Effect of Milk Fat Content on Dielectric Properties at 200 MHz – 20 GHz

Jessica Chong

April 2022

A report submitted as partial fulfilment
of the requirements of
PHYS 437B

University of Waterloo

Supervisors:
Prof. George Shaker, Prof. Stefan Idziak

Abstract

To understand the effect of milk fat content on dielectric properties to develop a butterfat detector for real-time monitoring, 5-35% creams are tested on the DAK-TL, an open-ended coaxial probe, connected to a vector network analyzer over the frequency range 200 MHz to 20 GHz. The results showed that the dielectric constants and losses decreased linearly with an increase in milk fat content and that the other constituents of milk also affect the dielectric properties of cream. The results indicated that the DAK-TL is not great for studying the effect of temperature and volumetric resonances arose in the data, suggesting a larger volume and different instrument should be used for more accurate measurements. Two regression models were developed to determine the amount of fat content of a given sample based on the dielectric constant and dielectric loss with promising results. If the dielectric properties are differentiable, then the fat content can be predicted, and a butterfat detector may be possible to be implemented at farms and fluid dairy manufacturing facilities.

Contents

List of Tables

List of Figures

Acknowledgements

First and foremost, I would like to express my sincerest appreciation to Professor George Shaker and Professor Stefan Idziak for allowing me to explore a joint topic within both of their fields of expertise. The completion of this research project would not have been possible if it were not for their continued support, constructive criticism, and guidance motivating me throughout the term. I have learned a lot from them about thinking critically and perseverance that I hope to carry forward with me as lifelong lessons.

I would also like to say thank you Ala Eldin Omer and Michael Riad, who both took the time to help me troubleshoot the DAK-TL and conduct the experiments on the salt/sugar solutions and milk/cream samples this term.

Finally, I must also thank my parents and pet dog for their support and encouragement throughout this project.

1 Introduction

With recent advancements in technology, the Internet of Things (IoT) has the potential to enable real-time monitoring of food products for quality control [1] from raw to finished product. Current quality control practices to detect constituents of a product in the fluid dairy industry are manual and time-consuming, because it requires bringing a sample into a lab to test using expensive and bulky equipment. The instruments most used for compositional quality checks are the MilkoScan FT120 and MilkoScan FT1 provided by FOSS Analytics which uses Fourier-transform mid-infrared spectrometry which take roughly 30 seconds to test a single sample [2]. Due to the nature of this instrument, it cannot be used for real-time monitoring on a production line, during transport, or to monitor milk during the pasteurization process. It can be used at the line, but the machine itself is quite costly. By examining the dielectric properties of milk, it may be possible to implement a radar for the real-time detection of the composition of a product that could be installed in-line or at-the-line [2]. This would help improve supply and production planning in the industry. However, there are many challenges that the concept faces because the dielectric properties are dependent on the chemical and physical properties of a sample, and milk has many components to its composition.

The purpose of this report is to outline the resolved issues experienced in PHYS 437A with the salt and sugar solution experiment. Next, the background knowledge gained from PHYS 437A will be applied to examine the effect on the dielectric properties of butterfat content over the frequency range 200 MHz to 20 GHz. Butterfat, or milk fat, is a major constituent of milk and cream and is the part that would vary the most among these products. Differentiability of the dielectric properties among the milk and cream samples at different concentrations will be the key factor when performing the measurements. This report is split up into 4 sections: PHYS 437A recap, theoretical background, methodology, and experimental results. In the theoretical background section, milk composition will be examined, and a few predictive mixing models will be described. Measurements are conducted using the DAK-TL (Dielectric Assessment Kit for Thin Layers) similarly to in PHYS 437A. Plotting and regression modelling will be done using Python.

1.1 PHYS 437A Recap

From PHYS 437A, the effect of an applied time varying electric field on the behaviour of dielectric properties was studied. The dielectric properties are comprised of the dielectric constant and the dielectric loss, where the dielectric constant is the contribution of total polarization mechanisms occurring in a sample and the dielectric loss is attributed to energy dissipation of the material due to internal friction and heat [3]. At the microwave frequency range, dipolar/orientational, ionic/atomic, and electronic polarization processes can be observed. As frequency increases, the dielectric properties of a material decrease because the polarization mechanisms cannot keep up with the oscillating field. To determine an operating frequency for the radar, the frequency at which the dielectric properties are most differentiable at should be chosen. This is an experimental step, since there is no universal model that can predict the dielectric properties as a function of frequency.

The Debye models for ionic aqueous polar solutions were introduced in [3] and are

$$\epsilon' = \epsilon_\infty + \frac{\epsilon_s - \epsilon_\infty}{1 + (\omega\tau)^2} \tag{1}$$

$$\epsilon'' = \frac{(\epsilon_s - \epsilon_\infty)\omega\tau}{1 + (\omega\tau)^2} + \frac{\sigma}{\omega\epsilon_0} \tag{2}$$

where ϵ_s is the static dielectric constant, ϵ_∞ is the high frequency dielectric constant, and τ is the relaxation time. These parameters can be fitted to a set of data at a given temperature.

Inconsistent results were obtained in PHYS 437A when measuring the dielectric properties of 2-10% concentrations of salt and sugar solutions using the DAK-TL 3.5 probe. Over the tested frequency range 200 MHz to 10 GHz, the values of the dielectric constant of salt solutions were incorrect compared to that of previous work done in the field. The dielectric loss was missing the contribution due to ionic conductivity, which is an important characteristic of the losses for saline solutions. As for the sugar solutions, the dielectric constant increased, and the dielectric loss decreased with an increase in concentration of sugar. These trends were opposite of what were expected based on the conducted literature survey in [3].

1.2 PHYS 437A Retest

Following the end of PHYS 437A, it was learnt that the major issue was the calibration regarding the short S11, where the black platform plate for liquids was being used to measure the S11 of the copper tape on. The metal platform plate was supposed to be used instead, because the black platform plate does not allow for good electrical contact and is supposed to only be used to put the metal petri-dish on top when testing liquids.

The salt and sugar samples from [3] were retested with the fixed calibration and calibrated according to SPEAG's recommended frequency and resolution settings for the DAK-TL 3.5 probe. The concentrations prepared are shown in Table 1 and Table 2, while the settings are shown in Table 3. A smaller resolution is used at lower frequencies because solutions are more dispersive due to the dipolar/orientational polarization in water being the major contributor.

Table 1: Salt Concentrations Prepared for Retest

Salt			
Concentration	**Molarity (mol/L)**	**Weight (g)**	**Weight of Distilled Water (g)**
2%	0.342	1.00	50.01
4%	0.684	2.00	50.01
6%	1.027	3.00	50.00
8%	1.369	4.00	50.00
10%	1.711	5.00	50.01

Table 2: Sugar Concentrations Prepared for Retest

Sugar		
Concentration	**Weight (g)**	**Weight of Distilled Water (g)**
2%	1.00	50.00
4%	2.00	50.01
6%	3.00	50.01
8%	4.00	50.01
10%	5.00	50.01

Table 3: Recommended Calibration Settings over 200 MHz to 20 GHz from SPEAG

Frequency Range	Resolution
200 – 295 MHz	5 MHz
300 – 6,000 MHz	50 MHz
6,250 – 20,000 MHz	250 MHz

Each sample was tested 2 times and the averaged dielectric constant and loss results for 2-10% concentrations of salt and sugar are shown in Figure 1 and Figure 2. With the fixed calibration, the results seem to align well with the literature survey described in [3]. In Figure 1, small oscillations as well as the spike towards infinity at low frequencies can be observed. The large spike was caused by the electrode polarization effect where the ions in the solution floating towards the electrode or sample interface, forming an ionic double layer. The voltage applied drops as it passes through the double layer, and it masks the true characteristic of the material at low frequencies. Electrode polarization is a common effect that arises when measuring conductive samples and is difficult to minimize because there are many factors that contribute to it such as the conductivity of a sample, temperature, structure and composition of an electrode, and the electrochemical interactions occurring [4]. With the current DAK-TL probe, only saline solutions with less than 0.1M of salt can be measured without any issues but SPEAG is currently designing new probes to address this issue as part of their ongoing research and development initiatives.

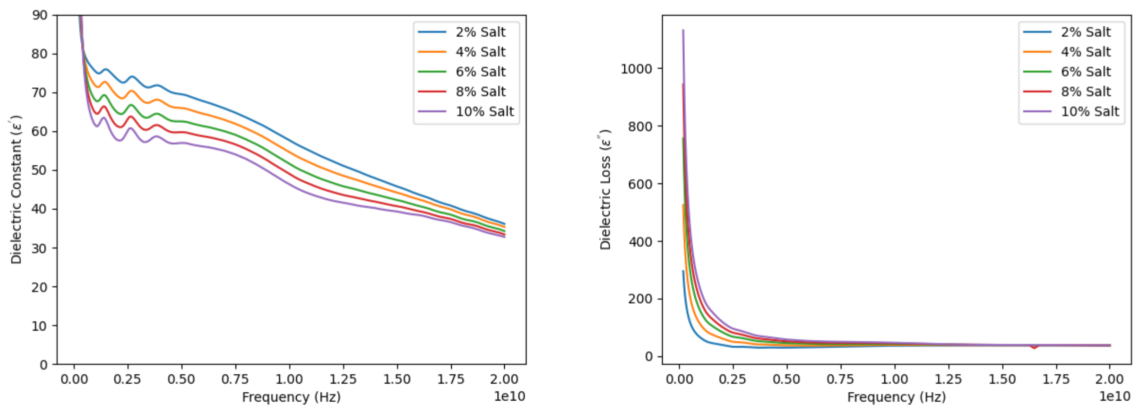

Figure 1: Dielectric Constant/Loss of 2-10% Salt Solutions from 200 MHz to 20 GHz

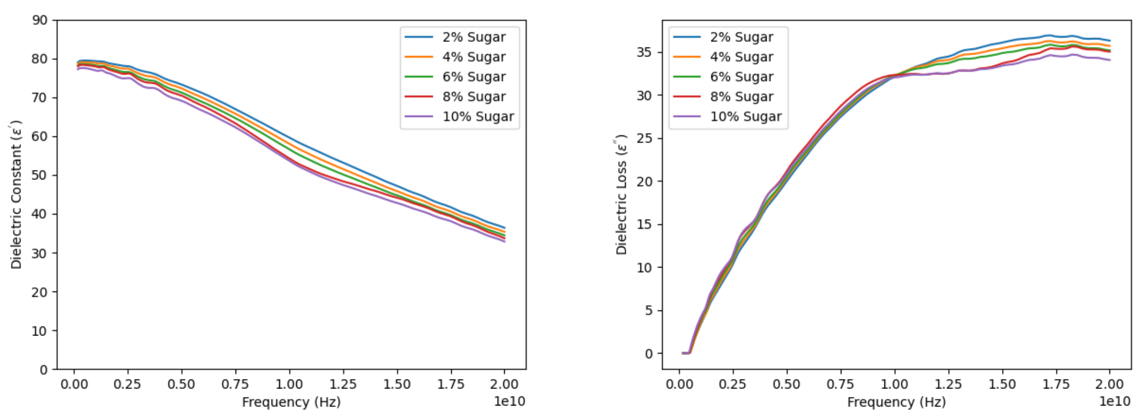

Figure 2: Dielectric Constant/Loss of 2-10% Sugar Solutions from 200 MHz to 20 GHz

Meanwhile, the ripples in the plots are caused by volumetric resonances that arise from testing small volumes of liquids. The resonances arise from the reflections at the boundaries of the sample, but these effects are negligible if a large enough volume of sample is used [5]. To improve the measurement accuracy and eliminate the resonances, the classic DAK system should be used instead, which has the capability of measuring liquids of larger volumes. The DAK-TL stands for the Dielectric Assessment Layers and has the capability of measuring about 10-12 mL of liquid in a small metal petri-dish or small solid specimens such as printed circuit boards. A comparison between the DAK and the DAK-TL can be seen in Figure 3.

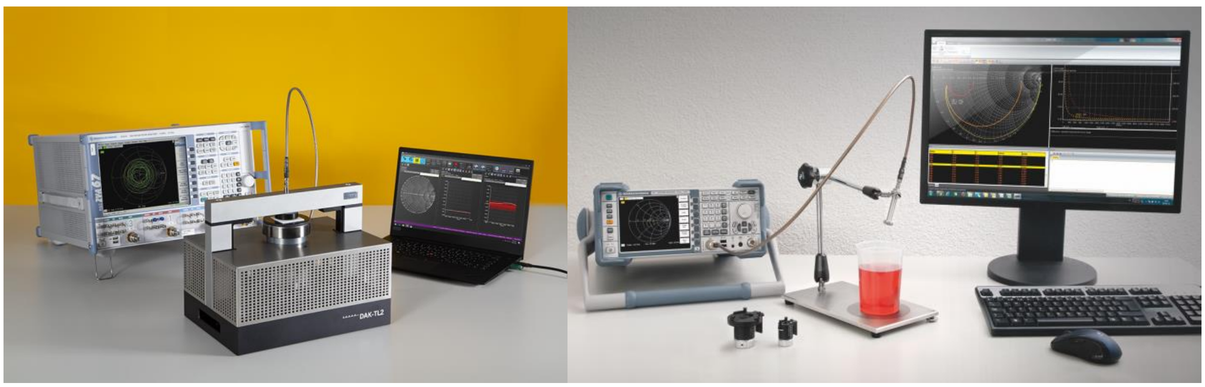

Figure 3: DAK-TL (Left) and DAK (Right) adapted from [6] and [7] respectively

2 Theoretical Background

2.1 Components of Milk

The dielectric properties have great potential for usage in compositional analysis of milk because they depend on the chemical and physical properties of a solution. However, this is precisely why implementing a radar based on dielectric properties is a complex process. Milk has many constituents, and each constituent can affect the dielectric properties of a sample. Raw milk is comprised of 87.4% water with various solids immersed in the aqueous solution making up the remaining 12.6% [8]. Milk fat found in raw milk accounts for about 3.6% of the total solids while the remaining constituents include caseins, whey proteins, lactose, and minerals [8]. The percentage that these constituents make up a sample of milk depend on various factors because no two cows will produce the exact same quality of milk. Milk quality and composition are subject to the age, nutrition, and species of a cow, as well as the lactation period, grazing, and seasonality [9]. In [10], the dielectric properties of butter were measured throughout the year, and it was shown that the properties did depend on seasonality. The dielectric constant was highest in mid April and lowest in mid February. Throughout the year, the dielectric constant fluctuated and gave a sawtooth trend. Interestingly, the percentage of total solids was reported to have little correlation to the fluctuations of dielectric constant with seasonality.

Milk fat itself is comprised of 97-98% triacylglyercides (TAGs), or more commonly known as triglycerides, and its remaining components are monoacylglycerol, diacylglycerol, cholesterol, phospholipids, and free fatty acids (FFA) [2], [11]. According to [12], no literature was found that measured the dielectric properties of pure milk fat. When Kudra et al. [12] measured pure milk fat, the measured data was determined to be similar in value to edible oil. This is expected because edible oil is a mixture of triacylglycerides, which is what makes up most of milk fat. Thus, milk fat can be treated as an emulsion, like that of oil-in-water [2], [12]-[13]. In section 4.2, an attempt to model milk as a two component solution of butterfat and water will be presented based on the measured dielectric properties of pure milk fat and water at 2450 MHz found in [12] using the mixture equations from [14] used to compare oil in water and water in oil.

Milk is a complex electrolyte solution due to its many components. Complex electrolytes are found in biological chemistry and understanding their effects will be of great importance to understanding the behaviour of these complex solutions [15]. The decrease of the relative permittivity of a solution with an increase of concentration of a solute is related to the number of solvent molecules bounded to the solute [15]. Whether this effect only arises due to bounded solvent molecules is unclear and is a controversial topic [15]. The behaviour of electrolytic solutions is currently a work in progress and was not studied intensively until about 30 years ago [15].

2.2 Behaviour of the Dielectric Properties

Since fats generally have a long nonpolar hydrocarbon chain, there is little interaction with an electromagnetic wave, making them dielectrically inert [16]. In both fats and oils, only electronic polarization processes should be observed and should exhibit very little loss [17]. In the experiment conducted by Gouw and Vlugter [18], the triglycerides used needed to be free of any moisture for accurate measurement results. The addition of water to oil causes the dielectric constant to spike upwards because water has a high dielectric constant and exhibits more polarization processes than oil. Lizhi et al. [17] examined the effect of increasing moisture content on the dielectric constant of corn oil. A regression model to determine the dielectric constant of corn oil based on moisture content was made. It was mentioned that by monitoring the changes in the dielectric constant of oil during the production process, the moisture content of a sample may be able to be determined.

Determining the butterfat content of a sample is the first step towards implementing a radar for the compositional analysis of milk, because it is the most variable factor and one of the major constituents in the composition of milk. In the study conducted by Nunes et al. [19], homo, low-fat, and skim milk were purchased from a local supermarket and tested over the frequency range 1 GHz to 20 GHz. Over this frequency range, with an increase in fat content, the dielectric constant and dielectric loss both decreased. In addition, the homo milk was diluted with water and measured. In comparison with the low-fat milk data, the dielectric constant and loss changed more rapidly with an increase of water than with an increase in butterfat. This suggests that the minor constituents and the non-fat components of milk play an important role in decreasing the dielectric constant or dielectric loss [19]. This also suggests that the fat content of milk can be estimated relative to the carbohydrate and protein content of a sample [19].

Zhu et al. [11] conducted a study using raw whole milk and raw skim milk over the frequency range 20 MHz to 4500 MHz. Over this frequency range, with an increase in butterfat, the dielectric constant decreased, and the dielectric loss decreased, similarly to the study conducted by Nunes et al. [19]. The decrease of the dielectric loss where the contribution due to ionic conductive losses dominates is due to a lack of mobility of the free-flowing ions caused by an increase in fat. It was determined that there was a linear relationship between the butterfat content and the dielectric properties. Using linear regression models, it is possible to determine the butterfat content from the dielectric constant and dielectric loss at a given frequency and temperature [11]. It was found that calculating the fat content from the dielectric loss of a sample gave a larger error than calculating from the dielectric constant. The error grew for predictions with less than 1% butterfat.

2.3 Modelling using Mixture Equations

The dielectric properties of a material are purely experimental and modelling them is a complicated topic of study. There are several different mixing models based on different assumptions that can be used to predict the dielectric properties of a material. Determining the model that fits best would be an important part in predicting the dielectric properties of milk because they can provide a basis for analysis. For the purposes of this report, the Fricke-Mudgett equation, the Landau and Lifshitz equation, and the Lichtenecker and Rother equation will be briefly examined and then applied in section 4.2. These three equations and many others were used in [14] to predict the dielectric properties of water-in-oil and oil-in-water emulsions at 2.45 GHz. It was found that out of the models used, the Fricke-Mudgett equation was the best for describing the oil-in-water emulsions while the Lichtenecker and Rother equation was the best for describing the water-in-oil emulsions.

2.3.1 Fricke-Mudgett Equation

The Fricke-Mudgett equation is given by

$$\epsilon_m = \epsilon_c \frac{\epsilon_d(1 + a\phi_d) + \epsilon_c a(1 - \phi_d)}{\epsilon_c(a + \phi_d) + \epsilon_d(1 - \phi_d)} \tag{3}$$

where ϵ_d is the dielectric constant or loss of the disperse phase, ϵ_c is the dielectric constant or loss of the continuous phase, ϕ_d is the volume fraction of the dispersed particles, and a is the shape coefficient of the dispersed particles [14], [20]. The shape equation takes on one of two values, either $a = 1$ for long cylinders or $a = 2$ for suspended spheres [14]. Fricke derived this equation based on the complex conductivity equations derived by Maxwell for solid spheres and Rayleigh for long cylinders [20]. As mentioned, the Fricke-Mudgett equation best predicted the dielectric properties of oil-in-water emulsions in [14] due to the condition that the dielectric constant of the continuous phase must be greater than that of the dispersed phase. Furthermore, this equation assumes that the suspended particles are relatively inert and not affected by electrochemical interactions in the solution [20], which gives rise to the alternative name noninteractive Fricke model. There is an interactive Fricke model which combines the Debye equations that can be used to describe alcohol-in-water mixtures mentioned in [20].

2.3.2 Landau and Lifshitz Equation

The Landau and Lifshitz Equation is given by

$$\epsilon_m = \epsilon_1 + 3\frac{\epsilon_1(\epsilon_2 - \epsilon_1)\phi_2}{2\epsilon_1 + \epsilon_2} \tag{4}$$

where ϵ_1 is the dielectric constant or loss of the medium, ϵ_2 is the dielectric constant or loss of the dispersed phase, and ϕ_2 is the volume fraction of the dispersed phase [14], [21]. The model assumes a finely dispersed solution with spherical suspended particles of low concentration, such that the inhomogeneity of the overall solution can be neglected. Additionally, the model is best used when the dielectric constant of the medium and the dielectric constant of the dispersed phase are similar. Due to these assumptions, the applications of the Landau and Lifshitz model are limited in the food industry [14].

2.3.3 Lichtenecker and Rother Equation

The Lichtenecker and Rother equation is given by

$$ln\epsilon_m = \phi_2 ln\epsilon_2 + (1 - \phi_2) ln\epsilon_1 \tag{5}$$

where the variables ϵ_m, ϵ_1, ϵ_2, and ϕ_2 are the same as the variables from the Landau and Lifshitz equation [14]. The equation requires that the relative permittivity is continuous, monotonic, and differentiable for ϵ_1, ϵ_2, and ϕ_2 [14]. Because the derivation of Lichtenecker-type mixing equations has a more mathematical approach than a physical one, there have been some criticisms of it for lacking rigour [22].

A summary of the 3 equations have been compiled in Table 4 based on the above descriptions from [14], [20]-[21].

Table 4: Summary of Predictive Modelling Equations

Model Name	**Equation**	**Conditions**
Fricke-Mudgett / Noninteractive Fricke	$\epsilon_m = \epsilon_c \frac{\epsilon_d(1 + a\phi_d) + \epsilon_c a(1 - \phi_d)}{\epsilon_c(a + \phi_d) + \epsilon_d(1 - \phi_d)}$	$a = 1$ (long cylinders) $a = 2$ (spheres) $\epsilon_c \gg \epsilon_d$
Landau and Lifshitz	$\epsilon_m = \epsilon_1 + 3\frac{\epsilon_1(\epsilon_2 - \epsilon_1)\phi_2}{2\epsilon_1 + \epsilon_2}$	ϕ_2 low ϵ_1, ϵ_2 similar in value
Lichtenecker and Rother	$ln\epsilon_m = \phi_2 ln\epsilon_2 + (1 - \phi_2)ln\epsilon_1$	ϵ_m continuous, monotonic, and differentiable for ϵ_1, ϵ_2, ϕ_2

3 Methodology

3.1 Sample Preparation

A few cartons of cream were purchased from a local No Frills of 5%, 10%, 18%, and 35% butterfat. Small samples were prepared in watertight Qorpak containers and transported to the microwave lab in Waterloo for testing. There was no access to a machine or device that could identify the exact concentration of each constituent in a sample. The butterfat content of a sample was taken as printed on the carton. However, there could be slight variations with the butterfat content of a sample, for example, 5% milk may have a bit more or less than 5% butterfat in it.

To test the validity of the regression model when applying to a new data set, new samples were mixed by weight from the 5%, 10%, 18%, and 35% creams using a Sartorius BP 2100 S scale to create 6%, 8%, 12%, 16%, and 20% creams. To obtain 6% cream, 30g of 5% cream and

7.5g of 10% cream were measured separately and then mixed. The measured values are displayed in Table 5 and the uncertainty per measurement was about ±0.02g.

Table 5: Mixed Cream Preparation

Mixed Percentage	**Sample 1**	**Required Weight of Sample 1 (g)**	**Measured Weight of Sample 1 (g)**	**Sample 2**	**Required Weight of Sample 2 (g)**	**Measured weight of Sample 2 (g)**
6%	5%	30	30.02	10%	7.5	7.51
8%	5%	16.8	16.80	10%	25.2	25.21
12%	10%	36	36.00	18%	12	12.01
16%	10%	10.5	10.50	18%	31.5	31.51
20%	18%	32.54	32.55	35%	4.34	4.35

3.2 Dielectric Measurements using DAK-TL

Similarly, to [3] and section 1.2, the DAK-TL 3.5 probe was calibrated with the Open-Short-Load (OSL) method over the frequency range 200 MHz to 20 GHz using the recommended resolution settings provided by SPEAG. The set-up of the DAK-TL is shown in Figure 4 where the instrument is connected to a Keysight PNA Network Analyzer N5227A. The DAK-TL is software controlled and the dielectric properties are calculated automatically through the DAK software from the measured reflection coefficients. A 10 mL syringe was used to draw 10 mL of a sample into a metal petri-dish. The distance between the platform and the bottom of the probe was set to 4 mm. This distance was determined after several repetitions with 10 mL of distilled water to ensure the bottom of the probe was fully submerged in the liquid without the liquid overflowing. Each sample was tested three times by lifting the probe and re-inserting it back into the liquid to ensure repeatability among the trials. After testing a sample, the sample was discarded, and the metal petri-dish was thoroughly cleaned with hand sanitizer.

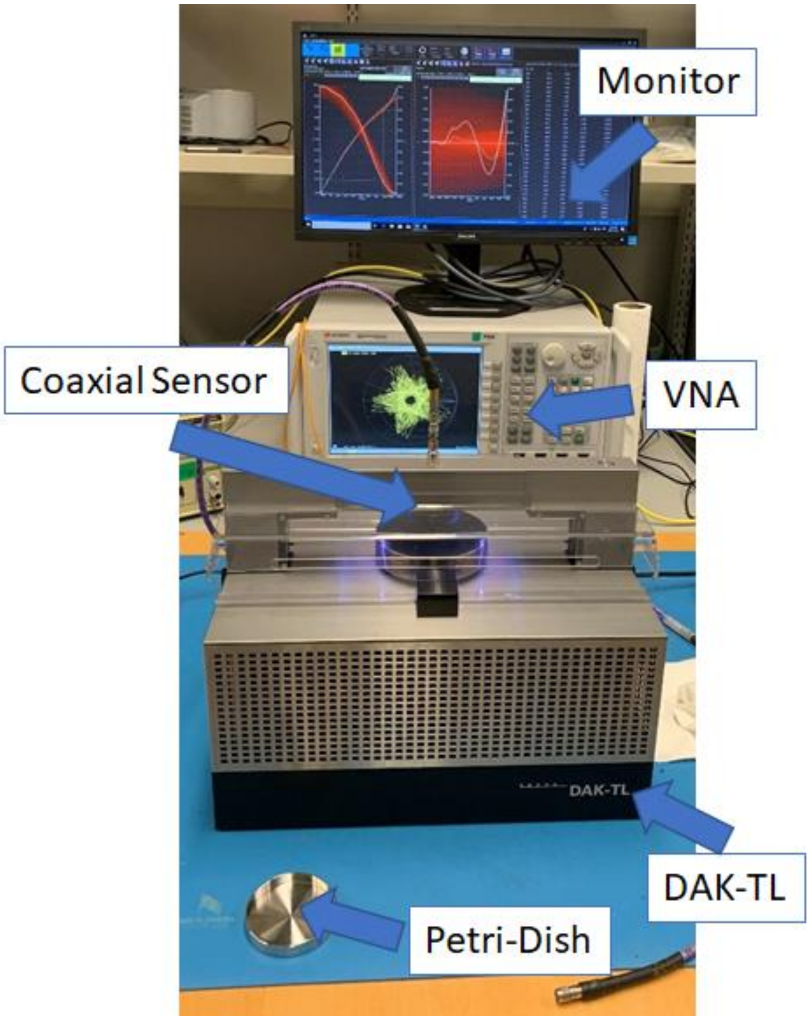

Figure 4: DAK-TL Set-Up

3.3 Temperature Study

Since the dielectric properties of a sample are temperature dependent, a brief temperature study was conducted on the DAK-TL before testing all the cream samples. Distilled water and two 5% cream samples at 9°C and 15°C were tested two times each. After, the calibration was wiped and re-calibrated with the exact same settings and procedure. This was done so that there would be confidence in the calibration of the machine and to eliminate the possibility of temperature affecting the results. Distilled water and the two 5% cream samples, this time at 6°C and 17.7°C, were tested again. The results are shown in Figure 5.

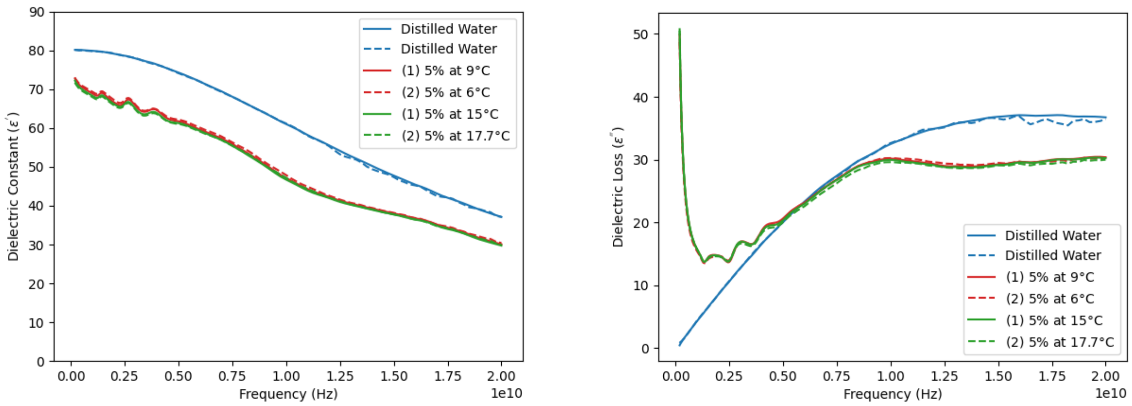

Figure 5: DAK-TL Calibration Comparison and Temperature Study

Interestingly, the dielectric properties between the 5% creams at different temperatures made very little difference to the measurements and were nearly unnoticeable. Zhu et al. [23] tested the dielectric properties of milk at different temperatures over the frequency range 10 MHz to 4,500 MHz, and there should be quite a difference. The reason why the temperature did not make much of a difference to the measurements may be because the TL controller or probe heats up the sample during testing. Before testing the cream samples, they were on average about 6°C and were about 20°C after testing. Thus, the DAK-TL is not great for studying the temperature-dependent properties of liquids.

4 Results and Discussion

4.1 Effect of Butterfat Concentration on Dielectric Properties

The measured dielectric constant and loss vs frequency of the cream products are plotted in Figure 6: Dielectric Constant/Loss of 5-35% Cream from 200 MHz to 20 GHzFigure 6. The plots seem to agree well with the plots presented in previous literature such as [11] and [19] as a function of frequency. With an increase of fat content, the dielectric constant and dielectric loss both decrease. It can be noted that there appears to be a linear relation between the fat content and dielectric properties, which Zhu et al. [11] pointed out. Using this as a basis for the relationship between the two, the butterfat content may be predicted using a linear regression model, which will be discussed in section 4.3. There are some small oscillations in the dielectric constant and loss at low frequencies and some waviness in the dielectric losses with higher frequencies after 10 GHz, as previously seen in section 1.2 regarding the salt and sugar solution experiment over the same frequency range. Currently, the reason why the dielectric losses are wavy is unclear, but the results show that the trials and sample are all repeatable. The contribution to the dielectric losses from the ionic conductivity of the sample decreases with an increase of fat concentration due to the fat particles immobilizing the free-flowing ions, as expected.

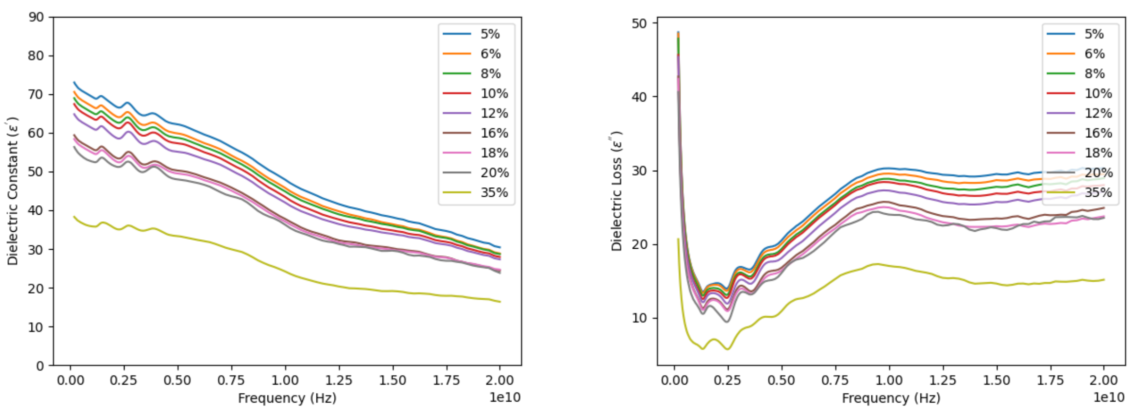

Figure 6: Dielectric Constant/Loss of 5-35% Cream from 200 MHz to 20 GHz

To investigate the oscillations at low frequencies, some of the filters in the DAK software were used and compared to the non-filtered data. For example, an attempt was made to smooth out the oscillations using the FR Filter. FR stands for Flange Resonance, where the flange is the boundary component of the coaxial sensor that meets the sample being tested (Figure 7). Flange resonance can be observed in low loss materials at measured frequencies that have high permittivity such that the loss tangent is less than 0.3 [5], [24]. Flange and volumetric resonances are interrelated because they both arise from the boundary effects of a sample [5].

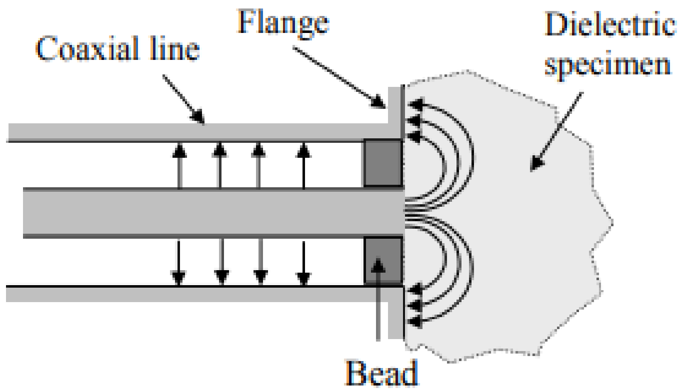

Figure 7:Coaxial Sensor with Flange. Adapted from [5]

Upon turning the FR Filter on, the dielectric losses changed more drastically than the dielectric constant as seen in Figure 8. Gregory [5] states that flange resonance is a common occurrence in water because of its low losses at low frequencies, and if it is not minimized in the calibration, the other measurements will be affected more greatly. In the calibrations performed in section 3.3, flange resonance was not noticeable in the distilled water measurements, nor was it visible during the calibration process. Also, milk and cream are quite lossy, especially at low frequencies due to conductive properties of milk. Therefore, the small oscillations seen are most likely due to volumetric resonances from the reflections of the electromagnetic wave at the boundaries of the sample rather than from flange resonances. Understanding exactly how flange resonance arises and how to minimize it is outside the scope of this project and may require a deeper understanding of sensor designs.

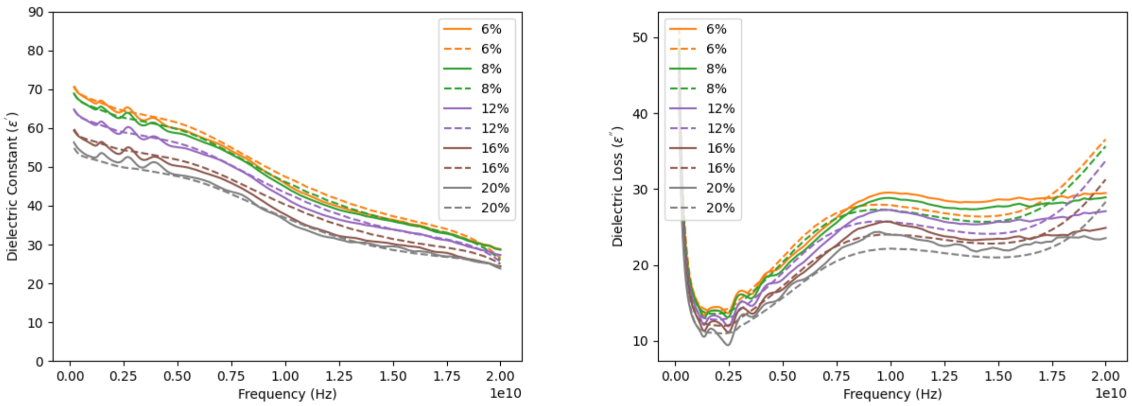

Figure 8: Dielectric Constant/Loss of 5-35% Cream from 200 MHz to 20 GHz with FR Filter

4.2 Modelling a Two-Component Solution of Butterfat and Water at 2450 MHz

Using the Fricke-Mudgett, Landau-Lifshitz, and Lichtenecker-Rother mixing equations described in section 2.3, an attempt to model milk and cream at a constant temperature and frequency was made. The dielectric constant and loss vs butterfat content are shown in Figure 9 at 2450 MHz. This frequency was chosen because [12] measured the dielectric properties of pure fat and distilled water at 2450 MHz. These values were used when computing the values for the predictive mixing models. Erle et al. [14] also used 2450 MHz for their comparison between the mixing models and their measured values. From [12], the measured dielectric constant of fat at 2450 MHz is 2.613 and the measured dielectric loss is 0.153. Meanwhile, the measured dielectric constant and loss of water at 2450 MHz are 78.00 and 13.4 respectively. The value of the volume fraction of the dispersed phase was taken to be the concentration of butterfat in a sample as printed on its carton.

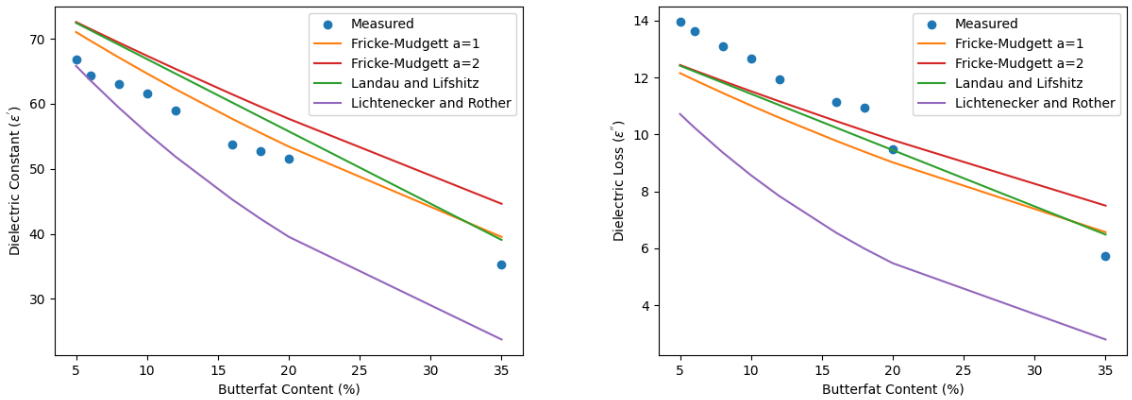

Figure 9: Modelling the Dielectric Constant/Loss vs. Butterfat Content at 2450 MHz

From the plotted measurements in Figure 9, the Fricke-Mudgett $a = 1$ model appears to best fit the measured data for both the dielectric constant and dielectric loss. The values are not exact, but this is to be expected because milk and cream have other constituents that would lower the dielectric constant. As for the dielectric loss, all the models begin to converge to a similar value for 0% butterfat, but the measured values of the dielectric loss are higher. This could be due to the treatment of milk as a two-component solution of emulsified fat in water. At 0% fat, the solution is not truly 100% water, as there are other components such as proteins, salts, lactose, etc. that are also in the solution that would decrease the dielectric constant and increase the dielectric losses. Dielectric losses increasing due to an increasing concentration of salt or sugar over this frequency range was explored in more detail in [3]. At lower concentrations, the effect on the dielectric constant and losses from the other constituents of milk becomes more apparent, especially in the case of the dielectric losses.

For both plots, the Lichtenecker-Rother model is very different the measured values. It was reported in [14], as previously mentioned, that the Lichtenecker-Rother model was best used for water-in-oil emulsions. Since milk is treated as an oil-in-water emulsion, it is expected to see a large deviation. In general, the Fricke-Mudgett models, especially $a = 1$, fit the measured data well considering that there were no fitted parameters used. This result is similar to the result obtained by Erle et al. [14] regarding oil-in-water emulsions. The reason why the Fricke-Mudgett $a = 1$ model fitted better than the Fricke-Mudgett $a = 2$ model was not explained in [14], but may be due to the fact triglycerides are made up of long chains of hydrocarbon molecules.

4.3 Butterfat Content Prediction using Linear Regression Analysis

The most accurate way to effectively model the dielectric constant or loss vs butterfat content would be to use regression analysis. Similarly, to the work presented by Zhu et al. [11], there is a linear relationship between the dielectric properties and the fat content. Thus, by fitting a linear equation with the form

$$\epsilon' = a_1 F + b_1 \tag{4}$$

$$\epsilon'' = a_2 F + b_2 \tag{5}$$

and inverting them for F, where a_1, b_1, a_2, and b_2 are the coefficients of best fit and F is the fat content, it is possible to obtain a prediction for the fat content of a sample [11]. Using the data collected, the two equations would each give a value for the fat content and can be compared to the milk fat percentage printed on the carton. The code used for this portion of the report can be found in the Appendix.

Using the sklearn package in Python, the collected dielectric measurement data of the 5%, 10%, 18%, and 35% creams were split into a 50:50 ratio and plotted at 2450 MHz shown in Figure 10. After inverting the equation for the fat content of a sample, the random_state of the train_test_split() function was adjusted several times to determine the coefficients that would give the most accurate predictions for the test data and best root mean square value.

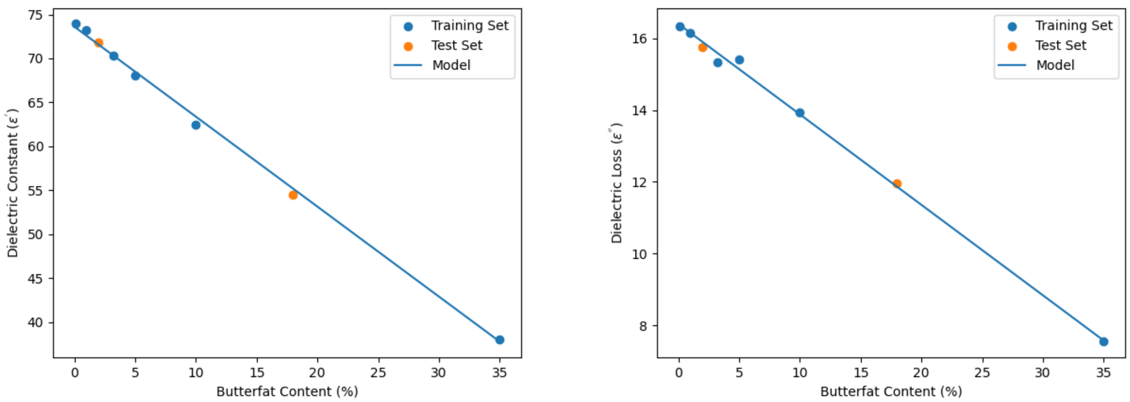

Figure 10: Dielectric Constant/Loss vs Butterfat Content at 2450 MHz

For the dielectric constant vs butterfat content plot, the equation of best fit obtained using this method, to three decimal places, is given by

$$\epsilon' = -1.030F + 71.342\,. \tag{6}$$

The root-mean-square error given by the training set and the test set are 5.024e-15 and 0.630 respectively. As for the dielectric loss vs butterfat content, the obtained equation is given by

$$\epsilon'' = -0.273F + 15.311 \tag{7}$$

with root-mean-square error 6.280e-16 for the training set and 0.389 for the test set. Using these two equations for the test data, which consisted of 5% and 10% for the dielectric constant model and 10% and 18% for the dielectric loss model, the calculated fat results are presented in Table 6 and Table 7. The results are within decent range but could be improved with a larger set of training data.

Table 6: Predicted Fat Content at 2450 MHz from Dielectric Constant using Test Data

BF %	**Predicted BF % from ϵ'**	**Error in Prediction from ϵ'**
5	4.336	13.28%
10	9.447	5.53%

Table 7: Predicted Fat Content at 2450 MHz from Dielectric Loss using Test Data

BF %	**Predicted BF % from ϵ''**	**Error in Prediction from ϵ''**
10	9.629	3.71%
18	16.021	10.99%

Applying the regression models to the mixed cream data (Table 8) showed promising results and had a 4.73% average error based on the dielectric constants and a 5.83% average error based on the dielectric losses. This is consistent with the work conducted by Zhu et al. [11], where the dielectric constant model provided a smaller error than the dielectric loss one, but at 41 MHz instead.

Table 8: Predicted Fat Content of Mixed Cream Samples at 2450 MHz

BF %	**Predicted BF % from ϵ'**	**Error in Prediction from ϵ'**	**Predicted BF % from ϵ''**	**Error in Prediction from ϵ''**
6	6.722	12.03%	6.108	1.80%
8	8.079	0.99%	8.118	11.80%
12	11.985	0.13%	12.368	3.07%
16	17.019	6.37%	15.237	4.77%
20	19.171	4.15%	21.540	7.70%

4.4 Possible Sources of Error

There were small noticeable oscillations from Figure 6 that may have arisen due to volumetric resonance. Unfortunately, the volumetric resonance issue would be difficult to address because the petri-dish only holds about maximum 12 mL of liquid. For the most accurate measurements, the classic DAK system should be used, but is unfortunately not available in the microwave lab.

Another issue that may have come into play during the development of the regression model is that the concentration of butterfat in a carton may not be exactly what is printed on the carton. There may be a slight deviance from 5%. It is also possible that the percentage of total solids in the cartons also ranged. To obtain a more accurate regression model, the exact concentrations of butterfat and total solids in the cartons should be known, since the dielectric constant and loss increase with water content for milk. This issue would also be difficult to address because the microwave lab does not have a machine that can be used for the compositional analysis of milk.

Since the determination of the regression model coefficients used a small range of data, 2 points, it potentially introduces a large error. This can be improved by increasing the number of points in both the training and the test sets. Even with the small training set, the fat content predictions are within good range and shows great promise towards determining the amount of butterfat a sample has given its dielectric properties.

5 Conclusion

The dielectric properties of 5-35% cream products were measured using the DAK-TL over the frequency range 200 MHz to 20 GHz. Comparing the obtained results with the results found in the literature survey conducted, the trends align well and are expected. The dielectric constant and loss both decreased linearly with an increase in fat content, and the other constituents play a role in lowering the measured properties. These were compared to predictive models of a two-component solution consisting of milk fat and water, and the differences were as expected. Two regression models were made, one based on the dielectric constants and the other based on the dielectric losses of cream, to determine the butterfat content of a given sample. Considering the small training set and test set that was used in the development of the models, they both show predictions within a good range. Based on the temperature study conducted on the DAK-TL prior to the measurements, it indicated that the instrument is not great for studying the effect of temperature on the dielectric properties of a liquid sample. There are also volumetric resonances that arise with this instrument when testing cream samples at lower frequencies. For future studies, it would be interesting to study the effect of homogenization, seasonality, and the dielectric properties at different frequencies other than 200 MHz to 20 GHz.

Appendix

Python Code for Regression Modelling

```
import pandas as pd
import matplotlib.pyplot as plt
from sklearn.model_selection import train_test_split
from sklearn.linear_model import LinearRegression
from sklearn.metrics import mean_squared_error, r2_score
import numpy as np

# read in the data file
df = pd.read_csv(r'D:\Jessica\Documents\School\Project
Winter\Jessica_march24\CompiledData.csv', skiprows=3)

# import the data for a given frequency
f = 2450
findex = df.loc[df["fmhz"] == f]

# import the regular cream data for the frequency given above
cream5dc = findex["5_dc"].values
cream5dl = findex["5_dl"].values

cream10dc = findex["10_dc"].values
cream10dl = findex["10_dl"].values

cream18dc = findex["18_dc"].values
cream18dl = findex["18_dl"].values

cream35dc = findex["35_dc"].values
cream35dl = findex["35_dl"].values

# import the mixed cream data for the frequency given above
cream6dc = findex["6_dc"].values
cream6dl = findex["6_dl"].values

cream8dc = findex["8_dc"].values
cream8dl = findex["8_dl"].values

cream10dc = findex["10_dc"].values
cream10dl = findex["10_dl"].values

cream12dc = findex["12_dc"].values
cream12dl = findex["12_dl"].values

cream16dc = findex["16_dc"].values
cream16dl = findex["16_dl"].values

cream20dc = findex["20_dc"].values
```

```
cream20dl = findex["20_dl"].values

# prepare data sets to plot
fat = np.array([5.00, 10.00, 18.00, 35.00]).reshape(-1, 1)
dc = np.concatenate([cream5dc, cream10dc, cream18dc, cream35dc])
dl = np.concatenate([cream5dl, cream10dl, cream18dl, cream35dl])

# split the data randomly for training and testing
sample_size = 0.5
randomdc = 4
randomdl = 8
fat_train, fat_test, dc_train, dc_test = train_test_split(fat, dc,
test_size=sample_size, random_state=randomdc)
fat_train2, fat_test2, dl_train, dl_test = train_test_split(fat, dl,
test_size=sample_size, random_state=randomdl)

dcmodel = LinearRegression().fit(fat_train, dc_train)

dctrainpredict = []
dctestpredict = []
for i in range(0, len(dc_train)):
    dctrainpredict.append(dcmodel.coef_*fat_train[i] +
dcmodel.intercept_)
for i in range(0, len(dc_test)):
    dctestpredict.append(dcmodel.coef_*fat_test[i] +
dcmodel.intercept_)

print("Dielectric Constant Equation: ", dcmodel.coef_ , "x + ",
dcmodel.intercept_)
print("R2 (Training Set): ", dcmodel.score(fat_train, dc_train))
print("RMSE (Training Set): ", np.sqrt(mean_squared_error(dc_train,
dctrainpredict)))
print("R2 (Test Set): ", dcmodel.score(fat_test, dc_test))
print("RMSE (Test Set): ", np.sqrt(mean_squared_error(dc_test,
dctestpredict)))

dlmodel = LinearRegression().fit(fat_train2, dl_train)

dltrainpredict = []
dltestpredict = []
for i in range(0, len(dl_train)):
    dltrainpredict.append(dlmodel.coef_*fat_train2[i] +
dlmodel.intercept_)
for i in range(0, len(dl_test)):
    dltestpredict.append(dlmodel.coef_*fat_test2[i] +
dlmodel.intercept_)

print("Dielectric Loss Equation: ", dlmodel.coef_ , "x + ",
dlmodel.intercept_)
print("R2 (Training Set): ", dlmodel.score(fat_train2, dl_train))
print("RMSE (Training Set): ", np.sqrt(mean_squared_error(dl_train,
dltrainpredict)))
```

```
print("R2 (Test Set): ", dlmodel.score(fat_test2, dl_test))
print("RMSE (Test Set): ", np.sqrt(mean_squared_error(dl_test,
dltestpredict)))

# perform fat content prediction based on dc_test data and compare
with fat_test
print("Fat Content Prediction using Test Data")
fatpredict = []
fatpredict2 = []
print("From Dielectric Constant")
for i in range(0, len(dc_test)):
    fatpredict.append((dc_test[i] - dcmodel.intercept_)/dcmodel.coef_)
    print("Measured Fat: ", fat_test[i], "Predicted Fat from DC: ",
fatpredict[i])
    i += 1
print("From Dielectric Loss")
for i in range(0, len(dl_test)):
    fatpredict2.append((dl_test[i] - dlmodel.intercept_) /
dlmodel.coef_)
    print("Measured Fat: ", fat_test2[i], "Predicted Fat from DC: ",
fatpredict2[i])
    i += 1

# predict fat based on the new set of collected data
print("Fat Content Prediction using Mixed Cream Data")
fatmix = np.array([6.00, 8.00, 12.00, 16.00, 20.00])
mixdc = np.concatenate([cream6dc, cream8dc, cream12dc, cream16dc,
cream20dc])
mixdl = np.concatenate([cream6dl, cream8dl, cream12dl, cream16dl,
cream20dl])

fatmixpredict =[]
fatmixpredict2 = []
for i in range(0, len(mixdc)):
    fatmixpredict.append((mixdc[i] -
dcmodel.intercept_)/dcmodel.coef_)
    fatmixpredict2.append((mixdl[i] - dlmodel.intercept_) /
dlmodel.coef_)
    print("Measured Fat: ", fatmix[i], "Predicted Fat from DC: ",
fatmixpredict[i], "Predicted Fat from DL: ", fatmixpredict2[i])
    i += 1

# plot the models, training sets, and test sets
fat_ = np.linspace(0, 35, 100)

plot1 = plt.figure(1)
plt.scatter(fat_train, dc_train, label="Training Set")
plt.scatter(fat_test, dc_test, label="Test Set")
plt.scatter(fatmix, mixdc, label="New Data")
plt.plot(fat_, dcmodel.coef_*fat_ + dcmodel.intercept_, label="Model")

handles, labels = plt.gca().get_legend_handles_labels()
```

```
order = [1, 2, 0, 3]
plt.legend([handles[i] for i in order], [labels[i] for i in order])

plt.xlabel("Butterfat Content (%)")
plt.ylabel("Dielectric Constant ($\epsilon^{'}$)")

plot2 = plt.figure(2)
plt.scatter(fat_train2, dl_train, label="Training Set")
plt.scatter(fat_test2, dl_test, label="Test Set")
plt.scatter(fatmix, mixdl, label="New Data")
plt.plot(fat_, dlmodel.coef_*fat_ + dlmodel.intercept_, label="Model")

handles, labels = plt.gca().get_legend_handles_labels()
order = [1, 2, 0, 3]
plt.legend([handles[i] for i in order], [labels[i] for i in order])

plt.xlabel("Butterfat Content (%)")
plt.ylabel("Dielectric Loss ($\epsilon^{''}$)")

plt.show()
```

References

[1] A. Yavari, D. Georgakopoulos, H. Agrawal, H. Korala, P. P. Jayaraman and J. Karabotić Milovac, "Internet of Things Milk Spectrum Profiling for Industry 4.0 Dairy and Milk Manufacturing," *2020 International Conference on Information Networking (ICOIN),* pp. 342-347, 2020.

[2] Z. Zhu and W. Guo, "Recent developments on rapid detection of main constituents in milk: a review," *Critical Reviews in Food Science and Nutrition,* vol. 61, no. 2, pp. 312-324, 2021.

[3] J. Chong, "Dielectric Properties of Salt and Sugar Solutions at 200 MHz to 10 GHz," PHYS 437A, December 2021.

[4] P. B. Ishai, M. S. Talary, A. Caduff, E. Levy and Y. Feldman, "Electrode polarization in dielectric measurements: a review," *Measurement Science and Technology,* vol. 24, 2013.

[5] A. P. Gregory and R. N. Clarke, "Dielectric Metrology with Coaxial Sensors," *Measurement Science and Technology,* March 2007.

[6] "DAK – Dielectric Assessment Kit Product Line," SPEAG, [Online]. Available: https://speag.swiss/products/dak/overview/. [Accessed 8 April 2022].

[7] "DAK (4 MHz - 67 GHz)," SPEAG, [Online]. Available: https://speag.swiss/products/dak/dak-probes/. [Accessed 8 April 2022].

[8] R. C. Chandan, Ed., Dairy Processing & Quality Assurance, John Wiley & Sons, Inc., 2008.

[9] D. Agranovich, I. Renhart, P. B. Ishai, G. Katz, D. Bezman and Y. D. Feldman, "A microwave sensor for the characterization of bovine milk," *Food Control,* December 2015.

[10] J. F. O'Connor and E. C. Synnott, "Seasonal Variation in Dielectric Properties of Butter at 15 MHz and 4°C," *Irish Journal of Food Science and Technology,* vol. 6, no. 1, pp. 49-59, 1982.

[11] X. Zhu, W. Guo and Z. Liang, "Determination of the Fat Content in Cow’s Milk Based on Dielectric Properties," *Food Bioprocess Technology,* vol. 8, pp. 1485-1494, 2015.

[12] T. Kudra, V. Raghaven, C. Akyel, R. Bosisio and F. van de Voort, "Electromagnetic Properties of Milk and its Constituents at 2.45 GHz," *Journal of Microwave Power and Electromagnetic Energy,* vol. 27, no. 4, pp. 199-204, 1992.

[13] H. D. Goff, A. Hill and M. A. Ferrer, Dairy Science and Technology eBook.

[14] U. Erle, M. Regier, H. Persch and H. Schubert, "Dielectric Properties of Emulsions and Suspensions: Mixture Equations and Measurement Comparisons," *Journal of Microwave Power and Electromagnetic Energy,* vol. 35, no. 3, pp. 185-190, 2000.

[15] M. R. Wright, An Introduction to Aqueous Electrolyte Solutions, John Wiley & Sons Ltd, 2007.

[16] V. Orsat and G. V. Raghavan, "Radio-Frequency Processing," in *Emerging Technologies for Food Processing*, 2nd ed., D. Sun, Ed., Academic Press, 2014.

[17] H. Lizhi, K. Toyoda and I. Ihara, "Dielectric properties of edible oils and fatty acids as a function of frequency, temperature, moisture, and composition," *Journal of Food Engineering,* vol. 88, pp. 151-158, 2008.

[18] I. Gouw and I. Vlugter, "Physical Properties of Triglycerides IV. Dielectric Constant," *European Journal of Lipid Science and Technology,* vol. 69, no. 4, pp. 223-226, 1967.

[19] A. C. Nunes, X. Bohigas and J. Tejada, "Dielectric study of milk for frequencies between 1 and 20 GHz," *Journal of Food Engineering,* vol. 76, pp. 250-255, 2006.

[20] R. E. Mudgett, D. I. C. Wang and S. A. Goldblith, "Prediction of Dielectric Properties in Oil-Water and Alcohol-Water Mixtures at 3,000 MHz, 25°C Based on Pure Component Properties," *Journal of Food Science,* vol. 39, pp. 632-635, 1974.

[21] L. D. Landau and E. M. Lifshitz, Course of Theoretical Physics Volume 8: Electrodynamics of Continuous Media, Pergamon Press, 1960.

[22] R. Simpkin, "Derivation of Lichtenecker's Logarithmic Mixture Formula From Maxwell's Equations," *IEEE Transactions on Microwave Theory and Techniques,* vol. 58, no. 3, pp. 545-550, March 2010.

[23] T. Truong, M. Palmer, N. Bansal and B. Bhandari, Effect of Milk Fat Globule Size on the Physical Functionality of Dairy Products, Springer, 2016.

[24] R. Vijay, R. Jain and K. S. Sharma, "Dielectric Properties of Water in Microwave Frequencies," *International Journal of Engineering Research & Technology,* vol. 3, no. 3, 2015.

[25] W. Guo, X. Zhu, H. Liu, R. Yue and S. Wang, "Effects of milk concentration and freshness on microwave dielectric properties," *Journal of Food Engineering,* vol. 99, pp. 344-350, 2010.