STA5075: Practical 15

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Maximum likelihood estimation

Assume these 13 observations are from a Poisson distribution, with rate parameter λ:

```
counts <- c(3, 1, 1, 3, 1, 4, 3, 2, 0, 5, 0, 4, 2)
```

The log-likelihood is:

```
pois.ll <- function(lambda, y) {
  sum(dpois(y, lambda, log = TRUE))
}
```

Question 1a

Use optim() and nlm() (non-linear minimization) to find the MLE for λ

The negative log-likelihood is:

```
# negative log likelihood
pois.negll <- function(lambda, y) {
  -sum(dpois(y, lambda, log = TRUE))
}
```

```
# using optim()

optim <- optim(par = 5, # Initial values for the parameters to be optimized over.
               fn = pois.negll, # A function to be minimized (or maximized), here we minimise
               y = counts,
               method = "L-BFGS-B", # use the quasi-Newton method
               lower = 0.0001,
               upper = Inf)
optim
```

```
## $par
## [1] 2.230769
##
## $value
## [1] 23.63712
##
## $counts
## function gradient
##        9        9
##
## $convergence
## [1] 0
##
```

```
## $message
## [1] "CONVERGENCE: REL_REDUCTION_OF_F <= FACTR*EPSMCH"
```

```r
# using nlm()
nlm <- nlm(f = pois.negll, # function to be minimized
           y = counts,
           p = 5) # Initial values for the parameters to be optimized over.
```

```
## Warning in dpois(y, lambda, log = TRUE): NaNs produced

## Warning in nlm(f = pois.negll, y = counts, p = 5): NA/Inf replaced by maximum
## positive value

## Warning in dpois(y, lambda, log = TRUE): NaNs produced

## Warning in nlm(f = pois.negll, y = counts, p = 5): NA/Inf replaced by maximum
## positive value
```

```r
nlm
```

```
## $minimum
## [1] 23.63712
##
## $estimate
## [1] 2.230768
##
## $gradient
## [1] 0
##
## $code
## [1] 1
##
## $iterations
## [1] 9
```

The Poisson negative log-likelihood minimisation was successful using both the optim() and nlm() functions:

- The optimal parameter estimate is 2.230769.
- The algorithm converged normally (convergence = 0).
- It found the minimum negative log-likelihood at 23.63712 after 9 iterations.

Question 1b

Use Newton's method (from scratch) to find the MLE for λ in the above problem.

```
# see page 119 for notes on Newton's method
pois.negll <- function(lambda, y) {
  -sum(dpois(y, lambda, log = TRUE))
}

# parameters: lambda, counts, tol (stopping value for when you find best lambda - use a while loop for
newton <- function(lambda, counts, tol){
  ratio <- Inf # initially set ratio to a large nr
  # keep iterating until our ratio < tol
  while(abs(ratio) > tol){
    ratio <- ((length(counts)*(lambda^2))/sum(counts)) - lambda # ratio of 1st to 2nd derivatives
    lambda <- lambda - ratio # update lambda value
  }
  print(lambda) # quit the loop when the optimum value is found
}

counts <- c(3, 1, 1, 3, 1, 4, 3, 2, 0, 5, 0, 4, 2)
newton(lambda = 3, counts = counts, tol = 0.001)
```

```
## [1] 2.230769
```

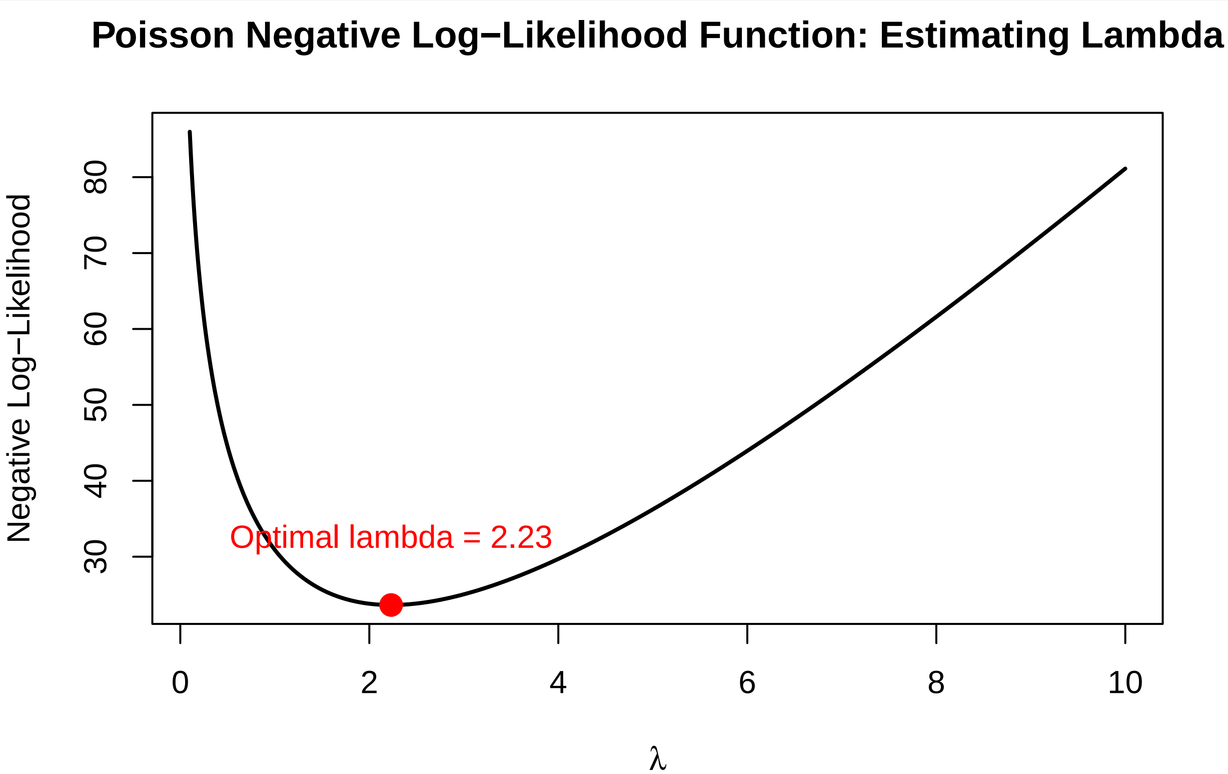
Question 1c

Plot the function and check your answer.

```r
# Define a range of lambda values
lambda <- seq(0.1, 10, length.out = 1000)

# Compute the negative log-likelihood for each value of lambda
negll <- sapply(lambda, pois.negll, y = counts)

# Plot the function
plot(lambda, negll,
     type = "l", lwd = 2,
     xlab = expression(lambda),
     ylab = "Negative Log-Likelihood",
     main = "Poisson Negative Log-Likelihood Function: Estimating Lambda")

# Add a point for the estimated lambda using Newton's method
lambda_hat <- newton(lambda = 3, counts = counts, tol = 0.001)
```

```
## [1] 2.230769
```

```r
points(lambda_hat, pois.negll(lambda_hat, counts), col = "red", pch = 19, cex = 1.5)

# Annotate the optimal lambda
text(lambda_hat, pois.negll(lambda_hat, counts) + 5,
     labels = paste0("Optimal lambda = ", round(lambda_hat, 2)), col = "red", pos = 3)
```



When we plot the function we can visualise where the MLE is minimised, at a lambda value of ~ 2.23, the same lambda value as estimated earlier using both the nlm() and optim() functions.