PS07

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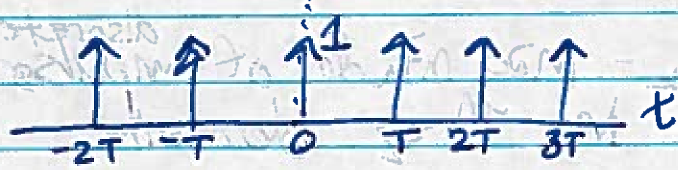
1) a) sketch representation of p(t)

$$p(t) = \sum_{k=-\infty}^{\infty} \delta(t-kT)$$



b) find Fourier series representation of p(t)

$$C_k = \frac{1}{T}\int x(t)\, e^{-j\frac{2\pi}{T}kt}\, dt$$

$$C_k = \frac{1}{T}\int_{-T/2}^{T/2}\left[\sum_{k=-\infty}^{\infty} \delta(t-kT)\right] e^{-j\frac{2\pi}{T}kt}$$

$$C_k = \frac{1}{T}\left[\cancel{\int_{-T/2}^{0} 0\, e^{-j\frac{2\pi}{T}kt}} \to 0 + \int_{0}^{0} 1\cdot e^{-j\frac{2\pi}{T}kt}\, dt + \cancel{\int_{0}^{T/2} 0\cdot e^{-j\frac{2\pi}{T}kt}} \to 0\right.$$

$$C_k = \frac{1}{T}\cancel{\int_{0}^{0} 1\, e^{-j\frac{2\pi}{T}kt}\, dt} \to 1$$

Unit impulse has integral = 1

$$\boxed{\tilde{x}_K(t) = \sum_{k=-K}^{K} \frac{1}{T} e^{j\frac{2\pi}{T}kt}}$$

① c) let a function $x(t) = \sum_{k=-\infty}^{\infty} c_k e^{j\frac{2\pi}{T}kt}$

find $X(\omega)$ in terms of C_k

~~tk~~ C_k - magnitude of discrete impulse in freq domain

~~kreet~~

kind of ignore infinite sum because infinite sum ⟺ infinite integral

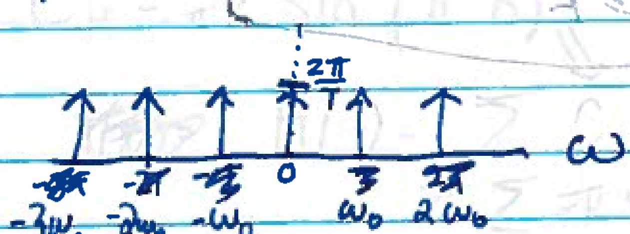
$$\boxed{X(\omega) = C_k \sum_{k=-\infty}^{\infty} 2\pi\,\delta(\omega-\omega_0)}$$

d) using your answers ↑, find $P(\omega)$

$$p(\omega) = \cancel{2\pi}\ \frac{1}{T} \cdot \sum_{k=-\infty}^{\infty} 2\pi\,\delta(\omega-\omega_0)$$

$$\boxed{P(\omega) = \frac{2\pi}{T} \sum_{k=-\infty}^{\infty} \delta(\omega-\omega_0)}$$

e) sketch $P(\omega)$, how does changing T affect $p(t)$ & $P(\omega)$? expected?



changing T

$p(t)$: ~~PR~~ T affects the spacing between impulses

$P(\omega)$: T affects the amplitude of frequency components

② consider LTI system w/impulse response $h(t)$, input signal $x(t)$, output $y(t)$

a) find $h(t)$

$$C_k = \frac{1}{T}\int_{-\omega_c}^{\omega_c} e^{-j\frac{2\pi}{2\omega_c}kt}\,dt$$

$$= \frac{1}{2\omega_c}\int_{-\omega_c}^{\omega_c} e^{-j\frac{\pi}{\omega_c}kt}\,dt$$

$$= \frac{1}{2\omega_c}\left[\frac{1}{-j\frac{\pi}{\omega_c}k}e^{-j\frac{\pi}{\omega_c}kt}\right]_{-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\omega_c}\left[-\frac{1}{j\frac{\pi}{\omega_c}k}e^{-j\frac{\pi}{\omega_c}k\omega_c} + \frac{1}{j\frac{\pi}{\omega_c}k}e^{+j\frac{\pi}{\omega_c}k\omega_c}\right]$$

$$= \frac{1}{\omega_c}\left[\frac{1}{\frac{\pi}{\omega_c}k}\cdot\frac{1}{-2j}e^{-j\frac{\pi}{\omega_c}k\omega_c} + \frac{1}{2j}\cdot\frac{1}{\frac{\pi}{\omega_c}k}e^{j\frac{\pi}{\omega_c}k\omega_c}\right]$$

$$= \frac{1}{\pi k}\left(-\frac{1}{2j}e^{-j\pi k} + \frac{1}{2j}e^{j\pi k}\right)$$

$$C_k = \frac{1}{\pi k}\sin(\pi k)$$

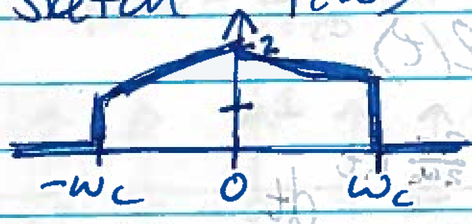
$$\Downarrow$$

$$H(\omega) = \sum C_k e^{-j\frac{2\pi}{2\omega_c}kt}$$

$$= \sum \frac{1}{\pi k}\sin(\pi k)e^{-j\frac{\pi}{\omega_c}kt}$$

$$h(t) = \frac{1}{2\pi}\int \sum \frac{1}{\pi k}\sin(\pi k)\, e^{-j\frac{\pi}{\omega_c}kt}$$

b) sketch $Y(\omega)$ $\quad Y(\omega) = X(\omega)H(\omega)$



c) explain why this system = ideal low pass filter

because all frequencies above and below ~~ω~~ $-\omega_c$ & ω_c are multiplied by 0, thus ~~p~~radically eliminated

d) modify code to implement low pass filter w/ cutoff frequency $\omega_c = 0.75\pi$

run & then do ~~w~~ $\omega_c = 1.75\pi$

turn in plots for both

(3) signal $x(t)$ range $[-\omega_m, \omega_m]$

$X(\omega)=0$ for $\omega < -\omega_m$ & $\omega > \omega_m$

$y(t) = x(t)\cos(\omega_c t)$ where $\omega_c \gg \omega_m$

sketch $Y(\omega)$

using property

$y(t) = x(t)h(t)$

$\Downarrow$

$Y(\omega) = \frac{1}{2\pi} X * H(\omega)$

$h(t) = \cos(\omega_c t)$

$H(\omega) = \pi\,\delta(\omega-\omega_c) + \pi\,\delta(\omega+\omega_c)$

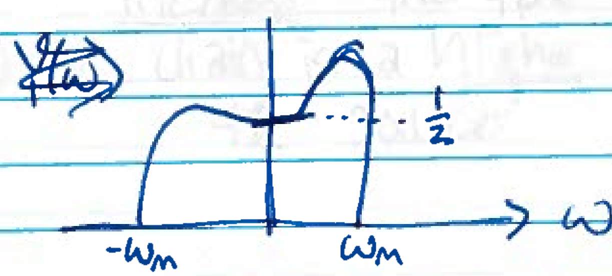
$Y(\omega) = \frac{1}{2\pi} \cdot X(\omega) * (\pi\,\delta(\omega-\omega_c) + \pi\,\delta(\omega+\omega_c))$

graph

magnifies the amplitude of $X(\omega)$ by π

Since/if $\omega_c \gg \omega_m$

you know ~~there are~~ only impulse within the ω values $-\omega_m \leftrightarrow \omega_m$ ~~are~~ is @ $\omega=0$ w/ amplitude $=\pi$



then

$\frac{1}{2\pi} \cdot (X(\omega) \cdot \pi)$

$= \frac{1}{2}(X(\omega))$