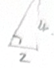
Sept 4



B.1-5 B.1-15 B.2-1, B.3-2 , B.4-2 1.1-2 1.1-4 3.1-2

B.1-5(a) $w_a = -je^{j\pi/4}$
$= -j(\cos\pi/4 + j\sin\pi/4)$
$= -j(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2})$
$= -j\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}$
$\mathrm{Im}(w_a) = \underline{-\frac{\sqrt{2}}{2}}$

B.1-5(b) $w_b = 1-2je^{2-4j}$
$= 1-2je^2 e^{-4j}$
$= 1-2je^2[\cos(-4) + j\sin(-4)]$
$= 1-2je^2[-0.653 + j0.7568]$
$= 1+9.6596j + 11.1841$
$\mathrm{Im}(w_b) = \underline{9.6596}$

B.1-5(c) $w_c = \tan(j)$
$= \frac{\sin(j)}{\cos(j)}$

$\sin(x) = \frac{e^{jx} - e^{-jx}}{2j}$
$\cos(x) = \frac{e^{jx} + e^{-jx}}{2}$

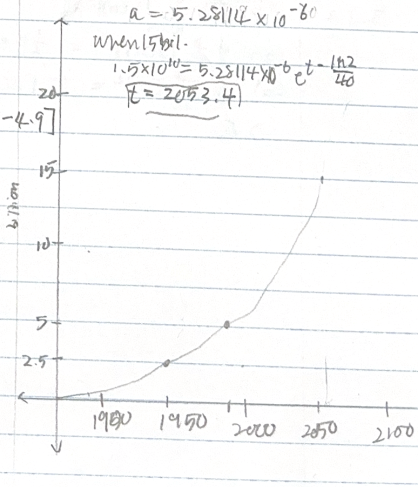
$w_c = \frac{e^{-1} - e}{2j} \cdot \frac{2}{e^{-1}+e}$
$= \underbrace{(-)(\frac{1}{e} - e)(\frac{1}{\frac{1}{e}+e})}_{\tanh(1)} j$
$\underline{\mathrm{Im}[\tan(j)] = \tanh(1)}$

B.1-15. Let $z=a+bj$ be a complex number,
solving $j = z^2$. $j = z^2 = (a^2-b^2) + 2abj$
set re and Im parts: $0 = a^2-b^2$
$1 = 2ab$
Solve for a, b — $a = \frac{1}{\sqrt{2}}$, $b = \frac{1}{\sqrt{2}}$
$\sqrt{j} = z = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}j$ or $\boxed{\frac{1}{\sqrt{2}}(1+j)}$

B.2-1 (a) $5\pi = w_0$. $f_0 = \underline{2.5\text{Hz}}$ $T_0 = \underline{0.4s}$
(b) $\frac{2}{3} = w_0$. $f_0 = \underline{\frac{1}{3\pi}\text{Hz}}$ $T_0 = \underline{3\pi s}$

$T = \frac{1}{f}$
$w_0 = 2\pi f$

B3-2. $p(t) = ae^{bt}$, a, b?
$p(1950) = 2.5$ $p(1990) = 5$
$5 = ae^{1990b}$
$2.5 = ae^{1950b}$
$2 = e^{40b}$, $b = \frac{\log 2}{40}$ ln2
Find a: $5\times10^9 = ae^{1990(\frac{\log 2}{40})}$
$a = 5.28114\times10^{-6}$
when 15bil.
$1.5\times10^{10} = 5.28114\times10^{-6} e^{t\cdot\frac{\ln 2}{40}}$
$\boxed{t = 2053.4}$



B4-2 $\begin{bmatrix}1&2&0\\0&3&4\\5&0&6\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}7\\8\\9\end{bmatrix}$

$\cos(\varphi + wt)$

$\begin{vmatrix}1&2&0\\0&3&4\\5&0&6\end{vmatrix} = 1\cdot\begin{vmatrix}3&4\\0&6\end{vmatrix} - 2\begin{vmatrix}0&4\\5&6\end{vmatrix} + 0 = 18 - 2(-20) = 58$

$D_x = \begin{bmatrix}7&2&0\\8&3&4\\9&0&6\end{bmatrix}$ $|D_x| = 7\begin{vmatrix}3&4\\0&6\end{vmatrix} - 2\begin{vmatrix}8&4\\9&6\end{vmatrix} + 0 = 7(18) - 2[8\cdot6 - 4\cdot9]$
$= 102$

$D_y = \begin{bmatrix}1&7&0\\0&8&4\\5&9&6\end{bmatrix}$ $|D_y| = 1\cdot\begin{vmatrix}8&4\\9&6\end{vmatrix} - 7\begin{vmatrix}0&4\\5&6\end{vmatrix} = 12 + 140 = 152$

$D_z = \begin{bmatrix}1&2&7\\0&3&8\\5&0&9\end{bmatrix}$ $|D_z| = \begin{vmatrix}3&8\\0&9\end{vmatrix} - 2\begin{vmatrix}0&8\\5&9\end{vmatrix} + 7\begin{vmatrix}0&3\\5&0\end{vmatrix} = 2$

$\boxed{x = \frac{D_x}{D} = \frac{102}{58} = \frac{51}{29}}$ $\boxed{y = \frac{D_y}{D} = \frac{76}{29}}$

$\boxed{z = \frac{D_z}{D} = \frac{+2}{58} = \frac{1}{29}}$

FIVE STAR.

$E = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$ $\quad P =$ $\quad 4x|_0^2 = 8 + 4x$

$x|_0^2 = 2$

1.1-7 Find the energies

(a) $\int_0^2 1^2 \, dt + \int_2^3 (-1)^2 \, dt = 3$. x(t) negative value becomes +ve E.

(b) $\int_0^2 (-1)^2 dt + \int_2^3 1^2 dt = 3$. Note: sign change : energy unchanged

(c) $\int_0^3 0 \, dt + \int_3^5 (1) \, dt + \int_5^6 (-1)^2 \, dt = 3$ NOTE: Time shift : energy unchanged.

(d) $\int_0^2 (2)^2 \, dt + \int_2^3 (-2)^2 \, dt = 12$

1.1-2

(a) $\int_0^1 t^2 dt = \frac{1}{3}t^3 |_0^1 = \frac{1}{3}$

(b) $\int_{-1}^0 (-t)^2 dt = \frac{1}{3}t^3 |_{-1}^0 = 0 - (-\frac{1}{3}) = \frac{1}{3}$ - unaffected by sign

$\frac{1}{3}t^2$ (c) $\int_0^1 (-t)^2 dt = \frac{1}{3}t^3 |_0^1 = \frac{1}{3}$ - unaffected by flip

(d) $\int_1^2 t^2 dt = \frac{1}{3}t^3 |_1^2 = \frac{1}{3}$ - unaffected by shift

(e) $\int_0^1 (2t)^2 \, dt = 4\left(\frac{1}{3}t^3\right)|_0^1 = 4/3$ - magnified by 4 when x(t) ×2.

1.1-4 Figure P1.1-4 $\quad P = \frac{1}{2T}\int_{-T}^{T} |x(t)|^2 \, dt$

(a) $-x(t)$

(b) $2x(t)$

(c) $c\,x(t)$

$= \frac{1}{4}\int_{-2}^{2} (t^3)^2 dt$

$= \frac{1}{4}\frac{1}{7} t^7 |_{-2}^{2}$

$= \frac{1}{4}\frac{1}{7}(256)$

$= \frac{64}{7}$

(a) $\frac{1}{4}\int_{-2}^{2} (-t^3)^2 \, dt = \frac{1}{4}\frac{1}{7}t^7|_{-2}^2 = \frac{64}{7}$

(b) $\frac{1}{4}\int_{-2}^{2} (2t^3)^2 \, dt = \frac{1}{7}t^7|_{-2}^2 = \frac{256}{7}$

(c) $\frac{1}{4}\int_{-2}^{2} (ct^3)^2 dt$

$= \frac{1}{4}c^2 \int_{-2}^{2} t^6 \, dt = \frac{1}{7}\frac{1}{4}c^2 t^7 |_{-2}^2 \, dt$

$= \frac{64}{7}c^2$

Changing sign and shifts do not affect power. Signal increases by c^2 when factoring increase by c.

3.1-2 $\quad P = \frac{1}{2N+1}\sum_{n=-N}^{N} |x(n)|^2$

(a) $\frac{1}{6}\sum_{-3}^{3} [x[n]]^2 = \frac{1}{6}[3^2 + 2(2)^2 + 2(1)^2] = 19/6$

(b) $\frac{1}{12}\sum_{-6}^{6} [x[n]]^2 = \frac{1}{12}[2(3)^2 + 2(2)^2 + 2(+1)^2] = 7/3$

(c) $\frac{1}{N_0}\sum_{0}^{N_0-1} a^n = \frac{1}{N_0}\left[\frac{a^{N_0}-1}{a-1}\right] = \frac{a^{N_0}-1}{N_0(a-1)}$