SYSC 4602 Assignment 3

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1. Given a channel speed of $56\,\frac{\text{kbit}}{\text{s}}$ and a frame size of 1000 bit, the frame transmission time is then:

$$F = \frac{1\,\text{kbit}}{56\,\frac{\text{kbit}}{\text{s}}}$$
$$F = 1/56\,\text{s} = 0.017\,857\,143\,\text{s}$$

Assuming no round-trip delay, the total frame transmission time, X, is simply equal to F. If N stations are outputting 1 frame every 100 s, then the load G, per X seconds is:

$$G = \frac{N \text{ per } 100 \text{ s}}{56 \times 100} = \frac{N}{5600} \text{ per } 1/56\,\text{s}$$

Given that the throughput of pure ALOHA is $S = Ge^{-2G}$, to find the optimal number of stations for this channel, we take the derivative of S, set it to zero, and solve for N:

$$\frac{dS}{dG} = 0 = e^{-2G} - 2Ge^{-2G}$$
$$0 = e^{-2G}(1 - 2G)$$
$$0 = 1 - 2G$$
$$G = \frac{1}{2}$$
$$\frac{N}{5600} = \frac{1}{2}$$
$$N = 2800$$

Therefore, the optimal maximum number of stations on a $56\,\frac{\text{kbit}}{\text{s}}$ pure ALOHA channel is 2800 stations.

2. Insert plot here

3. Let i represent the number of attempts made by the stations to contend for the channel. On the first round ($i = 1$), there is only one backoff period (0), and there is a $2^{-(1-1)} = 1.0$ chance of collision for that round. For the second round ($i = 2$), the backoff slots are uniformly distributed between 0 and $2^{2-1} - 1 = 1$, and there is a $2^{-(2-1)} = 0.50$ chance of collision for that round. For the $(k-1)$th round ($i = k-1$), the backoff slots are uniformly distributed between 0 and $(2^{(k-1)-1} - 1)$, and there is a $2^{-[(k-1)-1]}$ chance of collision for that round. So, the probability of $k-1$ collisions up to the kth contention round is given by the product of the chance of collision for each round up to the $(k-1)$th round:

$$\prod_{i=1}^{k-1} 2^{-(i-1)}$$

The probability of no collisions on the kth round ($i = k$) is given by 100% minus the probability of collision for that round:

$$(1 - 2^{-(k-1)})$$

So the expression for the probability that contention ends on round k is given by:

$$P_k = (1 - 2^{-(k-1)}) \times \prod_{i=1}^{k-1} 2^{-(i-1)}$$

Simplified, knowing that $\prod_{i=1}^{N} 2^{-i} = 2^{\frac{-N \times (1+N)}{2}}$:

$$P_k = (1 - 2^{-(k-1)}) \times 2^{-\frac{(k-1)(k-2)}{2}}$$

Finally, the mean number of rounds per contention period is given by the expected value for this formula:

$$E[X] = \sum_k k \times P_k$$

4. (a) When A is transmitting to B, no other communication can take place, because all other stations will have their transmissions interfered with by A's packet.

   (b) When B is transmitting to A, no other communications are possible. Although station D will not see B's packet, the other stations that could transmit to/from D (A, C, and E) will be interfered with by B's packet.

(c) When B is transmitting to C, E can transmit to D without interference. B would not see the packet from E, as its receiver is off when transmitting to A. E will shut off its receiver to transmit to D, and D cannot see B's packet, a transmission from E to D will be safe.

5. Uuhh he didn't teach us what the capacity of a protocol is... Read the textbook later I guess

6.