CS 325 Project 3: Linear Programming
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Project Report

Problem 1: Transshipment Model

Part A: Determine the number of refrigerators to be shipped plants to warehouses and then warehouses to retailers to minimize the cost.

i. Formulate the problem as a linear program with an objective function and all constraints.

The following is a breakdown of the linear program. Each plant is labeled from p1 through p4, each warehouse from w5 through w8, and each retailer from r9 through r14. This means a connection between, for example, the second plant and the 2nd warehouse would look like p2w6, or more simply, 26. I used this to form my variables which start with x, then the following numbers indicate the link, so x712 is an edge from 7 to 12, or W3 to R5.

USING LINDO:

```
min 10x15 + 15x16
+11x25+8x26+13x35+8x36+9x37+14x46+8x47+5x58+6x59+7x510+10x511+12x610+8x611+10
x612+14x613+14x711+12x712+12x713+6x714

ST
   x15+x16 <= 150
   x25+x26 <= 450
   x35+x36+x37 <=250
   x46+x47 <= 150

   x58 >= 100
   x59 >=150
   x510+x610 >=100
```

x511+x611+x711 >=200
x612+x712 >=200
x613+x713 >= 150
x714 >= 100

x15+x25+x35-x58-x59-x510-x511 = 0
x16+x26+x36+x46-x610-x611-x612-x613=0
x37+x47-x711-x712-x713-x714=0

x15 >=0
x16 >= 0
x25 >= 0
x26 >= 0
x35 >= 0
x36 >= 0
x37 >= 0
x46 >= 0
x47 >= 0

x58 >= 0
x59 >= 0
x510 >= 0
x69 >= 0
x610 >= 0
x611 >= 0
x612 >= 0
x710 >= 0
x711 >= 0
x712 >= 0
x713 >= 0

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

Using Lindo:

<u>LP OPTIMUM FOUND AT STEP</u>   <u>13</u>

<u>OBJECTIVE FUNCTION VALUE</u>

<u>1)</u>   <u>17100.00</u>

VARIABLE	VALUE	REDUCED COST
X15	150.000000	0.000000
X16	0.000000	8.000000
X25	200.000000	0.000000
X26	250.000000	0.000000
X35	0.000000	2.000000
X36	150.000000	0.000000
X37	100.000000	0.000000
X46	0.000000	7.000000
X47	150.000000	0.000000
X58	100.000000	0.000000
X59	150.000000	0.000000
X510	100.000000	0.000000
X511	0.000000	5.000000
X610	0.000000	2.000000
X611	200.000000	0.000000
X612	200.000000	0.000000
X613	0.000000	1.000000
X711	0.000000	7.000000
X712	0.000000	3.000000
X713	150.000000	0.000000
X714	100.000000	0.000000
X69	0.000000	0.000000
X710	0.000000	0.000000

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	1.000000
3)	0.000000	0.000000
4)	0.000000	0.000000
5)	0.000000	1.000000
6)	0.000000	-16.000000
7)	0.000000	-17.000000
8)	0.000000	-18.000000
9)	0.000000	-16.000000
10)	0.000000	-18.000000
11)	0.000000	-21.000000
12)	0.000000	-15.000000
13)	0.000000	-11.000000
14)	0.000000	-8.000000
15)	0.000000	-9.000000
16)	150.000000	0.000000
17)	0.000000	0.000000

18)	200.000000	0.000000
19)	250.000000	0.000000
20)	0.000000	0.000000
21)	150.000000	0.000000
22)	100.000000	0.000000
23)	0.000000	0.000000
24)	150.000000	0.000000
25)	100.000000	0.000000
26)	150.000000	0.000000
27)	100.000000	0.000000
28)	0.000000	0.000000
29)	0.000000	0.000000
30)	200.000000	0.000000
31)	200.000000	0.000000
32)	0.000000	0.000000
33)	0.000000	0.000000
34)	0.000000	0.000000
35)	150.000000	0.000000

NO. ITERATIONS= 13

iii. What are the optimal shipping routes and minimum cost.
The optimal routes are:

P1 to W1: 150
P2 to W1: 200
W1 to R1: 100
W1 to R2: 150
W1 to R3: 100
P2 to W2: 250
P3 to W2: 150
W2 to R4: 200
W2 to R5: 200
P3 to W3: 100
P4 to W3: 150
W3 to R6: 150
W3 to R7: 100

The minimum cost to ship is $17,100.00

Part B: Due to old infrastructure Warehouse 2 is going to close eliminating all of the associated routes. What is the optimal solution for this modified model? Is it feasible to

ship all the refrigerators to either warehouse 1 or 3 and then to the retailers without using warehouse 2? Why or why not?

It is not feasible to ship all the refrigerators to either warehouse 1 or 3 and then to the retailers without using warehouse 2. Without warehouse 2, retailers 5, 6 and 7 must receive all of their shipments from warehouse 3. The demand of these three retailers is 450 (200 + 150 + 100). Yet, warehouse 3 continues to receive shipments from plants 3 and 4 which have a supply of only 400 (250 + 150). Therefore, it is impossible to meet the demand of the retail stores without the use of warehouse 2.

Part C: Instead of closing Warehouse 2 management has decided to keep a portion of it open but limit shipments to 100 refrigerators per week. Is this feasible? If so what is the optimal solution when warehouse 2 is limited to 100 refrigerators?

Yes, it is feasible to ship all the refrigerators when warehouse 2 is limited to 100 refrigerators. This is achieved by adding the following constraint to the original system of equations:
x16+x26+x36+x46 <= 100

Lindo results:
P1 to W1: 150
P2 to W1: 350 (+150 from original)
W1 to R1: 100
W1 to R2: 150
W1 to R3: 100
W1 to R4: 150 (+150 from original)
P2 to W2: 100 (-150 from original)
P3 to W2: 0 (-150 from original)
W2 to R4: 50 (-150 from original)
W2 to R5: 50 (-150 from original)
P3 to W3: 250 (+150 from original)
P4 to W3: 150
W3 to R5: 150 (+150 from original)
W3 to R6: 150
W3 to R7: 100

The minimum cost to ship is $18,300.

Part D: Formulate a generalized linear programming model for the transshipment problem. Give the objective function and constraints as mathematical formulas.

<u>Objective function:</u>

(Minimize) $\sum_{i=1}^{p}\sum_{j=1}^{w} cp_{ij} * x_{ij} + \sum_{j=1}^{w}\sum_{k=1}^{r} cw_{jk} * x_{jk}$

where:
p is the number of plants
w is the number of warehouses
r is the number of retailers
cp_{ij} is the cost of shipping from plant p_i to warehouse w_j
x_{ij} is the number of refrigerators shipped from plant p_i to warehouse w_j
cw_{jk} is the cost of shipping from warehouse w_j to retailer r_k
x_{jk} is the number of refrigerators shipped from warehouse w_j to retailer r_k
s_i is the number of refrigerators produced by plant p_i
d_i is the number of refrigerators needed by retailer r_k

Constraints:
$cp_{ij} \geq 0$, for all plant i, warehouse j
$x_{ij} \geq 0$, for all plant i, warehouse j
$cw_{jk} \geq 0$, for all warehouse j, retailer k
$x_{jk} \geq 0$, for all warehouse j, retailer k
$\sum_{j=1}^{w} x_{ij} \leq s_i$, for each plant i
$\sum_{j=1}^{w} x_{jk} \geq d_k$, for each retailer k
$\sum_{i=1}^{p} x_{ij} - \sum_{k=1}^{r} x_{jk} = 0$, for each warehouse j

Problem 2: A mixture problem

Part A: Determine the combination of ingredients that minimizes calories but meets all nutritional requirements.

i. Formulate the problem as a linear program with an objective function and all constraints.

USING LINDO:

min 21x1 + 16x2 + 40x3 + 41x4 + 585x5 + 120x6 + 164x7 + 884x8
ST
0.85x1 + 1.62x2 + 2.86x3 + .93x4 + 23.4x5 + 16x6 + 9x7 + 0x8 >= 15

0.33x1 + 0.2x2 + 0.39x3 + 0.24x4 + 48.7x5 + 5x6 + 2.6x7 + 100x8 >= 2
0.33x1 + 0.2x2 + 0.39x3 + 0.24x4 + 48.7x5 + 5x6 + 2.6x7 + 100x8 <= 8
4.64x1 + 2.37x2 + 3.63x3 + 9.58x4 + 15x5 + 3x6 + 27x7 + 0x8 >= 4
9x1 + 28x2 + 65x3 + 69x4 + 3.8x5 + 120x6 + 78x7 + 0x8 <= 200
0.4x1 - 0.6x2 - 0.6x3 + 0.4x4 + 0.4x5 + 0.4x6 + 0.4x7 + 0.4x8 <= 0
x1 >= 0
x2 >= 0
x3 >= 0
x4 >= 0
x5 >= 0
x6 >= 0
x7 >= 0
x8 >= 0

Where x1 is the weight of tomatoes (in 100g), x2 is the weight of lettuce (in 100g), x3 is the weight of spinach (in 100g), x4 is the weight of carrots (in 100g), x5 is the weight of sunflower seeds (in 100g), x6 is the weight of smoked tofu (in 100g), x7 is the weight of chickpeas (in 100g), and x8 is the weight of oil (in 100g).

Objective: to minimize the number of calories in the salad
Constrains:

- Total grams of protein must be ≥ 15
- Total grams of fat must be ≥ 2
- Total grams of fat must be ≤ 8
- Total grams of carbohydrates must be ≥ 4
- Total milligrams of sodium must be ≤ 200
- Lettuce and spinach (leafy greens) must make up ≥ 40% of the total mass of the salad

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

The optimal solution is to include 58.548 grams of lettuce and 87.822 grams of smoked tofu in the salad.

The number of calories in this salad = (0.585480 x 16) + (0.878220 x 120) = (9.36768 + 105.3864) = 114.75 calories.

OBJECTIVE FUNCTION VALUE

1) 114.7541

VARIABLE	VALUE	REDUCED COST
X1	0.000000	16.901640

X2	0.585480	0.000000
X3	0.000000	14.513662
X4	0.000000	36.289616
X5	0.000000	408.387970
X6	0.878220	0.000000
X7	0.000000	97.551910
X8	0.000000	886.404358

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-7.650273
3)	2.508197	0.000000
4)	3.491803	0.000000
5)	0.022248	0.000000
6)	78.220139	0.000000
7)	0.000000	6.010929
8)	0.000000	0.000000
9)	0.585480	0.000000
10)	0.000000	0.000000
11)	0.000000	0.000000
12)	0.000000	0.000000
13)	0.878220	0.000000
14)	0.000000	0.000000
15)	0.000000	0.000000

iii. What is the cost of the low calorie salad?
The cost of the low calorie salad = (0.585480 x $0.75) + (0.878220 x $2.15) = ($0.44 + $1.89) = $2.33

Part B: Veronica realizes that it is also important to minimize the cost associated with the new salad. Unfortunately some of the ingredients can be expensive. Determine the combination of ingredients that minimizes cost.

i. Formulate the problem as a linear program with an objective function and all constraints.

USING LINDO:

min 1x1 + 0.75x2 + 0.5x3 + 0.5x4 + 0.45x5 + 2.15x6 + 0.95x7 + 2x8
ST

0.85x1 + 1.62x2 + 2.86x3 + .93x4 + 23.4x5 + 16x6 + 9x7 + 0x8 >= 15
0.33x1 + 0.2x2 + 0.39x3 + 0.24x4 + 48.7x5 + 5x6 + 2.6x7 + 100x8 >= 2
0.33x1 + 0.2x2 + 0.39x3 + 0.24x4 + 48.7x5 + 5x6 + 2.6x7 + 100x8 <= 8

4.64x1 + 2.37x2 + 3.63x3 + 9.58x4 + 15x5 + 3x6 + 27x7 + 0x8 >= 4
9x1 + 28x2 + 65x3 + 69x4 + 3.8x5 + 120x6 + 78x7 + 0x8 <= 200
0.4x1 - 0.6x2 - 0.6x3 + 0.4x4 + 0.4x5 + 0.4x6 + 0.4x7 + 0.4x8 <= 0
x1 >= 0
x2 >= 0
x3 >= 0
x4 >= 0
x5 >= 0
x6 >= 0
x7 >= 0
x8 >= 0

Where x1 is the weight of tomatoes (in 100g), x2 is the weight of lettuce (in 100g), x3 is the weight of spinach (in 100g), x4 is the weight of carrots (in 100g), x5 is the weight of sunflower seeds (in 100g), x6 is the weight of smoked tofu (in 100g), x7 is the weight of chickpeas (in 100g), and x8 is the weight of oil (in 100g).

Objective: to minimize the cost of the salad
Constrains:

- Total grams of protein must be ≥ 15
- Total grams of fat must be ≥ 2
- Total grams of fat must be ≤ 8
- Total grams of carbohydrates must be ≥ 4
- Total milligrams of sodium must be ≤ 200
- Lettuce and spinach (leafy greens) must make up ≥ 40% of the total mass of the salad

ii. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.

The optimal solution is to include 83.2298 grams of spinach, 9.6083 grams of sunflower seeds and 115.2364 grams of chickpeas in the salad.

The cost of this salad = (0.832298 x \$0.50) + (0.096083 x \$0.45) + (1.152364 x \$0.95) = (\$0.42 + \$0.04 + \$1.09) = \$1.55

OBJECTIVE FUNCTION VALUE

1) 1.554133

VARIABLE	VALUE	REDUCED COST
X1	0.000000	1.002081
X2	0.000000	0.402912
X3	0.832298	0.000000

X4	0.000000	0.486914
X5	0.096083	0.000000
X6	0.000000	0.405609
X7	1.152364	0.000000
X8	0.000000	7.281258

ROW	SLACK OR SURPLUS	DUAL PRICES
2)	0.000000	-0.131261
3)	6.000000	0.000000
4)	0.000000	0.051847
5)	31.576324	0.000000
6)	55.651089	0.000000
7)	0.000000	0.241358
8)	0.000000	0.000000
9)	0.000000	0.000000
10)	0.832298	0.000000
11)	0.000000	0.000000
12)	0.096083	0.000000
13)	0.000000	0.000000
14)	1.152364	0.000000
15)	0.000000	0.000000

iii. How many calories are in the low cost salad?

The number of calories in the low cost salad = (0.832298 x 40) + (0.096083 x 585) + (1.152364 x 164) = (33.29192 + 56.208555 + 188.987696) = 278.49 calories.

Part C: Compare the results from part A and B. Veronica's goal is to create a Very Veggie Salad that is both low calorie and low cost. She would like to sell the salad for $5.00 and still have a profit of at least $3.00. However if she can advertise that the salad has under 250 calories then she may be able to sell more.

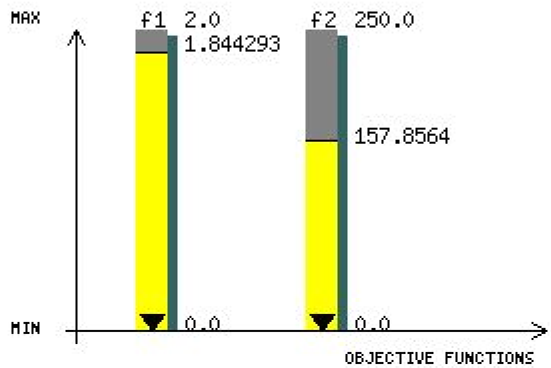
i. Suggest some possible ways that she select a combination of ingredients that is "near optimal" for both objectives. This is a type of multi-objective optimization.

Veronica should find the pareto optimal solution that takes into account both objectives: minimizing cost and minimizing calories. Her goal of keeping the cost under $2 and the calories under 250 are added as constraints to ensure an ideal solution that satisfies both goals. This optimal solution lies somewhere in between the optimal combinations found in parts a and c.

ii. What combination of ingredient would you suggest and what is the associated cost and calorie.

An optimal salad contains 53.46083 grams of spinach, 8.654325 grams of sunflower seeds, and 71.53693 grams of smoked tofu.

The cost of this salad is $1.84 and it contains 157.86 calories.



In the graph above, f1 represents cost while f2 represents calories.

Alternatively, a salad containing the following balance of ingredients would also meet Veronica's cost and calorie goals, with a total cost of $1.62:

Spinach	76.1996g
Sunflower Seeds	09.3830g
Smoked Tofu	16.8941g
Chickpeas	88.0222g

This combination is based on the following linear program, entered into Lindo:

```
min 1x1 + 0.75x2 + 0.5x3 + 0.5x4 + 0.45x5 + 2.15x6 + 0.95x7 + 2x8
ST
    0.85x1 + 1.62x2 + 2.86x3 + .93x4 + 23.4x5 + 16x6 + 9x7 + 0x8 >= 15
    0.33x1 + 0.2x2 + 0.39x3 + 0.24x4 + 48.7x5 + 5x6 + 2.6x7 + 100x8 >= 2
    0.33x1 + 0.2x2 + 0.39x3 + 0.24x4 + 48.7x5 + 5x6 + 2.6x7 + 100x8 <= 8
    4.64x1 + 2.37x2 + 3.63x3 + 9.58x4 + 15x5 + 3x6 + 27x7 + 0x8 >= 4
    9x1 + 28x2 + 65x3 + 69x4 + 3.8x5 + 120x6 + 78x7 + 0x8 <= 200
    0.4x1 - 0.6x2 - 0.6x3 + 0.4x4 + 0.4x5 + 0.4x6 + 0.4x7 + 0.4x8 <= 0
    21x1 + 16x2 + 40x3 + 41x4 + 585x5+ 120x6 + 164x7 + 884x8 <= 250
   1x1 + 0.75x2 + 0.5x3 + 0.5x4 + 0.45x5 + 2.15x6 + 0.95x7 + 2x8 <= 2
    x1 >= 0
    x2 >= 0
    x3 >= 0
    x4 >= 0
    x5 >= 0
```

x6 >= 0
x7 >= 0
x8 >= 0

iii. Note: There is not one "right" answer. Discuss how you derived your solution.

Approach 1

The first solution was achieved by using an interactive multiobjective optimization system called www-nimbus. By inputting the following constraints, the system produced this pareto optimal result:

min f1 = 1x1 + 0.75x2 + 0.5x3 + 0.5x4 + 0.45x5 + 2.15x6 + 0.95x7 + 2x8
min f2 = 21x1 + 16x2 + 40x3 + 41x4 + 585x5 + 120x6 + 164x7 + 884x8
ST
0.85x1 + 1.62x2 + 2.86x3 + .93x4 + 23.4x5 + 16x6 + 9x7 + 0x8 >= 15
0.33x1 + 0.2x2 + 0.39x3 + 0.24x4 + 48.7x5 + 5x6 + 2.6x7 + 100x8 >= 2
0.33x1 + 0.2x2 + 0.39x3 + 0.24x4 + 48.7x5 + 5x6 + 2.6x7 + 100x8 <= 8
4.64x1 + 2.37x2 + 3.63x3 + 9.58x4 + 15x5 + 3x6 + 27x7 + 0x8 >= 4
9x1 + 28x2 + 65x3 + 69x4 + 3.8x5 + 120x6 + 78x7 + 0x8 <= 200
0.4x1 - 0.6x2 - 0.6x3 + 0.4x4 + 0.4x5 + 0.4x6 + 0.4x7 + 0.4x8 <= 0
x1 >= 0
x2 >= 0
x3 >= 0
x4 >= 0
x5 >= 0
x6 >= 0
x7 >= 0
x8 >= 0
f1 <= 2
f2 <= 250

Approach 2

Our second solution was achieved using a slightly different approach, with the Lindo software program. Constraints were added to the list of constraints used for the part B optimal cost model, to account for the total-cost-under-$2 and total-calories-under-250 objectives.

Using Lindo:
min 1x1 + 0.75x2 + 0.5x3 + 0.5x4 + 0.45x5 + 2.15x6 + 0.95x7 + 2x8
ST
0.85x1 + 1.62x2 + 2.86x3 + .93x4 + 23.4x5 + 16x6 + 9x7 + 0x8 >= 15
0.33x1 + 0.2x2 + 0.39x3 + 0.24x4 + 48.7x5 + 5x6 + 2.6x7 + 100x8 >= 2

0.33x1 + 0.2x2 + 0.39x3 + 0.24x4 + 48.7x5 + 5x6 + 2.6x7 + 100x8 <= 8
4.64x1 + 2.37x2 + 3.63x3 + 9.58x4 + 15x5 + 3x6 + 27x7 + 0x8 >= 4
9x1 + 28x2 + 65x3 + 69x4 + 3.8x5 + 120x6 + 78x7 + 0x8 <= 200
0.4x1 - 0.6x2 - 0.6x3 + 0.4x4 + 0.4x5 + 0.4x6 + 0.4x7 + 0.4x8 <= 0
21x1 + 16x2 + 40x3 + 41x4 + 585x5+ 120x6 + 164x7 + 884x8 <= 250
1x1 + 0.75x2 + 0.5x3 + 0.5x4 + 0.45x5 + 2.15x6 + 0.95x7 + 2x8 <= 2
x1 >= 0
x2 >= 0
x3 >= 0
x4 >= 0
x5 >= 0
x6 >= 0
x7 >= 0
x8 >= 0

Approach 3
Yet another approach, not adopted here, could follow the same pattern as our second approach, but use an objective equation designed to minimize calories instead of cost.

Problem 3: Solving shortest path problems using linear programming.

The file Project3Problem3.txt contains a list of the edges and weights in a directed graph. Use linear programming to answer the following questions. Include a copy of the linear program code.

a) What are the lengths of the shortest paths from vertex a to all other vertices.

USING LINDO:

max a + b + c + d + e + f + g + h + i + j + k + l + m
ST
a = 0
b - a <= 2
c - a <= 3
d - a <= 8
h - a <= 9
a - b <= 4
c - b <= 5
e - b <= 7

f - b <= 4
d - c <= 10
b - c <= 5
g - c <= 9
i - c <= 11
f - c <= 4
a - d <= 8
g - d <= 2
j - d <= 5
f - d <= 1
h - e <= 5
c - e <= 4
i - e <= 10
i - f <= 2
g - f <= 2
d - g <= 2
j - g <= 8
k - g <= 12
i - h <= 5
k - h <= 10
a - i <= 20
k - i <= 6
j - i <= 2
m - i <= 12
i - j <= 2
k - j <= 4
l - j <= 5
h - k <= 10
m - k <= 10
m - l <= 2

VARIABLE	VALUE
A	0
B	2
C	3
D	8
E	9
F	6
G	8
H	9
I	8
J	10
K	14

L	15
M	17

b) If a vertex z is added to the graph for which there is no path from vertex a to vertex z, what will be the result when you attempt to find the lengths of shortest paths as in part a).

Adding a vertex z to the graph for which there is no path from vertex a to vertex z should not affect the shortest paths solution, because if there is no path from vertex a to vertex z then vertex z will necessarily not be be part of any solution for a shortest path from vertex to any other vertex.

If a vertex v is added to the objective function in the Lindo system of equations used above, then solving for the minimum distances from a will not work as expected, because the new vertex max value will not be constrained as the other vertices are, by a=0.

c) What are the lengths of the shortest paths from each vertex to vertex m. How can you solve this problem with just one linear program?

The lengths of the shortest paths from each vertex to vertex m can be found by reversing the direction of each link in the graph, and then finding the length of the shortest paths from vertex m to each vertex, as done for vertex a in part a.

USING LINDO:

```
max a + b + c + d + e + f + g + h + i + j + k + l + m
ST
    m = 0
    b - a >= -2
    c - a >= -3
    d - a >= -8
    h - a >= -9
    a - b >= -4
    c - b >= -5
    e - b >= -7
    f - b >= -4
    d - c >= -10
    b - c >= -5
    g - c >= -9
    i - c >= -11
    f - c >= -4
    a - d >= -8
    g - d >= -2
```

j - d >= -5
f - d >= -1
h - e >= -5
c - e >= -4
i - e >= -10
i - f >= -2
g - f >= -2
d - g >= -2
j - g >= -8
k - g >= -12
i - h >= -5
k - h >= -10
a - i >= -20
k - i >= -6
j - i >= -2
m - i >= -12
i - j >= -2
k - j >= -4
l - j >= -5
h - k >= -10
m - k >= -10
m - l >= -2

VARIABLE	**VALUE**
A	17
B	15
C	15
D	12
E	19
F	11
G	14
H	14
I	9
J	7
K	10
L	2
M	0

d) Suppose that all paths must pass through vertex i. How can you calculate the length of the shortest path from any vertex x to vertex y that pass through vertex i (for all x,y V)? Calculate the lengths of these paths for the given graph. (Note for some vertices x and y it may be impossible to pass through vertex i).

The length of the shortest path from any vertex x to vertex y that passes through vertex i is equal to the length of the shortest path from vertex x to vertex i, plus the length of the shortest path from vertex i to vertex y.

Min path to i from any other vertex:
USING LINDO:

```
max a + b + c + d + e + f + g + h + i + j + k
ST
    i = 0
    b - a >= -2
    c - a >= -3
    d - a >= -8
    h - a >= -9
    a - b >= -4
    c - b >= -5
    e - b >= -7
    f - b >= -4
    d - c >= -10
    b - c >= -5
    g - c >= -9
    i - c >= -11
    f - c >= -4
    a - d >= -8
    g - d >= -2
    j - d >= -5
    f - d >= -1
    h - e >= -5
    c - e >= -4
    i - e >= -10
    i - f >= -2
    g - f >= -2
    d - g >= -2
    j - g >= -8
    k - g >= -12
    i - h >= -5
    k - h >= -10
    a - i >= -20
    k - i >= -6
    j - i >= -2
    m - i >= -12
    i - j >= -2
```

k - j >= -4
l - j >= -5
h - k >= -10
m - k >= -10
m - l >= -2

There is no path from m or l to i, because m does not have any outgoing edges, and l's only outgoing edge connects to m. When these vertices are removed from consideration, the minimum paths become as follows.

VARIABLE	VALUE
A	8
B	6
C	6
D	3
E	10
F	2
G	5
H	5
I	0
J	2
K	15

Min path from i to any other vertex:
USING LINDO:

max a + b + c + d + e + f + g + h + i + j + k + l + m
ST
a = 0
b - a <= 2
c - a <= 3
d - a <= 8
h - a <= 9
a - b <= 4
c - b <= 5
e - b <= 7
f - b <= 4
d - c <= 10
b - c <= 5
g - c <= 9
i - c <= 11
f - c <= 4
a - d <= 8

g - d <= 2
j - d <= 5
f - d <= 1
h - e <= 5
c - e <= 4
i - e <= 10
i - f <= 2
g - f <= 2
d - g <= 2
j - g <= 8
k - g <= 12
i - h <= 5
k - h <= 10
a - i <= 20
k - i <= 6
j - i <= 2
m - i <= 12
i - j <= 2
k - j <= 4
l - j <= 5
h - k <= 10
m - k <= 10
m - l <= 2

VARIABLE	**VALUE**
A	20
B	22
C	23
D	28
E	29
F	26
G	28
H	16
I	0
J	2
K	6
L	7
M	9

Shortest total paths passing through i

The length of the min path between any two vertices, passing through i, is then the sum of the first vertex’s min distance to i, plus the second vertex’s min distance from i.

Listing source vertices on left, destination vertices on top, min path distances are given in the following table.

	A	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**	**K**	**L**	**M**
A	8+20=28	8+22=30	8+23=31	8+28=36	8+29=37	8+26=34	8+28=36	8+16=24	8+0=8	8+2=10	8+6=14	8+7=15	8+9=17
B	6+20=26	6+22=28	6+23=29	6+28=34	6+29=35	6+26=32	6+28=34	6+16=22	6+0=6	6+2=8	6+6=12	6+7=13	6+9=15
C	6+20=26	6+22=28	6+23=29	6+28=34	6+29=35	6+26=32	6+28=34	6+16=22	6+0=6	6+2=8	6+6=12	6+7=13	6+9=15
D	3+20=23	3+22=25	3+23=26	3+28=31	3+29=32	3+26=29	3+28=31	3+16=19	3+0=3	3+2=5	3+6=9	3+7=10	3+9=12
E	10+20=30	10+22=32	10+23=33	10+28=38	10+29=39	10+26=36	10+28=38	10+16=26	10+0=10	10+2=12	10+6=16	10+7=17	10+9=19
F	2+20=22	2+22=24	2+23=25	2+28=30	2+29=31	2+26=28	2+28=30	2+16=18	2+0=2	2+2=4	2+6=8	2+7=9	2+9=11
G	5+20=25	5+22=27	5+23=28	5+28=33	5+29=34	5+26=31	5+28=33	5+16=21	5+0=5	5+2=7	5+6=11	5+7=12	5+9=14
H	5+20=25	5+22=27	5+23=28	5+28=33	5+29=34	5+26=31	5+28=33	5+16=21	5+0=5	5+2=7	5+6=11	5+7=12	5+9=14
I	0+20=20	0+22=22	0+23=23	0+28=28	0+29=29	0+26=26	0+28=28	0+16=16	0+0=0	0+2=2	0+6=6	0+7=7	0+9=9
J	2+20=22	2+22=24	2+23=25	2+28=30	2+29=31	2+26=28	2+28=30	2+16=18	2+0=2	2+2=4	2+6=8	2+7=9	2+9=11
K	15+20=35	15+22=37	15+23=38	15+28=43	15+29=44	15+26=41	15+28=43	15+16=31	15+0=15	15+2=17	15+6=21	15+7=22	15+9=24
L	No path	No path	No path	No path	No path	No path	No path	No path	No path	No path	No path	No path	No path
M	No path	No path	No path	No path	No path	No path	No path	No path	No path	No path	No path	No path	No path