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The GARCH-EVT-Copula Approach to Investigating Dependence and Quantifying Risk in a Portfolio of Bitcoin and the South African Rand

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Abstract: This study uses a hybrid model of the exponential generalised auto-regressive conditional heteroscedasticity (eGARCH)-extreme value theory (EVT)-Gumbel copula model to investigate the dependence structure between Bitcoin and the South African Rand, and quantify the portfolio risk of an equally weighted portfolio. The Gumbel copula, an extreme value copula, is preferred due to its versatile ability to capture various tail dependence structures. To model marginals, firstly, the eGARCH(1, 1) model is fitted to the growth rate data. Secondly, a mixture model featuring the generalised Pareto distribution (GPD) and the Gaussian kernel is fitted to the standardised residuals from an eGARCH(1, 1) model. The GPD is fitted to the tails while the Gaussian kernel is used in the central parts of the data set. The Gumbel copula parameter is estimated to be $\alpha = 1.007$, implying that the two currencies are independent. At 90%, 95%, and 99% levels of confidence, the portfolio's diversification effects (DE) quantities using value at risk (VaR) and expected shortfall (ES) show that there is evidence of a reduction in losses (diversification benefits) in the portfolio compared to the risk of the simple sum of single assets. These results can be used by fund managers, risk practitioners, and investors to decide on diversification strategies that reduce their risk exposure.

Keywords: Gaussian kernel; generalised Pareto distribution; Gumbel copula; tail dependence; diversification effects; value at risk; expected shortfall

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1. Introduction

Systematic risk impacts the entire market or a large sector of the market, not just a single asset class or sector of industry. Examples of systematic risk include natural disasters, weather events, inflation, changes in interest rates, and global financial crises that affect all assets or the economy as a whole. It is known to be undiversifiable. Conversely, unsystematic risk refers to risk that is not common to all asset classes or wider markets. This kind of risk is specific to an individual asset or sector of industry. It can be reduced or eliminated by diversifying (spreading across asset classes and sectors of industry that are not positively correlated) one's investments.

The diversification effect is the notion that by investing in different (usually not positively correlated) classes of financial assets, the investor's overall portfolio risk is lowered (Evans and Archer 1968). By investing in these asset classes, sectors of industry, or maturities, an investor or trader is expected to be less likely to experience market shocks that impact a single asset/sector in their portfolio (Ford 1998). A well-diversified portfolio is expected to reduce portfolio risk for a given level of portfolio mean return (Lee et al. 2020).

Risk managers are required to search for safe haven assets as a means of reducing unsystematic risk. Baur and Lucey (2010) defined a safe haven asset "as one that is uncorrelated or negatively correlated with another asset or portfolio in time of market stress". Traditionally, gold has been viewed as a safe haven asset for most risky assets. However, Malandala and Olaomi (2020) found a strong positive correlation between gold, platinum prices, and the Rand exchange rate, casting doubt on gold's ability to act as a safe haven asset for the Rand.

On the other hand, several studies have shown that Bitcoin can act as a safe haven instrument for emerging economy markets, especially during a crisis period like the recent COVID-19 pandemic (Yan et al. 2022; Cocco et al. 2022). Is Bitcoin a safe haven asset for the Rand users? This study seeks to use a copula approach to provide a scientific answer to this question.

Investors and traders in the currency market in South Africa are exposed to both systematic and unsystematic risk. A sizeable number of traders are known to be moving their investment between the South African Rand and other fiat currencies, as well as cryptocurrencies like Bitcoin. This study seeks to quantify the diversification effects (DE), merits, and demerits of evenly spreading one's investment between Bitcoin and the South African Rand from a risk analytics point of view.

The Value at risk (VaR) and expected shortfall (ES) are the most widely used financial risk measures. Hotta et al. (2007) posits that VaR has become the standard measure used by investors and researchers to estimate the market risk of assets and portfolios due to its conceptual simplicity, ease of computation, and applicability. According to Daníelsson et al. (2013), VaR is not sub-additive and, hence, an incoherent risk measure.

The crucial stage in computing VaR lies in selecting the appropriate statistical distribution that describes the asset's loss distribution (growth rates), which is known to be non-normal, with volatility clustering, fat tails, and asymmetric conditional volatility. Following the global financial crisis of 2008, statistical models that reliably capture the fat tail property of financial growth rates are preferred in VaR modelling (Daníelsson et al. 2013). Extreme value theory (EVT) is a branch of statistics that specialises in describing the fat tail distribution of growth rates (Nadarajah and Chan 2016). The generalised Pareto distribution (GPD) is one of the models that belong to the EVT family of models.

Volatility clustering implies that the growth rate data are not identically and individually distributed (i.i.d), a feature that is required when one fits data to EVT models. To address this, McNeil and Frey (2000) proposed a two-stage approach to modelling volatility data. The first stage is to fit the data into a GARCH family model. GARCH models were developed to capture volatility clustering. The exponential GARCH (eGARCH) model captures volatility clustering and accommodates the presence of asymmetric conditional volatility (leverage effects) (Nelson 1991). EGARCH model residuals are extracted, standardised, and fitted to a mixture model where the tails are modelled using the generalised Pareto distribution, and the Gaussian kernel is used to model the central parts of the standardised residuals.

In financial risk management, one of the main challenges of estimating portfolio risk and diversification effects (DE) is the aggregation of individual asset risk factors. The problem becomes much more challenging when modelling fully dependent random variables or when one does not know what joint distribution to use to determine the dependence between the given individual risk factors. A copula is generally used to model the dependence between the individual risk factors. The main objective of this paper is to use the versatile Gumbel copula to determine the dependence structure between Bitcoin and the Rand and subsequently estimate portfolio risk. Investors and traders seek diversification to reduce the unsystematic risk inherent in investing in risky assets. This study uses the eGARCH-Gaussian kernel density-EVT-Archimedean copula model to estimate the portfolio risk and account for the DE of a portfolio of two equally weighted currency assets: Bitcoin and the South African Rand, both of which are measured against the United States Dollar (USD). The two currencies are assumed to be equal; however, this is optional, and they can be adjusted freely (Clemente and Romano 2005). The eGARCH-Gaussian kernel density-EVT-copula model is the model considered in this study due to its ability to account for the volatility clustering, asymmetric conditional volatility, and fat tail features prevalent in financial time series data.

2. Literature Review

A copula is a mathematical function that joins or couples a multidimensional distribution function with its single-dimensional marginal distribution functions (Sklar 1973; Nelsen 2007). Copulas have gained popularity in financial econometric research due to their ability to capture dependencies of assets with different distributional properties (Nelsen 1996). The main reason for their popularity is their ability to model multivariate distributions incorporating copulas coupled with convenient univariate marginal statistical distribution models. Furthermore, copulas can capture non-parametric dependence between non-normal random variables (Trivedi and Zimmer 2007).

Cryptocurrencies are virtual currencies that are traded without the regulations of a reserve bank and exist only in digital form. Bitcoin (BTC) is the most traded cryptocurrency in terms of volume. Since Bitcoin is not backed by any central bank or government, its users and traders are expected to be exposed to a higher risk. Despite the South African Rand (ZAR) being a central bank-backed currency, it is also considered risky since it is an emerging country's currency. Following its independence in 1994, the South African Reserve Bank (SARB) abandoned the fixed Rand–dollar exchange rate, adjusted weekly, in favour of a purely floating (flexible) exchange rate (Van Der Merwe 1996). This flexible exchange rate model is also characterised by high volatility (risk) and is sometimes expected to behave like a random walk (Eun et al. 2012). Both Bitcoin and the Rand are perceived to be risky assets.

By computing the skewness and the kurtosis of empirical cryptocurrency return distributions, Schmitz and Hoffmann (2021) concluded that cryptocurrencies are not normally distributed. Acereda et al. (2020) concluded that the returns of Bitcoin and other digital currencies follow a heavy-tailed distribution. Börner et al. (2021); Gkillas and Siriopoulos (2018); Omari and Ngunyi (2021); and Osterrieder and Lorenz (2017) applied extreme value theory (EVT) to model the extreme tail behaviour and derived overall risk characteristics of cryptocurrencies. All found that cryptocurrencies like Bitcoin showed fat-tailed behaviour. Gkillas and Katsiampa (2018) used EVT to compute the value at risk (VaR) and the expected shortfall (ES) as a measure of tail risk for five cryptocurrencies. Backtest results confirmed the suitability of EVT models in describing cryptocurrencies and other risky assets, hence the consideration of the GPD and an extreme value Gumbel copula to capture extreme risk characteristics in Bitcoin and the Rand. Tinungki et al. (2023) confirmed the adequacy of the Gumbel copula in modelling dependences among assets that possess heavy tails.

Regardless of the high volatility phenomena associated with Bitcoin, investors and researchers still consider it a noble investment and safe haven asset during economic crises. Jeribi and Fakhfekh (2021) investigated the best portfolio hedging strategy for a portfolio comprising a set of five cryptocurrencies and several traditional financial assets. They concluded that adding a position in cryptocurrency assets to a diversified portfolio significantly reduces the portfolio's overall risk and can offer opportunities for portfolio diversification. Huang et al. (2021) examined the performance of nine cryptocurrency asset categories and concluded that most of the categories provide diversification benefits, depending on an investor's risk aversion.

Cocco et al. (2022) studied the safe haven properties of Bitcoin and Ether against thirteen of the major stock market indices using daily data during the COVID-19 pandemic. Their findings confirmed Bitcoin's status as a safe haven asset during the pandemic period. A similar conclusion was reached by Yan et al. (2022), though they went further, describing Bitcoin as the new gold with stronger safe haven properties during economic turmoil. The same cannot be said about the South African Rand.

Several studies have recommended the GARCH-EVT-copula approach to investigate dependence among financial assets and quantify portfolio risk. Bruhn and Dietmar (2022) used the GARCH-EVT-copula model to investigate dependence among cryptocurrencies and found a strong positive dependence between the two largest cryptos, namely Bitcoin and Ethereum, and, hence, no diversification effects. Allen (2022) found a strong correlation between cryptocurrency (Bitcoin and Ethereum) and S&P 500, and, hence, no diversification benefits for S&P 500 investors using Bitcoin and Ethereum.

Subramoney et al. (2023) applied extreme value mixture models to model the tail risk of Bitcoin and Ethereum. They fitted mixture models of the Gaussian distribution with the generalised Pareto tails (GPD-Gaussian-GPD) and the kernel distribution estimator with the generalised Pareto tails (GPD-KDE-GPD) models in their study. They confirmed the adequacy of the two mixture models using Kupiec's likelihood ratios for the long positions.

Multivariate marginal statistical models based on normal distribution are not sufficient in the modelling of cryptocurrencies and other financial time series due to the presence of heavy tails (extreme risk) and tail dependence in the asset growth rates, hence the adoption of extreme value copula in this study. Furthermore, the use of the mixture model (GPD-Gaussian kernel-GPD), with GPD being used to model tails of marginal distribution, is motivated by the model's ability to sufficiently capture the fat tail characteristic of both exchange rates. The Gumbel copula is both an Archimedean and an extreme value copula and it is tractable.

3. Methodology

The concept of tail dependence deals with the joint probability of extreme events that can occur in the upper tail or lower tail and other extremities of the bivariate distribution. In this study, the extreme value Gumbel copula is used to analyse an equally weighted portfolio of two currencies.

3.1. The Gumbel Copula

The Gumbel copula is both an Archimedean and extreme value copula that effectively captures the upper tail dependence. Currency time series are known to exhibit fat tails and, therefore, the choice was made to use a Gumbel copula to capture the tail dependence of Bitcoin and the Rand (Danielsson 2011; Gkillas and Katsiampa 2018).

The Gumbel copula generator function is given by $\varphi(u) = (-\ln(u))^{\alpha}$, hence

$$\varphi^{-1}(u) = \exp -u^{-\frac{1}{\alpha}}.$$

where u represents standard uniform (0, 1) variates that are assumed to be i.i.d observations.

The bivariate Gumbel copula can be mathematically presented as follows:

$$C(\alpha; u_1, u_2) = \exp\left(-\left[\left(-\ln u_1\right)^{\alpha} + \left(-\ln u_2\right)^{\alpha}\right]^{\frac{1}{\alpha}}\right) \tag{1}$$

where α is the Gumbel copula parameter.

A Gumbel copula is preferred in this study due to its versatile properties, as shown in Figure 1. The parameter α estimates the degree of dependency. When $\alpha = 1$, independence is obtained, implying that the Gumbel copula is reduced to an independent copula, and when $\alpha \to \infty$ the Gumbel copula converges to perfect positive dependence. When $\alpha \to -\infty$, a rotated Gumbel copula can be used to model the margins. The rotation of the copula enables the modelling of negative dependence (Brechmann and Schepsmeier 2013).

The bivariate rotated Gumbel copula can be mathematically defined as follows:

$$C(\alpha; u_1, u_2) = u_1 + u_2 - 1 + \exp\left\{-\left[(-\ln(1-u_1))^{\alpha} + (-\ln(1-u_2)^{\alpha}\right]^{\frac{1}{\alpha}}\right\}. \tag{2}$$

The Gumbel copula parameter α, and the Kendall Tau τ are linked using the following equation:

$$\tau = 1 - \frac{1}{\alpha}. \tag{3}$$

The Gumbel copula's upper λ_U and lower λ_L tails are given theoretically as follows:

$$\begin{cases} \lambda_U = 2 - \frac{1}{2^{\alpha}} \\ \lambda_L = 0. \end{cases} \tag{4}$$

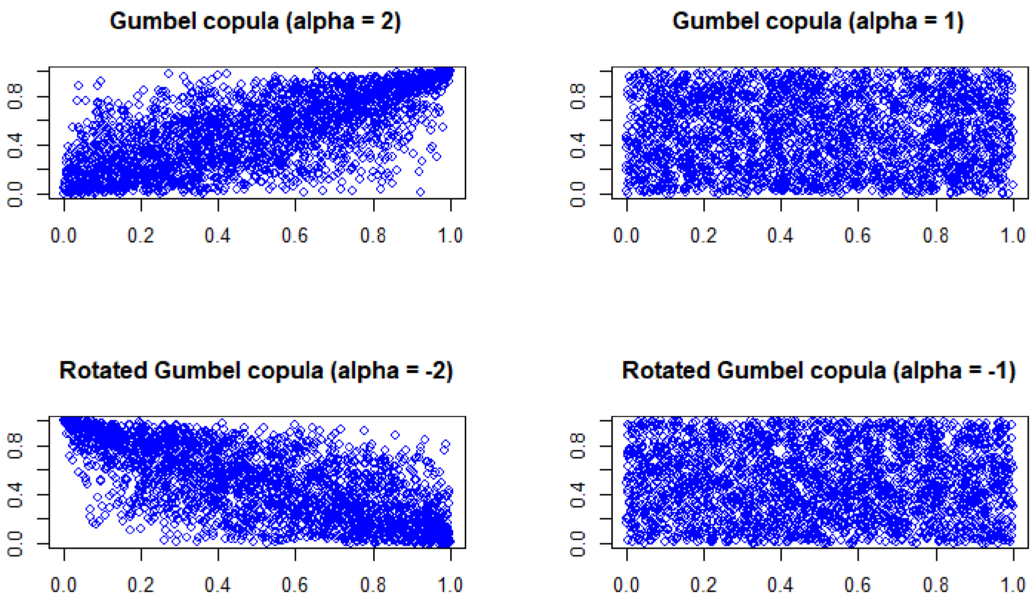

Figure 1. Scatter plots of simulated data of Gumbel copula ($\alpha = |2|$) with positive dependence (**top left**), no dependence ($\alpha = \pm 1$) (**top right** and **bottom right**), and negative dependence (**bottom left**).

3.2. Parameter Estimation

The Gumbel copula has one dependence parameter, α, that must be estimated. The inference for margins (IFM) approach is used to estimate this parameter.

3.3. Modelling Marginal Distribution

Financial asset time series data are known to portray some stylised facts like heteroscedasticity, fat tails, and clustering. A two-step approach to obtaining the marginal distributions of the currency growth rates is employed here. Firstly, the eGARCH(1, 1) model is fitted to capture clustering in both of the currencies' growth rates, and the residuals are extracted and standardised before the marginals of each currency growth rate are modelled.

3.4. The GARCH Model

The eGARCH $(p,\ q)$ model that describes the volatility dynamics of the exchange rates of Bitcoin and the Rand is mathematically defined as follows:

$$\ln\left(\sigma_t^2\right) = \omega + \sum_{i=1}^{p} \alpha_i [\frac{|\varepsilon_{t-i}|}{|\sigma_{t-i}|} + \gamma_i \frac{\varepsilon_{t-i}}{\sigma_{t-i}}] + \sum_{j=1}^{q} \beta_j \sigma_{t-1}^2). \tag{5}$$

An eGARCH (1, 1) model can be expressed as follows:

$$\ln\left(\sigma_t^2\right) = \omega + \alpha_1 \left(\frac{|\varepsilon_{t-1}|}{|\sigma_{t-1}|}\right) + \gamma_1 \left(\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right) + \beta_1 \ln\left(\sigma_{t-1}^2\right), \tag{6}$$

where $\omega,\ \alpha_1,\ \beta_1$ are model coefficients, and γ_1 is the leverage effect and is assumed to be negative in real applications.

3.5. The Generalised Pareto Distribution (GPD)

The second step of modelling marginal distribution involves fitting the semi-parametric cumulative distribution function to the standardised residuals obtained from the eGARCH(1, 1) model. The peak over the threshold (POT) approach is used to fit the GPD model to both lower and upper tails, while a kernel density estimation method (with Gaussian density as the kernel function) is used for the interior part of the distribution. Balkema and de Haan (1974) and Pickands (1975) showed that for large enough thresholds, u, the POT function of exceedances (observations) above the threshold can be estimated by the GPD. The GPD is defined as follows:

$$G_{\xi,\beta}(\varepsilon) = \begin{cases} 1 - \left(1 + \frac{\xi(\varepsilon - u)}{\beta}\right)^{\frac{-1}{\xi}} & \text{if } \xi \neq 0 \\ 1 - \exp\left(-\left(\frac{\varepsilon - u}{\beta}\right)\right) & \text{if } \xi = 0 \end{cases}, \tag{7}$$

where ξ is the extreme value index (EVI), β is the scale parameter, and u is the threshold. The value of ξ shows how heavy the tail is, with a bigger positive value indicating a heavy tail. When ξ is negative, the tail is short (bounded). $\xi = 0$ gives indicates a light tail.

To fit the GPD model, a threshold u must be selected. McNeil and Frey (2000) posited that a threshold that gives about 100 observations for fitting the GPD is sufficient for two-stage EVT modelling. McNeil and Frey (2000), Karmakar and Shukla (2015), Totić and Božović (2016), Li (2017), and Huang et al. (2017) used the 90th quantile of the loss distribution as a threshold. Therefore, in this research study, a 10th and a 90th quantile threshold of the standardised residual data points are used for the lower and upper tails, respectively, and the Gaussian kernel density estimator is used in the interior part. The semi-parametric CDF for the distribution is given by the following:

$$F_i(\varepsilon_i) = \begin{cases} \frac{N_{u_i^L}}{n}\left(1 + \xi_i^L \frac{u_i^L - \varepsilon_i}{\beta_i^L}\right)^{\frac{-1}{\xi_i^L}} & \varepsilon_i < u_i^L \\ \varphi(\varepsilon_i) & u_i^L < \varepsilon_i < u_i^R \\ 1 - \frac{N_{u_i^R}}{n}\left(1 + \xi_i^R \frac{\varepsilon_i - u_i^R}{\beta_i^R}\right)^{\frac{-1}{\xi_i^R}} & \varepsilon_i > u_i^R \end{cases} \tag{8}$$

where u_i^L and u_i^R are the lower and upper threshold values, respectively, $\varphi(\varepsilon_i)$ is the empirical Gaussian distribution of the interval $[u_i^L, u_i^R]$, n is the number of ε_i (residuals), $N_{u_i^L}$ is the number of residuals whose value is less than u_i^L, and $N_{u_i^R}$ is the number of residuals whose value is greater than u_i^R.

3.6. The Anderson–Darling (A–D) Test

The Anderson–Darling test for goodness of fit is used to confirm whether the data sample comes from a specific distribution (Stephens 1974). The null hypothesis claims that the data follow a specified distribution. The A–D test statistic A^2 is defined as follows:

$$A^2 = -n - \sum_{j=1}^{n} \frac{(2j-1)}{n}\left[\ln F(y_j) + \ln\left(1 - F(y_{n+1-j})\right)\right], \tag{9}$$

where $F(y_j)$ is the specified theoretical CDF and $\{y_1 < \ldots < y_n\}$ is the ordered sample data.

3.7. Value at Risk and Expected Shortfall

To quantify the diversification of the financial assets being studied, this research paper shall use *VaR* and *ES*.

Let $L = L_1 + L_2$ be the random variable that represents the total loss of a two-asset portfolio (in the bivariate case) with F_L as the distribution function. Then, *VaR* is defined as follows:

$$VaR_\alpha(L) = \inf\{I\epsilon\mathbb{R} : F_L(I) \geq \alpha\} = F_L^{-1}(\alpha), \tag{10}$$

where α is the confidence level and *VaR* is the quantile $(1-\alpha)$ of the loss variable, such that the probability of having a loss greater than that determined by *VaR* will be $(1-\alpha)$.

$$ES_\alpha(L) = \frac{1}{1-\alpha}\int_\alpha^1 VaR_u(L)\, du \tag{11}$$

3.8. Modelling Procedure

The modelling procedure follows a seven-step process, as follows.

First, fit the univariate eGARCH(1, 1) model to each currency's growth rate and obtain the standardised residuals.

1. Fit the eGARCH(1, 1) to the currency growth rate data and obtain the standardised residuals.
2. Model marginals by fitting a mixture model (semi-parametric empirical CDF) to each standardised residual. Firstly, estimate the tails of the distribution by applying extreme value theory to those residuals that fall in both tails. Specifically, find upper and lower thresholds such that 10% of the residuals are reserved for each tail. Then, fit the amount by which the extreme residuals in each tail fall beyond the associated threshold to a parametric GPD by maximum likelihood. Then use the Gaussian kernel estimation for the remaining central part.
3. The standardised residuals obtained in step (1) are transformed into standard uniform variates (0, 1) using the probability-integral transformation (PIT).
4. Fit a Gumbel copula model and estimate the dependence parameter using the transformed uniform variates (from step (3)).
5. Simulate N times from the estimated copula model to convert to N standardised residuals using the Inference Function for Margins (IFM) estimation for each model.
6. Use the inverse of the estimated marginal distribution function (from step 2) to obtain new standardised residuals for use in quantifying risk measures
7. Lastly, compute the portfolio VaR (using Equation (9)) and ES (using Equation (10)) of the equally weighted portfolio and the quantities to find the diversification effects.

3.9. Diversification Effects (DE)

According to Piwcewicz (2005), El-Gamal et al. (2006), and Yoshiba (2015), the DE formula is given as follows:

$$\mathrm{DE} = \frac{Simple\ Sum\ VaR/ES \quad - Aggregate\ VaR/ES}{Simple\ Sum\ VaR/ES} \times 100\%. \tag{12}$$

The formula above will be used to estimate the DE of a portfolio.

The simple sum *VaR*/*ES* is the sum of the *VaR* of the assets, i.e., for two assets, *A* and *B*, the simple sum *VaR* can be calculated using the following formula:

$$Simple\ Sum\ VaR = VaR(A) + VaR(B). \tag{13}$$

A similar formula can be used to compute simple sum *ES*. The aggregate *VaR* is the portfolio *VaR* estimated using Equations (9) and (10).

4. Data Analysis

The exchange rate data used in this research were obtained from the investing.com/currencies website, which serves the financial industry. R (R Core Team 2021), RStudio

(RStudio Team 2022), copula (Hofert et al. 2023), spd (Ghalanos 2022), and fitdistrplus (Delignette-Muller and Dutang 2023) statistical packages were used for the analysis.

The growth rates were calculated and used for modelling. The formula used to calculate growth rate is:

$$y_t = \frac{S_t}{S_{t-1}},$$

where S_t and S_{t-1} are today's and the previous day's adjusted values of Bitcoin and Rand exchange rates, both against the US dollar. The statistical marginal distribution models were fitted with the growth rates calculated from the daily adjusted closing prices from 1 January 2015 to 30 June 2021.

Table 1 shows the descriptive statistics, normality, and stationarity tests for the daily growth rates of Bitcoin and the Rand. The p-values of the normality and stationarity tests are shown in brackets. The Jarque–Bera test rejects the null hypothesis of normality at a 5% level of significance, suggesting the presence of heavy tails in the distributions of both currencies' growth rate series. The preliminary diagnosis suggests that that the rates are skewed and have excess kurtosis. The augmented Dickey–Fuller test (ADF) and Phillips–Perron test (PP) show that, at a 1% level of significance, the null hypothesis of a unit root is rejected, and it can be concluded that both exchange rate series are stationary.

Table 1. Descriptive statistics.

	Bitcoin Growth Rates	**Rand Growth Rates**
Observation	2372	2372
Mean	0.8028	0.5002
Maximum	1.2723	1.0493
Minimum	0.6082	0.4560
Skewness	−0.1903504	0.2957331
Kurtosis	8.623871	3.88743
Jarque–Bera Test (p-value)	< 0.001	< 0.001
ADF Test (p-value)	0.01	< 0.01
PP Test (p-value)	< 0.01	< 0.01

5. Marginal Distribution

A two-stage approach is employed as discussed in the Methodology section. The first stage involves fitting data to the eGARCH(1, 1) with Student's t distributed errors.

Table 2 presents the optimal parameters of the fitted model. The leverage effects parameter (γ_1) is negative, indicating that negatives (losses) have more impact than positives (gains).

Table 2. Parameter estimates of eGARCH(1, 1) with Student's t residual distribution.

Parameter	**Bitcoin**	**Rand**
ω	−7.95668	−8.019929
α_1	10.00000	9.999999
β_1	0.92921	0.009826
γ_1	−9.99946	−9.578614
$shape$	100.0000	10.0000

The second stage involves extracting and standardising residuals from the eGARCH(1, 1). A mixture model of GPD-Gaussian kernel-GPD is defined in Equation (7). The maximum likelihood estimation procedure is used to estimate the parameter of the generalised Pareto distribution. Table 3 summarises the output of the second stage of the marginal distribution modelling.

Table 3. Parameter estimates of eGARCH(1, 1)-EVT(GPD).

Parameter	Bitcoin	Rand
Upper Tail		
Exceedances	238	238
u	0.913	0.504
ξ	0.17656	0.05749
β	0.03068	0.00480
Lower Tail		
Exceedances	238	238
u	0.789	0.502
ξ	0.11106	0.06415
β	0.03025	0.00418

The extreme value index (EVI), ξ, measures the fatness of the tails. A high positive EVI implies that the tails are very heavy (fat), while a zero EVI suggests that the tails are light and extremes decay exponentially. A negative EVI implies that the tails are bounded.

As presented in Table 3, the EVIs (ξ) for both Bitcoin and the Rand are positive, confirming the presence of fat tails of varying degrees in both growth rates.

Figure 2 shows the mixture model's CDF and PDF plots of the Bitcoin marginals. The empirical CDF effectively traces the lines of best fit, implying that the mixture model successfully captures the distribution of the Bitcoin growth rate data.

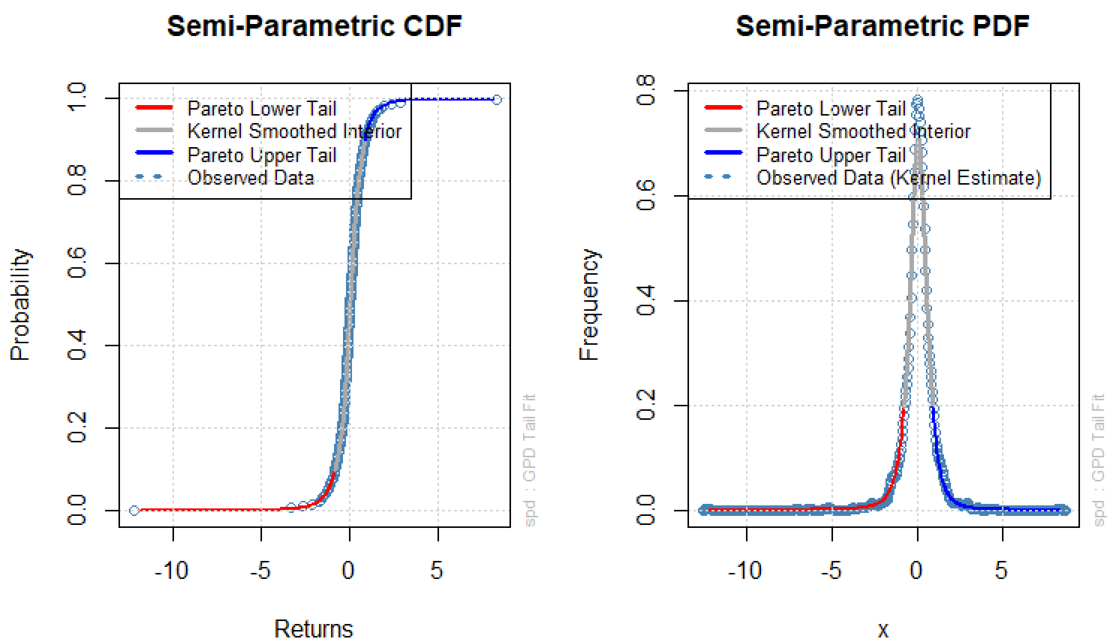

Figure 2. The empirical mixture model (semi-parametric) CDF and PDF plots of the Bitcoin standardised residuals.

Figures 3 and 4 show the diagnostic plots for the upper and lower tails of the Bitcoin residuals. Observations lie along the lines of best fit, confirming the suitability of the GPD model in describing the upper and lower tails of the Bitcoin data sets.

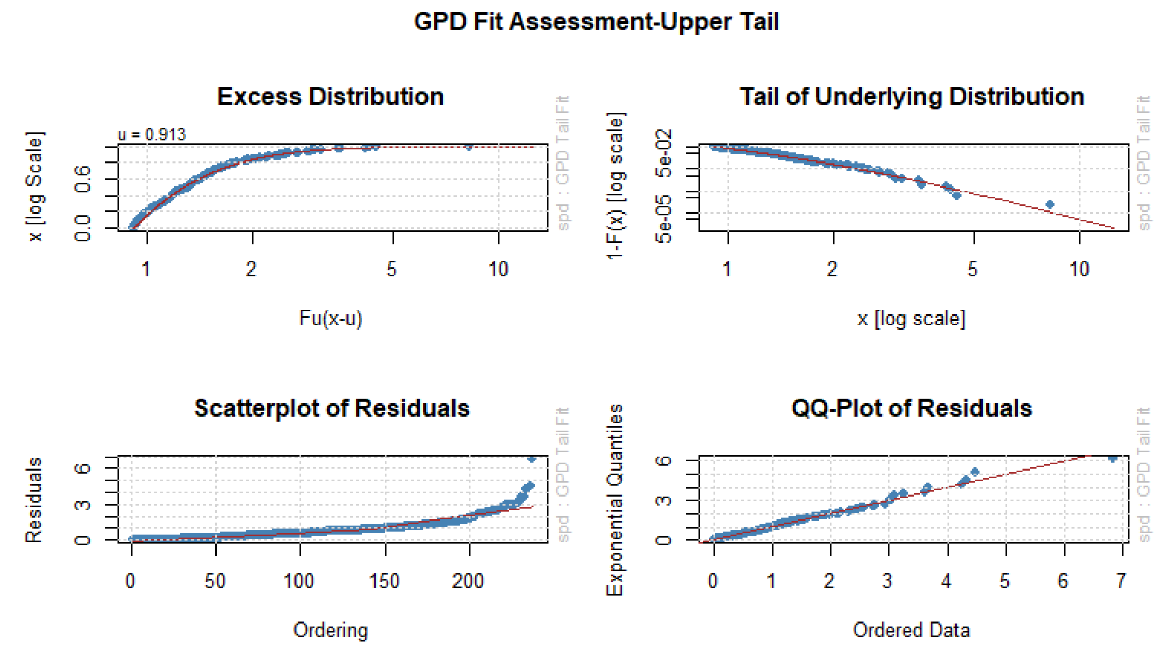

Figure 3. Diagnostic plots for the upper tails of the Bitcoin standardised residuals fitted using GPD.

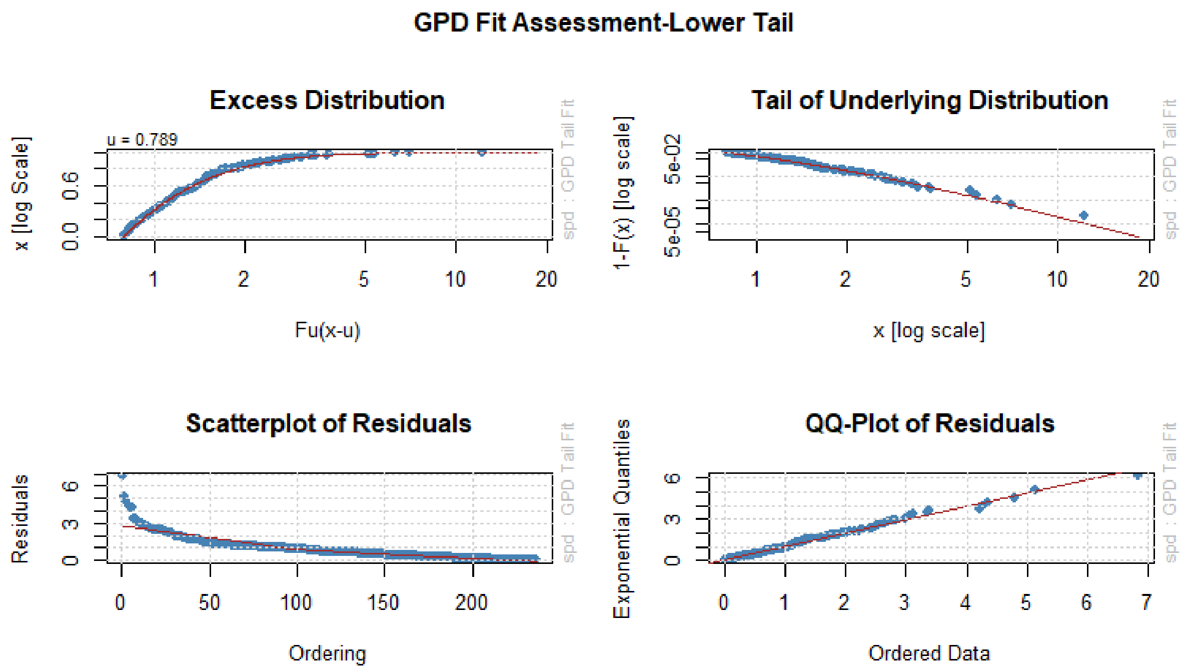

Figure 4. Diagnostic plots for the lower tails of the Bitcoin residuals fitted using GPD.

Figure 5 shows the mixture model's CDF and PDF plots of the Rand residuals' marginals. The empirical CDF successfully traces the lines of best fit, implying that the mixture model effectively describes the distribution of the Rand growth rate data.

Figures 6 and 7 show the diagnostic plots for the upper and lower tails of the Rand residuals. The plots lie along the lines of best fit, confirming the suitability of the GPD model in capturing and describing the upper and lower tails of the Rand growth rates. Risk needs to be modelled accurately in the tails to avoid the problems associated with wrong models that may have led to the GFC of 2008.

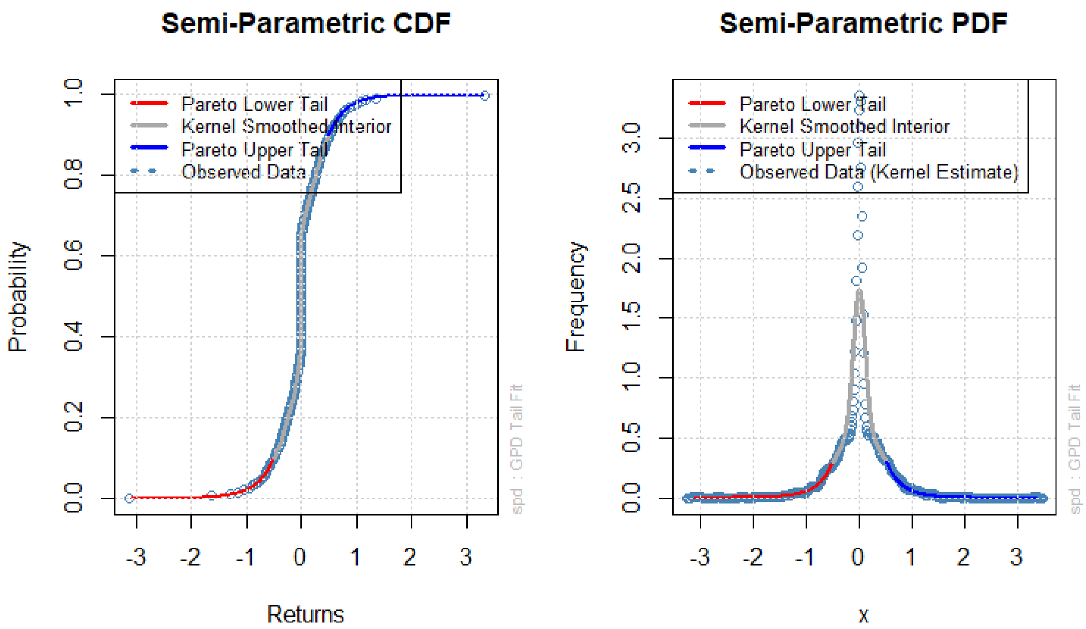

Figure 5. The empirical mixture model (semi-parametric) CDF and PDF plots of the Rand standardised residuals.

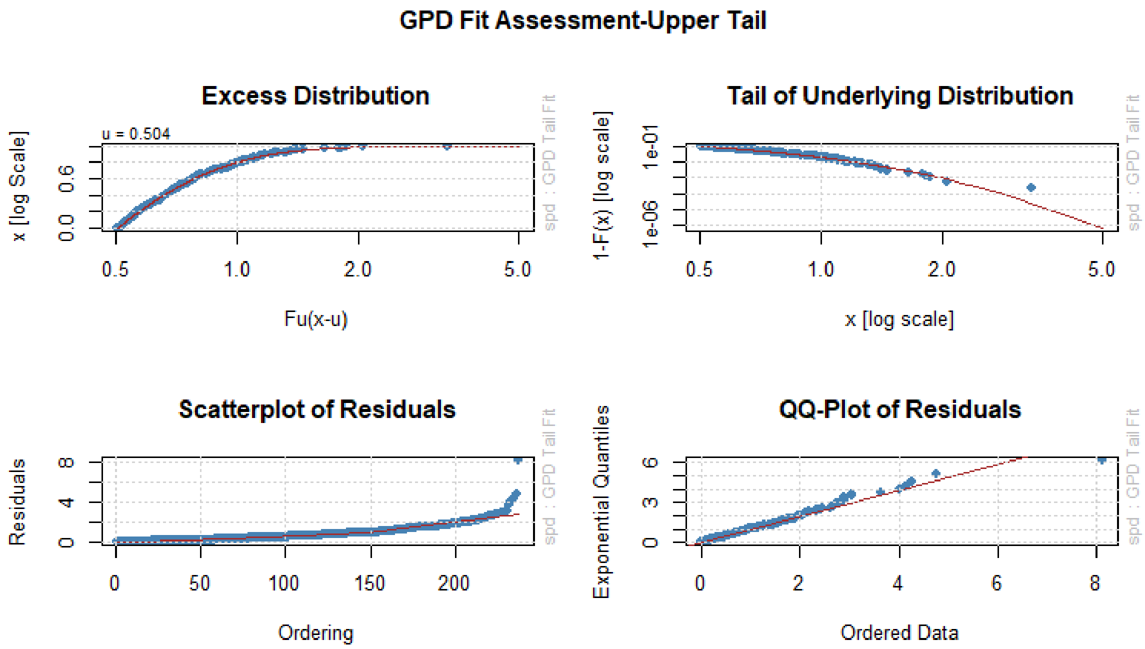

Figure 6. Diagnostic plots for the upper tails of the Rand residuals fitted using GPD.

Since all *p*-values in Table 4 are greater than 0.05, the mixture model's adequacy is confirmed at a 5% significance level. Therefore, the mixture model of GPD-Gaussian kernel-GPD adequately describes the marginal distribution of each currency asset.

Table 4. Anderson–Darling goodness of fit test.

	Bitcoin	**Rand**
p-value	0.0547	0.1704

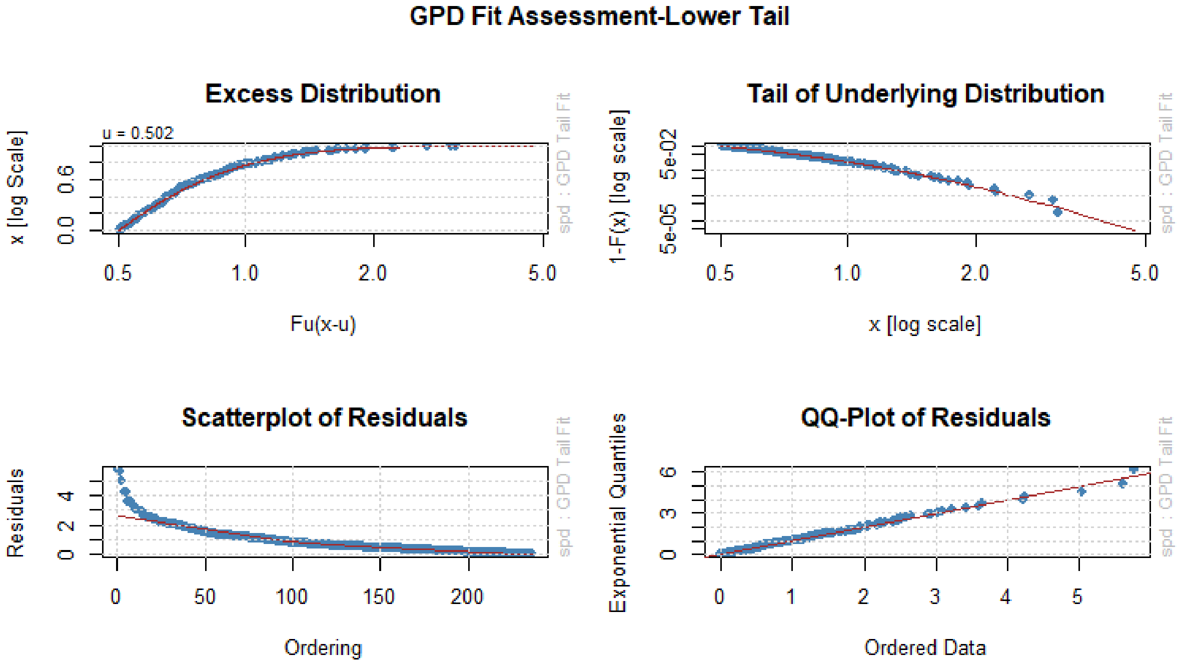

Figure 7. Diagnostic plots for the lower tails of the Rand residuals fitted using GPD.

6. Portfolio Risk Measures

To fit a copula, standardised residuals of growth rates are converted to marginal univariates using probability transform, and the bivariate distribution of the marginals is presented below in Figure 8.

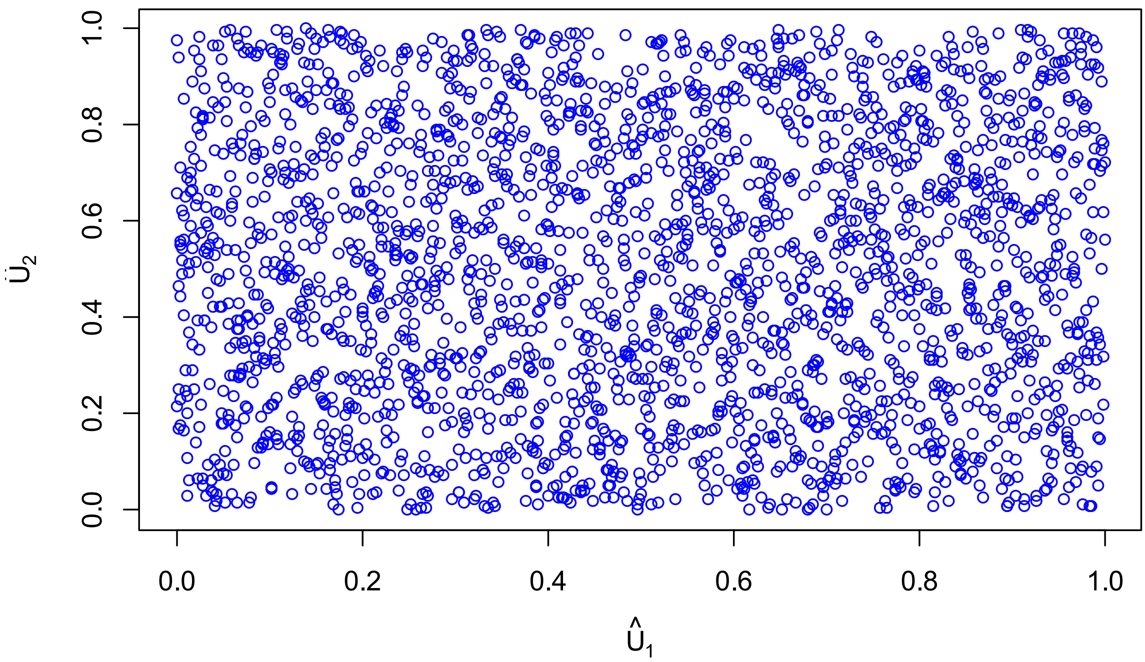

Figure 8. A scatter plot of the joint uniform marginal variates u_1 and u_2 for the bivariate rates of Bitcoin and the Rand, respectively.

Figure 8 shows a scatter plot of the distribution of the bivariate marginals for Bitcoin and the Rand. There appears to be no apparent or clear dependence between the two currencies' marginals. This could imply that the marginals are independent of each other.

As discussed in the Methodology section, a versatile Gumbel copula will be fitted to reveal or confirm the dependence structure between the two currencies.

As presented in Table 5, the Kendall Tau is very close to zero, implying that there is no significant dependence between the two currencies. The estimated parameter ($\hat{\alpha}$) for the Gumbel copula is $\hat{\alpha} = 1.007$ meaning that the Gumbel copula is reduced to an independent copula, confirming that there is no dependence between the bivariate uniform marginal distributions.

Table 5. Gumbel copula parameters.

	$\hat{\tau}$	$\hat{\alpha}$	Tail Dependence	
			Lower	**Upper**
Joint Marginals	0.007772646	1.007	0	0.009613426

The investor's objective when diversifying their investment is to minimise the portfolio risk compared to investing in, say, one risky asset. The diversification effect is measured by how the portfolio risk lowers the sum of individual asset risks (El-Gamal et al. 2006; Yoshiba 2015; Piwcewicz 2005). Table 6 summarises the risk measures (VaR and ES) for each currency, the sum of the two currencies' riskiness, and an equally weighted portfolio risk of Bitcoin and the Rand. The VaR quantities of an equally weighted portfolio's risk (2.021789, 2.043760, and 2.097417) are less than that of the simple sum of individual assets' risk (2.0289143, 2.0534, and 2.116704) at 90%, 95%, and 99% levels of significance, respectively. This resulted in positive diversification effects of 0.356%, 0.521%, and 0.991%, respectively. A similar trend can be observed using a more coherent risk measure called expected shortfall (ES). The ES quantities of equally weighted portfolio risk (2.055523, 2.079634, and 2.145787) are less than those of the simple sum of individual assets' risk (2.066548, 2.093598, and 2.169306), resulting in DE of 0.574%, 0.721%, and 1.088%, respectively.

Table 6. Estimated VaR, ES, and DE.

	Bitcoin/USD	**Rand/USD**	**Simple Sum**	**PORTFOLIO**	**Diversification Effects (%)**
			VaR		
90%	1.029736	0.9994753	2.0289143	2.021789	0.35606728
95%	1.051436	1.003017	2.0534	2.043760	0.52137918
99%	1.105283	1.013121	2.116704	2.097417	0.99154157
			ES		
	Bitcoin/USD	Rand/USD	Simple Sum	PORTFOLIO	Diversification Effects
90%	1.062252	1.00527	2.066548	2.055523	0.57443621
95%	1.08445	1.010282	2.093598	2.079634	0.72095980
99%	1.149116	1.021432	2.169306	2.145787	1.08785944

The diversification effects values imply that there is a risk reduction when an investor chooses an equally weighted portfolio instead of either of the individual assets.

7. Discussion

This study uses the eGARCH-Gaussian kernel density-EVT-Gumbel copula model to quantify the diversification effects of an equally weighted portfolio containing Bitcoin and the South African Rand. The eGARCH(1, 1)-EVT model was used to model the marginal distribution as the two currencies are known to possess somewhat heavy tails. The Gumbel copula is an extreme value copula (EV copula) (Hsu et al. 2012). The EV copula model is effective in quantifying the risk of a portfolio with heavy-tailed assets.

The scatter plot in Figure 8 suggests that there is no clear dependence between the two currencies. The Kendall Tau and Gumbel copula confirm the independence between the two data sets. Using the definition by Baur and Lucey (2010), Bitcoin can be classified as a safe haven asset for the Rand.

As shown in Table 6, at 90%, 95%, and 99% levels of confidence, using VaR and ES, the portfolio realises the DE of 0.356%, 0.521%, 0.991%, and 0.574%, 0.721%, 1.088%, respectively. This implies that there is a slight reduction in potential losses (diversification benefits) in the portfolio compared to the risk of the simple sum of single assets. These findings align with those of Ford (1998) and Lee et al. (2020), who observed that by diversifying investment, one is expected to lower their risk exposure.

These results can be used by fund managers, risk practitioners, and investors to decide on diversification strategies that can reduce their overall risk exposure. These findings suggest that Bitcoin can be classified as a safe haven asset and there is a small reduction in risk if one invests in an equally weighted portfolio of Bitcoin and the Rand.

8. Limitations and Further Related Studies

It is worth noting that while these findings transcend time, but not assets, risk managers must model every asset (including other currencies of emerging markets) to ascertain the safe haven properties of Bitcoin or any cryptocurrency of their choice. Similarities to and, hence, conclusions regarding other emerging economies' markets can be inferred, but as each emerging economy's market has a unique characterisation, studies must be conducted and conclusions should be reached separately.

In an upcoming article we will explore a study that incorporates investor sentiment in the portfolio using models like the Black–Litterman approach; this article can be considered in future studies.

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