_p(t) = x(t)\,p(t)$$

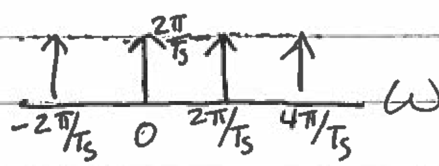


a) sketch a representation of $x_p(t)$

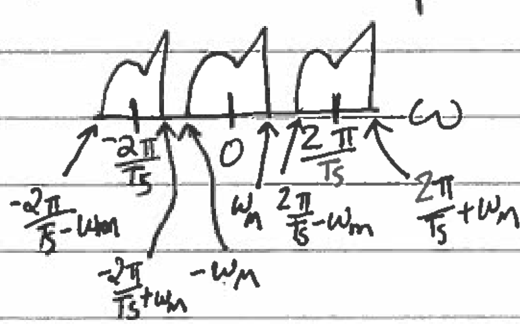




b) sketch $P(\omega)$



c) sketch $X_p(\omega)$



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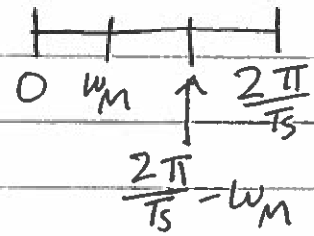
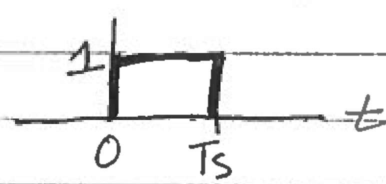
d) what is the relationship btwn T_s & ω_M which ensures $X_p(\omega)$ contains all info presented in $X(\omega)$?



$$\frac{2\pi}{T_s} > 2\omega_M$$

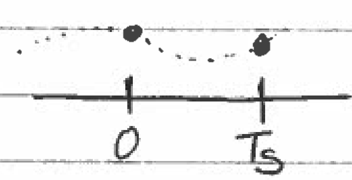
e) how can you recover $x(t)$ from $x_p(t)$ exactly?

f) consider new signal $z(t)$



rectangular pulse

g) sketch $x_2(t) = x_p * z(t)$



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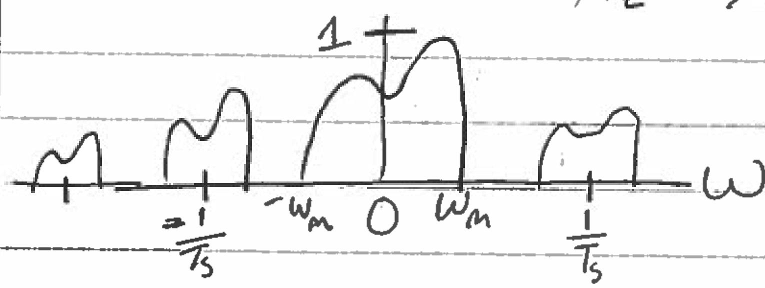
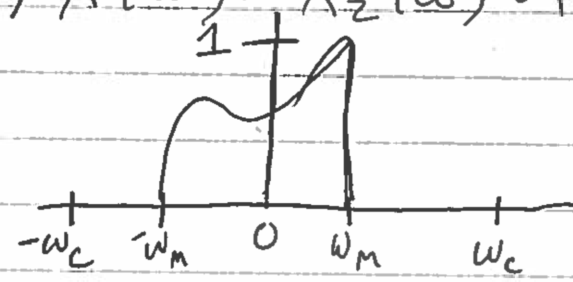
h) Sketch → $X_z(\omega)$

period=T
$\frac{1}{T}$ zeros @



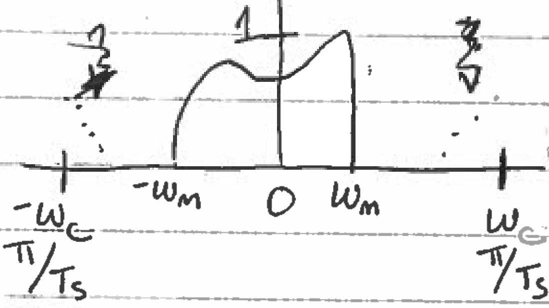
i) $\bar{X}(\omega) = X_z(\omega) \cdot H(\omega)$ $\omega_c = \frac{\pi}{T_s}$



$\hat{X}(\omega) = X_p(\omega) \cdot H(\omega)$ $\omega_c = \frac{\pi}{T_s}$

$\frac{2\pi}{T_s} > 2\omega_m$

$\frac{\pi}{T_s} > \omega_m$



I'm not 100% sure $2\omega_m > \frac{\pi}{T_s}$, ~~$2\omega_m < \frac{2\pi}{T_s}$~~ so there could be some leakage from the ~~signals~~ @ frequencies $2\omega_m$ & $-2\omega_m$

for both: I multiplied everything within the range $-\omega_c$ to ω_c by 1 and everything outside that range by 0

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1) j) ~~how are $\bar{X}(\omega)$ & $\hat{X}(\omega)$ different?~~

1) j) ~~see~~ $\hat{X}$ & $\bar{X}$ have very slight differences near the top a distortion

k) ratio of $\hat{X}(\omega_m)$ $\bar{X}(\omega_m)$

@ $\omega_m = \pi / T_s$

$\boxed{X_2(\omega) = X_p(\omega) \cdot Z(\omega)}$ $x_p \cdot z(t) = \cancel{(\ldots)}$ $x_2(t)$

$Z(\omega) =$ sinc · complex exponential

~~[illegible]~~

$e^{-j\omega\, t\text{shift}}$

X_2 is a function of x_p

$e^{-j\omega\, T_s/2}$

change

$\bar{X}(\omega) = X_p(\omega) Z(\omega) H(\omega)$ $\quad \hat{X} = X_p(\omega) H(\omega)$

$\bar{X}(\omega) = \hat{X}(\omega) \cdot Z(\omega)$

$$\frac{\bar{X}(\omega)}{\hat{X}(\omega)} = Z(\omega) = e^{-j\omega \frac{T_s}{2} \frac{\pi}{T_s}} \frac{2\sin(\omega \frac{T_s}{2})}{\omega}$$

$$e^{-j\pi/2} \frac{2\sin(\frac{\pi}{T_s}\frac{T_s}{2})}{\pi/T_s} = e^{-j\frac{\pi}{2}} \frac{2 \cancel{\sin \frac{\pi}{2}}=1}{\pi/T_s}$$

$$\boxed{e^{-j\frac{\pi}{2}} \frac{2}{(\pi/T_s)}}$$

PS08 continued

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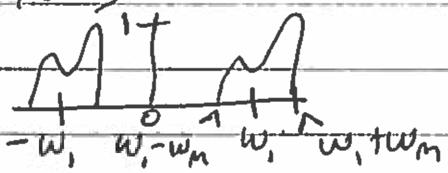
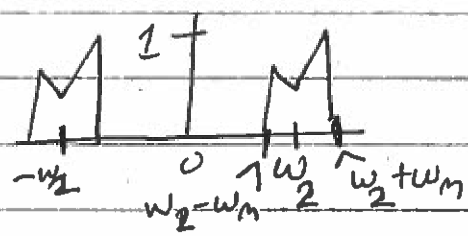
2 a) sketch $Y(\omega)$

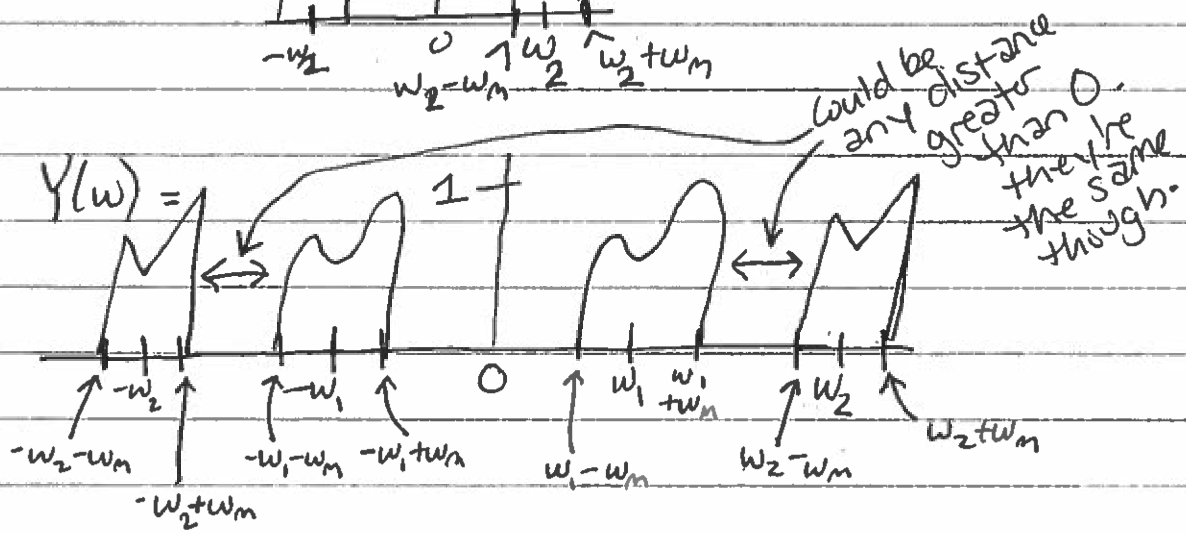
$X_1(\omega) =$



$X_2(\omega) =$

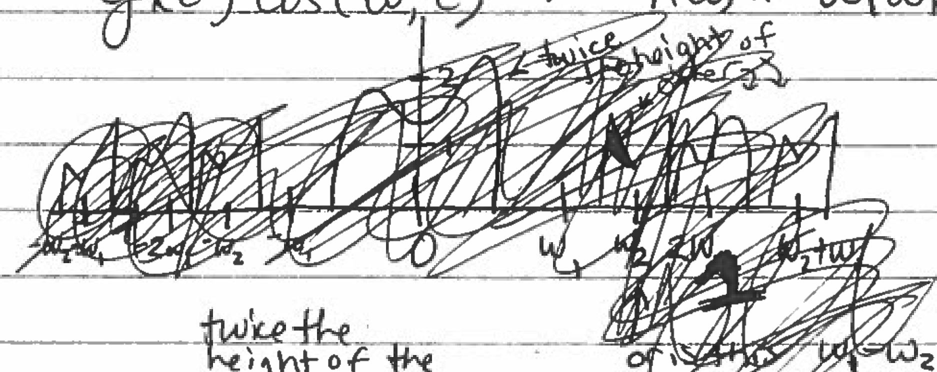


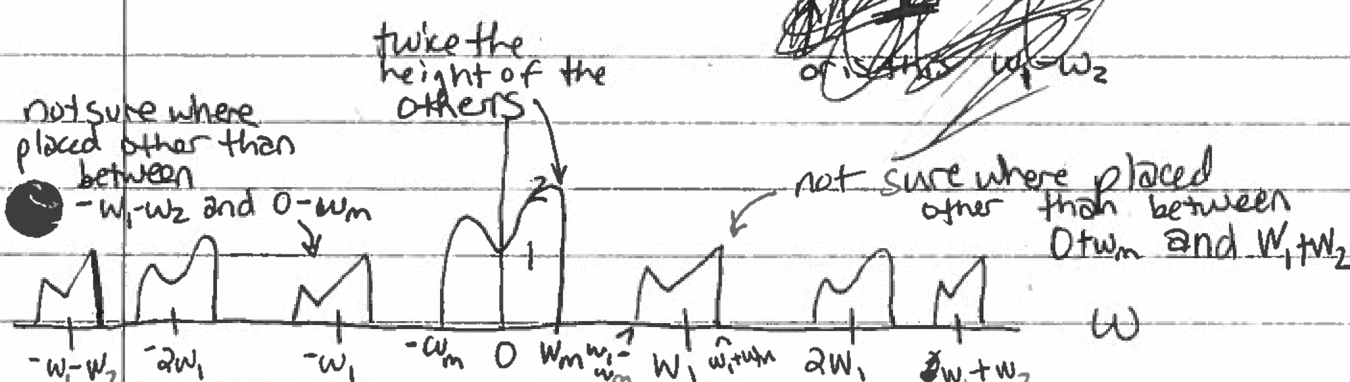
$Y(\omega) =$



b) sketch Fourier transform of

$y(t)\cos(\omega_1 t) \Rightarrow Y(\omega) * \delta(\omega_1)\ \tfrac{1}{2}\delta@\omega_1$





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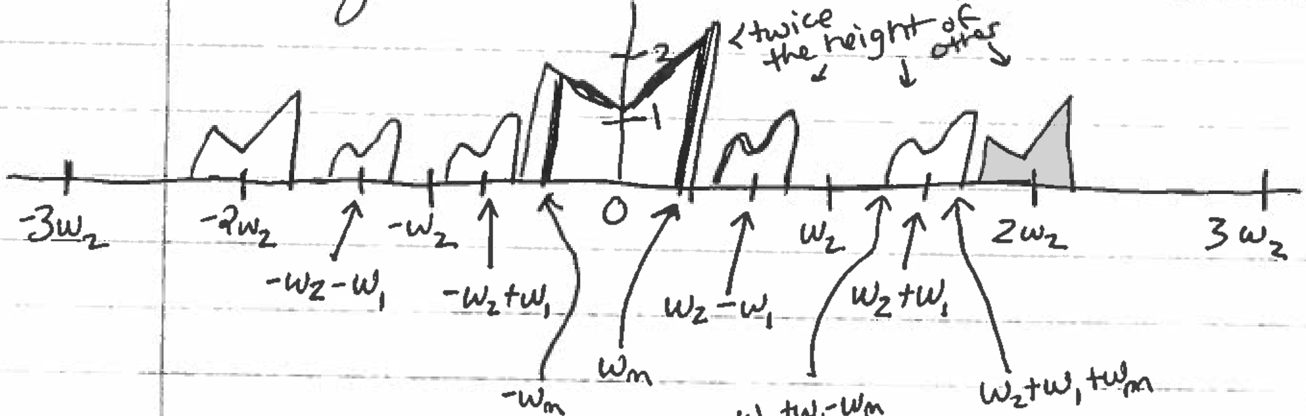
PS 08 (continued)

b) (continued) sketch Fourier transform of
$y(t)\cos(\omega_2 t) \Rightarrow Y(\omega) * \delta @ \omega_2$ & $-\omega_2$



spacing between each thing > 0

c) using appropriate sketches, describe how you can recover $x_1(t)$ and $x_2(t)$ from $y(t)$

$$\left.\begin{array}{l} x_1(t) \xrightarrow{\omega_1} \\ x_2(t) \xrightarrow[\omega_2]{} \end{array}\right\} y(t) \begin{array}{l} \nearrow ? \\ \searrow ? \end{array}$$

take $y(t)\cos(\omega_1 t)$ ~~and convolve~~ in freq domain and then apply a low pass filter w/ ω_m as "limit"
OR just take the thing in the middle @ $\omega = 0$ that's double in strength to all other ones

do again, but w/ $y(t)\cos(\omega_2 t)$

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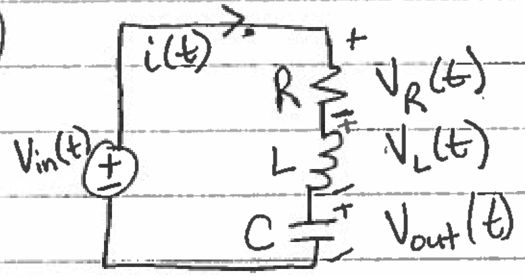
③



$i(t) = C\frac{d}{dt}V_{out}(t)$

$V_L(t) = L\frac{d}{dt}i(t)$

$V_R(t) = i(t)R$

$V_{in}(t) = V_R(t) + V_L(t) + V_{out}(t)$

a) write differential eq. relating $V_{out}(t)$ & $V_{in}(t)$

$$V_{in}(t) = i(t)R + L\frac{d}{dt}i(t) + V_{out}(t)$$

$$V_{in}(t) = rC\frac{d}{dt}V_{out}(t) + L\frac{d}{dt}C\frac{d}{dt}V_{out}(t) + V_{out}(t)$$

$$V_{in}(t) = rC\frac{d}{dt}V_{out}(t) + LC\frac{d^2}{dt^2}V_{out}(t) + V_{out}(t)$$

b) treating $V_{in}(t)$ and $V_{out}(t)$ as inputs & outputs of LTI system find expression for frequency response $H(\omega)$ of the system)

$$V_{in}(t) = \left(C\frac{d}{dt} + LC\frac{d^2}{dt^2} + 1\right)V_{out}(t)$$

$$\cancel{\left(\frac{1}{rC\frac{d}{dt} + LC\frac{d^2}{dt^2} + 1}\right)V_{in}(t) = V_{out}}$$

$$V_{in}(\omega) = rC\,V_{out}(\omega)j\omega + LC(j\omega)^2\,V_{out}(\omega) + V_{out}(\omega)$$

big   big

$$V_{in}(\omega) = (j\omega rC + L(j\omega)^2 + 1))V_{out}(\omega)$$

$$\frac{1}{j\omega rC + LC(j\omega)^2 + 1} = \frac{V_{out}(\omega)}{V_{in}(\omega)} = H(\omega)$$

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③ c) $H(\omega)$ magnitude

$$RC\,j\omega + LC(j\omega)^2 + 1$$

$$\underbrace{RC\,j\omega}_{\text{imaginary}} + \underbrace{-LC\omega^2 + 1}_{\text{real}}$$

$$\sqrt{(-LC\omega^2+1)^2 + (RC\omega)^2}$$

$$\sqrt{L^2C^2\omega^4 - 2LC\omega^2 + 1 + R^2C^2\omega^2}$$

$$\frac{1}{\sqrt{L^2C^2\omega^4 - 2LC\omega^2 + 1 + R^2C^2\omega^2}}$$

d) as function of R, L, C, find ω to max $|H(\omega)|$

smallest $\sqrt{L^2C^2\omega^4 - 2LC\omega^2 + 1 + R^2C^2\omega^2}$

$$(L^2C^2\omega^4 - 2LC\omega^2 + 1 + R^2C^2\omega^2)$$

$$4L^2C^2\omega^3 - 4LC\omega + 2R^2C^2\omega$$

$$0 = \omega(4L^2C^2\omega^2 + 2R^2C^2 - 4LC)$$

$$\boxed{\omega = 0}$$

$$4L^2C^2\omega^2 = 2R^2C^2 - 4LC$$

$$\omega^2 = \frac{2R^2C^2 - 4LC}{4L^2C^2}$$

$$\boxed{\cancel{\omega = \pm\sqrt{\frac{2R^2C^2 - 4LC}{4L^2C^2}}}} = \frac{2R^2C^2}{4L^2C^2} - \frac{4LC}{4L^2C^2}$$

Cameron told me my signs were wrong, but I didn't ~~want~~ feel it needed to check my algebra

$$\boxed{\omega = \pm\sqrt{\frac{R^2}{2L^2} - \frac{1}{LC}}}$$

Jessica Diller

PS08 Question Three

E)