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Example 2.2: If we toss two 6-sided dice, then

$$S = \{(i, j) \in \mathbb{Z}^2 \mid i = 1, 2, \ldots, 6, j = 1, 2, \ldots, 6\} \tag{2.1}$$

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Example 2.5: Two persons will meet. Each will arrive with a delay that is between 0 and 1 hour:

$$S = \{(x, y) \in \mathbb{R}^2 \mid 0 \leq x \leq 1, 0 \leq y \leq 1\}.$$

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is the event that both coins come up identical.

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Example 2.8: In roulette, $\text{even} = \{2,4,6,\ldots,36\}$ is called an even outcome and $\text{odd} = \{1,3,5,\ldots,35\}$ is called an odd outcome.

Example 2.9: In Example 2.5, the event that both arrive within 1/4 hour of each other is:

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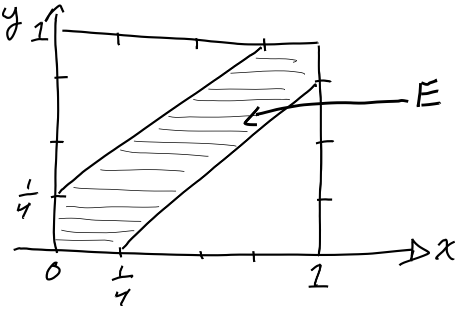

Definition 2.2: For 2 events E and F:

- $E \cup F$ is the event that either E <u>or</u> F occurs

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- $E \cap F$ is the event that both E <u>and</u> F occur
  $E \cap F = \{x \in S \,|x \in E \text{ and } x \in F\}$

  We also write EF.

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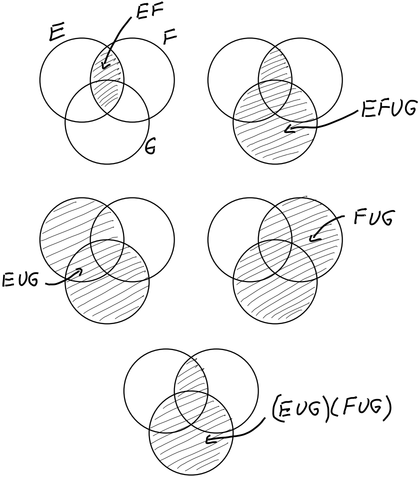
- Given F and $E_1, E_2, \ldots, E_n$, if
  - $E_1, E_2, \ldots, E_n$ are disjoint (i.e., $E_iE_j = \emptyset$ for $i \neq j$)
  - $F = \cup_{i=1}^{n} E_i$

then $E_1, E_2, \ldots, E_n$ are said to **partition** F.

Properties:

Commutative Laws:	$E \cup F = F \cup E$	$EF = FE$
Associative Laws:	$(E \cup F) \cup G = E \cup (F \cup G)$	$(EF)G = E(FG)$
Distributive Laws:	$(E \cup F)G = EG \cup FG$	$EF \cup G = (E \cup G)(F \cup G)$

Example 2.10: Venn diagram interpretation of $EF \cup G = (E \cup G)(F \cup G)$:



DeMorgan's Laws:

$$\left(\bigcup_{i=1}^{n} E_i\right)^c = \bigcap_{i=1}^{n} E_i^c$$

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Then $x \notin \cup_i E_i$

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Step 2: We will show $\cap_i E_i^c \subset (\cup_i E_i)^c$

Let $x \in \cap_i E_i^c$

Then, for each i, $x \in E_i^c$

Then, for each i, $x \notin E_i$

Then, $x \notin E_1 \cup E_2 \cup \cdots \cup E_n$

Then, $x \in \underbrace{(E_1 \cup E_2 \cup \cdots \cup E_n)^c}_{(\cup_i E_i)^c}$

Home Exercises: Verify other properties with Venn diagrams; prove 2nd DeMorgan Law.

Given two sets A and B, the Cartesian product $A \times B$ is:

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$\{0,1\}^{10} = \{$all binary strings of length 10$\}$