## Random Variables (rv)

**Discrete Random Variables** [Ross S4.2]

**Definition 9.1:** A random variable that can take at most a countable number of possible outcomes is called a discrete random variable.

**Definition 9.2:** For a discrete random variable X, we define its **Probability Mass Function** (PMF)  $p_X(a)$  by

$$p_X(a) = P[X = a].$$

Let  $\mathcal{X} = \{x_1, x_2, ...\}$  be the possible outcomes that X takes.

Then 
$$p_X(x) \ge 0$$
 for  $x \in \mathcal{X}$   $p_X(x) = 0$  for all other  $x$ 

and, since X must take one of its possible values:

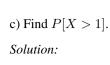
$$\sum_{x \in \mathcal{X}} p_X(x) = 1$$

**Example 9.1:** Say the PMF of the random variable X is

$$p_X(k) = C \frac{\lambda^k}{k!}, \quad \text{for } k = 0, 1, 2, \dots$$

and  $\lambda > 0$  is given.

- a) Find C in terms of  $\lambda$
- b) Find P[X=0]



Say  $\mathcal{X} \subset \mathbb{R}$ . Instead of specifying  $p_X(x)$  for every  $x \in \mathcal{X}$ , we can specify:

$$F_X(x) = P[X \le x] \qquad x \in \mathbb{R}$$

instead.

 $F_X(x)$  is called the **Cumulative Distribution Function** (CDF) of X.

**Example 9.2:** Let X be such that

$$p_X(1) = \frac{1}{4}$$
  $p_X(2) = \frac{1}{2}$   $p_X(3) = \frac{1}{8}$   $p_X(4) = \frac{1}{8}$ 

Plot the CDF  $F_X(x)$ .

Solution:

## Expected (Mean) Value [Ross S4.3]

**Definition 9.3:** The **expected** (or **mean**) value of a random variable X is

$$E[X] = \sum_{x \in \mathcal{X}} x p_X(x) \tag{9.1}$$

This is an "average" where each outcome is weighted by the probability that X assumes that outcome.

**Example 9.3:** Say  $p_X(0) = 1/2$ ,  $p_X(1) = 1/2$ .

Then 
$$E[X] = 0 \times 1/2 + 1 \times 1/2$$
$$= 1/2$$

**Example 9.4:** Say  $p_X(0) = 1/3$ ,  $p_X(1) = 2/3$ .

Then 
$$E[X] = 0 \times 1/3 + 1 \times 2/3$$
$$= 2/3$$

**Example 9.5:** : Let  $A \subset S$  be an event. Let the random variable I be such that

$$I = \begin{cases} 1 & A \text{ occurs} \\ 0 & A \text{ does not occur.} \end{cases}$$

Then

$$E[I] = 0 \times P[I = 0] + 1 \times P[I = 1]$$
  
=  $0 \times P[A^c] + 1 \times P[A]$   
=  $P[A]$ 

I is called an **indicator for event** A. We often write  $I_A$  or  $1_A$  for this kind of random variable.

**Example 9.6:** 120 students are driven in 3 buses with 36, 40 and 44 students each. One of the 120 students is chosen randomly.

Let X = # students on bus of randomly chosen student.

What is E[X]?

Solution: