The Living Influence Model (LIM): A Network-Theoretic, Physics-Based Theory of Space, Connectivity, and Off-Ball Threat in Football

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Abstract

We present the Living Influence Model (LIM), a theoretical framework to quantify tactical influence in association football as a function of player reachability, subgraph connectivity, and off-ball scoring threat. LIM integrates physics-based control times—time-to-intercept (TTI) and time-to-control (TTC)—with network-theoretic pass probabilities and off-ball scoring opportunity (OBSO) fields to construct a unified influence kernel over pitch locations and time. We define "living space" as regions that are both controllable (a team player holds a control advantage) and valuable (the region either supports a high-probability pass hub or confers significant off-ball threat). We introduce a notion of subgraph health based on spectral connectivity, edge volatility, and path redundancy, and we operationalize collapse risk as the short-horizon chance that structural connectivity or team influence falls below critical thresholds. LIM admits a high-fidelity implementation with tracking data and a credible events-only surrogate, enabling research-grade evaluation now and elite deployment with full tracking later.

Keywords: football analytics, spatiotemporal modeling, network theory, pitch control, off-ball threat, spectral graph theory

1 Introduction

Formation labels (e.g., 4–3–1–2) understate how teams actually control space, exchange influence, and rewire interactions across phases of play. We propose the Living Influence Model (LIM): a physics-grounded, graph-based theory where a player's value is defined relationally—by how their presence and movement modulate team connectivity and opponent behavior. LIM unifies (i) physics-based control times for reach and stabilization, (ii) probabilistic pass edges, and (iii) off-ball scoring fields into an interpretable influence framework applicable at frame rate. The central premise is that *space matters only insofar as it contributes to graph continuity and threat*; hence we distinguish living from dead space and evaluate formation not as a static shape but as a structural function of influence.

Contributions. (1) A unified influence kernel that fuses edge (pass) and node (off-ball threat) channels through control probabilities; (2) a definition of living space and subgraph health with spectral and redundancy metrics; (3) algorithmic pipelines for both tracking and events-only data; (4) testable predictions and a validation suite relating LIM signals to entries and shots; (5) an interpretable analytic vocabulary for coaches and analysts.

2 Preliminaries and Notation

Let $\mathcal{T}$ and $\mathcal{O}$ be our team and opponents. Players $i \in \mathcal{T} \cup \mathcal{O}$ have positions $p_i(t) \in \mathbb{R}^2$ and velocities $v_i(t)$; the ball is at $p_B(t)$. The pitch is discretized into cells $z \in \mathcal{G}$ with spatial weights $w(z)$ (e.g., expected threat prior). At time t, the directed team graph is $G_t = (V_t, E_t)$ with adjacency $A(t) = [A_{ij}(t)]$ over $i, j \in \mathcal{T}$.

2.1 Physics-based control

From the pass-physics literature, time-to-intercept (TTI) and time-to-control (TTC) give a control time

$$\mathrm{TTR}_i(z,t) = \mathrm{TTI}_i(z,t) + \mathrm{TTC}_i(z,t), \tag{1}$$

and a control advantage relative to the fastest opponent

$$\Delta_i(z,t) = \min_{k \in \mathcal{O}} \mathrm{TTR}_k(z,t) - \mathrm{TTR}_i(z,t). \tag{2}$$

We convert advantage to a soft control probability via a logistic:

$$\pi_i(z,t) = \sigma(\beta\, \Delta_i(z,t)), \quad \sigma(x) = \frac{1}{1+e^{-x}}. \tag{3}$$

2.2 Pass probabilities (edges)

Let $P_{\text{pass}}(i \to j \mid t) \in [0,1]$ be the pass success probability from i to j at time t, parameterized by ball-line geometry, receiver motion, and opponent interception physics. This induces the adjacency

$$A_{ij}(t) = P_{\text{pass}}(i \to j \mid t), \quad i \neq j,\ i,j \in \mathcal{T}. \tag{4}$$

Define the (symmetrized) Laplacian $\mathcal{L}(t) = (D(t) - A(t) + (D(t) - A(t))^\top)/2$, where $D_{ii}(t) = \sum_j A_{ij}(t)$.

2.3 Off-ball scoring (nodes)

Let $\Theta_i(z,t) \in [0,1]$ be the off-ball scoring opportunity (OBSO) field: probability that player i could generate a shot (or scoring opportunity) from z within horizon H given the spatiotemporal context. We will also use the prospective threat when i could establish control at z:

$$\tilde{\theta}_i(z,t) = \Theta_i(z,t) \cdot \pi_i(z,t). \tag{5}$$

3 Unified Influence Kernel and Living Space

We mix on-ball and off-ball channels using an instantaneous possession propensity $q_i(t) \in [0,1]$ (estimated from ball proximity, last touch, and role).

$$\Phi_i(z,t) = q_i(t) \left[\pi_i(z,t) \cdot \max_{j \in \mathcal{T} \setminus \{i\}} P_{\text{pass}}(i \to j \mid t; z)\right] + \big(1 - q_i(t)\big)\, \tilde{\theta}_i(z,t). \tag{6}$$

The player and team influence aggregates are

$$\mathrm{LIM}_i(t) = \sum_{z \in \mathcal{G}} w(z)\, \Phi_i(z,t), \tag{7}$$

$$\Phi_{\mathcal{T}}(z,t) = \max_{i \in \mathcal{T}} \Phi_i(z,t), \qquad \mathrm{LIM}_{\mathcal{T}}(t) = \sum_{z \in \mathcal{G}} w(z)\, \Phi_{\mathcal{T}}(z,t). \tag{8}$$

We define a **living space** indicator for team $\mathcal{T}$ at cell z:

$$\mathrm{Live}(z,t) = \nVdash\Big[\max_{i\in\mathcal{T}} \pi_i(z,t) > \tau_c\Big] \cdot \nVdash\Big[\max_{i\in\mathcal{T}} \Big(\lambda_E \max_j P_{\mathrm{pass}}(i \to j \mid t; z) + \lambda_N\, \tilde{\theta}_i(z,t)\Big) > \tau_v\Big]. \tag{9}$$

Intuitively, space is living if it is controllable and tactically valuable (either as a pass hub or a threat region).

4 Network Health, Stability, and Collapse Risk

We monitor structural stability via three axes:

(a) **Spectral connectivity**: the algebraic connectivity $\lambda_2(t)$ of $\mathcal{L}(t)$;

(b) **Edge volatility**: $\nu_{ij}(t) = \sqrt{\frac{1}{W}\sum_{\tau\in[t-W,t]}(A_{ij}(\tau) - \bar{A}_{ij})^2}$ over a window W;

(c) **Path redundancy**: the fraction of node pairs connected by at least two disjoint high-probability routes (above threshold η).

We define subgraph health

$$\mathcal{H}_{\mathcal{T}}(t) = \lambda_{\mathrm{conn}}\, \lambda_2(t) - \lambda_{\mathrm{vol}}\overline{\nu}(t) + \lambda_{\mathrm{red}}\, \bar{\rho}(t), \tag{10}$$

and a short-horizon collapse risk

$$\mathcal{R}(t) = \mathbb{E}\Big[\mathbf{1}\{\lambda_2(t+\delta) < \tau_\lambda \text{ or } \mathrm{LIM}_{\mathcal{T}}(t+\delta) - \mathrm{LIM}_{\mathcal{T}}(t) < -\tau_\Delta\} \mid \delta \le H\Big]. \tag{11}$$

We conjecture monotonic relationships between λ_2 and expected possession duration, and between redundancy and turnover resistance; these are testable with tracking.

5 Algorithms

Tracking-rich pipeline

1. Compute TTI/TTC for all players and grid cells z; obtain $\pi_i(z,t)$.
2. Fit/pass the physics-based pass model $P_{\mathrm{pass}}(i \to j|t)$ using tracking & events.
3. Estimate OBSO $\Theta_i(z,t)$; derive $\tilde{\theta}_i(z,t)$.
4. Evaluate $\Phi_i(z,t)$, $\Phi_{\mathcal{T}}(z,t)$, living space mask, and LIM aggregates.
5. Build $A(t)$, compute λ_2, volatility, redundancy; report $\mathcal{H}_{\mathcal{T}}(t)$ and $\mathcal{R}(t)$.

6 Validation and Testable Predictions

We outline three families of tests:

(a) **Predictive**: Windows with higher λ_2 and higher $\Phi_{\mathcal{T}}$ should show higher third-entry and box-entry rates; ablations removing node or edge channels degrade prediction.

(b) **Temporal lead**: Rising $\mathcal{R}(t)$ precedes turnovers; spikes in $\Phi_{\mathcal{T}}$ precede shots.

(c) **Analyst concordance**: Blinded analyst ratings of "structurally strong/fragile" sequences correlate with $\mathcal{H}_{\mathcal{T}}(t)$.

Calibration uses reliability curves (for pass model), Brier scores, and permutation baselines (role-shuffled teams).

7 Discussion: Analyst Semantics and Coaching Use

LIM provides interpretable overlays: living-space maps for training cues; subgraph-health timelines for in-game risk management; and player influence profiles that separate on-ball facilitation from off-ball gravity. The model is modular: clubs with tracking can enable the full physics channels; events-only environments can deploy LIM-lite for scouting and research.

8 Limitations and Future Work

Limitations include sensitivity to calibration of TTI/TTC and OBSO, discretization effects, and reliance on position imputation in events-only settings. Future work includes multi-horizon influence forecasting, opponent-specific suppression models, and learning morph kernels that capture phase-aware role shifts.

9 Conclusion

LIM formalizes formation analysis as dynamic, phase-aware network mechanics. By unifying control physics, pass probabilities, and off-ball threat into a single influence construct, LIM explains how teams create and deny living space, why subgraphs stabilize or collapse, and how player roles translate into structural outcomes. The framework is both implementable and interpretable, enabling rigorous evaluation with public data and high-fidelity deployment with tracking.

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A Parameterization and Hyperparameters

- Decay and logistic slope: α, β.
- Thresholds: $\tau_c, \tau_v, \tau_\lambda, \tau_\Delta$; edge cut η.
- Mixing weights: $\lambda_E, \lambda_N, \lambda_{\text{conn}}, \lambda_{\text{vol}}, \lambda_{\text{red}}$.
- Horizon H, volatility window W; grid $\mathcal{G}$ resolution and weights $w(z)$.

B Algorithmic Details

Pseudocode and complexity for computing π_i, Φ_i, λ_2, redundancy, and volatility over sliding windows; memory/performance notes for tracking vs. events-only modes.