10 Covering Spaces

Orbit Spaces

Exercise 10.1. We know by Theorem 10.54 that $\mathrm{Cov}(\tilde{X}/(\tilde{X}/H)) = H$, and so we can think of G as a subgroup of $\mathrm{Cov}(\tilde{X}/(\tilde{X}/H))$. Now use Theorem 10.52, with $X = \tilde{X}/H$. We know, in particular, that G is a subgroup of $\mathrm{Cov}(\tilde{X}/X)$, and thus is exactly a covering space $(\tilde{X}/G, v)$ of $X = \tilde{X}/H$, as desired.

Exercise 10.2.

(i) Suppose $gx = x$ and consider a proper neighborhood V of x. Then we know that $gV \cap V = \emptyset$, but $x = gx \in gV \cap V$, contradiction.

(ii) If $G = \{e, g_1, \ldots, g_n\}$ and $x \in X$, then, since X is Hausdorff and since $g_i x \neq x$, there exists a neighborhood V of X which does not contain any $g_i x$. Obviously, this V is a proper neighborhood.

Exercise 10.3. This is exactly the argument in the proof of Theorem 10.2, namely in the first full paragraph on p. 276.

Exercise 10.4.

(i)