Project 2 Report

MECHENG 5139: Applied Finite Element Method

By: Jessica Hudak

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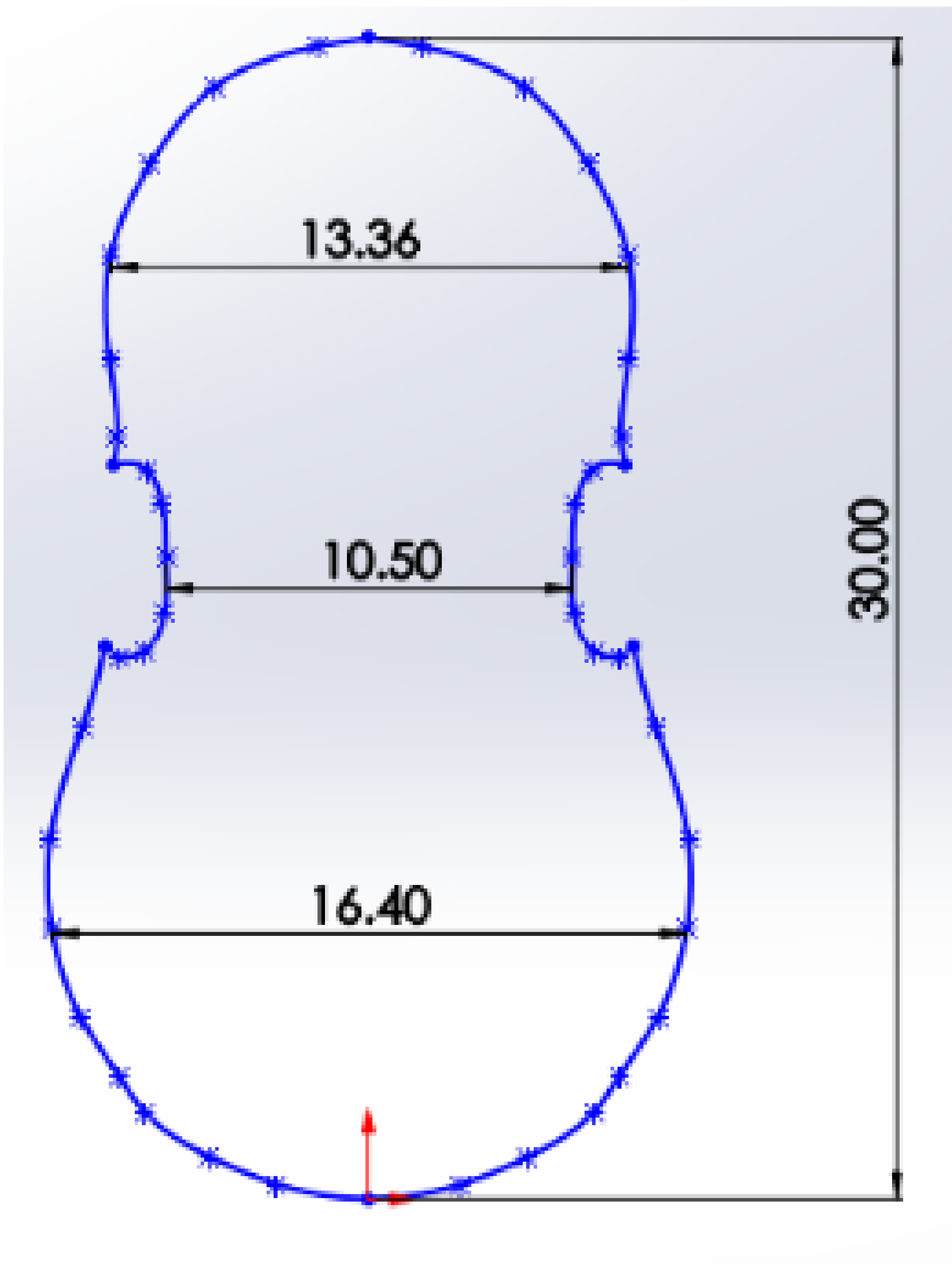
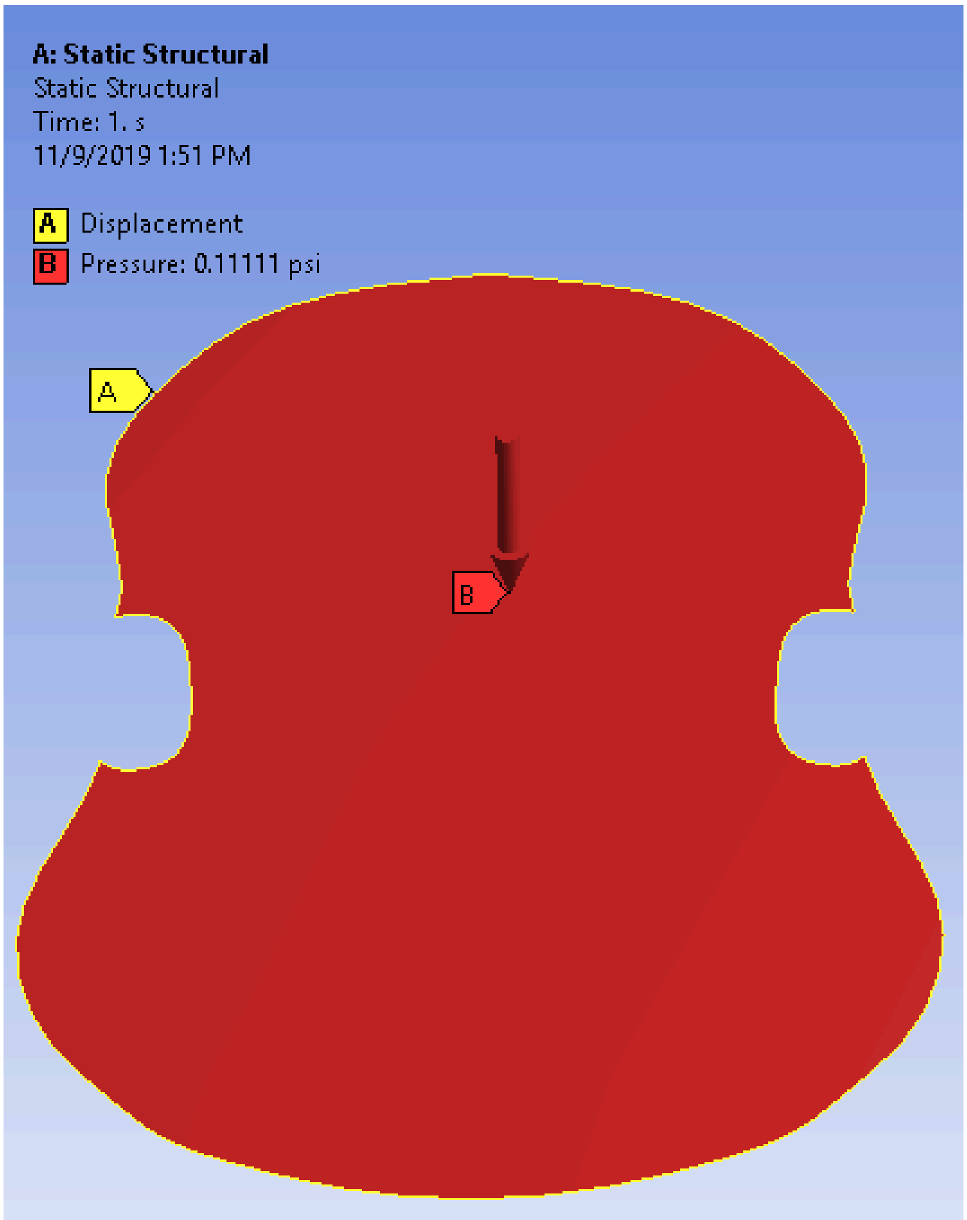
Overview

When Steven Sharp Nelson performs with the Trans-Siberian Orchestra, he gets carried away: he grabs his maple cello by the base of the neck and smashes the back face flat on the ground (similar to smashing a guitar). This exerts a uniform pressure of 0.1111 psi into the face. The dimensions in inches of this ¼-inch-thick face is shown below. Structural and modal analyses were done.



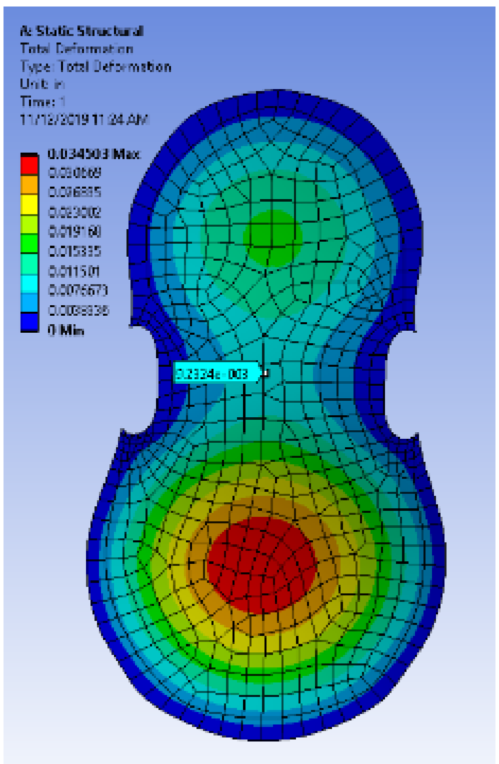
Description of FEA Model

Two quadrilateral mesh densities (of 1-in and 0.5-in elements) were analyzed. All outside edges were given a zero-displacement condition in the x, y, and z directions to mimic rib support. A normal pressure of 0.1111 psi was also added into the back face.

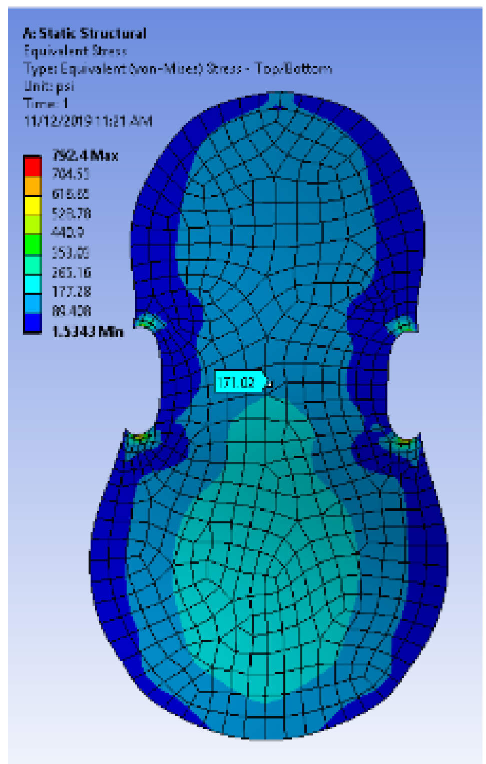


Results (Maple)

By comparing the displacement and stress values of the 1-in and 0.5-in meshes, convergence is ensured. In the modal analysis, the natural frequencies and modal shapes are very similar for the full and half models. Because of the strong alignment of maple's grain orientation, no antisymmetric frequencies were skipped in the half model.



Total Deformation



Equivalent Stress

Element Size	***Max Total Displacement***	***Max Von Mises Stress***
1-inch	0.034503 in	792.4 psi
0.5-inch	0.034534in	781.79 psi

<table>
<tr><th>Maple</th><th>1st Mode</th><th>2nd Mode</th><th>3rd Mode</th><th>4th Mode</th><th>5th Mode</th><th>6th Mode</th></tr>
<tr><td>Full Model
<table>
<tr><th></th><th>Mode</th><th>Frequency [Hz]</th></tr>
<tr><td>1</td><td>1.</td><td>92.201</td></tr>
<tr><td>2</td><td>2.</td><td>134.05</td></tr>
<tr><td>3</td><td>3.</td><td>159.39</td></tr>
<tr><td>4</td><td>4.</td><td>205.48</td></tr>
<tr><td>5</td><td>5.</td><td>245.6</td></tr>
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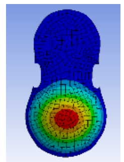
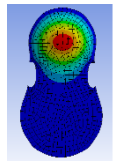
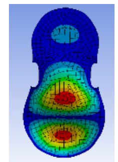
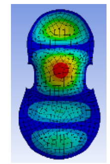
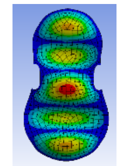
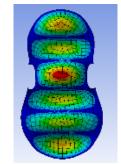
</table></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
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<tr><td>3</td><td>3.</td><td>159.35</td></tr>
<tr><td>4</td><td>4.</td><td>205.5</td></tr>
<tr><td>5</td><td>5.</td><td>245.57</td></tr>
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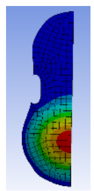
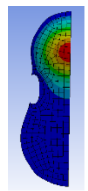
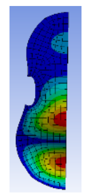
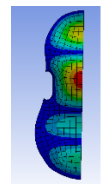
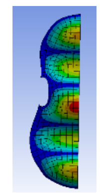
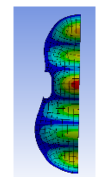
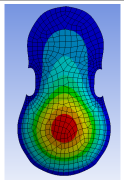
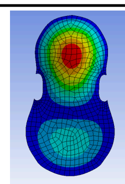
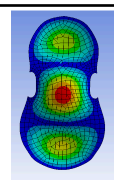
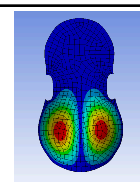
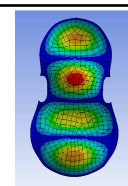
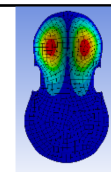
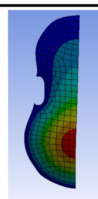
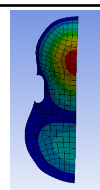
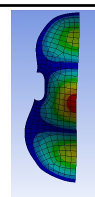
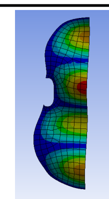
</table></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
</table>

utline Row 3: Maple

A	B	
Property	Value	
Material Field Variables	Table	
Density	0.02315	lb in^-3
Orthotropic Elasticity		
Young's Modulus X direction	1.62E+06	psi
Young's Modulus Y direction	1.08E+05	psi
Young's Modulus Z direction	2.27E+05	psi
Poisson's Ratio XY	0.49	
Poisson's Ratio YZ	0.354	
Poisson's Ratio XZ	0.434	
Shear Modulus XY	1.2E+05	psi
Shear Modulus YZ	32000	psi
Shear Modulus XZ	2.16E+05	psi

Results (Steel)

If the cello was made of (isotropic) structural steel, the half model would miss antisymmetric frequencies:

<table>
<tr><th>Steel</th><th>1st Mode</th><th>2nd Mode</th><th>3rd Mode</th><th>4th Mode</th><th>5th Mode</th><th>6th Mode</th></tr>
<tr><td>Full Model
<table><tr><th></th><th>Mode</th><th>Frequency [Hz]</th></tr><tr><td>1</td><td>1.</td><td>184.81</td></tr><tr><td>2</td><td>2.</td><td>249.39</td></tr><tr><td>3</td><td>3.</td><td>406.84</td></tr><tr><td>4</td><td>4.</td><td>494.72</td></tr><tr><td>5</td><td>5.</td><td>587.63</td></tr><tr><td>6</td><td>6.</td><td>705.35</td></tr></table></td><td></td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>Half Model
<table><tr><th></th><th>Mode</th><th>Frequency [Hz]</th></tr><tr><td>1</td><td>1.</td><td>184.77</td></tr><tr><td>2</td><td>2.</td><td>249.39</td></tr><tr><td>3</td><td>3.</td><td>406.8</td></tr><tr><td>4</td><td>4.</td><td>587.6</td></tr><tr><td>5</td><td>5.</td><td>817.09</td></tr><tr><td>6</td><td>6.</td><td>976.85</td></tr></table></td><td></td><td></td><td></td><td>N/A</td><td></td><td>N/A</td></tr>
</table>

Properties of Outline Row 3: Structural Steel

	A	B	C	D	E
1	Property	Value	Unit		
2	Material Field Variables	Table			
3	Density	0.2836	lb in^-3		
4	Isotropic Secant Coefficient of Thermal Expansion				
5	Coefficient of Thermal Expansion	1.2E-05	C^-1		
6	Isotropic Elasticity				
7	Derive from	Young's Modulus and Poisson...			
8	Young's Modulus	2.9008E+07	psi		
9	Poisson's Ratio	0.3			
10	Bulk Modulus	1.6667E+11	Pa		
11	Shear Modulus	7.6923E+10	Pa		
12	Strain-Life Parameters				
20	S-N Curve	Tabular			
24	Tensile Yield Strength	36259	psi		
25	Compressive Yield Strength	36259	psi		
26	Tensile Ultimate Strength	66717	psi		
27	Compressive Ultimate Strength	0	psi		

Verification

The cello was approximated as a rectangle of length a = 15 in, height b = 30 in, and thickness h = 0.25 in. The case of clamped edges and uniform pressure loading was used for all calculations, which can be found at the end of the appendix. The stress at the center of the cello was estimated to be 198 psi, which is close to the probed stress of 171 psi. The deflection at the center of the cello was estimated with lower and upper bounds (from the highest and lowest orthotropic Young's modulus). The bounds of 0.0061 in and 0.0916 in correctly encompass the probed value of 0.00923 in.

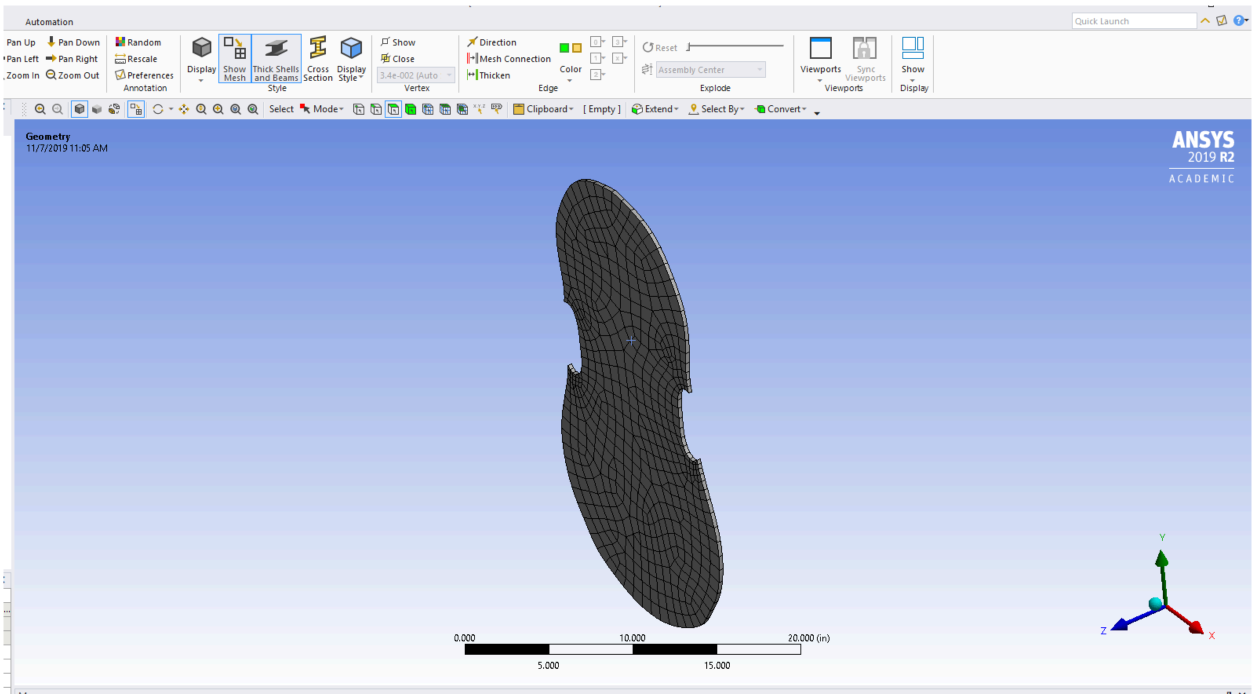
The accuracy of the modal analysis was checked by verifying the value of the first natural frequency. Matlab estimated the maple cello's first frequency to be 82.25 Hz, which is relatively close to ANSYS's calculation of 92.2 Hz. Additionally, the first frequency of the steel cello was estimated to be 251.57 Hz, which is of the same magnitude as ANSYS's 184.8 Hz. It is acceptable for these approximations to be slightly off, as the geometry of the cello is more complex than a mere rectangle. Also, the frequency table used estimated a strict 0.4 length-to-height ratio and was geared for isotropic materials (not orthotropic, like maple).

Conclusion

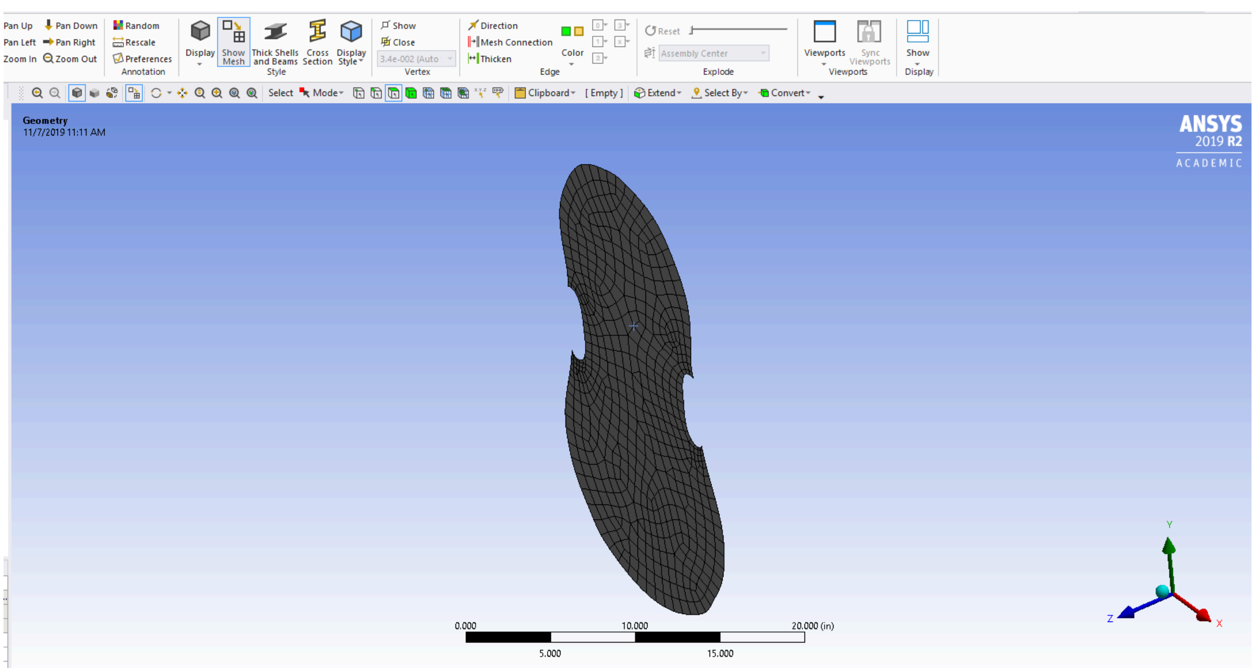
Although slamming a cello on the ground is unadvisable, performing this stunt with a cello that is thick (high h) and stiff (high E) would help minimize deformation. Maple is less dense ($\rho = 5.999 * 10^{-5} \frac{lb\, s^2}{in^4}$) and less stiff ($E_{max}$ = 1.62e6 psi) than steel, which has a density of $7.3395 * 10^{-4} \frac{lb\, s^2}{in^4}$ and Young's modulus of 2.9e7 psi. Maple is a much more common material selection for cellos than steel (due to acoustic reasons) and exhibited lower natural frequencies. However, the maple cello undesirably exhibited larger deformations during this stunt that likely diminished its playing abilities.

Appendix

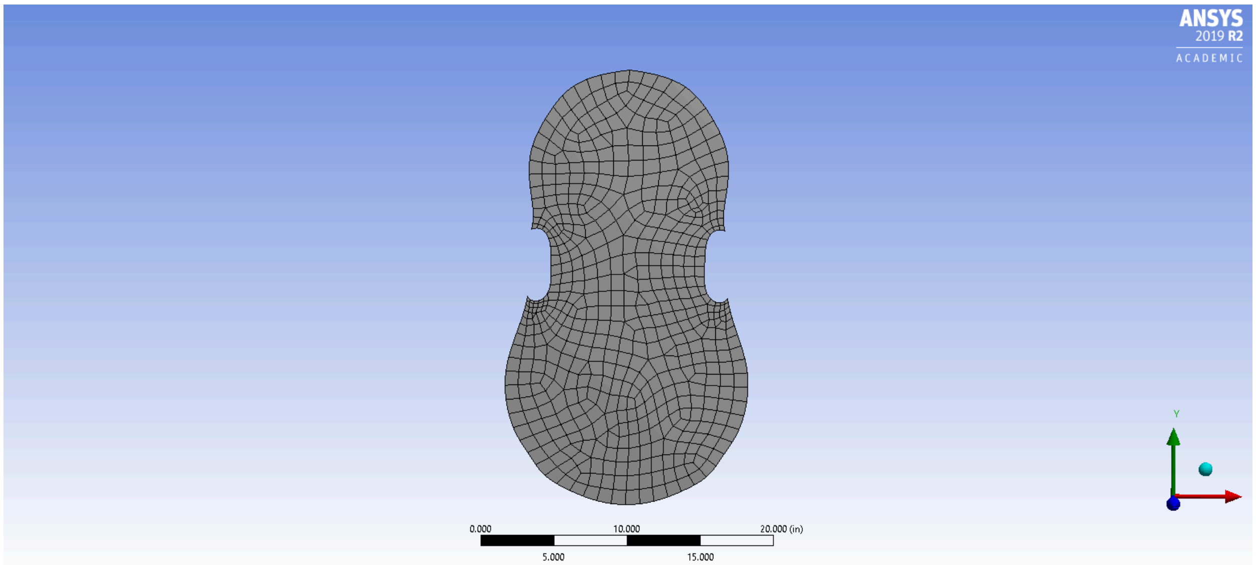
When the “Thick Shells and Beam” option is selected, the cello first looks like a 3D solid:



By unchecking this display option, one can see that the cello face really is meshed with shell elements.



First, a mesh of 1-inch quadratic elements was investigated.



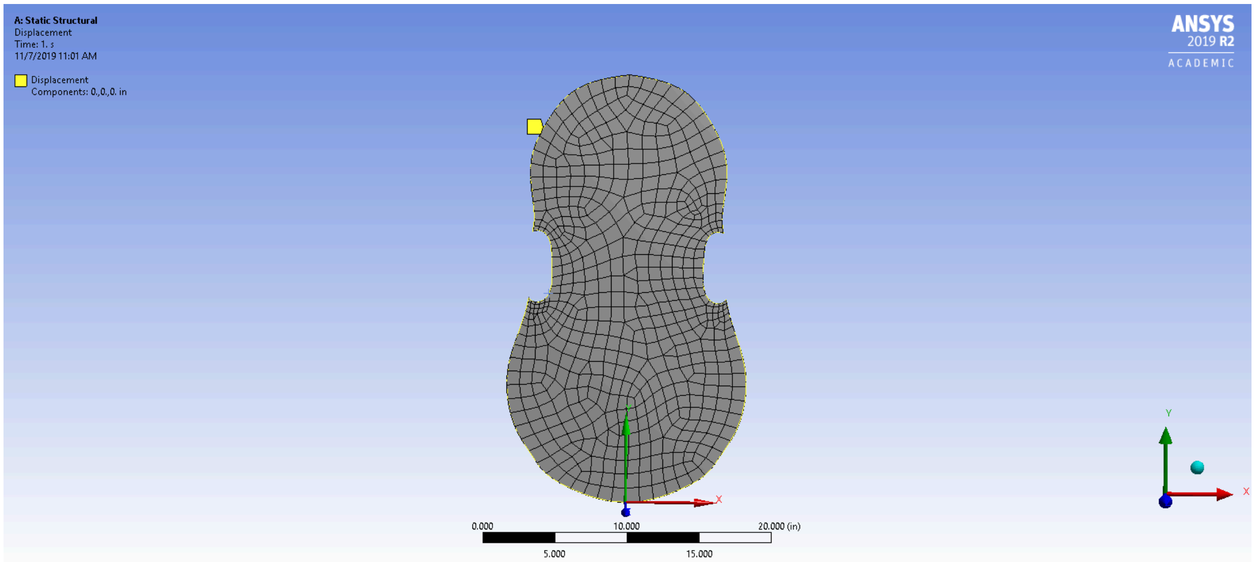
The material assignment for the cello is maple. The orthotropic properties of maple are found in the below table. As a note, the Poisson's ratio in the xy-direction was rounded down from 0.509 to 0.49, as ANSYS can only handle Poisson's ratios that are below 0.5.

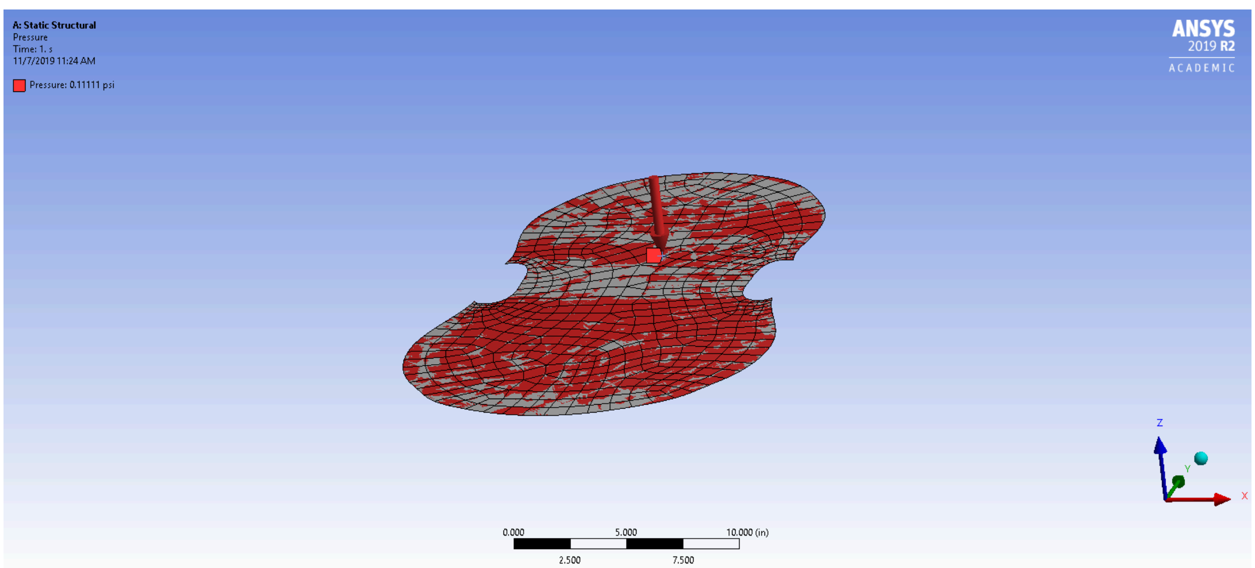
	E_x $\times 10^6$psi	E_y $\times 10^6$psi	E_z $\times 10^6$psi	G_{xy} $\times 10^6$psi	G_{yz} $\times 10^6$psi	G_{xz} $\times 10^6$psi	ν_{xy}	ν_{yz}	ν_{xz}
Spruce	1.43	0.061	0.112	0.087	0.0043	0.0915	0.467	0.245	0.372
Douglas Fir	1.83	0.0915	0.1244	0.143	0.0128	0.117	0.449	0.374	0.292
Maple	1.62	0.108	0.227	0.120	0.032	0.216	0.509	0.354	0.434
CFRP Composite	18.4	11.2	11.2	0.95	4.15	0.95	0.28	0.35	0.28

Table B.3 Anisotropic elastic properties in US Customary Units, grain or fibers are aligned with the x-direction [1].

Boundary conditions:

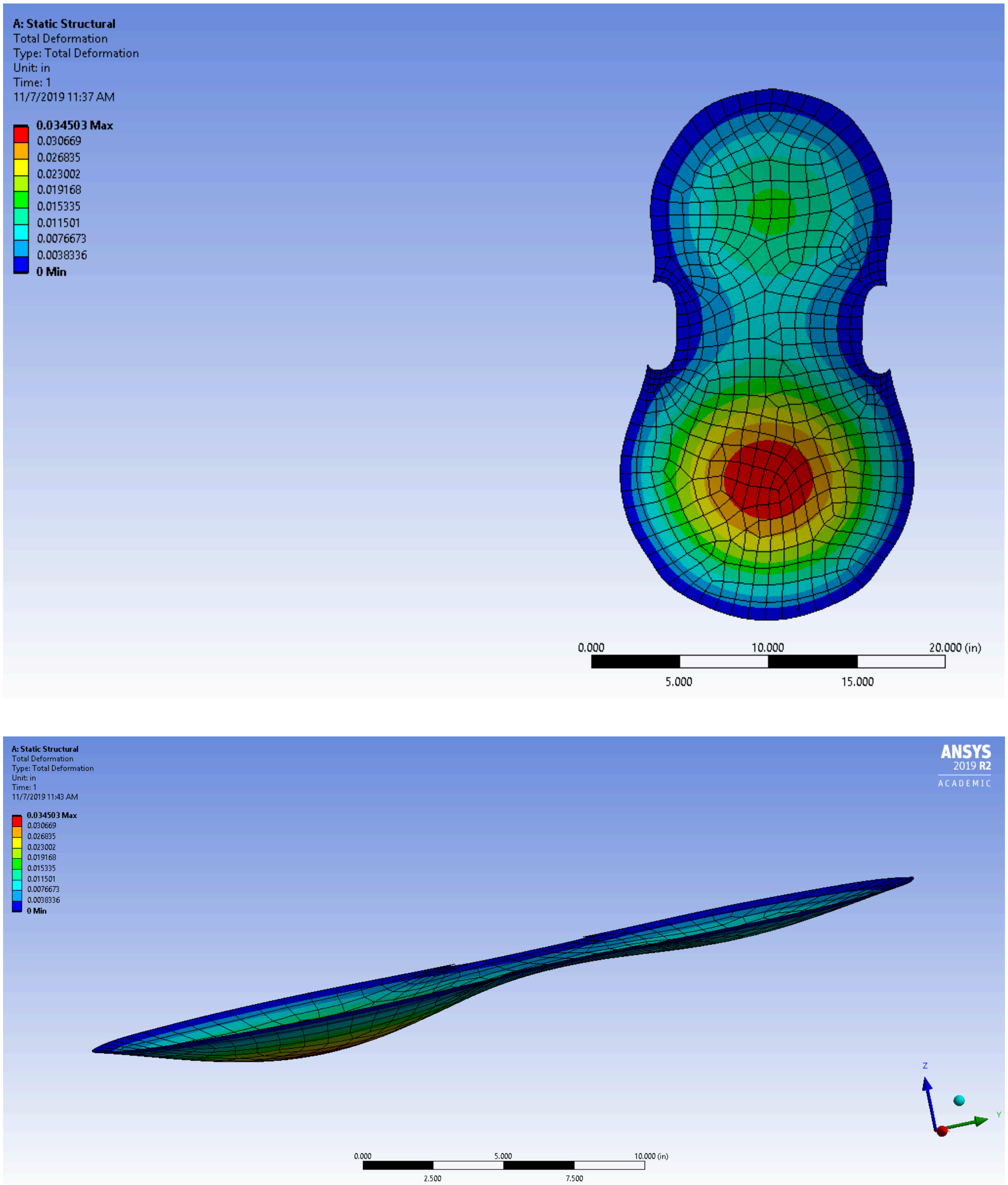
- 0-displacement on all sides in the x, y, and z directions to demonstrate the effect of ribs support
- Pressure of 0.1111 psi across entire back face (in the negative z direction)





Structural Analysis with the 1-inch Mesh:

Total deformation = 0.034503 inches



Total Deformation

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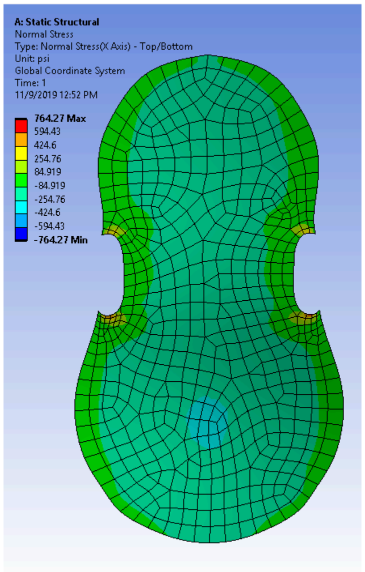
<td>
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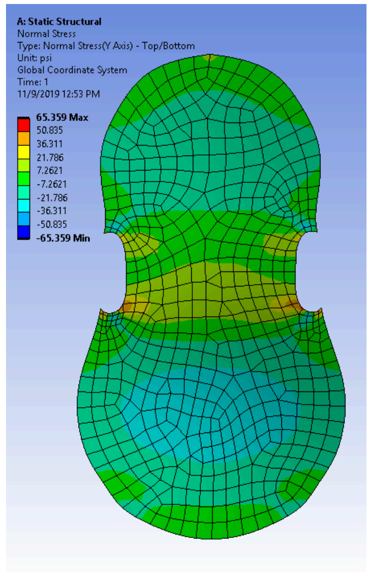
<td>
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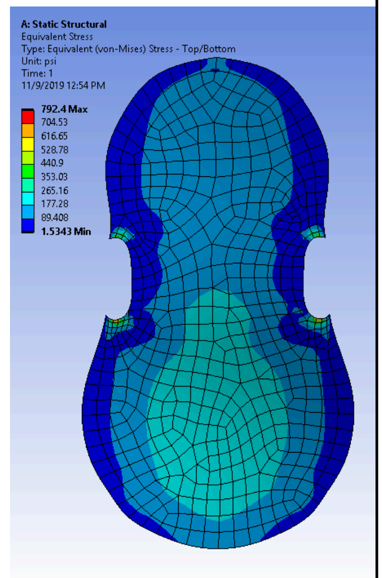
<td>
</td>
</tr>
<tr>
<td>X-Normal Stress</td>
<td>Y-Normal Stress</td>
<td>Von-Mises Stress</td>
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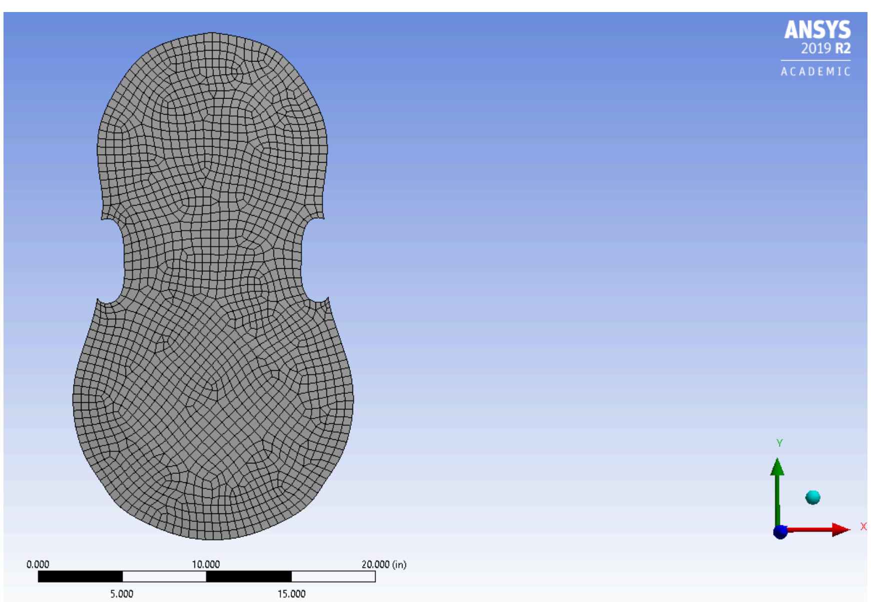
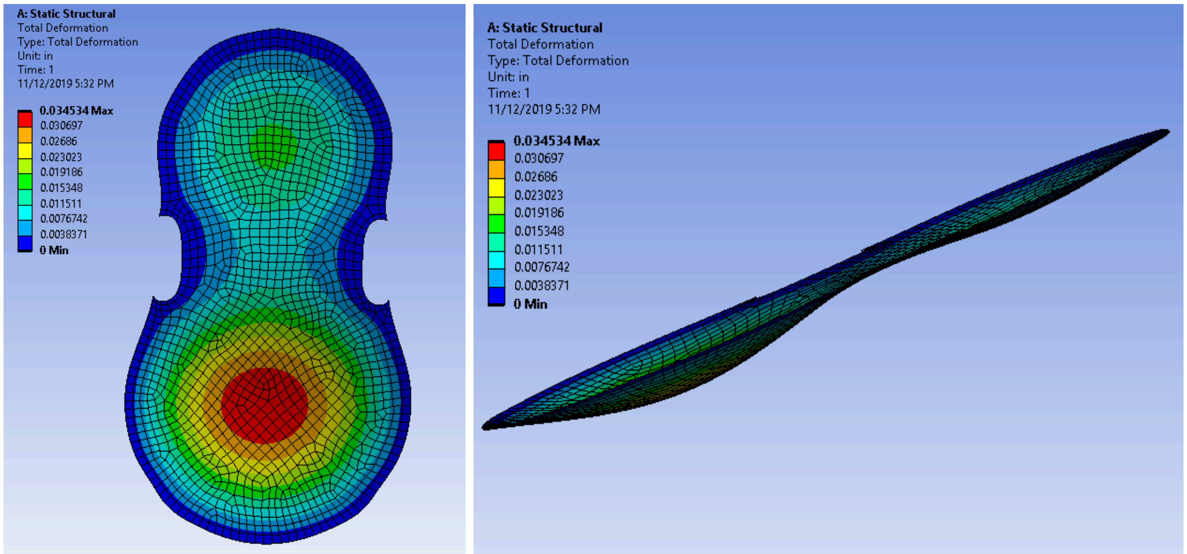
</table>

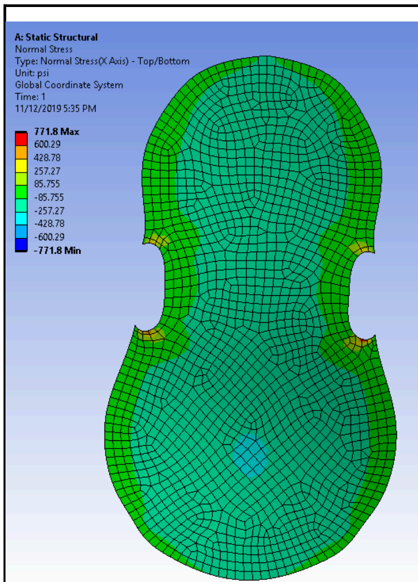
Structural Analysis with the 0.5-inch Mesh:

Now making the mesh size smaller (0.5-inch quadratic elements), total deformation = 0.034534 inches.

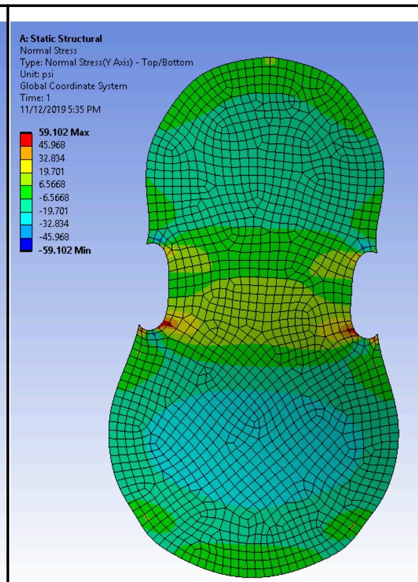




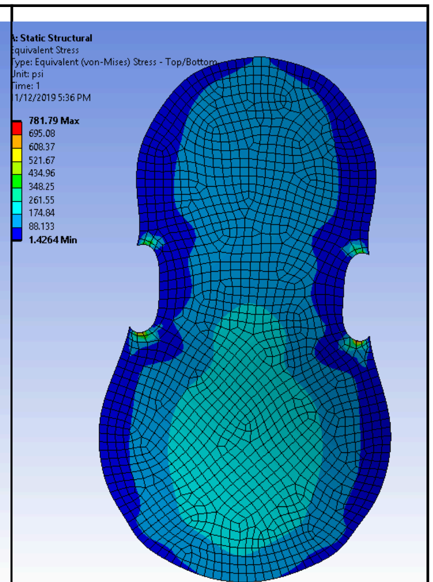
Total Deformation



X-Normal Stress



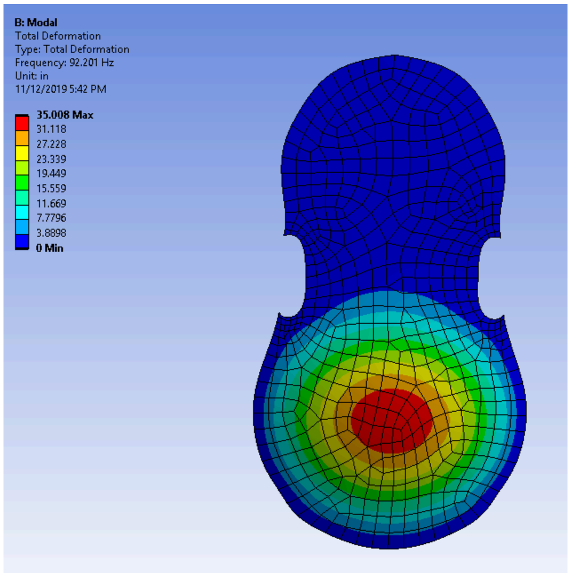
Y-Normal Stress

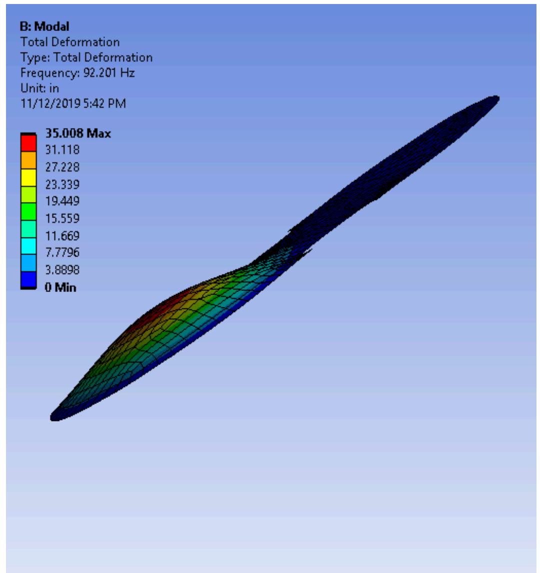


Von-Mises Stress

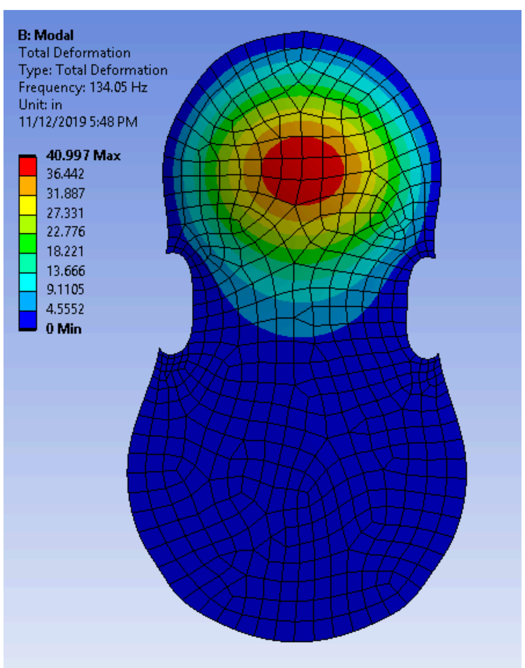
(Maple) Modal Analysis: Full Model

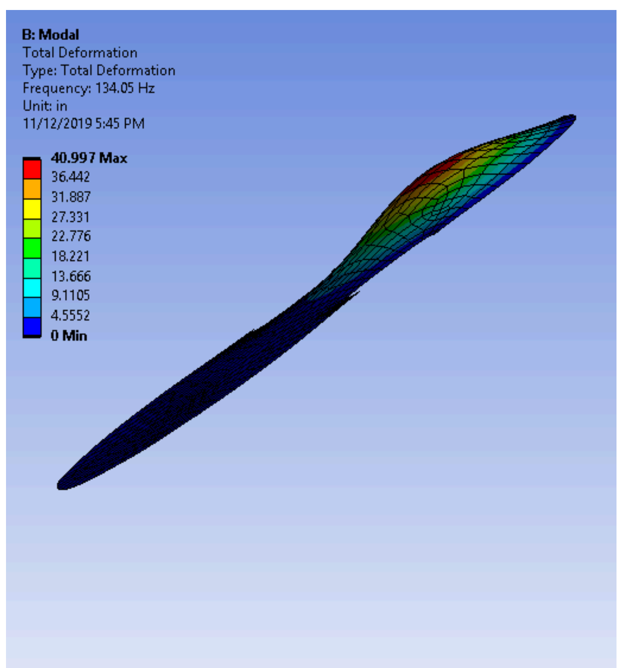
MODE 1:





MODE 2:





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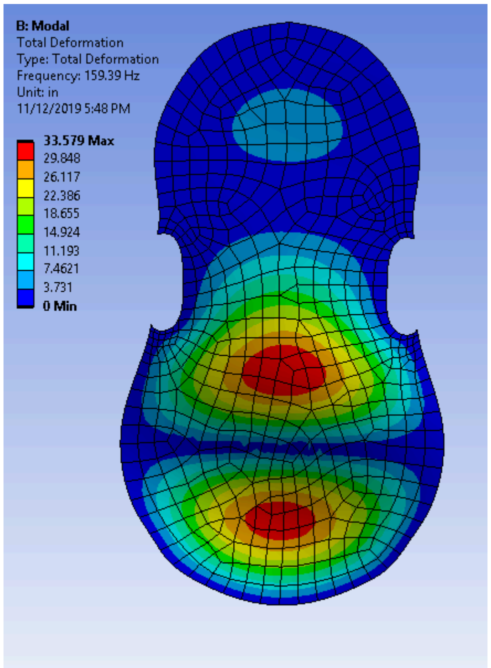
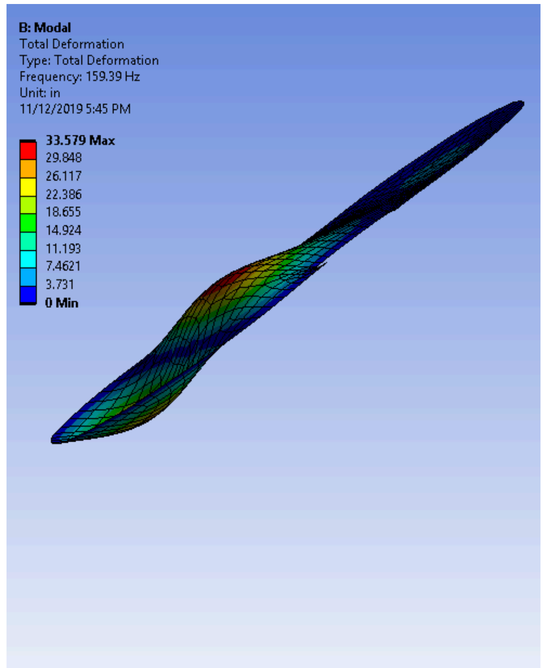
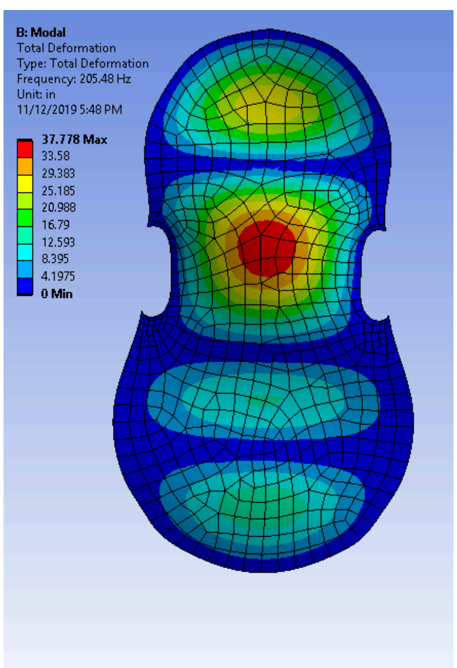
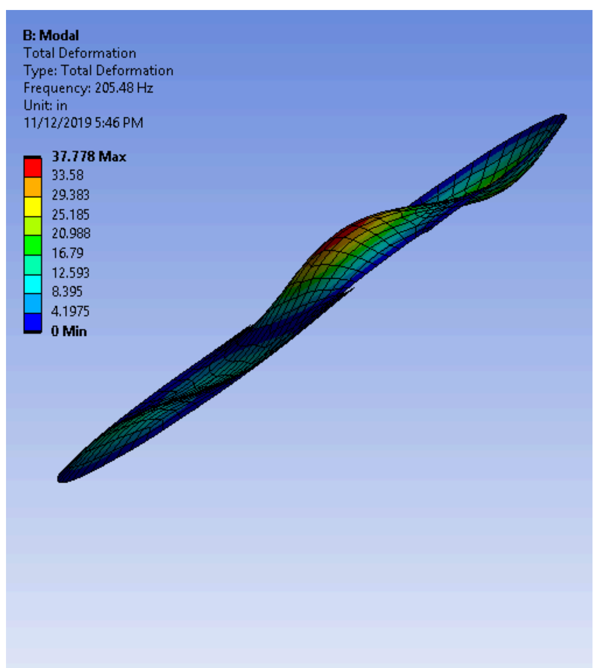

B: Modal
Total Deformation
Type: Total Deformation
Frequency: 159.39 Hz
Unit: in
11/12/2019 5:48 PM
33.579 Max
29.848
26.117
22.386
18.655
14.924
11.193
7.4621
3.731
0 Min

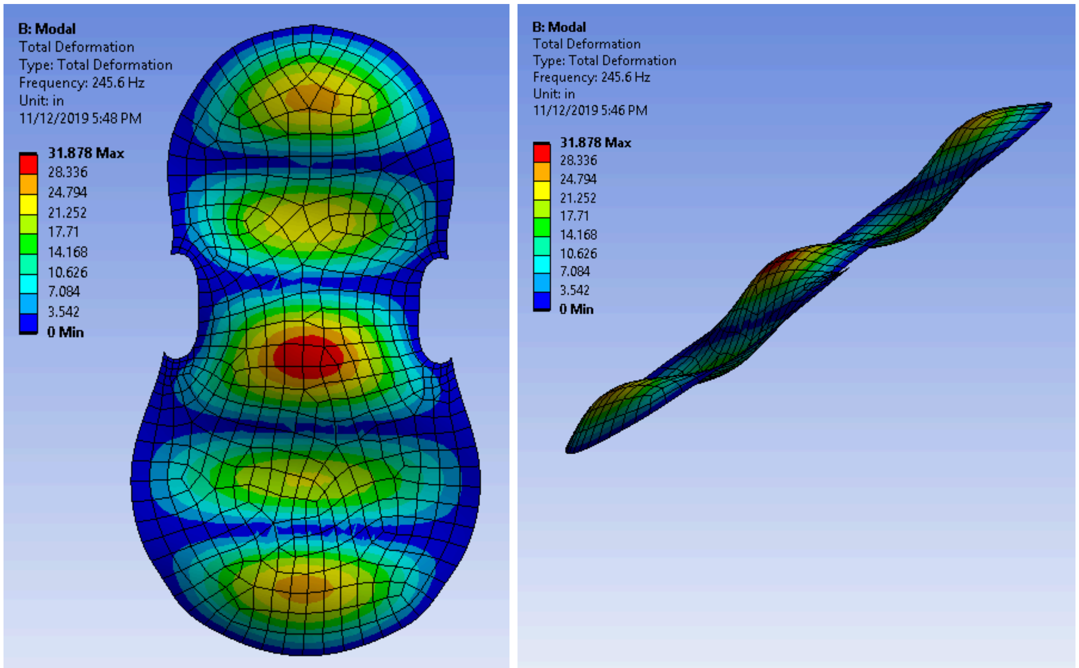

B: Modal
Total Deformation
Type: Total Deformation
Frequency: 159.39 Hz
Unit: in
11/12/2019 5:45 PM
33.579 Max
29.848
26.117
22.386
18.655
14.924
11.193
7.4621
3.731
0 Min

MODE 4:

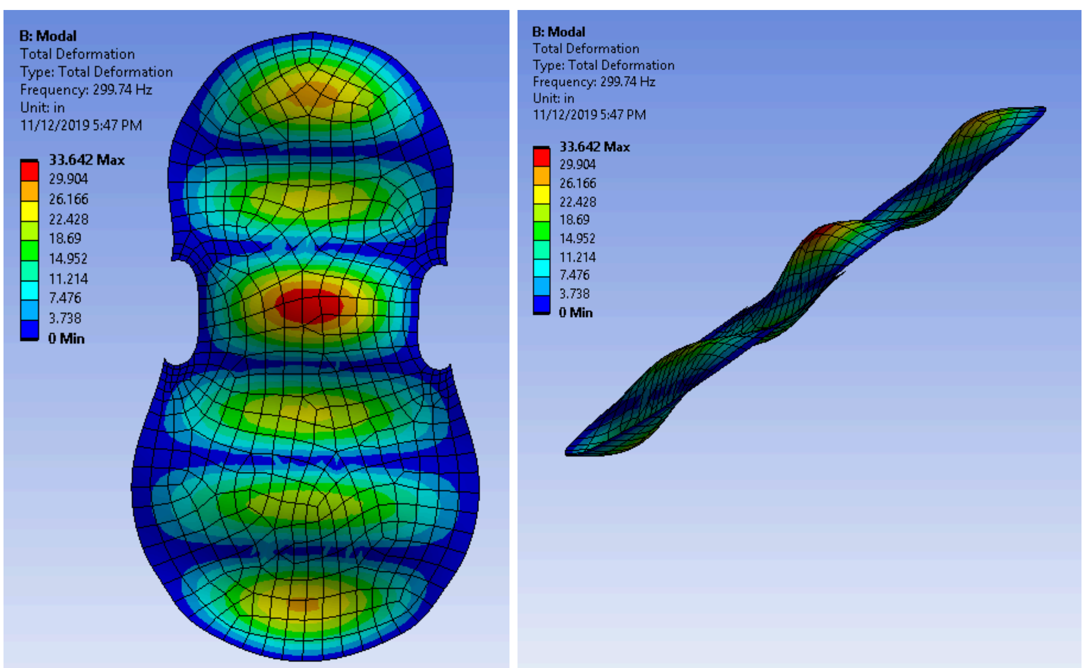

B: Modal
Total Deformation
Type: Total Deformation
Frequency: 205.48 Hz
Unit: in
11/12/2019 5:48 PM
37.778 Max
33.58
29.383
25.185
20.988
16.79
12.593
8.395
4.1975
0 Min


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 205.48 Hz
Unit: in
11/12/2019 5:46 PM
37.778 Max
33.58
29.383
25.185
20.988
16.79
12.593
8.395
4.1975
0 Min

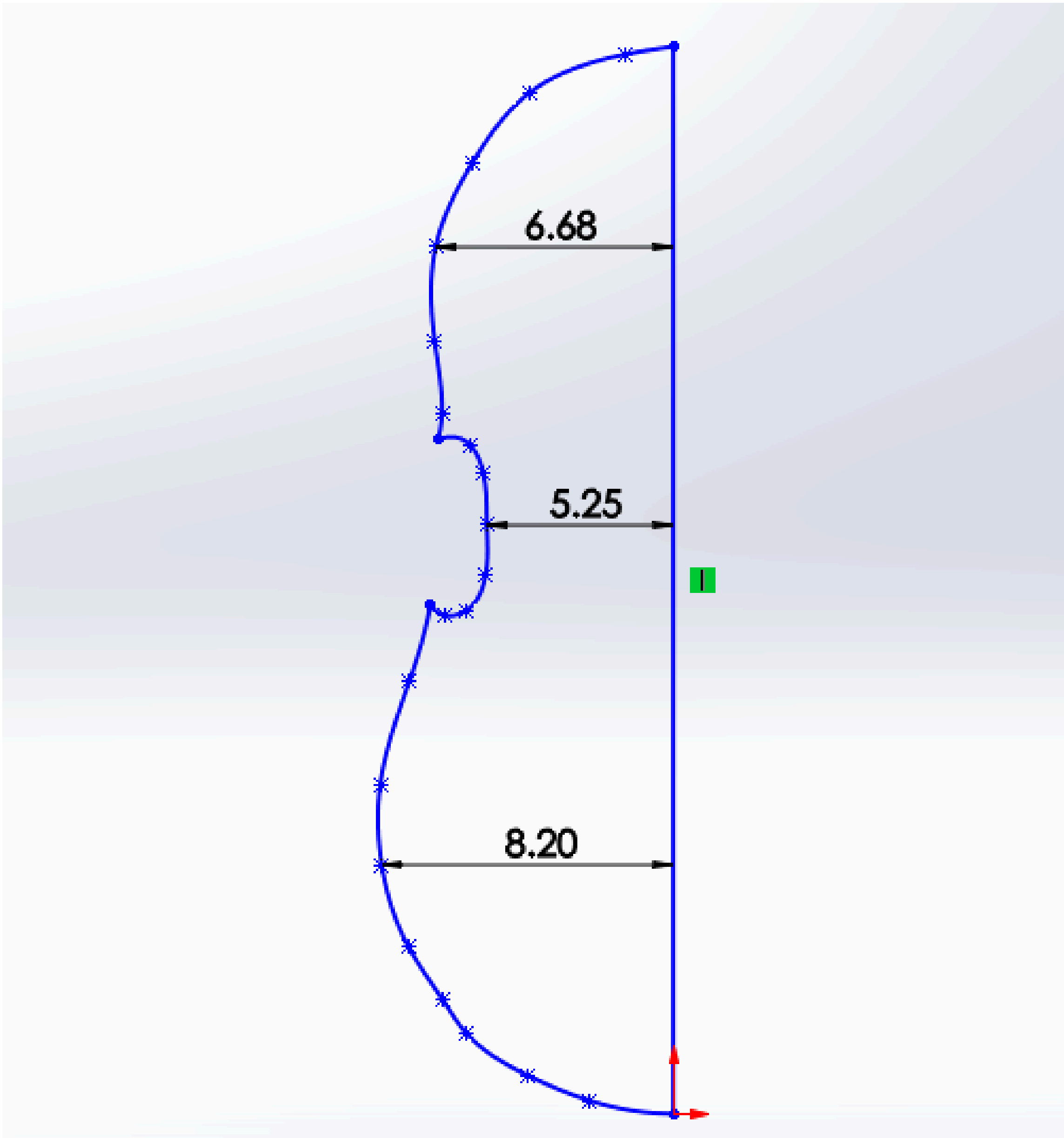
MODE 5:


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 245.6 Hz
Unit: in
11/12/2019 5:48 PM
31.878 Max
28.336
24.794
21.252
17.71
14.168
10.626
7.084
3.542
0 Min
B: Modal
Total Deformation
Type: Total Deformation
Frequency: 245.6 Hz
Unit: in
11/12/2019 5:46 PM
31.878 Max
28.336
24.794
21.252
17.71
14.168
10.626
7.084
3.542
0 Min

MODE 6:


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 299.74 Hz
Unit: in
11/12/2019 5:47 PM
33.642 Max
29.904
26.166
22.428
18.69
14.952
11.214
7.476
3.738
0 Min
B: Modal
Total Deformation
Type: Total Deformation
Frequency: 299.74 Hz
Unit: in
11/12/2019 5:47 PM
33.642 Max
29.904
26.166
22.428
18.69
14.952
11.214
7.476
3.738
0 Min

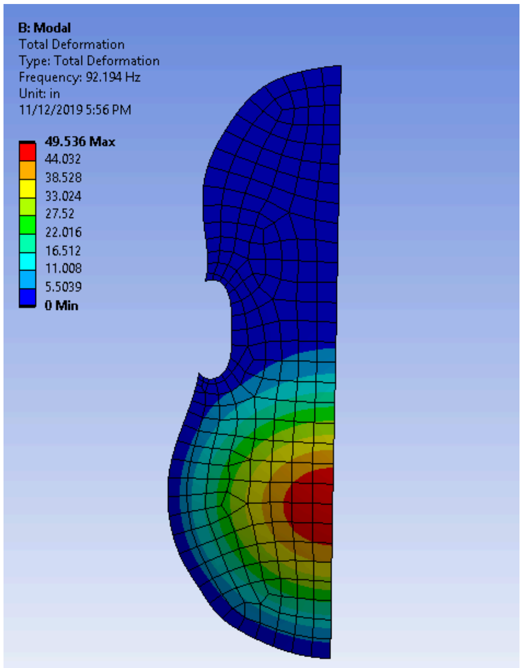
Now using a half model:

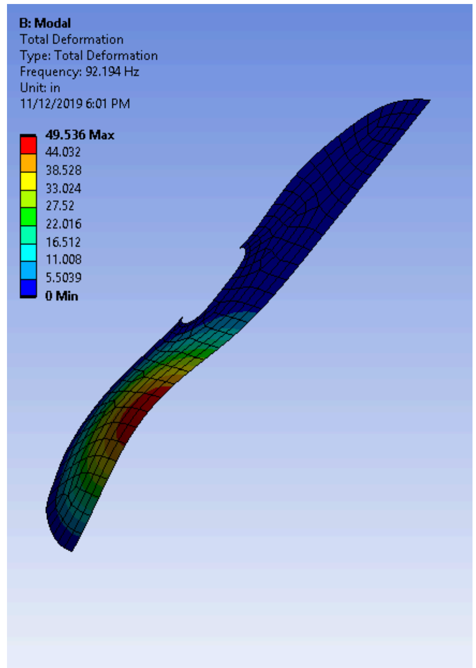


The same boundary conditions were applied on the half model as the full model (0 displacement in the x, y, and z directions on the outside edges, and a pressure of 0.1111 psi into the entire face), except one fixed rotation constraint was additionally subjected on the symmetry line.

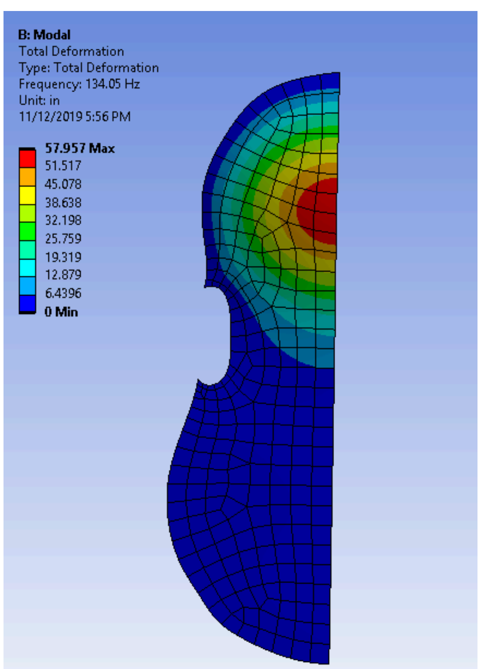
(Maple) Modal Analysis: Half Model

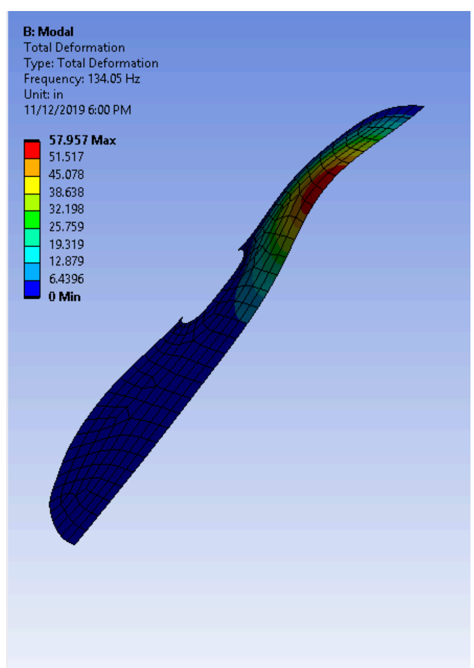
MODE 1:



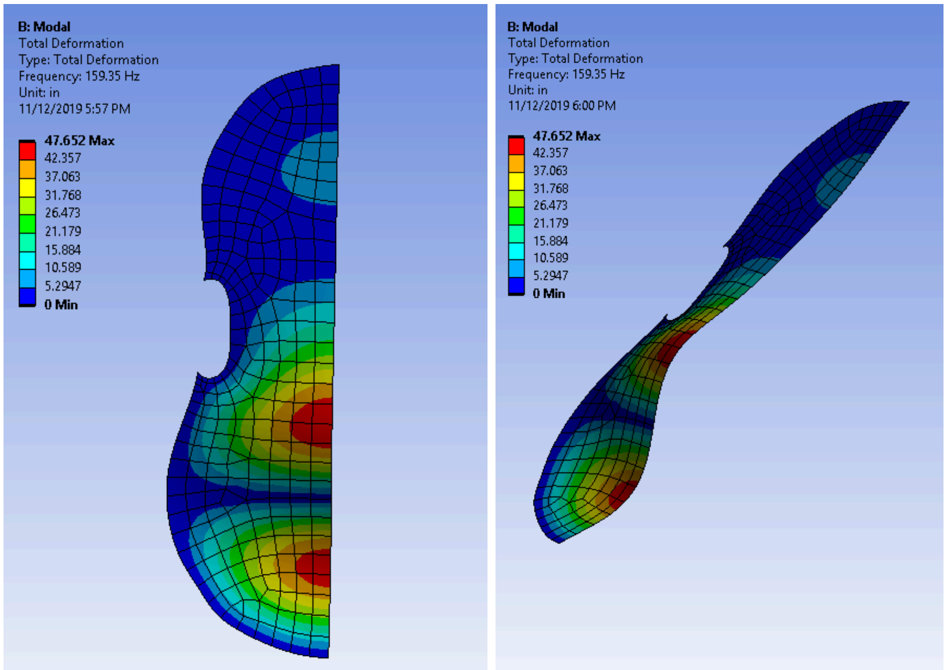


MODE 2:

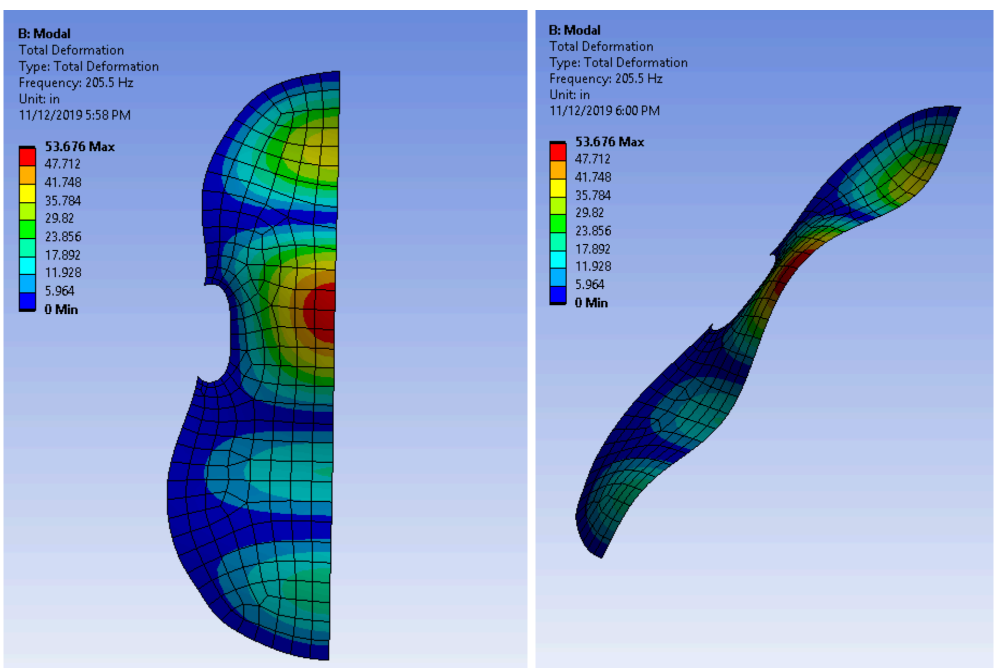




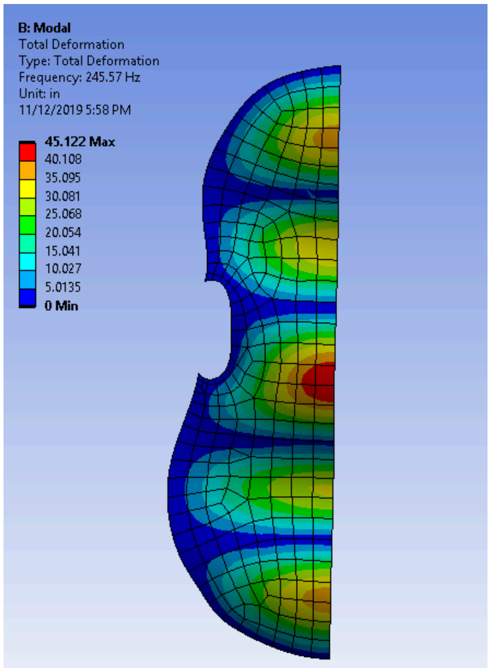
MODE 3:

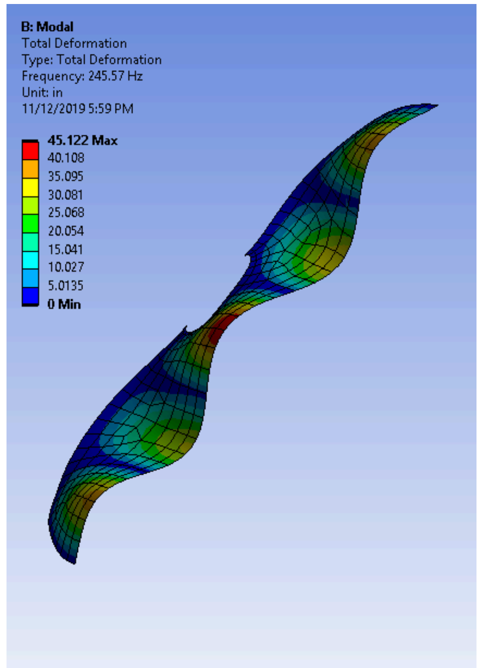

B: Modal
Total Deformation
Type: Total Deformation
Frequency: 159.35 Hz
Unit: in
11/12/2019 5:57 PM
47.652 Max
42.357
37.063
31.768
26.473
21.179
15.884
10.589
5.2947
0 Min
B: Modal
Total Deformation
Type: Total Deformation
Frequency: 159.35 Hz
Unit: in
11/12/2019 6:00 PM
47.652 Max
42.357
37.063
31.768
26.473
21.179
15.884
10.589
5.2947
0 Min

MODE 4:

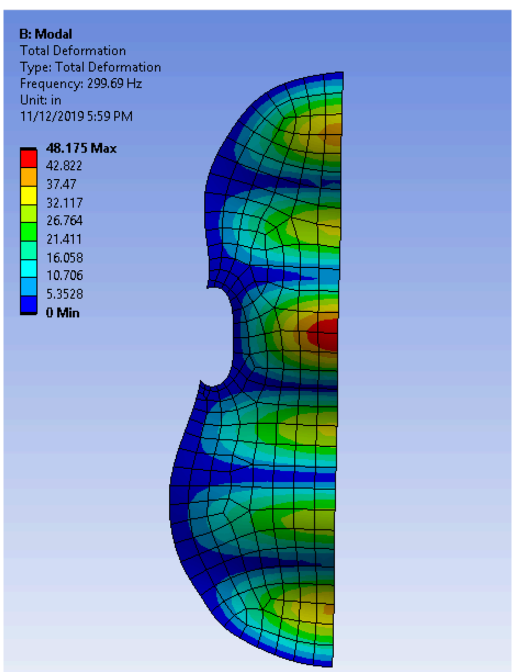

B: Modal
Total Deformation
Type: Total Deformation
Frequency: 205.5 Hz
Unit: in
11/12/2019 5:58 PM
53.676 Max
47.712
41.748
35.784
29.82
23.856
17.892
11.928
5.964
0 Min
B: Modal
Total Deformation
Type: Total Deformation
Frequency: 205.5 Hz
Unit: in
11/12/2019 6:00 PM
53.676 Max
47.712
41.748
35.784
29.82
23.856
17.892
11.928
5.964
0 Min

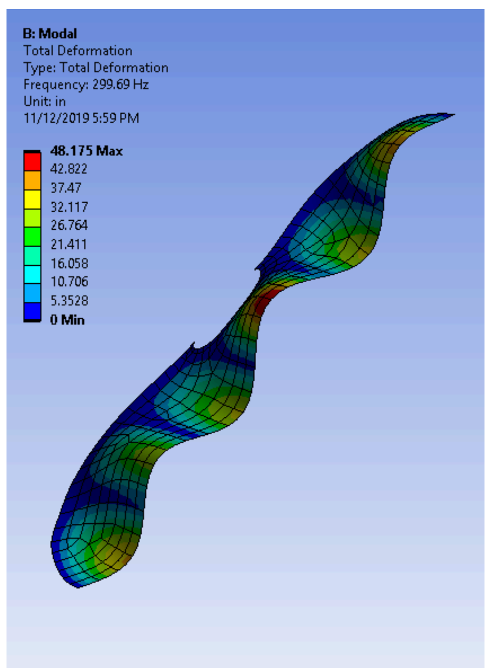
MODE 5:


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 245.57 Hz
Unit: in
11/12/2019 5:58 PM
45.122 Max
40.108
35.095
30.081
25.068
20.054
15.041
10.027
5.0135
0 Min


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 245.57 Hz
Unit: in
11/12/2019 5:59 PM
45.122 Max
40.108
35.095
30.081
25.068
20.054
15.041
10.027
5.0135
0 Min

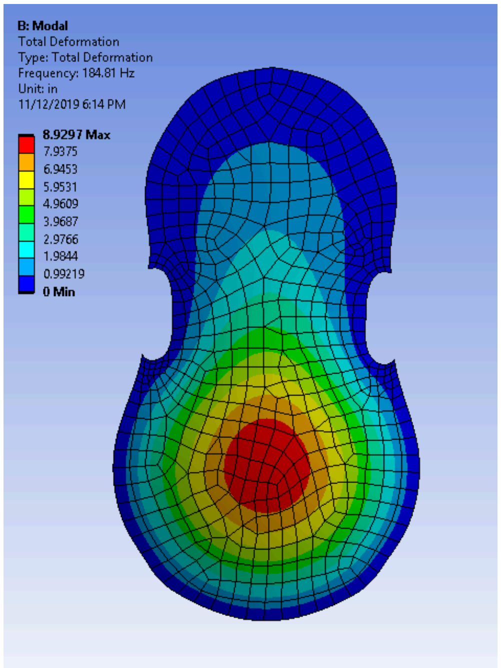
MODE 6:

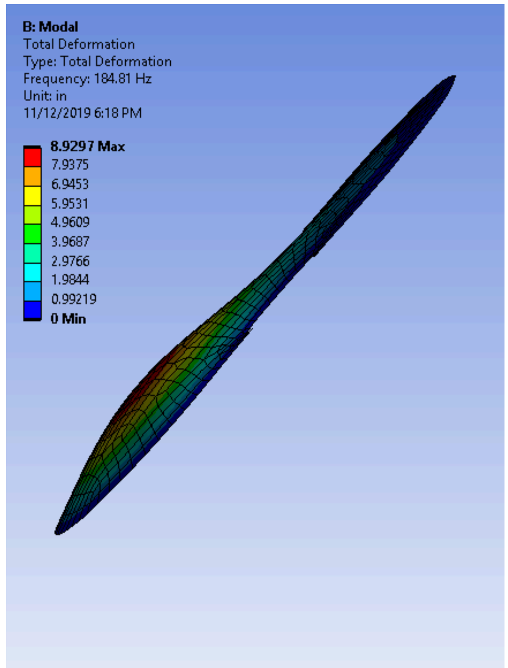

B: Modal
Total Deformation
Type: Total Deformation
Frequency: 299.69 Hz
Unit: in
11/12/2019 5:59 PM
48.175 Max
42.822
37.47
32.117
26.764
21.411
16.058
10.706
5.3528
0 Min


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 299.69 Hz
Unit: in
11/12/2019 5:59 PM
48.175 Max
42.822
37.47
32.117
26.764
21.411
16.058
10.706
5.3528
0 Min

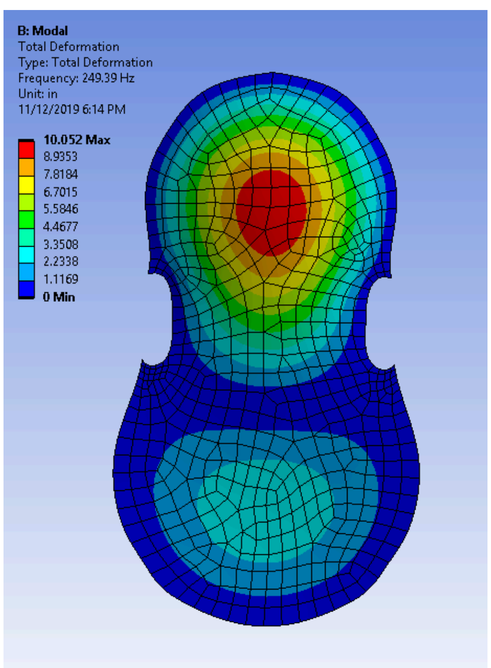
(Steel) Modal Analysis: Full Model

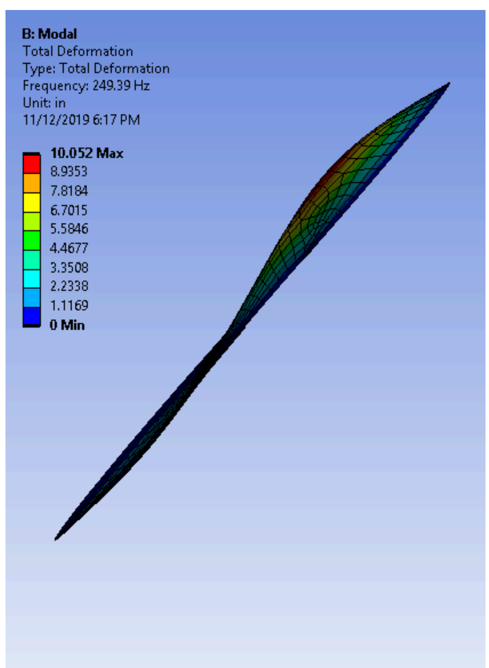
Mode 1:



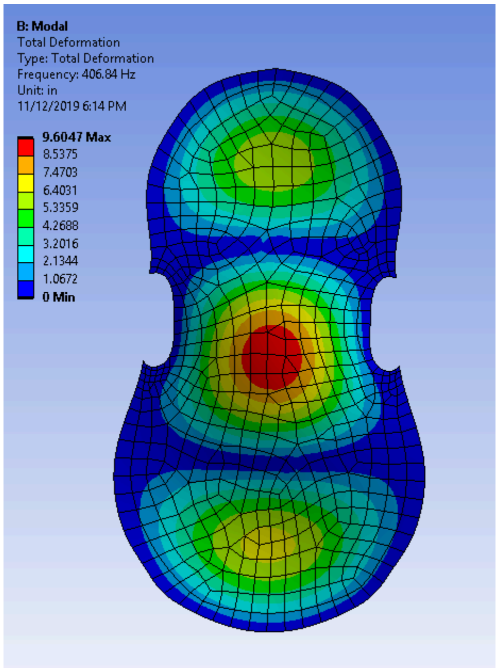


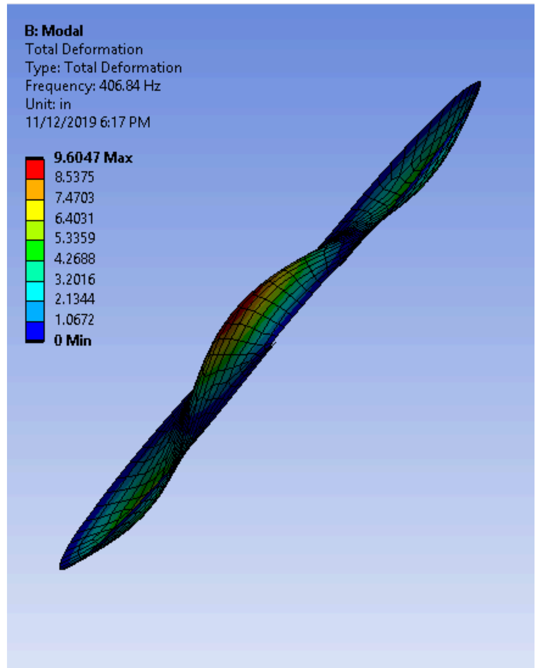
Mode 2:



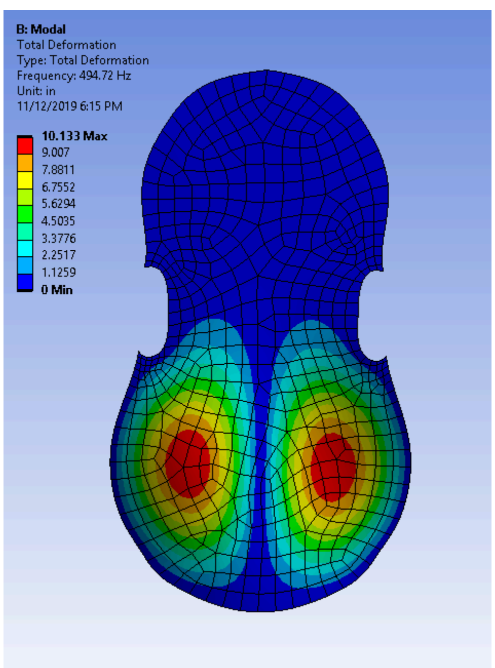


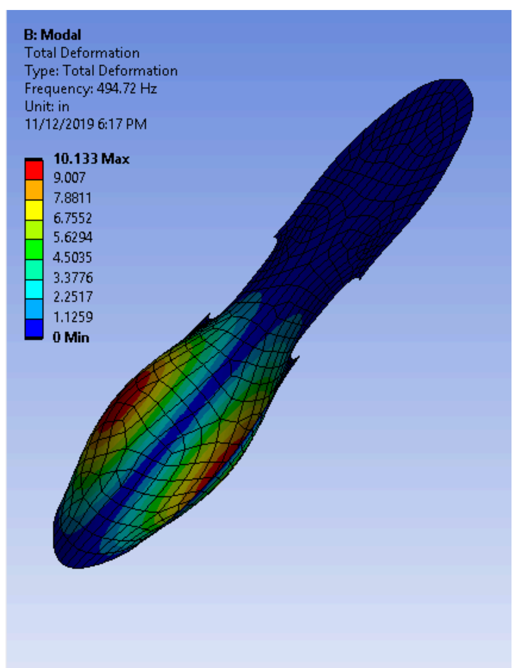
Mode 3:


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 406.84 Hz
Unit: in
11/12/2019 6:14 PM
9.6047 Max
8.5375
7.4703
6.4031
5.3359
4.2688
3.2016
2.1344
1.0672
0 Min

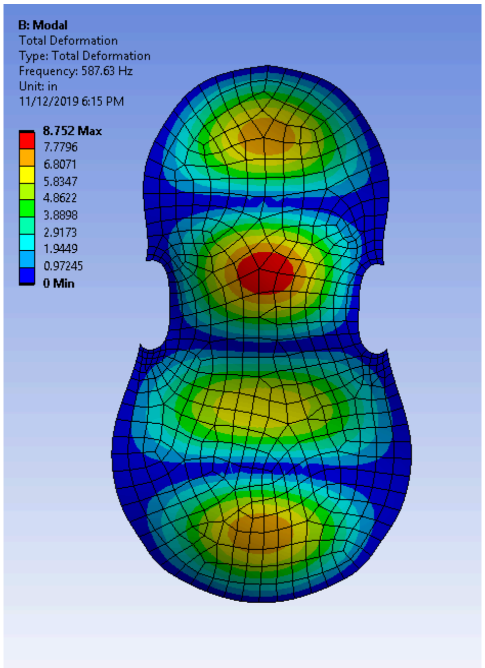

B: Modal
Total Deformation
Type: Total Deformation
Frequency: 406.84 Hz
Unit: in
11/12/2019 6:17 PM
9.6047 Max
8.5375
7.4703
6.4031
5.3359
4.2688
3.2016
2.1344
1.0672
0 Min

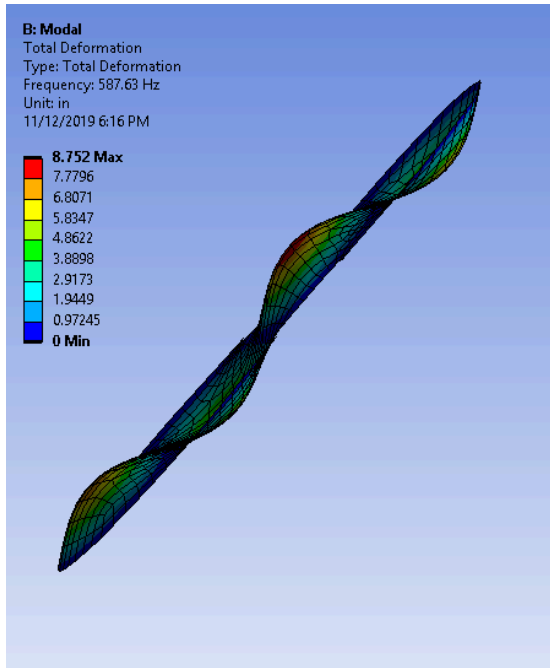
Mode 4:


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 494.72 Hz
Unit: in
11/12/2019 6:15 PM
10.133 Max
9.007
7.8811
6.7552
5.6294
4.5035
3.3776
2.2517
1.1259
0 Min

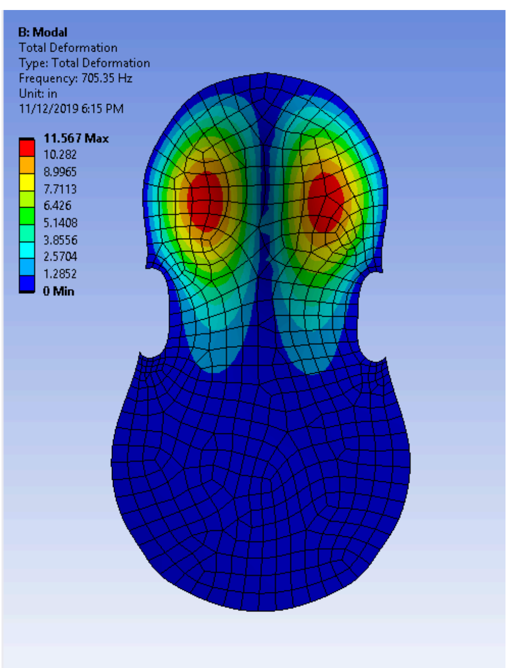

B: Modal
Total Deformation
Type: Total Deformation
Frequency: 494.72 Hz
Unit: in
11/12/2019 6:17 PM
10.133 Max
9.007
7.8811
6.7552
5.6294
4.5035
3.3776
2.2517
1.1259
0 Min

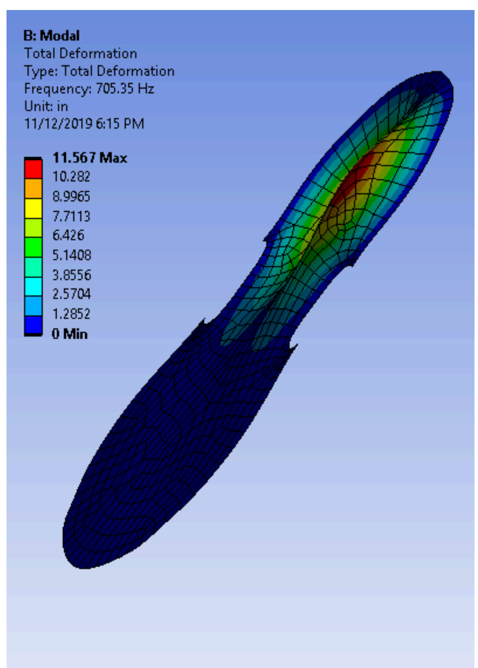
Mode 5:


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 587.63 Hz
Unit: in
11/12/2019 6:15 PM
8.752 Max
7.7796
6.8071
5.8347
4.8622
3.8898
2.9173
1.9449
0.97245
0 Min


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 587.63 Hz
Unit: in
11/12/2019 6:16 PM
8.752 Max
7.7796
6.8071
5.8347
4.8622
3.8898
2.9173
1.9449
0.97245
0 Min

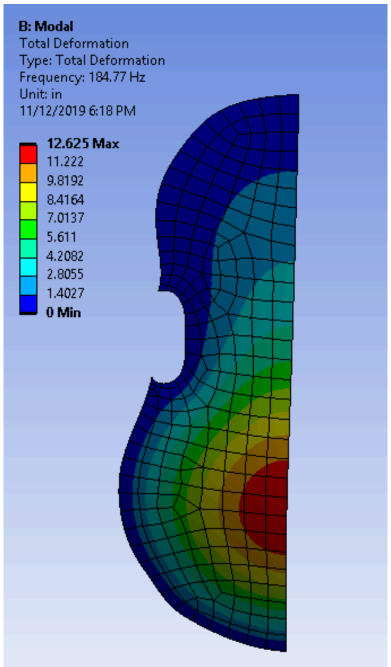
Mode 6:

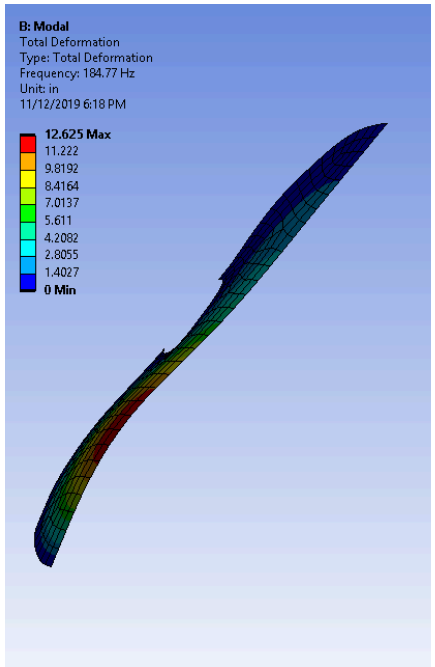

B: Modal
Total Deformation
Type: Total Deformation
Frequency: 705.35 Hz
Unit: in
11/12/2019 6:15 PM
11.567 Max
10.282
8.9965
7.7113
6.426
5.1408
3.8556
2.5704
1.2852
0 Min


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 705.35 Hz
Unit: in
11/12/2019 6:15 PM
11.567 Max
10.282
8.9965
7.7113
6.426
5.1408
3.8556
2.5704
1.2852
0 Min

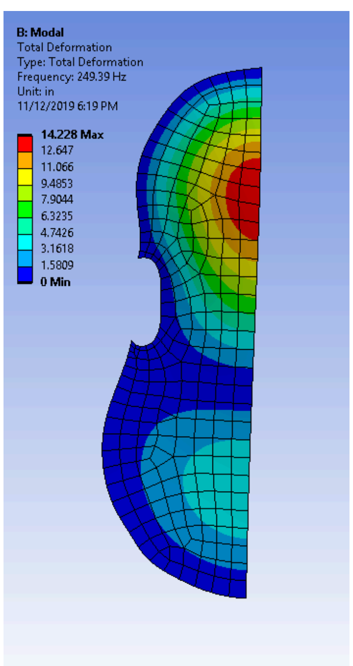
(Steel) Modal Analysis: Half Model

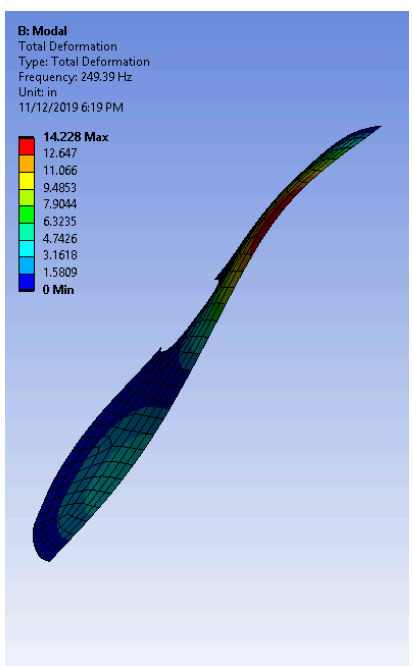
Mode 1:



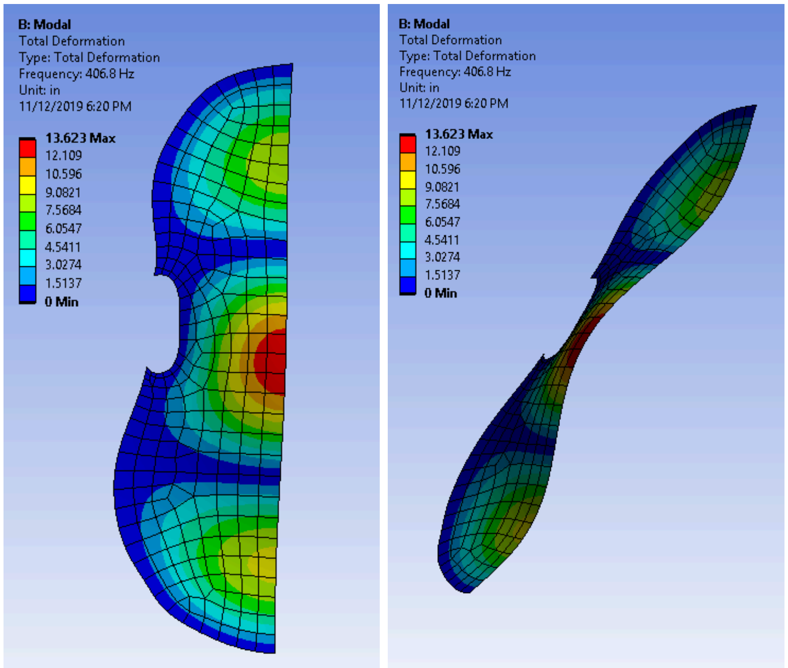


Mode 2:

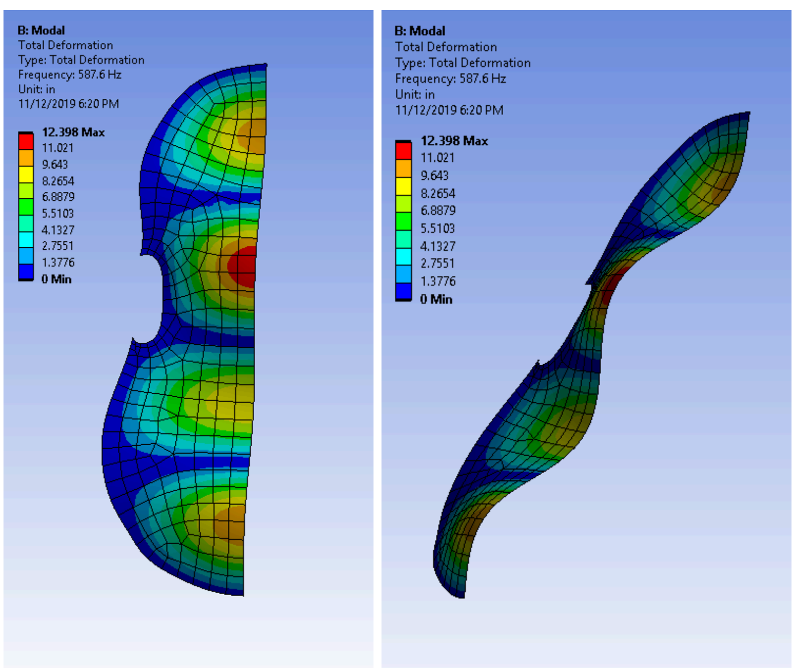




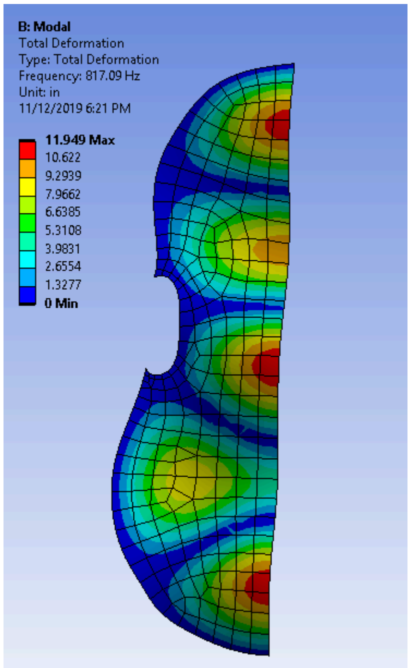
Mode 3:

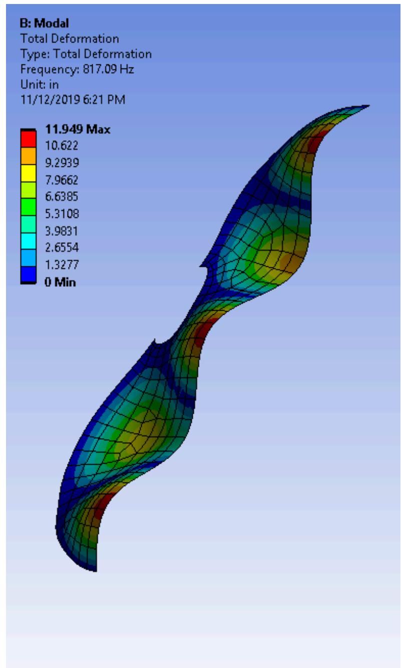

B: Modal
Total Deformation
Type: Total Deformation
Frequency: 406.8 Hz
Unit: in
11/12/2019 6:20 PM
13.623 Max
12.109
10.596
9.0821
7.5684
6.0547
4.5411
3.0274
1.5137
0 Min
B: Modal
Total Deformation
Type: Total Deformation
Frequency: 406.8 Hz
Unit: in
11/12/2019 6:20 PM
13.623 Max
12.109
10.596
9.0821
7.5684
6.0547
4.5411
3.0274
1.5137
0 Min

Mode 4:

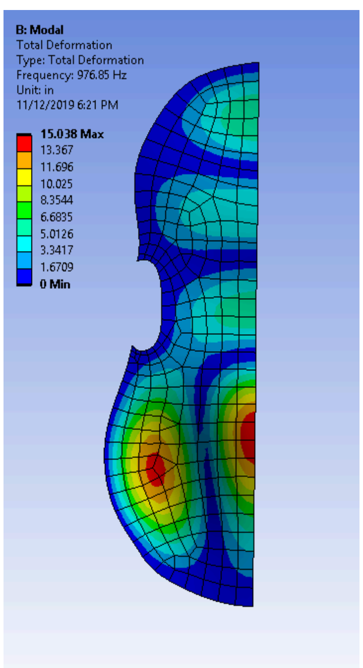

B: Modal
Total Deformation
Type: Total Deformation
Frequency: 587.6 Hz
Unit: in
11/12/2019 6:20 PM
12.398 Max
11.021
9.643
8.2654
6.8879
5.5103
4.1327
2.7551
1.3776
0 Min
B: Modal
Total Deformation
Type: Total Deformation
Frequency: 587.6 Hz
Unit: in
11/12/2019 6:20 PM
12.398 Max
11.021
9.643
8.2654
6.8879
5.5103
4.1327
2.7551
1.3776
0 Min

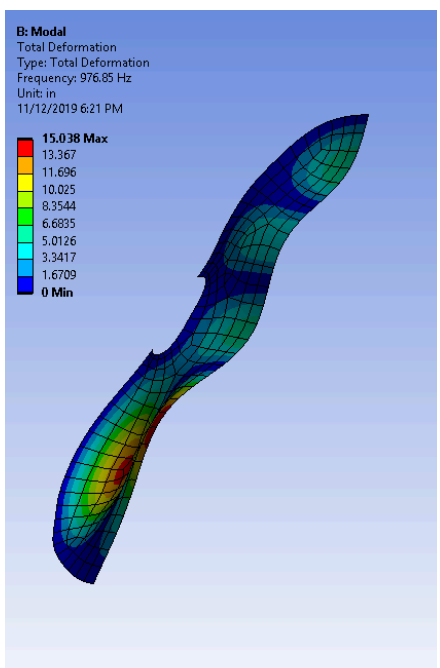
Mode 5:


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 817.09 Hz
Unit: in
11/12/2019 6:21 PM
11.949 Max
10.622
9.2939
7.9662
6.6385
5.3108
3.9831
2.6554
1.3277
0 Min


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 817.09 Hz
Unit: in
11/12/2019 6:21 PM
11.949 Max
10.622
9.2939
7.9662
6.6385
5.3108
3.9831
2.6554
1.3277
0 Min

Mode 6:


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 976.85 Hz
Unit: in
11/12/2019 6:21 PM
15.038 Max
13.367
11.696
10.025
8.3544
6.6835
5.0126
3.3417
1.6709
0 Min


B: Modal
Total Deformation
Type: Total Deformation
Frequency: 976.85 Hz
Unit: in
11/12/2019 6:21 PM
15.038 Max
13.367
11.696
10.025
8.3544
6.6835
5.0126
3.3417
1.6709
0 Min

Verification Computations:

The cello can be approximated as a rectangle of length a = 15 in, height b = 30 in, and thickness h = 0.25 in. The case of clamped edges and uniform pressure loading was used for the following calculations.

To verify the accuracy of the structural analysis, the stress and deflection at the center of the cello were checked.

$$\sigma_{maple} = \frac{p\,a^2}{2t^2[0.623\left(\frac{a}{b}\right)^6+1]} = \frac{\left(0.1111\frac{lb}{in^2}\right)(15in)^2}{2\,(0.25in)^2[0.623\left(\frac{15in}{30in}\right)^6+1]} = 198.0445\,psi$$

This is close to the probed value of 171.02 psi!

$$\delta_{lower,\ maple} = \frac{0.0284\,p\,a^4}{Et^3[1.056\left(\frac{a}{b}\right)^5+1]} = \frac{0.0284\left(0.1111\frac{lb}{in^2}\right)(15in)^4}{\left(1.62*10^6\frac{lb}{in^2}\right)(0.25in)^3[1.056\left(\frac{15in}{30in}\right)^5+1]} = 0.0061\,in$$

$$\delta_{upper,\ maple} = \frac{0.0284\,p\,a^4}{Et^3[1.056\left(\frac{a}{b}\right)^5+1]} = \frac{0.0284\left(0.1111\frac{lb}{in^2}\right)(15in)^4}{\left(1.08*10^5\frac{lb}{in^2}\right)(0.25in)^3[1.056\left(\frac{15in}{30in}\right)^5+1]} = 0.0916\,in$$

These values correctly surround the probed value of 0.00923 in!

The accuracy of the maple cello's modal analysis was checked by verifying the value of its first natural frequency.

$$\rho_{maple} = \left(0.02315 \frac{lb}{in^3}\right)\left(\frac{1}{\left(32.2 \frac{ft}{s^2}\right)\left(12 \frac{in}{ft}\right)}\right) = 5.999 * 10^{-5} \frac{lb\, s^2}{in^4}$$

Assume ν = 0.43, which is the average of maple's orthotropic Poisson's ratios. Also choose the Young's modulus, E, to be $2.27*10^5$ psi:

$$D_{maple} = \frac{E\, h^3}{12\,(1-\nu^2)} = \frac{\left(2.27*10^5 \frac{lb}{in^2}\right)(0.25\ in)^3}{12\left(1-0.43^2\right)} = 362.6217\ lb\ in$$

1026 **PLATES**

Table 1 Nondimensional free vibration frequencies $\omega a^2 \sqrt{\rho h / D}$ for rectangular plates ($\nu = 0.3$)

Edge conditions	*a/b = 1 (square)*			*a/b = 0.4*	*a/b = 2.5*
	Mode 1	*Mode 2*	*Mode 3*	*Mode 1*	*Mode 1*
SS–SS–SS–SS	19.739	49.348	49.348	11.449	71.556
SS–C–SS–C	28.951	54.743	69.327	12.135	145.484
SS–C–SS–SS	23.646	51.674	58.646	11.750	103.923
SS–C–SS–F	12.687	33.065	41.702	10.189	30.628
SS–SS–SS–F	11.684	27.756	41.197	10.126	18.801
SS–F–SS–F	9.631	16.135	36.726	9.760	9.484
C–C–C–C	35.99	73.41	73.41	23.65	147.80
C–C–C–SS	31.83	63.35	71.08	23.44	107.07
C–C–C–F	24.02	40.04	63.49	22.58	37.66
C–C–SS–SS	27.06	60.54	60.79	18.85	105.31
C–C–SS–F	17.62	36.05	52.06	15.70	33.58
C–C–F–F	6.942	24.03	26.68	3.986	24.91
C–SS–C–F	23.46	35.61	63.13	22.54	28.56
C–SS–SS–F	16.86	31.14	51.63	15.65	23.07
C–SS–F–F	5.364	19.17	24.77	3.854	10.10
C–F–C–F	22.27	26.53	43.66	22.35	22.13
C–F–SS–F	15.28	20.67	39.78	15.38	15.13
C–F–F–F	3.492	8.525	21.43	3.511	3.456
SS–SS–F–F	3.369	17.41	19.37	1.320	8.251
SS–F–F–F	6.648	15.02	25.49	2.692	14.94
F–F–F–F	13.49	19.79	24.43	3.463	21.64

Adapted from more extensive data given in Leissa AW (1973) The free vibration of rectangular plates. *Journal of Sound and Vibration* 31: 257–293.

In the above table, the C-C-C-C edge condition row and the “a/b = 0.4” column were used to find the value of 23.65. This number was set equal to the characteristic equation for rectangular plates to solve for the frequency of the first mode. In the attached Matlab code, the first frequency of the maple cello was found to be 82.2534 Hz, which is relatively close to ANSYS’s first frequency of 92.2 Hz.

Properties of Outline Row 3: Structural Steel

	A	B	C	D	E
1	Property	Value	Unit		
2	Material Field Variables	Table			
3	Density	0.2836	lb in^-3		
4	Isotropic Secant Coefficient of Thermal Expansion				
5	Coefficient of Thermal Expansion	1.2E-05	C^-1		
6	Isotropic Elasticity				
7	Derive from	Young's Modulus and Poisson...			
8	Young's Modulus	2.9008E+07	psi		
9	Poisson's Ratio	0.3			
10	Bulk Modulus	1.6667E+11	Pa		
11	Shear Modulus	7.6923E+10	Pa		
12	Strain-Life Parameters				
20	S-N Curve	Tabular			
24	Tensile Yield Strength	36259	psi		
25	Compressive Yield Strength	36259	psi		
26	Tensile Ultimate Strength	66717	psi		
27	Compressive Ultimate Strength	0	psi		

Additionally, the accuracy of the steel cello’s modal analysis was checked by verifying the value of its first natural frequency.

$$\rho_{steel} = \left(0.2836\frac{lb}{in^3}\right)\left(\frac{1}{\left(32.2\frac{ft}{s^2}\right)\left(12\frac{in}{ft}\right)}\right) = 7.3395 * 10^{-4}\ \frac{lb\, s^2}{in^4}$$

For this isotropic material, use $\nu = 0.30$ and $E = 2.9*10^7$ psi:

$$D_{steel} = \frac{E\, h^3}{12\,(1-\nu^2)} = \frac{\left(2.9*10^7\frac{lb}{in^2}\right)(0.25\ in)^3}{12(1-0.3^2)} = 41,495\ lb\ in$$

Using Matlab, the steel cello’s first natural frequency was estimated to be 251.57 Hz. This is of the same magnitude as ANSYS’s value of 184.8 Hz.