
generalized forces

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In[1]:= f = {dx0, dy0, dz0, dx1, dy1, dz1};  
MatrixForm[f]
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Out[2]//MatrixForm=

$$\begin{pmatrix} dx0 \\ dy0 \\ dz0 \\ dx1 \\ dy1 \\ dz1 \end{pmatrix}$$

function

```
In[3]:= V = (1/2 * k * (Sqrt[{Subscript[x, 1] - Subscript[x, 0], Subscript[y, 1] - Subscript[y, 0],  
Subscript[z, 1] - Subscript[z, 0]} . {Subscript[x, 1] - Subscript[x, 0],  
Subscript[y, 1] - Subscript[y, 0], Subscript[z, 1] - Subscript[z, 0]}] - l)^2)
```

Out[3]= $\frac{1}{2} k \left(-l + \sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2 + (-z_0 + z_1)^2} \right)^2$

derivatives

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In[4]:= dx0 = D[V, Subscript[x, 0]]
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dx0 = dx0 /.

Sqrt[(-Subscript[x, 0] + Subscript[x, 1])^2 + (-Subscript[y, 0] + Subscript[y, 1])^2 +
(-Subscript[z, 0] + Subscript[z, 1])^2] → n;

dx0 = dx0 /. 1/Sqrt[(-Subscript[x, 0] + Subscript[x, 1])^2 +
(-Subscript[y, 0] + Subscript[y, 1])^2 +
(-Subscript[z, 0] + Subscript[z, 1])^2] → 1/n

Out[4]=
$$-\frac{k(-x_0 + x_1) \left(-l + \sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2 + (-z_0 + z_1)^2} \right)}{\sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2 + (-z_0 + z_1)^2}}$$

Out[6]=
$$-\frac{k(-l + n)(-x_0 + x_1)}{n}$$

In[7]:= $dy0 = D[V, \text{Subscript}[y, 0]]$

$dy0 = dy0 /.$

$\text{Sqrt}[(-\text{Subscript}[x, 0] + \text{Subscript}[x, 1])^2 + (-\text{Subscript}[y, 0] + \text{Subscript}[y, 1])^2 + (-\text{Subscript}[z, 0] + \text{Subscript}[z, 1])^2] \rightarrow n;$

$dy0 = dy0 /. 1/\text{Sqrt}[(-\text{Subscript}[x, 0] + \text{Subscript}[x, 1])^2 +$

$(-\text{Subscript}[y, 0] + \text{Subscript}[y, 1])^2 +$

$(-\text{Subscript}[z, 0] + \text{Subscript}[z, 1])^2] \rightarrow 1/n$

$$\text{Out[7]} = - \frac{k(-y_0 + y_1) \left(-l + \sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2 + (-z_0 + z_1)^2} \right)}{\sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2 + (-z_0 + z_1)^2}}$$

$$\text{Out[9]} = - \frac{k(-l + n)(-y_0 + y_1)}{n}$$

In[10]:= $dz0 = D[V, \text{Subscript}[z, 0]]$

$dz0 = dz0 /.$

$\text{Sqrt}[(-\text{Subscript}[x, 0] + \text{Subscript}[x, 1])^2 + (-\text{Subscript}[y, 0] + \text{Subscript}[y, 1])^2 + (-\text{Subscript}[z, 0] + \text{Subscript}[z, 1])^2] \rightarrow n;$

$dz0 = dz0 /. 1/\text{Sqrt}[(-\text{Subscript}[x, 0] + \text{Subscript}[x, 1])^2 +$

$(-\text{Subscript}[y, 0] + \text{Subscript}[y, 1])^2 +$

$(-\text{Subscript}[z, 0] + \text{Subscript}[z, 1])^2] \rightarrow 1/n$

$$\text{Out[10]} = - \frac{k(-z_0 + z_1) \left(-l + \sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2 + (-z_0 + z_1)^2} \right)}{\sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2 + (-z_0 + z_1)^2}}$$

$$\text{Out[12]} = - \frac{k(-l + n)(-z_0 + z_1)}{n}$$

In[13]:= $dx1 = D[V, \text{Subscript}[x, 1]]$

$dx1 = dx1 /.$

$\text{Sqrt}[(-\text{Subscript}[x, 0] + \text{Subscript}[x, 1])^2 + (-\text{Subscript}[y, 0] + \text{Subscript}[y, 1])^2 + (-\text{Subscript}[z, 0] + \text{Subscript}[z, 1])^2] \rightarrow n;$

$dx1 = dx1 /. 1/\text{Sqrt}[(-\text{Subscript}[x, 0] + \text{Subscript}[x, 1])^2 +$

$(-\text{Subscript}[y, 0] + \text{Subscript}[y, 1])^2 +$

$(-\text{Subscript}[z, 0] + \text{Subscript}[z, 1])^2] \rightarrow 1/n$

$$\text{Out[13]} = \frac{k(-x_0 + x_1) \left(-l + \sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2 + (-z_0 + z_1)^2} \right)}{\sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2 + (-z_0 + z_1)^2}}$$

$$\text{Out[15]} = \frac{k(-l + n)(-x_0 + x_1)}{n}$$

```

In[16]:= dy1 = D[V, Subscript[y, 1]]
dy1 = dy1 /.
  Sqrt[(-Subscript[x, 0] + Subscript[x, 1])^2 + (-Subscript[y, 0] + Subscript[y, 1])^2 +
    (-Subscript[z, 0] + Subscript[z, 1])^2] -> n;
dy1 = dy1 /. 1/Sqrt[(-Subscript[x, 0] + Subscript[x, 1])^2 +
  (-Subscript[y, 0] + Subscript[y, 1])^2 +
  (-Subscript[z, 0] + Subscript[z, 1])^2] -> 1/n

Out[16]= 
$$\frac{k(-y_0 + y_1) \left( -l + \sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2 + (-z_0 + z_1)^2} \right)}{\sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2 + (-z_0 + z_1)^2}}$$


Out[18]= 
$$\frac{k(-l + n)(-y_0 + y_1)}{n}$$


In[19]:= dz1 = D[V, Subscript[z, 1]]
dz1 = dz1 /.
  Sqrt[(-Subscript[x, 0] + Subscript[x, 1])^2 + (-Subscript[y, 0] + Subscript[y, 1])^2 +
    (-Subscript[z, 0] + Subscript[z, 1])^2] -> n;
dz1 = dz1 /. 1/Sqrt[(-Subscript[x, 0] + Subscript[x, 1])^2 +
  (-Subscript[y, 0] + Subscript[y, 1])^2 +
  (-Subscript[z, 0] + Subscript[z, 1])^2] -> 1/n

Out[19]= 
$$\frac{k(-z_0 + z_1) \left( -l + \sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2 + (-z_0 + z_1)^2} \right)}{\sqrt{(-x_0 + x_1)^2 + (-y_0 + y_1)^2 + (-z_0 + z_1)^2}}$$


Out[21]= 
$$\frac{k(-l + n)(-z_0 + z_1)}{n}$$


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final

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In[22]:= MatrixForm[f]
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Out[22]//MatrixForm=
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$$\begin{pmatrix} -\frac{k(-l+n)(-x_0+x_1)}{n} \\ -\frac{k(-l+n)(-y_0+y_1)}{n} \\ -\frac{k(-l+n)(-z_0+z_1)}{n} \\ \frac{k(-l+n)(-x_0+x_1)}{n} \\ \frac{k(-l+n)(-y_0+y_1)}{n} \\ \frac{k(-l+n)(-z_0+z_1)}{n} \end{pmatrix}$$