Jessie Wu

PH235: Physics Simulations

Final Project Proposal

24 March 2017

Numerical Analysis of the Dynamics of Multiple Pendulum

**Member and Roles**

I will analyze the dynamics of a triple pendulum and create a video animation. I plan on working on this project independently. My responsibilities will include model development, implementation, and animation.

**Project Description**

The double pendulum is a classic example of chaotic dynamics. The system is deterministic, which means that the future behavior is fully determined by initial conditions with no randomness involved. However, the system is extremely sensitive to small differences in initial conditions; with a slightly different set-up, a double pendulum would have a completely different trajectory.

Many physicists have characterized the double pendulum and performed numerical analysis. For example, Shinbrot, Grebogi, Widom, and Yorke experimentally and numerically analyzed a double pendulum system using a fourth-order Runge-Kutta. In my project, I wish to verify their results, perform a similar analysis on a triple pendulum, and provide a visual representation of the system.

**Model and Method**

While previous work has been done on multiple pendulum, I want to model a triple pendulum modeled in the picture below. Because the pendulums have significant masses, they will be modeled as rods instead of point masses. To the best of my knowledge, a simulation of this configuration of the triple pendulum has not been done before.

The equations of motion can be calculated using the Lagrangian method. The Langrangian can be calculated from the potential and kinetic energy of the system. The Lagrangian equations produces a system of three second-order equations. These can be re-written as six simultaneous first-order equations. The system of differential equations will be solved using the 4th Order Runge-Kutta algorithm. Finally, the entire system can be modeled and visualized by using Python libraries.

**PH235 Topics**

4th Order Runge-Kutta for Second-Order Differential Equations

Visualization and Animation of Results

**Evaluation Plan**

The project will be evaluated on accuracy, computational speed, applicability, and complexity.

**Demo and Documentation**

For the final exam date, I will capture the animation for a triple pendulum and share via a YouTube link. The progress and code will also be hosted online through Jessie's Github.

**Plans and Challenges**

*Low Hanging Fruit:*

The first part of the project is to implement a Runge-Kutta method to solve the equations of motion that govern a double-pendulum. A similar problem was already conducted for a homework experiment, but with pendulums as point masses instead of rods.

*Goal:*

The second part would be to extend the analysis to a triple pendulum. This step includes developing a model, testing, optimizing, and outputting the simulation.

*Challenge:*

In order to increase the complexity of the project, depending on the time constraints, additional challenges can be added. The Lyapunov exponent for both pendulum systems can also be calculated. As an extra feature, the simulated results can be compared against the experimental results of a pendulum by borrowing equipment from Prof. Wolf.

**Timeline**

04/03: In-Class Presentation

04/10: Re-Create Double Pendulum; Triple Pendulum Model Development

04/17: Triple Pendulum Implementation (In-Class Update)

04/24: Visualization, Animation, and Documentation

05/01: Stretch Goals and Buffer Time

05/08: Final Presentation and Final Report

**References**

Morin, D. (2012). "The Lagrangian Method". Introduction to Classical Mechanics: with problems and solutions. Cambridge: Cambridge University Press. [http://www.people.fas.harvard.edu/~djmorin/chap6.pdf]

Shinbrot, T., Grebogi, C., Wisdom, J., Yorke, J. "Chaos in a double pendulum".

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