A Reconsideration of Money Growth Rules

Jessica van der Berg[a]

[a]*Stellenbosch Univeristy, South Africa*

Abstract

The reports builds on a research paper submitted by Johannes Coetsee, Jessica Van der Berg and Cassandra Pengelly in May 2021. This reports sets out to simulate, estimate, analyze and compare the Taylor rule, Constant Money Growth rule and Flexible Money Growth rule. The paper concludes that the flexible money growth rules preforms similarly to the Taylor rule during 1982 - 2019.

Keywords: MCMC, DSGE, Money Growth rules

JEL classification L250, L100

1. Introduction

Central banks implement monetary policy usually by manipulating the nominal interest rate, otherwise known as the Taylor rule. The Taylor rule offers many advantages including easy communication that boost investors and the publics confidence in the central bank. However, it has been argued that the Taylor rule should be used as a mere benchmark, and that not doing so can lead to poor economic outcomes. Belongia, Ireland & others (2019) argue that constant and flexible money growth rules can be as effective in stabilizing inflation and output in a low interest rate environment, and in certain cases, can outperformed the historically preferred Taylor rule.

Belongia, Ireland & others (2019) estimate a New Keynesian model using Bayesian methods. Bayesian methods create the opportunity for researchers to evaluate macroeconomic models that frequentist econometrics find challenging. A popular macroeconomic model that economist estimate using Bayesian methods is dynamic stochastic general equilibrium (DSGE) model. In the past, classical optimization methods were used to estimate DSGE models, however, Bayesian methods are preferred due to the wide variety of tools that research can make use of to estimate and evaluate DSGE models. Bayesian methods also have the advantage of increasing computing power meaning that more complicated DSGE models can be estimated Guerrón-Quintana & Nason (2013).

*Corresponding author: Jessica van der Berg
Email address: `20190565@sun.ac.za` (Jessica van der Berg)

By estimating a New Keynesian model, Belongia, Ireland & others (2019) reconsiders money growth rules. This is accomplished by identifying a parsimonious rule that dictates a systematic response of money growth to changes in the output gap. Further, simulations are performed to assess how the United States (US) economic have preformed over the sample period from 1983 through 2019 and the feasibility of the money growth rule. The simulations shows that the money growth rule preforms satisfactory during the entire sample period, including during the Great Recession and its aftermath. Belongia, Ireland & others (2019) found that money growth rules offer more flexibility that leads to a decrease in volatility in output growth and inflation. Implementing a flexible money growth rate rule would have allowed the US economy to recover faster from the Great Recession and the 2008/2009 financial crisis.

This report sets out to replicate the estimation results of Belongia, Ireland & others (2019). However, this paper deviates from the Belongia, Ireland & others (2019) paper by focusing on the full sample of quarterly data from 1983 to 2019, instead of examining low interest rate environments. Therefore, instead of estimating and comparing policy rules for low-interest rates environments specifically, this report compares different simulated models over the entire sample period, thereby assessing the relative strengths of policy rules regardless of the economic environment. This report sets out to simulate, analyse and compare three New Keynesian models to evaluate the Taylor rule, Constant Money Growth rule and a Flexible Money Growth rule. The comparisons will be visualized and summarized with, amongst others, Impulse Response Functions (IRFs), multivariate convergence diagnostics, and Monte Carlo Markov Chain (MCMC) diagnostics. My findings largely support the results of Belongia, Ireland & others (2019), with the main conclusion being that the flexible money growth rules preform similarly to the Taylor rule during the sample. This finding supports the argument of a possible reconsideration of incorporating flexible money growth rules in the Fed's monetary policy toolkit.

The rest of the paper is structured as followed: Section 2 briefly outlines the model[1]. Section 3 discusses the data as well as the calibration and the priors. Section 4 discusses the estimation results of the model, including the posterior distribution estimations and the impulse response functions. Section 5 provides an in-dept discussion regarding several diagnostic test that were carried out in order to asses the DSGE mode. Finally, section 6 concludes.

[1] The replication, analysis and discussion of the model can be found in the first submission that was submitted by Johannes Coetsee, Jessica Van der Berg and Cassandra Pengelly in May 2021.

2. The Model

The model economy includes four economic agents: a representative household, a representative finished goods-producing firm, an intermediate goods-producing firm, and a central bank. The equations are then simplified and linearized. An in-dept discussion about the model can be found in the first submission, however, this section describes the final equations that were log-linearized around the steady-state to describe how the economy responds to shocks.

1. Linearized Household Budget Constraint

$$\hat{y_t} = \hat{c_t}$$

2. Linearized Consumption Euler Equation:

$$(z-\beta\gamma)(z-\gamma)\hat{\lambda}_t = \gamma z\hat{y_{t-1}} - (z^2+\beta\gamma^2)\hat{y_t} + \beta\gamma E_t\hat{y_{t+1}} + (z-\beta\gamma\rho_a)(z-\gamma)\hat{a}_t - \gamma z\hat{z}_t$$

3. Linearized Bonds Euler Equation:

$$\hat{\lambda}_t = \hat{r}_t + E_t\hat{\lambda}_{t+1} - E_t\hat{\pi}_{t+1}$$

4. Linearized Efficient Level of Output:

$$0 = \gamma z\hat{q_{t-1}} - (z^2+\beta\gamma^2)\hat{q_t} + \beta\gamma z E_t\hat{q_{t+1}} + \beta\gamma(z-\gamma)(a-\rho_a)\hat{a}_t - \gamma z\hat{z}_t$$

5. Linearized Output Gap:

$$\hat{x}_t = \hat{y_t} - \hat{q_t}$$

6. Linearized Philips Curve:

$$(1+\beta\alpha)\hat{\pi}_t = \alpha\hat{\pi}_{t-1} + \beta E_t\hat{\pi}_{t+1} - \psi\hat{\lambda}_t + \psi\hat{a}_t + \hat{e}_t$$

7. Linearized Taylor Rule:

$$\hat{r}_t = \rho_r\hat{r}_{t-1} + \rho_\pi\hat{\pi}_t + \rho_x\hat{x}_t + \varepsilon_{rt}$$

8. Linearized Money Euler Equation:

$$\begin{aligned}
&\frac{1}{\delta}[-\hat{m}_t + \hat{u}_t] - \phi_m\hat{z}_t + \beta\phi_m E_t\hat{m}_{t+1} - (1+\beta)\phi_m\hat{m}_t + \phi_m\hat{m_{t-1}} \\
&= \frac{\delta_r}{\delta}(r-1)\hat{\lambda}_t + \frac{\delta_r\hat{r}_t}{\delta} - \frac{\delta_r}{\delta}(r-1)\hat{a}_t - \hat{m}_t + \hat{u}_t \\
&- \phi_m\delta\hat{z}_t + \beta\phi_m\delta E_t\hat{m}_{t+1} - (1+\beta)\phi_m\delta\hat{m}_t + \phi_m\delta\hat{m}_{t-1}
\end{aligned}$$

9. Linearized Nominal Money Growth:

$$\hat{\mu}_t = \hat{m}_t - \hat{m}_{t-1} + \hat{z}_t + \hat{\pi}_t$$

10. Linearized Output Growth:

$$\hat{g}_t = \hat{y}_t - \hat{y}_{t-1} + \hat{z}_t$$

11. Linearized Preference Shock:

$$\ln(a_t) = \rho_a \ln(a_{t-1}) + \varepsilon_{a_t}$$
$$\hat{a}_t = \rho_a \hat{a_{t-1}} + \varepsilon_{a_t}$$

12. Linearized Productivity Shock with a Random Walk:

$$\hat{z}_t = \varepsilon_{z_t}$$

13. Linearized Money Demand Shock:

$$ln(u_t) = \rho_u \ln(u_{t-1}) + \varepsilon_{ut}$$
$$\hat{u}_t = \rho_u \hat{u_{t-1}} + \varepsilon_{ut}$$

14. Linearized Cost Push Shock:

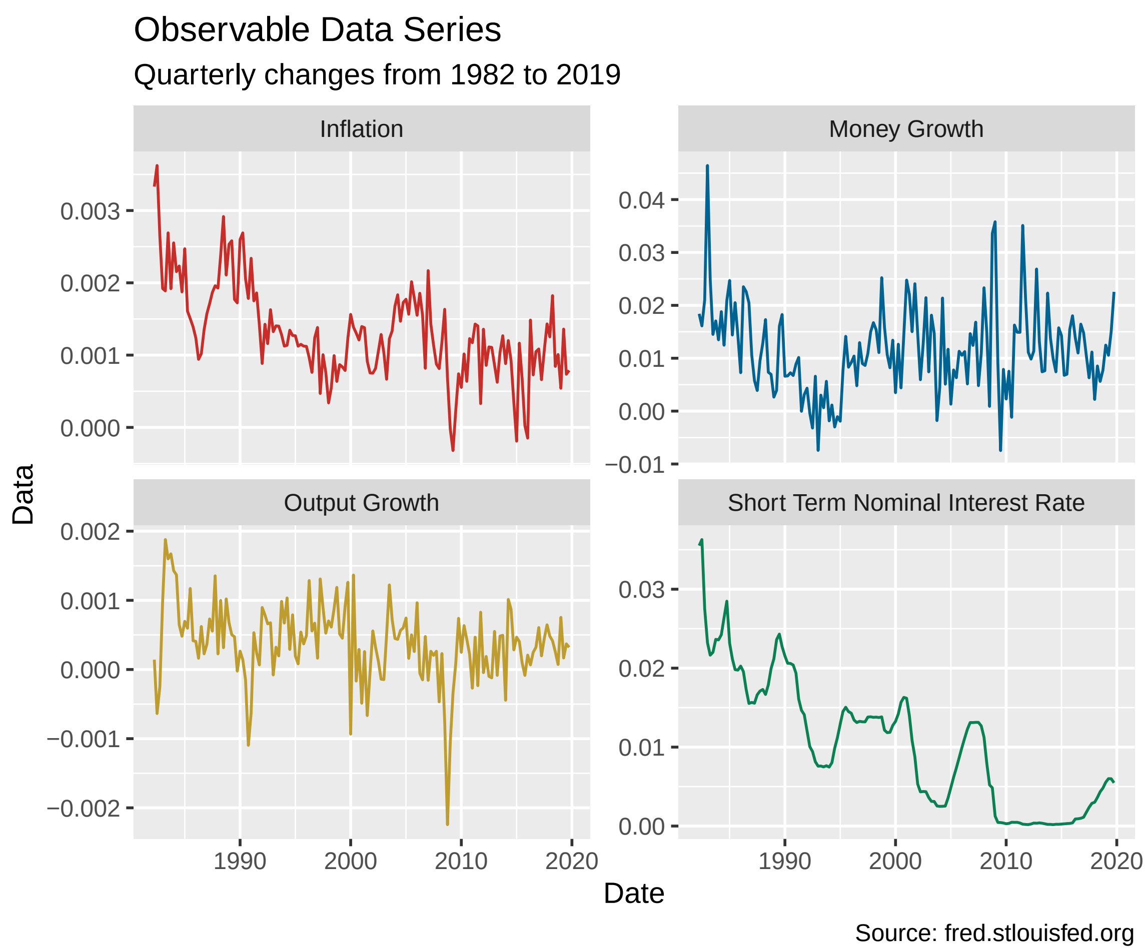
$$\widehat{e}_t = \rho_e \widehat{e}_{t-1} + \varepsilon_{et}$$

An explanation of the variable can be found in appendix A.

3. Data

This section will provide a brief discussion regarding the series of data and the subsequent transformation, as well as calibration of parameters and the estimated priors for the model.

The model presented in the previous section is estimated with Bayesian estimation techniques using four key macroeconomic variables: output growth, inflation, short term nominal interest rate and nominal money growth. The series span from 1983:Q1 to 2019:Q4, thereby amounting to 151 observations. Following Belongia, Ireland & others (2019), output is measured by the changes in the log of per capita real GDP; inflation is measured by changes in the log of the GDP deflator; nominal interest rates are measured by the annualized effective federal funds rate, which is convert to quarterly rates by dividing the series by 400 for each quarter, and last, the growth in nominal money is measured with changes in the per capita M2 index of monetary services. The data was collected on the FRED database of the Federal Reserve of St Louis. The data series used in the model are displayed below.



4. Estimation

4.1. Calibration and Priors

To calibrate the model, I first calculated the logged first difference of the quarterly data to get stationary series. The average values over the entire sample period were calculated in order to obtain the parameter values for inflation (π), interest rate (r) and output growth (z). These values represent the steady state values as we manipulated the data to capture changes from the steady state. Using these values, the subjective discount rate (β) was calculated using the following formula:

$$\beta = \left(\frac{z}{r}\right)\pi$$

Table 4.1 below summarizes the parameter values that we calculate using our data. The values are used when simulating the models for each of the monetary rules.

Table 4.1: Parameters calculater using own data

Parameter	Value
π	1.00128
r	1.00978
z	1.00039
β	0.99197

Figure 4.1 and Figure 4.2 plots the prior distributions of the structural parameters, including the standard deviations of the shocks. The five structural shocks: σ_a, σ_z, σ_u, σ_e and σ_r all have an inverse chi-square distribution with four degrees of freedom. Smets & Wouters (2007) argued in favour of rather lose prior for the five shocks, and assigned the shocks inverse gamma distributions with two degree of freedom. However, Belongia, Ireland & others (2019) adjust this to allow the New Keynesian model to capture the increase in volatility after the 2008/2009 financial crises.

Table 4.2: Exogenous Variables

Parameter	Description
σ_a	preference shock volatility
σ_z	productivity shock volatility
σ_u	money demand shock volatility
σ_e	cost push shock volatility
σ_r	monetary policy shock volatility

Figure 4.1: (a) Prior Distributions

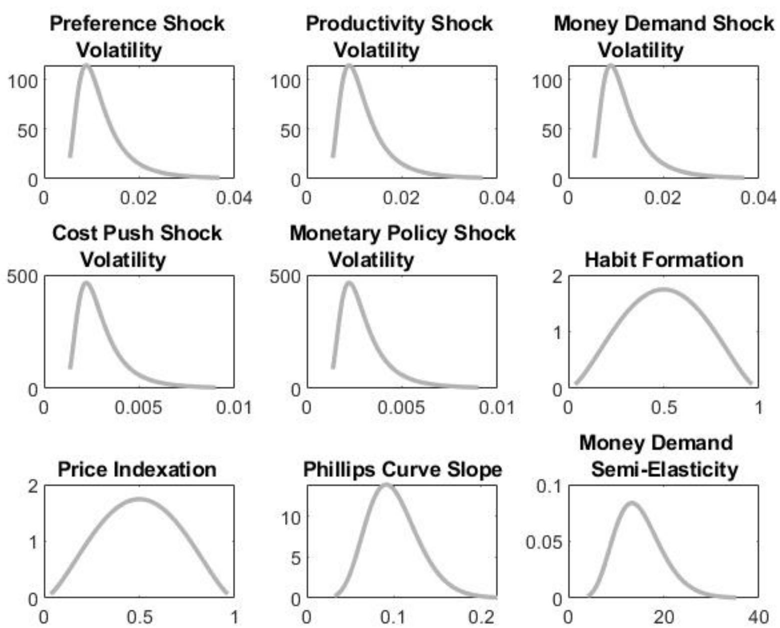


Figure 4.2: (b) Prior Distributions

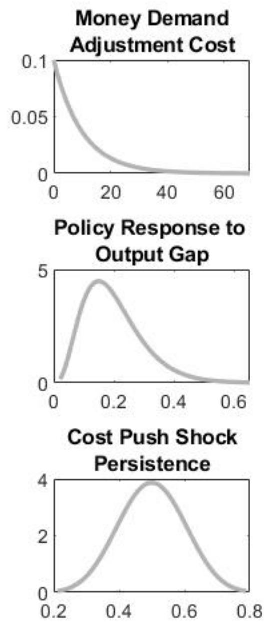


Table 4.3 displays statistical characteristics of the priors. The habit formation (γ) and price indexation (α) as well as the interest rate smoothing (ρ_r), preference shock persistence (ρ_a), money demand shock persistence (ρ_u) and cost push shock persistence (ρ_e) are constrained between zero and one by assigning a beta distribution to the parameters. To restrict the parameters to positive values, Belongia, Ireland & others (2019) assign gamma distributions for the Phillips curve slope (ψ), money demand adjustment semi-elasticity (δ_r), money demand adjustment cost (ϕ), policy response to inflation (ρ_π) and policy response to output gap (ρ_x).

Table 4.3: Prior information (parameters)

Symbol	Description	Distribution	Mean	Std.dev.	Bounds	
					Lower	Upper
σ_a	preference shock volatility	Inv. Chi-squared	0.0125	0.0066	0.0000	∞
σ_z	productivity shock volatility	Inv. Chi-squared	0.0125	0.0066	0.0000	∞
σ_u	money demand shock volatility	Inv. Chi-squared	0.0125	0.0066	0.0000	∞
σ_e	cost push shock volatility	Inv. Chi-squared	0.0031	0.0016	0.0000	∞
σ_r	monetary policy shock volatility	Inv. Chi-squared	0.0031	0.0016	0.0000	∞
γ	habit formation	Beta	0.5000	0.2000	0.0000	1.0000
α	price indexation	Beta	0.5000	0.2000	0.0000	1.0000
ψ	Phillips curve slope	Gamma	0.1000	0.0300	0.0000	∞
δ_r	money demand semi-elasticity	Gamma	15.0000	5.0000	0.0000	∞
ϕ	money demand adjustment cost	Gamma	10.0000	10.0000	0.0000	∞
ρ_r	interest rate smoothing	Beta	0.7500	0.1000	0.0000	1.0000
ρ_π	policy response to inflation	Gamma	0.4000	0.1000	0.0000	∞
ρ_x	policy response to output gap	Gamma	0.2000	0.1000	0.0000	∞
ρ_a	preference shock persistence	Beta	0.7500	0.1000	0.0000	1.0000
ρ_u	money demand shock persistence	Beta	0.7500	0.1000	0.0000	1.0000
ρ_e	cost push shock persistence	Beta	0.5000	0.1000	0.0000	1.0000

It is important to note that although the distribution for the shocks are inverse chi-square, in the actual estimation, they are simulated as having inverse gamma distribution. This is because the chi-square prior is not implemented in *dynare*. The chi-squared distribution is a special case of the gamma distribution, therefore it is possible to interchange between the two.

4.2. Posterior Estimation

Table 4.4 displays statistical information about the posterior estimations for the Taylor rule and the flexible money growth rate rule. The posterior mean and mode between the two rules are very similar with the only noticeably large difference being the money demand adjustment cost (ϕ), where under the flexible money growth rate rule it is twice as large. The standard deviation and the highest posterior denisty interval (HPDI) upper and lower bound remain relatively constant between the two rules, with, once again, the only exception being money demand adjustment cost (ϕ).

Table 4.4: Posterior Information - where HPD (L) and HPD (U) display the 90 percent Highest Posterior Density Interval Lower and Upper bound, respectively.

Taylor Rule								
		Prior		Posterior				
Symbol	Distribution	Mean	Std.dev.	Mean	Mode	Std. dev	HPD (L)	HPD (U)
σ_a	Inv. Chi-squared	0.0125	0.0066	0.0694	0.0665	0.0215	0.0064	0.0237
σ_z	Inv. Chi-squared	0.0125	0.0066	0.0086	0.0067	0.0019	0.0064	0.0237
σ_u	Inv. Chi-squared	0.0125	0.0066	0.0699	0.0336	0.0326	0.0064	0.0237
σ_e	Inv. Chi-squared	0.0031	0.0016	9.511e-04	7.294e-04	2.041e-04	0.0016	0.0058
σ_r	Inv. Chi-squared	0.0031	0.0016	0.0013	0.0013	8.628e-05	0.0016	0.0058
γ	Beta	0.5000	0.2000	0.9496	0.9383	0.0133	0.1718	0.8282
α	Beta	0.5000	0.2000	0.9006	0.9655	0.0590	0.1718	0.8282
ψ	Gamma	0.1000	0.0300	0.0350	0.0108	0.0174	0.0563	0.1539
δ_r	Gamma	15.0000	5.0000	12.5553	7.8858	5.7152	7.8254	24.0577
ϕ	Gamma	10.0000	10.0000	36.3493	12.0157	21.4019	0.5129	29.9573
ρ_r	Beta	0.7500	0.1000	0.9079	0.8985	0.0216	0.5701	0.8971
ρ_π	Gamma	0.4000	0.1000	0.9836	0.8030	0.1464	0.2509	0.5774
ρ_x	Gamma	0.2000	0.1000	0.2909	0.2948	0.1080	0.0683	0.3877
ρ_a	Beta	0.7500	0.1000	0.9889	0.9905	0.0048	0.5701	0.8971
ρ_u	Beta	0.7500	0.1000	0.9642	0.9919	0.0289	0.5701	0.8971
ρ_e	Beta	0.5000	0.1000	0.5926	0.2348	0.2444	0.3351	0.6649
Flexible Money Growth Rule								
σ_a	Inv. Chi-squared	0.0125	0.0066	0.0638	0.0716	0.0135	0.0462	0.0851
σ_z	Inv. Chi-squared	0.0125	0.0066	0.0051	0.0046	4.604e-04	0.0043	0.0058
σ_u	Inv. Chi-squared	0.0125	0.0066	0.0073	0.0066	0.0015	0.0048	0.0094
σ_e	Inv. Chi-squared	0.0031	0.0016	6.414e-04	635e-04	3.969e-05	5.760e-04	7.031e-04
σ_r	Inv. Chi-squared	0.0031	0.0016	-	-	-	-	-
γ	Beta	0.5000	0.2000	0.9132	0.9055	0.0114	0.8931	0.9304
α	Beta	0.5000	0.2000	0.8475	0.8436	0.0397	0.7789	0.9076
ψ	Gamma	0.1000	0.0300	0.0038	0.0046	9.218e-04	0.0024	0.0054
δ_r	Gamma	15.0000	5.0000	8.6681	8.1955	1.0702	6.9422	10.4597
ϕ	Gamma	10.0000	10.0000	68.35	65.6454	16.5853	39.4357	95.0324
ρ_r	Beta	0.7500	0.1000	0.7637	0.7817	0.0886	0.6258	0.9141
ρ_π	Gamma	0.4000	0.1000	0.3970	0.3750	0.0964	0.2480	0.5210
ρ_x	Gamma	0.2000	0.1000	0.2003	0.1500	0.0939	0.0484	0.3420
ρ_a	Beta	0.7500	0.1000	0.9902	0.9920	0.0019	0.9876	0.9932
ρ_u	Beta	0.7500	0.1000	0.8574	0.8548	0.0496	0.7652	0.9277
ρ_e	Beta	0.5000	0.1000	0.1189	0.1263	0.0247	0.0786	0.1579

4.3. MCMC acceptance ratio

Since the DSGE model specified has more than five parameters, we would expect to see an acceptance ration of around 23 percent Bedard (2008). In my *dynare* code, I set the number of Metropolis-Hastings Chains to two, which is the default and the number of iteration to 25 000. The acceptance ratios are all strictly positive and close the expected value. The Taylor rule and constant money growth rule are slightly above the expected average, and the flexible money growth rate rule acceptance ratio is slightly below.

Table 4.5: MCMC acceptance ratio (precent)

	Taylor Rule	Flexible Money Growth rate Rule	Constant Money Growth Rule
Chain 1	31.184	18.616	33.192
Chain 2	35.176	17.228	33.784

4.4. Blanchard-Khan Stability Conditions

The assumption that macroeconomic models assume perfect foresight has been widely critiqued. However, the assumption has been made possible due to improvement of simulation algorithms. For the model to have a unique solution, the Blanchard-Khan conditions have to be met. These conditions are easy to check, in terms of eigenvalues computed at the steady state of the model Laffargue & others (2000). This unique solution for the model is determined if and only if the number of unstable eigenvalues is equal to the number of non-predetermined variables. The Blanchard-Khan condition for the DSGE model of this project is satisfied as there are five eigenvalues larger than one in the modulus for five forward looking variables in the model.

4.5. Bayesian Impulse Response functions

Much research in macroeconomics have been done to understand the relative impact of different shock on aggregate economic activity. Analyzing the bayesian Impulse response functions, the Taylor rule seems to deliver better results given a shock to preferences and productivity, while the flexible money growth rule delivers similar-to-better results for the money demand and cost push shocks. The IRFs that we simulated look very similar to Belongia, Ireland & others (2019) in shape.

The gray shaded area reflects the highest posterior density interval (HPDI). A description of the variables can be found in table 6.1 under appendix A.

4.5.1. Preference Shock

Figure 4.3 and Figure 4.4 represents the impulse response functions as a results of a preference shock to the variables in the model for the Taylor rule and the flexible money growth rule, respectively. Preference shock can be interpreted as a shock to household production, since it will affect household behaviour by affecting savings-consumption decisions.

A positive preference shock leads to an increase in labour supply and decrease in leisure, which in turn leads to an increase in initial output and consumption. Consequently, we would expect to see that savings and investment decrease. After prices and wages adjust, the output growth slowly returns to its steady state.

In figure 4.3 and Figure 4.4, an initial spike in inflation can be seen. This is due to the increase in output and labour supply. However, inflation returns to its steady state faster under the Taylor rule. The Taylor rule and the flexible monetary policy growth rule have similar responses, with the flexible monetary policy rule being more volatile to changes in output gap (x) and money growth (μ) and therefore taking longer to return to its steady states values.

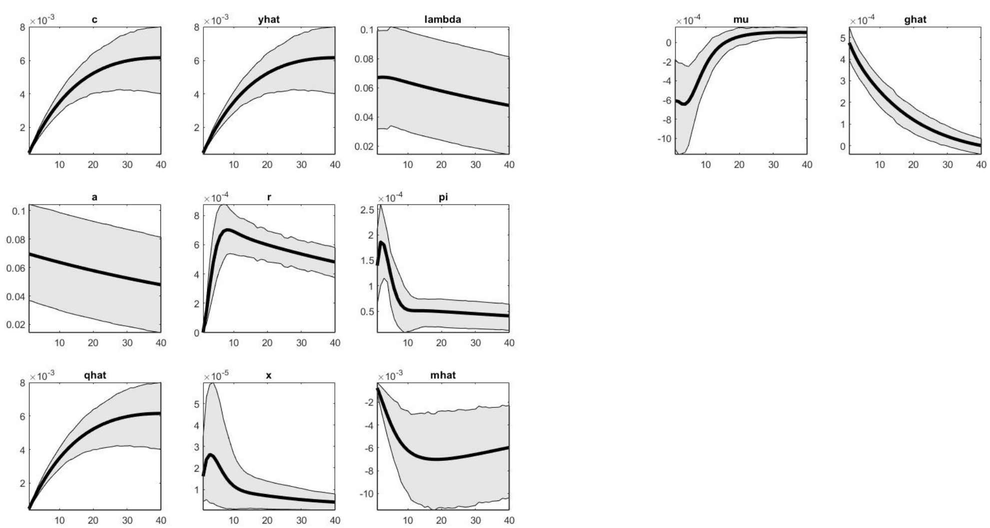


Figure 4.3: Orthogonalized Shock to Preference Shock - Taylor Rule

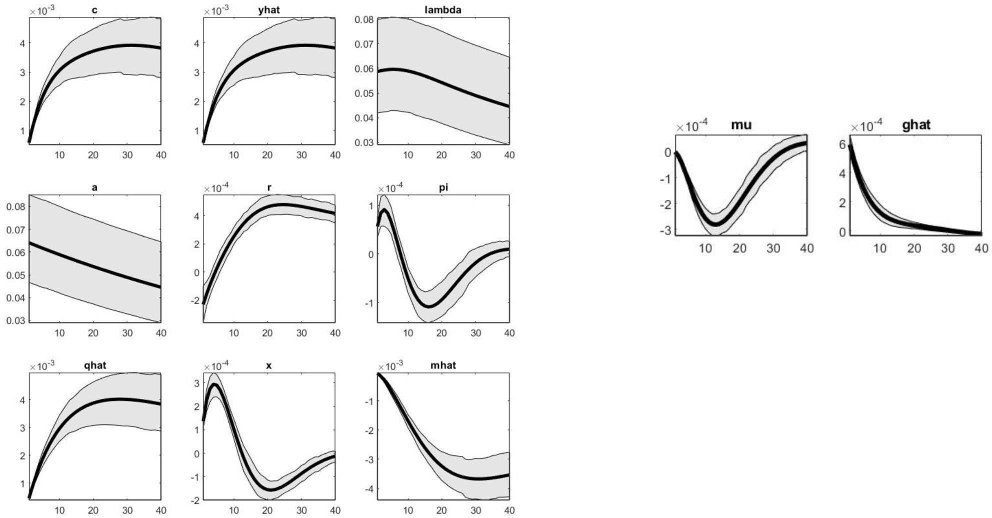


Figure 4.4: Orthogonalized Shock to Preference Shock - Flexible Money Growth Rule

4.5.2. Productivity Shock (z)

Figure 4.5 and Figure 4.6 represents the impulse response functions as a results of a productivity shock to the variables in the model for the Taylor rule and the flexible money growth rule, respectively. A positive productivity shock is expected to persist over time, although not permanently. Therefore, a productivity shocks leads to an increase in hours worked and real wage which in turn increase output, consumption and investment. The increase in output would then lead to an increase in money growth, as it would be necessary to increase money printing.

The flexible money growth rules better reflects what we theoretically would expect to see when a productivity shock occurs. Figure 4.5 and 4.6 support the view that productivity shocks play an important roles in accounting for fluctuations in output. Furthermore, it supports the view that technology shocks affect the marginal product of labour and the marginal product of capital.

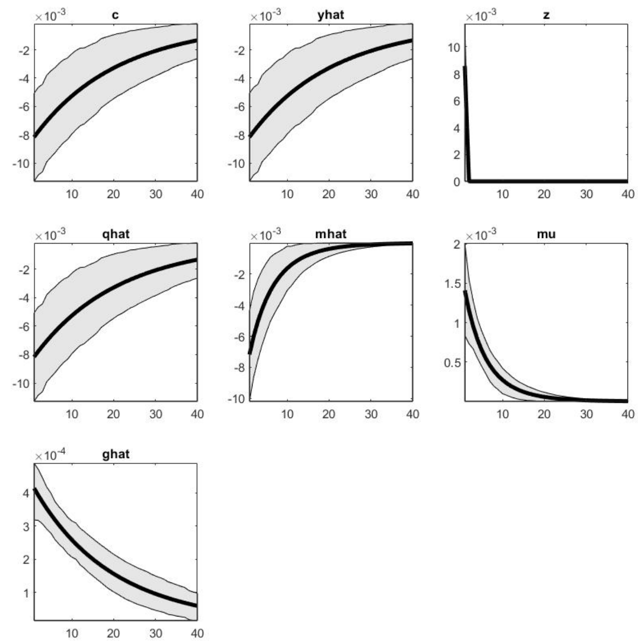


Figure 4.5: Orthogonalized Shock to Productivity Shock - Taylor Rule

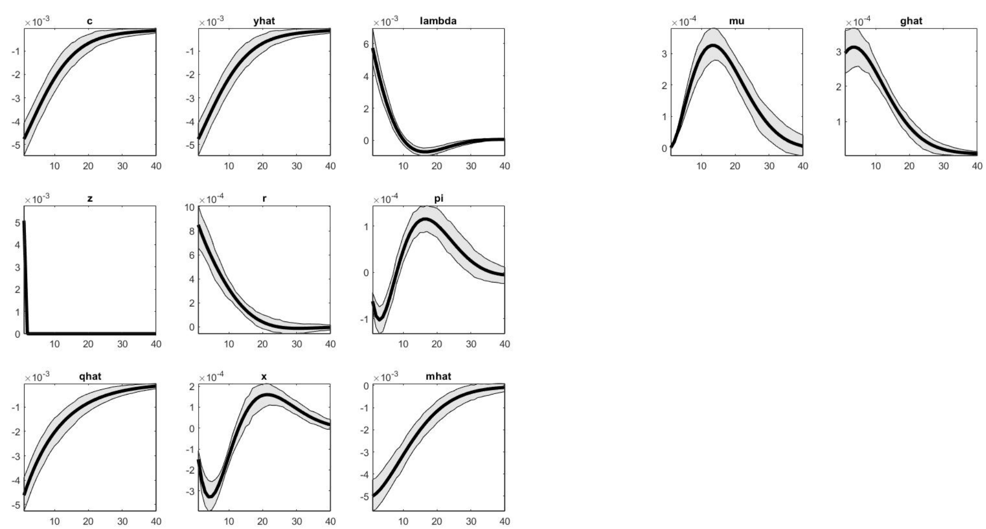


Figure 4.6: Orthogonalized Shock to Productivity Shock - Flexible Money Growth Rule

4.5.3. Money Demand Shock (u)

Figure 4.7 and Figure 4.8 represents the impulse response functions as a results of a money demand shock to the variables in the model for the Taylor rule and the flexible money growth rule, respectively. Money demand shocks are more likely to lead to volatility in inflation and output. This results can be seen in figure 4.8, which shows that output ($\hat{y}$), consumption (c), inflation (π), output gap (x) and money growth (μ) all follow volatility and similar paths. The rise in the demand for money is temporarily followed by an increase in the growth rate of money (μ), which in turn exerts upward pressure on inflation (π), as can be seen in in figure 4.8.

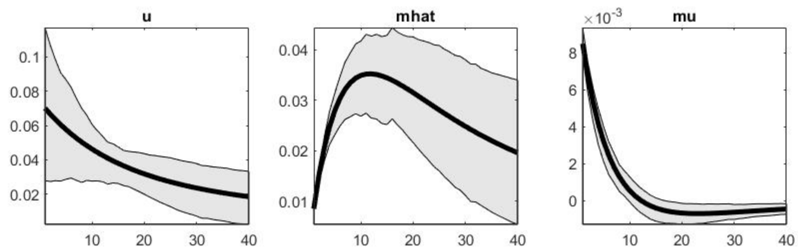


Figure 4.7: Orthogonalized Shock to Money Demand Shock - Taylor Rule

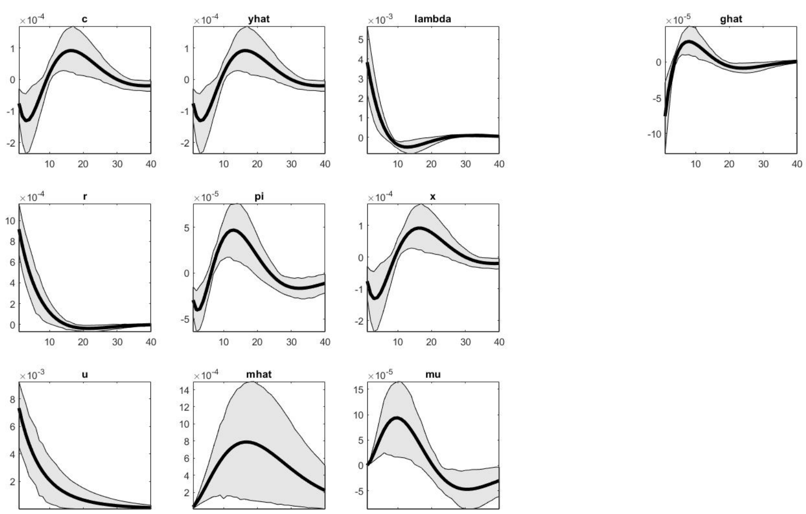


Figure 4.8: Orthogonalized Shock to Money Demand Shock - Flexible Money Growth Rule

4.5.4. Cost Push Shock (e)

Cost push shocks are transitory deviations between the flexible price equilibrium and efficient allocation. Cost push shocks create a increase in inflation and a decrease in output gap, simultaneous. Therefore, the optimal response of the central bank is extremely controversial. The central bank can make one of two contradicting choices; it can either relax its monetary policy in order to accommodate the negative output gap, or it can tighten its monetary policy in order to combat the increase inflation. Figure 4.9 and Figure 4.10 represents the impulse response functions as a results of a cost push shock to the variables in the model for the Taylor rule and the flexible money growth rule, respectively.

The Taylor rule and the flexible money growth rate rule are comparatively similar. This implies that the flexible money growth rate rule can be used as an alternative to the Taylor rule give most of the shocks.

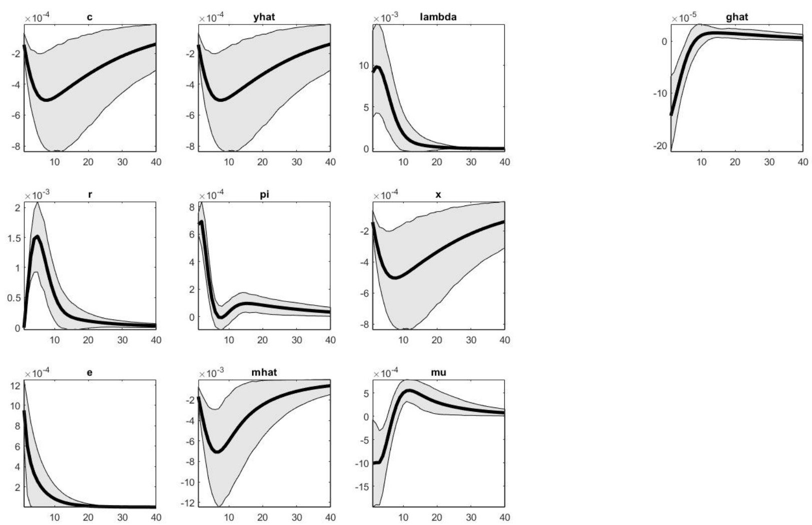


Figure 4.9: Orthogonalized Shock to Cost Push Shock - Taylor Rule

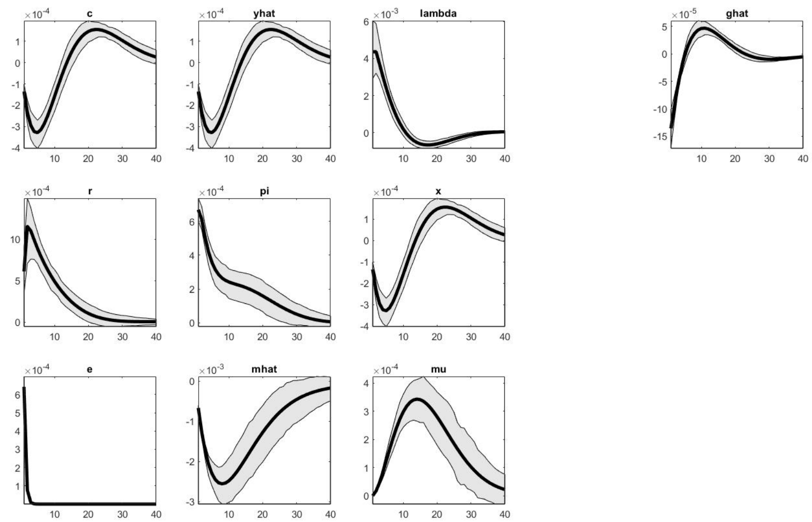


Figure 4.10: Orthogonalized Shock to Cost Push Shock - Flexible Money Growth Rule

5. Evaulation

Many resources and time has been dedicated to find the optimal rule to advise monetary policy rules. An optimal monetary policy rule is designed in such a manner as to stabilize prices and output as well as boost consumer confidence. In this section, several diagnostic test are carried out in order to analyze the empirical fit of the DSGE model in terms of the Taylor rule and the flexible money growth rule. The Taylor rule and flexible money growth rule will be analyzed and compared, with some reference being made to the constant money growth rule.

5.1. Mode Plots

Figure 5.1 and 5.2 display the mode check plots for the taylor rule and the flexible money growth rule, respectively. The difference in the shapes of the likelihood kernel (red line) and the posterior likelihood (blue line) indicates the role of the prior in influencing the curvature of the likelihood function. As can be seen in the plots, the mode is at the maximum point of the posterior likelihood. This implies that there are no identification problems. The red dots on the figures indicates a violation of the Blanchard-Kahn conditions and the model could not solve those parameter values.

There are slight difference between the taylor rule and the flexible money growth rule mode check plots. These differences are due to the observed money growth being included in the taylor rule but not the flexible money growth rate rule.

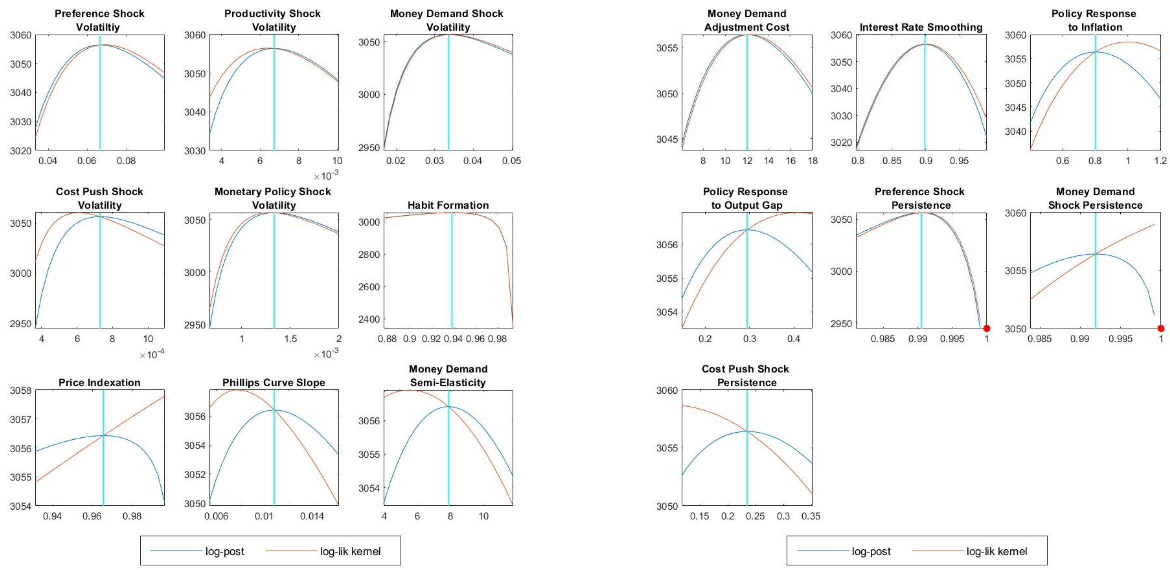


Figure 5.1: Mode Check Plots - Taylor Rule

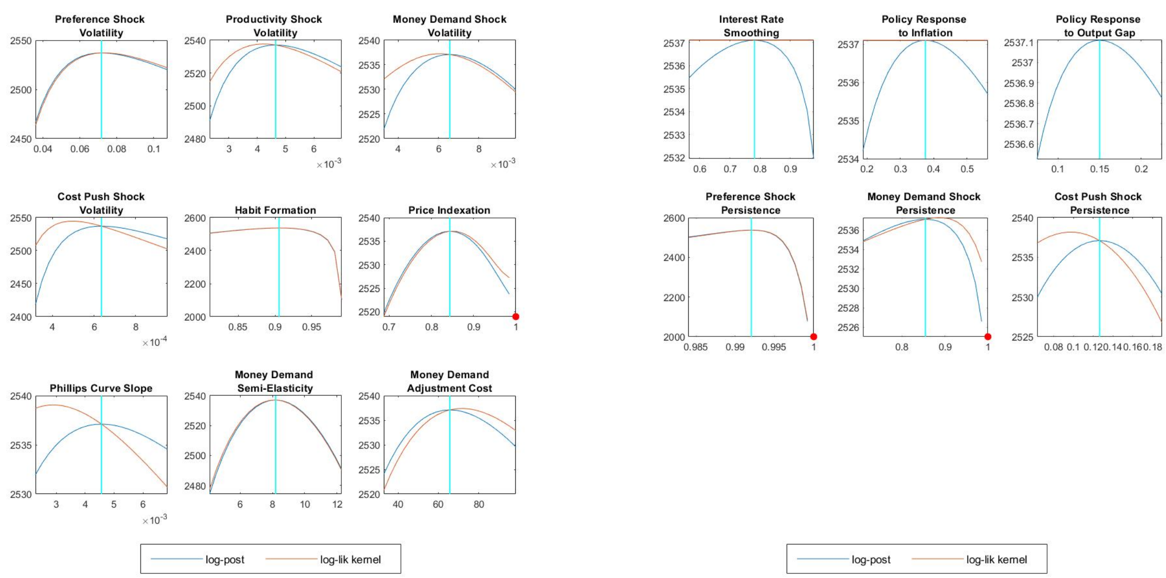


Figure 5.2: Mode Check Plots - Flexible Money Growth Rule

5.2. Monte Carlo Markov Chain univariate diagnostics

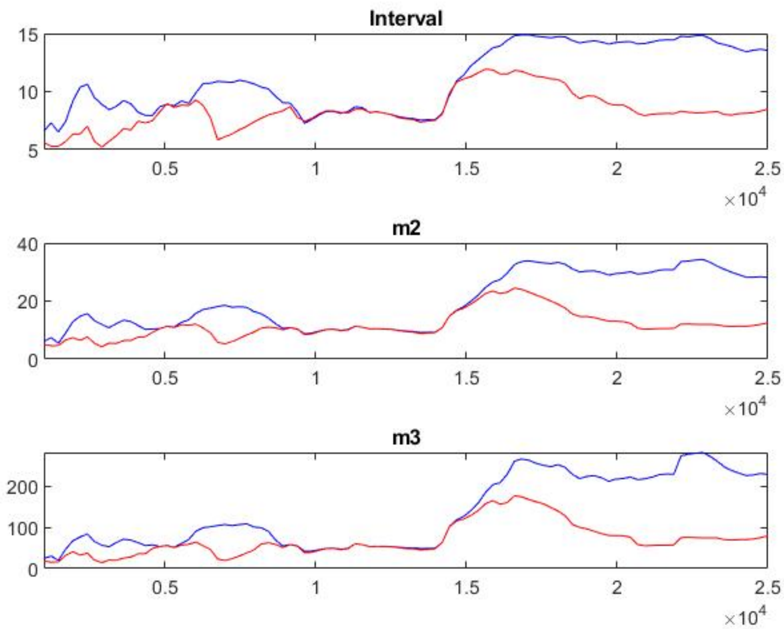
It is extremely challenging to test for the convergence of the posterior distribution. However, *dynare* provides us with Monte Carlo Markov Chain (MCMC) univariate diagnostics which makes it easier to analyze. Figure 5.3, 5.4, 5.5 and 5.6 as well as figure 5.7, 5.8, 5.9, 5.10 represents convergences indicators for all parameters considered for the Taylor rule and the flexible money growth rule.

There are three plots for each parameter. The first plot shows the convergence diagnostics for the 80 percent interval. The second plot shows the estimate of the second central moment (m2), the variance, and the third plot shows the estimate of the third central moment (m3). The red line shows the 80 percent quantile range based on the pooled draws from all sequences and the blue line shows the mean interval range based on the draws of the individual sequences. The plots can be interpreted as the chains having converged if the red and blue line stabilize horizontally and remain close to each other.

I expect to see that many of the iterations of the Metropolis-Hasting simulating to be similar, meaning that for the results to be sensible, I would expect to see the red and blue lines remain relatively constant and they should converge. Studying the results for the Taylor rule, I find mixed results. Preference shock volatility (σ_a), productivity shock volatility (σ_z), monetary policy shock volatility (σ_r), price indexation (α), Interest rate smoothing (ρ_r), policy response to inflation (ρ_π), policy response to output gap (ρ_x) and preference shock persistence (ρ_a) all seem to all converge, especially in relation *interval*. However, the other eight variables show greater variation during the process of convergence. This unsatisfactory performance for the estimation of these parameters is related to the prior values for each as they show weak identification. This is concerning since the the general results displayed in figure 5.3 is considered unsatisfactory, implying that the prior might need reconsideration. Figure 5.3 also shows that the parameters converge to the target distribution at around 10 000 iterations, after which they drastically diverge again.

However, the flexible money growth rate rule shows much more promising and satisfactory results. The only parameter that does not converge is the preference shock persistence (ρ_a). The results are extremely satisfactory, implying that that is no reasons to change the prior values. Figure 5.11 shows that the general results for the constant money growth rule is similar to that of the flexible money growth rate rule, however, it seems to diverge at 4000 iterations before it converges again at 7000 iterations. The divergence implies that the flexible money growth rate is more stable than the constant money growth rate. This suggest that the flexible money growth rule could lead to a more stable and predicable path of the parameters. Thus, taking only the MCMC univariate diagnostics into account, I conclude that the flexible Money growth rule is more optimal given the information in the observed data.

Figure 5.3: MCMC general - Taylor Rule



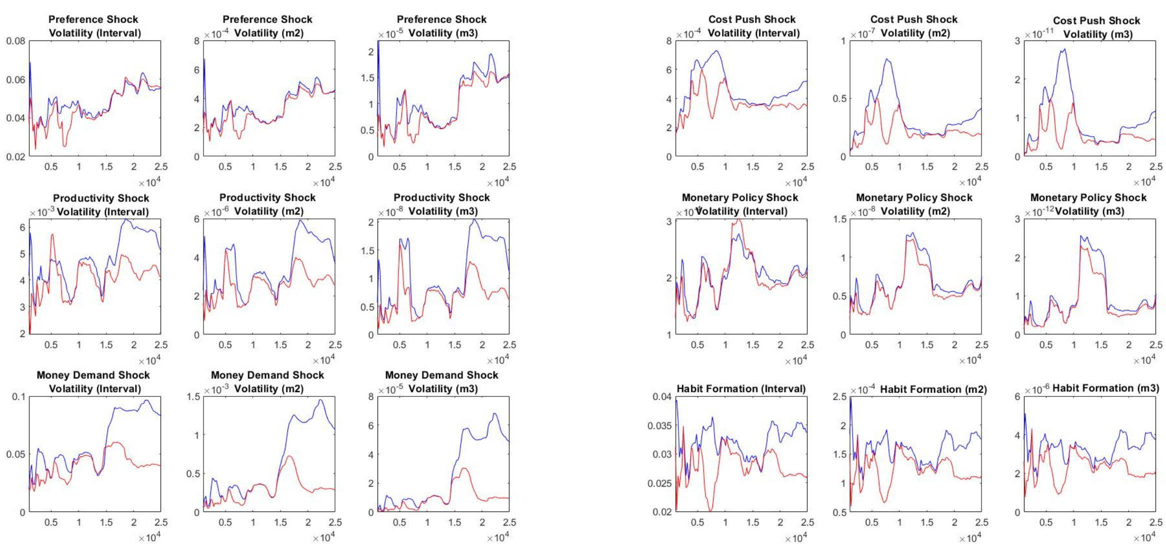


Figure 5.4: (a) MCMC - Taylor Rule

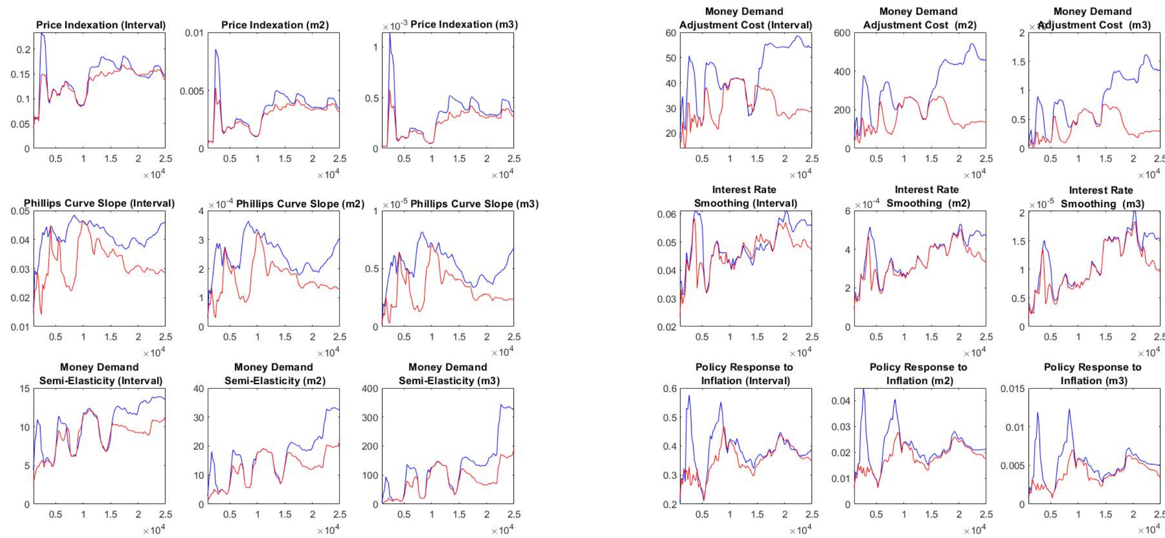


Figure 5.5: (b) MCMC - Taylor Rule

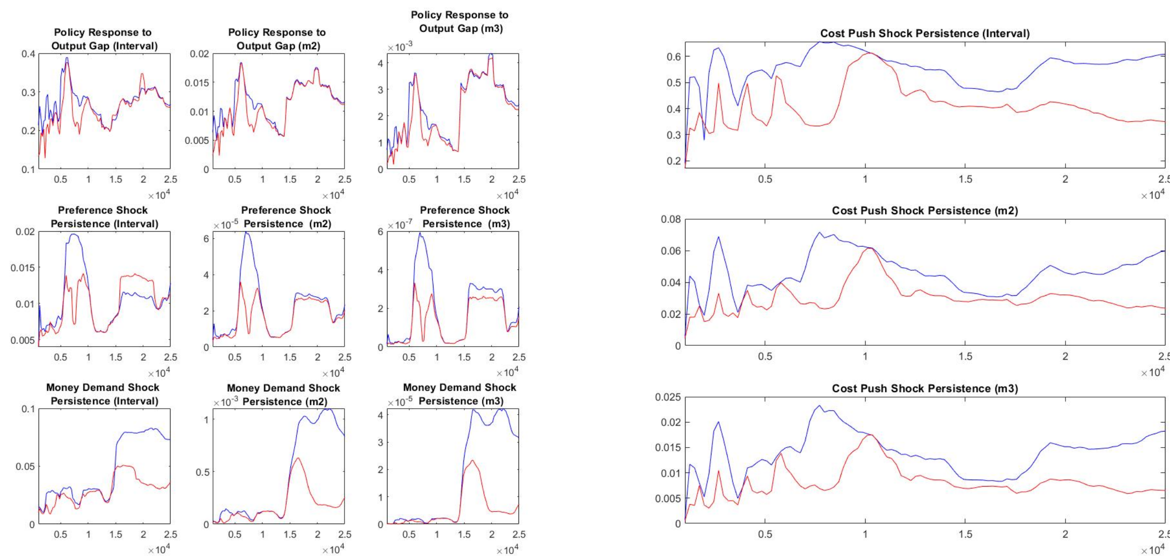
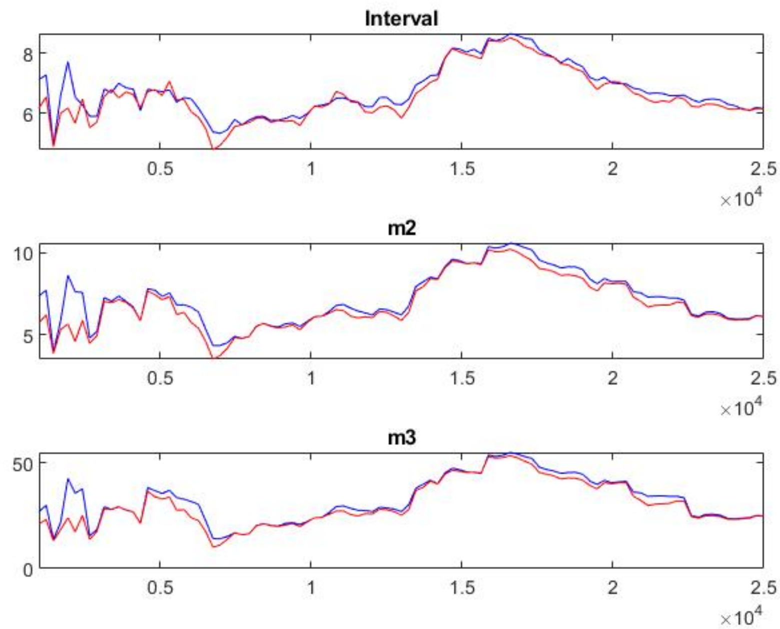


Figure 5.6: (c) MCMC - Taylor Rule

Figure 5.7: MCMC general - Flexible Money Growth Rate Rule



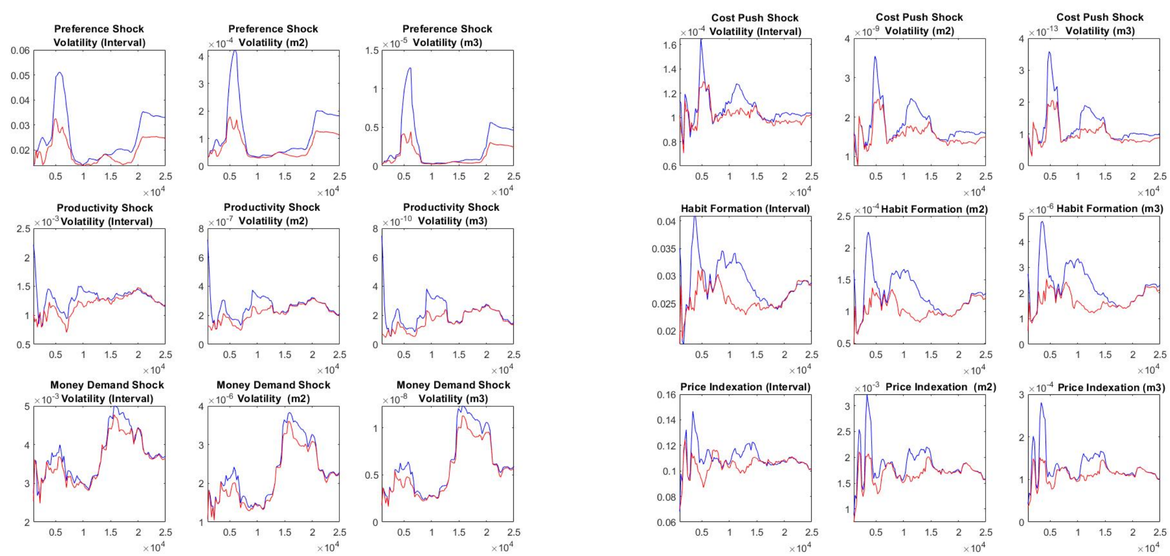


Figure 5.8: (a) MCMC - Flexible Money Growth Rate Rule

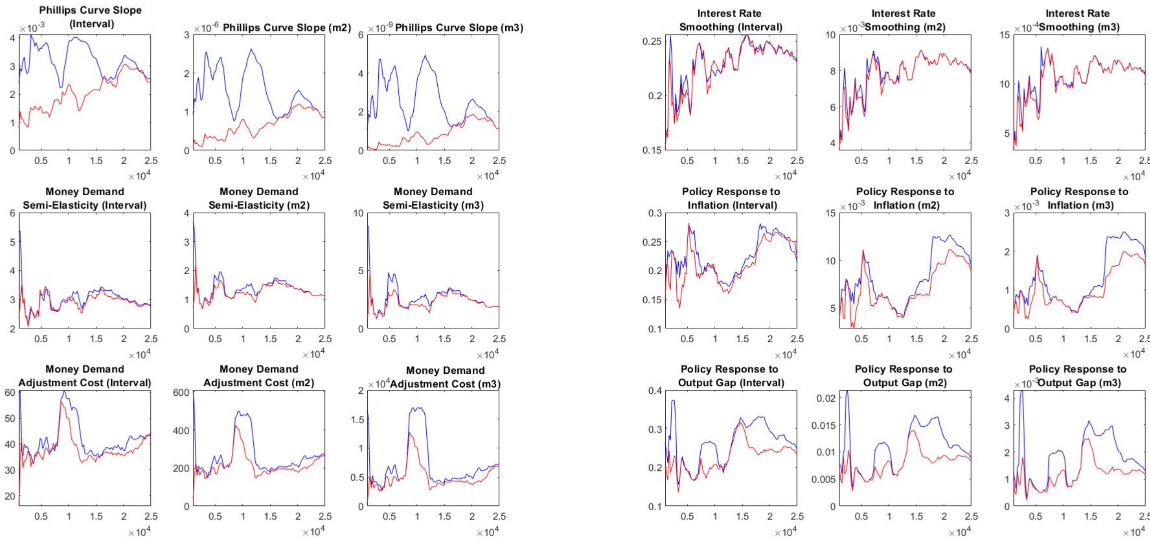


Figure 5.9: (b) MCMC - Flexible Money Growth Rate Rule

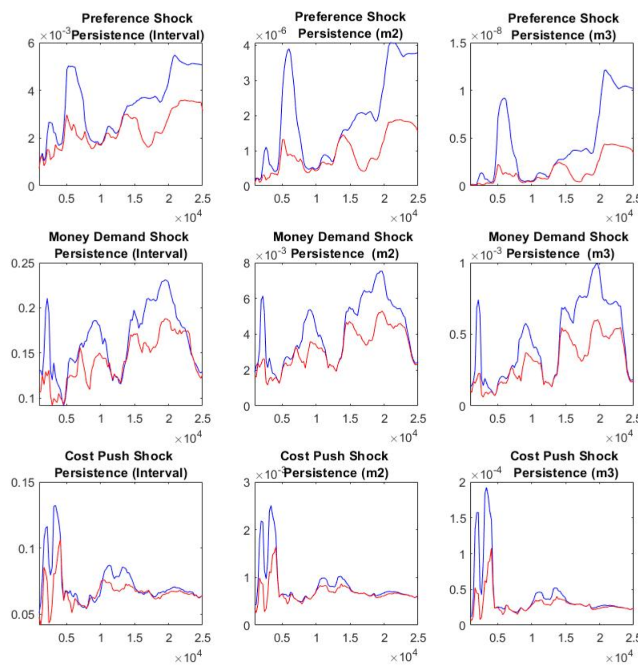
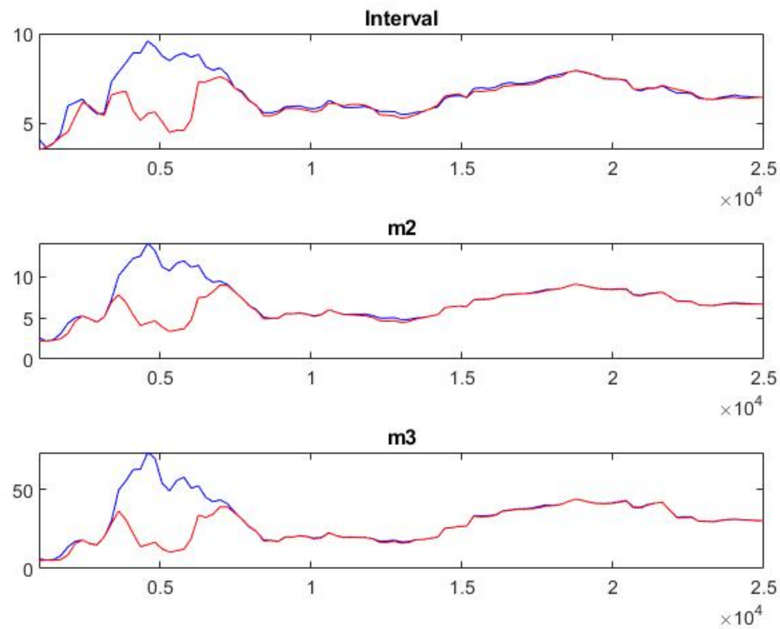


Figure 5.10: (c) MCMC general - Flexible Money Growth Rate Rule

Figure 5.11: MCMC general - Constant Money Growth Rule



5.3. Prior-Posterior Plot

Understanding the impact that the prior distribution has on the posterior density is very important. The impact of the priors is is extremely important to the model complexity and the structure of the data. Figure 5.12 and figure 5.13 display the relationship between the prior and the posterior. The gray line shows the prior distribution that is also displayed in figure 4.1 and figure 4.2. The black line shows the density of the posterior distribution and the green line shows the posterior mode. Figure 5.12 shows the prior and posterior plots for the Taylor rule. All the plots prior and posterior distributions are vastly different, except for money demand semi-elasticity (δ_r) and policy response to output gap (ρ_x). This can be due to the fact the prior is an very accurate reflection of the information in the data. However, more likely, it is due to that fact that (ρ_x) and (δ_r) are only weakly identified and that the data proves limited information to update the prior.

Figure 5.13 displays the prior and posterior plots for the flexible money growth rule. The prior and posterior distributions of interest rate smoothing (ρ_r), policy response to inflation (ρ_π) and policy response to output gap (ρ_x) are extremely similar. This is of concern as it is an indication that the parameters were only weakly identified.

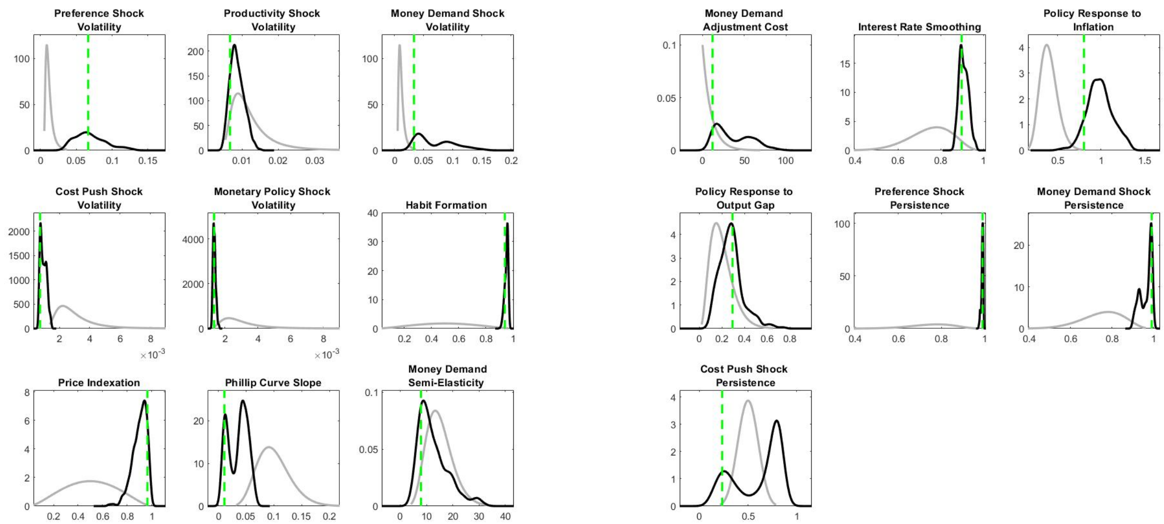


Figure 5.12: Priors and Posterior: Taylor Rule

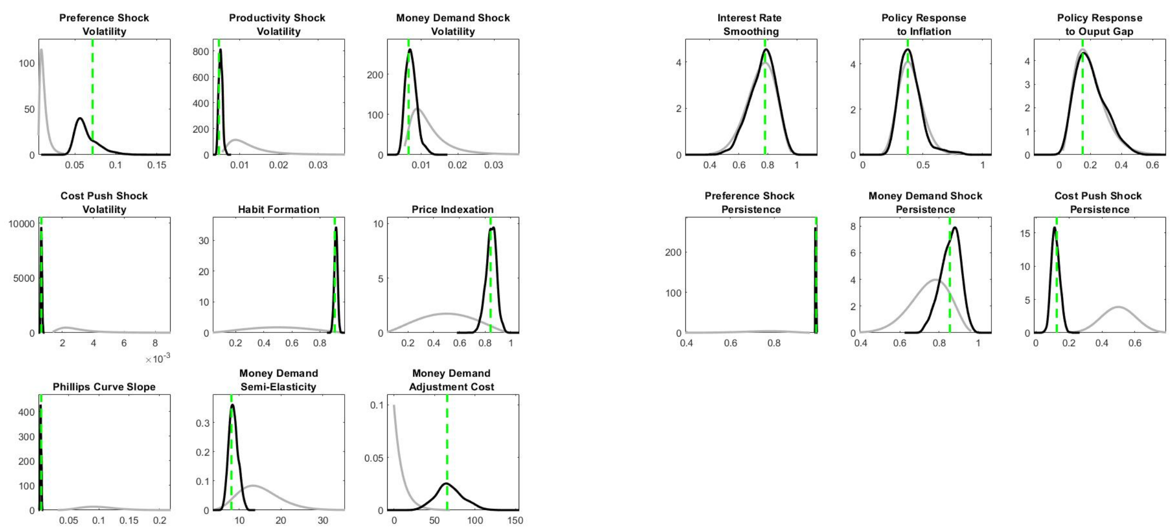


Figure 5.13: Prior and Posterior: Flexible Money Growth Rule

5.4. Smoothed Shocks

Smoothed shocks are a reconstructions of the best estimate of the values of the unobserved shocks over the entire sample period, using the observed data. Figure 5.16 show that the structural shocks to both the Taylor rule and the flexible money growth rule are stationary around a mean of zero (the red line).

Figure 5.16 shows a significant shock at around 75 periods for preference and productivity shock for both rules. This volatility represents the 2001 recession, where a decline in economic activity was short lived and occurred mainly in developed countries. Further volatility can be seen around 110 periods for money demand shock, cost push shock and preference shock, representing the 2007/2008 financial crisis. The monetary policy shock in the Taylor rule seems to be relatively stable. Preference shocks seem to be more volatile under the flexible money growth rule. However, the general results remain that the flexible money growth rule and Taylor rule estimate similar volatility of the respective shocks.

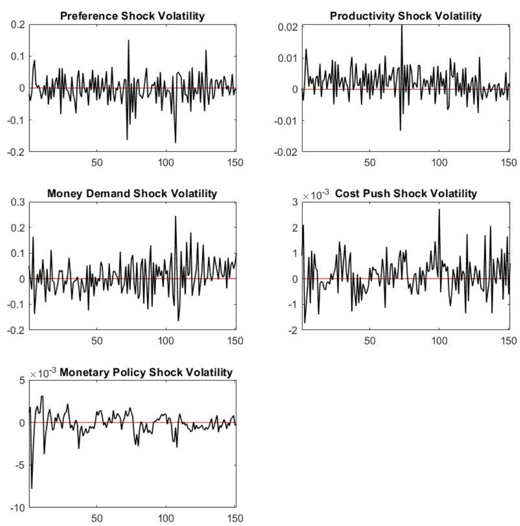


Figure 5.14: Taylor Rule

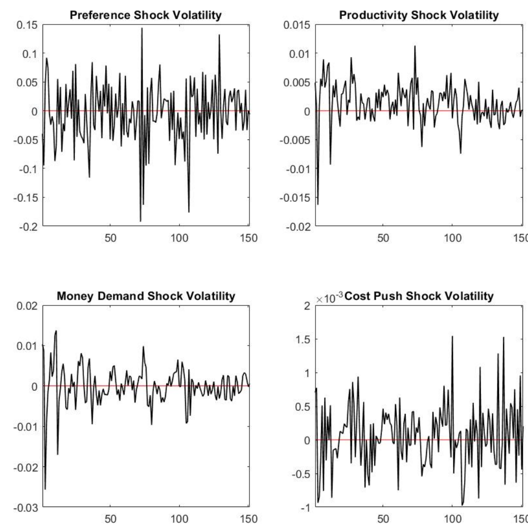


Figure 5.15: Flexible Monetary Policy Rule

Figure 5.16: Smoothed shocks

5.5. Historical and Smooth Variables

Figure 5.19 represent historical and smoothed variable plots. The analysis is done is order to determine whether measurement error exist in our model. The dotted black line represents the actual data that is observed and the red line represents the estimate of the smoothed variable. Under the Taylor rule and flexible money growth rate rule both series are identical as they overlap eachother, implying that there is no measurement error and that model is specified correctly. This is another indication that the flexible money growth rate rule preforms just as well as the Taylor rule.

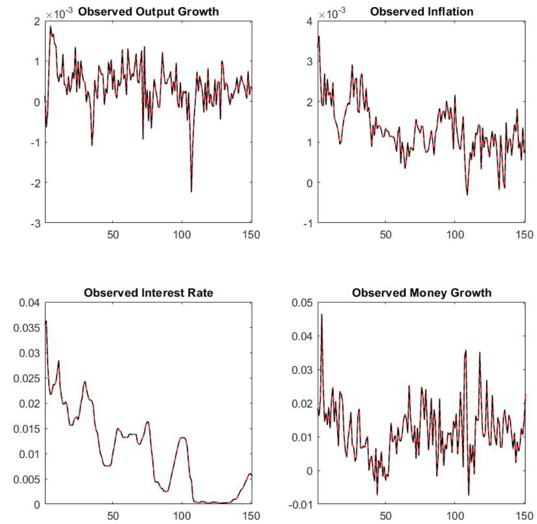


Figure 5.17: Taylor Rule

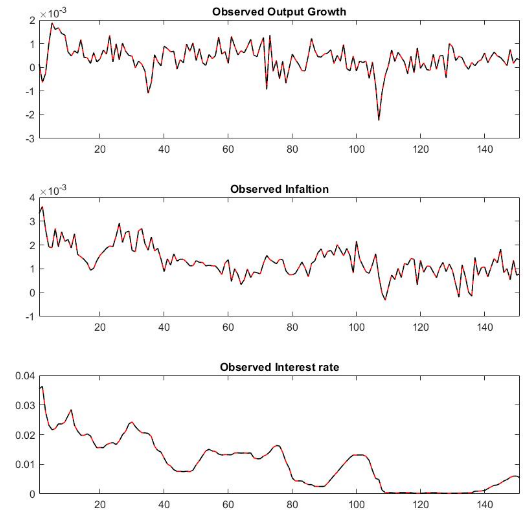


Figure 5.18: Flexible Monetary Policy Rule

Figure 5.19: Historical and Smooth Variables

5.6. Variance Decomposition

The variance decomposition displayed in table 5.1 and table 5.2 is used to analyze how variation in the model is generated by a specific shock. This method is informative in establishing the relative importance of a specific shock as a source of volatility to a macroeconomic variable. Productivity shocks accounts for most of the variation in Consumption (c), output ($\hat{y}$) and the efficient level of output ($\hat{q}$) under the Taylor rule and the flexible money growth rule. Whereas a cost push shock accounts for most of the variation in inflation (π). Variation in monetary policy shock (r) is explained mostly by preference shock [60.86 percent and 51.09 percent], with cost push shock also explaining a large portion of the variation in the Taylor rule and flexible money growth rule, respectively.

Under the Taylor rule, variation in money growth (μ) and money demand ($\hat{m}$) is largely explained by a monetary demand shock. However, under the flexible money growth rule, money demand ($\hat{m}$) is mostly explained by a preference shock and money growth (μ) is explained by similar by a preference shock, productivity shock and cost push shock.

Table 5.1: Taylor Rule: Posterior Mean Variance Decomposition (in percent)

	Description	σ_a	σ_z	σ_u	σ_e	σ_r
c	consumption	82.04	17.81	0.00	0.14	0.00
$\hat{y}$	output	82.04	17.81	0.00	0.14	0.00
λ	lambda	99.38	0.00	0.00	0.60	0.02
a	preference shock	100.00	0.00	0.00	0.00	0.00
z	productivity shock	0.00	100.00	0.00	0.00	0.00
r	monetary policy shock	60.86	0.00	0.00	30.33	8.80
π	inflation	17.85	0.00	0.00	72.23	9.91
$\hat{q}$	efficient level of output	82.13	17.87	0.00	0.00	0.00
x	output gap	0.99	0.00	0.00	98.21	0.80
e	cost push shock	0.00	0.00	0.00	100.00	0.00
u	money demand shock	0.00	0.00	100.00	0.00	0.00
$\hat{m}$	money demand	9.69	0.51	87.62	3.99	1.31
μ	money growth	1.83	3.25	89.62	3.99	1.31
$\hat{g}$	output growth	49.20	49.00	0.00	1.75	0.05

Table 5.2: Flexible Monetary Policy Rule: Posterior Mean Variance Decomposition (in percent)

	Description	σ_a	σ_z	σ_u	σ_e
c	consumption	89.54	10.36	0.02	0.08
$\hat{y}$	output	89.54	10.36	0.02	0.08
λ	lambda	99.87	0.06	0.02	0.05
a	preference shock	100.00	0.00	0.00	0.00
z	productivity shock	0.00	100.00	0.00	0.00
r	monetary policy shock	51.09	12.88	9.23	26.80
π	inflation	6.73	7.69	1.31	84.27
$\hat{q}$	efficient level of output	90.64	9.36	0.00	0.00
x	output gap	27.43	32.38	6.40	33.79
e	cost push shock	0.00	0.00	0.00	100.00
u	money demand shock	0.00	0.00	100.00	0.00
$\hat{m}$	money demand	74.40	17.01	1.52	7.08
μ	money growth	25.09	33.15	3.44	38.33
$\hat{g}$	output growth	52.27	44.91	0.68	2.13

5.7. Forecasting

Figure 5.20 and figure 5.21 displays the mean forecast plots of the Taylor rule and flexible money growth rule, respectively. The black line represents the mean forecasts of the macroeconomic endogenous variable, starting at the last observation in the sample and going 40 periods ahead. The green lines represent the mean forecast deciles. It is important to note that the forecasts only take parameter uncertainty into account and not the uncertainty about future shocks. Under the flexible money growth rule, the macroeconomic variable appear to stabilize to their steady state faster than the under the Taylor rule. This, once again, established that the flexible money growth rule could be more optimal than the Taylor rule.

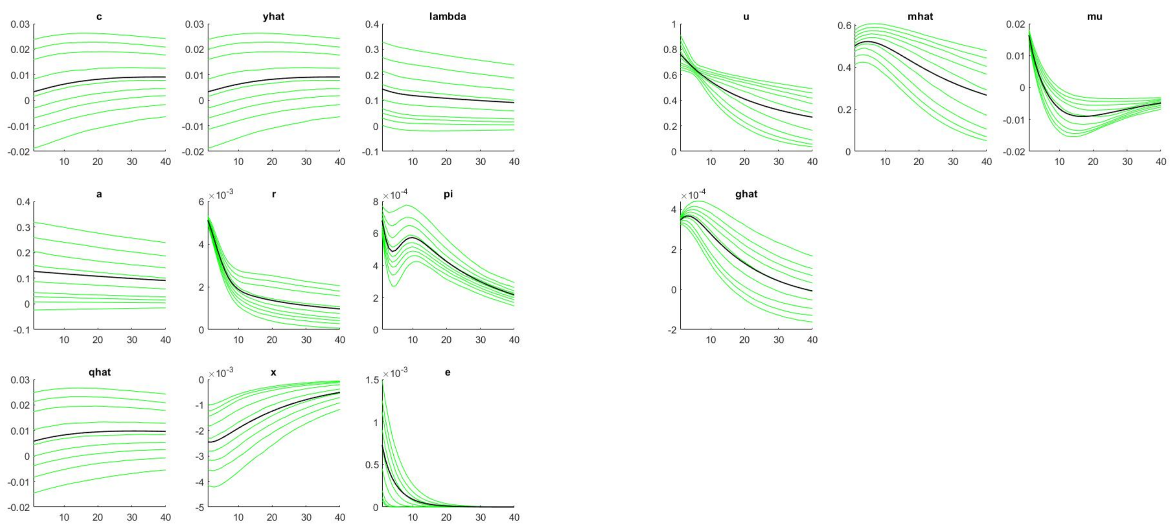


Figure 5.20: Forecasted Variables (mean) - Taylor Rule

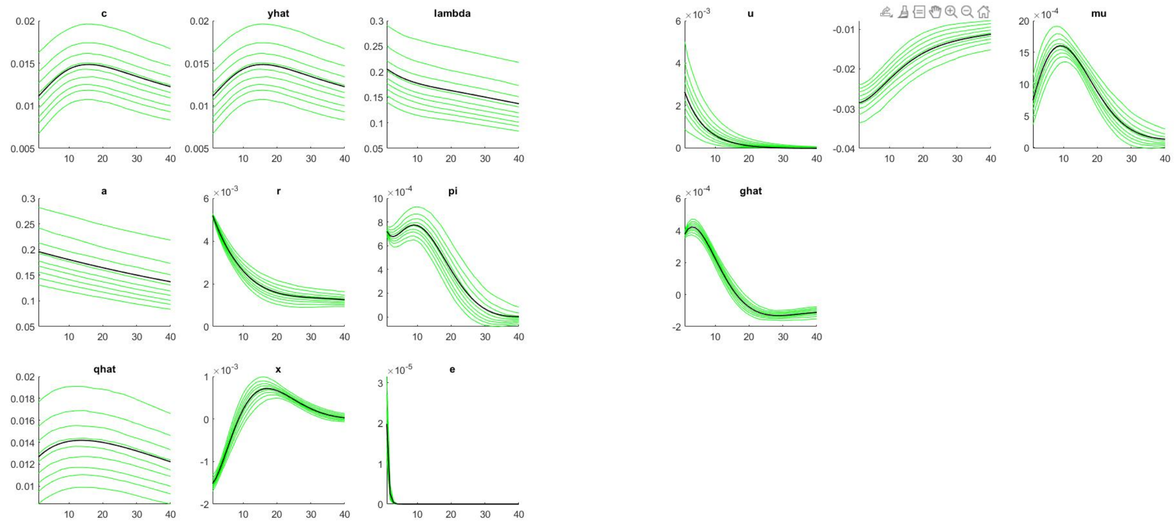


Figure 5.21: Forecasted Variables (mean) - Flexible Money Growth Rule

Figure 5.22 and figure 5.23 displays the point forecast plots for the Taylor rule and flexible money growth rule, respectively. In contrast to the mean forecast, the point forecast takes the parameter uncertainty and the uncertainty about future shocks into account. Both rules have very similar results, with the exception being the output gap (x) and the money demand shock (u), which both appear to stabilize around the steady state faster under the flexible money growth rule.

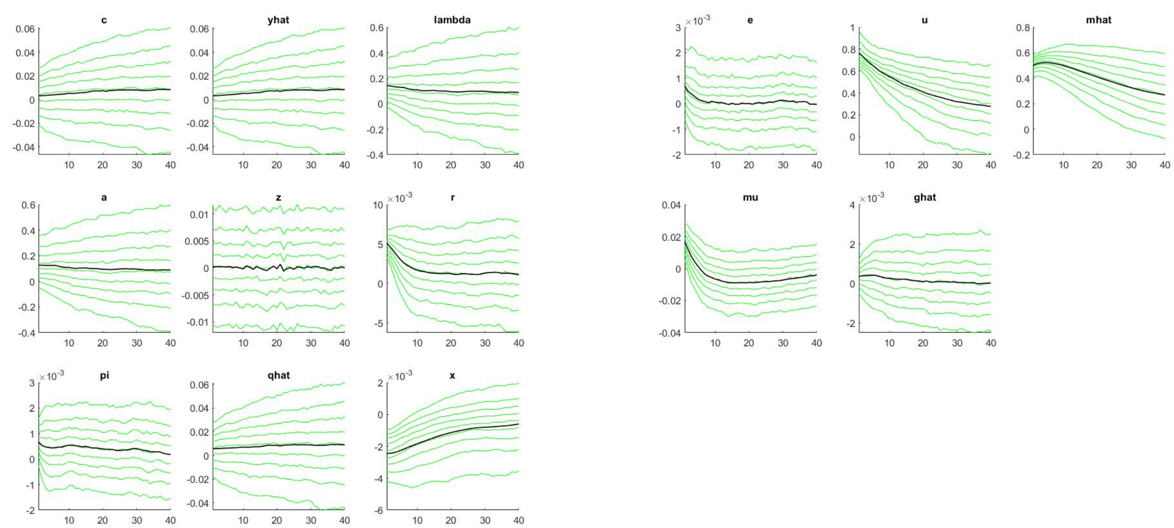


Figure 5.22: Forecasted Variables (point) - Taylor Rule

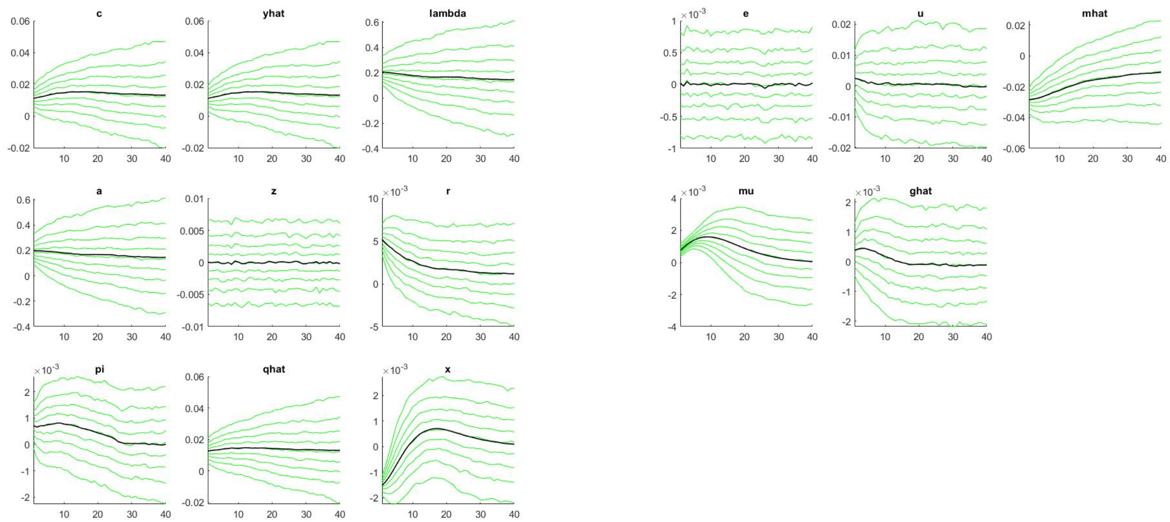


Figure 5.23: Forecasted Variables (point) - Flexible Money Growth Rule

6. Conclusion

The central object of this project was to estimate, simulate and analyze the effect of the Taylor rule and flexible money growth rate rule on macroeconomic variables. Following Belongia, Ireland & others (2019), this projects makes of the DSGE model and was estimated using a Bayesian approach. The overall findings indicate that the flexible money growth rule comparatively similar, and in some cases better, than the Taylor rule. This implies that policymakers should consider a flexible money growth rate rule since it successfully decreases macroeconomic volatility in inflation and output growth.

In the previous report, the conclusion was reach that the constant money growth rule preformed the poorest in the simulations, therefore, only it was only briefly mentioned. However, it remains clear that a reconsideration of the Fed's monetary policy toolkit is warranted, and further study along this avenue should be encouraged.

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Appendix

Appendix A

Table 6.1: Endogenous Variables

Variable	Symbol	Description
`c`	c	consumption
`yhat`	$\hat{y}$	output
`lambda`	λ	lambda
`a`	a	preference shock
`z`	z	productivity shock
`r`	r	monetary policy shock
`pi`	π	inflation
`qhat`	$\hat{q}$	efficient level of output
`x`	x	output gap
`e`	e	cost push shock
`u`	u	money demand shock
`mhat`	$\hat{m}$	money demand
`mu`	μ	money growth
`ghat`	$\hat{g}$	output growth
`gobs`	g^{obs}	observed output growth
`piobs`	π^{obs}	observed inflation
`robs`	r^{obs}	observed interest rate
`muobs`	μ^{obs}	observed money growth

Table 6.2: Parameters

Variable	Symbol	Description
`z_ss`	z_{ss}	output growth steady state
`beta`	β	discount factor
`gamma`	γ	habit formation
`rho_a`	ρ_a	preference shock persistence
`alpha`	α	price indexation
`psi`	ψ	Phillips curve slope
`rho_r`	ρ_r	interest rate smoothing
`rho_pi`	ρ_π	policy response to inflation
`rho_x`	ρ_x	policy response to output gap
`delta_r`	δ_r	money demand semi-elasticity
`phi`	ϕ	money demand adjustment cost
`r_ss`	$r_s s$	interest rate steady state
`rho_u`	ρ_u	money demand shock persistence
`rho_e`	ρ_e	cost push shock persistence
`rho_mm`	$\rho_m m$	money shock
`rho_mpi`	$\rho_m \pi$	money inflation shock
`rho_mx`	$\rho_m x$	output gap shock