Ratchet currents in activity landscapes

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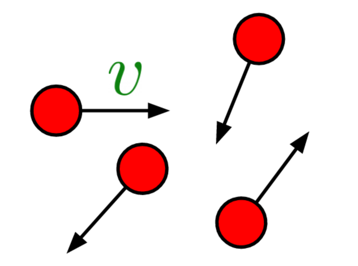
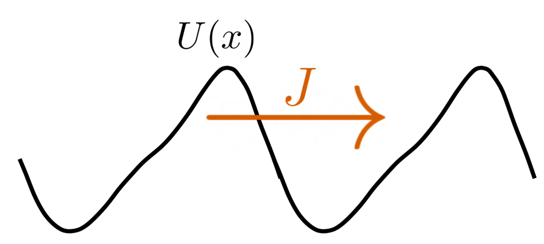
Introduction

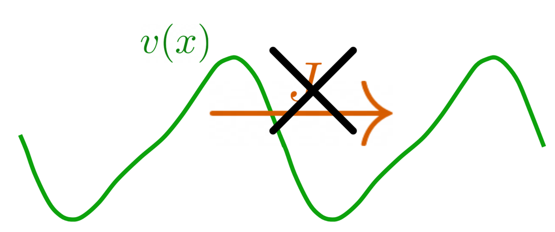
Breaking time-reversal and parity symmetries in stochastic dynamics generically leads to **steady-state density currents (ratchet currents)** J. In active matter, a typical example is active particles in an asymmetric potential $U(x)$ [1]. An interesting exception is non-interacting self-propelled particles in an asymmetric activity landscape $v(x)$, which does not lead to steady currents. Surprisingly, the emergence of ratchet currents is restored by including symmetric repulsive interactions between the particles [2].



TRS-breaking self-propulsion

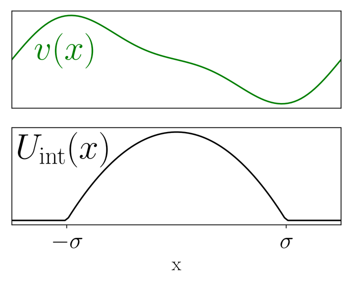


Asymmetric potential



Asymmetric activity

Interacting particles in activity landscape



Let us focus on RTPs in 1D, whose dynamics read

$$\dot{x}_i = v(x_i)\sigma_i(t) - \sum_{j\neq i} \partial_i U_{\rm int}(x_i - x_j)\,, \qquad \sigma_i \xrightarrow{\alpha} -\sigma_i$$

The dynamics for the density and magnetization fields, defined by $\rho(x) = \langle \frac{1}{N}\sum_i \delta(x-x_i)\rangle$ and $m(x) = \langle \frac{1}{N}\sum_i \sigma_i \delta(x-x_i)\rangle$, then read:

$$\partial_t \rho = -\frac{\partial}{\partial x}\Big(mv - \int dx' \partial_x U_{\rm int}(x-x')P_2(x,x')\Big)$$
$$\partial_t m = -\frac{\partial}{\partial x}\Big(\rho v - \int dx' \partial_x U_{\rm int}(x-x')M_2(x,x')\Big) - \alpha m\,,$$

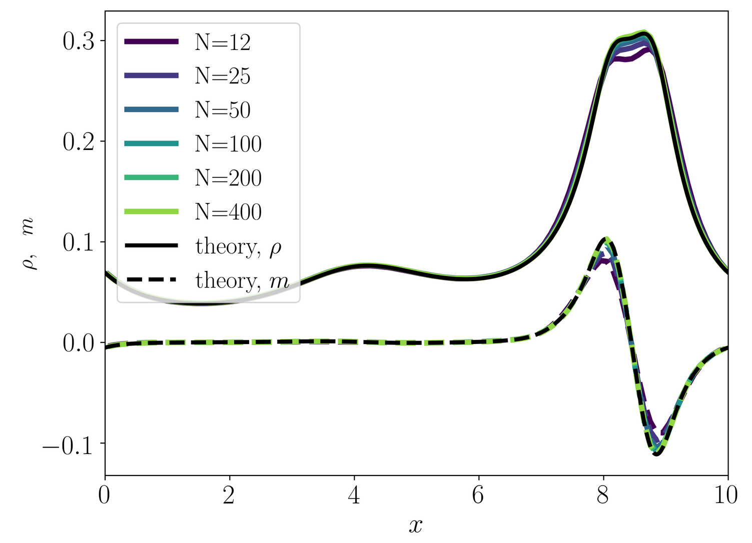
where P_2 and M_2 are two-point functions whose mean-field approximations are given by $P_2(x,x') \approx \rho(x)\rho(x')$ and $M_2(x,x') \approx m(x)\rho(x')$. This leads to a closed system:

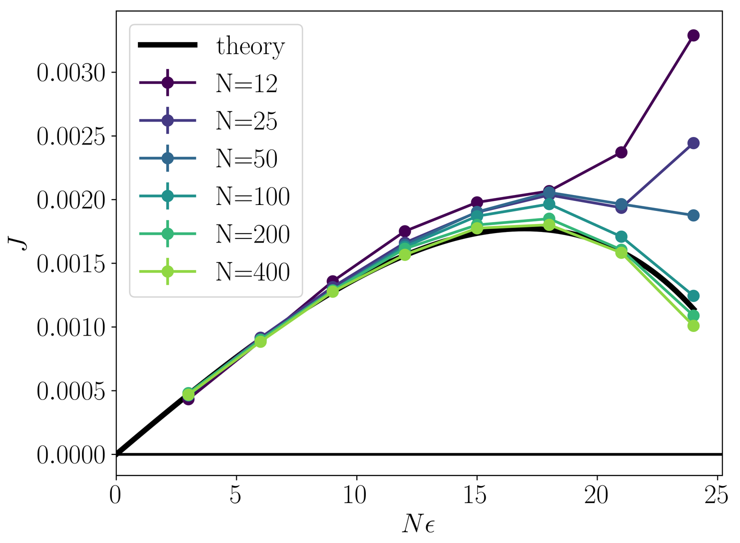
$$\partial_t \rho = -\frac{\partial}{\partial x}\Big(mv - \int dx' \partial_x U_{\rm int}(x-x')\rho(x')\rho(x)\Big) \equiv -\partial_x J$$
$$\partial_t m = -\frac{\partial}{\partial x}\Big(\rho v - \int dx' \partial_x U_{\rm int}(x-x')\rho(x')m(x)\Big) - \alpha m$$

To understand the emergence of currents, we rescale $U_{\rm int}(x) = \varepsilon \tilde{U}_{\rm int}(x)$ and solve the dynamics perturbatively in ε by expanding ρ, m, and J in powers of ε. This can be done to arbitrary order and the first orders read

$$J_0 = 0, \qquad J_1 = \frac{1}{N}\int_0^L \rho_0^2 \partial_x[(\rho_0 * \tilde{U}_{\rm int})(x)]dx, \quad \ldots$$
$$\rho_0 = \frac{C_0}{v}, \qquad \rho_1 = -\frac{\alpha}{v}\int_0^x \frac{1}{v}\big[J_1 + \rho_0(\rho_0 * \tilde{U}_{\rm int})\big]dx' + \frac{C_1}{v}, \quad \ldots$$

Numerical simulations carried out with a soft repulsive harmonic potential show a convergence to the theory as $N \to \infty$.





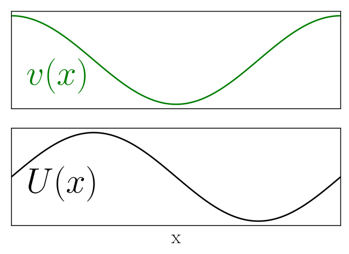
Effective potential picture

In this mean-field computation, the origin of the ratchet current can be understood intuitively as follows: each particle move in a mean-field effective potential:

$$U_{\rm eff}(x) = \int dx' U_{\rm int}(x-x')\rho(x') \qquad (1)$$

with a spatially varying activity $v(x)$. The problem is then equivalent to a single self-propelled particle experiencing both an activity landscape $v(x)$ and an external potential $U(x)$.

Activity landscape and external potential



This simpler problem, which has attracted attention recently for active Brownian particles [3], can be solved exactly for run-and-tumble particles.

Microscopic dynamics

$$\dot{x} = v(x)\sigma(t) - \partial_x U(x)$$
$$\sigma \xrightarrow{\alpha} -\sigma$$

Master equation

$$\partial_t \rho = -\partial_x(vm - \rho\mu\partial_x U) \equiv -\partial_x J$$
$$\partial_t m = -\partial_x(v\rho - m\mu\partial_x U) - \alpha m$$

In the steady-state, direct algebra shows that (with $f' \equiv \partial_x f$):

$$J = -\frac{\mu}{1 + \frac{\mu}{\alpha}U'' - \frac{\mu v'}{\alpha v}U'}\left[\left(U' + \frac{vv'}{\mu\alpha} + \frac{\mu (U')^2 v'}{\alpha v}\right)\rho - \left[\left(\frac{v^2}{\mu\alpha} - \frac{\mu (U')^2}{\alpha}\right)\rho\right]'\right]$$

Diffusion in inhomogenous media

The aforementioned computation is equivalent to an older problem, solved in [4, 5] and that applies to many active ratchets. If the steady-state Master equation can be written in the form

$$\partial_t \rho = 0 = -\partial_x J, \qquad J \equiv \mu_{\rm e}[\rho \partial_x U_{\rm e} + \partial_x(T_{\rm e}\rho)] \qquad (2)$$

Then, for ***any*** $\mu_{\rm e}(x)$, $U_{\rm e}(x)$, $T_{\rm e}(x)$, the steady-state solution in the presence of periodic boundary conditions is [5]:

$$J \propto 1 - e^{\Phi(L)}, \qquad \rho(x) \propto \frac{e^{-\Phi(x)}}{T(x)}, \qquad \Phi(x) = \int_0^x \frac{U'(t)}{T(t)}dt \qquad (3)$$

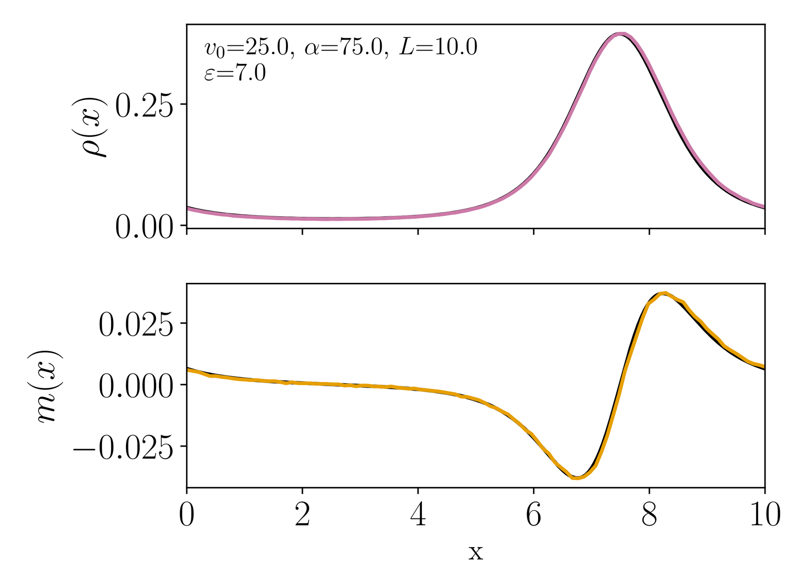
The existence of a ratchet current then boils down to the aperiodicity of the effective potential Φ.

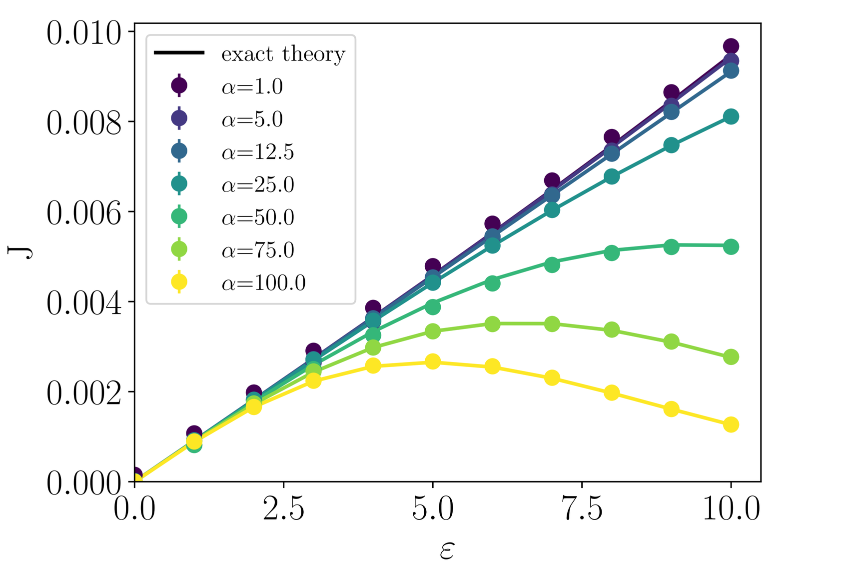
Nonzero ratchet current $\iff$ $\Phi(L) \neq \Phi(0)$

We find, from the expression for J, the following effective mobility, potential, and temperature fields:

$$\mu_{\rm e}(x) = \frac{\mu}{1 + \frac{\mu}{\alpha}U''(x) - \frac{\mu v'}{\alpha v}U'(x)}$$
$$U_{\rm e}(x) = U(x) - \frac{v^2}{2\mu\alpha} + \frac{\mu}{\alpha}\int_0^x \frac{U'(x')^2 v'(x')}{v(x')}dx'$$
$$T_{\rm e}(x) = \frac{v^2}{\mu\alpha} - \frac{\mu U'(x)^2}{\alpha}.$$

Direct simulations of the microscopic problems shows a perfect agreement with the predictions (3) for J and ρ.





References

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Acknowledgements

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