Random Variables (rv)

Random Variables [Ross S4.1]

After an experiment is done, we are often interested in a function of the outcome:

- e.g., sum of two dice rolls
- e.g., number of heads after flipping 10 coins

A function that maps each outcome $s \in S$ to a real number is called a **random variable** [often abbreviated as rv].

Example 8.1: Let $S = \{(1,1),(1,2),\ldots,(6,6)\}$ be outcomes of two dice rolls.

For $s = (a,b)$, if $X(s) = a + b$, then $X(s)$ is a random variable.

We often write X instead of $X(s)$ since s and S are clear from context, or don't matter.

Example 8.2: Toss 3 coins. Let X = # of heads. Then X is a rv that can only take values 0, 1, 2 or 3.

$\{X = 0\} = \{ttt\}$

$\{X = 1\} = \{tth, tht, htt\}$

$\{X = 2\} = \{hht, hth, thh\}$

$\{X = 3\} = \{hhh\}$

and

$$
\begin{aligned}
P[\{X=0\}] = P[X=0] &= 1/8 \\
P[X=1] &= 3/8 \\
P[X=2] &= 3/8 \\
P[X=3] &= 1/8
\end{aligned}
$$

Note: Since $\{X=0\}, \{X=1\}, \{X=2\}, \{X=3\}$ are disjoint and cover all possible outcomes for X:

$$\sum_{i=0}^{3} P[X=i] = 1.$$

Example 8.3: Let E and F be independent events with:

$$P[E] = 0.1, \qquad P[F] = 0.2$$

Let $Y =$ # events that have occured. Then

$$
\begin{aligned}
&P[Y=0] = P[E^cF^c] = P[E^c]P[F^c] = 0.9 \times 0.8 \\
&P[Y=1] = P[EF^c \cup E^cF] = P[EF^c] + P[E^cF] = 0.1 \times 0.8 + 0.9 \times 0.2 \\
&P[Y=2] = P[EF] = P[E]P[F] = 0.1 \times 0.2
\end{aligned}
$$

Example 8.4: A flipped coin has probability p of being heads. We flip the coin until a head occurs, up to a max of n flips. Let $Z =$ # of flips. Then

$$
\begin{aligned}
P[Z=1] &= P[h] = p \\
P[Z=2] &= P[th] = (1-p) \times p \\
P[Z=3] &= P[tth] = (1-p)^2 \times p \\
P[Z=n-1] &= P[n-2 \text{ tails followed by heads }] = (1-p)^{n-2} \times p \\
P[Z=n] &= P[n-1 \text{ tails followed by anything }] = (1-p)^{n-1}
\end{aligned}
$$